AI1103-Assignment 4

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Download all python codes from

https://github.com/asishcs2011010/demo/blob/main/ assignment−4/codes

and latex-tikz codes from

https://github.com/asishcs2011010/demo/blob/main/ assignment−4/assignment−4(11).tex

QUESTION NO

gov/stats/2015/statistics-I(1), Q.1(C)

QUESTION

1)(c) Let X have pdf

$$
f(x)=
\begin{cases}
\frac{1}{3} & -1 \le x < 2 \\
0 & otherwise
\end{cases}
$$

Obtain the cdf of $Y=X^2$

SOLUTION

Lemma 0.1. *The cdf of X is defined as,*

$$
F_X(x) = \Pr(X \le x) \tag{0.0.1}
$$

$$
\Pr(X \le x) = \int_{-\infty}^{x} f(x)dx \tag{0.0.2}
$$

$$
F_X(x) = \int_{-\infty}^{x} \frac{1}{3}dx \tag{0.0.3}
$$

$$
F_X(x)=
\begin{cases}
0 & x < -1 \\
\int_{-1}^{x} \frac{1}{3}dx & -1 \le x < 2 \\
\int_{-1}^{2} \frac{1}{3}dx & x \ge 2
\end{cases}
$$

$$
F_X(x) =
\begin{cases}
0 & x < -1 \\
\frac{x+1}{3} & -1 \le x < 2 \\
1 & x \ge 2
\end{cases}
$$

Lemma 0.2. *The cdf of $Y = X^2$ is given by $F_Y(y)$*

$$
F_Y(y) = Pr(X^2 \le y) = Pr(-\sqrt{y} \le X \le \sqrt{y}) \tag{0.0.4}
$$

$$
= F_X(\sqrt{y}) - F_X(-\sqrt{y}) \tag{0.0.5}
$$

$$
F_Y(y) =
\begin{cases}
0 & y < 0 \\
\left(\frac{\sqrt{y}+1}{3}\right) - \left(\frac{-\sqrt{y}+1}{3}\right) & 0 \le y < 1 \\
\left(\frac{\sqrt{y}+1}{3}\right) - 0 & 1 \le y < 4 \\
1 - 0 & y \ge 4
\end{cases}
$$

The cdf of Y is

$$
F_Y(y) =
\begin{cases}
0 & y < 0 \\
\frac{2\sqrt{y}}{3} & 0 \le y < 1 \\
\frac{\sqrt{y}+1}{3} & 1 \le y < 4 \\
1 & y \ge 4
\end{cases}
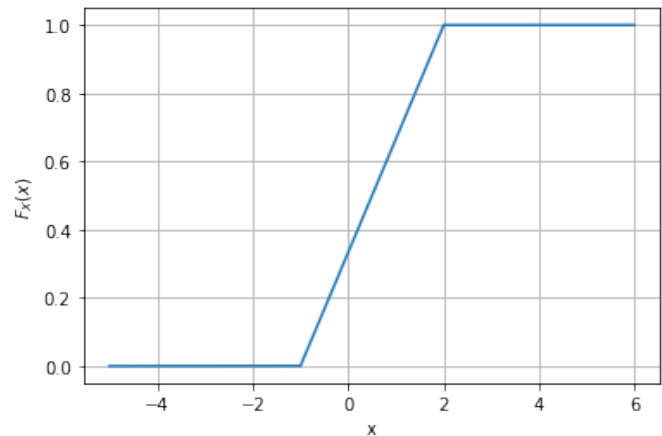$$

The plot of CDF of X is given in the Figure 0

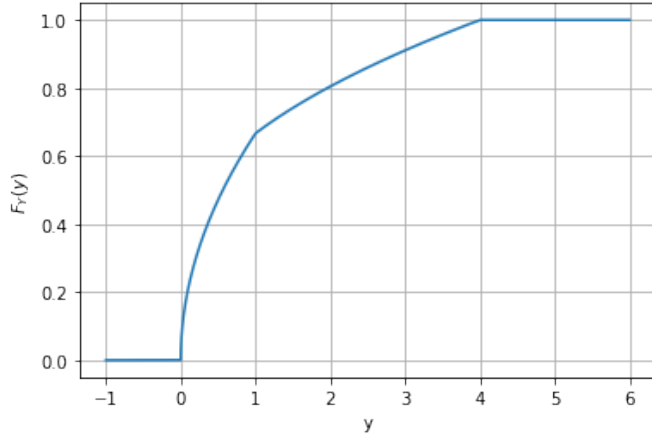Fig. 0: CDF of X

The plot of CDF of Y is given in the Figure 0

Fig. 0: CDF of Y