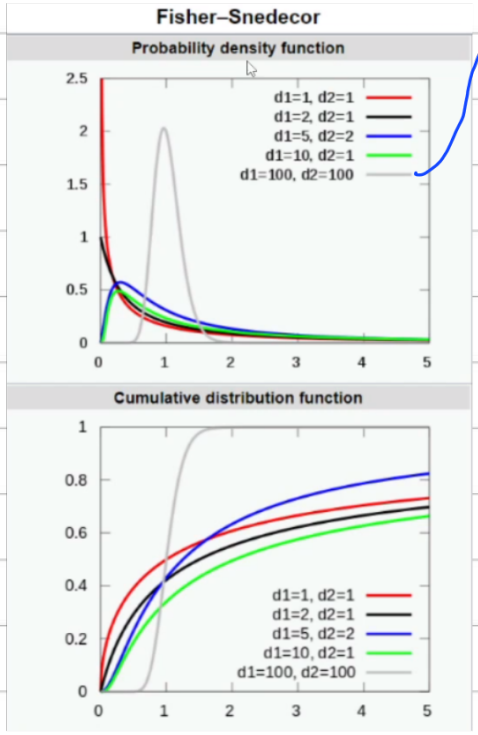
F-DISTRIBUTION

In probability theory and statistics f-distribution or f-ratio, also known as Sendecor's f-distribution or the Fisher-Sendecor distribution (after Ronald Fisher and George W. Sendecor) is a continous probability distribution that arises frequently as the null distribution of a statistic, most notably in the analysis of variance (ANOVA) and other f-tests.



☆ as d1 & d2 increases the distribution becomes on starts to look like normal distribution

$$f(x;d_1,d_2) = \frac{\sqrt{\frac{(d_1x)^{d_1}\, d_2^{d_2}}{(d_1x+d_2)^{d_1+d_2}}}}{x\,\mathrm{B}\left(\frac{d_1}{2},\frac{d_2}{2}\right)}$$

Beta function

$$\mathrm{B}(m,n) = \frac{(m-1)!\,(n-1)!}{(m+n-1)!} = \frac{m+n}{mn} \Big/ \binom{m+n}{m}.$$

☆ Parameter used : $d_1, d_2 > 0$: degree of freedom

☆ Supports : $x \in (0, +\infty)$

☆ Pdf = $$f(x, d_1, d_2) = \sqrt{\frac{(d_1 x)^{d_1}\, d_2^{d_2}}{(d_1 x + d_2)^{d_1+d_2}}} \Big/ x B\left(\frac{d_1}{2}, \frac{d_2}{2}\right)$$ — Beta function

☆ F-distribution with d_1 and d_2 degree of freedom is the distribution of Random variable such that:

$$X = \frac{S_1/d_1}{S_2/d_2}$$

, d_1 & d_2 → Respective dof for S_1 & S_2

, S_1 & S_2 → Independent random variables with chi square distribution

Chi sq distribution is a Right skewed distribution

F-test [Variance Ratio Test] :-

Q) The following data shows no. of bulbs produced daily for some days by 2 workers A and B

A	B
40	39
30	38
38	41
41	33
38	32
35	39
	40
	34

Can we consider based on the data that worker B is more sizable and efficient ($\alpha = 0.05$)

Sol^y

S1) $H_0 = \sigma_1^2 = \sigma_2^2$

S2) $H_1 = \sigma_1^2 \neq \sigma_2^2$

S3) Calculation of variance

$$s_1^2 = \frac{\Sigma (x_i - \bar{x})^2}{n-1} =$$

Worker A

x_1	$\bar{x}_1$	$(x_1 - \bar{x}_1)^2$
40	37	9
30	"	49
38	"	1
41	"	16
38	"	1
35	"	4
	$\bar{x}_1 = 37$	$\Sigma = 80$

$$s_1^2 = \frac{80}{n-1} = \frac{80}{8} = \boxed{13}$$

Worker B

x_2	$\bar{x}_2$	$(x_2 - \bar{x}_2)^2$
39	37	4
38	"	1
41	"	16
33	"	16
32	"	25
39	"	4
40	"	9
34	"	9
	$\bar{x}_2 = 37$	$\Sigma = 84$

$$s_2^2 = \frac{84}{7} = \boxed{12}$$

S4) f-test Calculation (i.e variance Ratio test)

$$f = \frac{S_1^2}{S_2^2} = \frac{13}{12} = 1.33$$

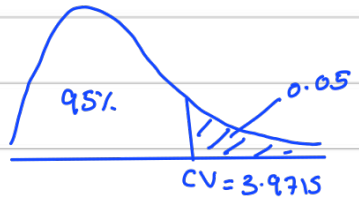
S5) Decision Boundary

$df_1 = 6-1 = 5$ & $df_2 = 8-1 = 7$ — Using this to soffer f-table

$\alpha = 0.05$

⇓

CV = 3.9715



∴, If f-tes is > 3.9715 ⇒ reject H_0.

f-test is < 3.9715 ⇒ accept H_0

So,

as 1.33 < 3.9715 ∴ accepting Null Hypothesis

S6) Conclusion :- Worker B is not more stable or efficient when compared to worker A.