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Research note

Dynamic multi-attribute decision making model based on triangular intuitionistic fuzzy numbers

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Abstract Triangular Intuitionistic Fuzzy Numbers (TIFNs) express more abundant and flexible information than Triangular Fuzzy Numbers (TFNs). The main purpose of this paper is to propose a Dynamic Multi-Attribute Decision Making (DMADM) model on the basis of TIFNs, to solve the DMADM problem, where all the decision information takes the form of TIFNs. A new distance measure between two TIFNs is developed to aid in determining attribute weights, using the entropy method. An aggregation operator, the weighted arithmetic averaging operator on TIFNs (TIFN-WAA), is presented to aggregate the decision information with TIFNs. Finally, the effectiveness and applicability of the proposed DMADM model, as well as analysis of comparison with another model, are illustrated with an investment example.

1. Introduction

Multiple-Attribute Decision Making (MADM) methods have been extensively applied to various areas, such as society, management science, economics, military research and public administration [1–5]. However, most MADM methods focus on decision making problems at the same period, such as those proposed by Ye [6] who developed a MADM model with interval-valued, intuitionistic, fuzzy numbers, and Jaskowski et al. [7] who presented an extended fuzzy AHP model for group decision making, at the same period. Greco et al. [8,9] and Blaszczynski et al. [10] extended the rough set theory into a multi-attribute decision making method, and Hu et al. [11] also extended a rough set MADM model to solve a multi-attribute decision making problem at the same period.

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In many decision areas, such as multi-period investment and personnel dynamic examination, the decision information is usually collected at different periods. Thus, it is necessary to develop some dynamic decision making models to deal with these multi-period and multi-attribute decision making problems (also known as Dynamic Multi-Attribute, Decision Making (DMADM) problems [12]). Recently, research on DMADM problems has received some attention [12–15]. Xu [12] developed a multi-period and multi-attribute decision making model based on a simple additive weighting method. Lin et al. [13] proposed a dynamic multi-attribute decision making model, where the attribute values are firstly aggregated into an overall evaluation value at each period, then all evaluation values are aggregated into an overall score of all alternatives. Xu and Yager [14] investigated a dynamic multi-attribute decision making problem where the decision information takes the form of the interval uncertain information. Wei [15] developed two aggregation operators to solve a dynamic multi-attribute decision making problem where the decision information also takes the form of the interval uncertain information. All existing research focuses on DMADM problems where the decision information takes the form of a real number or interval uncertain information. Nevertheless, in many practical cases, the available decision information is usually difficult to judge precisely; instead, they can be easily characterized by some fuzzy linguistic terms, such as "good", "poor" and so on. In addition, triangular intuitionistic fuzzy numbers in the Intuitionistic Fuzzy sets (IFs) can not only deal with vagueness information, but also express more abundant and flexible information than triangular fuzzy numbers [16,17]. Therefore, the main purpose of this paper is to propose a Dynamic Multi-Attribute Decision Making (DMADM) model on the basis of TIFNs to solve DMADM problems where the decision information takes the form of TIFNs. In addition, for the purpose of dealing with decision information with TIFNs, this paper also extends the entropy method to calculate attribute weights in the DMADM model.

2. Preliminaries

Some basic definitions of Intuitionistic Fuzzy Sets (IFSs) and Triangular Intuitionistic Fuzzy Numbers (TIFNs) are reviewed.

Definition 1. An intuitionistic fuzzy set, A, in the universe of discourse X is defined with the form [18]:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\},$$

where $\mu_A : X \to [0, 1]$, $\nu_A : X \to [0, 1]$, and with the condition:

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \quad \forall x \in X.$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ denote membership and non-membership degrees of x, with respect to A, respectively. In addition, we call:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x),$$

the degree of hesitancy of x, with respect to A [19]. Especially, if $\pi_A(x) = 0$, $\mu_A(x) = 1$, $\nu_A(x) = 0$, for all $x \in X$, then the IFSs A is reduced to a fuzzy set.

Definition 2. A triangular intuitionistic fuzzy number $\tilde{\alpha} = \langle (a, b, c); \mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}}\rangle$ is a special intuitionistic fuzzy set on a real number set $\mathfrak{R}$, whose membership function and non-membership function are defined as follows [18]:

$$\mu_{\tilde{\alpha}}(x) = \begin{cases} (x-a)\mu_{\tilde{\alpha}}/(b-a) & \text{if } a \le x < b, \\ \mu_{\tilde{\alpha}} & \text{if } x = b, \\ (c-x)\mu_{\tilde{\alpha}}/(c-b) & \text{if } b < x \le c, \\ 0 & \text{Otherwise,} \end{cases} \tag{1}$$

and:

$$\nu_{\tilde{\alpha}}(x) = \begin{cases} [b - x + \nu_{\tilde{\alpha}}(x-a)]/(b-a) & \text{if } a \ge x < b, \\ \nu_{\tilde{\alpha}} & \text{if } x = b, \\ [x - b + \nu_{\tilde{\alpha}}(c-x)]/(c-b) & \text{if } b < x \le c, \\ 1 & \text{Otherwise.} \end{cases} \tag{2}$$

The values $\mu_{\tilde{\alpha}}$ and $\nu_{\tilde{\alpha}}$ represent the maximum membership degree and the minimum non-membership degree, respectively.

