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AI1103-Assignment 4

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Download all python codes from

https://github.com/asishcs2011010/demo/blob/main/assignment-4/assignment-4.py

and latex-tikz codes from

https://github.com/asishcs2011010/demo/blob/main/assignment-4/assignment-4(7).tex

QUESTION NO

gov/stats/2015/statistics-I(1), Q.1(C)

QUESTION

1)(c) Let X have pdf

$$f(x) = \begin{cases} \frac{1}{3} & -1 \le x < 2\\ 0 & otherwise \end{cases}$$

Obtain the cdf of $Y=X^2$

Solution

CDF of X is defined as,

$$F_X(x) = \Pr(X \le x) \tag{0.0.1}$$

The cdf of $Y = X^2$ is given by $G(y)=F_X(y)$

$$F_X(y) = Pr(X^2 \le y) = Pr(-\sqrt{y} \le X \le \sqrt{y})$$
 (0.0.2)

$$G(y) = \int_{\max(-1, -\sqrt{y})}^{\min(2, \sqrt{y})} \frac{1}{3} dx$$
 (0.0.3)

As, $-1 \le -\sqrt{y} \le X \le \sqrt{y} \le 2$, we take limits as $\max(-1, -\sqrt{y})$ and $\min(2, \sqrt{y})$

if
$$y < 0$$
, $\max(-1, -\sqrt{y}) = \min(2, \sqrt{y}) = 0$
if $0 \le y < 1$, $\max(-1, -\sqrt{y}) = -\sqrt{y}$,
 $\min(2, \sqrt{y}) = \sqrt{y}$
if $1 \le y < 4$, $\max(-1, -\sqrt{y}) = -1$, $\min(2, \sqrt{y}) = \sqrt{y}$
if $y \ge 4$, $\max(-1, -\sqrt{y}) = -1$, $\min(2, \sqrt{y}) = 2$

$$G(y) = \begin{cases} 0 & y < 0 \\ \frac{2\sqrt{y}}{3} & 0 \le y < 1 \\ \frac{\sqrt{y}+1}{3} & 1 \le y < 4 \\ 1 & y \ge 4 \end{cases}$$

The plot of CDF is given in the Figure 0

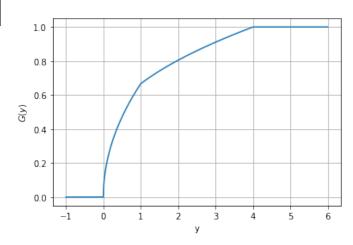


Fig. 0: CDF of Y