AI1103-Assignment 4

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Download all python codes from

https://github.com/asishcs2011010/demo/blob/main/assignment−4/codes

and latex-tikz codes from

https://github.com/asishcs2011010/demo/blob/main/assignment−4/assignment−4(12).tex

QUESTION NO

gov/stats/2015/statistics-I(1), Q.1(C)

QUESTION

1)(c) Let X have pdf

$$\text{f(x)}= \begin{cases} \frac{1}{3} & -1 \le x < 2 \\ 0 & otherwise \end{cases}$$

Obtain the cdf of $Y=X^2$

SOLUTION

Lemma 0.1. $F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{x+1}{3} & -1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$

Proof. The cdf of X is defined as,

$$\mathrm{F}_X(x) = \Pr(X \le x) \tag{0.0.1}$$

$$\Pr(X \le x) = \int_{-\infty}^{x} \mathrm{f}(x)dx \tag{0.0.2}$$

$$\mathrm{F}_X(x) = \int_{-\infty}^{x} \frac{1}{3}dx \tag{0.0.3}$$

$$\mathrm{F}_X(x)= \begin{cases} 0 & x < -1 \\ \int_{-1}^{x} \frac{1}{3}dx & -1 \le x < 2 \\ \int_{-1}^{2} \frac{1}{3}dx & x \ge 2 \end{cases}$$

□

Lemma 0.2. *The cdf of Y is*

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{2\sqrt{y}}{3} & 0 \le y < 1 \\ \frac{\sqrt{y}+1}{3} & 1 \le y < 4 \\ 1 & y \ge 4 \end{cases}$$

Proof. The cdf of $Y = X^2$ is given by $\mathrm{F}_Y(y)$

$$\mathrm{F}_Y(y) = \Pr(X^2 \le y) = \Pr(-\sqrt{y} \le X \le \sqrt{y}) \tag{0.0.4}$$

$$= \mathrm{F}_X(\sqrt{y}) - \mathrm{F}_X(-\sqrt{y}) \tag{0.0.5}$$

$$\mathrm{F}_Y(y) = \begin{cases} 0 & y < 0 \\ \left(\frac{\sqrt{y}+1}{3}\right) - \left(\frac{-\sqrt{y}+1}{3}\right) & 0 \le y < 1 \\ \left(\frac{\sqrt{y}+1}{3}\right) - 0 & 1 \le y < 4 \\ 1 - 0 & y \ge 4 \end{cases}$$

□

The plot of CDF of X is given in the Figure 0

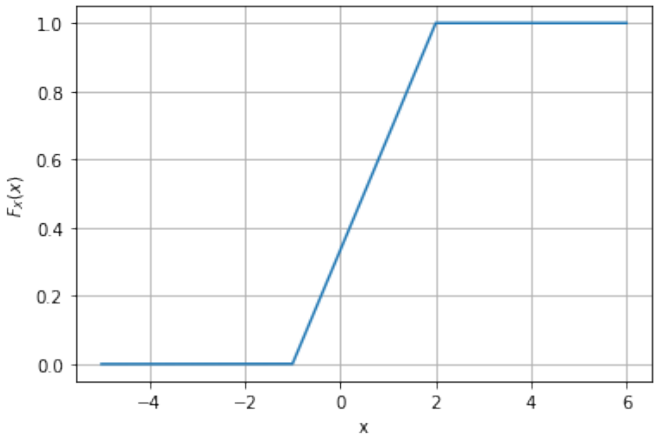

Fig. 0: CDF of X

The plot of CDF of Y is given in the Figure 0

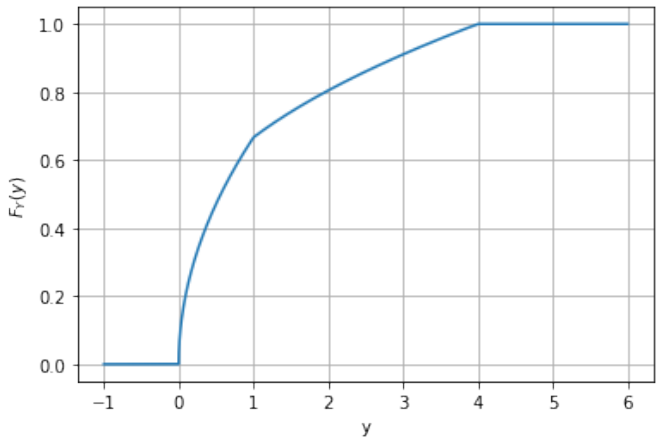

Fig. 0: CDF of Y