Definition 3. Let $\tilde{\alpha_1} = \langle (a_1, b_1, c_1); \mu_{\tilde{\alpha_1}}, \nu_{\tilde{\alpha_1}}\rangle$ and $\tilde{\alpha_2} = \langle (a_2, b_2, c_2); \mu_{\tilde{\alpha_2}}, \nu_{\tilde{\alpha_2}}\rangle$ be two triangular intuitionistic fuzzy numbers and λ is a real number. Some arithmetical operations are defined as follows [20]:

$$\tilde{\alpha_1} \oplus \tilde{\alpha_2} = \left\langle \begin{matrix} (a_1 + a_2, b_1 + b_2, c_1 + c_2); \\ \mu_{\tilde{\alpha_1}} + \mu_{\tilde{\alpha_2}} - \mu_{\tilde{\alpha_1}}\mu_{\tilde{\alpha_2}}, \nu_{\tilde{\alpha_1}}\nu_{\tilde{\alpha_2}} \end{matrix} \right\rangle, \tag{3}$$

$$\lambda\tilde{\alpha_1} = \begin{cases} \langle (\lambda a_1, \lambda b_1, \lambda c_1); 1 - (1 - \mu_{\tilde{\alpha_1}})^{\lambda}, (\nu_{\tilde{\alpha_2}})^{\lambda}\rangle & \text{if } \lambda \ge 0 \\ \langle (\lambda c_1, \lambda b_1, \lambda a_1); 1 - (1 - \mu_{\tilde{\alpha_1}})^{\lambda}, (\nu_{\tilde{\alpha_2}})^{\lambda}\rangle & \text{if } \lambda < 0. \end{cases} \tag{4}$$

Definition 4. Let $\tilde{\alpha_i} = (a_i, b_i, c_i)$ $(i = 1, \ldots, n)$ be a set of triangular fuzzy numbers, and $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector of $\tilde{\alpha_i}$ $(i = 1, \ldots, n)$, then we call:

$$\text{TFN-WAA}_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \sum_{i=1}^{n} w_i\tilde{\alpha}_i,$$

$$= \left(\sum_{i=1}^{n} w_i a_i, \sum_{i=1}^{n} w_i b_i, \sum_{i=1}^{n} w_i c_i\right), \tag{5}$$

the weighted averaging operator on TFNs (TFN-WAA).

Definition 5. Let $\tilde{\alpha}_i = \langle (a_i, b_i, c_i); \mu_{\tilde{\alpha}_i}, \nu_{\tilde{\alpha}_i}\rangle$ be a set of triangular intuitionistic fuzzy number and $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector of $\tilde{\alpha}_i (i = 1, \ldots, n)$, then we call:

$$\text{TFN-WAA}_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \left\langle \left(\sum_{j=1}^{n} w_j a_j, \sum_{j=1}^{n} w_j b_j, \sum_{j=1}^{n} w_j c_j\right); 1 - \prod_{j=1}^{n}(1 - \mu_{\tilde{\varepsilon}_j})^{w_j}, \prod_{j=1}^{n}(\nu_{\tilde{\varepsilon}_j})^{w_j} \right\rangle, \tag{6}$$

the weighted arithmetic averaging operator on TIFNs (TIFN-WAA).

3. Weight measures using entropy method

Weight measures have a direct relationship with the distance measure between two fuzzy numbers [21]. In order to especially deal with decision information with triangular intuitionistic fuzzy numbers, this paper proposes a new distance measure.

Definition 6. Let $\tilde{A} = \langle \tilde{a}; \mu_{\tilde{A}}, \nu_{\tilde{A}}\rangle$ and $\tilde{B} = \langle \tilde{b}; \mu_{\tilde{B}}, \nu_{\tilde{B}}\rangle$ be two arbitrary triangular intuitionistic fuzzy numbers where $\tilde{a}$ and $\tilde{b}$ are two triangular fuzzy numbers with λ-cut representations, $\tilde{a}_\lambda = [a^L(\lambda), a^R(\lambda)]$ and $\tilde{b}_\lambda = [b^L(\lambda), b^R(\lambda)]$. The distance between $\tilde{A}$ and $\tilde{B}$ is defined as follows:

$$d(\tilde{A}, \tilde{B}) = \sqrt{\int_0^1 \left[\left(a^L(\lambda) - b^L(\lambda)\right)^2 + (a^R(\lambda) - b^R(\lambda))^2\right] d\lambda} + \sqrt{\frac{1}{2}((\mu_{\tilde{A}} - \mu_{\tilde{B}})^2 + (\nu_{\tilde{A}} - \nu_{\tilde{B}})^2 + (\mu_{\tilde{A}} + \nu_{\tilde{A}} - \mu_{\tilde{B}} - \nu_{\tilde{B}})^2)}. \tag{7}$$

An MADM decision making problem usually includes a set of m possible alternatives A_i $(i = 1, 2, \ldots, m)$, which is based on a set of n evaluation attributes C_j $(j = 1, 2, \ldots, n)$. The final normalized decision matrix is expressed as $\tilde{D} = [\tilde{r}_{ij}]$ where $\tilde{r}_{ij} = \langle (a_{ij}, b_{ij}, c_{ij}); \mu_{ij}, \nu_{ij}\rangle$ is the normalized rating value of the alternative A_i with respect to the attribute C_j.

