

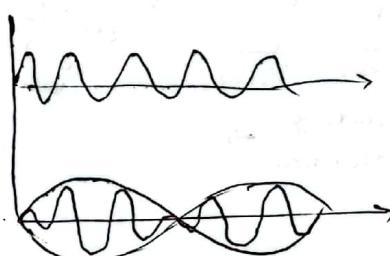
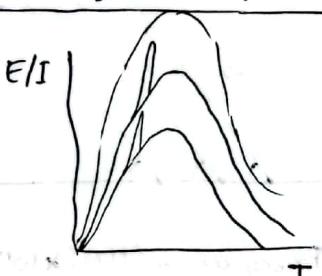
Lecture-1

Dr Md. Asaduzzaman → Assistant Prof

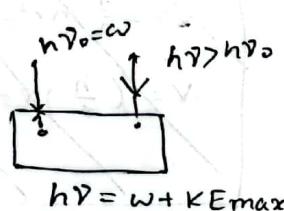
Md. Idris Ali → Assistant Prof

Concept of Modern Physics

- A Bein

Inadequacy of classical conceptQuantum Theory of LightLecture-2Date: 08.01.2023  
Sunday

Threshold energy → amount of energy needed to remove a  $e^-$  from its orbit



1 m<sup>2</sup> area  
1 atm thick →  $N_A$   
↓  
 $10^{19}$  atoms (roughly)

Stopping potential → potential energy = K.E.  
Retarding Potential

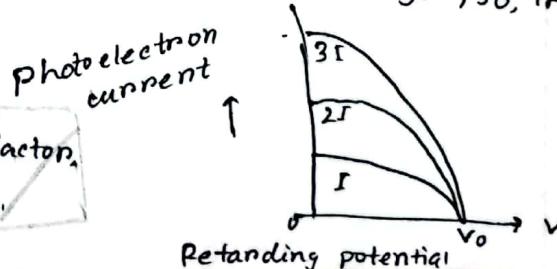
$w = h\nu_0$   
↓  
threshold energy  
threshold frequency



① Why individual metal has individual threshold energy?

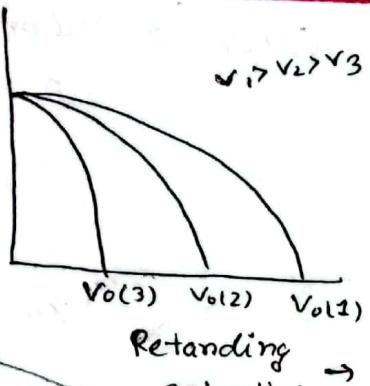
→ Bcoz individual metal has individual  $e^-$  energy, so,  $\nu_0$  is different.

frequency is the main factor,  
energy  $\propto \nu$  & number of  $e^-$



my whole life was a lie

photoelectron current



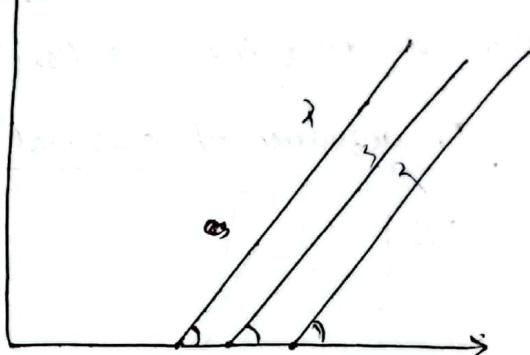
$$E = h\nu$$

$$\frac{E}{h} = \nu$$

$$y = mx$$

$$\textcircled{5} = m = \frac{eV}{h\nu}$$

Max photoelectron energy (eV) K.E.



Frequency (Hz) ( $\times 10^{14} \text{ Hz}$ ) Range

Individual metal needs individual ~~is~~ threshold energy

$$h\nu = KE_{\max} + \varphi / W_0 / w$$

$$= KE_{\max} + h\nu_0$$

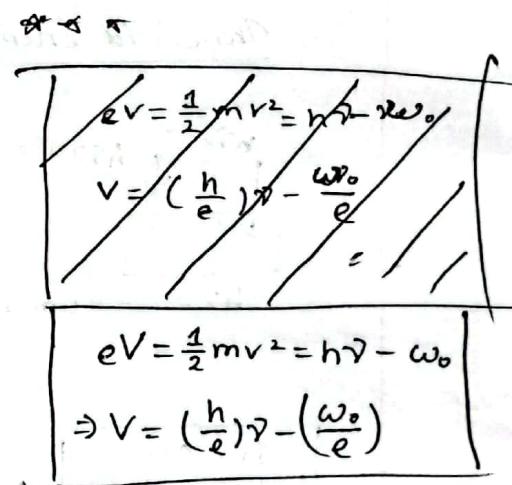
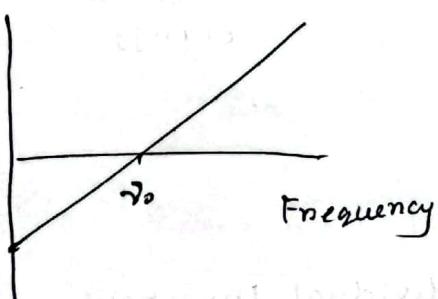
$$h\nu = KE_{\max} + \frac{he}{\lambda_0} \propto$$

$$\Rightarrow KE_{\max} = h(\nu - \nu_0)$$

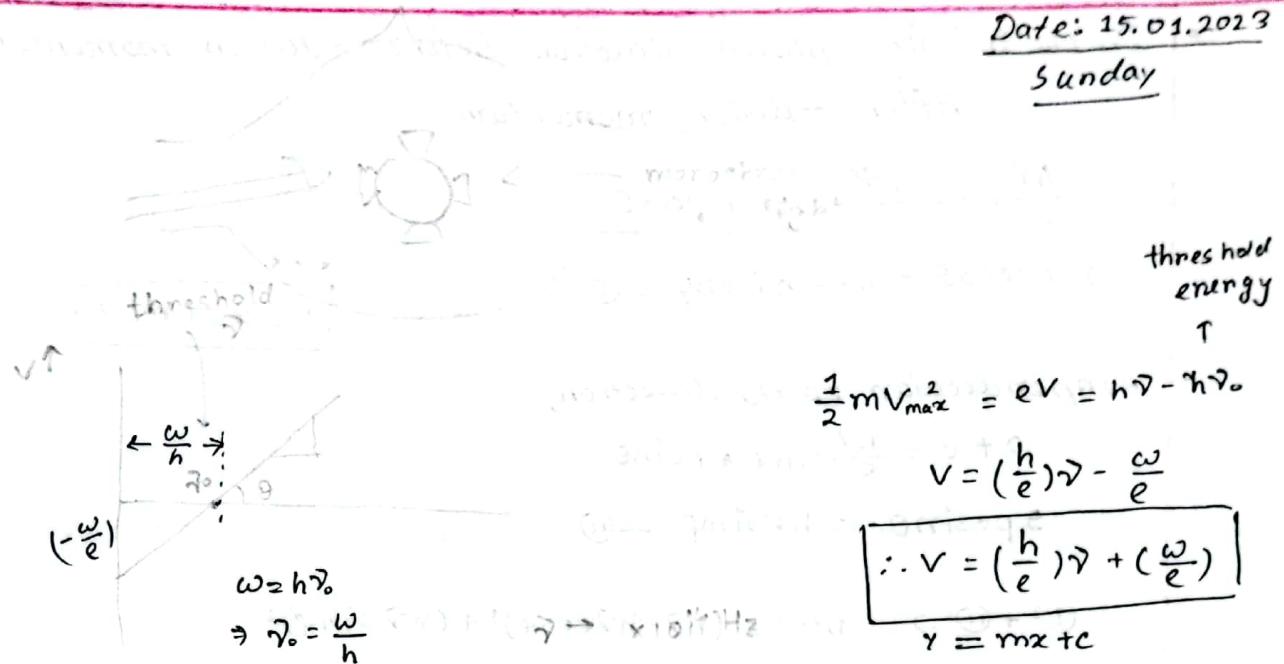
Millikan's equation

$$\begin{aligned} \text{Na} - 5890 \text{ Å} \\ 5896 \text{ Å} \end{aligned} \quad \left. \begin{array}{l} \text{monochromatic} \\ \text{light intensity} \end{array} \right\}$$

Systematic Potential



Date: 15.01.2023  
Sunday



- $\tan \theta = \frac{\omega/e}{\omega/h}$
- Slope  $e = \frac{dy}{dx} = \frac{h}{e} \rightarrow 6.55 \times 10^{34} \text{ Js}$

