dronePowertrainSizingNotes

Hover power

List of methods (non-CFD)

1. Momentum theory
2. Blade element theory

Momentum theory

- *Momentum* refers to the linear momentum of the airflow that is accelerated as the rotor (propeller) spins
- The theory is based on momentum conservation, which translates to the increase in momentum corresponding to equal and opposite reaction force (thrust) generated by the propeller
- Assumptions
  - propeller is an actuator disk
  - flow is incompressible
  - air is accelerated from still air to fully developed wake
    - $v_{wake} = v_{induced}$
- Links
  - link1
  - link2

Equation

$$P_{hover} = \frac{W^{3/2}}{\sqrt{2\rho(n_{prop}A)}} \times \frac{1}{\eta_{hover}}$$

Symbol	Description	Unit
W	weight of aircraft	N
ρ	air density	kg/m^3
n_{prop}	number of propellers	-
A	disk area of a single prop	m^2
η_{hover}	prop efficiency at hover	-

Cruise power

List of methods (non-CFD)

1. Lift-to-drag (L/D) ratio based
2. Drag based

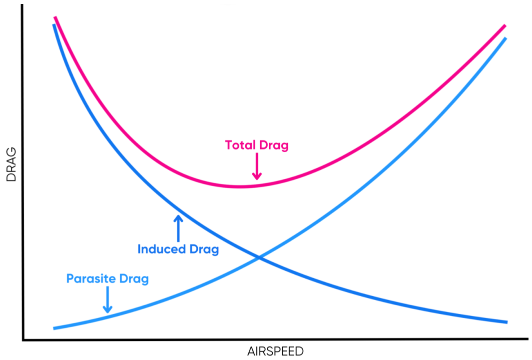
L/D method

1. Every aircraft will have a L/D curve
2. It is parabolic in nature with a minimum at the best cruise speed
3. Obviously, lift = weight at cruise (fully airborne $\rightarrow$ weight fully supported by the wings)

Equation

$$P_{cruise} = \frac{W}{L/D} v_{cruise} \times \frac{1}{\eta_{cruise}}$$

L/D (just drag, as lift is constant in wing-borne flight) has a parabolic dependence on cruise speed



For derivation [aero101 > Equations]

Battery sizing

Power sizing

Additional parameters / models necessary

1. $\eta_{powertrain}$ powertrain efficiency

$$P_{maxBattery} = \frac{1}{\eta_{powertrain}} \times max(P_{hover}, \; P_{cruise})$$

- Most likely $P_{hover} >> P_{cruise}$ around *best range* speed
- There will be slight difference (a percent or two) in $\eta_{powertrain}$ during hover and cruise

Energy sizing

Additional Parameters / models / requirements necessary

1. $\eta_{powertrain}$ powertrain efficiency
2. d total range requirement
3. t_{hover} hover time
4. v_{cruise} cruise speed

$$E_{total} = (t_{hover} \times P_{hover} + \frac{d}{v_{cruise}} \times P_{cruise}) \times \frac{1}{\eta_{powertrain}} \times \frac{1}{1000 \times 3600} \;\; kWh$$

Cell architecture

- number of cells in series is a simple

$$n_s = \frac{V_{bat_{min}}}{V_{cell_{min}}}$$

  $V_{bat_{min}}$ is a requirement, $V_{cell_{min}}$ is cell specification
- number of cells is parallel (#parallel branches of series cell) needs two main consideration
  - energy required for the mission
  - maximum discharge C-rate of the cell
- if the cell chemistry doesn't allow, we might need to add parallel branches just to be below the maximum allowable discharge C-rates and end up with higher than required energy

