Design of an Electromagnetic Railgun

1 Objective

An electromagnetic railgun is a device that uses the Lorentz force to accelerate a metallic projectile to a high velocity. It consists of two parallel metal rails with a voltage differential between them (frequently created by a capacitor to make it possible to have high currents), and a small metal projectile which passes between the rails. When the projectile is placed between the rails, it shorts the two rails and a current flows through the rails and the projectile. This causes a magnetic field to be generated perpendicular to the rails and projectile, as defined by the Biot-Savart law. The current running through the projectile, combined with the magnetic field generated by the rails, applies a Lorentz force which accelerates the projectile forward, shooting it out of the barrel.
The goals for this project are the following (in order of importance):

- Accelerate a 10g metallic projectile to a velocity of at least 2 m/s
- Be able to be fired multiple times without any physical damage to the device
- Cost as little as possible (materials and electronic components)

2 Theory

2.1 Factors Considered

The following are factors we calculate and take into account in this design:

- Force exerted on the projectile
- Capacitor discharge rate
- Heat generated by resistance of rails (Joule heating effect)
- Change in resistance as rail temperature increases

2.2 Simplifying Assumptions

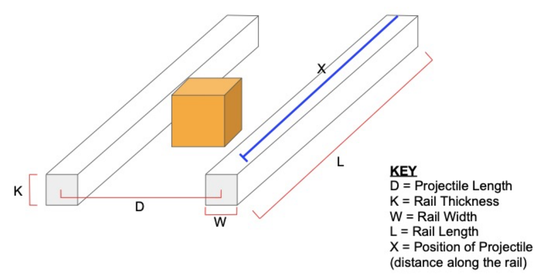
Following are some factors that are ignored/treated in a simplified manner in this design:

- Ignore physical thermal expansion of rails/projectile
- Ignore effects of friction on the projectile

- Ignore heat transfer between rails, environment, and projectile
- Assume resistance of capacitor circuit does not change
- Ignore all effects of induction in the rails, projectile, and wiring
- Assume no internal resistance/inductance in capacitors
- Assume no capacitance in rails and projectile
- Assume rails and projectile are infinitely thin when applying Lorentz force law and Biot-Savart law
- Ignore magnetic field created by the projectile

2.3 Design

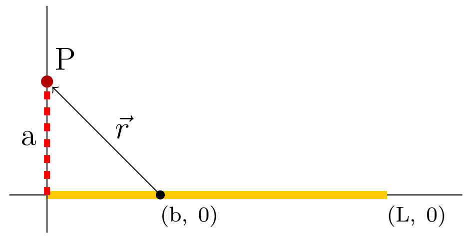
We begin with a general design, and then tune the parameters to optimize the design to the desired specifications. This design includes two solid extruded-rectangle rails and a small cuboid as the projectile. The rails are each connected to one terminal of a capacitor bank, which is charged to its maximum voltage before firing. The projectile is dropped in between the rails to fire it. Following is a diagram showing the general design:



3 Calculations

3.1 Force on the Projectile

The first step is to calculate the force F exerted on the projectile as a function of the position x of the projectile and the current running through the system (assuming the projectile is within the barrel). We begin by computing the magnitude of the magnetic field generated by one rail (the effective length of the rail is the position of the projectile, as no current flows through the remaining portion of the rail):



Given that point P is located at the coordinates $(0, a)$ (as the projectile is always at the end of the portion of the rail which current is running in):

$$d\vec{B} = \frac{\mu_0 I d\vec{s} \times \hat{r}}{4\pi \|\vec{r}\|^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{s} \times \hat{r}}{\|\vec{r}\|^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{s} \times \left(\frac{\vec{r}}{\|\vec{r}\|}\right)}{\|\vec{r}\|^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{s} \times \vec{r}}{\|\vec{r}\|^3} \tag{1}$$

We begin by solving for the infinitesimal portion of the rails:

$$d\vec{s} = db\hat{\boldsymbol{\imath}} \tag{2}$$

We then solve for the distance and direction to the point where the magnetic field is being observed:

$$\vec{r} = (0 - b)\hat{\boldsymbol{\imath}} + (a - 0)\hat{\boldsymbol{\jmath}} = -b\hat{\boldsymbol{\imath}} + a\hat{\boldsymbol{\jmath}} \tag{3}$$

$$\|\vec{r}\| = \sqrt{(-b)^2 + a^2} = \sqrt{b^2 + a^2} \tag{4}$$

$$\|\vec{r}\|^3 = (b^2 + a^2)^{3/2} \tag{5}$$

We then write and simplify the expression for $d\vec{B}$:

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{s} \times \vec{r}}{\|\vec{r}\|^3} = \frac{\mu_0 I}{4\pi} \cdot \frac{db\hat{\boldsymbol{\imath}} \times (-b\hat{\boldsymbol{\imath}} + a\hat{\boldsymbol{\jmath}})}{(b^2 + a^2)^{3/2}} \tag{6}$$

$$db\hat{\boldsymbol{\imath}} \times (-b\hat{\boldsymbol{\imath}} + a\hat{\boldsymbol{\jmath}}) = adb\hat{\boldsymbol{k}} \tag{7}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{adb\hat{\boldsymbol{k}}}{(x^2 + a^2)^{3/2}} \tag{8}$$

We take the norm of $d\vec{B}$ to find the magnitude of the field:

$$B = \frac{\mu_0 I}{4\pi} \cdot \frac{adb}{(x^2 + a^2)^{3/2}} \tag{9}$$

Finally, we integrate along the rail to find B at a given distance a from the rail, where x is the effective length of the rails:

$$B(a) = \frac{\mu_0 I a}{4\pi} \int_0^x \frac{db}{(b^2 + a^2)^{3/2}} = \frac{\mu_0 I x}{4\pi a \sqrt{x^2 + a^2}} \tag{10}$$

Now that we have computed the magnitude of the magnetic field of the rails, we proceed by calculating the force exerted on the projectile. By the right-hand rule, both rails produce a magnetic field such that the force applied on the projectile is forward. Therefore, we can use the Lorentz force law (for a wire) on the projectile (given that d is the separation between the rails and w is the width of the rails):

$$F = I \int B ds = I \int_{(w/2)}^{(d+w/2)} B(a) da \tag{11}$$

$$F = I \int_{(w/2)}^{(d-w/2)} \frac{\mu_0 I x}{4\pi a \sqrt{x^2 + a^2}} da = \frac{\mu_0 I^2 x}{4\pi} \int_{(w/2)}^{(d-w/2)} \frac{da}{a\sqrt{x^2 + a^2}} \tag{12}$$

Integrating (and doubling, because there are two rails) yields the final expression for force on the projectile when it is a given distance through the barrel (where $x = 0$ is the start of the barrel):

$$F = \frac{2\mu_0 I^2 (\log(d - w/2) - \log(x\sqrt{(d - w/2)^2 + x^2} + x^2) - \log(w/2) + \log(x\sqrt{(w/2)^2 + x^2} + x^2))}{4\pi} \tag{13}$$

3.2 Current in Rails and Projectile

It follows that the next step is to calculate the amount of current running through the projectile. We begin by calculating the total capacitance of the capacitor bank:

$$C = N \cdot C_{\text{each}} \tag{14}$$

Next, we find the resistance of the circuit as a function of time, temperature (which itself is a function of distance along the rails), and effective rail length (position of the projectile).

$$R_{\text{rail}} = \frac{\int \rho \; dl}{A} \tag{15}$$

$$\rho(l) = \rho_0(1 + \alpha(T(l) - 20)) = \rho_0(1 + \alpha T(l) - 20\alpha) = \rho_0(1 - 20\alpha) + \rho_0 \alpha \cdot T(l) \tag{16}$$

Given the width of the rail is w and the thickness of the rail is k:

$$R_{\text{rail}} = \frac{\int_0^x \rho(l) dl}{A} = \frac{\rho_0(1 - 20\alpha) \int_0^x dl + \rho_0 \alpha \int_0^x T(l) dl}{wk} \tag{17}$$

$$R_{\text{rail}} = \frac{\rho_0 x(1 - 20\alpha) + \rho_0 \alpha (\int_0^x T(l) dl)}{wk} \tag{18}$$