- Slope  $= \frac{h}{e}$

photon  $\rightarrow$  massless particle

### Compton effect:

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad P = mv = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow p^2 c^2 = \frac{m_0^2 v^2 c^2}{1 - \frac{v^2}{c^2}} \quad \text{--- (1)}$$

$$E^2 = m_0^2 c^4 + p^2 c^2$$

$$\bullet E^2 = \frac{m_0^2 c^4}{1 - \frac{v^2}{c^2}} \quad \text{--- (1)}$$

$$\Rightarrow \frac{m_0^2 c^4}{E^2} = \frac{m_0^2 v^2 c^2}{p^2 c^2}$$

$$\text{--- (1)} - \text{--- (1)} \Rightarrow$$

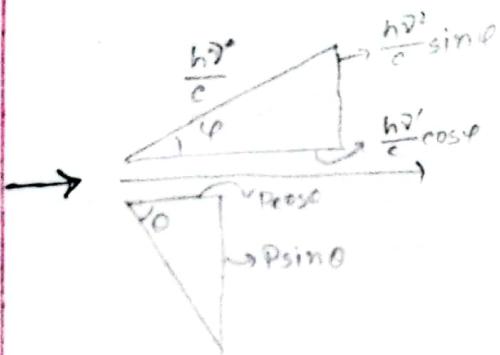
$$E^2 - p^2 c^2 = \frac{m_0^2 c^4 (1 - \frac{v^2}{c^2})}{(1 - \frac{v^2}{c^2})}$$

$$\Rightarrow E^2 = m_0^2 c^4 + p^2 c^2$$

$$\Rightarrow E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$$m_0 = 0, E = pc$$

$$\Rightarrow p = \frac{h\nu}{c}$$



## Particle properties of wave

- In the incident photon direction before collision momentum  
= After collision momentum

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c} \cos\theta + p \cos\theta$$

$$\Rightarrow p \cos\theta = h\nu - h\nu' \cos\varphi \quad \text{---(1)}$$

\*\*\*

$E = KE + m_e c^2$

- Perpendicular to the direction,

$$0 + 0 = \frac{h\nu'}{c} \sin\varphi + p \sin\theta$$

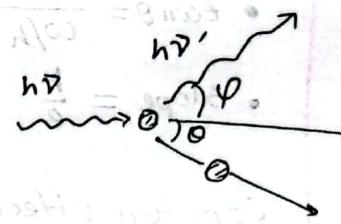
$$\Rightarrow p \sin\theta = h\nu' \sin\varphi \quad \text{---(2)}$$

$$(1)^2 + (2)^2 \Rightarrow p^2 c^2 = (h\nu - h\nu' \cos\varphi)^2 + (h\nu' \sin\varphi)^2$$

$$= (h\nu)^2 - 2h\nu h\nu' \cos\varphi + (h\nu')^2$$

$KE = h\nu - h\nu'$

electron



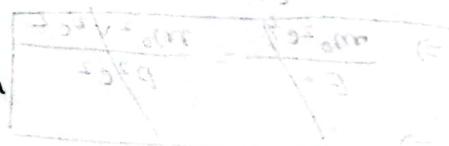
$$\frac{2m_e c^2 (h\nu - h\nu')}{2h^2 c^2} = \frac{2(h\nu)(1-\cos\varphi)}{2h^2 c^2}$$

$$\Rightarrow \frac{m_e c}{h} \left( \frac{\nu}{c} - \frac{\nu'}{c} \right) = \frac{\nu}{c} \frac{\nu'}{c} (1-\cos\varphi)$$

$$\Rightarrow \frac{m_e c}{h} \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right) = \frac{1}{\lambda} \frac{1}{\lambda'} (1-\cos\varphi)$$

$$\Rightarrow \lambda' - \lambda = \left( \frac{h}{m_e c} \right) (1-\cos\varphi)$$

↓  
scattered photon wavelength  
wavelength



$$\textcircled{1} \quad \frac{d^2y}{dt^2} + 2\lambda \frac{dy}{dt} + \omega^2 y = f_0 \sin pt - \textcircled{1}$$

$$\Rightarrow y = A \sin(pt - \theta) - \textcircled{2}$$

$$\frac{dy}{dt} = A p \cos(pt - \theta)$$

$$\frac{d^2y}{dt^2} = -A p^2 \sin(pt - \theta)$$

Putting the values in eq<sup>n</sup>(i),

$$\begin{aligned} & -A p^2 \sin(pt - \theta) + 2\lambda A p \cos(pt - \theta) + \omega^2 A \sin(pt - \theta) \\ & = f_0 \sin \underbrace{\{(pt - \theta) + \theta\}}_{= f_0 \sin(pt - \theta) \cos \theta + f_0 \cos(pt - \theta) \sin \theta} - \textcircled{3} \end{aligned}$$

- If this solution is to be hold good for all values of  $t$ , the respective low-efficiency of  $\sin(pt - \theta)$  and  $\cos(pt - \theta)$  on either side of eq<sup>n</sup>(iii) must be equal.

$$A(\omega^2 - p^2) = f_0 \cos \theta - \textcircled{4}$$

$$\textcircled{5} \Rightarrow 2\lambda A p = f_0 \sin \theta - \textcircled{5}$$

Squaring and adding eq<sup>n</sup>(iv) and (v)  $\rightarrow$

$$A^2 (\omega^2 - p^2)^2 + 4\lambda^2 A^2 p^2 = f_0^2$$

$$\Rightarrow A^2 [(\omega^2 - p^2) + 4\lambda^2 p^2] = f_0^2$$

$$\Rightarrow A^2 = \frac{f_0^2}{(\omega^2 - p^2)^2 + 4\lambda^2 p^2}$$

$$\Rightarrow A = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4\lambda^2 p^2}}$$

Dividing eq<sup>n</sup>(v) to (iv)  $\Rightarrow$

$$\frac{f_0 \sin \theta}{f_0 \cos \theta} = \frac{2\lambda A p}{A(\omega^2 - p^2)}$$

$$\Rightarrow \tan \theta = \frac{2\lambda p}{1(\omega^2 - p^2)}$$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{2\lambda p}{\omega^2 - p^2} \right)$$

\* Solution of forced vibration,

$$y = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4\lambda^2 p^2}} \cdot \sin \left( pt - \tan^{-1} \frac{2\lambda p}{\omega^2 - p^2} \right)$$

## Resonance

Maximum displacement of a driven oscillator,

$$A^2 = \frac{f_0^2}{(\omega^2 - \rho^2)^2 + 4\lambda^2\rho^2}$$

The amplitude is maximum when the denominator is minimum.

$$\frac{d}{dp} [(\omega^2 - \rho^2)^2 + 4\lambda^2\rho^2] = 0$$

$$\Rightarrow 2(-2\rho)(\omega^2 - \rho^2) + 4\lambda^2 2\rho = 0$$

$$\Rightarrow -4\rho(\omega^2 - \rho^2) + 8\lambda^2\rho = 0$$

$$\Rightarrow \rho(\rho^2 - \omega^2) + 2\lambda^2\rho = 0$$

$$\Rightarrow \rho(\rho^2 - \omega^2 + 2\lambda^2) = 0 \quad \because \rho \neq 0$$

$$\therefore \rho^2 - \omega^2 - 2\lambda^2 = 0$$

$$\Rightarrow \rho^2 = \omega^2 - 2\lambda^2$$

$$\therefore \rho = \sqrt{\omega^2 - 2\lambda^2}$$

Then the amplitude will be maximum when the driven frequency

$$\frac{P_n}{2\pi} = \frac{\sqrt{\omega^2 - 2\lambda^2}}{2\pi}$$

resonant frequency

The state of vibration when the amplitude of the driven oscillator is maximum is called resonance.

In the absence of damping, resonance take place when natural frequency of oscillator is equal to the frequency of driving force.

The maximum Amplitude,

$$A_{max} = \frac{f_0}{\sqrt{(\omega^2 - \rho^2)^2 + 4\lambda^2\rho^2}}$$

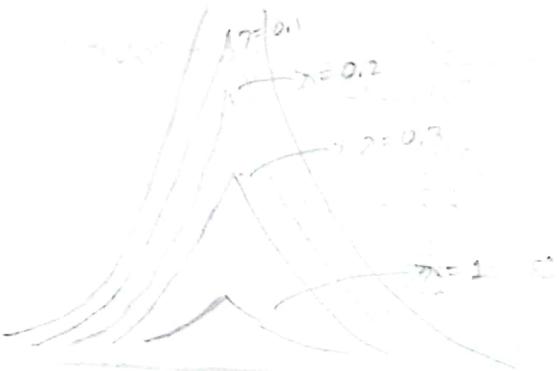
$$\therefore A_{max} = \frac{f_0}{2\sqrt{\omega^2 - \gamma^2}}$$

$$= \frac{f_0}{\sqrt{(\omega^2 - \omega^2 + \gamma^2)^2 + \gamma^2 \times 4(\omega^2 - \gamma^2)}}$$

$$= \frac{f_0}{\sqrt{4\gamma^4 + 4\gamma^2\omega^2 - 8\gamma^4}}$$

$$= \frac{f_0}{\sqrt{4\gamma^2\omega^2 - 4\gamma^4}}$$

$\omega = 0$ , the damping is zero



Pesec

Period = 3      Damping = 1

### Sharpness of Resonance :

It is a measure of the rate of fall amplitude from its maximum value at resonant frequency on either side of it. The sharper the fall in amplitude sharper the resonance.

- Forced vibration of quality ~~factor~~ factor.

