Name: _____

Please write your solutions in an organized and systematic manner; use scratch paper to solve the problems first and then write up a neat solution with the relevant work shown.

1. Consider the following sets [5 pts]

$$A = \{-1, 1, 2\}$$
$$B = \emptyset$$
$$C = \{x \in \mathbb{Z}; x^4 - 5x^2 + 4 = 0\}$$
$$D = \{x \in \mathbb{N}; \cos(x) = 0\}$$

(a) Which of these four sets are equal?

(b) What is the cardinality of $C$?

(c) Find two sets among these such that one will be a proper subset of the other.

(d) Find $A \cup C$ and $A \cap C$.

(e) Find $A \cap B$.

2. Give an example of three sets $A, B$ and $C$ such that $B \neq C$ but $B \setminus A = C \setminus A$.

[5 pts]

3. Give an example of four different subsets $A, B, C$ and $D$ of $\{1, 2, 3, 4\}$ such that all 6 intersections of two of them (i.e. $A \cap B$, $A \cap C$, $A \cap D$, $B \cap C$, etc.) will be distinct.

[5 pts]

4. Find an example of two infinite subsets $A_1$ and $A_2$ of $\mathbb{N}$, satisfying $A_1 \cap A_2 = \emptyset$ and $A_1 \cup A_2 = \mathbb{N}$. [5 pts]

5. For $A = \{1, 2\}$ and $B = \{4\}$, determine $\mathcal{P}(A \times B)$. [5pts]

6. (extra credit) [5 pts]

    (a) Find an example of three infinite subsets $A_1, A_2$ and $A_3$ of $\mathbb{N}$, such that $A_1 \cap A_2 = \emptyset$, $A_1 \cap A_3 = \emptyset$ and $A_2 \cap A_3 = \emptyset$ and $A_1 \cup A_2 \cup A_3 = \mathbb{N}$.

    (b) Find an example of a family $\{A_n\}_{n \in \mathbb{N}}$ satisfying the following conditions:

- each $A_n$ is an infinite subset of $\mathbb{N}$;
- $A_n \cap A_m = \emptyset$ for any two distinct indices $n$ and $m$;
- $\bigcup_{n \in \mathbb{N}} A_n = \mathbb{N}$.