Choice of cell chemistry

Comparison is limited to Li-Polymer and Li-ion

- Li-Po is more popular for drones
- Li-Po has more than twice the power density W/kg (higher C-rates) and about 25% less energy density Wh/kg
- Depending on the ratio of battery-to-drone mass the 25% might look even less
- But if the range requirements are *really* high, Li-ion batteries might be a better option as [this reasoning] would no longer be valid
- For *heavy* lifting Li-Po are a better option
- Other relevant differences - Li-Po $\rightarrow$
  - life span is lower by about 50% (300 cycles)
  - safety
    - more prone to swelling (flexible casing)
    - less prone to thermal runaway
  - higher lowest operating voltage (part of reason why less energy dense)
  - comes mostly in pouch configuration (as opposed to cylindrical Li-ion), but pouch cells might need structural reinforcements that might take away the volumetric density advantage of Li-Po (pouch cell) when it comes to making a battery with multiple cells
    - note that when making a battery, the volumetric density drops for both types of cells (wasted space for cylindrical), (structural reinforcements for punch), but the mass density will also decrease for pouch cells

Comparative table

Chemistry$\rightarrow$ Property$\downarrow$	**Li-ion**	**Li-Po**
Energy density	1.25x	
Power density		2x
Life		2x
Thermal runaway		lower
Swelling	lower	
Cost	lower	
Popular AR	cylindrical	pouch
Use case	long range	heavy lifting

Thermals

Motor sizing

Rating

Power rating

This is same as battery power rating excluding $1/\eta_{powertrain}$

$$P_{maxMotor} = max(P_{hover},\ P_{cruise})$$

Torque rating (τ)

Data needed $\rightarrow$ Thrust vs. prop speed curve
Generally we have an upper limit on the radius of the propeller
And we have the maximum thrust required as about 1.3 x weight of the drone (extra 30% for controllability)

$$T_{max_{prop}} = \frac{W + 0.3W}{n_{prop}}$$

Propeller speed required for T_{max} can be found by

$$\omega_{maxProp}\ (rad/s) = \frac{1}{R} \times \sqrt{\frac{T_{max_{prop}}}{0.5 C_T \rho (\pi R^2)}}$$

Symbol	Description	Unit
T_{max}	maximum required thrust (per prop)	N
R	radius of the prop	m
C_T	thrust coefficient (0.1 - 0.2)	-
ρ	air density	kg/m^3

Motor's torque rating can be found from Power rating and Speed rating (max) as

$$\tau_{max} = \frac{P_m ax Motor}{\omega_{maxProp}}$$

Note: C_T for a *particular* (fixed diameter, number of blades) propeller will vary wrt:

- advance ratio (forward speed / propeller speed)
- blade pitch
- angle of attack (of the drone)
- *low* propeller speeds where Reynold's number changes are significant

Thermals

Heat generation

$$Q_{gen} = Q_{stator} + Q_{mech}$$

$$= (Q_{copper} + Q_{iron}) + (Q_{friction} + Q_{windage})$$

$$= (Q_{winding} + Q_{hysteresis} + Q_{eddycurrent}) + (Q_{friction} + Q_{windage})$$

Q_{rotor} is minimal

Heat loss

$$Q_{cool} = Q_{cond} + Q_{conv} + Q_{rad}$$

- $Q_{cond} \rightarrow$ motor to motor housing (Fourier's law)
  - depends on material property
- $Q_{conv} \rightarrow$ airflow (forced conv.) over motor housing (Newton's law of cooling)
  - doesn't depend on material property
  - depends on fluid property
  - depends on motor geometry
- $Q_{rad} \rightarrow$ (Stefan-Boltzmann's law)

For small/mid-sized air-cooled drones $\rightarrow$ forced convection is the main heat transfer means

Steady state analysis

This gives a steady state temperature for a given set of operating conditions (torque, speed, voltage, air flow, ambient temperature)

Dynamic model

Assumptions

1. Lumped capacitance can be used for the motor housing becuase Biot number $B_i = hL_c/k << 0.1$
   h convective HT coefficient
   k thermal conductivity (of motor housing)
   L_c characteristic length (volume / cooled surface area)
   $L_c = L/2$ for a fat short cylinder ($R >> L$)
2. The air-flow is turbulent
   1. this has implications on the constants in the Nuesselt number N_u calculation equation
3. The air flows perpendicular to the spin-axis of the motor
   1. This has implications on the characteristic length used for Reynold number R_e calculation that is, in turn used in the Nusselt number calculation

