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Lecture-1

Kinetic theory of gases - 35
Theromodyniamics - 35
Waves & Oscillations - 35
Laser Physics - 35

Gnieeman

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105 marks

Lecture-1

Date: 04.01.2023
Wednesday

Simple Harmonic motion:

mobile, Vibration / vibrating motion - solar cell → periodic motion
pendulum

When the particle, undergoing periodic motion, covers the same path back and forth about a mean position, it is said to be executing an oscillatory or vibrating motion.

A motion which repeats itself over and over again after a regular interval of time is referred to a peridic motion. The time required for it's repeatation is called time period.

• Simple Harmonic motion:

Whenever a force acting on a particle, and hence the acceleration of the particle is proportional to it's displacement from it's equilibrium position but always directed in a direction opposite to the direction of the displacement and if the maximum displacement of the particle is same in either side of the main position, the particle is said to executing a simple Harmonic motion.

eqⁿ

Differential equation of SHM (Simple Harmonic Motion)

From the defination of SHM,

$$f \propto -y$$

$$\Rightarrow F = -ky \quad \text{— (I)}$$

[where k is restoring force constant]

$\Rightarrow \cancel{F = -ky}$

we know that, $F = ma$ — (II)

$$\Rightarrow ma = -ky$$

$$\Rightarrow m\frac{dv}{dt} = -ky$$

$$\Rightarrow m\frac{d^2y}{dt^2} + ky = 0$$

$$\Rightarrow \frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$

$$\boxed{\Rightarrow \frac{d^2y}{dt^2} + \omega^2 y = 0} \rightarrow \omega = \sqrt{\frac{k}{m}}$$

ω ↓ angular velocity

$\therefore \frac{d^2y}{dt^2} = -\omega^2 y = -\mu y$

when, $y = 1$,

$\frac{d^2y}{dt^2} = -\mu$

↓ acceleration per unit displacement of the particle

Solution of Defferential eqⁿ of SHM:

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \quad \text{— (I)}$$

$$\Rightarrow \frac{d^2y}{dt^2} = -\omega^2 y$$

$\Rightarrow$ Multipdying both side by $\frac{2dy}{dt}$

$$\frac{2dy}{dt} \times \frac{d^2y}{dt^2} = -\omega^2 y \times 2\frac{dy}{dt}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{dy}{dt}\right)^2 = -\omega^2 \frac{d}{dt}(y^2) \quad \text{— (II)}$$

Integrating eqⁿ no. 2.

$$\Rightarrow \int \frac{d}{dt}\left(\frac{dy}{dt}\right)^2 dt = \int -\omega^2 \frac{d}{dt}(y^2)\, dt$$

$$\Rightarrow \left(\frac{dy}{dt}\right)^2 = -\omega^2 y^2 + c \quad \text{— (III)}$$

When ~~if~~ $y = a$, $\frac{dy}{dt} = 0 = v$

↓ maximum displacement

From (III),

$$c = \omega^2 a^2$$

puting this in eq^n (iv),

$$\left(\frac{dy}{dt}\right)^2 = -\omega^2y^2 + \omega^2a^2$$

$$\Rightarrow \left(\frac{dy}{dt}\right)^2 = \omega^2(a^2-y^2)$$

$$\Rightarrow \frac{dy}{dt} = \pm\,\omega\sqrt{a^2-y^2}$$

$$\Rightarrow \frac{dy}{dt} = \omega\sqrt{a^2-y^2}$$ neglecting negative term because velocity cannot be negative, and $v \propto \omega$.

$$\Rightarrow \cancel{\frac{dy}{dt}\frac{1}{\sqrt{a^2-y^2}} = \omega}$$

$$\cancel{\Rightarrow \int \frac{d}{dt}\left(\frac{y}{\sqrt{a^2-y^2}}\right)dt = \int \omega\, dt}$$

$$\Rightarrow \int \frac{1}{\sqrt{a^2-y^2}}\,dy = \omega\int dt$$

$$\Rightarrow \sin^{-1}\frac{y}{a} = \omega t + \varphi$$

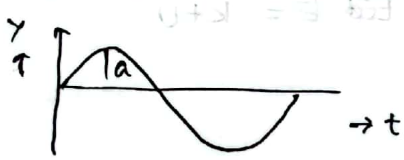
$$\Rightarrow \frac{y}{a} = \sin(\omega t + \varphi)$$

$$\Rightarrow \boxed{y = a\sin(\omega t + \varphi)}$$

Independent variable (time)

Graphical Representation of displacement:

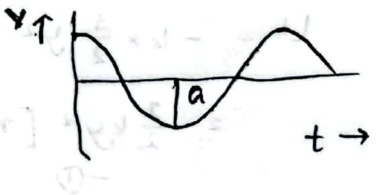
$y = a\sin(\omega t + \varphi)$, when $\varphi = 0$;



Velocity,

$v = \omega a\cos(\omega t + \varphi)$

when $\varphi = 0$



$$v = a\omega\cos(\omega t + \varphi)$$

$$= a\omega \cancel{\sin} \sqrt{1-\sin^2(\omega t+\varphi)}$$

$$= a\omega\sqrt{1-\frac{y^2}{a^2}}$$

$$\therefore \boxed{v = \omega\sqrt{a^2-y^2}}$$

when, $y = 0$, $v_{max} = \omega a$ → max

when, $y = a$, $v_{min} = 0$ → min

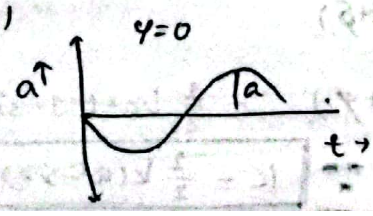
$$a_c = \frac{dv}{dt}$$

$$= \frac{d}{dt}\, a\omega\cos(\omega t + \delta)$$

$$= -a\omega^2\sin(\omega t + \delta)$$

$$\Rightarrow a_c = -\omega^2 y$$

$\varphi = 0$



when, $y = 0$, $a_{cmin} = 0$

$y = a$, $a_{cmax} = \omega^2 a$

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Time period of body executing SHM:

$$y = a\sin(\omega t + \varphi)$$

If the time is increased by $\frac{2\pi}{\omega}$

$$y = a\sin\left\{\omega\left(t + \frac{2\pi}{\omega}\right) + \varphi\right\}$$
$$= a\sin(\omega t + 2\pi + \varphi)$$
$$= a\sin(2\pi + \omega t + \varphi) = a\sin(\omega t + \delta\varphi)$$

Time period,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

Frequency, $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

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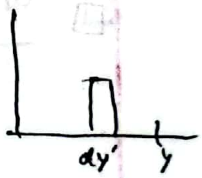
Tuesday

Energy of a particle executing SHM:

* kinetic energy – K
* Potential energy – U

Total Energy ~~Ea~~ $E = K + U$

$$-ky' = F$$
$$\Rightarrow dU = F\,dy'$$



$$U = \int_0^y F\,dy'$$
$$= \int_0^y -ky'\,dy'$$
$$= -k \times \left[\frac{y'^2}{2}\right]_0^y$$
$$U = -k \times \frac{1}{2} y^2$$
$$= \frac{1}{2}ky^2 \quad \text{[neglecting negative sign]} \quad -(1)$$

$$U = \frac{1}{2}ky^2$$
$$y = a\sin(\omega t + \varphi)$$
$$U = \frac{1}{2}ka^2\sin^2(\omega t + \varphi) \quad -(v)$$

$$K = \frac{1}{2}mv^2 = \qquad v = \frac{dy}{dt}$$
$$= \frac{1}{2}m\left(\frac{dy}{dt}\right)^2 \qquad = \frac{d}{dt}a\sin(\omega t + \varphi)$$
$$= \frac{1}{2}m\omega^2\cos^2(\omega t + \varphi) \times a^2 = \qquad \omega a\,\cancel{\sin}\cos(\omega t + \varphi)$$
$$= \frac{1}{2}m\omega^2 a^2\cos^2(\omega t + \delta)$$
$$K = \frac{1}{2}ka^2\cos^2(\omega t + \varphi) = \frac{1}{2}ka^2(1 - \sin^2(\omega t + \varphi))$$
$$\therefore \boxed{K = \frac{1}{2}k(a^2 - y^2)}$$

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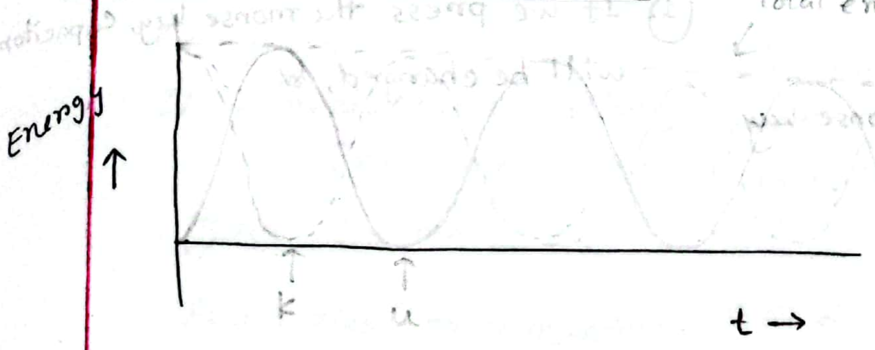
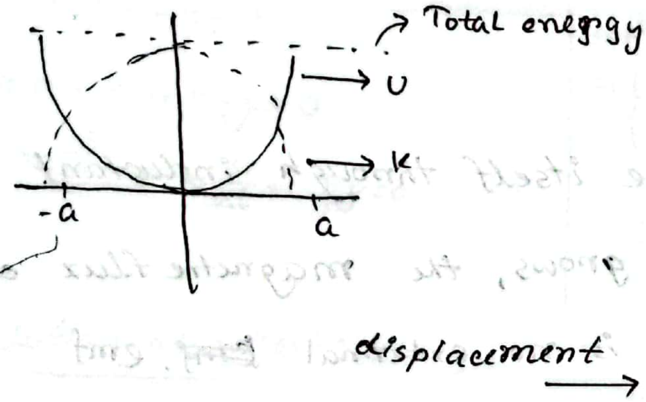
Graphical representation:



Total energy, $E = U + K$

$= \frac{1}{2}ka^2\sin^2(\omega t+\varphi) + \frac{1}{2}ka^2\cos^2(\omega t+\varphi)$

$= \frac{1}{2}ka^2$



Average Potential energy,

$$U = \frac{1}{2}ka^2\sin^2(\omega t+\varphi)$$

$$U_{avg} = \frac{1}{T}\int_0^T \frac{1}{2}ka^2\sin^2(\omega t+\delta) \quad [\text{with respect to } T]$$

$$= \frac{ka^2}{4T}\int_0^T 2\sin^2(\omega t+\varphi)$$

$$= \frac{ka^2}{4T}\int_0^T \{1-\cos\{2(\omega t+\varphi)\}\} \rightarrow \int_0^T / \int_0^\lambda / \int_0^t 2\sin^2(\quad)$$

Meaning

$$= \frac{ka^2T}{4T}$$

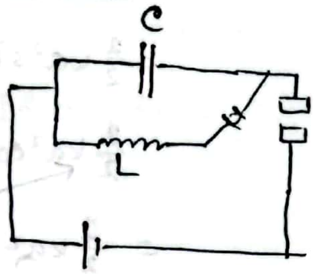
$$= \frac{ka^2}{4}$$

$$= \frac{1}{2}(\text{total energy})$$

$$K_{avg} = \frac{1}{T}\int_0^T \frac{1}{2}ka^2\cos^2(\omega t+\varphi) \rightarrow \frac{1}{2}\times(\text{total energy})$$

$\boxed{\text{• SHM} \rightarrow \text{2nd order} \rightarrow \text{sinusoidal wave}}$

L.C. circuit :



(1) If we press the monse key, capaciton will be changed,

- Capacitor discharge itself through inductant coil and current in the latter grows, the magnetic flux due to the current increase. There is no external ~~emf~~. emf

$$\boxed{\frac{Q}{C} = -L\frac{di}{dt}}$$

$$\Rightarrow L\frac{di}{dt} + \frac{Q}{C} = 0$$

$I = \frac{dQ}{dt}$

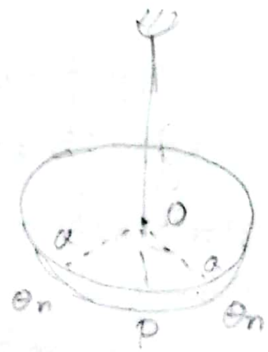
$$\Rightarrow \frac{d}{dt}\left(\frac{dQ}{dt}\right) + \frac{Q}{LC} = 0$$

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0 \qquad \omega^2 = \frac{1}{LC}$$

$$\Rightarrow \boxed{\frac{d^2Q}{dt^2} + \omega^2 Q = 0}$$

$$\boxed{Q = Q_0 \sin(\omega t + \varphi)}$$

Torsional Pendulum:



- $\tau = -k\theta$, $\tau = I\alpha$

$\Rightarrow -k\theta = I\alpha$

$$\Rightarrow I\alpha + k\theta = 0$$

$$\Rightarrow I \times \frac{d^2\theta}{dt^2} + k\theta = 0$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{k}{I}\theta = 0$$

$$\therefore \boxed{\frac{d^2\theta}{dt^2} + \omega^2\theta = 0}$$

2nd Order

$$\omega^2 = \frac{k}{I}$$

- Prob - Compute the kinetic & potential energies of a body when it has moved in halfway from its initial position towards the center of motion.

