AI1103-Assignment 4

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Download all python codes from

```
https://github.com/asishcs2011010/demo/blob/main/
    assignment−4/codes
```

and latex-tikz codes from

```
https://github.com/asishcs2011010/demo/blob/main/
    assignment−4/assignment−4(8).tex
```

Question No

gov/stats/2015/statistics-I(1), Q.1(C)

Question

1)(c) Let X have pdf

$$\text{f(x)=}\begin{cases} \frac{1}{3} & -1 \le x < 2 \\ 0 & otherwise \end{cases}$$

Obtain the cdf of $Y=X^2$

Solution

CDF of X is defined as,

$$F_X(x) = \Pr(X \le x) \tag{0.0.1}$$

The cdf of X is given by $G(x)=F_X(x)$

$$F_X(x) = Pr(X \le x) = G(x) \tag{0.0.2}$$

$$G(x) = \int_{-\infty}^{x} \frac{1}{3} dx \tag{0.0.3}$$

$$x < -1 \to G(x) = 0$$

$$-1 \le x < 2 \to G(x) = \int_{-1}^{x} \frac{1}{3} dx$$

$$x > 2 \to G(x) = \int_{-\infty}^{-1} \frac{1}{3} dx + \int_{-1}^{2} \frac{1}{3} dx + \int_{2}^{y} \frac{1}{3} dx$$

$$G(x) = \begin{cases} 0 & x < -1 \\ \frac{x+1}{3} & -1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

The cdf of $Y = X^2$ is given by $G(y)=F_X(y)$

$$F_X(y) = Pr(X^2 \le y) = Pr(-\sqrt{y} \le X \le \sqrt{y}) \tag{0.0.4}$$

$$= F_X(\sqrt{y}) - F_X(-\sqrt{y}) \tag{0.0.5}$$

$$y < 0 \to F_X(y) = 0$$

$$\begin{aligned} 0 \le y < 1 \to F_X(y) &= F_X(\sqrt{y}) - F_X(-\sqrt{y}) \\ &= \left(\frac{\sqrt{y}+1}{3}\right) - \left(\frac{-\sqrt{y}+1}{3}\right) \\ &= \left(\frac{2\sqrt{y}}{3}\right) \end{aligned}$$

$$1 \le y < 4 \to F_X(y) = \left(\frac{\sqrt{y}+1}{3}\right) - 0 = \left(\frac{\sqrt{y}+1}{3}\right)$$

$$y \ge 4 \to F_X(y) = 1 - 0 = 1$$

The cdf of Y is

$$\text{G(y)} = \begin{cases} 0 & y < 0 \\ \frac{2\sqrt{y}}{3} & 0 \le y < 1 \\ \frac{\sqrt{y}+1}{3} & 1 \le y < 4 \\ 1 & y \ge 4 \end{cases}$$

The plot of CDF of X is given in the Figure 0

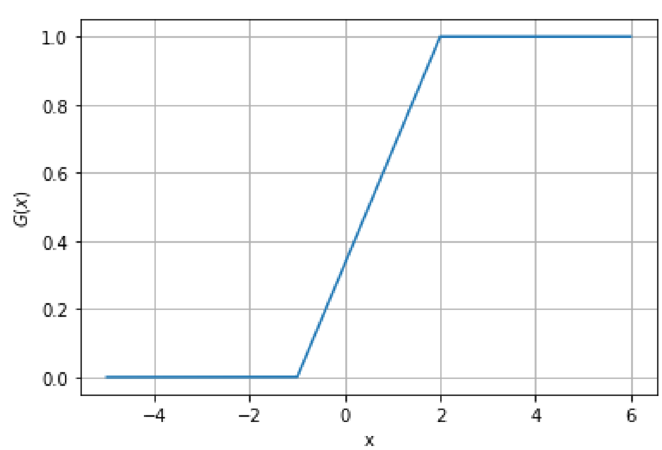

Fig. 0: CDF of X

The plot of CDF of Y is given in the Figure 0

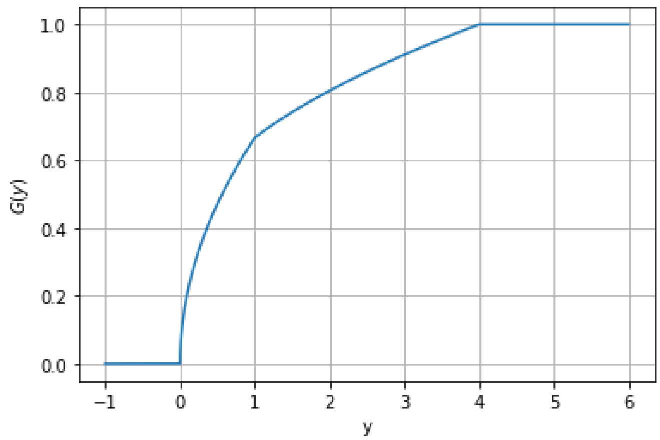

Fig. 0: CDF of Y