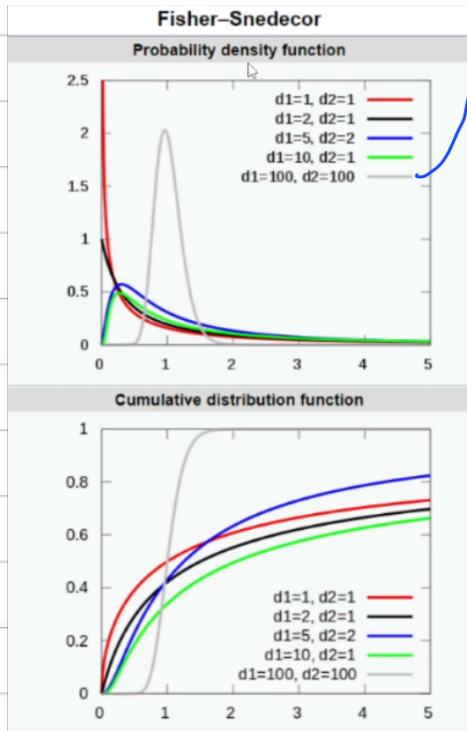


F-DISTRIBUTION

In probability theory and statistics F-distribution or f-ratio, also known as Sendecon's F-distribution or the Fisher-Sendecon distribution (after Ronald Fisher and George W. Sendecon) is a continuous probability distribution that arises frequently as the null distribution of a statistic, most notably in the analysis of variance (ANOVA) and other F-tests.



☆ as d_1 & d_2 increases the distribution becomes on stands to look like normal distribution

$$f(x; d_1, d_2) = \frac{\sqrt{\frac{(d_1 x)^{d_1} d_2^{d_2}}{(d_1 x + d_2)^{d_1 + d_2}}}}{x B\left(\frac{d_1}{2}, \frac{d_2}{2}\right)}$$

Beta function

$$B(m, n) = \frac{(m-1)!(n-1)!}{(m+n-1)!} = \frac{m+n}{mn} \left/ \binom{m+n}{m} \right.$$

☆ Parameter used : $d_1, d_2 > 0$: degree of freedom

☆ Supports : $x \in (0, +\infty)$

☆ Pdf = $f(x, d_1, d_2) = \frac{(d_1 x)^{d_1} d_2^{d_2}}{(d_1 x + d_2)^{d_1 + d_2}} \cdot \frac{1}{x B\left(\frac{d_1}{2}, \frac{d_2}{2}\right)}$ — Beta function

☆ F-distribution with d_1 and d_2 degree of freedom is the distribution of Random variable such that:

$$X = \frac{S_1/d_1}{S_2/d_2}, \quad d_1, d_2 \rightarrow \text{Respective dof for } S_1, S_2$$

$S_1, S_2 \rightarrow$ Independent random variables with Chi Square distribution

Chi Sq distribution is a Right skewed distribution

F-test [Variance Ratio Test] :-

Q. The following data shows no. of bulbs produced daily for some days by 2 workers A and B

A	B
40	39
30	38
38	41
41	33
38	32
35	39
	40
	34

Can we consider based on the data that worker B is more sizable and efficient
($\alpha = 0.05$)

Soln

S1y $H_0 = \sigma_1^2 = \sigma_2^2$
S2y $H_1 = \sigma_1^2 \neq \sigma_2^2$

S3y Calculation of variance

$$s_1^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

Worker A

x_1	\bar{x}_1	$(x_1 - \bar{x}_1)^2$
-------	-------------	-----------------------

40	37	9
----	----	---

30	"	49
----	---	----

38	"	1
----	---	---

41	"	16
----	---	----

38	"	1
----	---	---

35	"	4
----	---	---

$\bar{x}_1 = 37$	$\Sigma = 80$
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$$s_1^2 = \frac{80}{n-1} = \frac{80}{8} = 10$$

Worker B

x_2	\bar{x}_2	$(x_2 - \bar{x}_2)^2$
-------	-------------	-----------------------

39	37	4
----	----	---

38	"	1
----	---	---

41	"	16
----	---	----

33	"	16
----	---	----

32	"	25
----	---	----

39	"	4
----	---	---

40	"	9
----	---	---

34	"	9
----	---	---

$\bar{x}_2 = 37$	$\Sigma = 84$
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$$s_2^2 = \frac{84}{7} = 12$$

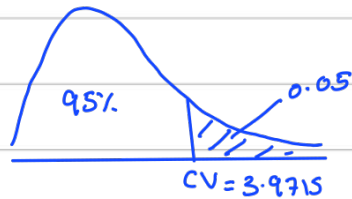
S47 F-test Calculation (i.e. variance Ratio test)

$$F = \frac{S_1^2}{S_2^2} = \frac{13}{12} = 1.33$$

S51 Decision Boundary

$$\left[\begin{array}{l} df_1 = 8-1 = 5 \quad \& \quad df_2 = 8-1 = 7 \\ \alpha = 0.05 \end{array} \right] \quad \text{--- \# Using this to refer F-table}$$

$$\Downarrow \\ CV = 3.9715$$



\therefore , If F-test is $> 3.9715 \Rightarrow$ reject H_0 .

F-test is $< 3.9715 \Rightarrow$ accept H_0

So,

as $1.33 < 3.9715 \therefore$ accepting Null Hypothesis

S57 Conclusion :- Worker B is not more stable or efficient when compared to worker A.