Damped Harmonic Oscillations:

Date : 11.01.2023
Wednesday

$F = -ky$ — (i)

If the medium is resistive,

$F_r = -bv$ — (ii) b is resisting force constant/damping force

→ constant of proportionality
→ damping coefficient

Again, $F = ma$ — (iii)

From these three equations,

$ma = -ky - bv$

$\Rightarrow ma + bv + ky = 0$

$\Rightarrow m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$

$\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{k}{m}y = 0$

$$\Rightarrow \boxed{\frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + \omega^2 y = 0} \quad \text{— (iv)}$$

when, $2\lambda = \frac{b}{m}$

differential equation of damped harmonic oscillation

Solution of eq^n (iv):

Let the solution is, $y = Ae^{kt}$ — (i)

$\Rightarrow \frac{dy}{dt} = Ake^{kt}$

$\Rightarrow \frac{d^2y}{dt^2} = Ak^2e^{kt}$

⊛ From eq^n (iv),

$Ak^2e^{kt} + 2\lambda \times Ake^{kt} + \omega^2 Ae^{kt} = 0$

$\Rightarrow Ae^{kt}(k^2 + 2\lambda k + \omega^2) = 0$

$\Rightarrow k^2 + 2\lambda k + \omega^2 = 0$ — (ii)

$\Rightarrow k = \frac{-2\lambda \pm \sqrt{4\lambda^2 - 4\omega^2}}{2.1}$

$\Rightarrow k = -\lambda \pm \sqrt{\lambda^2 - \omega^2}$

from eq^n(i),

$y = Ae^{(-\lambda + \sqrt{\lambda^2-\omega^2})t}$

$y = Ae^{(-\lambda - \sqrt{\lambda^2-\omega^2})t}$

⊛ In general case,

$$\boxed{y = A_1 e^{(-\lambda + \sqrt{\lambda^2-\omega^2})t} + A_2 e^{(-\lambda - \sqrt{\lambda^2-\omega^2})t}} \quad \text{— (iii)}$$

$$\frac{dy}{dt} = A_1(-\lambda + \sqrt{\lambda^2-\omega^2})e^{(-\lambda + \sqrt{\lambda^2-\omega^2})t} + A_2(-\lambda - \sqrt{\lambda^2-\omega^2})e^{(-\lambda + \sqrt{\lambda^2-\omega^2})t}$$

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Let,

When $t=0$, $y=a_0$ → maximum displacement

From eq^n (iii),

$$y = A_1 + A_2 = a_0$$

when, $y = a_0$ → maximum displacement, $\frac{dy}{dt} = 0, t=0.$

$$(-\lambda + \sqrt{\lambda^2-\omega^2})A_1 + (-\lambda - \sqrt{\lambda^2-\omega^2})A_2 = 0$$

$$\Rightarrow -\lambda(A_1+A_2) + \sqrt{\lambda^2-\omega^2}(A_1-A_2) = 0$$

$$\Rightarrow -\lambda a_0 + \sqrt{\lambda^2-\omega^2}(A_1-A_2) = 0$$

$$\Rightarrow A_1 - A_2 = \frac{\lambda a_0}{\sqrt{\lambda^2-\omega^2}} \quad \text{—(v)}$$

From eq^n (iv) and (v),

$$A_1 + A_2 = a_0$$

$$\Rightarrow A_1 - A_2 = \frac{\lambda a_0}{\sqrt{\lambda^2-\omega^2}}$$

$$A_1 = \frac{1}{2}\left(a_0 + \frac{a_0\lambda}{\sqrt{\lambda^2-\omega^2}}\right) \quad \text{—(vi)}$$

Again, $A_2 = a_0 - A_1 = a_0 - \frac{1}{2}a_0 - \frac{1}{2}\times\frac{a_0\lambda}{\sqrt{\lambda^2-\omega^2}}$

$$= \frac{1}{2}\left(a_0 - \frac{a_0\lambda}{\sqrt{\lambda^2-\omega^2}}\right) \quad \text{—(vii)}$$

The general solution is,

$$y = \frac{1}{2}a_0\left(1+\frac{\lambda}{\sqrt{\lambda^2-\omega^2}}\right)e^{(-\lambda+\sqrt{\lambda^2-\omega^2})t} + \frac{1}{2}a_0\left(1-\frac{\lambda}{\sqrt{\lambda^2-\omega^2}}\right)e^{(-\lambda-\sqrt{\lambda^2-\omega^2})t} \quad \text{—(viii)}$$

When $(\lambda^2 > \omega^2)$,

equation → over damped Harmonic Oscillation

$e^{-\square}$ → negative exponential

Three important cases:

i) $\lambda^2 > \omega^2$: Damping is large $\sqrt{\lambda^2-\omega^2}$ is clearly a real quantity with positive value, less than λ. Then each of the two terms on the right hand side of eqn (viii) ~~and~~ has an exponential term with a negative power and hence each decreases exponentially with time. In this case particle doesn't vibrate. There is no oscillations and the motion is therefore called overdamped. Dead beat galvanometer, pendulum oscillating in a viscous fluid like oil.

(ii) When $\lambda^2 = \omega^2$, In this case $\sqrt{\lambda^2-\omega^2}$ is obviously equal to zero. In this case motion break down.

consider, $\sqrt{\lambda^2-\omega^2} = h$; very small value

$$y = A_1 e^{(-\lambda+h)t} + A_2 e^{(-\lambda-h)t}$$

$$\Rightarrow y = e^{-\lambda t} \left[A_1 e^{ht} + A_2 e^{-ht} \right]$$

$$= e^{-\lambda t} \cancel{(A_1+A_2)} \left[A_1 \left[1 + ht + \frac{h^2t^2}{2!} + \ldots \right] + A_2 \left[1 - ht + \frac{h^2t^2}{2!} - \ldots \right] \right]$$

$$= e^{-\lambda t} \left[(A_1+A_2) + ht(A_1-A_2) \right] \text{ (neglecting higher sign)}$$

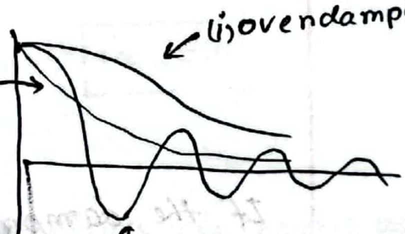
$$= e^{-\lambda t} \left[a_0 + h(A_1-A_2)t \right] \text{ where } \left[\begin{array}{l} A_1 + A_2 = a_0 \\ A_1 - A_2 = \dfrac{\lambda a_0}{\sqrt{\lambda^2-\omega^2}} \doteq \dfrac{\lambda a_0}{h} \end{array} \right]$$

$$= e^{-\lambda t} \left[a_0 + \lambda a_0 t \right] \quad \text{—(ix)}$$

****

$$\boxed{y = a_0 e^{-\lambda t} [1 + \lambda t]}$$

***



(iii) The second term ($\lambda t a_0 e^{-\lambda t}$) in eqn (ix) decays less rapidly than the first term $a_0 e^{-\lambda t}$ and the displacement of the oscillation first increases but as time increases exponential factor $e^{-\lambda t}$ becomes more important and the displacement decreases rapidly reaching the value of zero at the finite value of t. This is called critical damping

(iii) $\lambda^2 < \omega^2$, The quantity is $\sqrt{\lambda^2-\omega^2}$ is clearly imaginary, say equal to ig, → imaginary

$$y = A_1 e^{(-\lambda + ig)t} + A_2 e^{(-\lambda - ig)t}$$

$$= e^{-\lambda t}\left[A_1 e^{igt} + A_2 e^{-igt}\right]$$

$$= e^{-\lambda t}\left[A_1(\cos gt + i\sin gt) + A_2(\cos gt - i\sin gt)\right]$$

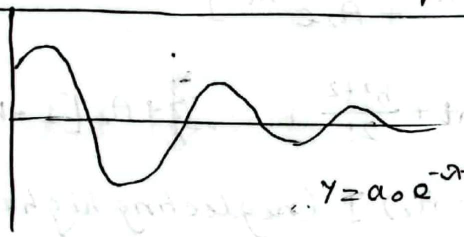
$$= e^{-\lambda t}\left[(A_1 + A_2)\cos gt + i(A_1 - A_2)\sin gt\right]$$

$$= e^{-\lambda t}\left[A \cos gt + iB \sin gt\right]$$

simplified
$\boxed{y = a_0 e^{-\lambda t}\sin(gt+\varphi)}$

Power Dissipation of damped Harmonic Oscillations

Date: 17.01.2023
Tuesday



$y = a_0 e^{-\lambda t}\sin(gt+\varphi)$

$$\frac{dy}{dt} = a_0 (-\lambda) e^{-\lambda t}\sin(gt+\varphi) + a_0 e^{-\lambda t} g\cos(gt+\omega)$$

Kinetic energy, $E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dy}{dt}\right)^2$

$$= \frac{1}{2}m \times a_0^2 e^{-2\lambda t}\left[-\lambda\sin(gt+\varphi) + g\cos(gt+\varphi)\right]^2$$

$$= \frac{1}{2}m a_0^2 e^{-2\lambda t}\left[\lambda^2\sin^2(gt+\varphi) + g^2\cos^2(gt+\varphi) - 2\lambda g\sin(gt+\varphi)\cos(gt+\varphi)\right]$$

If the damping is less than the term $\boxed{a_0^2 e^{-2\lambda t}}$ doesn't apparently, i.e., amplitude doesn't change apparently

Average Kinetic energy, $E_{k\,av} = \frac{1}{2}m a_0^2 e^{-2\lambda t}\left[\frac{1}{2}\lambda^2 + \frac{1}{2}g^2 + 0\right]$

$$= \frac{1}{2}m a_0^2 e^{-2\lambda t}\left[\frac{1}{2}\lambda^2 + \frac{1}{2}g^2\right]$$

$g = \sqrt{\omega^2-\lambda^2}$

$g^2 >> \omega^2$

For average value, $\sin^2(gt+\varphi)$ and $\cos^2(gt+\varphi)$ are $\frac{1}{2}$

Average ~~ϕ~~ value of $2\sin(gt+\varphi)\cos(gt+\varphi) = \sin 2(gt+\varphi)$ is zero

$$= \frac{1}{4} m a_0^2 g^2 e^{-2\lambda t}$$

Potential energy, $E_p = \int_0^y m g^2 y \, dy = m g^2 \left[\frac{y^2}{2}\right]_0^y$

$$= \frac{1}{2} m g^2 y^2$$

$$\therefore E_p = \frac{1}{2} m g^2 a_0^2 e^{-2\lambda t} \sin^2(gt+\varphi)$$

average value of $\sin^2(gt+\varphi)$ is $\frac{1}{2}$

So, average Potential energy, $E_{P_{av}} = \frac{1}{4} m g^2 a_0^2 e^{-2\lambda t}$

Total Energy, $E = E_{P_{av}} + E_{k_{av}}$

$$= \boxed{\frac{1}{2} m a_0^2 g^2} e^{-2\lambda t}$$

*** $$= \boxed{E_{undamped}} e^{-2\lambda t}$$

Average power dissipation, = rate of loss energy

$$\Rightarrow -\frac{dE}{dt} = -\frac{d}{dt} \frac{1}{2} m a_0^2 g^2 e^{-2\lambda t}$$

$$= 2\lambda \frac{1}{2} m a_0^2 g^2 e^{-2\lambda t}$$

$$= 2E\lambda = 2\lambda E \qquad \therefore \boxed{P = 2E\lambda}$$

Quality factor: $\left(\frac{g}{2\lambda}\right)$

The quality factor, Q is defined as 2π times the ratio ~~of~~ between the energy stored and the energy lost per period,

$$\boxed{Q = 2\pi \frac{\text{energy stored}}{\text{Energy lost period}}}$$

$$= 2\pi \frac{E}{pT}$$

$$= \frac{Eg}{p}$$

$$= \frac{Eg}{2\lambda E} = \boxed{\frac{g}{2\lambda}} \; ***$$

$$g = \frac{2\pi}{T}$$

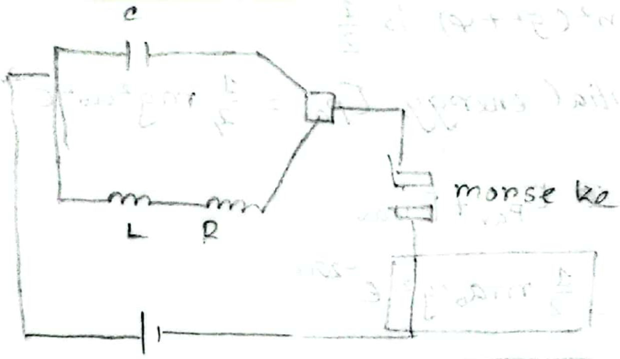
$$g >> \lambda$$

$$Q = \frac{\sqrt{\omega^2 - \lambda^2}}{2\lambda} = \frac{\omega}{2\lambda} = \frac{\sqrt{\frac{k}{m}}}{\frac{b}{m}}$$

$$\omega^2 > \lambda^2$$

$$2\lambda = \frac{b}{m}$$

$$\therefore \boxed{Q = \frac{\sqrt{km}}{b}} \; ***$$



$$\frac{Q}{C}, -L\frac{di}{dt}, -iR$$

$$\frac{Q}{C} = -L\frac{di}{dt} - iR$$

$$\Rightarrow L\frac{di}{dt} + iR + \frac{Q}{C} = 0$$

$$\Rightarrow L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1Q}{C} = 0$$

$$\Rightarrow \frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{1}{LC}Q = 0$$

$$\Rightarrow \boxed{\frac{d^2Q}{dt^2} + 2\lambda\frac{dQ}{dt} + \omega^2 Q = 0}$$

$$2\lambda = \frac{R}{L}, \quad \omega^2 = \frac{1}{LC}$$

$$Q = Q_0 e^{-\lambda t}\sin(gt + \varphi)$$

Freequency, $n = \frac{g}{2\pi}$

$$= \frac{\sqrt{\omega^2 - \lambda^2}}{2\pi}$$

$$= \frac{1}{2\pi}\sqrt{\omega^2 - \lambda^2}$$

$$= \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right) + \left(\frac{R}{2L}\right)^2}$$

$$\therefore \boxed{n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}$$

$$\boxed{Q = \frac{g}{2\lambda} = \frac{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}{\frac{R}{L}}}$$