Assuming the resistance of the projectile is negligible, we find that the total resistance is (given that R_{circuit} is the additional resistance of the capacitor circuit and wiring):

$$R_{\text{total}} = 2R_{\text{rail}} + R_{\text{circuit}} = 2\frac{\rho_0 x(1 - 20\alpha) + \rho_0 \alpha (\int_0^x T(l) dl)}{wk} + R_{\text{circuit}} \tag{19}$$

Now, we can use the capacitor discharge formulas to find the amount of current running through the circuit:

$$V = \frac{Q}{C} \tag{20}$$

$$\frac{Q}{C} = IR \tag{21}$$

We write this in terms of time and solve for the rate of change of charge:

$$\frac{Q}{C} = \frac{dQ}{dt}R \tag{22}$$

$$I = \frac{dQ}{dt} = \frac{Q}{R_{\text{total}}C} \tag{23}$$

In addition, we can find the initial charge as a function of voltage and capacitance:

$$Q = CV \tag{24}$$

Because resistance is constantly changing, we will use Euler's method to approximate the current and charge over time.

3.3 Temperature Increase of Rails

By the joule-heating effect, we know that the heat q added to a resistor can be found by the following formula:

$$q = Pt = I^2Rt \tag{25}$$

Therefore, we can calculate the amount of heat added to a system in a given amount of time:

$$q = Pt = \left(\frac{Q}{R_{\text{rail}}C}\right)^2 R_{\text{rail}}t = \frac{Q^2t}{C^2R_{rail}} \tag{26}$$

Next, we need to calculate the change in temperature due to the added heat:

$$q = mc\Delta T \tag{27}$$

$$\Delta T = \frac{q}{mc} \tag{28}$$

The specific heat c will depend on the metal used, and we can calculate the volume and mass as a function of the density p of the metal and the effective length x of the rails:

$$V = xwk \tag{29}$$

$$m = pV = pxwk \tag{30}$$

$$\Delta T = \frac{q}{pxwkc} \tag{31}$$

$$\Delta T = \frac{\frac{Q^2 t}{C^2 R_{total}}}{pxwkc} = \frac{Q^2 t}{pxwkc \cdot C^2 R_{rail}} \tag{32}$$

We must note that the increase in temperature only applies between the start of the rail and the point x meters into the rail. Therefore, it must be calculated for every point along the rail.

4 Results

4.1 Implementation

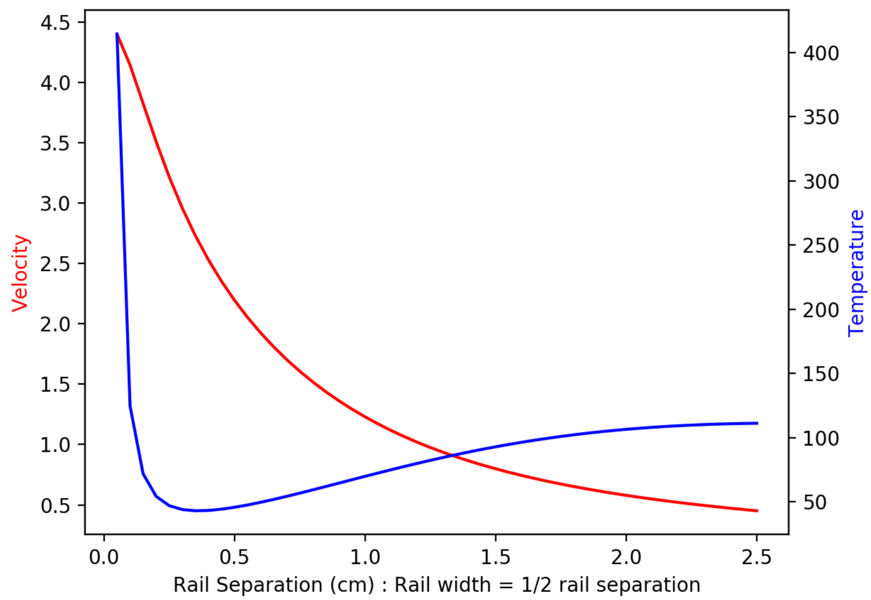
We implement a Python simulation of our proposed railgun system using equations (13) (18) (19) (23) (24) (32). Code can be viewed at: `https://github.com/asinghani/railgun/blob/master/simulate.py`

