Kennesaw State University

College of Computing and Software Engineering

Department of Computer Science

CS 5070, Mathematical Structures for Computer Science, Section W01

Assignment #5

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Problem Statement

Problem of this document is to solve the following problem set for assignment #5.

Module 5 Assignment #5 (Chapter 3)

1. Problem #1, page 228
2. Problem #2, page 228
3. Problem #3, page 228
4. Problem #6, page 229

Summary / Purpose

Purpose of this document is to provide solutions to the problem set outlined in the Problem Statement section.

Solutions

1. Problem #1, page 228: Complete a truth table for the statement $\neg P \rightarrow (Q \wedge R)$.

   Suppose we have two propositions, P and Q. For the implication, $P \rightarrow Q$, the only time it is true is if P is false or Q is true (or both), and false otherwise. Therefore, our truth table is the following:

P	Q	R	¬P	Q ∧ R	$\neg P \rightarrow (Q \wedge R)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	F	F
F	T	F	T	F	F
F	F	F	T	F	F

2. Problem #2, page 228: Suppose you know that the statement "if Peter is not tall, then Quincy is fat and Roberts is skinny" is false. What, if anything, can you conclude about Peter and Robert if you know that Quincy is indeed fat? Explain (you may reference problem 3.3.1).

Suppose we can represent our statements here into 3 propositions, P, Q, and R. They are the following:

- P is that Peter is tall
- Q is that Quincy is fat
- R is that Roberts is skinny

Then our implication to the statement of "if Peter is not tall, then Quincy is fat and Roberts is skinny" is the following:

$$\neg P \rightarrow (Q \wedge R)$$

Which is false. So, what can we conclude? Using the truth table from problem 1, the only times this statement is false is below:

P	Q	R	¬P	Q ∧ R	$\neg P \rightarrow (Q \wedge R)$
F	F	T	T	F	F
F	**T**	**F**	**T**	**F**	**F**
F	F	F	T	F	F

If we say that Quincy is fat, then we say that Peter is not tall and Roberts is not skinny using row 2 above.

3. Problem #3, page 228: Are the statements $P \rightarrow (Q \vee R)$ and $(P \rightarrow Q) \vee (P \rightarrow R)$ logically equivalent? Explain your answer.

   To say if the statements $P \rightarrow (Q \vee R)$ and $(P \rightarrow Q) \vee (P \rightarrow R)$ logically equivalent, we can construct a truth table.

P	Q	R	Q ∨ R	$P \rightarrow Q$	$(P \rightarrow R)$	$P \rightarrow (Q \vee R)$	$(P \rightarrow Q) \vee (P \rightarrow R)$
T	T	T	T	T	T	**T**	**T**
T	T	F	T	T	F	**T**	**T**
T	F	T	T	F	T	**T**	**T**
T	F	F	F	F	F	**F**	**F**
F	T	T	T	T	T	**T**	**T**
F	F	T	T	T	T	**T**	**T**
F	T	F	T	T	T	**T**	**T**
F	F	F	F	T	T	**T**	**T**

Using the truth table, we can see that the two statements are logically equivalent.

4. Problem #6, page 229: Consider the statements: for all integers *n*, if *n* is even and $n \leq 7$ then *n* is negative or $n \in \{0, 2, 4, 6\}$.

These statements can be the following implication:

P = *n* is even and $n \leq 7$

Q = *n* is negative or $n \in \{0, 2, 4, 6\}$

$$P \rightarrow Q$$

(a) Is the statement true? Explain why.

The statement is true because (1) there are only 4 possible positive even integer values that n can be, i.e. 0, 2,4,6, that are less than or equal to 7 and (2) it is implied that *n* can be any negative even integer, all of whom are also less than or equal to 7.

(b) Write the negation of the statement. Is it true? Explain.

The negation of the implication is the following:

$$\neg(P \rightarrow Q) = P \wedge \neg Q$$

This means that there exists some *n* integer that is even and $n \leq 7$ AND *n* is not negative or $n \notin \{0, 2, 4, 6\}$. This is not true as there is no positive even integer that is less than or equal to 7 that is also not in the set {0, 2, 4, 6}.

(c) State the contrapositive of the statement. Is it true? Explain.

The negation of the implication is the following:

$$\neg Q \rightarrow \neg P$$

This means that for all *n* integers, if *n* is not negative and $n \notin \{0, 2, 4, 6\}$, then *n* is not even or $n \nleq 7$. We can use a couple examples. Suppose n = 1; it’s not negative and not 0, 2, 4, or 6. It’s not even, but it is less than 7. Implication holds. What about n = 10? It’s not negative, and not 0,2,4, or 6. It’s not less than or equal to 7 but it is even. Implication holds. Finally, let’s say n = 9. It’s not negative and not 0, 2, 4, 6. It’s also not even nor is it not less than or equal to 7. It works. All of these examples make the implication true

(d) State the converse of the statement. Is it true? Explain.

The converse of the statement is the following:

$$P \rightarrow Q$$

This means that for all integers *n*, if *n* is negative or $n \in \{0, 2, 4, 6\}$ then = *n* is even and $n \leq 7$. Let’s use *n* = 4. This value is in the set of *n* as defined in the hypothesis so left hand side is true, as it is not negative. It is also even and less than or equal to 7. The implication holds. If n = -1, we see it is negative so the hypothesis in the left hand side is true even though it’s not in the set. It is less than or equal to 7 but it is not even. The implication does not hold. Therefore it is false.

References

[1] Garrido, J. (2021, August 14). *CS5070 Mathematical Structures for Computer Science - Notes 5* [Slide show; Powerpoint]. D2L.

[2] Kennesaw State University, College of Computing and Software Engineering, Department of Computer Science, Mathematical Structures for Computer Science. (n.d.). CS5070 Assignment 5.

[3] Levin, O. (2016). *Discrete mathematics: An Open Introduction*.