

Name: _____

Please write your solutions in an organized and systematic manner.

Please show all work for partial credit. I may give no credit for solutions with no supporting work.

This exam is scored out of 80 points.

Useful definitions:

- An integer n is *even* if there is an integer k such that $n = 2k$.
- An integer n is *odd* if there is an integer k such that $n = 2k + 1$.
- If a, b are integers and $a \neq 0$ then a *divides* b (written $a \mid b$) if there is an integer k such that $b = ak$. We write $a \nmid b$ if a does not divide b .
- If a, b, n are integers and $n \neq 0$ then we say a and b are *congruent modulo* n (written $a \equiv b \pmod{n}$) if n divides $a - b$.
- A natural number $p > 1$ is *prime* if the only positive divisors of p are 1 and p . If a natural number $n > 1$ is not prime we call it *composite*.

You may use the following facts without proof or reference:

- Any logical equivalence or equality of sets proved in class or given as homework.
- The axioms for integers given in class.
- The sum of two integers is even iff they have the same parity.
- The product of two integers is even iff at least one of them is even.
- A natural number $p > 1$ is prime iff whenever p divides a product $a \cdot b$ it either divides a or it divides b .
- If p is prime then \sqrt{p} is irrational. In particular, $\sqrt{2}$ is irrational.
- (the fundamental theorem of arithmetic) Any natural number $n > 1$ can be written uniquely as a product of (not necessarily distinct) primes. In particular, every natural $n > 1$ has a prime factor.

If you need to use some other fact you need to either give a proof for it or give a reference to either the lectures, homework or the textbook.