TIFNs can enable decision makers to assess the alternatives in different dimensions. For example, a triangular intuitionistic fuzzy number $\tilde{r}_{ij} = \langle G; 0.7, 0.2\rangle = \langle (0.65, 0.8, 0.95); 0.7, 0.2\rangle$ can be interpreted as "the expert can not only use the linguistic variable ("Good") to rate the ith alternative, with respect to the jth attribute, but can also provide the confidence level of 0.7 at which the expert believes the ith alternative belongs to "Good", as well as the non-confidence level of 0.2, at which the expert does not believe the ith alternative belongs to "Good".

Vector M_j, as shown in Eq. (8), can be used to express deviations of the rating values, with respect to their average rating values:

$$M_j = \lfloor d(\tilde{r}_{1j}, \tilde{r}_j), \ldots, d(\tilde{r}_{ij}, \tilde{r}_j), \ldots, d(\tilde{r}_{mj}, \tilde{r}_j)\rfloor, \tag{8}$$

where $d(\tilde{r}_{ij}, \tilde{r}_j)$ is the distance between two TIFNs defined in Definition 6, and the average rating value $\tilde{r}_j = \langle(\alpha_j, \beta_j, \eta_j); \mu_j \nu_j\rangle$ can be obtained as follows:

$$\begin{aligned}\tilde{r}_j &= \langle(\alpha_j, \beta_j, \eta_j); \mu_j \nu_j\rangle \\ &= \left(\frac{1}{m}\right) \otimes (\tilde{r}_{1j} \oplus \cdots \oplus \tilde{r}_{ij} \oplus \cdots \oplus \tilde{r}_{mj}) \quad (j = 1, 2, \ldots, n),\end{aligned} \tag{9}$$

where:

$$\alpha_j = \left(\frac{1}{m}\right)\left(\sum_{i=1}^{m} a_{ij}\right), \qquad \beta_j = \left(\frac{1}{m}\right)\left(\sum_{i=1}^{m} b_{ij}\right),$$

$$\eta_j = \left(\frac{1}{m}\right)\left(\sum_{i=1}^{m} c_{ij}\right),$$

$$\mu_j = 1 - \prod_{i=1}^{m}(1-\mu_{ij})^{1/m}, \qquad \nu_j = \prod_{i=1}^{m}(\nu_{ij})^{1/m}.$$

The normalized vector for vector M_j is calculated as follows:

$$M'_j = [\rho_i] = \left[\frac{d(\tilde{r}_{ij}, \tilde{r}_j)}{\max_i d(\tilde{r}_{ij}, \tilde{r}_j)}\right], \quad i = 1, 2, \ldots, m. \tag{10}$$

The entropy measure with the jth attribute, C_j can be described as follows:

$$e_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m}\left[\frac{\rho_i}{\sum_{i=1}^{m}\rho_i} \ln\left(\frac{\rho_i}{\sum_{i=1}^{m}\rho_i}\right)\right]. \tag{11}$$

Then the entropy weight of the jth attribute can be obtained as follows:

$$w_j = \frac{1-e_j}{n - \sum_{k=1}^{n} e_k} \quad (j = 1, 2, \ldots, n). \tag{12}$$

4. Dynamic multi-attribute decision making model

A dynamic, multi-attribute decision making problem can be described as follows: the most desirable alternative needed to be found from a set of m feasible alternatives, A_i ($i = 1, 2, \ldots, m$), evaluated, with respect to a set of n attributes, C_j ($j = 1, 2, \ldots, n$), by a group of p decision makers for the periods, k. Let $w_j(t_k)$ represents the attribute weight of the jth attribute, C_j, at the kth period. And let $\lambda(t_k)$ be the period weight of the kth period. The average decision matrix at different periods can be denoted as follows:

$$\tilde{D}(t_k) = \{\tilde{x}_{ij}(t_k)\} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{bmatrix} C_1(t_k) & C_2(t_k) & \cdots & C_n(t_k) \\ \tilde{x}_{11}(t_k) & \tilde{x}_{12}(t_k) & \cdots & \tilde{x}_{1n}(t_k) \\ \tilde{x}_{21}(t_k) & \tilde{x}_{22}(t_k) & \cdots & \tilde{x}_{2n}(t_k) \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1}(t_k) & \tilde{x}_{m2}(t_k) & \cdots & \tilde{x}_{mn}(t_k) \end{bmatrix}, \tag{13}$$

where $\tilde{x}_{ij}(t_k)$ indicates the average rating values of the ith alternative, A_i, with respect to the jth attribute, $C_j(t_k)$, at the kth period, and $\tilde{x}_{ij}(t_k) = \langle(a_{ij}(t_k), b_{ij}(t_k), c_{ij}(t_k)); \mu_{ij}(t_k), \nu_{ij}(t_k)\rangle$.

