Impulse response

- Theoretically, we impulse the system ONLY at the first time instant $(t = 0)$ and measure the outputs
    - exampe would be whacking a MSD or flexible body with a hammer and measuring displacement
- Mathematically, we start with a unit IC in the $input\ u$ direction

For a linear system $\dot{x} = Ax + Bu$, the *initial condition* (x_0) respose ($u = 0$) in given by

$$x(t)_{IC} = e^{At}x_0$$

For the same linear system, the *impulse* response will be given by

$$x(t)_{impulse} = e^{At}B$$

Comparing the above two equations we see $B = x_0$. This is giving a unit IC in the B direction.

Question

1. How does this change when we have multiple inputs u? (i.e., the B matrix has more than one column)

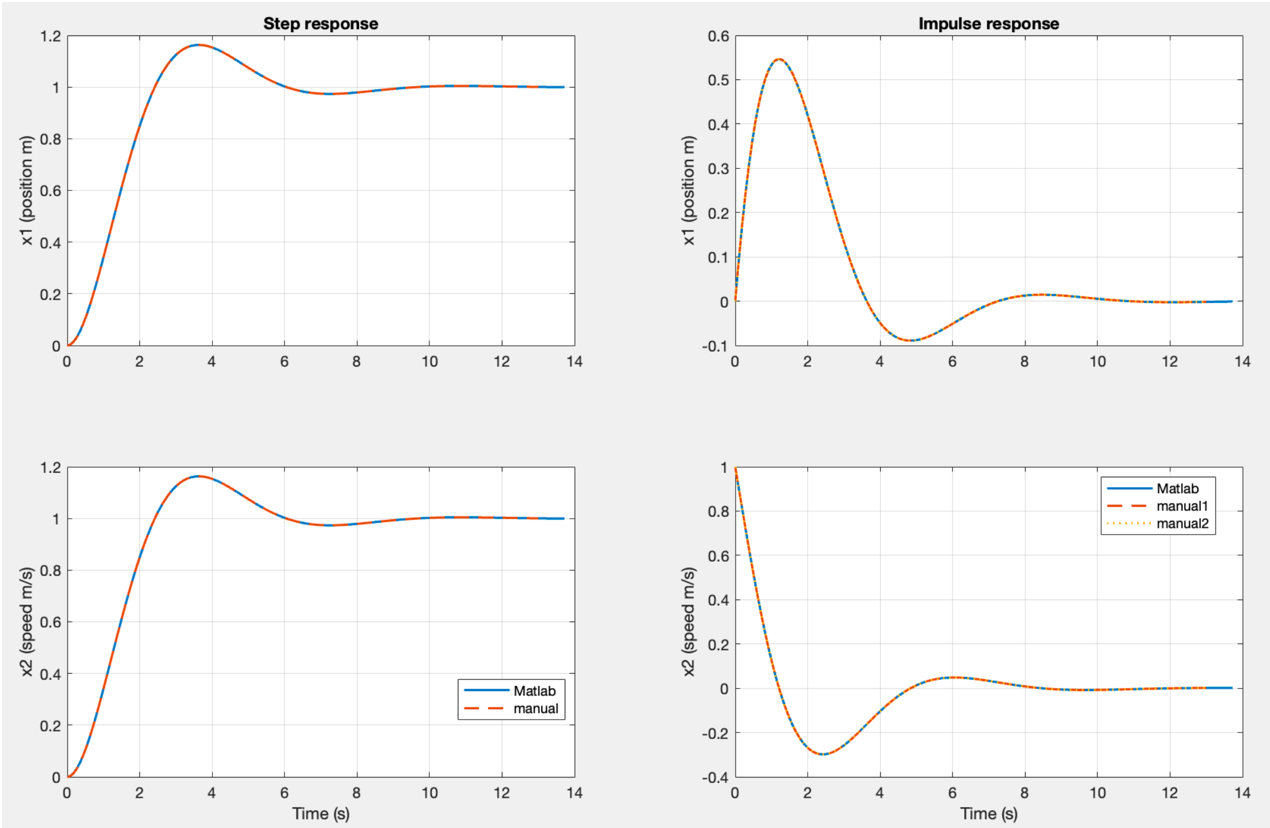
Step response

- Step response is giving a unit input u to a system from rest $x_0 = 0$ for all time
- Skipping the mathematical expression here as it involves convolution integrals and all

Matlab simulation

Method	Impulse	Step	Code format
Built-in	`impulse(linSys)`	`step(linSys)`	
Manual1	`lsim(linSys, 0, t, B)`	`lsim(linSys, 1, t, 0)`	`lsim(sys, u, t, x0)`
Manual1	`expmv(A, B, t)`		$= e^{At}B$

For a simple MSD sytem here's a comparison plot of the above mentioned methods



Other insights

1. Writing out impulse response for discrete systems generates the format of a controllability matrix. For a system with n states, the ctrollability matrix C

$$C = [B \;\; AB \;\; A^2B \;\; \ldots \;\; A^{n-1}B]$$

2. This is why the *impule* response is tied to the how well we can control the system, i.e., are there any directions in $\mathbb{R}^n$ that are not touched by the designed input system Bu

Future work

1. Use Matlab's `impulse` command to back out the mathematical definition of impulse response of a *multi-input* system