

Name: _____

Please write your solutions in an organized and systematic manner; use scratch paper to solve the problems first and then write up a neat solution with the relevant work shown.

You may use any results proved in class or in chapters 1–6 of the textbook in your proofs. Be sure to provide a reference: either a theorem number from the textbook or the statement from class.

1. Disprove the following statement: Let n be an integer. If $\frac{(n+1)(n+2)}{2}$ is odd then $\frac{n^2(n+1)^2}{2}$ is odd. [5 pts]

2. Suppose that x is a rational number and y is an irrational number. Show that $x + y$ is irrational. [5 pts]

3. Show that there are a rational number a and an irrational number b such that a^b is irrational. [5 pts]

(Hint: We proved in class that $\sqrt{2}$ is irrational and this easily implies that $\frac{1}{2\sqrt{2}}$ is also irrational. Now consider the number $2^{\frac{1}{2\sqrt{2}}}$. Depending on whether this is rational or irrational find a and b .)

4. Let n be a natural number. Show that $\sum_{k=0}^n (3k - 2) = \frac{3n^2 - n - 4}{2}$. [5 pts]

5. Show that $n! > 2^n$ for any natural number $n \geq 4$.

[5 pts]

6. (extra credit) Let $x > -1$ be a real number and n a natural number. Prove Bernoulli's inequality:

[5 pts]

$$(1 + x)^n \geq 1 + nx$$

(Hint: Argue by induction on n .)