4.1. DMADM model with TIFNs

This paper attempts to propose a DMADM model with TIFNs based on priority attributes. The DMADM model firstly aggregates the rating values of each alternative, with respect to each attribute at the kth period, into an overall rating vector of each alternative, with respect to the kth period, then builds all overall rating vectors for each period into a new decision matrix of each alternative, with respect to period. Finally, the final evaluation value is calculated based on the new decision matrix (the flowchart is shown in Figure 1).

Based on a priority attribute, the DMADM model can obtain not only the final evaluation value of each alternative, but also the middle evaluation value of each alternative at each period. The middle evaluation value will provide the assistant function of determining the most desirable alternative. A procedure for the DMADM model with TIFNs, based on the priority attribute, is discussed in the following section.

4.2. Procedure for DMADM model based on priority attribute

Step 1: Calculate the average rating values evaluated by p decision makers for each period, and obtain the normalized decision matrix at each period.

Let $\tilde{\zeta}_{ijs}(t_k) = \langle(a'_{ijs}(t_k), b'_{ijs}(t_k), c'_{ijs}(t_k), \mu'_{ijs}(t_k), \nu'_{ijs}(t_k))\rangle$ be the orig of the ith alternative, with respect to the jth attribute, evaluated by sth decision maker at the kth period. The average rating value of the ith alternative, A_i, with respect to the jth attribute, C_j, for p decision makers can be obtained as follows:

$$\begin{aligned}\tilde{x}_{ij}(t_k) &= \langle(a_{ij}(t_k), b_{ij}(t_k), c_{ij}(t_k)); \mu_{ij}(t_k), \nu_{ij}(t_k)\rangle \\ &= \left(\frac{1}{p}\right) \otimes \left(\tilde{\zeta}_{ij1}(t_k) \oplus \tilde{\zeta}_{ij2}(t_k) \oplus \cdots \oplus \tilde{\zeta}_{ijp}(t_k)\right).\end{aligned} \tag{14}$$

To obtain the normalized decision matrix $\tilde{N}(t_k) = \{\tilde{r}_{ij}(t_k)\}_{m\times n}$, one can employ the following normalized transformation:

$$\tilde{r}_{ij}(t_k) = \left\langle\left(\frac{a_{ij}(t_k)}{c_j^+}, \frac{b_{ij}(t_k)}{c_j^+}, \frac{c_{ij}(t_k)}{c_j^+}\right); \mu_{ij}(t_k), \nu_{ij}(t_k)\right\rangle, \quad j \in \Omega_B,$$

$$\tilde{r}_{ij}(t_k) = \left\langle\left(\frac{c_j^-}{c_{ij}(t_k)}, \frac{c_j^-}{b_{ij}(t_k)}, \frac{c_j^-}{a_{ij}(t_k)},\right); \mu_{ij}(t_k), \nu_{ij}(t_k)\right\rangle, \quad j \in \Omega_C, \tag{15}$$

where $c_j^+ = \max_i\left(c_{ij}(t_k)\right)$, $c_j^- = \min_i\left(a_{ij}(t_k)\right)$, and Ω_B, Ω_C are the benefit and cost attribute sets, respectively.

Step 2: Calculate the attribute weights at each period as $w_j(t_k)$ ($j = 1, 2, \ldots, n$) based on the normalized decision matrices, using the entropy method with TIFNs.

Step 3: Aggregate the normalized decision matrices, $\tilde{N}(t_k) = \{\tilde{r}_{ij}(t_k)\}_{m\times n}$, for all periods into a final decision matrix, $\tilde{R} = \{\tilde{\xi}_{ik}\}_{m\times k}$. Utilizing the TIFN-WAA operator in Definition 5;

$$\begin{aligned}\tilde{\xi}_{ik} &= \text{TIFN-WAA}_{w_j(t_k)}(\tilde{r}_{i1}(t_k), \\ &\qquad \tilde{r}_{i2}(t_k), \ldots, \tilde{r}_{in}(t_k)) \\ &= w_1(t_k)\tilde{r}_{i1}(t_k) \oplus w_2(t_k)\tilde{r}_{i2}(t_k) \\ &\qquad \oplus \cdots \oplus w_n(t_k)\tilde{r}_{in}(t_k),\end{aligned}$$

to aggregate all normalized decision matrices $\tilde{N}(t_k) = \{\tilde{r}_{ij}(t_k)\}_{m\times n}$, into a final decision matrix, $\tilde{R} = \{\tilde{\xi}_{ik}\}_{m\times k}$.

