Math 156.01 — Summer 2016 Homework set #3

Name:

Please write your solutions in an organized and systematic manner; use scratch paper to solve the problems first and then write up a neat solution with the relevant work shown.

1. Prove that the sum of two odd integers is even. [5 pts]

2. Let A and B be sets. Prove that if $A \subseteq B$ then $A \cap B = A$. [5 pts]

   (Hint: Remember, to prove two sets X, Y are equal you need to prove $X \subseteq Y$ and $Y \subseteq X$).

3. Let x be an integer. Prove that if 2^{2x} is an odd integer then 2^{-2x} is an odd integer.

[5 pts]

4. Here is a definition:

   A subset A of $\mathbb{N}$ is *bounded* if there is a natural number n such that $A \subseteq \{0, 1, \ldots, n\}$.

   Prove that the union of two bounded sets is bounded. [5 pts]

5. Let A, B be sets contained in some universal set. Show that if $A \subseteq B$ then $B^{\mathsf{c}} \subseteq A^{\mathsf{c}}$. [5 pts]

6. (extra credit) [5 pts]
   Let $A = \{0, 1, 2, 3, 4, 5\}$. Prove that if x is an element of every subset of A then $x = 1$.