AI1103-Assignment 4

Name: Asish sashank reddy, Roll Number: CS20BTECH11010

Download all python codes from

https://github.com/asishcs2011010/demo/blob/main/assignment−4/codes

and latex-tikz codes from

https://github.com/asishcs2011010/demo/blob/main/assignment−4/assignment−4(10).tex

QUESTION NO

gov/stats/2015/statistics-I(1), Q.1(C)

QUESTION

1)(c) Let X have pdf

$$\text{f(x)}= \begin{cases} \frac{1}{3} & -1 \le x < 2 \\ 0 & otherwise \end{cases}$$

Obtain the cdf of $Y=X^2$

SOLUTION

Lemma 0.1. *the cdf of X is defined as,*

$$F_X(x) = \Pr(X \le x) \tag{0.0.1}$$

$$\Pr(X \le x) = \int_{-\infty}^{x} f(x)dx \tag{0.0.2}$$

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{3}dx \tag{0.0.3}$$

$$\text{F}_X(x)= \begin{cases} 0 & x < -1 \\ \int_{-1}^{x} \frac{1}{3}dx & -1 \le x < 2 \\ \int_{-1}^{2} \frac{1}{3}dx & x \ge 2 \end{cases}$$

$$\text{F}_X(x) = \begin{cases} 0 & x < -1 \\ \frac{x+1}{3} & -1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

Lemma 0.2. *The cdf of $Y = X^2$ is given by $F_Y(y)$*

$$F_Y(y) = Pr(X^2 \le y) = Pr(-\sqrt{y} \le X \le \sqrt{y}) \tag{0.0.4}$$

$$= F_X(\sqrt{y}) - F_X(-\sqrt{y}) \tag{0.0.5}$$

$$y < 0 \rightarrow \text{F}_Y(y) = 0 \tag{0.0.6}$$

$$0 \le y < 1 \rightarrow \text{F}_Y(y) = \text{F}_X(\sqrt{y}) - \text{F}_X(-\sqrt{y}) \tag{0.0.7}$$

$$= \left(\frac{\sqrt{y}+1}{3}\right) - \left(\frac{-\sqrt{y}+1}{3}\right) \tag{0.0.8}$$

$$= \left(\frac{2\sqrt{y}}{3}\right) \tag{0.0.9}$$

$$1 \le y < 4 \rightarrow \text{F}_Y(y) = \left(\frac{\sqrt{y}+1}{3}\right) - 0 = \left(\frac{\sqrt{y}+1}{3}\right) \tag{0.0.10}$$

$$y \ge 4 \rightarrow \text{F}_Y(y) = 1 - 0 = 1 \tag{0.0.11}$$

The cdf of Y is

$$\text{F}_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{2\sqrt{y}}{3} & 0 \le y < 1 \\ \frac{\sqrt{y}+1}{3} & 1 \le y < 4 \\ 1 & y \ge 4 \end{cases}$$

The plot of CDF of X is given in the Figure 0

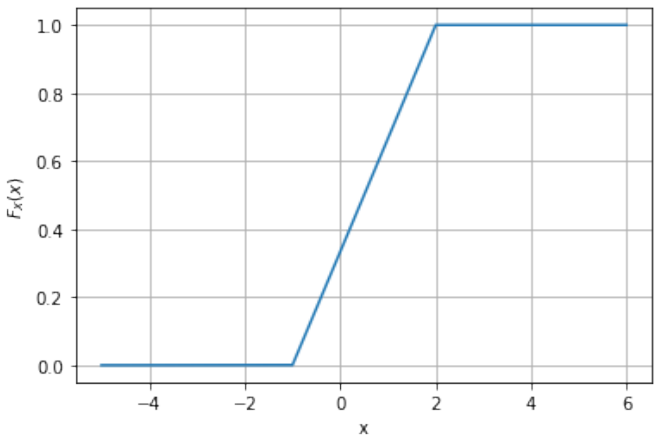

Fig. 0: CDF of X

The plot of CDF of Y is given in the Figure 0

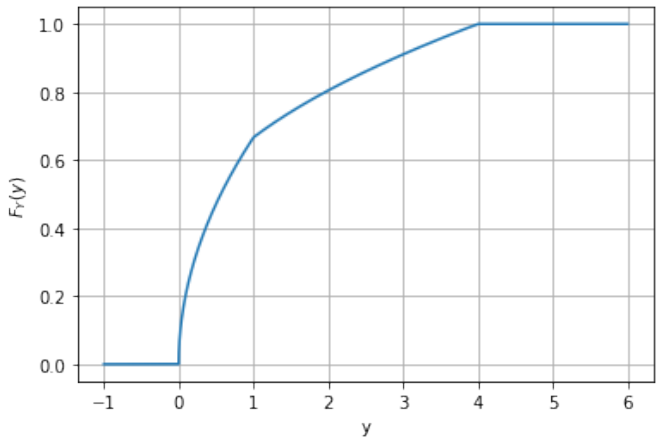

Fig. 0: CDF of Y