### Quality factor of forced vibration :

The ratio of the response of the oscillation when the driven frequency is equal to the resonant frequency to the response when the driven frequency is zero is called the quality factor of the forced vibration.

$$A = \frac{f_0}{\sqrt{(\omega^2 - \rho^2) + 4\gamma^2\rho^2}}$$

when  $\rho = 0$

$$A_{\rho=0} = \frac{f_0}{\omega^2}$$

when,  $\rho = \rho_0$        $A_{max} = \frac{f_0}{2\sqrt{\omega^2 - \eta^2}}$

Quality factor =  $\frac{A_{max}}{A_{\rho=0}} = \frac{f_0 / 2\sqrt{\omega^2 - \eta^2}}{\frac{f_0}{\omega^2}}$

$$= \frac{\omega^2}{2\sqrt{\omega^2 - \eta^2}} \quad [\omega^2 \gg \eta^2]$$

$$= \frac{\omega^2}{2\eta\omega}$$

$$\alpha = \frac{\omega}{2\eta}$$

### Phase of the driven Oscillator :

$$Y = A \sin(pt - \theta) \quad \theta = \tan^{-1} \frac{2\eta p}{\omega^2 - \rho^2}$$

The phase angle  $\theta$  depends upon damping and relative values of  $\omega$  and  $p$ , following cases are,

(i) when  $p < \omega$ ; i.e

$\tan\theta$  will positive and values of  $\theta$  lies between  $0$  to  $\frac{\pi}{2}$

for all values of  $\eta$ .

(ii) when  $p > \omega$ ; i.e, the  $\tan\theta$  will be a negative quantity and the values of  $\theta$  lies between  $\frac{\pi}{2}$  to  $\pi$  for all values of  $\eta$ .

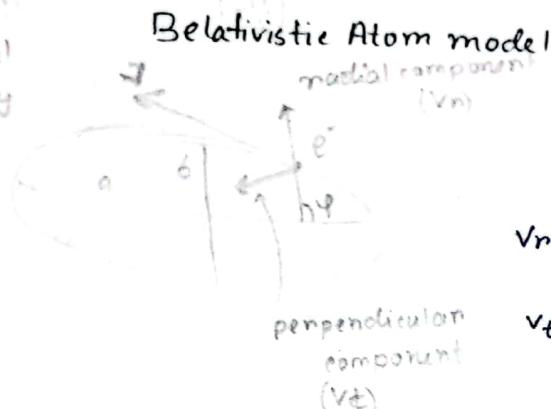
(iii) when,  $p = \omega$  (one case of lossless or pure resistive)

$$\tan\theta = \alpha, \quad \theta = \frac{\pi}{2}$$

Date: 22-07-2023  
Sunday

Atomic Physics → J.B. Rajan

tangential velocity



$$\oint p_r \cdot dr = n_r \cdot h$$

$$\oint p_\varphi \cdot dr = n_\varphi \cdot h$$

$$\text{Total energy } E = p.E + \text{radial K.E.} + \text{angular K.E.} = mvr_n \cdot \left( \frac{d\varphi}{dt} \right)$$

$$= -\frac{Ee}{r} + \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}mr_n^2\left(\frac{d\varphi}{dt}\right)^2$$

$$W = -\frac{2\pi^2 m E^2 e^2}{h^2} \left( \frac{1}{n_\varphi + n_r} \right)^2$$

$$p_r = m\left(\frac{dr}{dt}\right) \quad p_r = mv_r$$

$$p_\varphi = mr^2 \left( \frac{d\varphi}{dt} \right)$$

$$\begin{cases} 1 - \epsilon^2 = \frac{b^2}{a^2} \\ \Rightarrow \epsilon^2 = 1 - \frac{b^2}{a^2} \quad a > b \end{cases}$$

Now,

$$\sqrt{1 - \epsilon^2} = \frac{b}{a} = \frac{n_\varphi}{n_\varphi + n_r} = \frac{n_\varphi}{n}$$

A special case  
 $a = b, \frac{n_\varphi}{n} = 1$

$$W = -\frac{2\pi^2 m E^2 e^2}{n^2 h^2}$$

Relativistic variation of mass

$$W = -\frac{2\pi^2 m E^2 e^2}{n^2 h^2} \left[ \frac{1}{n^2} + \frac{h n^2 E^2 e^2}{c^2 h^2} \left( \frac{n}{n_\varphi} - \frac{3}{4} \right) \frac{1}{n^4} + \dots \right]$$

$n = 3$

3 kinds of ellipses

$$\therefore -\frac{2\pi^2 m e^2}{h^2} \times \frac{2^4 \alpha^2}{n^4} \left( \frac{n}{n_\varphi} - \frac{3}{4} \right)$$

$$\text{where, } E = Ze \quad \alpha = \frac{2\pi e^2}{ch}$$

$$n=3 \quad \underline{\underline{\quad}} \quad \underline{\quad}$$

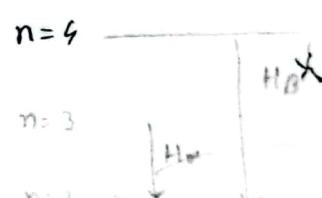
6 transitions possible

$$3_3 \rightarrow 2_2, 3_3 \rightarrow 2_1,$$

$$3_2 \rightarrow 2_2, 3_2 \rightarrow 2_1,$$

$$3_1 \rightarrow 2_2, 3_1 \rightarrow 2_1$$

$$n=2 \quad \underline{\underline{\quad}} \quad \underline{\quad}$$



$n = 1$

## De Broglie Waves :

A moving body behaves in certain ways as though it has a wave nature.

$$E = h\nu \therefore p = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

↓  
De Broglie wave-length

Probability  $|\Psi|^2$

→ probability density

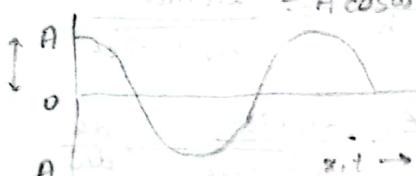
$$\Psi = A + iB$$

$$\Psi^* = A - iB$$

$$|\Psi\Psi^*| = A^2 + B^2$$

group velocity:

$$y = A \cos 2\pi \nu t$$



electrical  
signals  
mechanical  
signals  
Modulated  
wave

## Wave-particle duality

particle

• Waves of what?

matter wave

- light wave → electric & magnetic
- spherical wave → sinusoidal
- Sound wave → Pressure

■ Quantum mechanics basic equation

Schrödinger's equation

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{8\pi^2 m}{h^2} (E - u) \Psi = 0$$

Describing a wave

$$\nu_p = \frac{c}{\lambda} \quad h\nu = mc^2$$

↓ phase velocity  $\Rightarrow \nu = \frac{mc^2}{h}$   
wave velocity

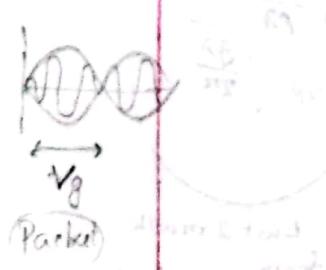
$$= \left( \frac{mc^2}{h} \right) \times \left( \frac{h}{mv} \right)$$

$$N_p = \frac{c^2}{\nu} \quad \# \#$$

$$\lambda = \frac{h}{(mv)} \quad \rightarrow \text{Relativistic calculation}$$

$\rightarrow p \rightarrow \text{momentum}$

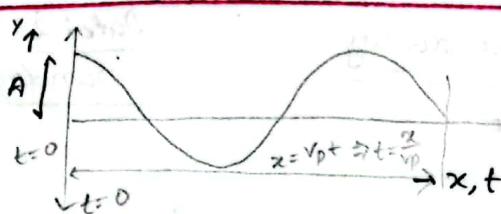
The greater the particle's momentum,  
the shorter its wavelength



decay

Date : 05.02.2023

Sunday



$$y = A \cos 2\pi f t = A$$

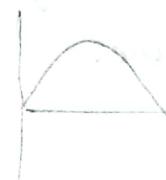
$$= A \cos \omega t$$

$$y = A \cos 2\pi f (t - \frac{x}{v_p})$$

$$= A \cos (2\pi f t - \frac{2\pi f x}{v_p})$$

$$= A \cos (\omega t - \frac{2\pi f x}{\lambda})$$

$$\boxed{y = A \cos(\omega t - kx)} \text{ Wave formula}$$



$$v_p = \frac{\lambda}{T}$$
$$\Rightarrow \frac{1}{T} = \frac{\lambda}{v_p}$$

$$\boxed{k = \frac{2\pi}{\lambda}}$$

$$y_1 = A \cos(\omega t - kx)$$

$$y_2 = A \cos \{(\omega t + \Delta\omega)t - (k + \Delta k)x\}$$

$$y = y_1 + y_2 = A_2 \left\{ \cos \frac{\omega t + \beta}{2} \cdot \cos \frac{\omega t - \beta}{2} \right\}$$

$$E = h\nu$$

$$\boxed{\lambda = \frac{h}{mv}}$$

$$E = mc^2$$

$$\Rightarrow h\nu = mc^2$$

$$\Rightarrow \nu = \frac{mc^2}{h}$$

$$\omega = 2\pi f = \frac{2\pi m c^2}{h} = \frac{2\pi m_0 c^2}{h\sqrt{1 - v^2/c^2}}$$

