Financial Derivatives Pricing & Risk Analysis

Project Overview

This project demonstrates a full workflow that mirrors the responsibilities of a quantitative analyst or risk manager:

1. Collect real market data for a publicly traded stock (Apple Inc., AAPL).
2. Model the price behavior using historical and conditional volatility (GARCH).
3. Price derivatives (European call options) using Black-Scholes and calculate sensitivities (Greeks).
4. Construct a portfolio combining the stock and options.
5. Measure and simulate portfolio risk using parametric and Monte Carlo Value at Risk (VaR).
6. Visualize results and interpret findings for practical decision-making.

Objective: Show the ability to integrate data analysis, statistical modeling, financial theory, and risk management into a coherent, quantitative workflow.

In [1]:
```
# Import necessary libraries
import yfinance as yf
import pandas_datareader.data as web
from arch import arch_model

import pandas as pd
import numpy as np
import warnings
warnings.filterwarnings("ignore")

import matplotlib.pyplot as plt
from scipy.stats import norm
import seaborn as sns
```

Step 1: Choose a Stock and Download Market Data

In [2]:
```
# Choose a stock
ticker = "AAPL" # (Apple Inc.)
```

In [3]:
```
# Download last 1 year of daily prices (from Jan 1, 2020 to Jan 1,
data = yf.download(ticker, start="2020-01-01", end="2025-01-01", in
```

```
[*********************100%***********************]  1 of 1 completed
```

In [4]:
```
data.columns = data.columns.get_level_values(0)
```

```
data
```

Out[4]:

Price	Close	High	Low	Open	Volume
Date					
2020-01-02	72.538521	72.598899	71.292311	71.545897	135480400
2020-01-03	71.833298	72.594063	71.608692	71.765674	146322800
2020-01-06	72.405678	72.444321	70.703012	70.954188	118387200
2020-01-07	72.065147	72.671341	71.845369	72.415337	108872000
2020-01-08	73.224403	73.526295	71.768079	71.768079	132079200
...	...	...	...	...	...
2024-12-24	257.286682	257.296626	254.386957	254.586262	23234700
2024-12-26	258.103729	259.179926	256.718662	257.276679	27237100
2024-12-27	254.685883	257.784897	252.164833	256.917949	42355300
2024-12-30	251.307877	252.603281	249.863009	251.337769	35557500
2024-12-31	249.534180	252.384064	248.547676	251.547039	39480700

1258 rows × 5 columns

In [5]:
```
data.isna().sum()
```

Out[5]:
```
Price
Close     0
High      0
Low       0
Open      0
Volume    0
dtype: int64
```

Step 2: Calculate Daily Returns

In [6]:
```
data['Returns'] = data['Close'].pct_change()
```

Step 3: Calculate Volatility

3a: Historical Volatility

It is a standard deviation of returns, showing how "noisy" or "risky" the asset is.

In [7]:
```
# Calculate annualized volatility
daily_vol = data['Returns'].std()
annual_vol = daily_vol * np.sqrt(252)  # 252 trading days in a year

print(f"\nAnnualized Volatility: {annual_vol:.4f}")
```

```
Annualized Volatility: 0.3168
```

This shows that Apple's stock fluctuates ~31.7% per year on average

3b: GARCH Volatility Modeling

Real markets exhibit volatility clustering (calm perdios vs. high-stress periods). Hence, we use statistical models to model how volatility changes over time. We compare historical constant volatility with conditional volatility from GARCH.

Notes about ARCH:

- ARCH stands for Autoregressive Conditional Heteroskedasticity
  - *Autoregressive* → depends on past values.
  - *Conditional* → today’s volatility depends on what happened yesterday.
  - *Heteroskedasticity* → variance (volatility) is not constant; it changes.
- This model says, "If yesterday's return was big (up or down), then today's volatility is likely to be high."
- Mathematically, $\sigma_t^2 = \omega + \alpha \cdot \epsilon_{t-1}^2$ where,
  - σ_t^2 = today's variance (volatility squared)
  - ϵ_{t-1}^2 = yesterday's squared return (big return $\longrightarrow$ big volatility)
  - ω, α = constants estimated from data (model fitting)
- ARCH(1) means it looks back one day While this model is useful, this only looks at the last day's return. That can be too simplistic since volatility tends to 'persist' for a while.

