Honour and glory problem #2

In problem 4 on Homework set 2 we saw a polynomial $p(n) = 2n^2 + 11$ with the peculiar property that $p(n)$ was prime for any $n \in \{0, 1, 2, \ldots, 10\}$. In the solutions I alluded to a result of Goldbach that no (nonconstant) polynomial with integer coefficients can output prime values for all natural number inputs.

1. Prove an easy version of Goldbach's theorem for linear polynomials: if $p(n) = an + b$ and a, b are integers and $a \neq 0$ then it is not true that $p(n)$ is prime for every natural number n.

   (Hint: This is very easy; think of an appropriate n to plug in and get a contradiction.)

2. Prove a slightly harder version for quadratic polynomials: if $p(n) = an^2 + bn + c$ and a, b, c are integers and $a \neq 0$ then it is not true that $p(n)$ is prime for every natural number n.

   (Hint: Almost the same thing works here as in part (1), but instead of plugging in that number, plug in a large enough multiple of that number.)

3. (after having done part (2)) Try to prove Goldbach's theorem in general, i.e. if $p(n) = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0$ and the $a_0, a_1, \ldots, a_k$ are integers and $a_k \neq 0$ then it is not true that $p(n)$ is prime for every natural number n.

   (Hint: This is practically the same as part (2).)