$$\begin{array}{c} \begin{array}{c|cccc} & C_1(t_k) & C_2(t_k) & \cdots & C_x(t_k) \\ \hline A_1 & \tilde{x}_{11}(t_k) & \tilde{x}_{12}(t_k) & \cdots & \tilde{x}_{1n}(t_k) \\ A_2 & \tilde{x}_{21}(t_k) & \tilde{x}_{22}(t_k) & \cdots & \tilde{x}_{2n}(t_k) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_m & \tilde{x}_{m1}(t_k) & \tilde{x}_{m2}(t_k) & \cdots & \tilde{x}_{mn}(t_k) \end{array} \\ (k = 1, 2, \ldots, K) \end{array} \xrightarrow{\text{Aggregate}} \begin{array}{c} \begin{array}{c|c} & t_k \\ \hline A_1 & \tilde{r}_{1k} \\ A_2 & \tilde{r}_{2k} \\ \vdots & \vdots \\ A_m & \tilde{r}_{mk} \end{array} \\ (k = 1, 2, \ldots, K) \end{array} \xrightarrow{\text{Build}} \begin{array}{c|cccc} & t_1 & t_2 & \cdots & t_K \\ \hline A_1 & \tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1K} \\ A_2 & \tilde{r}_{21} & \tilde{r}_{22} & \cdots & \tilde{r}_{2K} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_m & \tilde{r}_{m1} & \tilde{r}_{m2} & \cdots & \tilde{r}_{mK} \end{array}$$

Figure 1: Flowchart of the DMADM model based on priority attribute.

Table 1: Original rating values evaluated by three decision makers at the t_1 period.

	C_1		
	DM_1	DM_2	DM_3
A_1	⟨G; 0.7, 0.2⟩	⟨G; 0.8, 0.1⟩	⟨G; 0.9, 0.1⟩
A_2	⟨VG; 0.8, 0.1⟩	⟨VG; 0.9, 0.1⟩	⟨VG; 0.8, 0.2⟩
A_3	⟨G; 0.7, 0.2⟩	⟨VG; 0.6, 0.4⟩	⟨G; 0.8, 0.1⟩
A_4	⟨VG; 0.6, 0.3⟩	⟨VG; 0.7, 0.2⟩	⟨VG :, 0.6, 0.4⟩
A_5	⟨G; 0.7, 0.2⟩	⟨VG; 0.7, 0.3⟩	⟨G; 0.6, 0.3⟩
	C_2		
A_1	⟨G; 0.6, 0.3⟩	⟨VG; 0.7, 0.2⟩	⟨G; 0.6, 0.4⟩
A_2	⟨G; 0.5, 0.4⟩	⟨MG; 0.7, 0.2⟩	⟨MG; 0.7, 0.2⟩
A_3	⟨VG; 0.8, 0.1⟩	⟨VG; 0.9, 0.1⟩	⟨VG; 0.8, 0.2⟩
A_4	⟨G; 0.8, 0.1⟩	⟨G; 0.7, 0.2⟩	⟨G; 0.8, 0.1⟩
A_5	⟨G : 0.7, 0.3⟩	⟨VG; 0.7, 0.2⟩	⟨G; 0.8, 0.1⟩
	C_3		
A_1	⟨F; 0.8, 0.2⟩	⟨F; 0.8, 0.1⟩	⟨F; 0.7, 0.2⟩
A_2	⟨F; 0.7, 0.2⟩	⟨MP; 0.5, 0.4⟩	⟨F; 0.8, 0.1⟩
A_3	⟨MP; 0.8, 0.1⟩	⟨MP; 0.9, 0.1⟩	⟨MP; 0.8, 0.2⟩
A_4	⟨MP; 0.7, 0.2⟩	⟨F; 0.6, 0.3⟩	⟨MP; 0.5, 0.4⟩
A_5	⟨F; 0.6, 0.3⟩	⟨MG; 0.7, 0.2⟩	⟨F; 0.8, 0.1⟩

Table 2: Original rating values evaluated by three decision makers at the t_2 period.

	C_1		
	DM_1	DM_2	DM_3
A_1	⟨G; 0.8, 0.1⟩	⟨G; 0.9, 0.1⟩	⟨G; 0.8, 0.2⟩
A_2	⟨VG; 0.8, 0.1⟩	⟨VG; 0.7, 0.2⟩	⟨VG; 0.9, 0.1⟩
A_3	⟨G; 0.9, 0.1⟩	⟨VG; 0.5, 0.4⟩	⟨G; 0.8, 0.1⟩
A_4	⟨G; 0.8, 0.2⟩	⟨G; 0.7, 0.2⟩	⟨VG; 0.5, 0.4⟩
A_5	⟨VG; 0.8, 0.2⟩	⟨VG; 0.8, 0.1⟩	⟨VG; 0.7, 0.2⟩
	C_2		
A_1	⟨VG; 0.8, 0.1⟩	⟨G; 0.5, 0.4⟩	⟨VG; 0.6, 0.3⟩
A_2	⟨G; 0.7, 0.2⟩	⟨G; 0.8, 0.1⟩	⟨VG; 0.5, 0.4⟩
A_3	⟨G; 0.9, 0.1⟩	⟨G; 0.8, 0.2⟩	⟨G; 0.7, 0.2⟩
A_4	⟨G; 0.6, 0.3⟩	⟨MG; 0.5, 0.3⟩	⟨G; 0.7, 0.2⟩
A_5	⟨VG; 0.8, 0.1⟩	⟨VG; 0.8, 0.2⟩	⟨G; 0.5, 0.4⟩
	C_3		
A_1	⟨F; 0.8, 0.1⟩	⟨MP; 0.5, 0.4⟩	⟨F; 0.7, 0.2⟩
A_2	⟨MP; 0.8, 0.2⟩	⟨F; 0.5, 0.2⟩	⟨MP; 0.9, 0.1⟩
A_3	⟨MP; 0.8, 0.2⟩	⟨F; 0.5, 0.4⟩	⟨MP; 0.7, 0.2⟩
A_4	⟨F; 0.9, 0.1⟩	⟨F; 0.8, 0.1⟩	⟨F; 0.8, 0.2⟩
A_5	⟨MG; 0.7, 0.2⟩	⟨F; 0.8, 0.1⟩	⟨MG; 0.7, 0.2⟩

