Kennesaw State University

College of Computing and Software Engineering

Department of Computer Science

CS 5070, Mathematical Structures for Computer Science, Section W01

Quiz

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Problem Statement

Problem of this document is to solve the following problem set.

- 1. Question: Determine the 10th term of the sequence defined by $(a_n = 3n + 2)$.
- 2. Question: Find the sum of the first 20 terms of the arithmetic sequence where the first term (a₁) is 5 and the common difference d is 3.
- 3. Question: Given a geometric sequence with $(a_1 = 2)$ and common ratio (r = 3), find the 6th term.
- 4. Question: Translate the following statement into symbolic logic: "If it rains, then the ground will be wet."
- 5. Question: Prove by contradiction: "If (n) is an odd integer, then (n²) is odd."

Summary / Purpose

Purpose of this document is to provide solutions to the problem set outlined in the Problem Statement section.

Solutions

1. Question: Determine the 10th term of the sequence defined by $(a_n = 3n + 2)$.

The 10th term in the sequence can be determined by plugging in for n.

If n = 0;
$$a_0 = 3(0) + 2 = 2$$

If n = 1; $a_1 = 3(1) + 2 = 5$
If n = 2; $a_2 = 3(2) + 2 = 8$
...
If n = 2; $a_{10} = 3(10) + 2 = 32$

Therefore, the 10th term is 32.

2. Question: Find the sum of the first 20 terms of the arithmetic sequence where the first term (a_1) is 5 and the common difference d is 3.

If the terms of a sequence differ by a constant, then the sequence is an arithmetic sequence. If the initial term is 5 and the common difference, *d*, between the terms is 5. We can use the closed formula to determine the 20th term. Note that n here is going to be 19 because we already have the initial term, thus equaling 20 terms

$$a_n = a + d * n$$

$$a_{20} = 5 + 3 * (20 - 1)$$
$$a_{20} = 62$$

Now, we can use the technique of reversing and adding to get the sum of the sequence.

$$2S = 20 * (62 + 5)$$
$$S = \frac{1340}{2}$$
$$S = 670$$

Therefore, the sum of the first 20 terms is 670.

3. Question: Given a geometric sequence with $(a_1 = 2)$ and common ratio (r = 3), find the 6th term.

A sequence is called geometric if the **ratio** between successive terms is constant. For this geometric sequence, we know the initial term is 2 and the common ratio, r, is 3. We can use the closed formula to determine the 6^{th} term. Note again that n here is 5 since we already have the initial term.

$$a_n = a * r^n$$
 $a_6 = 2 * 3^{(6-1)}$
 $a_6 = 486$

Therefore, the 6th term is 486.

4. Question: Translate the following statement into symbolic logic: "If it rains, then the ground will be wet."

Suppose we have two propositions, P and Q. This statement can be broken down into two propositions and can translate to an implication. We can write this this way:

P = it rains

Q = the ground is wet

Therefore, an implication can be made such that

$$P \rightarrow Q$$

If it rains \rightarrow the ground will be wet

5. Question: Prove by contradiction: "If (n) is an odd integer, then (n^2) is odd."

A proof by contradiction is when we prove that the statement is true by proving the negation of the proposed implication is false. Here we can say that P is if n is an odd integer, and Q is that n^2 is odd. So, we try to prove the following is true: if n is an odd integer, then n^2 is even.

Suppose n is an odd number, then n=2k for some integer k. Then we can see that $n^2=(2k+1)^2=4k^2+4k+1=2(2k^2+2k)+1$. Here we can see that $(2k^2+2k)$ is just an integer, so let's call it m. Then $n^2=2m+1$, which we can see is odd. Therefore, n^2 must be odd if n is odd.

References

- [1] Garrido, J. (2021, August 14). *CS5070 Mathematical Structures for Computer Science Notes* [Slide show; Powerpoint]. D2L.
- [2] Kennesaw State University, College of Computing and Software Engineering, Department of Computer Science, Mathematical Structures for Computer Science. (n.d.). Quiz.
- [3] Levin, O. (2016). Discrete mathematics: An Open Introduction.