Notes about GARCH:

- GARCH stands for Generalized ARCH (Adds a term for past volatility itself)
- Mathematically, $\sigma_t^2 = \omega + \alpha \cdot \epsilon_{t-1}^2 + \beta \cdot \sigma_{t-1}^2$
- This model says that today's volatility depends on two factors: 1. how big yesterday's shock was (ARCH part), and 2. how high volatility was yesterday (GARCH part)

- Can smoothly describe how volatility clusters

Intuitive explanation

Let's say you're watching daily stock returns.

- If yesterday had a big move (e.g. +5% or −5%), today the market feels uncertain → volatility goes up. → That's the ARCH(1) effect (reacting to last shock).
- If volatility was already high yesterday, even if today's move wasn't huge, the market tends to "stay nervous." → That's the GARCH(1) effect (volatility persistence).

So, GARCH(1,1) mixes both effects:

"Volatility today = constant base + reaction to yesterday's surprise + continuation of yesterday's volatility."

In [8]:
```
returns = data['Returns'].dropna() * 100
garch = arch_model(returns, vol='Garch', p=1, q=1)
res = garch.fit(disp='off')
forecast = res.forecast(horizon=1)
sigma_t = np.sqrt(forecast.variance.values[-1,0]) / 100 # Forecaste
sigma_t_annualized = sigma_t * np.sqrt(252)
```

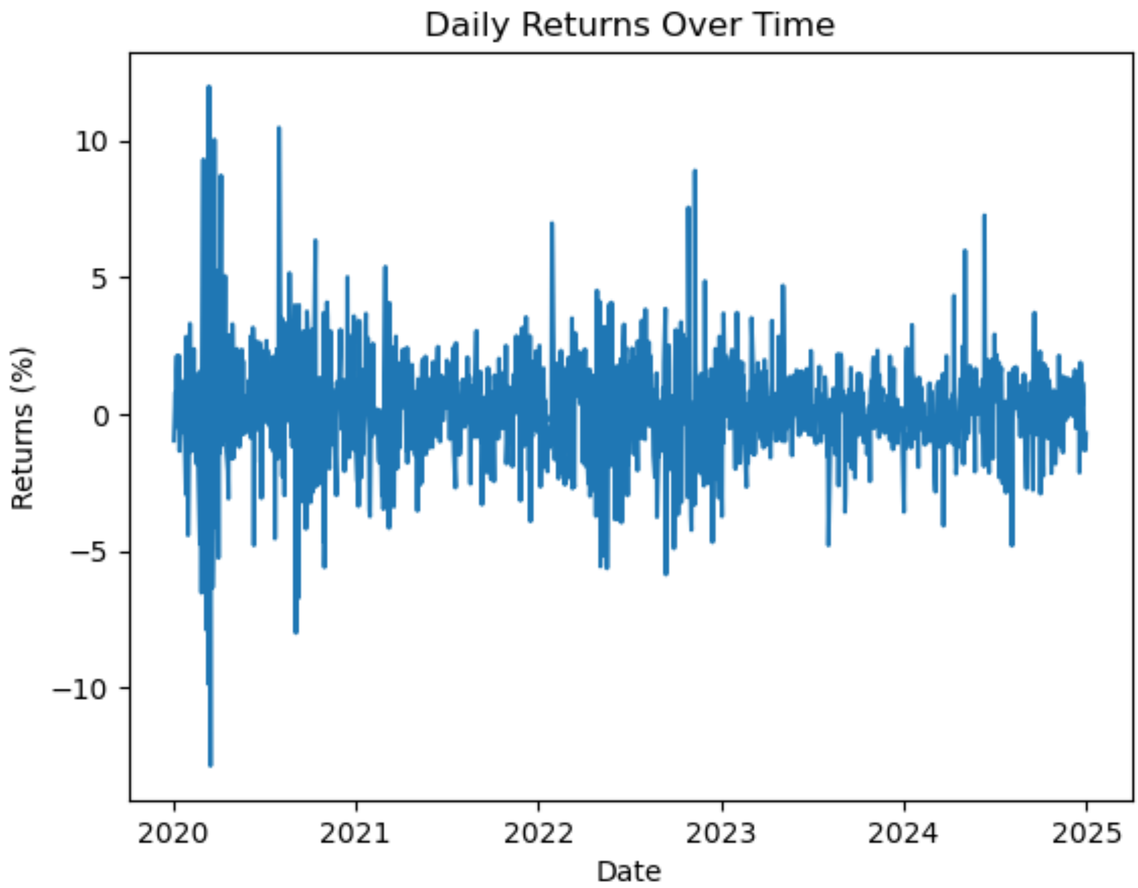
In [9]:
```
print("One day volatility")
print("*-------------------*")
print(f"Historical: {daily_vol:.4f}")
print(f"GARCH: {sigma_t:.4f}")

print("\n\nAnnualized volatility")
print("*-------------------*")
print(f"Historical: {annual_vol:.4f}")
print(f"GARCH: {sigma_t_annualized:.4f}")
```

```
One day volatility
*-------------------*
Historical: 0.0200
GARCH: 0.0134


Annualized volatility
*-------------------*
Historical: 0.3168
GARCH: 0.2131
```