Forced Vibration:

Let the applied sinusoidal force is $F = F_0 \sin pt$

$\boxed{m \frac{d^2y}{dt^2}}$ According to the defination of forced vibration

$$m\frac{d^2y}{dt^2} = -b\frac{dy}{dt} - ky + F_0 \sin pt$$

$$\Rightarrow m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = F_0 \sin pt$$

$2\lambda = \frac{b}{m}, \omega^2 = \frac{k}{m}, \boxed{f_0 = \frac{F_0}{m}}$

$$\Rightarrow \frac{d^2y}{dt^2} + \frac{b}{m}\cdot\frac{dy}{dt} + \frac{k}{m}y = F_0 \sin pt$$

$$\Rightarrow \frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + \omega^2 y = f_0 \sin pt \quad \text{---(1)}$$

Let the solution of equation (i),

$$y = A\sin(pt - \theta)$$

A is the amplitude and θ is the possible phase difference. frequency $= \frac{p}{2\pi}$

Date: 18.01.2023
Wednesday

(*) $\frac{d^2y}{dt^2} + 2\lambda \frac{dy}{dt} + \omega^2 y = f_0 \sin pt$ — (i)

$\Rightarrow y = A\sin(pt-\theta)$ — (ii)

$\frac{dy}{dt} = Ap\cos(pt-\theta)$

$\frac{d^2y}{dt^2} = -Ap^2\sin(pt-\theta)$

Putting the values in eqⁿ (i),

$-Ap^2\sin(pt-\theta) + 2\lambda Ap\cos(pt-\theta) + \omega^2 A\sin(pt-\theta)$
$= f_0\sin\{(pt-\theta)+\theta\}$

$= f_0\sin(pt-\theta)\cos\theta + f_0\cos(pt-\theta)\sin\theta$ — (iii)

- If this solution is to be hold good for all values of it, the respective co-efficient of $\sin(pt-\theta)$ and $\cos(pt-\theta)$ on either side of eqⁿ (iii) must be equal.

$A(\omega^2-p^2) = f_0\cos\theta$ — (iv)

$\Rightarrow 2\lambda Ap = f_0\sin\theta$ — (v)

Squaring and adding eqⁿ (iv) and (v) →

$A^2(\omega^2-p^2)^2 + 4\lambda^2A^2p^2 = f_0^2$

$\Rightarrow A^2[(\omega^2-p^2) + 4\lambda^2p^2] = f_0^2$

$\Rightarrow A^2 = \frac{f_0^2}{(\omega^2-p^2)^2 + 4\lambda^2p^2}$

$\Rightarrow \boxed{A = \frac{f_0}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}}}$ ***

Dividing eqⁿ (v) to (iv) ⇒

$\frac{f_0\sin\theta}{f_0\cos\theta} = \frac{2\lambda Ap}{A(\omega^2-p^2)}$

$\Rightarrow \tan\theta = \frac{2\lambda p}{1(\omega^2-p^2)}$

$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{2\lambda p}{\omega^2-p^2}\right)}$

(*) Solution of forced vibration,

$\boxed{y = \frac{f_0}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}} \cdot \sin\left(pt - \tan^{-1}\frac{2\lambda p}{(\omega^2-p^2)}\right)}$ ***

CamScanner

Resonance

Maximum displacement of a driven oscillator,

$$A^2 = \frac{f_0^2}{(\omega^2-p^2)^2 + 4\lambda^2 p^2}$$

The amplitude is maximum when the denominator is minimum,

$$\frac{d}{dp}\left[(\omega^2-p^2)^2 + 4\lambda^2p^2\right] = 0$$

$$\Rightarrow 2(-2p)(\omega^2-p^2) + 4\lambda^2 2p = 0$$

$$\Rightarrow -4p(\omega^2-p^2) + 8\lambda^2 p = 0$$

$$\Rightarrow p(p^2-\omega^2) + 2\lambda^2 p = 0$$

$$\Rightarrow p(p^2-\omega^2+2\lambda^2) = 0 \qquad \because p \neq 0$$

$$\therefore p^2-\omega^2-2\lambda^2 = 0$$

$$\Rightarrow p^2 = \omega^2 - 2\lambda^2 \qquad \therefore \boxed{p = \sqrt{\omega^2-2\lambda^2}}$$

- Then the amplitude will be maximum when the driven frequency $\boxed{\frac{p_r}{2\pi} = \frac{\sqrt{\omega^2-2\lambda^2}}{2\pi}}$ resonant frequency

- The state of vibration when the amplitude of the driven oscillator is maximum is called resonance.

In the absence of damping, ~~resp~~ resonance take place when natural frequency of oscillator, is equal to the freequency of driving force,

- The maximum Amplitude,

$$A_{max} = \frac{f_0}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}}$$

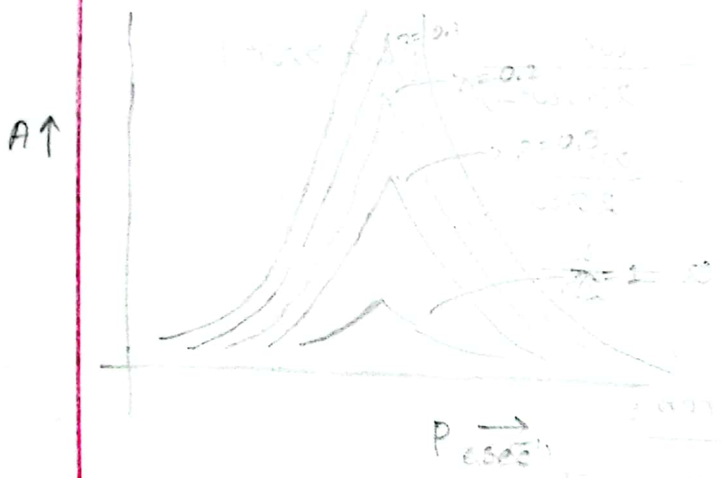
$$= \frac{f_0}{\sqrt{(\omega^2-\omega^2+2\lambda^2)^2+\lambda^2\times 4(\omega^2-2\lambda^2)}}$$

$$= \frac{f_0}{\sqrt{4\lambda^4+4\lambda^2\omega^2-8\lambda^4}}$$

$$\therefore A_{max} = \frac{f_0}{2\lambda\sqrt{\omega^2-\lambda^2}}$$

$$= \frac{f_0}{\sqrt{4\lambda^2\omega^2-4\lambda^4}}$$

$\lambda = 0$, No damping, $A_{max} = \infty$



Sharpness of Resonance :

It is a measure of the rate of fall amplitude from it's maximum value at resonant frequency on either side of it. The sharper the fall in amplitude sharper the resonance.

• Forced vibration of quality ~~vector~~ factor.

Quality factor of forced vibration :

The ratio of the response of the oscillation when the driven frequency is equal to the resonant frequency to the response when the driven frequency is zero is called the quality factor of the forced vibration.

$$A = \frac{f_0}{\sqrt{(\omega^2-p^2)+4\lambda^2p^2}}$$

when $p = 0$ ($p \to$ driving frequency)

$$A_{p=0} = \frac{f_0}{\omega^2}$$

when, $p = p_r$ $\qquad A_{max} = \dfrac{f_0}{2\lambda\sqrt{\omega^2-\lambda^2}}$

$$\text{Quality factor} = \frac{A_{max}}{A_{p=0}} = \frac{f_0/2\lambda\sqrt{\omega^2-\lambda^2}}{\frac{f_0}{\omega^2}}$$

$$= \frac{\omega^2}{2\lambda\sqrt{\omega^2-\lambda^2}} \quad [\omega^2 >> \lambda^2]$$

$$= \frac{\omega^2}{2\lambda\omega}$$

$$Q = \frac{\omega}{2\lambda}$$

Phase of the driven Oscillator :

$$Y = A\sin(pt-\theta) \qquad \theta = \tan^{-1}\frac{2\lambda p}{\omega^2-p^2}$$

The phase angle θ depends upon damping and relative values of ω and p, following cases are,

(i) when $p < \omega$; i.e

$\tan\theta$ will positive and values of θ lies between 0 to $\frac{\pi}{2}$ for all values of λ.

(ii) when $p > \omega$; i.e, the $\tan\theta$ will be a negative quantity and the values of θ lies between $\frac{\pi}{2}$ to π for all values of λ.

(iii) when, $p = \omega$

$\tan\theta = \infty$, $\quad \theta = \frac{\pi}{2}$

Date: 24.01.2023

Tuesday

absorption

Power of driven Oscillator:

$$F = F_0 \sin pt$$

$\Rightarrow dt \longrightarrow dy \longrightarrow$ displacement

$$dE = F dy$$

$$\Rightarrow \frac{dE}{dt} = F\frac{dy}{dt} \quad \text{—(i)}$$

$$y = A\sin(pt-\theta)$$

$$= \frac{f_0}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}} \times \sin(pt-\theta)$$

$$\boxed{\frac{dy}{dt} = \frac{pf_0}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}}\cos(pt-\theta)}$$

$$v = v_0\cos(pt-\theta)$$

$$\boxed{v_0 = \frac{pf_0}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}}}$$

$\frac{F_0}{m} = f_0$

$F = F_0 \sin pt$

$\Rightarrow F = m f_0 \sin pt$

$$\Rightarrow \boxed{P \Rightarrow \frac{dE}{dt} = F\frac{dy}{dt}}$$

$$P_{ab} = m f_0 \sin pt \cdot \frac{p f_0}{\sqrt{(\omega^2-p^2)^2 + 4\lambda^2 p^2}} \cos(pt-\theta)$$

$$\Rightarrow P_{ab} = \frac{m f_0^2 p}{\sqrt{(\omega^2-p^2)^2+4\lambda^2 p^2}} \sin pt \cos(pt-\theta)$$

Average power absorbtion:

$$P_{average} = \frac{1}{T}\int_0^T \frac{m f_0^2 p}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}} \times \sin pt \cos(pt-\theta)$$

$$= \frac{m f_0^2 p}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}} \times \frac{1}{T}\int_0^T \sin pt \cos(pt-\theta)$$

$\omega = 2\pi f$

Now, $\frac{1}{T} \times \frac{1}{2}\int_0^T 2\sin pt \cos(pt-\theta)$

$$= \frac{1}{2T}\int_0^T [\sin(2pt-\theta) + \sin\theta]\, dt$$

$$= \frac{1}{2T} \sin\theta\, T = \frac{1}{2}\sin\theta$$

$$P_{average} = \frac{m f_0^2 p}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}} \times \frac{1}{2}\sin\theta$$

$$\boxed{f_0 \sin\theta = 2\lambda A P}$$

We know that,

$$f_0 \sin\theta = 2\lambda A P$$

$$\Rightarrow \sin\theta = \frac{2\lambda A P}{f_0}$$

$$\therefore P_{average} = \frac{m f_0^2 p}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}} \cdot \frac{1}{2} \cdot \frac{2\lambda A P}{f_0}$$

$$= \frac{m f_0^2 p^2 \lambda}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}} \cdot \frac{f_0}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}}$$

$$\boxed{\therefore P_{average} = \frac{m f_0^2 p^2 \lambda}{(\omega^2-p^2)^2+4\lambda^2p^2}}$$

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C.T.

The average power absorbed is maximum when, $p = \omega$

$$P_{max} = \frac{m f_0^2 p^2 \lambda}{4\lambda^2 p^2}$$

C.T.

$$\boxed{P_{max} = \frac{1}{4} \cdot \frac{m f_0^2}{\lambda}}$$

Superposition :

Composition of two simple harmonic vibration at right angles to each other having equal frequencies but different in ~~phase~~ phase and amplitude.

$$x = a\sin(\omega t + \varphi) \quad \text{— (i)}$$

$$y = b\sin\omega t \quad \text{— (ii)}$$

$$\frac{x}{a} = \sin(\omega t + \varphi)$$

$$= \sin(\omega t)\cos\varphi + \cos\omega t \sin\varphi$$

$$= \sin\omega t\cos\varphi + \sqrt{1-\sin^2\omega t}\,\sin\varphi \quad \text{— (iii)}$$

From eq^n(ii),

$$\frac{y}{b} = \sin\omega t$$

From, eq^n(iii),

$$\frac{x}{a} = \frac{y}{b}\cos\varphi + \sqrt{1-\frac{y^2}{b^2}}\,\sin\varphi$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\varphi - 2\frac{x}{y}\cdot\frac{a}{b}\cos\varphi = \left(1-\frac{y^2}{b^2}\right)\sin^2\varphi$$

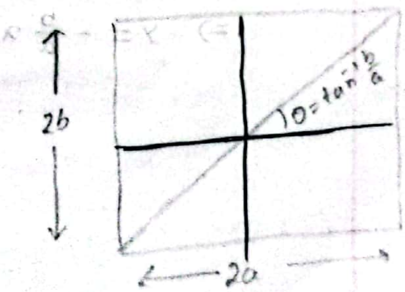
$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\varphi = \sin^2\varphi \quad \text{— (iv)}$$

This is the general equation of resultant vibrations.

