

AI1103-Assignment 4

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Download all python codes from

<https://github.com/asishcs2011010/demo/blob/main/assignment-4/codes>

and latex-tikz codes from

[https://github.com/asishcs2011010/demo/blob/main/assignment-4/assignment-4\(9\).tex](https://github.com/asishcs2011010/demo/blob/main/assignment-4/assignment-4(9).tex)

QUESTION NO

gov/stats/2015/statistics-I(1), Q.1(C)

QUESTION

1)(c) Let X have pdf

$$f(x) = \begin{cases} \frac{1}{3} & -1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Obtain the cdf of $Y = X^2$

SOLUTION

CDF of X is defined as,

$$F_X(x) = \Pr(X \leq x) \quad (0.0.1)$$

$$\Pr(X \leq x) = \int_{-\infty}^x f(x) dx \quad (0.0.2)$$

$$F_X(x) = \int_{-\infty}^x \frac{1}{3} dx \quad (0.0.3)$$

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \int_{-1}^x \frac{1}{3} dx & -1 \leq x < 2 \\ \int_{-1}^2 \frac{1}{3} dx & x \geq 2 \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{x+1}{3} & -1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

The cdf of $Y = X^2$ is given by $F_Y(y)$

$$F_Y(y) = \Pr(X^2 \leq y) = \Pr(-\sqrt{y} \leq X \leq \sqrt{y}) \quad (0.0.4)$$

$$= F_X(\sqrt{y}) - F_X(-\sqrt{y}) \quad (0.0.5)$$

$$y < 0 \rightarrow F_Y(y) = 0 \quad (0.0.6)$$

$$0 \leq y < 1 \rightarrow F_Y(y) = F_X(\sqrt{y}) - F_X(-\sqrt{y}) \quad (0.0.7)$$

$$= \left(\frac{\sqrt{y} + 1}{3} \right) - \left(\frac{-\sqrt{y} + 1}{3} \right) \quad (0.0.8)$$

$$= \left(\frac{2\sqrt{y}}{3} \right) \quad (0.0.9)$$

$$1 \leq y < 4 \rightarrow F_Y(y) = \left(\frac{\sqrt{y} + 1}{3} \right) - 0 = \left(\frac{\sqrt{y} + 1}{3} \right) \quad (0.0.10)$$

$$y \geq 4 \rightarrow F_Y(y) = 1 - 0 = 1 \quad (0.0.11)$$

The cdf of Y is

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{2\sqrt{y}}{3} & 0 \leq y < 1 \\ \frac{\sqrt{y} + 1}{3} & 1 \leq y < 4 \\ 1 & y \geq 4 \end{cases}$$

The plot of CDF of X is given in the Figure 0

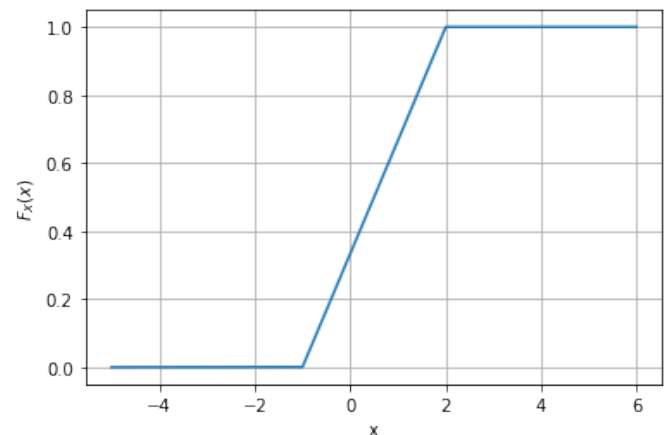


Fig. 0: CDF of X

The plot of CDF of Y is given in the Figure 0

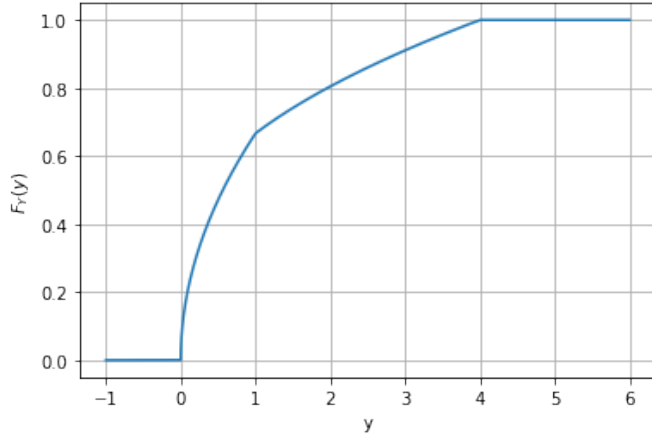


Fig. 0: CDF of Y