1

AI1103-Assignment 4

Name: Asish sashank reddy, Roll Number: CS20BTECH11010

Download all python codes from

https://github.com/asishcs2011010/demo/blob/main/assignment-4/assignment-4.py

and latex-tikz codes from

https://github.com/asishcs2011010/demo/blob/main/assignment-4/assignment-4(4).tex

QUESTION NO

gov/stats/2015/statistics-I(1), Q.1(C)

QUESTION

1)(c) Let X have pdf

$$f(x) = \begin{cases} \frac{1}{3} & -1 \le x < 2\\ 0 & otherwise \end{cases}$$

Obtain the cdf of $Y=X^2$

Solution

CDF of X is defined as,

$$F_X(x) = \Pr(X \le x) \tag{0.0.1}$$

The cdf of $Y = X^2$ is given by G(y)

$$G\left(y\right) = F_X\left(y\right) = \Pr(X^2 \le y) = \Pr(X \le \sqrt{y}) = F_X\left(\sqrt{y}\right)$$

(0.0.2)

$$G(y) = \int_{\max(-1, -\sqrt{y})}^{\min(2, \sqrt{y})} \frac{1}{3} dx$$
(0.0.3)

As bounds of integral should also remain within the support of the random variable X.

so when $0 \le y < 1$, we have $\max(-1, -\sqrt{y}) = -\sqrt{y}$, and when $1 \le y$ it is -1. similarly the upper limit is \sqrt{y} when y < 9 and 3 otherwise

$$G(y) = \begin{cases} 0 & y < 0 \\ \frac{2\sqrt{y}}{3} & 0 \le y < 1 \\ \frac{\sqrt{y}+1}{3} & 1 \le y < 4 \\ 1 & y \ge 4 \end{cases}$$

The plot of CDF is given in the Figure 0

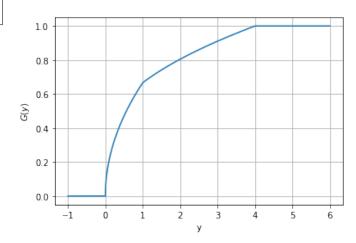


Fig. 0: CDF of Y