$$\bullet k = \frac{2\pi}{\lambda} = \frac{2\pi m \omega}{h} = \frac{2\pi m_0 \nu}{h\sqrt{1 - v^2/c^2}} \rightarrow \frac{dk}{d\omega}$$

$$\bullet v_g = \frac{d\omega}{dk} = \frac{d\omega/d\nu}{dk/d\nu}$$

$$\boxed{v_g = v_p - \frac{dv_p}{d\lambda}}$$

$$v_{p0} = \frac{\omega}{f}$$

$$\Rightarrow \omega = v_p k$$

$$\Rightarrow \frac{d\omega}{dk} = v_p + k \frac{dv_p}{dk}$$

$$\Rightarrow v_g = v_p + k \frac{dv_p}{dk}$$

$$\boxed{\Rightarrow v_g = v_p - \frac{dv_p}{d\lambda}}$$

$$k = \frac{2\pi}{\lambda}$$

$$\Rightarrow dk = -\frac{2\pi}{\lambda^2} d\lambda$$

$$\Rightarrow \frac{k}{dk} = \frac{\frac{2\pi}{\lambda}}{-\frac{2\pi}{\lambda^2} d\lambda} = -\frac{\lambda}{d\lambda}$$

$$\boxed{\sqrt{\frac{2\pi s}{\rho\lambda}}}$$

$$\boxed{v_p = \sqrt{\frac{g\lambda}{2\pi}}}$$

Last 2 math  
from ... book.

F = Force  $\propto \frac{q_1 q_2}{r^2}$

## Uncertainty Principle

$$\Psi(x) = \int_0^\infty g(k) \cos kx dk$$

Date: 12.02.2023

Sunday

## Electron Orbits:

Thompson, Rutherford  
Walzight experiment  
Bohr Model  $\rightarrow (1, 2, 3, 4)$  point

Angular momentum  $= n\hbar$

- $\Delta E = E_2 - E_1$   $\rightarrow$  ground state  
excited state

- electron radiate energy why?

$$F_c = F_e$$

$$\Rightarrow \frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \times \frac{e^2}{r^2}$$

$$\therefore e = \frac{e}{\sqrt{4\pi\epsilon_0 mn}}$$

$$\bullet KE = \frac{1}{2} mv^2$$

electron velocity

Total energy of Hydrogen

$$\bullet PE = -\frac{e^2}{4\pi\epsilon_0 r} \rightarrow \text{doesn't need extra force}$$

$$= +\frac{e^2}{4\pi\epsilon_0 r}$$

comes automatically

↑ needed extra

force to move 1C

charge inside a

electrical field

$$\text{Atom, } E = KE + PE$$

$$= \frac{mv^2}{2} - \frac{e^2}{4\pi\epsilon_0 r}$$
$$= -\frac{e^2}{8\pi\epsilon_0 r}$$

- The wavelength is exactly the same as the circumference of the electron orbit.

## 4.4. The Bohr Atom:

$$\lambda = \frac{h}{mv}$$

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 mn}}$$

$$\Rightarrow \lambda = \frac{h}{m \times \frac{e}{\sqrt{4\pi\epsilon_0 mn}}}$$

=

$$\left\{ \begin{array}{l} \text{Circumference} = n\lambda \\ \Rightarrow 2\pi r_n = n\lambda \end{array} \right.$$

$$\frac{nh}{e} \sqrt{\frac{4\pi\epsilon_0 n}{m}} = 2\pi r_n$$

$$\Rightarrow r_n = \frac{n^2 h^2 \epsilon_0}{nm e^2}, n=1, 2, 3, \dots$$

↑  
Orbital radii in Bohr model

$$r_n = n^2 a_0$$

when saying say only  
just magnitude, no operator

## \* Energy levels and spectra:

$$E_n = -\frac{e^2}{8\pi\epsilon_0 n}$$

$$2x^3 + 2x^2 + 5x + 13$$

$$2x^4 + 3x^3 + \dots$$

$$= 2x^3 - 2x^2 + 2x^3 - 4x^2 + 4x + 13$$

$$= 2x^2(2x - 2) + 2x^3 - 4x^2 + 4x + 13$$

$$= 2x^2(x - 1)^2 + 2x^3 - 4x^2 + 4x + 13$$

$$= 2x^2(x - 1)^2 + 2x^3 - 4x^2 + 4x + 13$$

## Origin of Line Spectra :

Initial energy - Final energy = photon energy

$$E_i - E_f = h\nu$$

$$= E_i \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

$$= - E_i \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = h\nu$$

$$\Rightarrow \nu = \frac{E_i - E_f}{h}$$

$$= \frac{E_i}{h} \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

$$= (2-2)(2+2+2+2+3)-31$$

derivations of Bohr's model  
of atom

work function

$$10^{-10} \times \frac{1}{2} \times R_H^2$$

$$= \frac{3}{16} R_H^2$$

specific to pairs of atoms

## Correspondence Principle :

The greater the quantum number, the closer quantum physics approaches classical physics.

$$\text{frequency of revolution, } f = \frac{\text{electron speed}}{\text{orbit circumference}} = \frac{v}{2\pi r}$$

$$\text{frequency of photon is } \nu = -\frac{E_i}{h} \left( \frac{2p}{n^3} \right)$$

$$2np^2 = 2np \\ 2np^2 \leq n^2 \\ (2p)^2 \leq n^2 \\ (2p)^2 R_m = n^2 R_m$$

$$2np^2 = \frac{n^2 R_m}{m} \cdot \frac{2\pi r}{S}$$

$$2np^2 = \frac{n^2 R_m}{m} \cdot \frac{2\pi r}{S}$$

laboratory reference frame

$$2np^2 = n^2 R_m$$

$$-\frac{E_i}{ch^3} = \frac{mv^4}{8\epsilon_0^2 c^2 h^3}$$

$$R_H = 1.099 \times 10^7 \text{ m}^1$$

$$\text{क्रियाकारी काल } = E K = \frac{1}{2} m v^2$$

$$= \frac{e}{2\pi \sqrt{4\pi \epsilon_0 m n^2}}$$

$$= -\frac{E_i}{h} \left( \frac{2}{n^3} \right)$$

$$\frac{d}{2\pi r} = R_m$$

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$$\frac{2\pi r}{n^2 R_m} = 1$$

परिवर्तन ग्राफ में



## The vector Atom Model

- ① Spatial quantization
- ② Spinning electron

shell  
Model

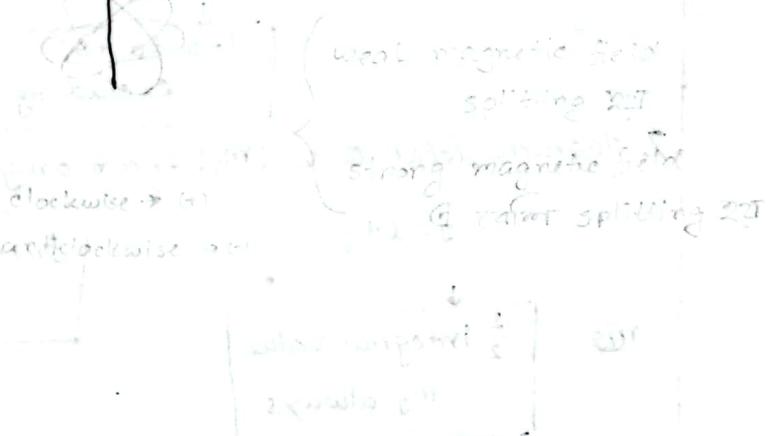
Orbital Q.N. Spin  
O.Q.N.

$$J = l \pm s$$

$$1 \pm \frac{1}{2}$$

$$\frac{3}{2}, \frac{1}{2}$$

(H) magnetic field direction



1.  $n \rightarrow$  Principal Q.N./Total Q.N.

2.  $l \rightarrow$  Orbital Q.N. (0 to  $n-1$ )  
Subshell

3.  $s \rightarrow$  Spin Q.N.

