Math 156.01 — Summer 2016 Homework set #2

Name: ____________________

Please write your solutions in an organized and systematic manner; use scratch paper to solve the problems first and then write up a neat solution with the relevant work shown. Each problem is worth 2 points.

1. Show that $(P \implies Q) \vee (Q \implies P)$ is a tautology. [5 pts]

2. Show, using the logical equivalences given in class (or as Theorems 2.17 & 2.18 in the textbook), that the statements $P \implies (Q \implies R)$ and $(P \wedge Q) \implies R$ are logically equivalent. Do not use truth tables. [5 pts]

3. (a) Show that $P \implies Q$ and $\sim Q \implies \sim P$ are logically equivalent. [5 pts]
   (These two implications are *contrapositives* of one another).

   (b) Show that $P \implies Q$ and $Q \implies P$ are not logically equivalent.
   (These two implications are *converses* of one another).

4. Consider the statement [5 pts]

$$P(n) : 2n^2 + 11 \text{ is prime}$$

   (a) Is the statement $\forall n \in \{0, 1, 2, 3, 4, 5\}\colon P(n)$ true or false? Justify your answer.

   (b) Is the statement $\sim \exists n \in \mathbb{N}\colon P(n)$ true or false? Justify your answer.

   (c) Is the statement $\forall n \in \mathbb{N}\colon P(n)$ true or false? Justify your answer.
   (You may want to find a list of primes online.)

5. Using the logical equivalences from class we can see that any statement is logically equivalent to another statement which only involves negations, conjunctions and disjunctions. [5 pts]

   Argue that in fact any statement is logically equivalent to one which only involves negations and conjunctions.

   (Hint: you need to show that the statement $P \vee Q$ is logically equivalent to a statement involving only negations and conjunctions).

6. (extra credit) Consider the following truth table [5 pts]

| $P$ | $Q$ | $P \uparrow Q$ |
|---|---|---|
| $T$ | $T$ | $F$ |
| $T$ | $F$ | $T$ |
| $F$ | $T$ | $T$ |
| $F$ | $F$ | $T$ |

   The connective $\uparrow$ is called the *Sheffer stroke* or *NAND*.

   Show that any statement is logically equivalent to one which only involves Sheffer strokes.

   (Hint: following problem 5, you need to show that the statements $\sim P$ and $P \wedge Q$ are logically equivalent to statements involving only Sheffer strokes).