

Name: _____

Please write your solutions in an organized and systematic manner; use scratch paper to solve the problems first and then write up a neat solution with the relevant work shown.

You may use any results proved in class or in chapters 1, 2 and 3 of the textbook in your proofs. Be sure to provide a reference: either a theorem number from the textbook or the statement from class.

1. Let x and y be integers. Show that if x is even or $y = 0$ then xy is even. [5 pts]

2. Let a and b be integers. Show that $(a + b)^2 = a^2 + b^2$ if and only if at least one of a and b is 0. [5 pts]

3. Let a and b be nonzero integers. [5 pts]

(a) Show that $a \mid a$.

(b) Assume additionally that a and b are positive. Show that if $a \mid b$ and $b \mid a$ then $a = b$.

4. Let a, b, c be integers and $a \neq 0$. Show that if $a \nmid bc$ then $a \nmid b$ and $a \nmid c$. [5 pts]

5. A positive natural number p is *prime* if $p \mid ab$ implies $p \mid a$ or $p \mid b$ for any integers a, b .
Show that 2 is prime. [5 pts]
(Hint: This is really just an application of a theorem from class.)

6. (extra credit) Let n be an integer. Show that $2 \mid (n^4 - 3)$ iff $4 \mid (n^2 + 3)$. [5 pts]