# stepAndImpulseResponseNotes

# Step and impulse response notes

## Impulse response

- Theoretically, we impulse the system ONLY at the first time instant $(t = 0)$ and measure the outputs
  - exampe would be whacking a MSD or flexible body with a hammer and measuring displacement
- Mathematically, we start with a unit IC in the $input\ u$ direction

For a linear system $\dot{x} = Ax + Bu$, the *initial condition* $(x_0)$ respose $(u = 0)$ in given by

$$x(t)_{IC} = e^{At}x_0$$

For the same linear system, the *impulse* response will be given by

$$x(t)_{impulse} = e^{At}B$$

Comparing the above two equations we see $B = x_0$. This is giving a unit IC in the B direction.

### Question

1. How does this change when we have multiple inputs $u$? (i.e., the $B$ matrix has more than one column)

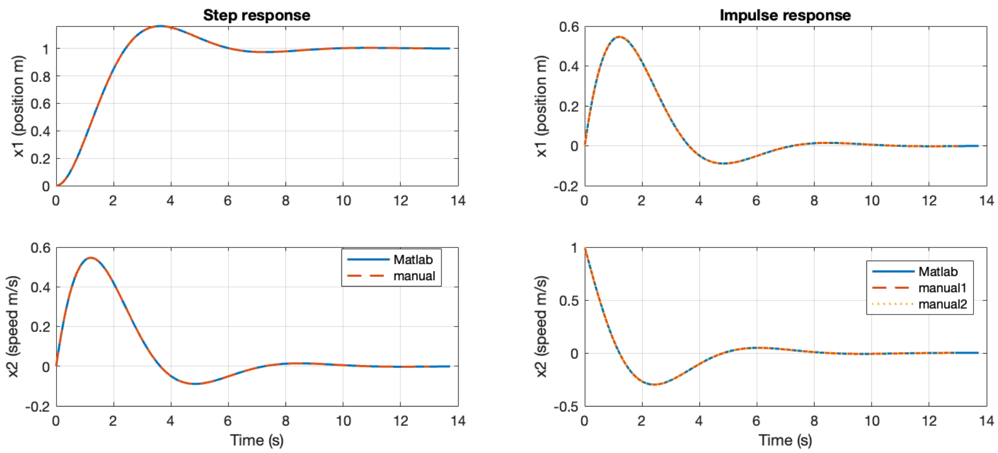
## Step response

- Step response is giving a unit input $u$ to a system from rest $x_0 = 0$ for all time
- Skipping the mathematical expression here as it involves convolution integrals and all

## Matlab simulation

| Method | Impulse | Step | Code format |
|---|---|---|---|
| Built-in | `impulse(linSys)` | `step(linSys)` | |
| Manual1 | `lsim(linSys, 0, t, B)` | `lsim(linSys, 1, t, 0)` | `lsim(sys, u, t, x0)` |
| Manual2 | `expmv(A, B, t)` | | $= e^{At}B$ |

For a simple MSD sytem here's a comparison plot of the above mentioned methods




# Other insights

1. Writing out impulse response for discrete systems generates the format of a controllability matrix. For a system with $n$ states, the ctrollability matrix $C$

$$C = [B \; AB \; A^2B \; \dots \; A^{n-1}B]$$

2. This is why the *impule* response is tied to the how well we can control the system, i.e., are there any directions in $\mathbb{R}^n$ that are not touched by the designed input system $Bu$

# Future work

1. Use Matlab's `impulse` command to back out the mathematical definition of impulse response of a *multi-input* system