Math 156.01 — Spring 2017 Homework set #3

Name:

Please write your solutions in an organized and systematic manner; use scratch paper to solve the problems first and then write up a neat solution with the relevant work shown.

1. Translate the following formulas into natural language and determine whether they are true or false. The domain of all quantifiers is the set of all people and $P(x, y)$ means "x is a parent of y". [5 pts]

   (a) $\exists x \forall y \colon P(x, y)$

   (b) $\forall x \exists y \colon P(x, y)$

   (c) $\forall y \exists x \colon P(x, y)$

   (d) $\sim \exists x \exists y \colon P(x, y)$

   (e) $\exists x \sim \exists y \colon P(x, y)$

   (f) $\exists x \exists y \colon \sim P(x, y)$

2. Consider the statement scheme [5 pts]

$$P(n) : \text{“}2n^2 + 11 \text{ is prime”}$$

(a) Is the statement $\forall n \in \{0, 1, 2, 3, 4, 5\}$: $P(n)$ true or false? Justify your answer.

(b) Is the statement $\sim \exists n \in \mathbb{N}$: $P(n)$ true or false? Justify your answer.

(c) Is the statement $\forall n \in \mathbb{N}$: $P(n)$ true or false? Justify your answer.
(You can find a list of primes online.)

3. Let A, B be sets. Show that $A \cap B \subseteq B$. [5 pts]
(Write out the argument carefully.)

4. Let A, B be sets. Show that $A \setminus B \subseteq A$. [5 pts]
   (Write out the argument carefully.)