In [10]:
```
plt.plot(returns.index, returns, label='Daily Returns')
plt.xlabel("Date")
plt.ylabel("Returns (%)")
plt.title("Daily Returns Over Time")
plt.show()
```



Step 4: Black-Scholes Option Pricing

Now we move into derivatives, specifically options. An **option** gives you the right but not the obligation to buy/sell an asset in the future at a fixed price (the strike price, K).

Types

- Call option → right to buy at K
- Put option → right to sell at K

We'll price a European call option, which can only be exercised at expiry (not before).

The Black-Scholes formula for a European call option is:

$$C = S_0 \cdot N(d_1) - Ke^{-rT} \cdot N(d_2)$$

Where:

- C → Call option price
- S0 → current stock price
- K → strike price
- T → time to expiry/maturity in years
- r → risk-free interest rate

- σ → volatility of the stock
- N() → cumulative normal distribution

Here, $d_1 = \frac{\ln(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$

- d_1: “moneyness” adjusted for risk and time — how far current price is from strike in standard deviation units.
- d_2: same, but excludes half the variance drift.

In [11]:
```
def black_scholes_call(S, K, T, r, sigma):
    d1 = (np.log(S/K) + (r + 0.5*sigma**2)*T) / (sigma*np.sqrt(T))
    d2 = d1 - sigma*np.sqrt(T)
    call_price = S * norm.cdf(d1) - K * np.exp(-r*T) * norm.cdf(d2)
    return call_price
```

To get risk-free interest rate, we fetch data from Federal Reserve Economic Data (FRED) website, run by U.S. central bank. Some pointers to note:

- `DGS1MO` is the 1-month U.S. Treasury Constant Maturity Rate
- U.S. Treasury - The U.S. government borrows money by selling 'Treasury securities' (like bonds)
- Constant Maturity Rate - standardized interest rate the government publishes every day. It’s like saying, “If we borrowed money for 1 month today, this would be the interest rate.”

Why we use this? This rate is often treated as the risk-free interest rate because lending to the U.S. government for 1 month is considered almost zero risk (they always pay back). That’s why in finance and investing, people use `DGS1MO` (or other Treasury rates like 3-month, 1-year, etc.) as a baseline to compare other investments.

Hence, if a stock gives you 7% return per year, and the risk-free rate is 5%, you can say that you earned extra reward of 2% for taking risk.

Note: One could also use `DGS3MO`, `DGS6MO`, `DGS1`, etc. but here, we shall assume that the option expiry is 30 days and therefore use `DGS1MO`.