Case - 1:

$\varphi = 0, 2\pi, 4\pi, \ldots\ldots \quad n = 0, 1, 2, \ldots\ldots$

$\sin\varphi = 0 , \quad \cos\varphi = 1$



$$\text{(iv)} \rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$\Rightarrow \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$$\Rightarrow \frac{x}{a} = \frac{y}{b} \qquad \Rightarrow y = \frac{b}{a}x$$

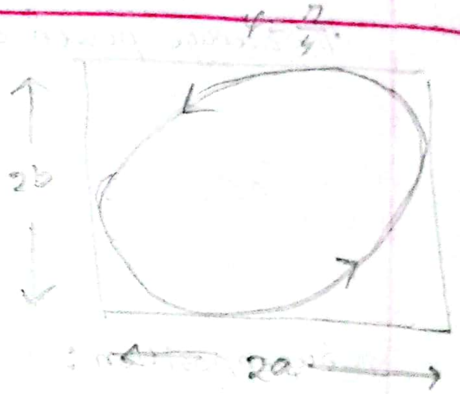
Assignment

Case - 2:

$\varphi = \frac{\pi}{4}$ $\qquad \cos\varphi = \sin\varphi = \frac{1}{\sqrt{2}}$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{\sqrt{2}\,xy}{ab} = \frac{1}{2}}$$

anticlockwise

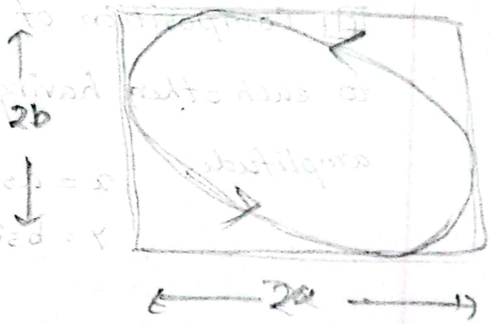


case - 4 :

$\varphi = \frac{3\pi}{4}$

$$\frac{x^2}{a^2} + \frac{y^2}{b} + \frac{\sqrt{2}\,xy}{ab} = \frac{1}{2}$$

clockwise



case - 3 :

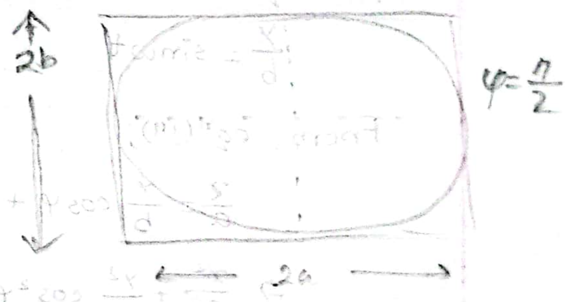
$\varphi = \frac{\pi}{2}$

$\sin\varphi = 1, \ \cos\varphi = 0.$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

when, $a = b$,

$$x^2 + y^2 = a$$



Case-5:

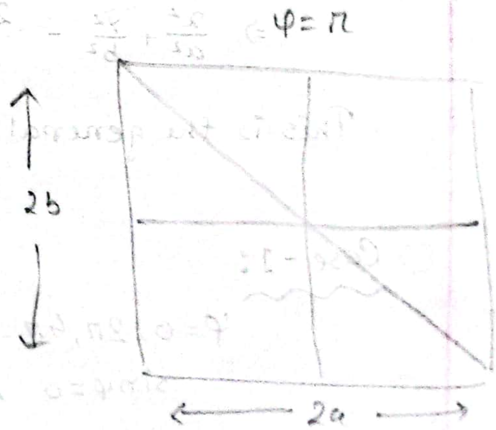
$\varphi = \pi,$

$\sin\varphi = 0, \quad \cos\varphi = -1$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

$$\Rightarrow \frac{x}{a} = -\frac{y}{b}$$

$$\Rightarrow y = -\frac{b}{a}x$$



Question Hard

Sun/Monday
C.T. probability 50%

Lissasous figure :

The composition of two ~~SHB~~ SHB in mutually perpendicular directions gives rise to an elliptical path. The actual shape of the wave will depend upon the phase difference φ between the two vibrations, and also on the ratio of the frequencies of the component of vibrations. These figures are called lissasous figure.

Composition of two SHB of same frequency but different phase and amplitude,

$$y_1 = a_1 \sin(\omega t + \alpha_1)$$

$$y_2 = a_2 \sin(\omega t + \alpha_2)$$

Resultant displacement,

$$y = y_1 + y_2$$

$$\Rightarrow a_1 \sin(\omega t + \alpha_1) + a_2 \sin(\omega t + \alpha_2)$$

wave equation, amplitude, other values.

Thermodynamics

Date: 25.01.2023
Wednesday

Thermodynamic System

A system may be defined as a definite quantity of matter (solid, liquid or gases) bounded by some closed surface.

→ Homogenous System → phase uniform

When a system is completely uniform throughout, such as gas or mixture of a gas.

4) Heterogenous System :

When a system is ~~s~~ consists of two or more phases.

A system maybe separated from its surrounding by a real or imaginary boundary through which heat or mechanical energy may pass.

Thermodynamic Variables :

The thermodynamic state or macroscopic state of a system is determined by four observable properties. These properties are the composition, pressure, volume & temperature which are called the variable of state.

- For system, in general, there is a equation of state,

$$f(P, V, T) = 0$$

For, ideal gas,

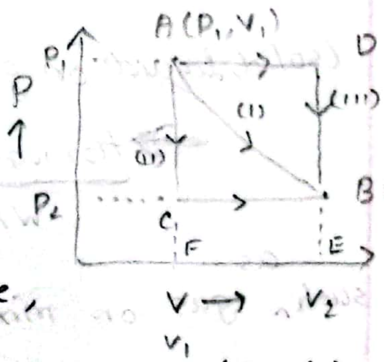
$$PV = RT$$

For, Real gas,

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$, when a and b vandenwal's constant

Work : Exam Must

A path dependant function :



A system can be taken from state A to state B by many ways in figure,

Let the co-ordinates of state A and B $(P_1 V_1)$ and (P_2, V_2)

respectively,

(i) starting from A, the pressure is continuously decreased from P_1 to P_2 along the wave AB so that volume increased from V_1 to V_2.

- The work done by the system,

$$W = \int_{V_1}^{V_2} PdV = \text{area } ABEF$$

(ii) Starting from point A, the volume V_1 is kept constant in going from A to C, the pressure decreases from P_1 to P_2, and then P_2 kept constant from C to B.

- The work done in this process $W_2 = P_2(V_2 - V_2) = \text{area } CBEF$

(iii) Again, starting from point A, the pressure P_1 is kept constant going from A to D and volume V_2 is kept constant from D to B.

The work done in this process,

$$W_3 = P_1(V_2 - V_1) = \text{area } ADEF$$

$\therefore W_1 \neq W_2 \neq W_3$

Work is a path dependant system ***

Thermodynamic Equilibrium:

(i) Mechanical equilibrium:

For a system to be mechanical equilibrium, there should be no macroscopic movement with in the system on of the system with respect to it's surrounding.

(Bridgelal)

• Explain zenoth law of thermondymics

(ii) Thermal Equilibrium:

For a system to be thermal equilibrium, there should be no temp. between the parts of the system or between the system and surrounding.

(iii) Chemical Equilibrium:

Zenoth Law of thermodynamics:

If two bodies A and B are each ~~s~~ separately in thermal equilibrium with a third body C, then A and B are also in their thermal equilibrium with each other.

Internal Energy:

The internal energy of a system is called interval energy. It is the sum of following form of energy of the system,

(i) Kinetic energy due to translational, rotational and vibrational motion of the molecular, all which depends on the temp.

ii) Potential energ due to inter-molecular force

iii) the energy of electron and nuclei.

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First law of thermodynamics:

When a certain amount of heat(Q) is supplied to a system which does external work W in passing from state 1 to state 2, The amount of heat is equal to sum of the increase of the internal energy of the system and the external work done by the system. $\boxed{Q = (U_2 - U_1) + W}$

For small change, $\boxed{dQ = dU + dW}$

Specific heat:

Heat capacity per unit mass is called specific heat.

Specific heat $= \frac{Q}{m\Delta T} = c$

The specific heat of materials is defined as the quantity of heat required to raise the temp. of unit mass of the material, through 1 degree.

$C_p \rightarrow$ at constant pressure $\qquad C_p = \frac{dQ}{dT}$

$C_v \rightarrow$ at constant volume $\qquad C_v = \frac{dQ}{dT}$

* * *

$\boxed{C_p > C_v}$

↓

Reason ?

↓

Question

May come

* Adiabatic/Isothermal/Isotopic curve,

* Slope graph(γ)

Application of first law of thermodynamics:

Date: 31.01.2023
Tuesday

Specific heat of a gas:

V, T, are ~~indepent~~ independent variable. $U = f(V,T)$ —(i)

Differenciating this equation

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \quad \text{—(ii)}$$

From 1st low of thermodynamics,

$$dQ = dU + dW = dU + pdV \quad \text{—(iii)}$$

Putting the value of dU in eq^n(iii),

$$dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + pdV$$

$$\Rightarrow \frac{dQ}{dT} = \left(\frac{\partial U}{\partial T}\right)_V + \left(\frac{\partial U}{\partial V}\right)_T \frac{dV}{dT} + p\frac{dV}{dT}$$

$$= \left(\frac{\partial U}{\partial T}\right)_V + \left[P + \frac{\partial U}{\partial V}\right]\frac{dV}{dT} \quad \text{—(iv)}$$

At constant volume, $(\partial V = 0)$

$$\frac{dQ}{dT} = \left(\frac{\partial U}{\partial T}\right)_V = C_V$$

At constant pressure,

$$\frac{dQ}{dT} = C_P$$

Putting this value in eq^n (iv),

$$C_P = C_V + \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right]\frac{dV}{dT}$$

$$\Rightarrow \boxed{C_P - C_V = P\frac{dV}{dT}} \qquad \because \left(\frac{\partial U}{\partial V}\right)_T = 0$$

We know that,

$$PV = RT$$

$$\Rightarrow P\frac{dV}{dT} = R \qquad \therefore \boxed{C_P - C_V = R}$$

For reversible adibatic process,

$$dQ = 0$$

From equation,

$$0 = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + p\,dV$$

$$= C_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] dV$$

$$= \frac{C_V dT}{dV} + \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right]$$

$$\therefore \frac{C_V dT}{dV} = -P \qquad \left[\left(\frac{\partial U}{\partial V}\right)_T = 0\right]$$

$$\Rightarrow \boxed{C_V \frac{dT}{dV} = \frac{C_V - C_P}{\alpha V}}$$

$$\Rightarrow \boxed{\frac{dT}{dV} = \frac{C_V - C_P}{\alpha V C_V}}$$

- This expantion holds for adibatic reversible process.

We know, Isobaric volume co-efficient expantion

Memorise $\boxed{\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P}$ ***

$$\Rightarrow \alpha V = \left(\frac{\partial V}{\partial T}\right)_P \quad ?$$

$$\Rightarrow \boxed{\alpha V = \frac{C_P - C_V}{P}} \ \text{***}$$

$$\therefore P = \frac{C_P - C_V}{\alpha V}$$

Isochoric Process:

$$dV = 0,$$

$$\boxed{dQ = dU}$$

Isobaric Process:

Pressure constant

$$\boxed{dQ = dU + p\,dV}$$

Adiabatic Process:

No heat transfer

$$dQ = 0$$

$$\boxed{dU = -p\,dV}$$

Isothermal process:

Temp constant

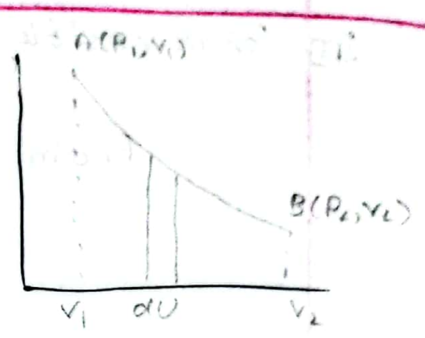
$$dU = 0$$

$$\boxed{dQ = p\,dV}$$

Isentropic Process:

No change of entropy

Work done during Isothermal Process



From the defination of work

$$dW = pdV$$

$$\Rightarrow \int dW = \int_{V_1}^{V_2} pdV$$

$$\Rightarrow W = \int_{V_1}^{V_2} pdV$$

$PV = RT$

$\Rightarrow p = \frac{RT}{V}$

$$\boxed{W = RT\ln\left(\frac{V_2}{V_1}\right)}$$

$$\boxed{W = RT\ln\left(\frac{P_1}{P_2}\right)}$$

$P_1V_1 = P_2V_2$

$\Rightarrow \frac{P_1}{P_2} = \frac{V_2}{V_1}$

Work done during Adiabatic process

$PV^{\gamma} = K$

$$\int dW = \int_{V_1}^{V_2} PdV$$

$$\Rightarrow W = \int_{V_1}^{V_2} PdV$$

$$= \int_{V_1}^{V_2} K \frac{dV}{V^{\gamma}}$$

$$= K\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma}\left[V_2^{1-\gamma} - V_1^{1-\gamma}\right]$$

$$= \frac{1}{1-\gamma}\left[\frac{P_2V_2^{\gamma}}{V_2^{\gamma-1}} - \frac{P_1V_1^{\gamma}}{V_1^{\gamma-1}}\right]$$

$$= \frac{1}{1-\gamma}\left[P_2V_2 - P_1V_1\right]$$

$$= \frac{1}{1-\gamma}\left[RT_2 - RT_1\right]$$

$$= \frac{R}{1-\gamma}\left[T_2 - T_1\right]$$

Reversible Process :

A process which can be retracked in the opposite direction so that the working substance passes through exactly the same states in all respects as in the direct process is called reversible process.

