Eigen values

For the system to be stable the

- real parts of Eigen values of a continuous system must be < 0
- absolute value of Eigen values of a discrete system must be < 1

Note: this is only to get an intuition, not establish mathematical rigor.

For a continuous system $\dot{x} = Ax$

The general solution is given by

$$x(t) = e^{At}$$

If A it's a single state system

$$x(t) = e^{at}$$

$x(t)$ will decay (stable) onyl if $a < 0$

For a discrete system $x_{k+1} = \tilde{A}x_k$

Note: for the same system, $\tilde{A} \neq A$ Writing the solution

$$x_1 = \tilde{A}x_0$$

$$x_2 = \tilde{A}x_2 = \tilde{A}^2 x_0$$

$$x_3 = \tilde{A}x_2 = \tilde{A}^3 x_0$$

$$\vdots$$

$$x_n = \tilde{A}^n x_0$$

Or, for a system with one state

$$x_n = \tilde{a}^n x_0$$

x_n will decay (stable) only if $|\tilde{a}| < 1$