1. Consider the following sets

$$\begin{aligned} A &= \{-1, 1, 2\} \\ B &= \emptyset \\ C &= \{x \in \mathbb{Z}; x^4 - 5x^2 + 4 = 0\} \\ D &= \{x \in \mathbb{N}; \cos(x) = 0\} \end{aligned}$$

The sets A and B are already given in a simple way. We can simplify C and D as well by listing their elements.

The set C consists of integer zeros of the polynomial $x^4 - 5x^2 + 4 = 0$. A calculation shows that these are ± 1 and ± 2, so $C = \{1, -1, 2, -2\}$.

The set D consists of natural number zeros of the function cos. There are no such zeros, so $D = \emptyset$.

(a) Which of these four sets are equal?

We have $B = D$, since both are empty. All other pairs are distinct.

(b) What is the cardinality of C?

We listed the four elements of C above, so $|C| = 4$.

(c) Find two sets among these such that one will be a proper subset of the other.

There are many possible solutions here. For example $B \subset A$ or $B \subset C$, since both A and C are nonempty (but not $B \subset D$ as $B = D$). Another possibility is $A \subset C$ (since every element of A is an element of C and $A \neq C$).

(d) Find $A \cup C$ and $A \cap C$.

Going by the definitions of union and intersection we get $A \cup C = \{1, -1, 2, -2\}$ and $A \cap C = \{1, -1, 2\}$.

(e) Find $A \cap B$.

We see that A and B have no elements in common (because B has no elements at all) and so, going by the definition of intersection, $A \cap B = \emptyset$.

2. Give an example of three sets A, B and C such that $B \neq C$ but $B \setminus A = C \setminus A$.

There are literally infinitely many possible solutions here. For example, we could take

$$A = \{2, 3\}, \quad B = \{1, 2\}, \quad C = \{1, 3\}$$

Then obviously $B \neq C$ but $B \setminus A = \{1\} = C \setminus A$.

Comments: A clever solutions is to let A be any nonempty set and take $B = \emptyset$ and $C = A$.

3. Give an example of four different subsets A, B, C and D of $\{1, 2, 3, 4\}$ such that all 6 intersections of two of them (i.e. $A \cap B$, $A \cap C$, $A \cap D$, $B \cap C$, etc.) will be distinct.

   Again, there are several possible solutions here. For example, we can take

   $$A = \{1, 2\}, \quad B = \{1, 2, 3\}, \quad C = \{2, 3, 4\}, \quad D = \{1, 3, 4\}$$

   The pairwise intersections are then as follows:

   $$\begin{array}{lll} A \cap B = \{1, 2\} & A \cap C = \{2\} & A \cap D = \{1\} \\ B \cap C = \{2, 3\} & B \cap D = \{1, 3\} & \\ C \cap D = \{3, 4\} & & \end{array}$$

   and they are all distinct.

   Comments: If you think hard there is a strategy of finding combinations of sets like these, but for these small examples it is usually better to try a few times and see what works.

4. Find an example of two infinite subsets A_1 and A_2 of $\mathbb{N}$, satisfying $A_1 \cap A_2 = \emptyset$ and $A_1 \cup A_2 = \mathbb{N}$.

   One possibility here is to let A_1 be the set of odd numbers and A_2 the set of even numbers. Another is to take A_1 to be the set of prime numbers and A_2 the set of composite numbers.

5. Compute $\mathcal{P}(\{1, 2, 4\})$.

   We just list out all of the subsets of $\{1, 2, 4\}$. We know there have to be $2^3 = 8$. We get

   $$\mathcal{P}(\{1, 2, 4\}) = \{\emptyset, \{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 4\}, \{2, 4\}, \{1, 2, 4\}\}$$

6. (extra credit)

   Let A, B and C be sets. "Prove" (i.e. convince me) that if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

   Here is an "intuitive" argument; we will learn how to do this proof properly in class.

   If A is a subset of B then every element in A is an element of B. On the other hand, since B is a subset of C, every element of B is an element of C. Putting this together, every element of A is in B, and then it is also in C. So A is a subset of C.