CS 535 - Design and Analysis of Algorithms, Fall Semester - 2016

HomeWork-0

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Consider the function, useless:

```
1: function useless(n)
2: if n=1 then
3:       return 1
4:else
5:       return useless(random(1,n))
6:end if
7:endfunction
```

where random(1, n) returns a uniformly distributed random integer in the range 1…..n. Assume an initial call useless(m).

1. What value is returned? Prove your answer formally.

Solution: Value returned will be **1**.

Explanation: The terminating condition for the useless function is when we have the value of n=1. Which comes in two ways. First, caller of the useless function passes value of n=1. Second, the random function at the line 5 generate n as 1. In both the cases, condition at line number 2 will pass and the value returned will be "1" from line number 3.

Point to remember is that the given function "useless" will make recursive call till the "random" function at line 5 produces output as 1.

2. Calculate exactly the expected number of calls to random in line 5.

Solution: **$Hm-1 + 1$.**

Explanation: As we know that the random returns uniformly distributed integer between 1- n. Suppose random function makes m calls, so we can say

$$f(m) = f(m)+1$$

$$\text{or } f(m) = f(m-1) + 1$$

$$\text{or } f(m) = f(m-2) + 1$$

.

.

$$\text{similarly, } f(m) = f(2) + 1$$

Adding all we will get,

$$f(m)(m-1) = m + \sum_{i=2}^{m-1} f(i) \qquad \text{--- eq. (1)}$$

Subtract 1 from m, we get

$$f(m-1)\ (m-2)=(m-1) + \sum_{i=2}^{m-2} f(i) \qquad \text{---eq. (2)}$$

Now, Subtracting eq. (1) & eq. (2)

$$(m-1)f(m) - (m-2)f(m-1) = m + \sum_{i=2}^{m-1} f(i) - (m-1) - \sum_{i=2}^{m-2} f(i)$$

$$(m-1)f(m) - mf(m-1) + 2f(m-1) = 1 + f(m-1)$$

$$f(m) - f(m-1) = 1/(m-1) \text{ ---eq. (3)}$$

So, we can get

$$f(m-1) - f(m-2) = 1/(m-2) \text{ ---eq. (4)}$$

$$f(m-2) - f(m-3) = 1/(m-3) \text{ ---eq. (5)}$$

Adding eq. (3), (4), (5) we get

$$f(m) - f(1) = 1/(m-1) + 1/(m-2) + 1/(m-3) \ldots.. + 1$$

which we can write as Harmonic mean of m-1 plus 1,

$$f(m) = H\ m-1 + f(1)$$

So, Hm-1 +f (1) Is the total number of calls at line 5

3. In the worst case, what is the number of calls to random in line 5?

Solution: There will be **infinite** number of calls to random in line 5, in worst case.

Explanation: At line 5 useless function is doing a recursive call which is dependent on random(1,n) function and as random function returns a uniformly distributed random integer in the range 1…..n, so in worst case It won't generate "1"ever which is a terminating condition for the useless function.