In [12]:
```
rf = web.DataReader('DGS1MO', 'fred', '2024-01-01', '2025-01-01')
rf
```

Out[12]:

DATE	DGS1MO
2024-01-01	NaN
2024-01-02	5.55
2024-01-03	5.54
2024-01-04	5.56
2024-01-05	5.54
...	...
2024-12-26	4.45
2024-12-27	4.44
2024-12-30	4.43
2024-12-31	4.40
2025-01-01	NaN

263 rows × 1 columns

In [13]:

```
rf_ffill = rf.fillna(method='ffill')  # forward fill missing values
r = rf_ffill.iloc[-1].values[0] / 100  # latest rate as decimal
```

In [14]:

```
# For example, taking the last closing price as S0
days_to_expiry = 30  # days until option expiry
K = 170  # strike price

S0 = data['Close'].iloc[-1]  # latest price
T = days_to_expiry/365  # 30 days to expiry
sigma = sigma_t_annualized  # using GARCH forecasted volatility

call_price = black_scholes_call(S0, K, T, r, sigma)
print("European Call Option Price:", call_price)
```

```
European Call Option Price: 80.14786386392103
```

This means that a 1-month call option with strike 170 costs $80.23.

How to use this information?

1. Compare to market price: * If market call price > Black-Scholes price → option may be “expensive” * If market call price < Black-Scholes price → option may be “cheap”

2. Estimate risk-adjusted hedging: * Use it to determine how many shares to buy/sell to hedge an option (delta-hedging).

3. Feed into trading strategies: * You can combine Black-Scholes price with

volatility forecast to make trading decisions.

Step 5: The Greeks - Sensitivities

Greeks are derivatives of the option price with respect to key inputs.

Think of Greeks as risk measures:

1. Delta (Δ) – How much the option price changes if the stock price moves by
   $1. * Measures sensitivity to Stock Price (S) * Call option : 0 \to 1 * Put$
   1 increase in stock → $0.60 increase in call price.
2. Gamma (Γ) – How much delta changes if the stock price moves by
   $1. * Measures curvature of the option price. * Measures sensitivity to D$
   \Delta$ * High gamma → option price reacts sharply to stock moves.
3. Theta (Θ) – How much the option price decreases per day due to time decay. * Measures sensitivity to Time * Options lose value as expiry approaches.
4. Vega (ν) – How much the option price changes if volatility changes by 1%. * Measures sensitivity to Volatility (σ) * Higher volatility → more expensive options.
5. Rho (ρ) – How much the option price changes if risk-free interest rate r changes by 1%. * Measures sensitivity to Interest Rate (r)

For European call options, the formulas are:

$$\Delta = N(d_1)$$

$$\Gamma = \frac{N'(d_1)}{S\sigma\sqrt{T}}$$

$$\Theta = -\frac{SN'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

Vega = $S\sqrt{T}N'(d_1)$

Rho = $KTe^{-rT}N(d_2)$

Where $N'(d1)$ is the standard normal PDF.

In [15]:
```
def greeks_call(S, K, T, r, sigma):
    d1 = (np.log(S/K) + (r + 0.5*sigma**2)*T) / (sigma*np.sqrt(T))
    d2 = d1 - sigma*np.sqrt(T)

    delta = norm.cdf(d1)
    gamma = norm.pdf(d1) / (S * sigma * np.sqrt(T))
    theta = (-S * norm.pdf(d1) * sigma / (2 * np.sqrt(T))
             - r * K * np.exp(-r*T) * norm.cdf(d2)) / 365  # per da
    vega = S * np.sqrt(T) * norm.pdf(d1) / 100  # per 1% change
```

```
    rho = K * T * np.exp(-r*T) * norm.cdf(d2) / 100  # per 1% chang

    return delta, gamma, theta, vega, rho

delta, gamma, theta, vega, rho = greeks_call(S0, K, T, r, sigma)
print(f"Delta: {delta:.4f}, Gamma: {gamma:.4f}, Theta: {theta:.4f},
```

```
Delta: 1.0000, Gamma: 0.0000, Theta: -0.0204, Vega: 0.0000, Rho: 0.1
392
```

- Delta = 1.0 → the option moves almost 1-for-1 with the stock (deep in the money).
- Gamma ≈ 0 → small curvature, consistent with deep ITM option.
- Theta negative → loses value each day as time passes.
- Vega ≈ 0 → volatility doesn't matter much for deep ITM options.
- Rho positive → higher rates slightly increase the option value.

Step 6: Portfolio Setup

We setup up a portfolio combining shares and call option.