++a++

Irreversible Process:

A process which can not retracked in the opposite direction is called an irreversible process.

⊛ Slope of adibatic and isothermal process:

Adiabatic slope is more steeper than isothermal slope.

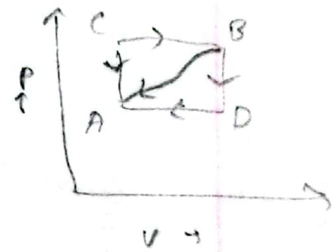
$PV = \cancel{RT}\ k$

$\Rightarrow \frac{d}{dV}(PV) = k$

$\Rightarrow P\frac{dV}{dV} + V\frac{dP}{dV} = \frac{d}{dV}(K)$

Assignment

⊛ When a system is taken from the state A to the state B along the Path ACB, 80 joules of the heat flows into the system, and system does 30 joules of work:



(a) How much heat flows into the system along the path ADB, if the workdone is 10 Joules.

(b) The system is ~~retracked~~ retarded from B, the state B to the state A along carved path, The workdone on the system is 20 joules Does the system absorb liberate heat and how much

(c) If $U_A = 0$, $U_D = 40$ joules, ~~final~~ find the heat absorbed in the process AD and DB

Armstrong

Heat Engine:

Date: 01.02.2023
Wednesday

A practical machine which converts heat into mechanical work is called a heat engine. Heat engines in their operation absorb heat at a higher temperature, converts parts of it into mechanical work, reject remainedy heat at a low temperature.

Defination of Efficiency:

The efficiency η of a heat engine is defined as the ratio of the mechanical work done by the engine in one cycle to heat absorbed from the high temp. source, $\eta = \frac{Q_1 - Q_2}{Q_1}$

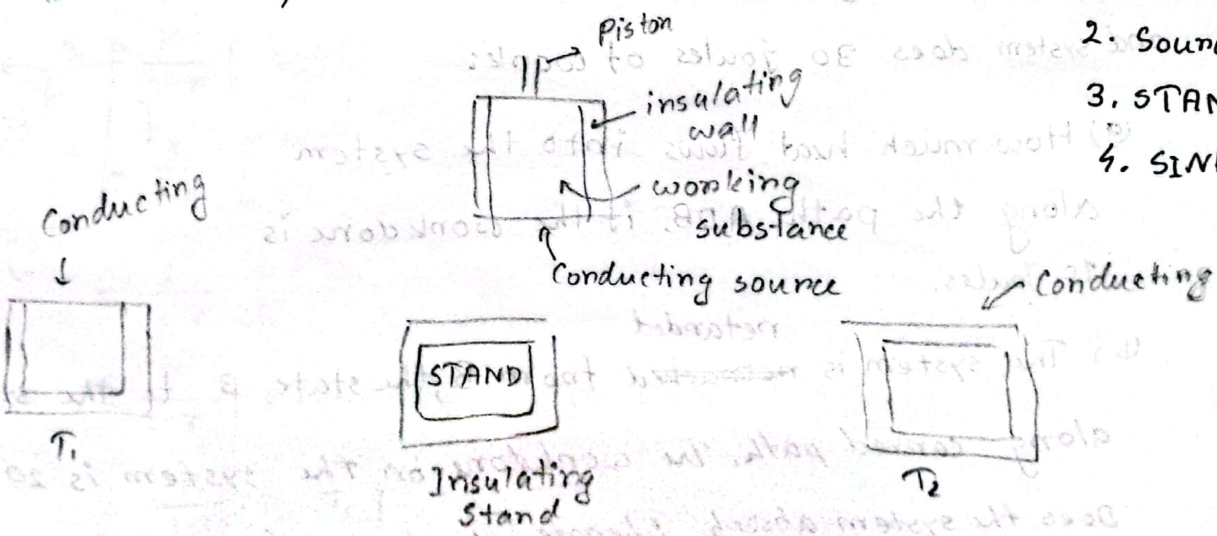
Carnot's Ideal Heat Engine:

Sadi Carnot conceived a theoretical energy which is from all practical imperfections

1. Cylinder
2. Source
3. STAND
4. SINK

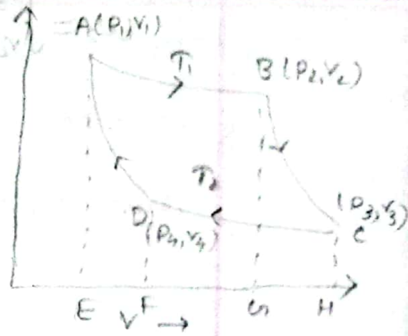
Adnan Hossain

$T_1 > T_2$



The Cycle

Cannot's $\boxed{\text{Ideal Heat Engine}}$:



A cycle in which the working substance starting from a given condition of temperature, pressure, volume is made to undergo two successive expansion and then two successive compression at two end of which the working substance is brought back to it's initial condition is called carnot cycle.

Isothermal Expansion:

The cylinder is first placed on the source, so that the gas acquires the temperature T_1 of the source. Then the gas undergoes show isothermal expansion at T_1.

Let the working substance during isothermal expansion goes from its initial state $A(P_1,V_1,T_1)$ to state $B(P_1 V_2 T_1)$ along AB.

In this process substance absorb heat Q_1 and does work, w

$$W_1 = Q_1 = \int_{V_1}^{V_2} PdV = RT_1 \ln\frac{V_2}{V_1}$$

Adiabatic Expansion:

The cylinder is now removed from the source and is placed on the insulating stand.

The gas is allowed to undergo slow ~~adib~~ adiabatic expansion. This operation is represented by adiabatic BC. The temperature fall to T_2. The work done in this process is,

$$W_2 = \int_{V_2}^{V_3} PdV = K\int_{V_2}^{V_3} \frac{dV}{V^\gamma} = \frac{KV_3^{1-\gamma} - KV_2^{1-\gamma}}{1-\gamma}$$

$$= \frac{P_3V_3 - P_2V_2}{1-\gamma} = \frac{RT_2 - RT_1}{1-\gamma}$$

$$W_2 = \frac{R}{1-\gamma}[T_2 - T_1]$$

3. *Isothermal Expansion:*

The cylinder is now removed from the insulating stand and is placed on the sink which is at T_2 temperature, Thus gas undergoes isothermal compression at a constant temperature T_2.

This operation is represented by CD.

In this process substance reject heat Q_2 and does work W_3

$$Q_2 = W_3 = \int_{V_4}^{V_3} PdV = RT_2 \ln\left(\frac{V_4}{V_3}\right)$$

$$\therefore W_3 = -RT_2 \ln\left(\frac{V_3}{V_4}\right)$$

-ve sign indicates that work is done on the working substance.

4. *Adiabatic Compression:*

The cylinder is now removed from sink and again placed on the insulating stand. The adiabatic compression is continued till the gas is back to its original condition, i.e. state $A(P_1, V_1)$ thus completing one full cycle, $W_4 = -\int_{V_1}^{V_4} PdV$

$$= \frac{-R(T_1 - T_2)}{\gamma - 1}$$

Work done by the engine per cycle:

During the above cycle, the working substance absorbs an amount of heat Q_1 from source and reject Q_2 to the sink.

The net amount heat absorbed $= Q_1 - Q_2$

net amount of work $= W_1 + W_2 + W_3 + W_4 = W_1 + W_3 \; [W_2 = -W_4]$

We know that,

Net heat absorbed = net work done

$$Q_1 - Q_2 = W_1 + W_3 = RT_1 \ln \frac{V_2}{V_1} - RT_2 \ln \frac{V_3}{V_4} \quad -①$$

Since, A and D lie on same adiabatic line.

$$T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} \quad -(ii)$$

Again,

B and C also lie on the same adibatic line.

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\Rightarrow T_2 = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \quad -(iii)$$

From (ii) and (iii) $\Rightarrow \left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \Rightarrow \boxed{\frac{V_1}{V_2} = \frac{V_4}{V_3}} \quad \boxed{\Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}}$

$$Q_1 - Q_2 = RT_1 \ln \frac{V_2}{V_1} - RT_2 \ln \frac{V_2}{V_1}$$

$$W = R(T_1 - T_2) \ln \frac{V_2}{V_1}$$

$$\text{Efficiency}, \eta = \frac{\text{net work done}}{\text{absorbed heat}} = \frac{Q_1 - Q_2}{Q_1} = \boxed{\frac{R(T_1 - T_2) \ln \frac{V_2}{V_1}}{RT_1 \ln \frac{V_2}{V_2}}}$$

$$\boxed{\eta = \frac{T_1 - T_2}{T_1} \times 100\%}$$

Second law of Thermodynamics

Date: 07.02-2023
Tuesday

It is impossible for a self-acting machine working in a cyclic process, unaided by external agency, to transfer heat from a body at a lower temperature to a body at higher temperature.

Kelvin's statement:

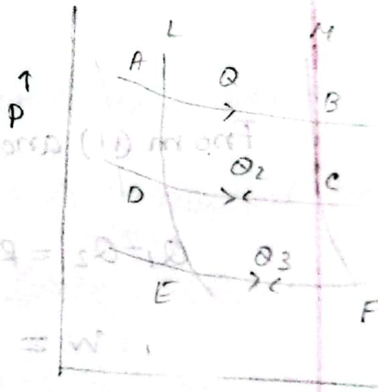
It is impossible to an engine which, working in complete cycle will produce no effort other than the raising of weight and the cooling of the heat reservoir.

Concent Entropy:

The quantity of entropy found to remain constant in adiabatic process just as temperature remain constant in an isothermal process. The entropy can be defined as the thermal property of a working substance which is constant in adiabatic process.

Change in Entropy:

Let us consider ~~resen~~ reversible cannot's cycles bounded by the same two adiabatic L and isothermal T_1, T_2, T_3 as shown in Figure.



During ABCD cycle, an amount of heat Q_1 is absorbed in T_1 temperature and Q_2 heat reject at T_2 temperature

Then the effectively,

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

$$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad \text{---(i)}$$

Similarly for DCEF

$$\frac{Q_2}{T_2} = \frac{Q_3}{T_3} \quad \text{---(ii)}$$

From eq^n (i) and (ii)

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3} = \text{constant}$$

$$\frac{Q}{T} = \text{constant}$$

If δQ a small amount of heat is reject or absorbed

$$\frac{\delta Q}{T} = \text{constant}$$

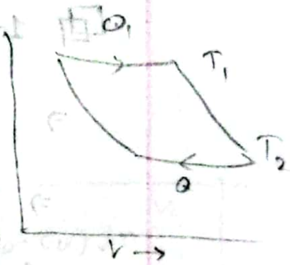
Constant ratio is called the change of entropy.

$$\delta S = \frac{\delta Q}{T}$$

$$\Rightarrow \int_{S_A}^{S_B} dS = \int_A^B \frac{dQ}{T}$$

$$\Rightarrow \boxed{S_B - S_A = \int_A^B \frac{dQ}{T}}$$

Change in entropy in reversible process:



In reversible cycle, there are two isothermal and two adiabatic process, there is neither absorbtion or rejection of heat and ..... there no change in entropy.

During expansion an amount of heat Q_1 is absorbed at temperature T_1 and in compression there is ... heat reject at T_2 temperature.

Change in entropy,

$$dS = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} \quad \text{---(i)}$$

We know that for reversible cycle,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow dS = 0$$

$\therefore S$ is constant

The entropy of a body in all reversible cycle remain constant.

Change in entropy in irreversible process :

Let us consider an engine performing reversible cycle in which working substance ... absorbs heat Q_1 at T_1 temperature and reject heat Q_2 at T_2 temperature.

$$\text{Efficiency}, \eta' = \frac{Q_1 - Q_2}{Q_1} = \boxed{1 - \frac{Q_2}{Q_1}} \rightarrow \text{Irreversible}$$

But according to carnot's theorem, this efficiency is less that of ~~resen~~ reversible engine working in the same temperature

Then $\boxed{\eta = 1 - \frac{T_2}{T_1}}$ Reversible

Thus $\boxed{\eta' < \eta}$

$$1 - \frac{Q_2}{Q_1} < 1 - \frac{T_2}{T_1}$$

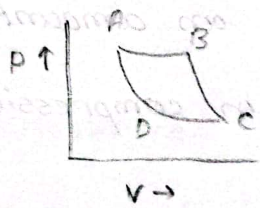
$$\Rightarrow \frac{Q_2}{Q_1} < \frac{T_2}{T_1}$$

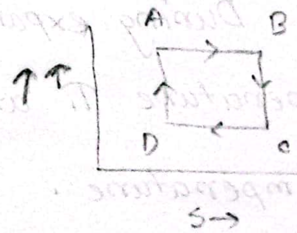
$$\Rightarrow \frac{Q_2}{T_2} > \frac{Q_1}{T_1}$$

$$\Rightarrow \left(\frac{Q_2}{T_2} - \frac{Q_1}{T_1}\right) > 0$$

There is an increase in entropy of the system during irreversible process.

T-S Diagram;





Explain

Third law of thermodynamics:

At absolute zero temperature the entropy tends to zero and molecular a substance or a system are in the perfect order.

