

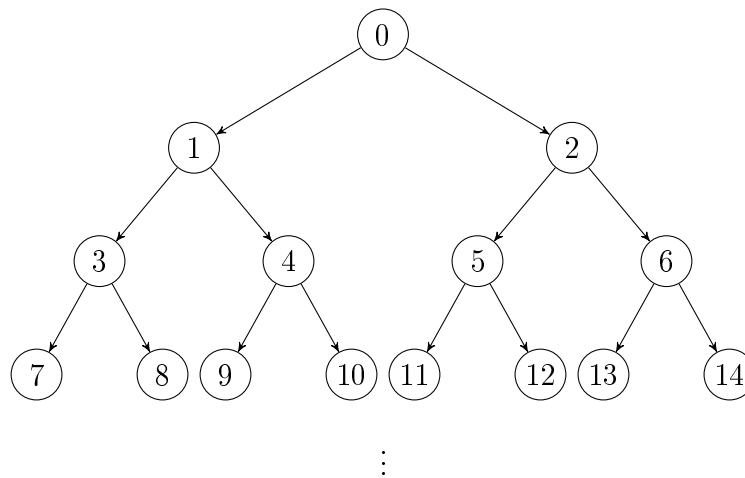
Honour and glory problem #1

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Find an example of a family  $\{A_i\}_{i \in I}$  where  $I$  is the set of infinite sequences of 0s and 1s (e.g.  $111\cdots \in I$  and  $010101\cdots \in I$  but  $0123\cdots \notin I$ ) satisfying the following conditions:

- each  $A_i$  is an infinite subset of  $\mathbb{N}$ ;
- $A_i \cap A_j$  is finite for any two distinct indices  $i$  and  $j$ ;
- $\bigcup_{i \in I} A_i = \mathbb{N}$ .

Hint 1: Instead of thinking of the natural numbers as being arranged in a line, think of them as being arranged in a tree structure like so:



Hint 2: Think of infinite binary sequences as a series of instructions for moving through the tree (e.g. interpret  $01100\cdots$  as moving left, then right twice, then left twice etc.).