ML CS 403/603 Lab : Linear Regression

Solution: (1) Numerical Part.

X_1	X_2	Y
2104	5	460
1416	3	232
1534	3	315

(a) Design matrix : X

$$X = \begin{bmatrix} 1 & 2104 & 5 \\ 1 & 1416 & 3 \\ 1 & 1534 & 3 \end{bmatrix}_{3\times 3} \qquad Y = \begin{bmatrix} 460 \\ 232 \\ 315 \end{bmatrix}_{3\times 1}$$

(b) $\theta = (X^T X)^{-1} X^T y$

$$(X^T X) = \begin{bmatrix} 3 & 5054 & 11 \\ 5054 & 8785028 & 19370 \\ 11 & 19370 & 43 \end{bmatrix}$$

$$|X^T X| = \begin{vmatrix} 3 & 5054 & 11 \\ 5054 & 8785028 & 19370 \\ 11 & 19370 & 43 \end{vmatrix}$$

$= 55695.999 \neq 0$, so inverse exists.

$$(X^TX)^{-1} = \begin{bmatrix} 45.95 & -0.07 & 22.63 \\ -0.07 & 0.00014 & -0.045 \\ 22.63 & -0.045 & 14.058 \end{bmatrix}$$

$$(X^Ty) = \begin{bmatrix} 1007 \\ 1779562 \\ 3941 \end{bmatrix}_{3\times 1}$$

$$\theta = (X^TX)^{-1} X^Ty = \begin{bmatrix} -380.10169491 \\ 0.70338983 \\ -127.96610169 \end{bmatrix}_{3\times 1}$$

© Time complexity for closed form of Linear Regression = $O(n^3)$ because for calculation inverse, it will take Order of n^3.

n = # features.

Solution:- 2(a) Numerical Part

x_1	x_2	x_3	y
2104	5	10	460
1416	3	6	232
1534	3	6	315

(a) Design Matrix

$$X = \begin{bmatrix} 1 & 2104 & 5 & 10 \\ 1 & 1416 & 3 & 6 \\ 1 & 1534 & 3 & 6 \end{bmatrix}_{3\times 4}$$

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \end{bmatrix}_{3\times 1}$$

$$\theta = (X^T X)^{-1} X^T y$$

$$X^T X = \begin{bmatrix} 1 & 1 & 1 \\ 2104 & 1416 & 1534 \\ 5 & 3 & 3 \\ 10 & 6 & 6 \end{bmatrix}_{4\times 3} \begin{bmatrix} 1 & 2104 & 5 & 10 \\ 1 & 1416 & 3 & 6 \\ 1 & 1534 & 3 & 6 \end{bmatrix}_{3\times 4}$$

$$= \begin{bmatrix} 3 & 5054 & 11 & 22 \\ 5054 & 8785028 & 19370 & 38740 \\ 11 & 19370 & 43 & 86 \\ 22 & 38740 & 86 & 172 \end{bmatrix}_{4\times 4}$$

$$(X^T X)^{-1} = \frac{\text{Adj}\,(X^T X)}{|X^T X|}$$

$$|X^T X| = \text{np.linalg.det}(X^T X) = 0.0$$

(b) Since, here features are Linearly Dependent on each other, so we can not find the inverse, our Design matrix X is non-invertible.

So, parameter θ can not be find.

Solution:-3 (Numerical Part).

x_1	x_2	y
2104	5	460
1416	3	232
1534	3	315

$$X = \begin{bmatrix} 1 & 2104 & 5 \\ 1 & 1416 & 3 \\ 1 & 1534 & 3 \end{bmatrix}_{3\times 3} \qquad Y = \begin{bmatrix} 460 \\ 232 \\ 315 \end{bmatrix}_{3\times 1}$$

Mean Normalization :-

$$X = \frac{(X - X.\text{mean}())}{X.\text{std}()}$$

$$X = \begin{bmatrix} -0.6924 & 1.8981 & -0.6874 \\ -0.6924 & 1.0506 & -0.6899 \\ -0.6924 & 1.1959 & -0.6899 \end{bmatrix}_{3\times 3}$$

$$\theta = (X^T X)^{-1} X^T y$$

$$X^T X = \begin{bmatrix} 1.4383 & -2.8698 & 1.4315 \\ -2.8698 & 6.1369 & -2.8549 \\ 1.4215 & -2.8549 & 1.4247 \end{bmatrix}_{3\times 3}$$

$$(X^T X)^{-1} = np.linalg.pinv(X^T X)$$

$$= \begin{bmatrix} 9.63766157e+06 & 2.98183125e+04 & -9.62382132e+06 \\ 2.98183125e+04 & 9.46603293e+01 & -2.97706736e+04 \\ -9.62382132e+06 & -2.97706736e+04 & 9.60011e+06 \end{bmatrix}$$

$$(X^T y) = \begin{bmatrix} -697.2694 \\ 1493.5989 \\ -693.6552 \end{bmatrix}$$

$$\theta = (X^T X)^{-1} . X^T y = \begin{bmatrix} 104045.07610607 \\ 571.01445216 \\ -103883.35234165 \end{bmatrix}_{3\times1}$$