Semi-definite relaxations for Optimal Power Flow

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Abstract

Problem. Given a task of finding an optimal solution for the Optimal Power Flow (OPF) problem. We have methods to solve it (AC-OPF, DC-OPF), which give local optima and depend on a starter point. Our team wants to try and reformulate this problem as convex one using SDP-relaxation.

Results. We managed to:

- ☑ Learn SDP with solving the problem at 3-bus case
- ☑ Compare result with Newton-Rhapson methods

OPF

Optimal power flow is an optimization problem widely employed in analysis and operation of power systems. It is based on power flow equations: the relationship between voltage input and power injections at buses in an electric power system; the OPF problem itself is stated as minimization of operating cost for an electric power system with network-related and engineering constraints.

classical OPF problem formulation

$$\min_{P_G,Q_G,S,V_d,V_q} \sum_{k\in\mathbb{G}} f_k(P_{G\,k})$$

subject to:

$P_{G_k}^{min} \leq P_{G_k} \leq P_{G_k}^{max}, \forall k \in \mathbb{G}$

$Q_{G_k}^{min} \leq Q_{G_k} \leq Q_{G_k}^{max}, \forall k \in \mathbb{G}$

$(V_k^{min})^2 \leq {V_d}_k^2 + {V_q}_k^2 \leq (V_k^{max})^2, \forall k \in \mathbb{N}$

$\|S_{lm} \leq S_{lm}^{max}\|, \forall (l,m) \in \mathbb{L}$

$P_{G\,k} - P_{D\,k} =$

$V_{dk} \sum_{i=1}^{n} (G_{k\,i} V_{d\,i} - B_{k\,i} V_{q\,i}) + V_{qk} \sum_{i=1}^{n} (B_{k\,i} V_{d\,i} + G_{k\,i} V_{q\,i})$

$Q_{G\,k} - Q_{D\,k} =$

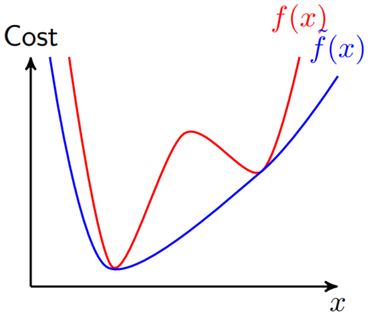
$V_{dk} \sum_{i=1}^{n} (-B_{k\,i} V_{d\,i} - G_{k\,i} V_{q\,i}) + V_{qk} \sum_{i=1}^{n} (G_{k\,i} V_{d\,i} - B_{k\,i} V_{q\,i})$

$\forall k \in \mathbb{N}$

SDP

Convex relaxation transforms OPF to convex Semi-Definite Program (SDP).

Under certain conditions, the obtained solution is the global optimum to the original OPF problem.



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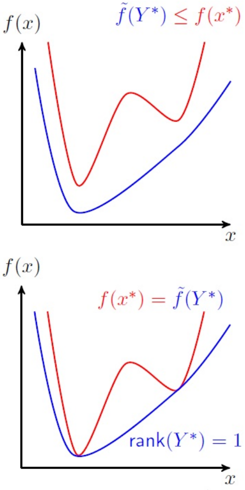
SDP

EXACT: $W = VV^T$

RELAX: $W0$

For the objective functions EXACT RELAX

If W^* happens with $rank(W) = 1$ then EXACT $=$ RELAX



OPF problem formulation in SDP

$$\min_{W} \sum_{k \in \mathbb{G}} \alpha_k$$

$\alpha_k = c_{k,2} f^2(W) + c_{k,1} f(W) + c_{k,0}$, $f(W) = tr(Z_k W) - P_{Dk}$

subject to:

$\mathrm{P}_{G_k}^{min} - P_{Dk} \leq tr(Z_k W) \leq P_{G_k}^{max} - P_{Dk}, \forall k \in \mathbb{G}$

$\mathrm{Q}_{G_k}^{min} - Q_{Dk} \leq tr(\overline{Z_k} W) \leq Q_{G_k}^{max} - Q_{Dk}, \forall k \in \mathbb{G}$

$(\mathrm{V}_k^{min})^2 \leq tr(M_k W) \leq (V_k^{max})^2, \forall k \in \mathbb{N}$

$$\begin{bmatrix} -(S_{lm}^{max})^2 & tr(Z_{lm} W) & tr(\overline{Z_{lm}} W) \\ tr(Z_{lm} W) & -1 & 0 \\ tr(\overline{Z_{lm}}) W & 0 & -1 \end{bmatrix} \succeq 0, \forall (l, m) \in \mathbb{L}$$

$\mathrm{W} \succeq 0$

Expectations and delivery

☑ Reformulate OPF problem as convex using SDP

☑ Implement SDP for OPF and apply it to artificial case

Results: OPF-SDP solved

We compared our solution with another result obtained via the use of the Newton-Raphson method, namely, $+5707.11$.

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Status: Solved
Optimal value (cvx_optval): +5.80543
```

Challenges

These are the challenges we've faced during this work:

```
rhs: 90.0
vexity: Convex.AffineVexity()
                Constraint:
<= constraint
lhs: AbstractExpr with
head: sum
size: (1, 1)
sign: Convex.NoSign()
vexity: Convex.AffineVexity()

rhs: -95.0
vexity: Convex.AffineVexity()
current status: not yet solved
```

```
WARNING: Problem status Infeasible;
solution may be inaccurate.

6×6 Array{Float64,2}:
 NaN  NaN  NaN  NaN  NaN  NaN
 NaN  NaN  NaN  NaN  NaN  NaN
 NaN  NaN  NaN  NaN  NaN  NaN
 NaN  NaN  NaN  NaN  NaN  NaN
 NaN  NaN  NaN  NaN  NaN  NaN
 NaN  NaN  NaN  NaN  NaN  NaN
```

Thank you for your attention!

References

- Daniel K. Molzahn, Bernard C. Lesieutre, Christopher L. DeMarco, *Investigation of Non-Zero Duality Gap Solutions to a Semidefinite Relaxation of the Optimal Power Flow Problem*
- A. Castillo, R. O'Neill, *Survey of Approaches to Solving the ACOPF (OPF Paper 4)*. [link]
- R. Zimmerman, C. Murillo-Sanchez, R. Thomas, *MATPOWER: Steady-State Operations, Planning, and Analysis Tools for Power Systems Research and Education*. [link]
- W. Tinney and C. Hart, *Power Flow Solution by Newton's Method*. [link]
- A. Trias, *The Holomorphic Embedding Load Flow Method*. [link]