This is a sketch of an elementary proof, substantially due to Fourier, showing that e is irrational. See if you can follow along and fill in the gaps in the argument.
Depending on how much mathematics you have seen in your life you might know different definitions of the number e. For the purposes of this proof we will work with

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \cdots$$

Theorem. *e is irrational.*

Proof. The proof will proceed in several steps.
First we will show that e is not an integer. Clearly $e > 2$ since the definition gives us that $e = 2 + \sum_{n=2}^{\infty} \frac{1}{n!}$ and all the terms of the sum on the right are strictly positive.

Gap. Show that $e < 3$. To do this, rewrite the series for e as follows

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1+1+\frac{1}{2}\left(1 + \frac{1}{3} + \frac{1}{3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \cdots\right) \leq 1+1+\frac{1}{2}\left(1 + \frac{1}{3} + \frac{1}{3 \cdot 3} + \frac{1}{3 \cdot 3 \cdot 3} + \cdots\right)$$

The sum on the right is a geometric series. Compute its value to get a bound on e.

Since $2 < e < 3$, we now know that e is not an integer. Now assume, toward a contradiction, that e is rational. Then there are positive natural numbers a, b such that $e = \frac{a}{b}$. We can immediately say that $b > 1$ since otherwise e would be an integer.
Consider the number

$$x = b!\left(e - \sum_{n=0}^{b} \frac{1}{n!}\right) = b! \sum_{n=b+1}^{\infty} \frac{1}{n!}$$

Gap. Show that x is an integer. To do this, work with the middle expression for x, use the assumption that e is rational, multiply through and check that all the denominators cancel.

It is clear from the second expression for x that $x > 0$, so we now know that x is a positive natural number.
Now consider the number $(b+1)x$.

Gap. Show that $(b+1)x \leq 1 + x$. To do this, observe that

$$(b+1)x = (b+1)! \sum_{n=b+1}^{\infty} \frac{1}{n!} = 1 + \frac{1}{b+2} + \frac{1}{(b+2)(b+3)} + \cdots \leq 1 + \frac{1}{b+1} + \frac{1}{(b+1)(b+2)} + \cdots$$

The rightmost side is related to x somehow.

The inequality $(b+1)x \leq 1 + x$ simplifies to give $bx \leq 1$. But this is impossible: both b and x were positive natural numbers and b was strictly greater than 1. $\square$