Bard College at SIMON'S ROCK the Early College

VASSAR

A minimal reaction-diffusion neural model generates *C. elegans* undulation

Anshul Singhvi [1, 3] Harold Hastings [1] Jenny Magnes [2] Cheris Congo [2] Miranda Hulsey-Vincent [2] Rifah Tasnim [1] Naol Negassa [1]

[1]Bard College at Simon's Rock [2]Vassar University [3]Columbia University

Abstract

The small (1 mm) nematode *Caenorhabditis elegans* has become widely used as a model organism; in particular the *C. elegans* connectome has been completely mapped, and *C. elegans* locomotion has been widely studied (c.f. http://www.wormbook.org). We describe a minimal reaction-diffusion model for the *C. elegans*. This may be considered a simple model for Xu et al.'s "descending pathway" description of the *C. elegans* central pattern generator (CPG) [4]. Olivares *et al* [3] present a likely more realistic model which relies on small networks of neurons, and presents a distributed model of the CPG. In particular, we use simulation methods to show that a small network of FitzHugh-Nagumo neurons (one of the simplest neuronal models) can generate key features of *C. elegans* undulation, and thus locomotion. Finally, we recreate the required oscillations and coupling with a network of coupled Keener [2] analog neurons.

The FitzHugh-Nagumo model

The FitzHugh-Nagumo equations have the form:

$$\frac{dv}{dt} = f(v) - w + I_{ext} + D \cdot (v_{\text{driving}} - v)$$
$$\frac{dw}{dt} = \epsilon(v - \gamma w + \beta)$$
$$f(v) = v - \frac{v^3}{3}$$

where v is the membrane potential, w is a slow inhibitor variable, AND ϵ, γ and β are constants. It turns out that $f(v)$ can be any cubic-like function which sufficiently approximates $v - \frac{v^3}{3}$.

A method for diffusive inter-neuron coupling has been introduced in green. D is the diffusion coefficient, and can be positive (excitatory synapses, gap junctions) or negative (inhibitory junctions). The quantity scaled by D is simply the voltage difference between the driving neuron and the driven one.

The central pattern generator

Caenorhabditis elegans is a small nematode with a well-known neuronal layout. Its central pattern generator can be sufficiently approximated by a simple neuronal network, arranged as such:

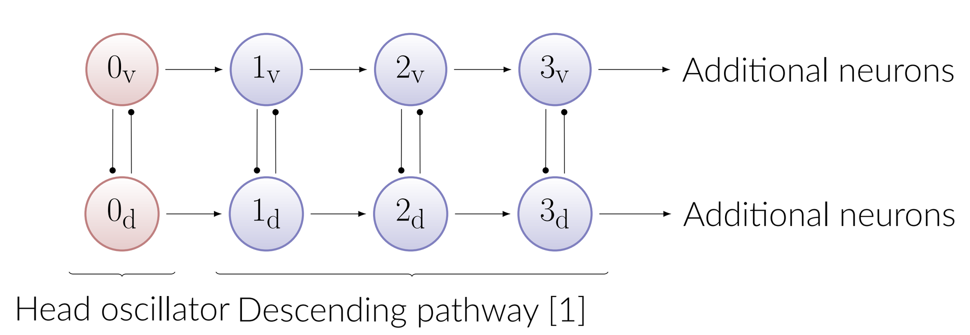

Figure 1. The central pattern generator, simplified.

wherein $0 \to 1$ represents unidirectional diffusion coupling, and $0 \leftrightarrow 1$ represents bidirectional diffusion coupling.

Simulation and experimental data

Vivamus congue volutpat elit non semper. Praesent molestie nec erat ac interdum. In quis suscipit erat. **Phasellus mauris felis, molestie ac pharetra quis**, tempus nec ante. Donec finibus ante vel purus mollis fermentum. Sed felis mi, pharetra eget nibh a, feugiat eleifend dolor. Nam mollis condimentum purus quis sodales. Nullam eu felis eu nulla eleifend bibendum nec eu lorem. Vivamus felis velit, volutpat ut facilisis ac, commodo in metus.

1. **Morbi mauris purus**, egestas at vehicula et, convallis accumsan orci. Orci varius natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus.
2. **Cras vehicula blandit urna ut maximus**. Aliquam blandit nec massa ac sollicitudin. Curabitur cursus, metus nec imperdiet bibendum, velit lectus faucibus dolor, quis gravida metus mauris gravida turpis.
3. **Vestibulum et massa diam**. Phasellus fermentum augue non nulla accumsan, non rhoncus lectus condimentum.

Fusce aliquam magna velit

Et rutrum ex euismod vel. Pellentesque ultricies, velit in fermentum vestibulum, lectus nisi pretium nibh, sit amet aliquam lectus augue vel velit. Suspendisse rhoncus massa porttitor augue feugiat molestie. Sed molestie ut orci nec malesuada. Sed ultricies feugiat est fringilla posuere.

