

AI1103-Assignment 2

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Download latex-tikz codes from

[https://github.com/asishcs2011010/demo/blob/main/Assignment-2/assignment-2\(4\).tex](https://github.com/asishcs2011010/demo/blob/main/Assignment-2/assignment-2(4).tex)

similarly,

$$P_Y(z) = q + (1 - q)z \quad (0.0.5)$$

Let Z be the convolution of X,Y.

$$Z = X + Y, \quad (0.0.6)$$

QUESTION NO

Gate-EC Q-38

The probability mass function of Z is

QUESTION

Let $X \in \{0, 1\}$ and $Y \in \{0, 1\}$ be two independent binary random variables. if $P(X = 0) = p$ and $P(Y = 0) = q$, then $P(X + Y \geq 1)$ is equal to

$$Pr(Z = z) = \sum_k Pr(X = K) \times Pr(Y = z - k) \quad (0.0.7)$$

equation (0.0.7) can be written as following using convolution operation

$$P_Z(z) = P_X(z) \times P_Y(z) \quad (0.0.8)$$

1) $pq + (1 - p)(1 - q)$

2) pq

3) $p(1 - q)$

4) $1 - pq$

$$P_Z(z) = (p + (1 - p)z) \times (q + (1 - q)z) \quad (0.0.9)$$

$$P_Z(z) = pq + (p + q - 2pq)z + (1 - p)(1 - q)z^2 \quad (0.0.10)$$

The pmf of Z is

$$p_Z(n) = Pr(Z = n) = \begin{cases} pq & n = 0 \\ p + q - 2pq & n = 1 \\ (1 - p)(1 - q) & n = 2 \end{cases} \quad (0.0.11)$$

SOLUTION

Given $X, Y \in \{0, 1\}$ be two independent random variables. The probability mass function (pmf) is expressed as

$$p_X(n) = Pr(X = n) = \begin{cases} p & n = 0 \\ 1 - p & n = 1 \end{cases} \quad (0.0.1)$$

$$p_Y(n) = Pr(Y = n) = \begin{cases} q & n = 0 \\ 1 - q & n = 1 \end{cases} \quad (0.0.2)$$

The Z-transform of $p_X(n)$ is defined as

$$P_X(z) = \sum_{n=-\infty}^{\infty} p_X(n)z^{-n}, \quad z \in \mathbb{C} \quad (0.0.3)$$

$$P_X(z) = p + (1 - p)z \quad (0.0.4)$$

$$Pr(Z < 1) = P(Z = 0) \quad (0.0.12)$$

From equation (0.0.12), we get

$$Pr(X + Y \geq 1) = 1 - Pr(X + Y < 1) \quad (0.0.13)$$

$$Pr(Z \geq 1) = 1 - Pr(Z < 1) = 1 - pq \quad (0.0.14)$$