Here are some extra practice problems for you to enjoy.

1. Let A, B be sets. Show that $A \cup (B \cap A) = A$ and $A \cap (B \cup A) = A$.

2. Let A, B be sets. Show that if $A \subseteq B$ then $A \setminus B \subseteq B \setminus A$.

3. If A, B are sets, define $A \triangle B = (A \setminus B) \cup (B \setminus A)$.
   Show that $A \triangle B$ and $A \cap B$ are disjoint.

4. If A, B are sets, show that $A \cup B = (A \triangle B) \cup (A \cap B)$.

5. If A, B are sets, show that $A \triangle B = \emptyset$ iff $A = B$.

6. Let a, b be integers and $a \neq 0$. Show that if $a \mid b$ then $a^{10} \mid b^{10}$.

7. Let a be an integer. Show that if $6 \mid 5a$ then $6 \mid a$.

8. Let x, y be integers. Show that x and y have opposite parities iff $(x - y + 3)^3$ is even.

9. Let x be an integer. Show that $2 \mid (x^4 + 1)$ iff $4 \mid (x^2 - 1)$.

10. Let x be an integer. Show that if $x^2 - 1$ is even then it is divisible by 8. Give a counterexample to disprove the same claim about $x^2 - 5$.

11. If a, b, c are integers and $c \neq 0$ define $a \equiv b \pmod{c}$ if $c \mid (a - b)$.
    Show that if $a \equiv b \pmod{c}$ and $d \equiv e \pmod{c}$ then $(a + d) \equiv (b + e) \pmod{c}$.

12. Show that there is no integer x such that $x^4 \equiv 2 \pmod{5}$.

13. Show that two integers x, y have the same parity iff $x \equiv y \pmod{2}$.

14. Let a be an integer and $n \geq 2$ a natural number. Show that if $a \equiv 0 \pmod{n}$ then $a^2 \equiv 0 \pmod{n^2}$. Give a counterexample (i.e. values for a, b, n) to show that if a, b are integers then $a \equiv b \pmod{n}$ does not imply that $a^2 \equiv b^2 \pmod{n^2}$.

15. Let a be an integer. Show that $a^4 \equiv (5 - a)^4 \pmod{5}$.

16. Show that $\sqrt{2} + \sqrt{3}$ is irrational.

17. Show that $\log_6 7$ is irrational.

18. Let a_n be the terms of the Fibonacci sequence (i.e. $a_0 = a_1 = 1$, $a_n = a_{n-2} + a_{n-1}$). Prove Cassini's identity: $a_{n-1}a_{n+1} - a_n^2 = (-1)^n$.

19. Define a sequence by $a_1 = -3, a_2 = 0$ and $a_{n+1} = 7a_n - 10a_{n-1}$ for $n \geq 2$. Show that $a_n = 2 \cdot 5^{n-1} - 5 \cdot 2^{n-1}$ for all positive natural n.

20. Let $x > -1$ be a real number and n a natural number. Prove Bernoulli's inequality: $(1 + x)^n \geq 1 + nx$.

21. Let $0 < x < 1$ be a real number. Show that $(1 + x)^n < 1 + 2^n x$ for all natural n.

22. Define a sequence by $a_0 = 1, a_1 = 3$ and $a_n = 2a_{n-1} + 8a_{n-2}$ for $n \geq 2$. Show that $a_n \leq 4^n$ for all natural n.