AI1103-Assignment 2

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QUESTION NO

Gate-EC Q-38

QUESTION

Let $X \in \{0, 1\}$ and $Y \in \{0, 1\}$ be two independent binary random variables. if P(X = 0) = p and P(Y = 0) = q, then P($X + Y \geqslant 1$) is equal to

1) $pq + (1 - p)(1 - q)$

2) pq

3) $p(1 - q)$

4) $1 - pq$

SOLUTION

Given Pr (X = 0) = p , Pr(Y=0) = q and X and Y are independent binary random variables.

X	$X = 0$	$X = 1$
Pr	p	$1 - p$

Y	$Y = 0$	$Y = 1$
Pr	q	$1 - q$

.
Let Z be the convolution of X,Y.

$$Z = X + Y, \qquad (0.0.1)$$

$$Pr(Z = z) = \sum_{k \in Z} Pr(X = k) \times Pr(Y = z - k) \quad (0.0.2)$$

$$Pr(Z = 0) = Pr(X = 0) \times Pr(Y = 0) = pq \quad (0.0.3)$$

Pr(Z = 1)= Pr(X = 0) ×Pr(Y = 1) + Pr(Z = 1) × Pr(Y = 0) = $p(1 - q) + q(1 - p) = p + q - 2pq$

Pr(Z=2) = Pr(X = 1) ×Pr(Y = 1) = $(1 - p)(1 - q) = 1 - p - q + pq$

Z	$Z = 0$	$Z = 1$	$Z = 2$
Pr	pq	$p + q - 2pq$	$1 - p - q + pq$

$$Pr(Z < 1) = P(Z = 0)(Z = 0, 1, 2) \qquad (0.0.4)$$

From equation (0.0.3), we get

$$Pr(Z = 0) = Pr(X = 0) \times Pr(Y = 0) = pq \quad (0.0.5)$$

$$Pr(X + Y \geq 1) = 1 - Pr(X + Y < 1) \quad (0.0.6)$$

$$Pr(Z \geq 1) = 1 - Pr(Z < 1) = 1 - pq \quad (0.0.7)$$

The correct option is option(4)