Spin angular momentum

$$p_s = s \cdot \frac{h}{2\pi}, \text{ where } s = \frac{1}{2}$$

but according to wave mechanics,

$$p_s = \sqrt{s(s+1)} \cdot \frac{h}{2\pi}$$

$$= \sqrt{s(s+1)} \cdot \hbar$$

4. A total angular Q.N. ( $j$ )

$J$  value always  $\frac{1}{2}$  [integer] value for single  $e^-$

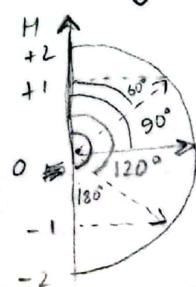
Total angular momentum  $p_j = \sqrt{j(j+1)} \cdot \frac{h}{2\pi}$

5. A magnetic orbital Q.N. ( $m_l$ ) (- $l$  to + $l$ )

$$[m_l = l \cos \theta]$$

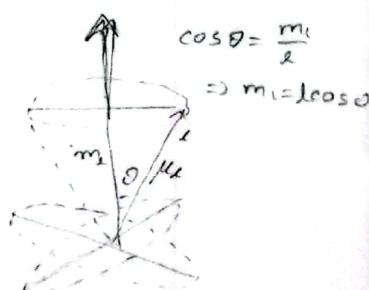
$$\therefore m_l = 2 \cos 180^\circ = -2$$

magnetic field direction  
orbital



$$\cos \theta = \frac{m_l}{l}$$

$$\Rightarrow m_l = l \cos \theta$$



Date: 26.02.2023

Sunday

6. Magnetic spin Q.N.  $(2s+1) = m_s$

value always at two  $(+\frac{1}{2}, -\frac{1}{2})$

$(-s \text{ to } +s)$

excluding zero

7. Magnetic Total Q.N.  $(m_J) \rightarrow$  can only have  $(2j+1)$  values

$$j = l \pm \frac{1}{2}$$

⇒

$\frac{1}{2}$  integral value  
 $m_j$  always

from  $j$  to  $-j$   
excluding zero

### Coupling Schemes

#### ⊗ L.S. Coupling

$$L = l_1 + l_2$$

$$S = s_1 + s_2$$

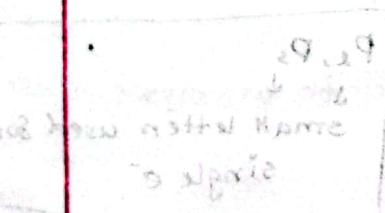
$$J = L + S$$

#### $J = j_1 + j_2$ coupling

$$j_1 = l_1 + s_1$$

$$j_2 = l_2 + s_2$$

$$J = j_1 + j_2$$



$$\frac{1}{n^2} \cdot (2l+1)^2 = 29$$

$$\sqrt{2l+1} =$$

(t) MO aligned total S

→ alignment of individual spins ↓ opposite sides

$$\frac{1}{n^2} \cdot (2l+1)^2 = 29 \quad \text{maximum resulting total S}$$

(at least  $l=1$ ) (one) MO aligned along one axis

Date: 01.03.2023

Wednesday

### J-J coupling

$$J = j_1 + j_2$$

$$J_1 = l_1 + s_1$$

$$\left(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots\right)$$

L-S coupling much effective

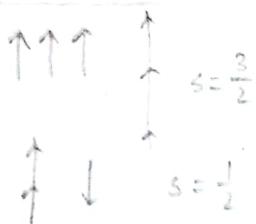
$$J_2 = l_2 + s_2$$

$\frac{1}{2}$  Integral value

$$J = L + S, L = l_1 + l_2$$

$$S = s_1 + s_2$$

3 electron



$Y_X$

$$\begin{matrix} 1 \\ -1 \\ 0 \end{matrix}$$

$n=2$

$$\frac{r = (2s+1)}{(2L+1)}$$

value?

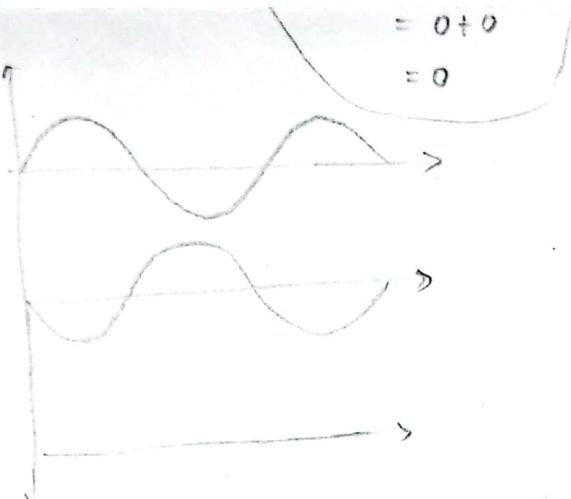
$$2L = 0, 1, 2, 3$$

$$2s+1 = 1$$

$$\Rightarrow S = 0$$

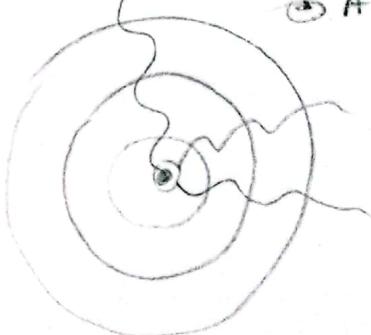
$L = 0$  because  $S = 0$

$$\begin{aligned} \text{Total } J &= l_1 + s_1 \\ &= 0 + 0 \\ &= 0 \end{aligned}$$



Wave, A disturbance produced by a periodic vibration

② A locus of all points vibrating in phase.

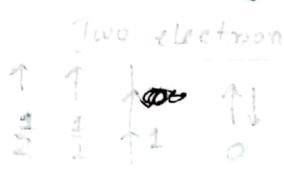


wavefront

crest

through

trough



$$\begin{array}{c} 5 \\ D \\ \frac{3}{2} \end{array} \rightarrow (2s+1) = 3 \rightarrow S = 2$$

D  $\rightarrow L = 2$  so, it is not possible

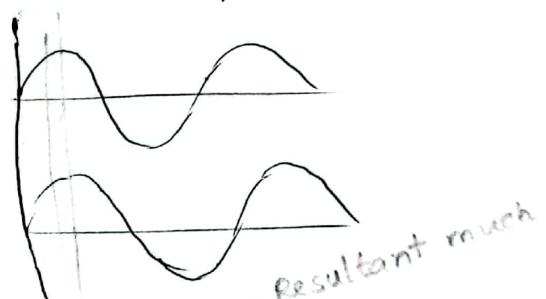
Interference

Light + light = more light

Light + light = no light

### Coherent source

1. Amplitude nearly same
2. Freq/wavelength nearly same
3. Same phase/constant phase diff



- Only one source there  $2\pi$  virtual source possible amplitude, frequency same
- $2\pi$  independent sources  $\Rightarrow$  dark place  $\Rightarrow$  fringes  $\Rightarrow$  alternating bright and dark regions
- Amplitude not same

$(a,b) \in R$   
 $\left( b,a \right) \in R$   
 $\Rightarrow (a,b) \neq (b,a)$

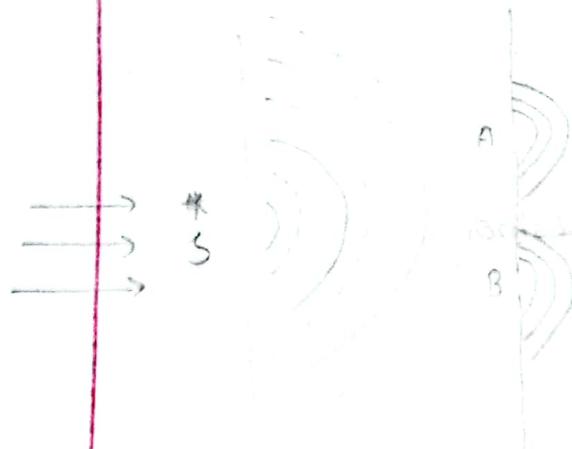
$(a,b) \in A$   
 $(b,a) \notin A$

- Draw spherical waves how do they interact and made?

Date: 05.03.2023

Sunday

## Interference:



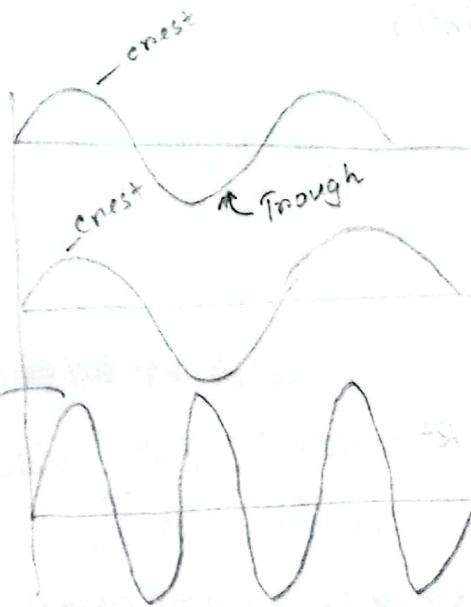
Two waves meeting at point P

Constructive interference

Bright

Destructive interference

Dark



Crest + crest = Bright

Trough + Trough = Bright

Crest + Trough = Dark

- Show the Interference using spherical waves

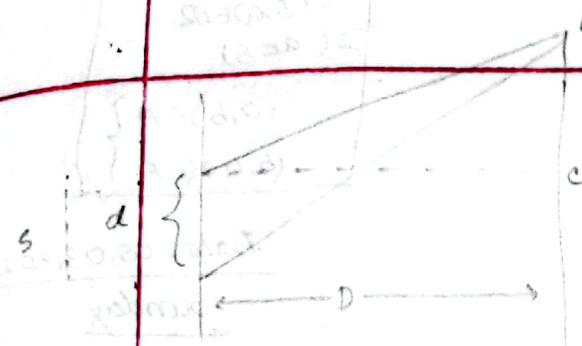
## Phase Difference and Path Differences:

~~at~~

Dark position  $\rightarrow$  Intensity 0  $\rightarrow$  ?