In [16]:
```
shares = 100
option_units = 1  # 1 call option contract = 100 shares

stock_price = S0
option_price = call_price

portfolio_value = shares * stock_price + option_units * option_pric
print(f"Portfolio Value: ${portfolio_value:.2f}")
```

```
Portfolio Value: $25033.57
```

Step 7: Risk Measurement (VaR)

Tells "How much can I lose in a day, with 95% confidence?"

Think of VaR as a “worst-case loss” indicator:

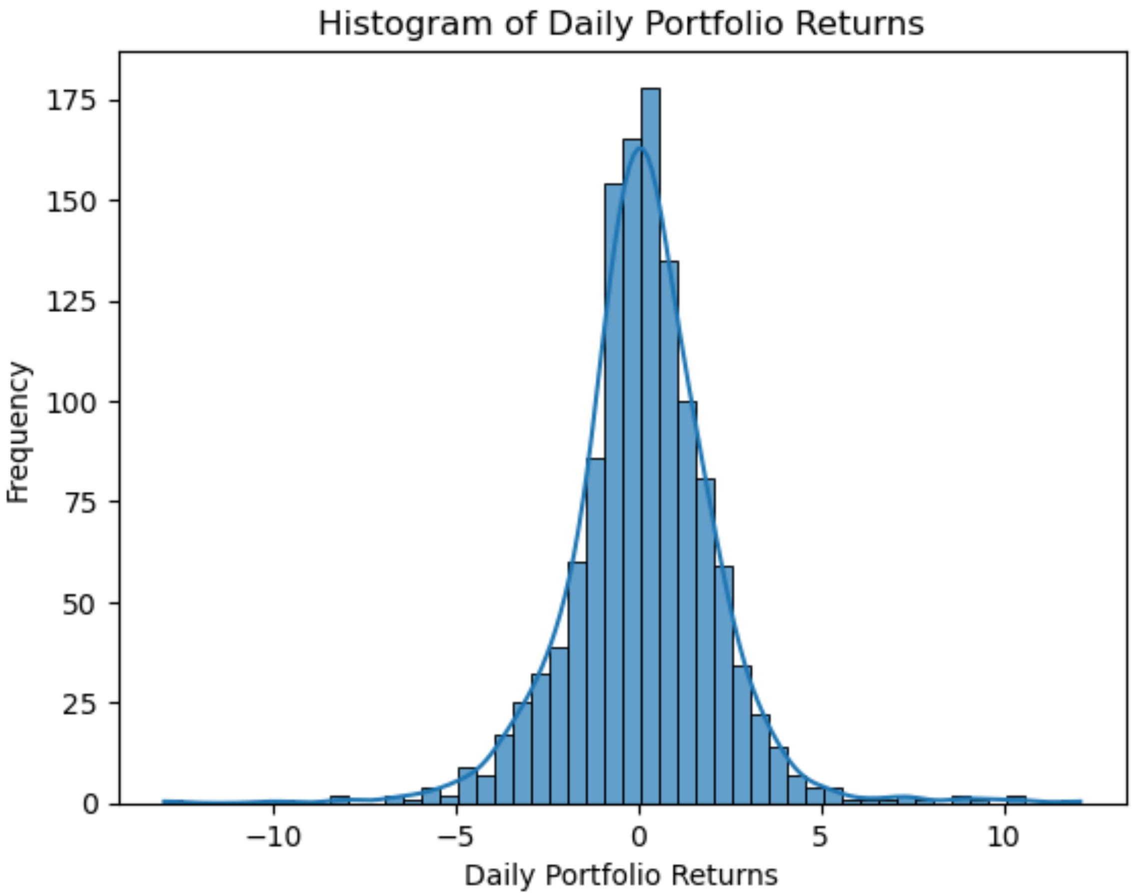
- Example: “Our 1-day 5% VaR is $1,000.”
- This means there’s a 5% chance that the portfolio will lose more than $1,000 in one day.

There are three main methods to calculate VaR:

1. Historical Simulation – Use past returns of assets.
2. Variance-Covariance (Parametric) – Assume returns are normally distributed.
3. Monte Carlo Simulation – Simulate thousands of possible future outcomes.

