Midterm Exam (Take-Home)

12/24-703 Numerical Methods in Engineering

1. Solve the following problems from the textbook:

   A. 6.5.14 (Hint: consult the proof of accuracy for the Simplified RK scheme that was done in class, or use the Telescopic Identity as suggested).

   B. 6.5.15

   C. 6.5.16 - stability here refers to showing whether or not the solution magnitude grows in time.

   D. Suppose the pde $u_t = -u_x$, $-\infty < x < \infty$, was approximated by the semi-discrete scheme mentioned in C) above. Is the pointwise magnitude of the exact solution to this problem for the initial condition e^{ikx} bounded by a constant for all times? Can the solution of the discrete problem for the initial condition e^{ikx} be similarly bounded for all times? Why or why not?

   E. 6.5.19

Note: For all the following problems, please include appropriate plots of your solutions to communicate your results clearly.

2. Make an 'upwind' modification to the difference approximation in 1.B above and code it up to generate approximate solutions to the following problem

$$u_t = -u_x, \quad -1 < x < +1, \quad 0 \le t \le 2$$

$$u(x,0) = \exp\left[-\left(\frac{x}{0.2}\right)^2\right] - \exp\left[-\left(\frac{1}{0.2}\right)^2\right] \quad \text{Initial condition}$$

$$u(-1,t) = 0 \quad \text{Boundary condition.}$$

3. Use the Lax-Wendroff scheme to solve the problem posed in 2. above. Are you able to obtain comparable accuracy for larger time steps (while maintaining stability)?

4. Code the scheme suggested in problem 1.E above for $\theta = 0$ to generate approximate solutions to the problem

$$u_t = u_{xx}, \quad -1 < x < +1, \quad 0 \le t \le 2$$

$$u(x,0) = \exp\left[-\left(\frac{x}{0.2}\right)^2\right] - \exp\left[-\left(\frac{1}{0.2}\right)^2\right] \quad \text{Initial condition}$$

$$u(-1,t) = 0 = u(+1,t) \quad \text{Boundary condition.}$$

5. Code up Friedrichs' scheme to solve the problem

$$u_{tt} = u_{xx}, \qquad -1 < x < +1, \quad 0 \le t \le 2$$

$$\left.\begin{aligned} u(x,0) &= \exp\left[-\left(\frac{x}{0.2}\right)^2\right] - \exp\left[-\left(\frac{1}{0.2}\right)^2\right] \\ u_t(x,0) &= 0 \end{aligned}\right\} \quad \text{Initial conditions}$$

$$u(-1,t) = 0 = u(+1,t) \quad \text{Boundary condition.}$$