Equations

$$mc_p\dot{T} = hA(T_a - T)$$

T is temperature of cylinder surface
T_a is air temperature

$$h = \frac{N_u k}{D}$$

k is thermal conductivity of air $0.026W.m^{-1}K^{-1}$ @room temperature
D is outer diameter of the cylinder m

$$N_u = CR_e^m P_r^n$$

$C, m, n = 0.023, 0.8, 0.33$ for turbulent flow
P_r is Prantl number $= 0.7$ for air

$$R_e = \frac{\rho V_{air} L_c}{\mu}$$

V_{air} is air velocity m/s
L_c is characteristic length of the cylinder m
μ is dynamic viscosity of air $Pa.s$

Note L_c depends on flow configuration and smoothness of the flow surface

- for cross-flow on short-fat smooth cylinder (drone motor) $L_c = 2r_{cylinder}$
- for cross-flow on short-fat finned cylinder $L_c = D_{hydraulic}$

$$D_{hydraulic} = \frac{4A_{flow}}{P_{wet}}$$

  A_{flow} is cross-sectional area of flow m^2
  P_{wet} is wetter perimeter m

Addition of fins

1. It increases effective surface area $A_{eff} = A + n_{fin} A_{fin}$
2. It might increase h too if the fins induce more turbulence

Although cooling might not increase proportionally with increase in area due to fins. It depends on fin efficiency. Heat transfer equation might become

$$mc_p\dot{T} = hA_{eff}\eta_{fin}(T_a - T)$$

Natural vs. forced convection

[link1]
[link2-Ansys]

- Natural convection is the fluid motion driven by the density difference occurring due to temperature gradients
- Forced convection is the fluid motion driven by an external force (fan, pump, wind)
- Both are modeled by Newton's law of cooling
  - $Q_{conv} = hA(T_{surface}. - T_{ambient})$
- **The key difference is how the heat transfer coefficient h is determined**
  - calculation of heat transfer coefficient h is same for for both cases

$$h = \frac{N_u k}{L}$$

  - but calculation of N_u varies

```
    - For natural convection
```

$$N_u = CR_a^n = C(G_r P_r)^n$$

```
    - For forced convection
```

$$N_u = CR_e^m P_r^n$$

k is thermal conductivity of the fluid ($W.m^-1K^-1$)
P_r is Prantl number
G_r is Grasshof number
C, m, n are constants $\rightarrow$ geometry dependent (not fluid)*
For more details look at basicThermals > Dimensionless numbers

	Nat. conv.	**Forced conv.**
Key dimensionless number	Grashof's number Gr	Reynolds number Re
Driving force	Buoyancy ($\Delta\rho$)	External force
Cooling η	Low	High

Things to know

- heat transfer is better in turbulent flow
  - better fluid mixing increasing the convective heat transfer h
  - mathematically R_e is bigger for turbulent flow
    - but note - N_u dependence of R_e also varies from laminar to turbulent flow

Cooling efficiency

$$\eta_{cooling} = \frac{T_{init} - T_{final}}{T_{init} - T_{ambient}}$$

Aside on prop sizing

By propeller sizing $\rightarrow$
find the

1. propeller radius
2. number of blades
3. possibly pitch of the blades too

such that

1. thrust > $T_{max_{prop}}$
2. radius < geometrical constraints
3. noise < requirements

Thrust

$$T \propto \omega^2 R^4$$

This allows lower prop speed for the same thrust requirement, e.g., 10% increase in radius will reduce speed requirement by ~ 18%. This effect *may* be even more pronounced as coefficient of thrust also increases with propeller radius.

Torque is supposed to increase with R^3 (to be researched)