Math 156.01 — Spring 2017 Homework set #8

Name:

Please write your solutions in an organized and systematic manner; use scratch paper to solve the problems first and then write up a neat solution with the relevant work shown.
You may use any results proved in class or on previous homeworks in your proofs. Be sure to clearly state when you do.

1. Let a_n be the terms of the Fibonacci sequence, defined by $a_0 = 1, a_1 = 1$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 2$.

   Let n be a natural number. Show that $\sum_{i=0}^{n} a_i = a_{n+2} - 1$. [5 pts]

2. Show that $\log_{10} 16$ is irrational. [5 pts]

3. We have often used the fact that if A is a set with n elements then $\mathcal{P}(A)$ has 2^n elements. Prove that this is actually true. [5 pts]

   (Hint: You may assume that $A = \{1, 2, \ldots, n\}$. Argue by induction on n. In the induction step you can break up $\mathcal{P}(A)$ into two parts: the subsets which don't contain $n + 1$ and the subsets which do.)

4. Let p be a prime number. Show that $\sqrt{p}$ is irrational. [5 pts]

   You may either prove this by hand or find a clever use of the fundamental theorem of arithmetic. You may **not** use the last theorem proved in class, that the square root of a natural number is either natural or irrational.

5. Let $n > 1$ be a composite number. Prove that n does not divide $(n-1)! - 1$.

[5 pts]

(Hint: Factor n as $n = ab$ with $a, b < n$. Argue by contradiction.)