Math 156.01 — Spring 2017 Homework set #6

Name:

Please write your solutions in an organized and systematic manner; use scratch paper to solve the problems first and then write up a neat solution with the relevant work shown.
You may use any results proved in class or on previous homeworks in your proofs. Be sure to clearly state when you do.

1. Suppose that x is a rational number and y an irrational number. Show that $x+y$ is irrational. [5 pts]

2. Show that if x and y are positive real numbers then $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$. [5 pts]

3. Prove the following statements (two thirds of them are easy): [5 pts]

   (a) There is an irrational number a and a rational number b such that a^b is irrational.

   (b) There are rational numbers a and b such that a^b is irrational.

   (c) There is a rational number a and an irrational number b such that a^b is irrational.

   (Hint for the hard statement: we proved in class that $\sqrt{2}$ is irrational and it follows easily that $\frac{1}{2\sqrt{2}}$ is also irrational. Now consider the number $2^{\frac{1}{2\sqrt{2}}}$; depending on whether this is rational or not find a and b.)

4. Show that $\sqrt{3}$ is irrational. [5 pts]

   (Hint: there are a few different ways to prove this. Depending on which strategy you pick, you may need to, along the way, prove that if a^2 is divisible by 3 then a is as well.)

5. Follow these steps to prove that $\sqrt{6}$ is irrational: [10 pts]

   (a) Let a be an integer. Show that if $3 \mid a^2$ then $3 \mid a$ (i.e. prove the hint from problem 4).

   (b) Let a be an integer. Show that if $2 \mid a$ and $3 \mid a$ then $6 \mid a$.

   (c) Prove that $\sqrt{6}$ is irrational, using parts (a) and (b).

   (You can treat the parts independently, e.g. if you can't do part (b), you can still use it in part (c).)