Entropy of a perfect gas:

Consider 1 gm of perfect gas occupies a volume V at P and T,
Let dQ be the heat supplied to the gas,

$$dQ = dU + dW \quad \text{— (i)}$$

$$\cancel{dU = dQ - dW} \quad dU = C_v dT$$

$$dQ = C_v dT + PdV$$

Change of Entropy,

$$\Delta S = \int \frac{dQ}{T} = \int \frac{1}{T}(C_v dT + PdV)$$

$$\Delta S = \int C_v \frac{dT}{T} + \int \frac{P}{T} dV \quad \text{— (ii)}$$

(i) S in terms of temperature and volume,

$$PV = RT$$

$$\Rightarrow \frac{P}{T} = \frac{R}{V}$$

$$\Delta S = \int_{1}^{2} C_v \frac{dT}{T} + \int \frac{R}{V} dV$$

$$= C_v [\ln T]_{T_1}^{T_2} + R[\ln V]_{V_1}^{V_2}$$

$$\boxed{\therefore \Delta S = C_v \ln \frac{T_2}{T_1} + (C_p - C_v)\ln \frac{V_2}{V_1}}$$

(ii) S in terms of temperature and ......,

$$PV = RT$$

$$\Rightarrow V = \frac{RT}{P}$$

$$PdV + VdP = RdT$$

$$\Rightarrow PdV = RdT - VdP$$

$$\Delta S = \int C_v \frac{dT}{T} + \int \frac{RdT - VdP}{T}$$

$$= \int C_v \frac{dT}{T} + R\int \frac{dT}{T} - \int \frac{VdP}{T}$$

$$= \int C_v \frac{dT}{T} + (C_p - C_v)\int \frac{dT}{T} - R\int \frac{dP}{T}$$

$$= C_p \int \frac{dT}{T} + (C_p - C_v)\int \frac{dP}{P}$$

CamScanner

$$\therefore \Delta S = C_p[\ln T]_{T_1}^{T_2} - (C_p - C_v)[\ln P]_{P_1}^{P_2}$$

$$\therefore \Delta S = C_p \ln\frac{T_2}{T_1} - (C_p - C_v)\ln\frac{P_2}{P_1}$$

(iii) S in terms of pressure and volume,

$$PV = RT \qquad PV = RT$$

$$\Rightarrow T = \frac{PV}{R} \qquad \Rightarrow \frac{P}{T} = \frac{R}{V}$$

$$\Rightarrow dT = \frac{PdV + VdP}{R}$$

$$\Delta S = C_v\int\frac{dT}{T} + \int\frac{P}{T}\,dV$$

$$= C_v\int\frac{PdV + VdP}{RT} + \int\frac{R}{V}\,dV$$

$$= C_v\int\frac{PdV}{RT} + \int C_v\frac{VdP}{RT} + R\int\frac{dV}{V}$$

$$= C_v\int\frac{dV}{V}\times\frac{T}{R} + C_v\int\frac{dP}{P}\times\frac{1}{R} + (C_p - C_v)\int\frac{dV}{V}$$

$$= \frac{C_vT}{R}\ln\frac{V_2}{V_1} + \frac{C_v}{R}\ln\frac{P_2}{P_1} + (C_p - C_v)\times\ln\frac{V_2}{V_1}$$

NOT RIGHT

LASER PHYSICS

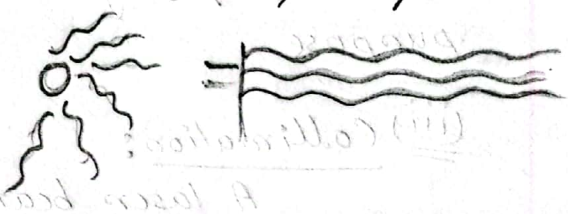
Date: 08.02.2023
Wednesday

- Quantum Optics
- LASER is a photonic device
  Photonics

Light Amplification through Stimulated emission of Radiation

not amplify
- Light Oscillation through Stimulated Emission of Radiation, LOSER

- Normal light → tungsten light, mercury lamps



- The unique property of laser is that its light waves travel very long distance with a very little divergence. In case of conventional light source, the light is emitted in jumble of separate waves that cancel each other at random and hence can travel very short distance.

In laser, the light waves are exactly in step with each other and have a fixed phase relationship.

Defination of Laser:

Laser is an optical electronic device that generate an intense beam of single color light by pumping photon with more energy through collision with other photon.

Properties of laser :

(i) Monochromativity :

The light emitted from ~~[illegible]~~ laser is monochromatic. It means that LASER light has single color.

(ii) Directionality :

LASER emit light that is a highly directed or undirected light, because of its directional nature, it can carry energy and data to very long distance for remote diagnosis and communication purpose.

(iii) Collimation :

A laser beam is collimated, meaning it consists of waves travelling parallel to each other in a single direction with very little divergence.

(iv) Coherent :

The light from a laser is said to be coherent which means the wavelength of the laser light are in phase in space and time.

(v) Intensity :

A laser beam is extremely ~~extremly~~ intense and brighter source compared to other conventional sources such as sun ~~sun~~. Brightness is defined as the power emitted per unit area per solid angle.

CamScanner

If $n_2 > n_1$
laser will emit

E_2 ——————— N_2

E_1 ——————— N_1

$N_1 >> N_2$

Thermal equation:

Boltzman

$N = e^{-\frac{E}{kT}}$

A material medium is composed of identical atoms, which are characterized by a specific system of energy levels. These energy levels are common to all atoms in the medium. We can therefore say that certain number of atoms occupy a certain energy level.

- The number of atoms per unit volume that occupy a given energy level is called population of that energy level.

Let E_1 be the ground level and E_2 the excited level. Atoms are distributed ~~efficiently~~ differently in ~~these~~ this two energy levels. Let the population at the levels E_1 and E_2 are N_1 and N_2,

E_2 ——————— N_2

E_1 ——————— N_1

At thermal equilibrium, the population at the energy levels can be found with the Boltzman statistic,

$$N_1 = e^{-\frac{E_1}{kT}}$$

$$N_2 = e^{-\frac{E_2}{KT}}$$

- The Relative population (N_2/N_1),

$$\left(\frac{N_2}{N_1}\right) = e^{-(E_2-E_1)/kT}$$

- $\left(\frac{N_2}{N_1}\right)$ is dependent on two factor,

(i) The temp. (T) and
(ii) The energy difference (E_2-E_1)

Effect of Temperature:

In case of hydrogen atom,

$E_1 = -13.6\,eV$, $E_2 = -3.39\,eV$, $E_2 - E_1 = 10.21\,eV$

① At room temperature, ($T = 300K$)

$kT = 0.025\,eV$

$$\frac{N_2}{N_1} = e^{-\frac{10.21}{0.025}} = e^{-408.4} \simeq 0$$

$\boxed{N_2 \simeq 0}$

$\Rightarrow N_2 \simeq 0$

(ii) At 6000K ($T = 6000K$)

$kT =$ ~~0.516~~ $0.516\,eV$

$\boxed{N_1 >> N_2}$

$$\frac{N_2}{N_1} = e^{-\frac{10.21}{0.516}} = \frac{4}{10^{10}}$$

(iii) when $T = \infty$

$$\frac{N_2}{N_1} \simeq e^0 = 1$$

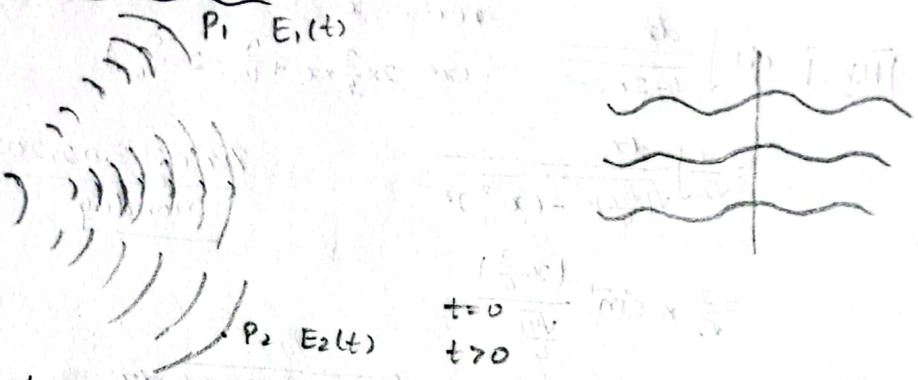
$\boxed{N_2 \simeq N_1}$

When, $E_2 - E_1 = 0$ $\frac{N_2}{N_1} \simeq 0 \simeq 1$

$\Rightarrow \boxed{N_2 \simeq N_2}$

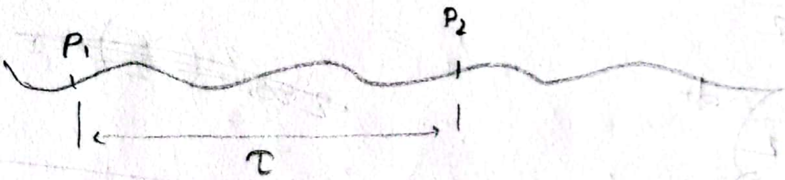
Both this limiting cases indicate, as long as medium is in thermal equilibrium, the population in ~~high~~ higher energy level can not exceed the lower energy level.

Spatial Coherence:



Let us, consider two points P_1 and P_2 that, at time $t=0$, lie on the same ~~the / phase~~ wavefront of same given em.f. wave and let $E_1(t)$ and $E_2(t)$ be the correnponding electric field at $t=0$, If the phase difference of two electric field is remain zero after $t>0$ then we will say that there is a perfect coherence between the two points.

If this occur for any two points of e.m. wavefront, we say that wave has perform spatial coherence.



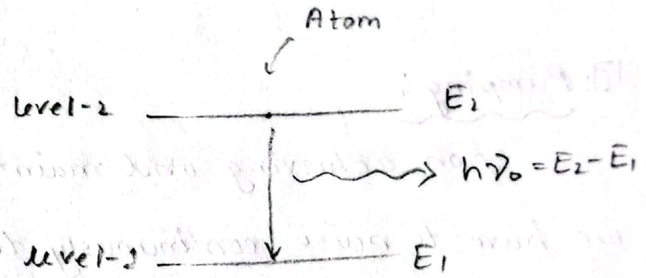
Temporal Coherence:

We now consider the electric field of the e.m. wave at a given point p_1 at times t and $(t+\tau)$. If for a given time delay τ, the phase difference between two field remain the same for any time. then we will say there is a temporal coherence.

Active medium:

Atoms in medium which can produce more ~~than~~ stimulated emissions then spontaneous emission and came amplification of light is called active center. The rest of the medium acts as host and supports active center. The medium which resting the active center is called the active medium.

a) Spontaneous emission:

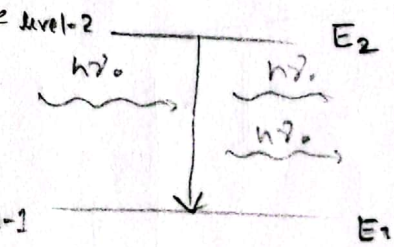


We assume that the atom is initially at level-2. Since $E_2 > E_1$ the atoms tends to be decay to level-1. The corresponding energy difference must be released by the atom. When the energy is delivered in the form of emt wave, the process is called spontaneous emission.

The frequency of the radioactive wave,

$$\boxed{\begin{aligned} h\nu_0 &= E_2 - E_1 \\ \nu_0 &= \frac{E_2 - E_1}{h} \end{aligned}}$$ * *
*

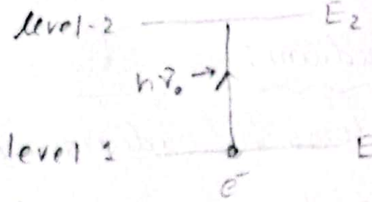
(b) Stimulated Emission:



Let the atom be initially at level-2. An em wave frequency ν_0 which is equal to that of spontaneous emission is incident on the material. The incident wave will force the atom to make a transition from level-2 to level-1. This is characterized by an emission of a photon with energy equal to $(E_2 - E_1)$. This phenomenon is known as Stimulated Emission.

(e)

Absorption:

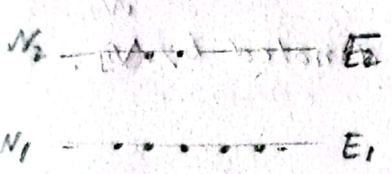


Let the atom is now in initially at level-1, The atom remains in this level unless some external energy is applied to the material when an incident wave with energy equal to $E_2 - E_1$ is applied to the material then the atom will be raised to level-2. This process is absorption.

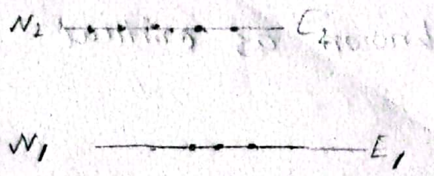
(f) Pumping:

For achieving and maintaining the condition of population, we have to raise continuously the atoms in the lower energy level to upper energy level. It requires energy to be supplied to the system. Pumping is the process of supplying energy to the ~~laser~~ lower medium with a view to transfer it into the state of population inversion.

Date : 28.02.2023
Tuesday



(a) Normal state
Thermal equilibrium
$(N_1 > N_2)$



(b) Inverted State
Non equilibrium condition
$(N_2 > N_1)$

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT}$$

Population inversion:

We know from thermal equilibrium condition, $\boxed{\frac{N_2}{N_1} = e^{\cancel{-(E_2-E_1)/kT}}}$

We may somehow enhance the number of atoms in the excited level than the lower such that the population ration $\left(\frac{N_2}{N_1}\right)$ momentarily increase without change in temperature.