- If the path difference between 2 waves  $\lambda$ , the phase difference =  $2\pi$

$$\delta = \frac{2\pi}{\lambda} \times x$$



$$Y_1 = a \sin \omega t$$

$$Y_2 = a \sin(\omega t + \delta)$$

$$\begin{aligned} Y &= Y_1 + Y_2 \\ &= a \sin \omega t + a \sin(\omega t + \delta) \\ &= a \sin \omega t + a \sin \omega t \cos \delta + a \cos \omega t \sin \delta \\ &= a \sin \omega t (1 + \cos \delta) + a \cos \omega t \cdot \sin \delta \end{aligned}$$

$$\text{Taking, } a(1 + \cos \delta) = R \cos \theta \quad \text{--- (1)}$$

$$a \sin \delta = R \sin \theta \quad \text{--- (2)}$$

$$\begin{aligned} Y &= \cancel{R \sin \omega t \cos \theta} R \sin \omega t \cos \theta + R \cos \omega t \sin \theta \\ &= R \sin(\omega t + \theta) \quad \text{--- (3)} \end{aligned}$$

$$(1)^2 + (2)^2 \Rightarrow$$

$$\begin{aligned} R^2 &= a^2 (1 + \cos \delta)^2 + a^2 \sin^2 \delta \\ &= a^2 + 2a^2 \cos \delta + a^2 (\cos^2 \delta + \sin^2 \delta) \\ &= 2a^2 + 2a^2 \cos \delta \\ &= 2a^2 (1 + \cos 2 \cdot \frac{\delta}{2}) \\ &= 2a^2 (2 \cos^2 \frac{\delta}{2}) \\ &= 4a^2 \cos^2 \frac{\delta}{2} \end{aligned}$$

Intensity,  $I = (\text{Energy/area/amplitude})^2$

$$\Rightarrow I = R^2$$

$$\Rightarrow I = 4a^2$$

$R^2 \rightarrow$  why  $R$  is regarded as amplitude  
Shouldn't it be  $a$ ?

Normally,  $I = a + a = 2a$

But,  $I = 4a^2$

Reason:

Dark  $\oplus$  energy  $\ominus$  bright  $\oplus$   
energy transfer  $\oplus$ ,  $\ominus$  transfer  
 $2a \times 2a = 4a^2 = \text{Intensity.}$

$$\text{i) } I = R^2$$

$$I = 4a^2 \cos^2 \frac{\delta}{2}$$

Maximum

Intensity is maximum when the phase difference is a whole num multiple of  $2\pi$ / path difference is a whole num of multiple of  $\lambda$ .

when  $\underline{\delta} = 0, 2\pi, 2(2\pi), \dots, h(2\pi)$

or path difference,  $\underline{x} = 0, \lambda, 2\lambda, \dots, n\lambda$

$$\boxed{I = 4a^2}$$

$$\text{ii) } S = n, 3n, \dots, (2n+1)\pi$$

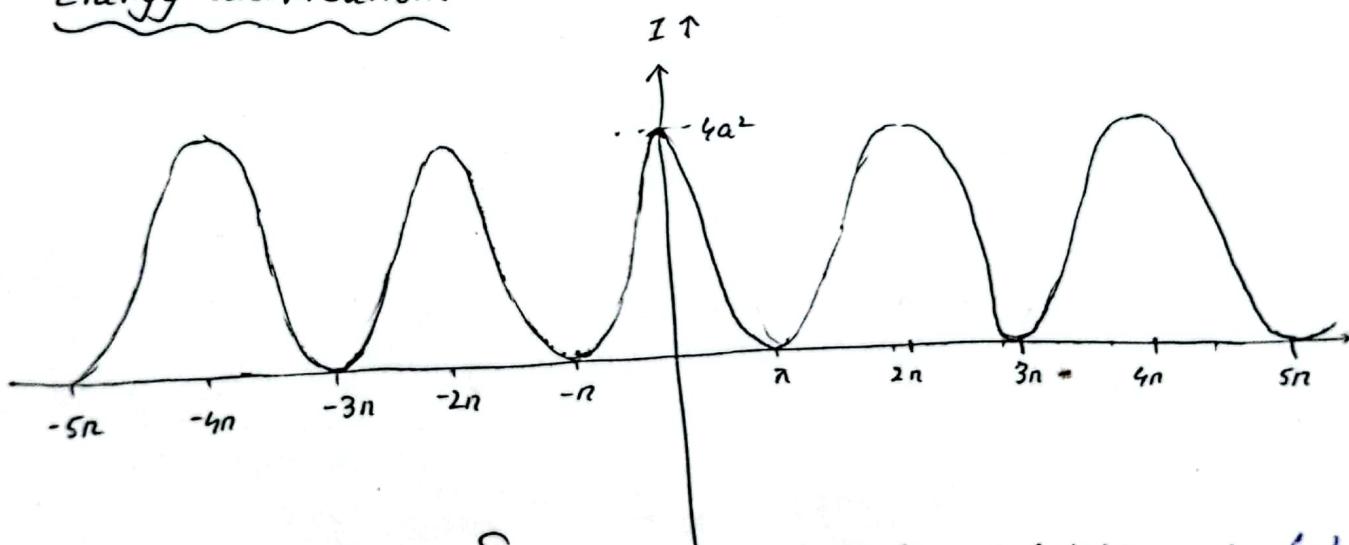
$$\text{Path diff, } \underline{x} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots, (2n+1)\frac{\pi}{2}$$

$$\boxed{I = 0}$$

Minimum

Intensity is minimum when the path difference is an odd num multiple of half  $\lambda$ .

Energy distribution:

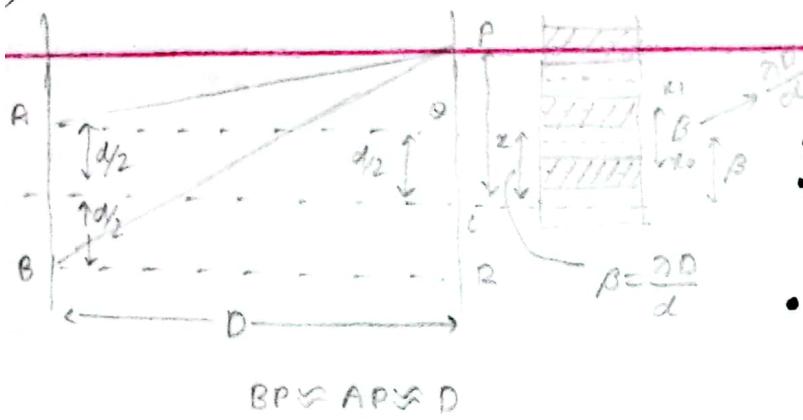


from eq(iv)  $I = 4a^2 \cos^2 \frac{\delta}{2}$ , we can say intensity at bright points  $4a^2$  and at dark points it is zero.

In the fig it is shown that the intensity varies from 0 to  $4a^2$ . and the average is still  $2a^2$ . It is equal to the uniform intensity  $2a^2$  which will be present in the absence of the interference phenomenon due to the two waves, therefore the formation of interference fringes is in accordance with the law of conservation of energy.

Fringe = Bright Position +  
Dark position

Date: .03.2023



$$PQ = x - \frac{d}{2}$$

$$PR = x + \frac{d}{2}$$

$$\bullet BP^2 - AP^2 = (BR^2 + PR^2) - (AQ^2 + PQ^2)$$

$$= D^2 + (x + \frac{d}{2})^2 - D^2 - (x - \frac{d}{2})^2$$

$$= 4x \cdot \frac{d}{2}$$

$$= 2xd$$

$$\therefore BP - AP = \frac{2xd}{BP + AP}$$

$$= \frac{2xd}{2D}$$

$$\therefore \text{Path Difference} = \frac{xd}{D}$$

$$\frac{x_n d}{D} = n\lambda$$

$$\Rightarrow x_n = \frac{n\lambda D}{d}$$

$$x_1 = \frac{\lambda D}{d}, x_2 = \frac{2\lambda D}{d}$$

$$\text{Hence, } x_2 - x_1 = \frac{\lambda D}{d} = \beta$$

$$x_3 - x_2 = \frac{3\lambda D}{d} = \beta$$

Dark fringe:

$$\frac{x_n d}{D} = (2n+1) \frac{\lambda}{2} \Rightarrow x_n = \frac{(2n+1)\lambda D}{2d}$$

$$\Rightarrow x_0 = \frac{\lambda D}{2d}$$

$$x_1 = \frac{3\lambda D}{2d}$$

$$x_1 - x_0 = \frac{\lambda D}{d} = \beta$$

Fringe Width,  $\beta$

Dark + Bright

$$= \frac{\lambda D}{d} + \frac{\lambda D}{d}$$

$$= \frac{2\lambda D}{d}$$

Fresnel's

Fringe width  $\frac{\lambda D}{d}$  is independent  
of the order of fringes, equal  
for both bright & dark.