In [17]:
```
data['Portfolio_Returns'] = shares * data['Returns'] + option_units
```

In [18]:
```
sns.histplot(data['Portfolio_Returns'].dropna(), bins=50, alpha=0.7
plt.xlabel("Daily Portfolio Returns")
plt.ylabel("Frequency")
plt.title("Histogram of Daily Portfolio Returns")
plt.show()
```



Note that this is a linear approximation, but it works well enough for small moves. We also see a bell-shaped curve, signifying that we can use parametric VaR to approximate risk

7a: 1-Day VaR (Parametric)

Assuming normal distribution of returns:

$$\text{VaR} = z_\alpha \cdot \sigma_p \cdot \text{Portfolio Value}$$

- z_α = critical value from standard normal (e.g., 1.65 for 5%, in other words z-score for 5th percentile)
- σ_p = standard deviation of portfolio returns

In [19]:
```
confidence_level = 0.05
z = norm.ppf(confidence_level)

# Portfolio standard deviation
sigma_p = data['Portfolio_Returns'].std()
```

```
VaR_1day = -z * sigma_p * portfolio_value
print(f"1-Day 5% VaR: ${VaR_1day:.2f}")
```

```
1-Day 5% VaR: $82992.25
```

Interpretation: There's a 5% chance your portfolio could lose more than this amount tomorrow.

7b: Monte Carlo Simulation for VaR

Idea: We simulate thousands of possible future stock prices, recalculate the option price using Black-Scholes, then compute the portfolio value for each simulation. Finally, we use the distribution of portfolio changes to estimate VaR.

Simulate Future Stock Prices

We assume stock returns are normally distributed, using historical mean and volatility:

$$S_{t+1} = S_t \cdot e^{(\mu - 0.5\sigma^2)\Delta t + \sigma\sqrt{\Delta t} \cdot Z}$$

Where:

- $Z \sim N(0, 1)$
- μ = average daily return
- σ = daily volatility
- Δt = 1 day

In [20]:
```
num_simulations = 100_000
mu = data['Returns'].mean()
sigma_daily = sigma_t
S_t = S0
```

In [21]:
```
# Simulate future stock prices
np.random.seed(42)
Z = np.random.normal(0, 1, num_simulations)
S_future = S_t * np.exp((mu - 0.5 * sigma_daily**2) + sigma_daily *
```

Recalculate Option Prices for Each Simulation

We recalculate European call option price for 1-day forward using Black-Scholes.

In [22]:
```
T_1day = 1/365  # 1 day to expiry
option_prices_future = [black_scholes_call(S, K, T_1day, r, sigma)
option_prices_future = np.array(option_prices_future)
```

Calculate Portfolio Value Changes

In [23]:
```
portfolio_future = shares * S_future + option_units * option_prices
portfolio_change = portfolio_future - portfolio_value
```