* Population inversion is the non-equilibrium condition of the material in which population of the upper energy level N_2 unmomentarily ~~exceeds~~ exceeds the population lower energy level N_1, $\boxed{N_2 > N_1}$

Three level laser:

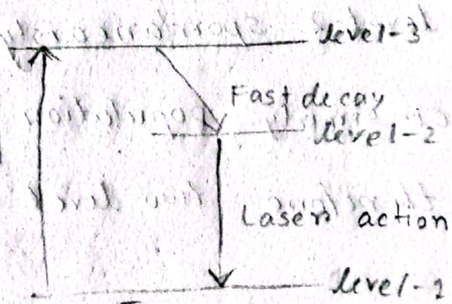


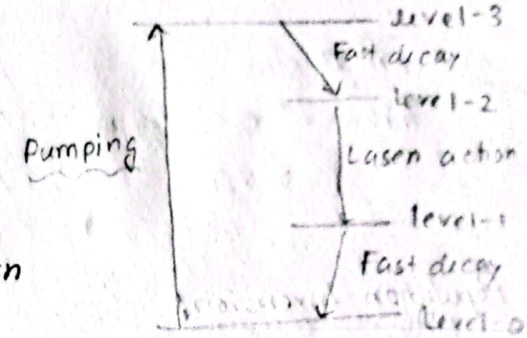
Fig: 3 level laser

In a three level laser the atoms are in some way raised from ground level 1 to level 3 if the material is such that after an atom has been raised to level 3, it decays rapidly to level 2, then the population inversion can be be obtained between level 2 to 1, in this case $N_3 \approx 0$ due to fast decays

For $N_2 > N_1$, $(N_2 - N_1)$ has a definite value, there is a minimum value of $(N_2 - N_1)$ for which we get laser action between level-1 and level-2.

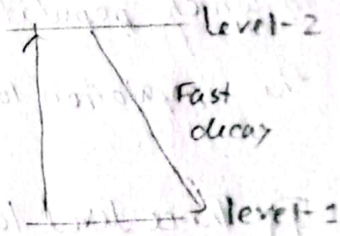
This minimum value is known as critical inversion denoted by N_1.

Four level laser:



In a foun level lasen atoms are naised from ground level-0 to level-3, if the atom then napidly decay to level-2, a population inversion obtain from level-2 to level 1. Once oscillation stants in foun level lasen, the atom will than be transfenned to level-1, though stimulated emission, Fon continuous wave openation it is thenefone necessany that the transition $1 \to 0$ should be veny fast

Two level laser: (Reason why it is not possible)

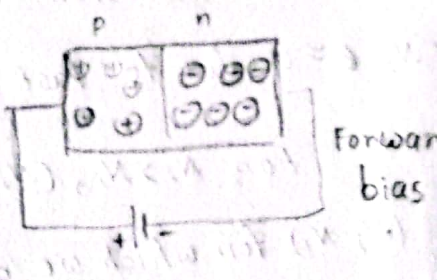


In two level laser population invension is not possible after pumping pnocess, the popoulation from level-2 spontaneously decays to level-1. Critical inversion or simply population invension is not possible, in two level system, thenefone two level lasen laser is possible

- Semiconducton laser
- Ruby laser (Solid state laser)
- He-Ne (Gas level) laser

Semiconducton diode laser:

It is specifically fabnicated p-n junction diode. This diode emits lasen light when it is forwand bias.



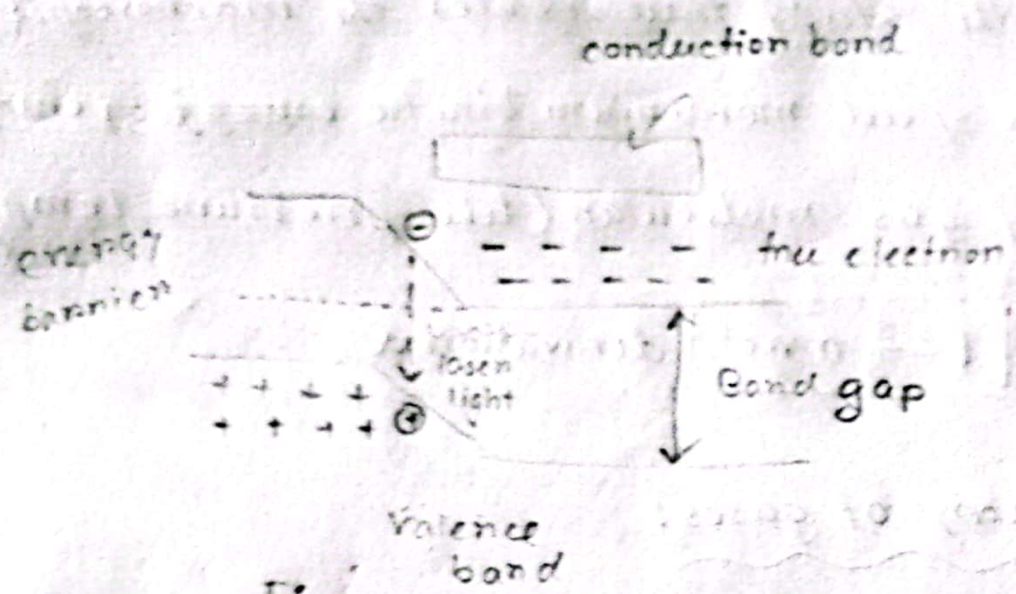


Fig: Solid state laser

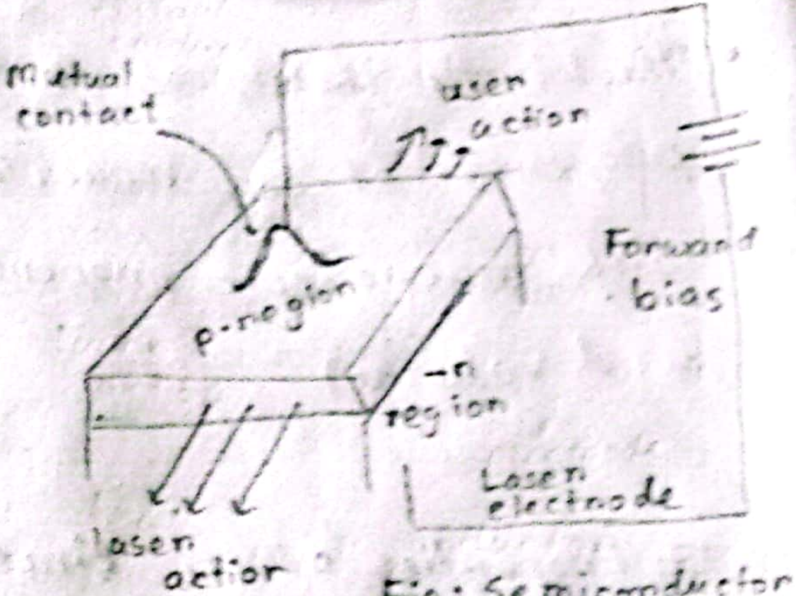


Fig: Semiconductor laser

$$\boxed{\begin{aligned} E_g &= h\nu = h\frac{c}{\lambda} \\ \lambda &= \frac{hc}{E_g} \end{aligned}}$$

Characteristics:

1. It is a solid state laser
2. A p-n junction made from single crystal of gallium arsenide is used in active medium.
3. Pumping method: The direct conversion by method is used in pumping action.
4. The power out of this level is ~~5mW~~ 1mW.

Date: 14.03.2023
Tuesday

Kinetic Theory of gases:

- volume, Pressure, Temperature

macroscopic property

Volume → The volume is ~~the~~ a result of the freedom of the ~~of~~ atoms have to spread throughout the container.

Pressure → The pressure is a result of the emission of the atom with the container wall.

Temperature → The temperature has to do with the kinetic energy of the atoms.

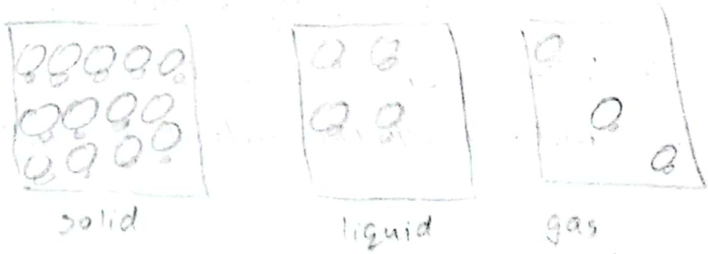
- The kinetic theory of gases is the study that relates the macroscopic properties of gas molecule (like speed, momentum, kinetic ~~energies~~) energies with macroscopic properties of gas molecular (like pressure, temp and volume).

$\boxed{VP = \frac{1}{3} mn\bar{c}^2}$ derivation

Properties of the kinetic theory of gases:

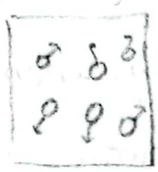
1. The gas is composed of small indivisible particles called molecules. The properties of the individual molecules are the same as that of the gas as a whole.

2. The distance between the molecules is large as compared to that of solid and liquid hence force of inter molecular attraction is negligible.

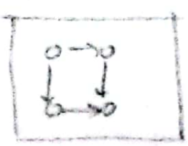


3. Any infinitely small volume of a gas contains many molecules under standard condition of temperature and pressure there are nearly $\boxed{3 \times 10^{25} \text{ molecules}}$ in a cubic meter.

4. The molecules continuously move about in random direction. Every directions is equally probable.



5. The size of the molecules is much less than the average distance between them.



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(6) The molecules of ~~the~~ a gas exert no forces on each other except when they collide i.e., there are no intermolecular forces. In the absence of external force the molecules will move in uniform rectilinear motion and their direction will ~~ca~~ change only by collision with walls of the container or with other molecules

(7) Collision between molecules and with the walls are perfectly elastic i.e., there is no loss in their kinetic energy in a collision.

elastic collision

(8) The molecules are distributed uniformly in the absence of a external field and there is no preferential direction of motion.

inelastic collision

(9) The molecules move will all speeds from 0 to infinity. This means that all molecules do not have the same speed.

(10) The time of impact is negligible in comparison with to the time taken to traverse the free path.

Date :

Brownian motion:

Light suspended particles in a gas or in low viscous fluid are found to move continuously in a random manner in short zigzag path such a motion if suspended particle is known as Brownian Motion.

Characteristics:

1. Brownian motion is ~~continuous~~ spontaneous, perpetual and random
2. With the rise of temp, the motion becomes more vigorous.
3. The smaller particles are agitated move.
4. The motion depends on the chemical nature of the particle.
5. At the same temp., particles of the same size are equally agitated.
6. The less the viscosity of the fluid, the more vigorous the motion.
7. The motion is independent of the movement of the containing vessel.
8. Brownian motion is not affected by electric and magnetic field.

Degree of freedom:

The degree of freedom for a dynamic system is the number direction in which a particle can move freely or the minimum number of independent coordinates needed to specify the position and configuration of a thermodynamic system in space. It is denoted f.

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Total number of freedom,

$$f = f_t + f_r + f_v$$

suppose, we have N number of gas molecules, degree of freedom.

$$f = 3N$$

But if the system has q number of constraints, then degrees of freedom, $f = 3n - q$

Monoatomic gas: (He,

f_{trans} ~~f_{trans}~~ $= 3$

↓

transition $f_{rot/rotation} = 0$ No bond, no vibration

$f_{vib} = 0$

$f_{total} = 3$

Diatomic gas (H_2, N_2)

$f_{trans} = 3$

$f_{rot} = 2$ rotation of freedom

$f_{vib} = 1$ vibration of freedom

Below 100 K → f_r → no rotation

100 K < T < 1000 K → $f_{trans} + f_{rot}$

T > 1000 K → $f_{trans} + f_{rot} + f_{vib}$

Triatomic gas

1. Linear triatomic (CO_2)

$f_{trans} = 3$

$f_{rot} = 2$

$f_{vib} = (3N - 5)$

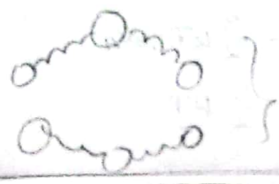
$= 4$

Symmetric stretch mode

asymmetric stretch mode

$f_{total} = 4 + 2 + 3$

$= 9$



bending mode

2. Non-linear triatomic gas (H_2O)

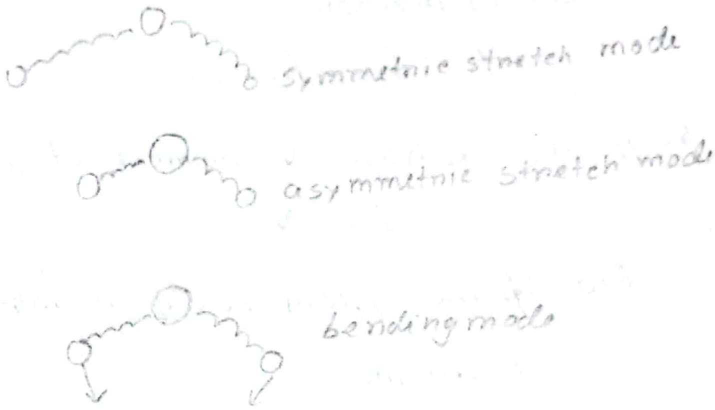
$f_{trans} = 3$

$f_{rot} = 3$

$f_{vib} = 3N-6 = 3$

$f_{total} = 3+3+3$

$= 9$



Principle of Equipartition of Energy:

According to this law, the total energy of a dynamic system in thermal equilibrium is shared equally by all its degrees of freedom, the energy associated per molecules per degree of freedom being a constant equal to $\frac{1}{2}kT$.