①  $\beta \propto \lambda$

$\beta \propto \frac{1}{d}$

②  $(\beta \propto D)$

two acute angle prism joint  
प्रिस्म जॉइंट

Prism:

$$\frac{\theta}{2} - x = 90^\circ$$

$$\frac{\theta}{2} + x = 90^\circ$$

$$30^\circ = \frac{1}{2}\theta$$

$$9A$$

$$179^\circ$$

$$9B$$

$$30^\circ = \frac{1}{2}\theta$$

$$9C =$$

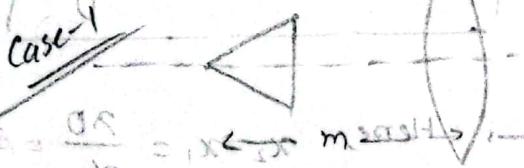
$$9D =$$

$$\boxed{\frac{9A+9B}{9A+9B} = 9A - 9B}$$

$$\frac{d_1}{d} = \frac{v}{u} = \frac{n}{m} \quad (1) \quad \frac{d_2}{d} = \frac{v}{u} = \frac{m}{n} \quad (2)$$

प्रिस्म अंतराल

Case 1



Case 2



$$\frac{d_1 d_2}{d} = \frac{n^2 - m^2}{n^2 + m^2} = m^2 \quad (3)$$

$$\frac{d_1 d_2}{d} = \frac{m^2}{n^2 + m^2} = \frac{m^2}{n^2} = \frac{m^2}{(1 + \frac{n^2}{m^2})} = \frac{m^2}{1 + \frac{n^2}{m^2}} = \frac{m^2}{1 + \frac{1}{\tan^2 A}} = \frac{m^2}{\sec^2 A} = \frac{m^2}{\cos^2 A}$$

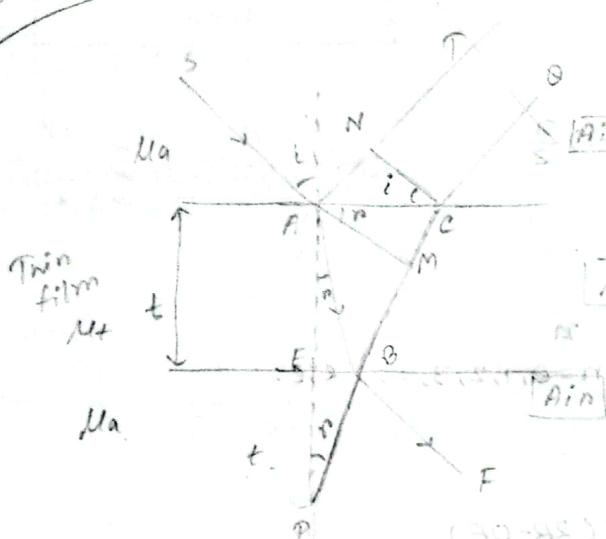
$$\frac{d_1 d_2}{d} = 1$$

$$\boxed{d = \sqrt{d_1 d_2}}$$

Date: 22.03.2023  
Wednesday

## ① Interference in thin films

### ① Interference due to reflected light (Thin films)



$$\text{Path diff} = \mu_f(AB + BC) - AN$$

$$= \mu_f(AB + BC) - AN$$

$$[AN = \mu_f CM]$$

$\Delta^s ACN$  and  $ACM$

$$\mu_f = \frac{\sin i}{\sin r} = \frac{AN/AC}{CM/AC}$$

$$= \frac{AN}{CM}$$

eq(i)  $\Rightarrow$

$$\text{Path diff} = \mu_f(AB + BC) - AN$$

$$\Rightarrow AN = \mu_f CM$$

$$= \mu_f(AB + BC) - \mu_f CM$$

$$= \mu_f(PB + BC) - CM$$

$$= \mu_f(PC - CM)$$

$$= \mu_f PM$$

$$\boxed{\text{Path diff } (\alpha) = 2\mu_f t \cos r}$$

$\Delta APM$

$$\cos r = \frac{PM}{AP} = \frac{PM}{AE + EP}$$

$$= \frac{PM}{2t}$$

$$\Rightarrow PM = 2t \cdot \cos r \quad (ii)$$

$$\boxed{\text{Connected Path diff. } (\alpha) = 2\mu_f t \cos r \pm \frac{\lambda}{2}}$$

$$\Rightarrow \boxed{2\mu_f t \cos r = (2n+1)\frac{\lambda}{2}} \rightarrow \text{bright position}$$

For bright position

$$2\mu_f t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow 2\mu_f t \cos r = (n+1)\lambda$$

$$\Rightarrow \boxed{2\mu_f t \cos r = n\lambda} \rightarrow \text{dark position}$$

We can write  $(n+1)\lambda$  as  $n$

## Interference

### Newton's Rings

For light ring

$$2\mu t \cos\theta = (2n-1) \frac{\lambda}{2} \quad n=1, 2, 3, \dots \text{etc.}$$

Hence,  $\theta$  is small,  $\cos\theta \approx 1$

$$\mu \approx 1.$$

$$2t = (2n-1) \frac{\lambda}{2}$$

For the dark ring,

$$2\mu t \cos\theta = n\lambda$$

$$\Rightarrow 2t = n\lambda, \quad n=0, 1, 2, 3, \dots \text{etc.}$$

Now some values

In the fig,

$$EP \times HE = OE \times (2R - OE)$$

$$EP = HE = r, \quad OE = PQ = t$$

$$2R - t = 2R$$

$$\boxed{n^2 = 2Rt} \Rightarrow \boxed{t = \frac{n^2}{2R}}$$

For bright

rings,

$$\boxed{n^2 = \frac{(2n-1)\lambda R}{2}}$$

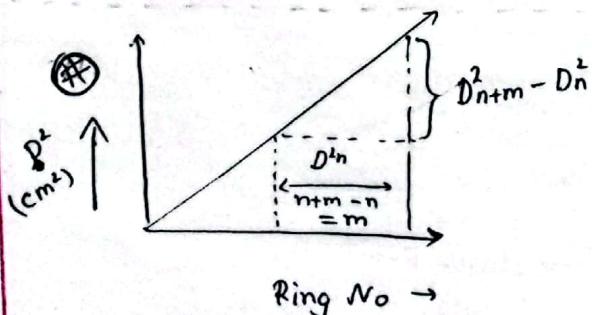
$$r = \sqrt{\frac{(2n-1)\lambda R}{2}}$$

For dark

rings,

$$n^2 = n\lambda R \quad \begin{matrix} \rightarrow \text{Radius of} \\ \text{curvature} \end{matrix}$$

$$\Rightarrow r = \sqrt{n\lambda R}$$



$$r_n^2 = n\lambda R$$

$$D_n^2 = 4n\lambda R$$

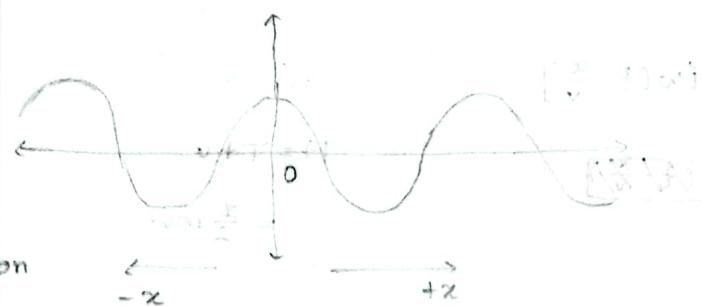
■ Determination of the wavelength of sodium light using Newton's ring

$$D^2n+m = 4(n+m)\lambda R$$

$$D^2n+m - D^2n = 4m\lambda R$$

$$\Rightarrow \lambda = \frac{D^2n+m - D^2n}{4mR}$$

### Quantum Mechanics:



$$\Psi = A + iB$$

↓  
wave function

$$\Psi^* = A - iB$$

$$|\Psi|^2 = \Psi * \Psi^* = A^2 + B^2$$

Probability

Probability:

$$P_{x_1, x_2} = \int_{x_1}^{x_2} |\Psi|^2 dx$$

Wave equation,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 y}{\partial t^2}$$

$$y = F(t + \frac{x}{v})$$

-x direction

$$\therefore \boxed{y = F(t \pm \frac{x}{v})} \quad \text{for both directions}$$

Normalization

Integration of  $\Psi^* \Psi$  over all space = 1

maximum probability exists

Date: 29.03.2023

wednesday

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2}$$

classical mechanics is an approximation of quantum mechanics

$$\int_{-\infty}^{\infty} |\Psi|^2 dv = 0.$$

To summarise:

1.  $\Psi$  must be continuous and single valued everywhere
2.  $\frac{\partial \Psi}{\partial x}, \frac{\partial \Psi}{\partial y}, \frac{\partial \Psi}{\partial z}$  must be continuous and single valued everywhere.
3.  $\Psi$  must be normalizable, which means that  $\Psi$  must go to 0 as  $x \rightarrow \pm\infty, y \rightarrow \pm\infty, z \rightarrow \pm\infty$  in order that  $\int |\Psi|^2 dv$  over all space be a finite constant

$$y = A \cos(\omega t - kx)$$

$$= A \cos 2\pi v (t - \frac{x}{v})$$

+x direction

$$= F (+ - \frac{x}{v})$$

$$\boxed{F \propto t^{\frac{1}{2}}}$$

$$\boxed{\Psi = A e^{i\omega t - \frac{x}{v}}} \rightarrow \text{Solution}$$

$$\Psi = A \cos(\omega(t - \frac{x}{v})) - i A \sin(\omega(t - \frac{x}{v}))$$

Schrodinger equation (Time dependent form):