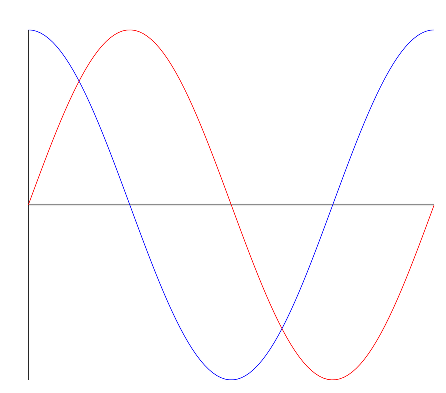

Figure 2. Another figure caption.

Nam cursus consequat egestas

Nulla eget sem quam. Ut aliquam volutpat nisi vestibulum convallis. Nunc a lectus et eros facilisis hendrerit eu non urna. Interdum et malesuada fames ac ante *ipsum primis* in faucibus. Etiam sit amet velit eget sem euismod tristique. Praesent enim erat, porta vel mattis sed, pharetra sed ipsum. Morbi commodo condimentum massa, *tempus venenatis* massa hendrerit quis. Maecenas sed porta est. Praesent mollis interdum lectus, sit amet sollicitudin risus tincidunt non.

Etiam sit amet tempus lorem, aliquet condimentum velit. Donec et nibh consequat, sagittis ex eget, dictum orci. Etiam quis semper ante. Ut eu mauris purus. Proin nec consectetur ligula. Mauris pretium molestie ullamcorper. Integer nisi neque, aliquet et odio non, sagittis porta justo.

- **Sed consequat** id ante vel efficitur. Praesent congue massa sed est scelerisque, elementum mollis augue iaculis.
  - In sed est finibus, vulputate nunc gravida, pulvinar lorem. In maximus nunc dolor, sed auctor eros porttitor quis.
  - Fusce ornare dignissim nisi. Nam sit amet risus vel lacus tempor tincidunt eu a arcu.

The circuit

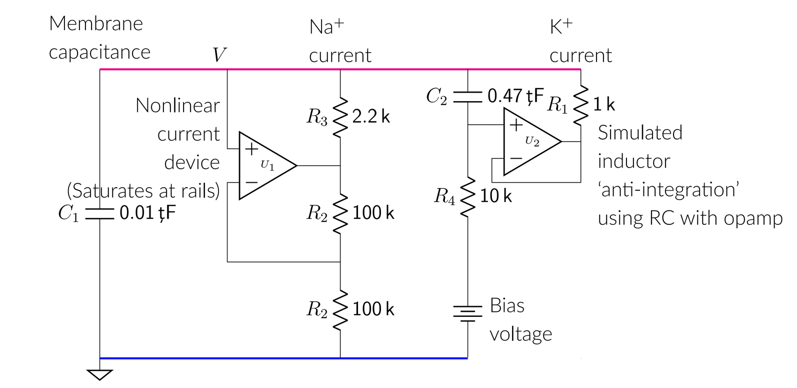

Figure 3. Our circuit (modified from [2]), simulating one Keener neuron.

Nullam non est elit. In eu ornare justo. Maecenas porttitor sodales lacus, ut cursus augue sodales ac.

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$$

Interdum et malesuada fames $\{1, 4, 9, \ldots\}$ ac ante ipsum primis in faucibus. Cras eleifend dolor eu nulla suscipit suscipit. Sed lobortis non felis id vulputate.

A heading inside a block

Praesent consectetur mi x^2+y^2 metus, nec vestibulum justo viverra nec. Proin eget nulla pretium, egestas magna aliquam, mollis neque. Vivamus dictum $\mathbf{u}^\intercal\mathbf{v}$ sagittis odio, vel porta erat congue sed. Maecenas ut dolor quis arcu auctor porttitor.

Another heading inside a block

Sed augue erat, scelerisque a purus ultricies, placerat porttitor neque. Donec $P(y \mid x)$ fermentum consectetur $\nabla_x P(y \mid x)$ sapien sagittis egestas. Duis eget leo euismod nunc viverra imperdiet nec id justo.

Nullam vel erat at velit convallis laoreet

Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos himenaeos. Phasellus libero enim, gravida sed erat sit amet, scelerisque congue diam. Fusce dapibus dui ut augue pulvinar iaculis.

Donec quis posuere ligula. Nunc feugiat elit a mi malesuada consequat. Sed imperdiet augue ac nibh aliquet tristique. Aenean eu tortor vulputate, eleifend lorem in, dictum urna. Proin auctor ante in augue tincidunt tempor. Proin pellentesque vulputate odio, ac gravida nulla posuere efficitur. Aenean at velit vel dolor blandit molestie. Mauris laoreet commodo quam, non luctus nibh ullamcorper in. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos himenaeos.

Nulla varius finibus volutpat. Mauris molestie lorem tincidunt, iaculis libero at, gravida ante. Phasellus at felis eu neque suscipit suscipit. Integer ullamcorper, dui nec pretium ornare, urna dolor consequat libero, in feugiat elit lorem euismod lacus. Pellentesque sit amet dolor mollis, auctor urna non, tempus sem.