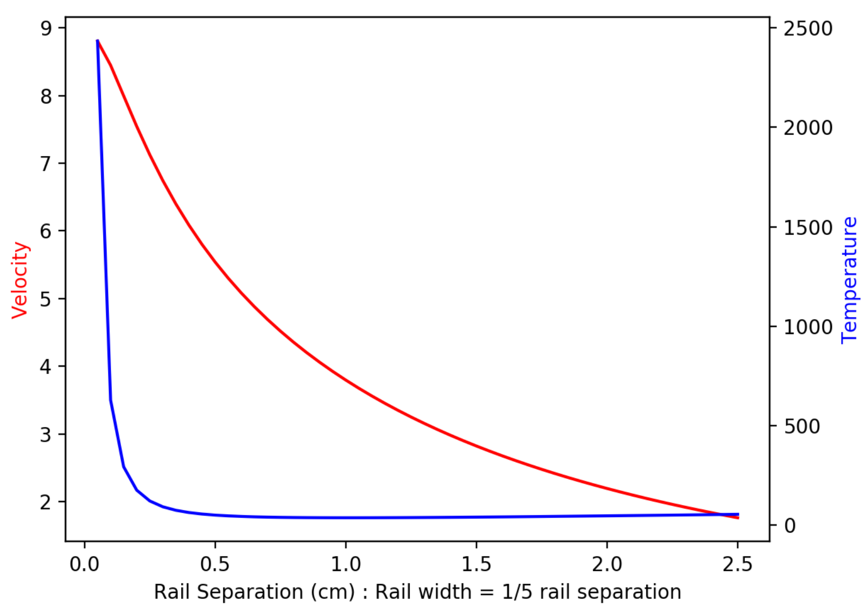
The system is simulated with a timestep of 0.001 seconds and the simulation results are discussed below.

4.2 Discussion of Results

Experimental results show that the main parameter that affects the velocity and temperature is the separation between the rails, as well as the capacitor bank's capacitance and voltage. Two options were examined for the capacitor bank: high capacitance (400+ F) at a low voltage (2.7 V), and low capacitance (0.0528 F) at a high voltage (400 V). Experimental results showed that the low capacitance at high voltage was much more effective in the force it created on the projectile.

Following are two graphs showing the velocity and temperature at different rail-separation distances, as well as the parameters used for the test:


Velocity
Temperature
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
400
350
300
250
200
150
100
50
0.0
0.5
1.0
1.5
2.0
2.5
Rail Separation (cm) : Rail width = 1/2 rail separation


Velocity
Temperature
9
8
7
6
5
4
3
2
2500
2000
1500
1000
500
0
0.0
0.5
1.0
1.5
2.0
2.5
Rail Separation (cm) : Rail width = 1/5 rail separation

```
# Projectile mass - kg
m = 0.01

# Rail separation
d = centi(z)

# Rail width
w = d / 5.0 #centi(0.3)

# Rail thickness
k = centi(2)

# Barrel length
L = 0.1

# Resistivity at 20C (metal)
rho = 1.59e-8

# Resistivity expansion coefficient (metal)
a = 0.0038

# Power circuit resistance
R_circuit = 0.03

# Capacitor charge voltage
V = 400.0

# Total capacitance
C = 16 * micro(3300)

# Density (metal, gram per cubic meter)
p = 10500000

# Specific heat (metal, per gram)
c = 0.240
```

From these results, we find our optimal design, with the same parameters as above, other than the separation between the rails (which is 2.0 cm), and the width of each rail (0.4 cm). This design is able to achieve a velocity of 2.5 m/s at a temperature of less than 100deg C (while the melting point of silver is around 960deg C).

We also observed that the temperature distribution in the rails appeared to be a gradient, with the highest temperature near the entry-point of the projectile and the lowest temperature at the exit:

4.3 Material Cost

- Rails: 18 cubic centimeters of pure silver at $5.36 / cubic cm = $100 (`https://www.moneymetals.com/precious-metals-charts/silver-price`)
- Aluminum sheet (for wiring and capacitor bank connections) = $100
- 16x 3300uF 400V Capacitor: $800 (`https://www.mouser.com/ProductDetail/KEMET/ALS31A332NF400?qs=sGAEpiMZZMtZ1n0r9vR22WsHOALhvbhWkC86km%252bsu2w%3d`)