Schrodinger eqn is a special kind of eigen value eqn:

→ Hamiltonian operator

$$\hat{H}\Psi = a\Psi$$

$$\hat{A}\Psi = a\Psi$$

$$\left( \frac{d}{dx} \right) \Psi = A e^{-i\omega(t - \frac{x}{v})}$$

$$= A e^{-i\omega(t - \frac{x}{v})} \left[ -i\omega(t - \frac{x}{v}) \right]$$

$$= \frac{i\omega}{v} \Psi - \boxed{-\frac{1}{2} \nabla^2 \Psi + V \Psi}$$

$$H = T + V$$

$$= \frac{1}{2} m v^2 + V$$

$$= \frac{p^2}{2m} + V$$

wave function → matter wave

Eigenfunction, E.P.

slagende Welle entstehend aus Teilchen + Elektronenwellen

$$\text{Zusammensetzung aus Teilchen } \frac{p_x}{2m}, \frac{p_y}{2m}, \frac{p_z}{2m}$$

$$\hat{H}\Psi = E\Psi \quad \text{Eigene Welle} \quad \hat{A}\Psi = a\Psi$$

Date: 02.04.2023

Sunday

$$P = -i\hbar \vec{\nabla} \quad \Rightarrow \quad \vec{\nabla} = i \hat{i} \frac{\partial}{\partial x} + i \hat{j} \frac{\partial}{\partial y} + i \hat{k} \frac{\partial}{\partial z}$$

$$\vec{\nabla}^2 \Psi = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$= \left( \frac{i^2 \hbar^2 \nabla^2}{2m} + V \right) \Psi$$

$$E = \left( -\frac{\hbar^2}{2m} \cdot \frac{d^2}{dx^2} + V \right) \Psi$$

$$\therefore \boxed{H\Psi = -\frac{\hbar^2}{2m} \cdot \frac{d^2}{dx^2} \Psi + V\Psi = E\Psi}$$

## Schrodinger's equation:

(Time dependent Form)

A basic physical principle that can not be derived from anything else.

$$\Psi = Ae^{-i\omega(t-\frac{x}{v})} \quad \text{---(1)}$$

$$\hbar = \frac{h}{2\pi c}$$

Putting  $\omega = 2\pi\nu$ ,  $v = \lambda\nu$  gives  $\underline{\text{eqn}}$

$$\Psi = Ae^{-i(2\pi\nu t - \frac{\lambda x}{\lambda\nu})}$$

$$= Ae^{-i(2\pi\nu t - \frac{x}{\nu})}$$

$$= Ae^{-i(\frac{E}{\hbar}t - \frac{p_x x}{2\pi\hbar})}$$

$$= Ae^{-(\frac{i}{\hbar})(Et - px)}$$

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{2\pi\hbar}{p}} \\ \boxed{h = 2\pi\hbar}$$

$$\left. \begin{array}{l} \text{And,} \\ E = h\nu \\ = 2\pi\hbar \\ \lambda = \frac{2\pi\hbar}{p} \end{array} \right\}$$

$$\text{And, } \frac{\delta^2\Psi}{\delta x^2} = -\frac{p^2}{\hbar^2}\Psi$$

$$\Rightarrow p^2\Psi = -\hbar^2 \frac{\delta^2\Psi}{\delta x^2}$$

Differentiating once with respect to  $t$

$$\frac{\partial\Psi}{\partial t} = -\frac{iE}{\hbar}\Psi \quad \frac{p^2}{2m}$$

$$\Rightarrow E\Psi = -\frac{\hbar}{i} \cdot \frac{\partial\Psi}{\partial t}$$

$$\boxed{E\Psi = \frac{p^2\Psi}{2m} + U\Psi}$$

Date: 03.05.2023

Wednesday

$$\Psi = A e^{-i/h(Et - px)} \quad (1)$$

Differentiating,

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{p^2}{\hbar^2} \Psi$$

$$\Rightarrow p^2 \Psi = -\hbar^2 \frac{\partial^2 \Psi}{\partial x^2}$$

$$\begin{aligned} & -\frac{\hbar}{i} \frac{\partial \Psi}{\partial t} \\ &= i^2 \frac{\hbar}{\hbar i} \frac{\partial \Psi}{\partial t} \\ &= i\hbar \frac{\partial \Psi}{\partial t} \end{aligned}$$

Differentiating equation (1) with respect  
to  $t$ ,

$$\frac{\partial \Psi}{\partial t} = -\frac{iE}{\hbar} \Psi$$

$$④ \boxed{i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \Psi}{\partial x^2} + U\Psi}$$

Time dependent Schrödinger  
equation in one dimension.

Q. Derive time dependent  
Schrödinger eq. in one dimension  
three dimension

$$\Psi = A e^{-i/h(Et - px)}$$

$$= A e^{-(iE/\hbar)t} e^{(ip/\hbar)x}$$

$$= \Psi e^{-(iE/\hbar)t} e^{(ip/\hbar)x}$$

$$⑤ E\Psi = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \Psi}{\partial x^2} + U\Psi$$

$$\Rightarrow \frac{\hbar^2}{2m} \cdot \frac{\partial^2 \Psi}{\partial x^2} + E\Psi - U\Psi = 0$$

$$\Rightarrow \boxed{\frac{\partial^2 \Psi}{\partial x^2} + \frac{2m}{\hbar^2} (E-U)\Psi = 0} \quad \text{Steady state Schrödinger eq.} \quad (2)$$

$$\boxed{3\text{dimension}} \quad \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{2m}{\hbar^2} (E-U)\Psi = 0$$

$$\Psi = e^{mx}$$

$$\Rightarrow \frac{\partial \Psi}{\partial x} = m e^{mx}$$

$$\Rightarrow \frac{\partial^2 \Psi}{\partial x^2} = m^2 e^{mx} \quad (3)$$

$$\begin{aligned} & (2) \Rightarrow \\ & \Rightarrow m^2 e^{mx} + \left( \frac{2mE}{\hbar^2} \right) e^{mx} = 0 \\ & \Rightarrow (m^2 + k^2) e^{mx} = 0 \end{aligned}$$

$$\lambda = 2L$$

$$m^2 + k^2 = 0$$

$$\Rightarrow m^2 = -k^2 = i2k^2$$

$$\Rightarrow m = \pm ik$$

(H.S) PS

IS

(H.S) PS

## Eigenvalue & Eigenfunctions

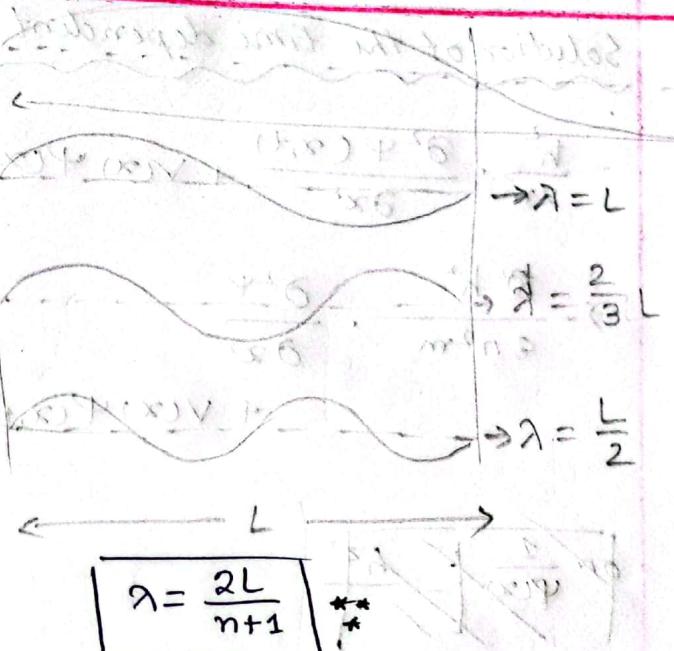
Proper

$$\hat{A} \Psi = a \Psi$$

Expectation value ( $\bar{x}$ )

How to extract information

from a wave function



$$\lambda = \frac{2L}{n+1}$$

Expectation value  
N<sub>i</sub> Particle in x<sub>i</sub> position

$$\bar{x} = \frac{N_1 x_1 + N_2 x_2 + \dots}{N_1 + N_2 + \dots} = \frac{\sum N_i x_i}{\sum N_i}$$

$$P_i = |\Psi_i|^2 dx$$

$$\langle x \rangle = \frac{\int_{-\infty}^{\infty} x |\Psi|^2 dx}{\int_{-\infty}^{\infty} |\Psi|^2 dx}$$

Expectation value  
for position

$\int \Psi^2 dx = 1 \rightarrow$  Particle exists  
(Probability)

$\int \Psi^2 dx = 0 \rightarrow$  Particle doesn't exist

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\Psi|^2 dx$$

↳ Some book

Introducing to negative