Step 4: Define $\gamma^+ = (\gamma_1^+, \ldots, \gamma_k^+, \ldots, \gamma_K^+)^T$ and $\gamma^- = (\gamma_1^-, \ldots, \gamma_k^-, \ldots, \gamma_K^-)^T$ as the triangular intuitionistic fuzzy positive ideal solution (TIFPIS) and the triangular intuitionistic fuzzy negative ideal solution (TIFNIS), respectively. According to the normalized decision matrix, we know that $\gamma_k^+ = \langle(1, 1, 1); 1, 0\rangle$ are the largest TIFNs, and $\gamma_k^+ = \langle(0, 0, 0); 0, 1\rangle$ are the smallest TIFNs. In addition, for convenience of depiction, we denote the alternative, $A_i(i = 1, 2, \ldots, m)$, by $A_i = (\tilde{\xi}_{i1}, \tilde{\xi}_{i2}, \ldots, \tilde{\xi}_{iK})$.

Step 5: Calculate the period weights in the decision matrix, $\tilde{R} = \{\tilde{\xi}_{ik}\}_{m\times k}$, as $\lambda(t_k)$, using the entropy method with TIFNs.

Step 6: Calculate the distance between the alternative and the TIFPIS (γ^+), and the distance between the alternative and the TIFNIS (γ^-) according to the following distance measure defined in Definition 6:

$$D(A_i, \gamma^+) = \sum_{k=1}^{K} \left[\lambda(t_k) d\left(\tilde{\xi}_{ik}, \tilde{\gamma}_k^+\right)\right], \quad (i = 1, 2, \ldots, m),$$
$$D(A_i, \gamma^-) = \sum_{k=1}^{K} \left[\lambda(t_k) d\left(\tilde{\xi}_{ik}, \tilde{\gamma}_k^-\right)\right], \quad (i = 1, 2, \ldots, m). \tag{16}$$

Step 7: Calculate the closeness coefficient, $CC_i(i = 1, 2, \ldots, m)$, of all alternatives and rank all alternatives, $A_i(i = 1, 2, \ldots, m)$, according to the closeness coefficient:

$$CC_i = \frac{D(A_i, \gamma^-)}{D(A_i, \gamma^+) + D(A_i, \gamma^-)}, \quad (i = 1, 2, \ldots, m). \tag{17}$$

5. Illustrative example

One example [14] is provided to demonstrate the effectiveness and applicability of the DMADM model based on a priority attribute. An investment company wants to invest an amount of money. There are five possible companies, $A_i(i = 1, 2, \ldots, 5)$, in which to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is an arms company; and (5) A_5 is a TV company. A group of decision makers is formed with three decision makers, $DM_s(s = 1, 2, 3)$. Each possible company will be evaluated across three attributes with regard to: (1) economic benefit (C_1); (2) social benefit (C_2); and (3) environmental pollution (C_3), where C_1 and C_2 are benefit attributes and C_3 is a cost attribute. The rating values of five possible companies, with respect to three attributes, are represented by TIFNs, and the three decision makers construct the original rating values at three periods, as listed in Tables 1–3.

5.1. Ranking alternatives using the DMADM model with TIFNs

The calculation procedure for the DMADM model with TIFNs, based on a priority attribute, is described step by step, as below:

Step 1: Calculate the average rating values evaluated by three decision makers, according to Eq. (14), and obtain the normalized decision matrices using Eq. (15). The normalized decision matrices, $\tilde{N}(t_k) = \{\tilde{r}_{ij}(t_k)\}_{5\times 3}$ $(k = 1, 2, 3)$ for three periods are shown in Tables 4–6.

Step 2: Calculate the attribute weights at each period. Using Eq. (12), the attribute weights, $w_j(t_k)$ $(j = 1, 2, 3)$, for three periods, are shown in Tables 4–6.

Table 3: Original rating values evaluated by three decision makers at the t_3 period.

