AI1103-Assignment 2

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Download latex-tikz codes from

https://github.com/asishcs2011010/demo/blob/main/Assignment−2/assignment−2(7).tex

Question No

Gate-EC Q-38

Question

Let $X \in \{0,1\}$ and $Y \in \{0,1\}$ be two independent binary random variables. if P(X = 0) = p and P($Y = 0$) = q, then P($X + Y \geqslant 1$) is equal to

1) $pq + (1-p)(1-q)$
2) pq
3) $p(1-q)$
4) $1-pq$

Solution

Given X,Y$\in$ $\{0,1\}$ be two independent random variables. The probability mass function (pmf) is expressed as

$$p_X(n) = \Pr(X = n) = \begin{cases} p & n = 0 \\ 1-p & n = 1 \end{cases} \tag{0.0.1}$$

$$p_Y(n) = \Pr(Y = n) = \begin{cases} q & n = 0 \\ 1-q & n = 1 \end{cases} \tag{0.0.2}$$

Let Z be the convolution of X,Y.

$$Z = X + Y, \tag{0.0.3}$$

Characteristic function of random variable is defined as

$$\phi_X(\omega) = E[e^{iwX}] \tag{0.0.4}$$

For discrete random variable X

$$\phi_X(\omega) = \sum_k e^{iwk} P_X(k) \tag{0.0.5}$$

$$\phi_X(\omega) = p + (1-p)e^{iw} \tag{0.0.6}$$

similarly For discrete random variable Y

$$\phi_Y(\omega) = \sum_k e^{iwk} P_Y(k) \tag{0.0.7}$$

$$\phi_Y(\omega) = q + (1-q)e^{iw} \tag{0.0.8}$$

if X and Y are independent random variable, then

$$\phi_{X+Y}(\omega) = E[e^{iw(X+Y)}] \tag{0.0.9}$$
$$= E[e^{iwX}.e^{iwY}] \tag{0.0.10}$$
$$= \phi_X(\omega) \times \phi_Y(\omega) \tag{0.0.11}$$

$$\phi_Z(\omega) = (p + (1-p)e^{iw}).(q + (1-q)e^{iw}) \tag{0.0.12}$$

$$\phi_Z(\omega) = pq + (p + q - 2pq)e^{iw} + (1-p)(1-q)e^{2iw} \tag{0.0.13}$$

From equation (0.0.5), we get
The pmf of Z is

$$p_Z(n) = \Pr(Z = n) = \begin{cases} pq & n = 0 \\ p + q - 2pq & n = 1 \\ (1-p)(1-q) & n = 2 \end{cases} \tag{0.0.14}$$

$$\Pr(Z < 1) = \mathrm{P}(Z = 0) \tag{0.0.15}$$

From equation (0.0.15), we get

$$\Pr(X + Y \geq 1) = 1 - \Pr(X + Y < 1) \tag{0.0.16}$$
$$\Pr(Z \geq 1) = 1 - \Pr(Z < 1) = 1 - pq \tag{0.0.17}$$