Here are some extra practice problems for you to enjoy.

1. Let A, B be sets. Show that $A \cup (B \cap A) = A$ and $A \cap (B \cup A) = A$.
2. Let A, B be sets. Show that if $A \subseteq B$ then $A \setminus B \subseteq B \setminus A$.
3. If A, B are sets, define $A \triangle B = (A \setminus B) \cup (B \setminus A)$.

   Show that $A \triangle B$ and $A \cap B$ are disjoint.
4. If A, B are sets, show that $A \cup B = (A \triangle B) \cup (A \cap B)$.
5. If A, B are sets, show that $A \triangle B = \emptyset$ iff $A = B$.
6. Let a, b be integers and $a \neq 0$. Show that if $a \mid b$ then $a^{10} \mid b^{10}$.
7. Let a be an integer. Show that if $6 \mid 5a$ then $6 \mid a$.
8. Let x, y be integers. Show that x and y have opposite parities iff $(x - y + 3)^3$ is even.
9. Let x be an integer. Show that $2 \mid (x^4 + 1)$ iff $4 \mid (x^2 - 1)$.
10. Let x be an integer. Show that if $x^2 - 1$ is even then it is divisible by 8. Give a counterexample to disprove the same claim about $x^2 - 5$.
11. If a, b, c are integers and $c \neq 0$ define $a \equiv b \pmod{c}$ if $5 \mid (a - b)$.

    Show that if $a \equiv b \pmod{c}$ and $d \equiv e \pmod{c}$ then $(a + d) \equiv (b + e) \pmod{c}$.
12. Show that there is no integer x such that $x^4 \equiv 2 \pmod{5}$.
13. Show that two integers x, y have the same parity iff $x \equiv y \pmod{2}$.
14. Let a be an integer and $n \geq 2$ a natural number. Show that if $a \equiv 0 \pmod{n}$ then $a^2 \equiv 0 \pmod{n^2}$. Give a counterexample to show that if a, b are integers then $a \equiv b \pmod{n}$ does not imply that $a^2 \equiv b^2 \pmod{n^2}$.
15. Let a be an integer. Show that $a^4 \equiv (5 - a)^4 \pmod{5}$.
16. Show that $\sqrt{6}$ is irrational.
17. Show that $\sqrt{2} + \sqrt{3}$ is irrational.
18. Show that $\log_6 7$ is irrational.
19. Recall the Fibonacci sequence, defined by $a_0 = 0, a_1 = 1$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 2$. Let n be a natural number. Show that $\sum_{k=0}^{n} a_k = a_{n+2} - 1$.
20. Define a sequence by $a_1 = -3, a_2 = 0$ and $a_{n+1} = 7a_n - 10a_{n-1}$ for $n \geq 2$. Show that $a_n = 2 \cdot 5^{n-1} - 5 \cdot 2^{n-1}$ for all positive natural n.
21. Let $0 < x < 1$ be a real number. Show that $(1 + x)^n < 1 + 2^n x$ for all natural n.
22. Define a sequence by $a_0 = 1, a_1 = 3$ and $a_n = 2a_{n-1} + 8a_{n-2}$ for $n \geq 2$. Show that $a_n \leq 4^n$ for all natural n.