	DM_1	DM_2	DM_3
	C_1		
A_1	⟨VG; 0.8, 0.1⟩	⟨G; 0.6, 0.3⟩	⟨VG; 0.8, 0.1⟩
A_2	⟨G; 0.7, 0.2⟩	⟨VG; 0.5, 0.4⟩	⟨G; 0.8, 0.2⟩
A_3	⟨VG; 0.9, 0.1⟩	⟨VG; 0.8, 0.2⟩	⟨VG; 0.8, 0.1⟩
A_4	⟨VG; 0.8, 0.2⟩	⟨VG; 0.5, 0.4⟩	⟨G; 0.8, 0.1⟩
A_5	⟨VG; 0.7, 0.2⟩	⟨VG; 0.9, 0.1⟩	⟨VG; 0.8, 0.2⟩
	C_2		
A_1	⟨G; 0.7, 0.1⟩	⟨VG; 0.5, 0.3⟩	⟨G; 0.8, 0.1⟩
A_2	⟨VG; 0.8, 0.1⟩	⟨G; 0.6, 0.3⟩	⟨VG; 0.8, 0.2⟩
A_3	⟨G; 0.8, 0.2⟩	⟨G; 0.8, 0.1⟩	⟨G; 0.7, 0.2⟩
A_4	⟨G; 0.8, 0.1⟩	⟨G; 0.9, 0.1⟩	⟨G; 0.8, 0.2⟩
A_5	⟨G; 0.7, 0.2⟩	⟨VG; 0.8, 0.1⟩	⟨G; 0.8, 0.1⟩
	C_3		
A_1	⟨MP; 0.7, 0.2⟩	⟨P; 0.5, 0.4⟩	⟨MP; 0.7, 0.2⟩
A_2	⟨MP; 0.8, 0.1⟩	⟨MP; 0.8, 0.1⟩	⟨MP; 0.8, 0.1⟩
A_3	⟨F; 0.7, 0.2⟩	⟨MP; 0.5, 0.4⟩	⟨F; 0.8, 0.2⟩
A_4	⟨MP; 0.5, 0.4⟩	⟨F; 0.8, 0.1⟩	⟨F; 0.8, 0.1⟩
A_5	⟨F; 0.8, 0.1⟩	⟨F; 0.8, 0.1⟩	⟨F; 0.8, 0.1⟩

Step 3: Utilize the TIFN-WAA operator to aggregate the normalized decision matrices $\tilde{N}(t_k) = \{\tilde{r}_{ij}(t_k)\}_{5\times 3}$ into a final decision matrix, $\tilde{R} = \{\tilde{\xi}_{ik}\}_{5\times 3}$, as shown in Table 7.

Step 4: Define the TIFPIS, γ^+, and TIFNIS, γ^-, by:

$$\gamma^+ = (((1, 1, 1); 1, 0), ((1, 1, 1); 1, 0), ((1, 1, 1); 1, 0))^T;$$

$$\gamma^- = (((0, 0, 0); 0, 1), ((0, 0, 0); 0, 1), ((0, 0, 0); 0, 1))^T.$$

Step 5: Calculate the period weights, $\lambda(t_k)$ $(k = 1, 2, 3)$, in the decision matrix $\tilde{R} = \{\tilde{\xi}_{ik}\}_{5\times 3}$, as shown in Table 7.

Step 6: Calculate the distance between the alternative and the TIFPIS (γ^+), and the distance between the alternative and the TIFNIS (γ^-) using Eq. (16), respectively.

Step 7: Using Eq. (17), the closeness coefficient, CC_i $(i = 1, 2, 3)$, can be obtained.

The distances, closeness coefficient and ranking order of five alternatives are tabulated in Table 8. We can see that the ranking order is "$A_3 \succ A_5 \succ A_2 \succ A_1 \succ A_4$", where "$\succ$" indicates the relation "preferred to".

5.2. Comparison analysis

If we do not consider the membership and non-membership degrees in TIFNs, i.e. $\mu_{ij} = 1$ and $\nu_{ij} = 0$, in the original rating values, then the rating values with TIFNs in Tables 1–3 will be reduced to values with TFNs, and the DMADM problem with TIFNs will also be reduced to the DMADM problem with TFNs. To verify the effectiveness and applicability of the proposed DMADM model with TIFNs, we also rank the alternatives using the DMADM model with TFNs, which is based on the normalized Euclidean distance and the weighted averaging operator on TFNs (TFN-WAA) in Definition 4. Table 9 shows the final ranking results obtained by the two DMADM models.

As shown in Table 9, it is not very difficult to observe that the most desirable alternative, A_3, presented by the DMADM model with TFNs, is the same as the result obtained through the DMADM model with TIFNs, but the ranking order obtained by the DMADM model with TFNs is somewhat different from the order presented through the DMADM model with TIFNs. These results show that the membership and non-membership degrees in TIFNs play an important role in determining the ranking order in the DMADM model with TIFNs, and they also provide more exact and abundant decision information. Therefore, we can draw a conclusion that the ranking order obtained by the DMADM model with TIFNs is more reliable than the order presented through the DMADM model with TFNs.

