uncertaintyAnalysis

What is uncertainty analysis?

Two methods to analysis

1. analytical (using calculus)
2. numerical (Monte-carlo)

Example problem used

Let's say we have a model to find certain output from given inputs and parameters, e.g., density of an iron cylinder. We measure the following parameters with measurement uncertainties given as:

- weight $W \rightarrow 100 \pm 1N$
- height $h \rightarrow 30 \pm 0.001m$
- diameter $d \rightarrow 7.4 \pm 0.001m$

$$\rho = \frac{M}{V} = \frac{W/g}{\pi(d/2)^2 h} = \frac{4W}{\pi g d^2 h}$$

Analytical method

Propagation of error

Propagation of error is a technique used to estimate uncertainty in a result when it depends on multiple measured variables, each with its own uncertainty.

If we have a model

$$Y = f(x1,\ x2,\ \ldots,\ x_n)$$

and each variable x_k has an uncertainty of Δx_k with the assumptions

- errors Δx_k in input variables are small
- errors of each variables are independent (uncorrelated)

then uncertainty ΔY is estimated as

$$\Delta Y = \sqrt{\left(\frac{\partial Y}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial Y}{\partial x_2}\Delta x_2\right)^2 + \ \ldots\ + \left(\frac{\partial Y}{\partial x_n}\Delta x_n\right)^2}$$

Numerical method

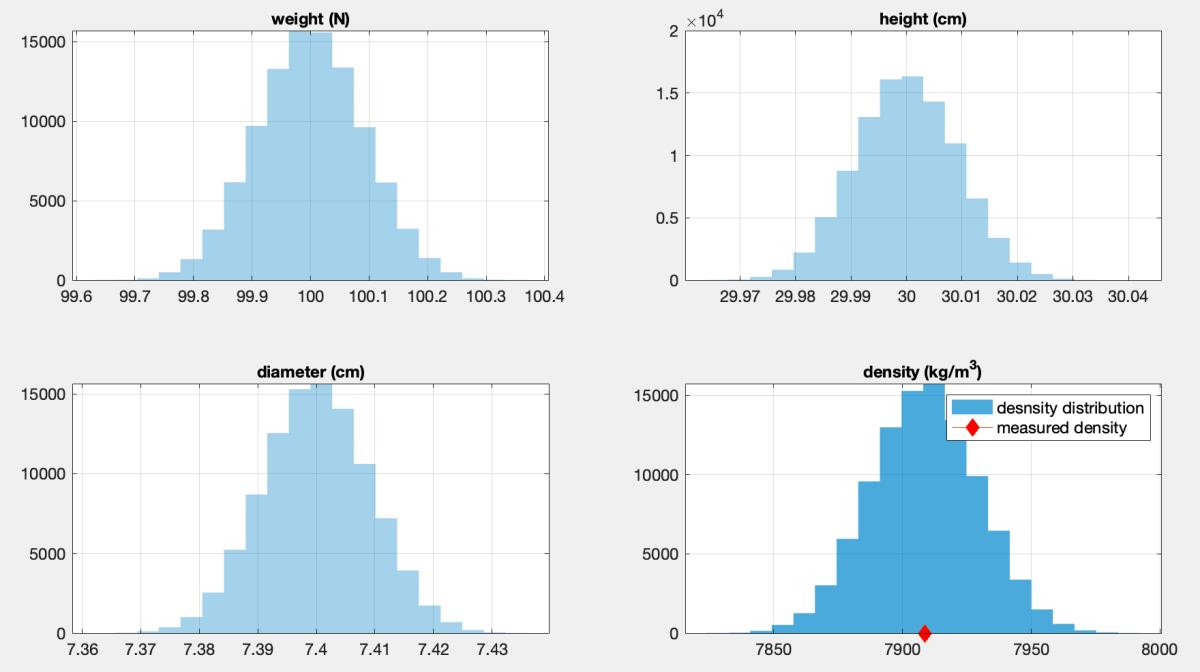
Monte-Carlo simulations

Find ρ_{mc} for a whole bunch of weight, height and diameter *normally* varying within the measurement uncertainty bounds and plot the distribution.

So with uncertainty taken into account, density with 95% confidence interval

$$\rho = mean(\rho_{mc}) \pm 2 \times std(\rho_{mc})$$

For the above-mentioned numbers (for iron)
$\rho = 7909 \pm 42\ kg/m^3$



Note: in Matlab it is not straightforward to generate normally distributed random numbers *between two bounds*. The following method is used to generate `n` (e.g., 10000) normally distributed random numbers Link

```
normRandNumber = xmin + (xmax - xmin) * sum(rand(n, p), 2)/p
```