According to kinetic theory,

$$\frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT$$

$\bar{v}^2 = \bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2$

$\Rightarrow \bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2$

So, $\frac{1}{2}m\bar{v}_x^2 = \frac{1}{2}m\bar{v}_y^2 = \frac{1}{2}m\bar{v}_z^2$

$\Rightarrow \frac{1}{2}m(\bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2) = \frac{1}{2}m\bar{v}^2$

$\Rightarrow \frac{1}{2}m(3\bar{v}_x^2) = \frac{1}{2}m\bar{v}^2$

$\Rightarrow \frac{1}{2}m(3\bar{v}_x^2) = \frac{3}{2}kT$

$\Rightarrow \boxed{\frac{1}{2}m\bar{v}_x^2 = \frac{1}{2}kT}$

$\frac{1}{2}m\bar{v}_y^2 = \frac{1}{2}kT$

$\frac{1}{2}m\bar{v}_z^2 = \frac{1}{2}kT$

Ratio of specific heat:

1. monoatomic gas-

$E_{trans} = \frac{1}{2}m\bar{v}_x^2 + \frac{1}{2}m\bar{v}_y^2 + \frac{1}{2}m\bar{v}_z^2$

$E_{trans} = \frac{1}{2}kT + \frac{1}{2}kT + \frac{1}{2}kT = \frac{3}{2}kT$

$U = \frac{3}{2}kT \cdot N_A$

$U = \frac{3}{2}RT$

$C_V = \frac{dU}{dT} = \frac{3}{2}R$

$C_P = \frac{3}{2}R + R$

$= \frac{5}{2}R$

$\gamma = \frac{C_P}{C_V} = \frac{\frac{5R}{2}}{3R/2} = 1.67$

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2. Diatomic gas: (a) At normal temp,

$E_T = \frac{5}{2}kT$

$V = \frac{5}{2}RT$

$C_V = \frac{dV}{dT} = \frac{5}{2}R$

$C_p = C_V + R = \frac{5}{2}R + R = \frac{7}{2}R$

$\gamma = \frac{C_P}{C_V} = \frac{7/2 R}{5/2 R} = 1.40$

(b) High temp:

3. Triatomic gas:

non linear, $E_T = \frac{6}{2}kT$, $V = \frac{6}{2}RT$

$C_V = \frac{dV}{dT} = \frac{6}{2}R = 3R$ $\quad C_p = C_V + R = 4R$ $\quad \gamma = \frac{4}{3} = 1.33$

Date: 27.03.2023
Tuesday

Distribution of velocity:

Determination of num of molecules having a certain velocity but range of velocities infinite, num of molecules finite.

$$f(c) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} c^2 e^{-\frac{mc^2}{2kT}}$$

$\boxed{dN \to c \quad c+dc}$

Let dN = num of molecules having velocities within range c and $(c+dc)$

$dN \propto dc$

$\Rightarrow dN = a\,dc$ ——(1)

a is proportionately constant and a is a function of c.

$a = f(c)$ ——(2)

But, $dN \propto N$

$\boxed{\text{Hence } f(c) \text{ is a distribution function}}$

We can write,

$dN = Nf(c)\,dc$ ——(3)

Considering $dc = 2$, $f(c) = \frac{dN}{N}$;

fraction of molecule whose velocities are within unit interval near c,

$$f(c) = \frac{dN}{N} \quad \text{--- (4)}$$

$\frac{dN}{N}$ is the probability of any molecule in unit volume having velocity that is within unit interval near velocity c.

(u, v, w) describes a velocity point.

Probability of molecules having velocities u and $u+du$, $f(u)du$

" " " " " v and $v+dv$, $f(v)dv$

" " " " " w and $w+dw$, $f(w)dw$

$$dN = N \underline{f(u) f(v) f(w)} \, du dv dw$$

$$\Rightarrow dN = N \underline{\varphi(c^2)} \, du dv dw$$

$$\Rightarrow \frac{dN}{du\,dv\,dw} = N f(u) f(v) f(w) = N \varphi(c^2)$$

$\frac{dN}{du dv dw}$ is ρ which is the num density in velocity space.

$$\rho = N f(u) f(v) f(w) = N \varphi(c^2)$$

$$\Rightarrow d\rho = d[N f(u) f(v) f(w)] = d[N \varphi(c^2)] \quad \text{--- (5)}$$

For a fixed c, RHS $= 0$.

$$d[N f(u) f(v) f(w)] = 0$$

$$\Rightarrow f'(u) f(v) f(w) du + f(u) f'(v) f(w) dv + f(u) f(v) f'(w) dw = 0$$

Dividing by $[f(u) f(v) f(w)]$ we get,

$$\Rightarrow \frac{f'(u)}{f(u)} du + \frac{f'(v)}{f(v)} dv + \frac{f'(w)}{f(w)} dw = 0 \quad \text{--- (6)}$$

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We know,

$$c^2 = u^2 + v^2 + w^2$$

$$\Rightarrow 2udu + 2vdv + 2wdw = 0$$

$$\Rightarrow udu + vdv + wdw = 0$$

$$\Rightarrow \beta udu + \beta vdv + \beta wdw = 0 \quad \text{—(7)}$$

where, β is undermined multiplier of language

• Adding equation (6) & (7)

$$\left(\frac{f'(u)}{f(u)} + \beta u\right)du + \left(\frac{f'(v)}{f(v)} + \beta v\right)dv + \left(\frac{f'(w)}{f(w)} + \beta w\right)dw = 0$$

$\boxed{f(x,y,z) + \lambda\varphi(x,y,z)}$ $\boxed{F(x,y,z) = f(x,y,z) + \varphi(x,y,z)}$

$$\frac{f'(u)}{f(u)} = -\beta u$$

$$\frac{f'(v)}{f(v)} = -\beta v$$

$$\frac{f'(w)}{f(w)} = -\beta w$$

→ Integrating, $\ln f(u) = -\frac{\beta u^2}{2} + \ln a$

[a is constant]

$$\Rightarrow f'(u) = a e^{-\frac{\beta u^2}{2}}$$

$$= a e^{-bu^2}$$

$\boxed{b = \frac{\beta}{2}}$

Similarly, $f(v) = a e^{-bv^2}$

$f(w) = a e^{-bw^2}$

$$f(u) f(v) f(w) = a^3 e^{-b(u^2+v^2+w^2)}$$

$$dN = N f(u) f(v) f(w) \, du dv dw$$

$$= N a^3 e^{-b(u^2+v^2+w^2)} du dv dw$$

$$\boxed{dN = N a^3 e^{-b(c^2)} du dv dw}$$

Date: 03.04.2023
Monday

$$* \; dN = Na^3 e^{-b(c^2)} \, du\,dv\,dw \quad \text{——} (9)$$

Finding the volume in velocity space,

$(x, y, z) \to (v, \theta, t)$

$dx\,dy\,dz \to r^2 dr \sin\theta \, d\theta \, d\varphi$

$du\,dv\,dw \to c^2 dc \; \sin\theta \, d\theta \, d\varphi$

$$du\,dv\,dw = c^2 dc \sin\theta \, d\theta \sin\varphi \quad \text{—} (10)$$

$$dN = Na^3 e^{-b(c^2)} c^2 \sin\theta \, dc \; d\theta \, d\varphi$$

$$\Sigma dN = Na^3 \int_0^\infty e^{-b(c^2)} c^2 dc \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\varphi \quad \text{——} (11)$$

4π

$$\Sigma dN = 4\pi N a^3 \int_0^\infty e^{-bc^2} c^2 dc$$

$$\Rightarrow \frac{\Sigma dN}{N} = 4\pi a^3 \int_0^\infty e^{-bc^2} c^2 dc$$

$$\Rightarrow \int f(c) dc = 4\pi a^3 \int_0^\infty e^{-bc^2} c^2 dc = 1$$

$$\Rightarrow 4\pi a^3 \int_0^\infty e^{-bc^2} c^2 dc = 1$$

Standard integral,

$$\int_0^\infty e^{-\boxed{b} c^2} c^2 dc = \frac{1}{4}\sqrt{\frac{\pi}{b^3}}$$

$$\bullet \int_0^\infty c^{-bc^2} c^2 dc = \frac{1}{4}\sqrt{\frac{\pi}{b^3}}$$

$$\Rightarrow 4\pi a^3 \frac{1}{4}\sqrt{\frac{\pi}{b^3}} = 1$$

$$\Rightarrow \pi^{3/2} a^3 \sqrt{\left(\frac{2}{\beta}\right)^3} = 1$$

$$\Rightarrow \pi^{3/2} a^3 . 2^{3/2} \beta^{-\frac{3}{2}} = 1$$

$$\Rightarrow a^3 = \frac{\beta^{3/2}}{(2\pi)^{3/2}}$$

$$\Rightarrow a = \sqrt{\frac{\beta}{2\pi}} \quad \left[b = \frac{\beta}{2}\right]$$

Expectation value of mean square velocity,

$$\langle c^2 \rangle = \frac{N_1 c_1^2 + N_2 c_2^2 + N_3 c_3^2 + \dots}{N_1 + N_2 + N_3 + \dots}$$

$$= \frac{1}{N}\int_0^\infty c^2 dN$$

$$= \int_0^\infty c^2 f(c)\, dc$$

$$= \int_0^\infty c^2 . \, 4\pi a^3 e^{-\frac{\beta}{2}c^2} c^2 dc$$

$$= 4\pi a^3 \int_0^\infty e^{-\frac{\beta}{2}c^2} c^4 dc$$

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$$\int_0^\infty e^{-sx^2} x^4 dx = \frac{3}{8}\sqrt{\frac{\pi}{s^5}} \qquad \int_0^\infty e^{-sx^2} x^2 dx = \frac{1}{4}\sqrt{\frac{\pi}{s^3}}$$

$$= 4\pi a^3 \frac{3}{8}\sqrt{\frac{\pi}{(\beta/2)^5}}$$

$$\langle c^2\rangle = 4\pi \left(\sqrt{\frac{\beta}{2\pi}}\right)^3 \frac{3}{8}\sqrt{\frac{\pi}{(\beta/2)^5}} = \frac{\beta}{3}$$

We know,

$$\frac{1}{2} m\langle c^2\rangle = \frac{3}{2} kT$$

$$\Rightarrow \frac{1}{2} m\left(\frac{\beta}{3}\right) = \frac{3}{2} kT$$

$$\Rightarrow \beta = \frac{m}{kT}$$

$$\Rightarrow a = \sqrt{\frac{\beta}{2\pi}} = \sqrt{\frac{m}{2\pi kT}}$$

$$f(c)\,dc = 4\pi a^3 e^{-bc^2} c^2 dc$$

$$\boxed{f(c) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} c^2 e^{-\frac{mc^2}{2kT}}}$$

Average velocity / mean velocity

$$\text{mean} = \frac{N_1c_1 + N_2c_2 + \dots}{N_1 + N_2 + \dots}$$

$$= \frac{1}{N}\int_0^\infty c\,dN$$

$$\int_0^\infty e^{-ax^2} x^3 dx = \frac{1}{2a^2}$$

$$= \int_0^\infty e^{-\frac{mc^2}{2kT}} c^3 dc$$

$$\frac{1}{2\left(\frac{m}{2kT}\right)^2}$$

$$dN = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} c^2 e^{-\frac{mc^2}{2kT}} dc$$

$$= \frac{1}{N}\int_0^\infty 4\pi N \left(\frac{m}{2\pi kT}\right)^3 c^3 e^{-\frac{mc^2}{2kT}}$$

$$= 4\pi\left(\frac{m}{2\pi kT}\right)^3 \int_0^\infty e^{-\frac{mc^2}{2kT}} c^3 dc$$

$$= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} \frac{1}{2\left(\frac{m}{2kT}\right)^2}$$

$$= 2\times \frac{m^{3/2} \cdot \pi\, 4 \cdot k^2T^2}{2^{3/2} \cdot \pi^{3/2} \cdot k^{3/2} T^{3/2}}$$

$$= \cancel{\frac{\pi \cdot 8\sqrt{2}\sqrt{2} \cdot x}{x\sqrt{2}}} = \sqrt{\frac{8kT}{m\pi}}$$

Root mean square velocity

$$c^2_{rms} = \frac{N_1c_1^2 + N_2c_2^2 + N_3c_3^2 + \dots}{N_1 + N_2 + N_3 + \dots}$$

$$c^2_{rms} = \frac{1}{N}\int_0^\infty c^2 dN$$

$$= \frac{1}{N}\int_0^\infty 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} c^2 e^{-\frac{mc^2}{2kT}} c^2 dc$$

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$=$

$\vdots$

$$c_{rms} = \sqrt{\frac{3kT}{m}}$$

$\therefore$ Most Probable velocity, $\frac{dN}{dc} = F$

$$\Rightarrow F = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} c^2 e^{-\frac{mc^2}{2kT}}$$

$$\Rightarrow \frac{dF}{dc} = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} \left\{ c^2 e^{-\frac{mc^2}{2kT}} \left(-\frac{2m}{2kT}\right) c + e^{-\frac{mc^2}{2kT}} 2c \right\}$$

$$\Rightarrow 0 = c^3 e^{-\frac{mc^2}{2kT}} \left(-\frac{m}{kT}\right) + e^{-\frac{mc^2}{2kT}} 2c$$

$$\Rightarrow c \left\{ c^2 \left(-\frac{m}{kT}\right) + 2 \right\} = 0$$

$c \neq 0$, $c_m = \sqrt{\frac{2kT}{m}}$

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