Estimate 1-Day Monte Carlo VaR

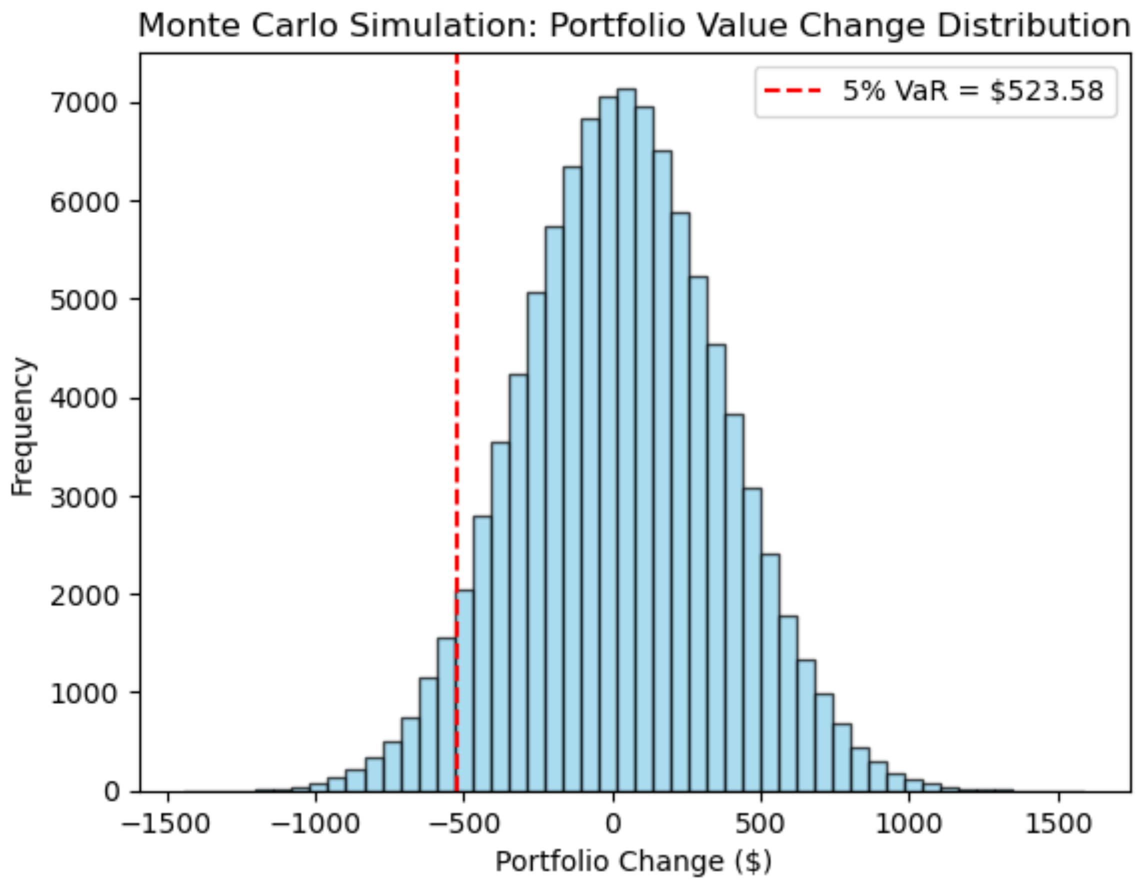
Take the 5th percentile of simulated losses:

In [24]:
```
VaR_mc = -np.percentile(portfolio_change, 5)
print(f"1-Day 5% Monte Carlo VaR: ${VaR_mc:.2f}")
```

```
1-Day 5% Monte Carlo VaR: $523.58
```

Visualize the distribution

In [25]:
```
plt.hist(portfolio_change, bins=50, color='skyblue', edgecolor='bla
plt.axvline(-VaR_mc, color='red', linestyle='--', label=f'5% VaR =
plt.title("Monte Carlo Simulation: Portfolio Value Change Distribut
plt.xlabel("Portfolio Change ($)")
plt.ylabel("Frequency")
plt.legend()
plt.show()
```



Step 8: Conclusions

Key Takeaways:

- GARCH models provide a more realistic volatility estimate than historical σ.
- Black-Scholes allows pricing and hedging of European options; Greeks

quantify risk exposures.
- Portfolio VaR estimates demonstrate the impact of derivatives on risk.
- Monte Carlo simulation captures nonlinear portfolio effects that parametric VaR may miss.

Overall: This project demonstrates the ability to go from market data → quantitative modeling → derivatives pricing → risk measurement — the core workflow of a financial analyst or quant.

Step 9: Future Work & Extensions

1. Multi-Asset Portfolio: * Include correlated stocks to model portfolio diversification effects
2. Delta-Hedging Simulation: * Track P&L while rebalancing to maintain delta-neutral portfolio
3. Expected Shortfall (CVaR): * Complement VaR with average loss beyond the threshold
4. Alternative Pricing Models: * Binomial trees for American options, or Monte Carlo for exotic options
5. Implied Volatility Analysis:
    - Compare Black-Scholes implied vol with market option prices
    - Visualize volatility smile or surface

✅ What we've achieved:

1. Simulated thousands of possible future stock prices.
2. Repriced the option for each scenario.
3. Calculated the portfolio change distribution.
4. Computed Monte Carlo 1-day VaR.
5. Visualized portfolio risk with a histogram.

This completes a full workflow from option pricing to risk management.