Kennesaw State University

College of Computing and Software Engineering

Department of Computer Science

CS 5070, Mathematical Structures for Computer Science, Section W01

Assignment #1

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Problem Statement

Problem of this document is to solve the following problem set for assignment #1.

1. If $P(x) = 7x + 3$ is even:
   (a) Find values of n for $P(x)$ true
   (b) With universal and existential quantifiers write a mathematical expression that defines two complete predicates
2. Write the complete mathematical logical expression of the following: there is an odd number beween any two even numbers
3. Write the complete mathematical logical expression of the following description: there is an even number beween any two odd numbers
4. Write the complete mathematical logical expression of the following description: there is no number beween any two consecutive numbers
5. Write the complete mathematical logical expression of the following informal description: If P is in set A, then Q is in set B. If Q is not in set B, then P is in set A. Therefore, P is not in set A or Q is not in set B.
6. Construct a truth table of the resulting logical expression.
7. Use a truth table to prove or disprove the following expression:

$$\neg(P \wedge (Q \vee R)) = \neg P \vee (\neg Q \vee \neg R)$$

Summary / Purpose

Purpose of this document is to provide solutions to the problem set outlined in the Problem Statement section.

Solutions

1. If P(x) = 7x + 3 is even:
   a. Find values of n for P(x) true

      If *P(x)* = 7*x* + 3 is even, then there is some set of n such that it makes this predicate even. Consider the following:

P(-3) = 7(-3) + 3 = -18 which is even
P(-2) = 7(-2) + 3 = -11 which is odd
P(-1) = 7(-1) + 3 = 4 which is even
P(0) = 7(0) + 3 = 3 which is odd
P(1) = 7(1) + 3 = 10 which is even
P(2) = 7(2) + 3 = 17 which is odd
P(3) = 7(3) + 3 = 24 which is even
P(4) = 7(4) + 3 = 31 which is odd
P(5) = 7(5) + 3 = 38 which is even
....
P(2n - 1) = 7(2n - 1) + 3 = 14n - 4 which is even
P(2n) = 7(2n) + 3 = 14n + 3 which is odd

Therefore, for all odd n in the integer set, we find an x such that x = 2n - 1 where *P(x)* = 7*x* + 3 is even.

b. With universal and existential quantifiers, write a mathematical expression that defines two complete predicates.

Considering what we've learned above, we can denote that there is a set A where there exists some n in the integer set such that 2n + 1 and there exists x in the integer set such that 7x + 3. This can be denoted as the following predicates:

P(x) = 7x + 3 = is even = 2n
Q(X) = x = is odd = 2n - 1

Which can be written as the following mathematical expression:

$A = \{ \forall x \in Z : \exists n \in Z\ (7x + 3 = 2n) \land (x = 2n - 1) \}$

2. Write the complete mathematical logical expression of the following:

There is an odd number between any two even numbers

This can be written as: between any two even numbers, there is an odd number.

$A = \{\forall x{\in}Z\ \forall y{\in}Z\ \forall z{\in}Z{:}\ \exists n{\in}N\ ((x = 2n) \land (y = 2n) \land (z = 2n -1)\ \wedge\ (x < z < y))\}$

For all x and y integers, there exists some number n, an element of natural numbers, and there exists some number z, an element of natural numbers, where x is even, y is even, z is odd, and x < z < y.

3. Write the complete mathematical logical expression of the following description:

   There is an even number between any two odd numbers

   This description can be written as between any two odd numbers, there is an even number.

   $A = \{\forall x \in Z\ \forall y \in Z\ \forall z \in Z\text{: } \exists n \in N\ ((x = 2n - 1) \wedge (y = 2n - 1) \wedge (z = 2n) \wedge (x < z < y))\}$

   For all x and y integers, there exists some number n, an element of natural numbers, and there exists some number z, an element of natural numbers, where x is odd, y is odd, z is even, and $x < z < y$.

4. Write the complete mathematical logical expression of the following description:

   There is no number between any two consecutive numbers

   This description can also be written as between any two consecutive numbers, there is no number.

   $A = \{\forall x \in Z\text{: } \neg(\exists n \in Z, (x < n < x + 1))\}$

   which is equivalent to….

   $A = \{\forall x \in Z\text{: } (\forall n \in Z, \neg (x < n < x + 1)\}$

   For all x, an element of integers, such that for all z integers there is not a z that is less than x and greater than x +1.

5. Write the complete mathematical logical expression of the following informal description:

   If P is in set *A*, then Q is in set *B*.

   If Q is not in set *B*, then P is in set *A*.

   Therefore, P is not in set *A* or Q is not in set *B*.

   These three statements mean the following:

   $P \in A \rightarrow Q \in B$

   $\neg(Q \in B) \rightarrow P \in A$

   Therefore, $\neg(P \in A) \vee \neg(Q \in B)$

   The complete mathematical expression is then:

   Let $R = P \in A$

Let $S = Q \in B$

$$((R \rightarrow S) \wedge (\neg S \rightarrow R)) \rightarrow (\neg R \vee \neg S)$$

6. Construct a truth table of the resulting logical expression.

R	S	$R \rightarrow S$	¬R	¬S	$\neg S \rightarrow R$	$(R \rightarrow S) \wedge (\neg S \rightarrow R)$	¬R ∨ ¬S	((R → S) ∧ (¬S →R)) → (¬R ∨ ¬S)
T	T	T	F	F	T	T	F	F
T	F	F	F	T	T	F	T	T
F	T	T	T	F	T	T	T	T
F	F	T	T	T	F	F	T	T

7. Use a truth table to prove or disprove the following expression:

¬(P ∧ (Q ∨ R)) = ¬P ∨ (¬Q ∨ ¬R)

P	Q	R	¬P	¬Q	¬R	Q ∨ R	¬(P ∧ (Q ∨ R))
T	T	T	F	F	F	T	F
T	T	F	F	F	T	T	F
T	F	T	F	T	F	T	F
T	F	F	F	T	T	F	T
F	T	T	T	F	F	T	T
F	F	T	T	T	F	T	T
F	T	F	T	F	T	T	T
F	F	F	T	T	T	F	T

(¬Q ∨ ¬R)	¬P ∨ (¬Q ∨ ¬R)
F	F
T	T
T	T
T	T
F	T
T	T
T	T
T	T

Using the truth table above, we can disprove the expression above because it is not equal in two cases, one where P = True, Q = True, R = False and two where P = True, Q = False, and R = True.

References

[1] Garrido, J. (2021, August 14). *CS5070 Mathematical Structures for Computer Science - Notes 1* [Slide show; Powerpoint]. D2L. https://kennesaw.view.usg.edu/d2l/le/content/3550928/viewContent/55786708/View?ou=3550928

[2] Garrido, J. (2022, May). *CS5070 Mathematical Structures for Computer Science - Additional notes* [Slide show; Powerpoint]. D2L. https://kennesaw.view.usg.edu/d2l/le/content/3550928/viewContent/55786706/View

[3] Kennesaw State University, College of Computing and Software Engineering, Department of Computer Science, Mathematical Structures for Computer Science. (n.d.). CS5070 Assignment 1. In *https://kennesaw.view.usg.edu/d2l/le/content/3550928/viewContent/55786667/View*.

[4] Levin, O. (2016). *Discrete mathematics: An Open Introduction*.