# AI1103-Assignment 2

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Download latex-tikz codes from

https://github.com/asishcs2011010/demo/blob/main/Assignment−2/assignment−2(4).tex

## Question No

Gate-EC Q-38

## Question

Let $X \in \{0,1\}$ and $Y \in \{0,1\}$ be two independent binary random variables. if P($X = 0$) = p and P($Y = 0$) = q, then P($X + Y \geqslant 1$) is equal to

1) $pq + (1-p)(1-q)$
2) $pq$
3) $p(1-q)$
4) $1 - pq$

## Solution

Given X,Y$\in$ $\{0,1\}$ be two independent random variables. The probability mass function (pmf) is expressed as

$$p_X(n) = \Pr(X = n) = \begin{cases} p & n = 0 \\ 1-p & n = 1 \end{cases} \tag{0.0.1}$$

$$p_Y(n) = \Pr(Y = n) = \begin{cases} q & n = 0 \\ 1-q & n = 1 \end{cases} \tag{0.0.2}$$

The Z-transform of $p_X(n)$ is defined as

$$P_X(z) = \sum_{n=-\infty}^{\infty} p_X(n) z^{-n}, \quad z \in \mathbb{C} \tag{0.0.3}$$

$$P_X(z) = p + (1-p)z \tag{0.0.4}$$

similarly,

$$P_Y(z) = q + (1-q)z \tag{0.0.5}$$

Let Z be the convolution of X,Y.

$$Z = X + Y, \tag{0.0.6}$$

The probability mass function of Z is

$$\Pr(Z = z) = \sum_k \Pr(X = K) \times \Pr(Y = z - k) \tag{0.0.7}$$

equation (0.0.7) can be written as following using convolution operation

$$\mathrm{P}_Z(z) = \mathrm{P}_X(z) \times \mathrm{P}_Y(z) \tag{0.0.8}$$

$$\mathrm{P}_Z(z) = (p + (1-p)z) \times (q + (1-q)z) \tag{0.0.9}$$

$$\mathrm{P}_Z(z) = pq + (p + q - 2pq)z + (1-p)(1-q)z^2 \tag{0.0.10}$$

The pmf of Z is

$$p_Z(n) = \Pr(Z = n) = \begin{cases} pq & n = 0 \\ p + q - 2pq & n = 1 \\ (1-p)(1-q) & n = 2 \end{cases} \tag{0.0.11}$$

$$\Pr(Z < 1) = \mathrm{P}(Z = 0) \tag{0.0.12}$$

From equation (0.0.12), we get

$$\Pr(X + Y \geq 1) = 1 - \Pr(X + Y < 1) \tag{0.0.13}$$

$$\Pr(Z \geq 1) = 1 - \Pr(Z < 1) = 1 - pq \tag{0.0.14}$$