

# Step and impulse response notes

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## Impulse response

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- Theoretically, we impulse the system ONLY at the first time instant ( $t = 0$ ) and measure the outputs
  - exemple would be whacking a MSD or flexible body with a hammer and measuring displacement
- Mathematically, we start with a unit IC in the *input*  $u$  direction

For a linear system  $\dot{x} = Ax + Bu$ , the *initial condition* ( $x_0$ ) response ( $u = 0$ ) is given by

$$x(t)_{IC} = e^{At}x_0$$

For the same linear system, the *impulse* response will be given by

$$x(t)_{impulse} = e^{At}B$$

Comparing the above two equations we see  $B = x_0$ . This is giving a unit IC in the  $B$  direction.

## Question

1. How does this change when we have multiple inputs  $u$ ? (i.e., the  $B$  matrix has more than one column)

## Step response

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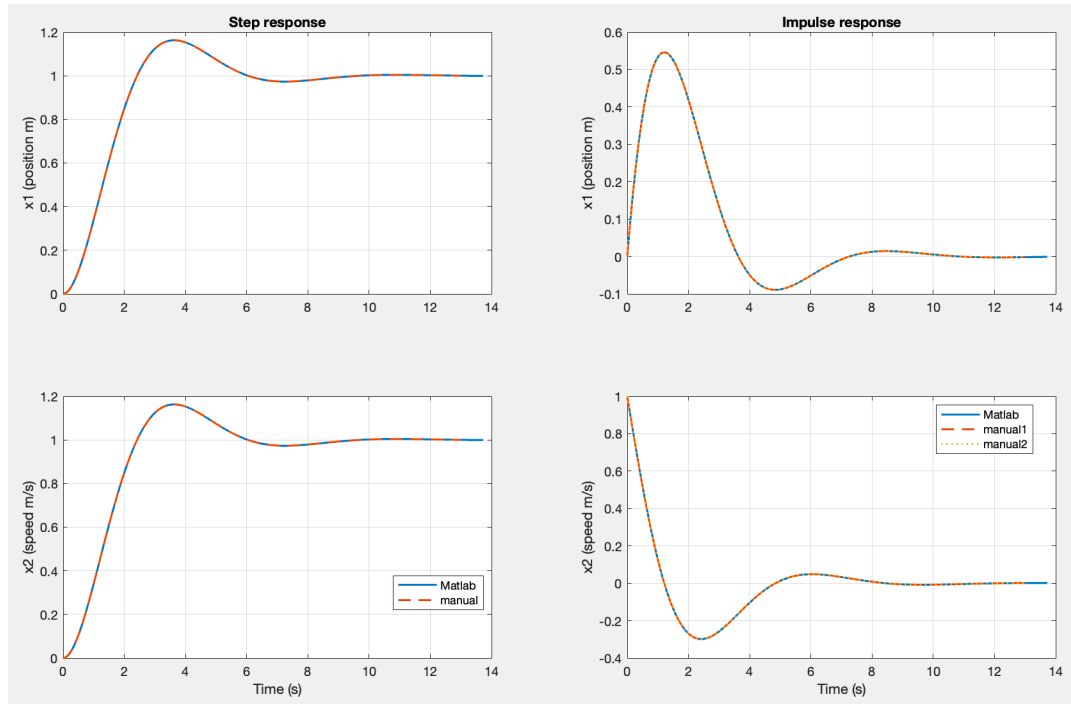
- Step response is giving a unit input  $u$  to a system from rest  $x_0 = 0$  for all time
- Skipping the mathematical expression here as it involves convolution integrals and all

## Matlab simulation

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Method	Impulse	Step	Code format
Built-in	<code>impz(linSys)</code>	<code>step(linSys)</code>	
Manual1	<code>lsim(linSys, 0, t, B)</code>	<code>lsim(linSys, 1, t, 0)</code>	<code>lsim(sys, u, t, x0)</code>
Manual2	<code>expmv(A, B, t)</code>		$= e^{At}B$

For a simple MSD system here's a comparison plot of the above mentioned methods



## Other insights

1. Writing out impulse response for discrete systems generates the format of a controllability matrix. For a system with  $n$  states, the controllability matrix  $C$

$$C = [B \ AB \ A^2B \ \dots \ A^{n-1}B]$$

2. This is why the *impulse* response is tied to the how well we can control the system, i.e., are there any directions in  $\mathbb{R}^n$  that are not touched by the designed input system  $Bu$

## Future work

1. Use Matlab's `impz` command to back out the mathematical definition of impulse response of a *multi-input* system