

Name: _____

Please write your solutions in an organized and systematic manner; use scratch paper to solve the problems first and then write up a neat solution with the relevant work shown.

You may use any results proved in class in your proofs. Be sure to clearly state when you do.

1. Let a, b be integers. Prove that $a + b$ is even if and only if a and b have the same parity (i.e. they are both even or both odd). [5 pts]

(Hint: depending on how you approach this problem, you may need to do two cases for each of the two directions. The cases will be very similar to one another.)

2. Let a, b be integers. Prove that ab is odd if and only if both a and b are odd.

[5 pts]

Here is a new property of integers that we can prove things about.

Definition. If a and b are integers and $a \neq 0$, we say that a *divides* b (in symbols $a \mid b$; note that the line is straight, not slanted) if there is an integer k such that $b = ak$. We also say that b is divisible by a or that a is a factor or a divisor of b . We write $a \nmid b$ to say that a does not divide b .

For example, saying that an integer is even (according to the definition from class) means exactly that 2 divides it (according to this definition). The divisibility relation allows us to talk about more things than just evenness and oddness.

3. Let a, b, c be integers and $a \neq 0$ and $b \neq 0$. Show that if $a \mid b$ and $b \mid c$ then $a \mid c$.

[5 pts]

4. Let a be an integer. Show that if 2 divides a^2 then 4 divides a^2 . [5 pts]
(Hint: use a theorem from class.)

5. Let a and b be integers and $a \neq 0$. [5 pts]
- (a) Show that $a \mid a$.
 - (b) Assume that a and b are positive. Show that if $a \mid b$ and $b \mid a$ then $a = b$.