1. Translate the following formulas into natural language and determine whether they are true or false. The domain of all quantifiers is the set of all people and $P(x, y)$ means "x is a parent of y".

   (a) $\exists x \forall y \colon P(x, y)$

   This is saying that there exists a person x such that for every person y you might pick, x is a parent of y. This means that the person x is a parent of every person in the world.

   Most naturally, the statement can be expressed as: there is a person who is a parent of everyone. This statement is *false*.

   (b) $\forall x \exists y \colon P(x, y)$

   This is saying that for any person x you might pick there is some person y such that x is a parent of y. In other words, whatever person x you choose, that person has a child.

   Most naturally, the statement can be expressed as: everyone has a child (or everyone is a parent). This statement is *false*.

   Comment: Note the order of quantifiers: we're saying that for every person x there is another person y such that .... The choice of person y can depend on the choice of person x. In particular, the statement isn't saying that there is some y that is everyone's child.

   (c) $\forall y \exists x \colon P(x, y)$

   This is saying that for any person y you might pick there is some person x such that x is a parent of y. In other words, whatever person y you choose, that person has a parent.

   Most naturally the statement can be expressed as: everyone has a parent (or everyone is a child). This statement is *true*.

   (d) $\sim \exists x \exists y \colon P(x, y)$

   This is saying that there do not exists people x and y such that x is a parent of y. In other words, whatever people x and y you pick, they will not be related.

   Most naturally the statement can be expressed as: no one has children (or noone has parents). This statement is *false*.

   Comment: Having the negation in front of the two quantifiers doesn't mean that just the first one gets negated, i.e. the statement should not be read as "there does not exist an x and there exists a y such that...". Instead the whole statement after the negation should be negated or, alternatively, the negation should be pushed through the quantifiers to get the equivalent statement $\forall x \forall y \colon \sim P(x, y)$.

(e) $\exists x \sim \exists y \colon P(x,y)$

This is saying that there exists a person x with the property that there isn't any person y such that x would be the parent of y. In other words, no one is x's child.

Most naturally the statement can be expressed as: there is a person with no children. This statement is *true*.

(f) $\exists x \exists y \colon \sim P(x,y)$

This is saying that there exist people x and y with the property that x is not a parent of y.

There isn't really a much simpler way to say this, but we could say something like: there are two people who are not related (in a parent-child way). This statement is *true*.

2. Consider the statement scheme

$$P(n) : \text{“}2n^2 + 11 \text{ is prime”}$$

(a) Is the statement $\forall n \in \{0,1,2,3,4,5\} \colon P(n)$ true or false? Justify your answer.

By plugging in the given values for n we see that $2n^2+11$ gives prime values in all of those cases. So this statement is true.

(b) Is the statement $\sim \exists n \in \mathbb{N} \colon P(n)$ true or false? Justify your answer.

The statement says that there is no natural number n for which $2n^2 + 11$ would be prime. But we just gave examples of such numbers n in part (a), like $n = 0$. The statement is therefore false.

(c) Is the statement $\forall n \in \mathbb{N} \colon P(n)$ true or false? Justify your answer.

Computing for a bit we see that for $n = 11$ the number $2(11)^2 + 11 = 253$ isn't prime (it is divisible by 11, for example). So the statement is false.

3. Let A, B be sets. Show that $A \cap B \subseteq B$.

Let x be an arbitrary element of $A \cap B$. By definition this means that $x \in A$ and $x \in B$. Therefore $x \in B$, which is what we needed to prove.

Comment: A better way of doing this is to observe that $A \cap B = B \cap A$. We showed in class that $A \cap B \subseteq A$, so putting it together gives $A \cap B = B \cap A \subseteq B$.

4. Let A, B be sets. Show that $A \setminus B \subseteq A$.

Let x be an arbitrary element of $A \setminus B$. By definition this means that $x \in A$ and $x \notin B$. Therefore $x \in A$, which is what we needed to prove.