AI1103-Assignment 4

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Download all python codes from

https://github.com/asishcs2011010/demo/blob/main/assignment−4/assignment−4.py

and latex-tikz codes from

https://github.com/asishcs2011010/demo/blob/main/assignment−4/assignment−4(3).tex

QUESTION NO

gov/stats/2015/statistics-I(1), Q.1(C)

QUESTION

1)(c) Let X have pdf

$$\text{f(x)=}\begin{cases}\frac{1}{3} & -1 \le x < 2\\ 0 & otherwise\end{cases}$$

SOLUTION

CDF of X is defined as,

$$F_X(x) = \Pr(X \le x) \tag{0.0.1}$$

The cdf of Y = X^2 is given by G(y)

$$G(y) = F_X(y) = \Pr(X^2 \le y) = \Pr(X \le \sqrt{y}) = F_X(\sqrt{y}) \tag{0.0.2}$$

$$G(y) = \int_{max(-1,-\sqrt{y})}^{min(2,\sqrt{y})} \frac{1}{3}dx \tag{0.0.3}$$

As bounds of integral should also remain within the support of the random variable X.

so when $0 \le y < 1$, we have
$\max(-1, -\sqrt{y}) = -\sqrt{y}$, and when $1 \le y$ it is -1. similarly the upper limit is $\sqrt{y}$ when $y < 9$ and 3 otherwise

$$\text{G(y)} = \begin{cases}0 & y < 0\\ \frac{2\sqrt{y}}{3} & 0 \le y < 1\\ \frac{\sqrt{y}+1}{3} & 1 \le y < 4\\ 1 & y \ge 4\end{cases}$$

The plot of CDF is given in the Figure 0

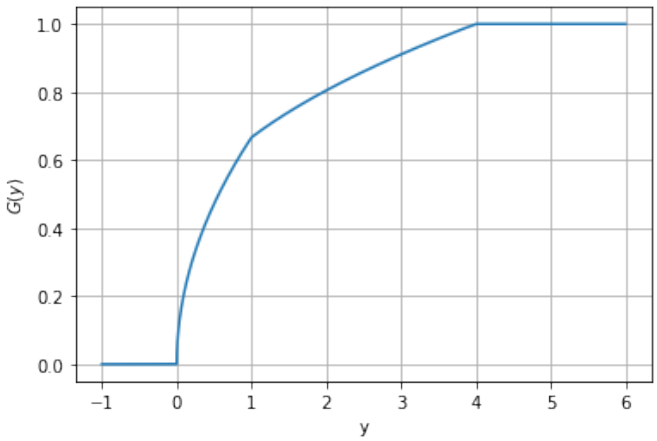

Fig. 0: CDF of Y