Math 156.01 — Spring 2017 Homework set #7

Name:

Please write your solutions in an organized and systematic manner; use scratch paper to solve the problems first and then write up a neat solution with the relevant work shown.
You may use any results proved in class or on previous homeworks in your proofs. Be sure to clearly state when you do.

1. Disprove the following statement: Let n be an integer. If $\frac{(n+1)(n+2)}{2}$ is odd then $\frac{n^2(n+1)^2}{2}$ is odd. [5 pts]

2. Let n be a natural number. Show that $\sum_{i=0}^{n}(3i-2) = \frac{3n^2-n-4}{2}$. [5 pts]

3. Let n be a natural number. Show that $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. [5 pts]

4. Define a sequence recursively by letting $a_0 = 1$ and $a_n = 2a_{n-1}$ for $n \geq 1$. Find a closed form expression for this sequence and prove that it is correct. [5 pts]

5. Let n be a natural number. Show that $3 \mid n^3 - n$. [5 pts]