AI1103-Assignment 2

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QUESTION NO

Gate-EC Q-38

QUESTION

Let $X \in \{0, 1\}$ and $Y \in \{0, 1\}$ be two independent binary random variables. if $P(X = 0) = p$ and $P(Y = 0) = q$, then $P(X + Y \geqslant 1)$ is equal to

1) $pq + (1 - p)(1 - q)$

2) pq

3) $p(1 - q)$

4) $1 - pq$

SOLUTION

Given $X,Y \in \{0, 1\}$ be two independent random variables. The probability mass function (pmf) is expressed as

$$p_X(n) = \Pr(X = n) = \begin{cases} p & n = 0 \\ 1 - p & n = 1 \end{cases} \quad (0.0.1)$$

$$p_Y(n) = \Pr(Y = n) = \begin{cases} q & n = 0 \\ 1 - q & n = 1 \end{cases} \quad (0.0.2)$$

Let Z be the convolution of X,Y.

$$Z = X + Y, \quad (0.0.3)$$

Characteristic function of random variable is defined as

$$\phi_X(\omega) = E[e^{iwX}] \quad (0.0.4)$$

For discrete random variable X

$$\phi_X(\omega) = \sum_k e^{iwk} P_X(k) \quad (0.0.5)$$

$$\phi_X(\omega) = p + (1 - p)e^{iw} \quad (0.0.6)$$

similarly For discrete random variable Y

$$\phi_Y(\omega) = \sum_k e^{iwk} P_Y(k) \quad (0.0.7)$$

$$\phi_Y(\omega) = q + (1 - q)e^{iw} \quad (0.0.8)$$

if X and Y are independent random variable, then

$$phi_{X+Y}(\omega) = E[e^{iw(X+Y)}] \quad (0.0.9)$$

$$= E[e^{iwX} . e^{iwY}] \quad (0.0.10)$$

$$= \phi_X(\omega) \times \phi_Y(\omega) \quad (0.0.11)$$

$$phi_Z(\omega) = (p + (1 - p)e^{iw}).(q + (1 - q)e^{iw}) \quad (0.0.12)$$

$$phi_Z(\omega) = pq + (p + q - 2pq)e^{iw} + (1 - p)(1 - q)e^{2iw} \quad (0.0.13)$$

From equation (0.0.5), we get
The pmf of Z is

$$p_Z(n) = \Pr(Z = n) = \begin{cases} pq & n = 0 \\ p + q - 2pq & n = 1 \\ (1 - p)(1 - q) & n = 2 \end{cases} \quad (0.0.14)$$

$$\Pr(Z < 1) = P(Z = 0) \quad (0.0.15)$$

From equation (0.0.15), we get

$$\Pr(X + Y \geq 1) = 1 - \Pr(X + Y < 1) \quad (0.0.16)$$

$$\Pr(Z \geq 1) = 1 - \Pr(Z < 1) = 1 - pq \quad (0.0.17)$$