5.3. Sensitivity analysis

Sensitivity analysis (SA) is the investigation of some potential changes and errors of rating values and their impact on the final ranking order [22]. In this paper, some sensitivity analyses are conducted to investigate the impact of changing the membership and non-membership degrees in the rating values on the alternatives' ranking order. A slight variation in the original rating values evaluated by decision makers goes as follows:

$$\tilde{x}_{ijs}(t_k) = \left\langle \begin{array}{c} (a_{ijs}(t_k), b_{ijs}(t_k), c_{ijs}(t_k)); \\ \mu_{ijs}(t_k) + q.h, \nu_{ijs}(t_k) - q.h \end{array} \right\rangle, \tag{18}$$

where $q = -\Delta\sigma_j/h, \ldots, -1, 0, 1, \ldots, \Delta\sigma_j/h$, h is the step size, and $\lfloor -\Delta\sigma_j, \Delta\sigma_j \rfloor$ $(j = 1, 2, 3)$ are the variation intervals of the membership and non-membership degrees, with respect to three attributes.

Table 4: Normalized decision matrix, $\tilde{N}(t_1)$, at the t_1 period.

	C_1	C_2	C_3
A_1	⟨(0.65, 0.8, 0.95); 0.8183, 0.126⟩	⟨(0.7, 0.8667, 0.9667); 0.6366, 0.2884⟩	⟨(0.3077, 0.4, 0.5714); 0.7711, 0.1587⟩
A_2	⟨(0.8, 1, 1); 0.8413, 0.126⟩	⟨(0.55, 0.7, 0.85); 0.6443, 0.252⟩	⟨(0.3333, 0.4444, 0.6667); 0.6893, 0.2⟩
A_3	⟨(0.7, 0.8667, 0.9667); 0.7116, 0.2⟩	⟨(0.8, 1, 1); 0.8413, 0.126⟩	⟨(0.4, 0.5714, 1); 0.8413, 0.126⟩
A_4	⟨(0.8, 1, 1); 0.6366, 0.2884⟩	⟨(0.65, 0.8, 0.95); 0.7711, 0.126⟩	⟨(0.3636, 0.5, 0.8); 0.6085, 0.2884⟩
A_5	⟨(0.7, 0.8667, 0.9667); 0.6698, 0.2621⟩	⟨(0.7, 0.8667, 0.9667); 0.7379, 0.1817⟩	⟨(0.2857, 0.3636, 0.5); 0.7116, 0.1817⟩
$w(t_1)$	0.19698	0.47834	0.32468

Table 5: Normalized decision matrix, $\tilde{N}(t_2)$, at the t_2 period.

	C_1	C_2	C_3
A_1	⟨(0.65, 0.8, 0.95); 0.8413, 0.126⟩	⟨(0.7627, 0.9492, 1); 0.658, 0.2289⟩	⟨(0.4167, 0.5556, 0.8333); 0.6893, 0.2⟩
A_2	⟨(0.8, 1, 1); 0.8183, 0.126⟩	⟨(0.7119, 0.8814, 0.9831); 0.6893, 0.2⟩	⟨(0.4545, 0.625, 1); 0.7846, 0.1587⟩
A_3	⟨(0.7, 0.8667, 0.9667); 0.7846, 0.1587⟩	⟨(0.661, 0.8136, 0.9661); 0.8183, 0.1587⟩	⟨(0.4545, 0.625, 1); 0.6893, 0.252⟩
A_4	⟨(0.7, 0.8667, 0.9667); 0.6893, 0.252⟩	⟨(0.6102, 0.7627, 0.9153); 0.6085, 0.2621⟩	⟨(0.3846, 0.5, 0.7143); 0.8413, 0.126⟩
A_5	⟨(0.8, 1, 1); 0.7711, 0.1587⟩	⟨(0.7627, 0.9492, 1); 0.7286, 0.2⟩	⟨(0.3333, 0.4167, 0.5556); 0.7379, 0.1587⟩
$w(t_2)$	0.24862	0.47255	0.27883

Table 6: Normalized decision matrix, $\tilde{N}(t_3)$, at the t_3 period.

	C_1	C_2	C_3
A_1	$\langle(0.75, 0.9333, 0.9833); 0.748, 0.1442\rangle$	$\langle(0.7119, 0.8814, 0.9831); 0.6893, 0.1442\rangle$	$\langle(0.3333, 0.5, 1); 0.6443, 0.252\rangle$
A_2	$\langle(0.7, 0.8667, 0.9667); 0.6893, 0.252\rangle$	$\langle(0.7627, 0.9492, 1); 0.748, 0.1817\rangle$	$\langle(0.3, 0.4286, 0.75); 0.8, 0.1\rangle$
A_3	$\langle(0.8, 1, 1); 0.8413, 0.126\rangle$	$\langle(0.661, 0.8136, 0.9661); 0.7711, 0.1587\rangle$	$\langle(0.25, 0.3333, 0.5); 0.6893, 0.252\rangle$
A_4	$\langle(0.75, 0.9333, 0.9833); 0.7286, 0.2\rangle$	$\langle(0.661, 0.8136, 0.9661); 0.8413, 0.126\rangle$	$\langle(0.25, 0.3333, 0.5); 0.7286, 0.1587\rangle$
A_5	$\langle(0.8, 1, 1); 0.8183, 0.1587\rangle$	$\langle(0.7119, 0.8814, 0.9831); 0.7711, 0.126\rangle$	$\langle(0.2308, 0.3, 0.4286); 0.8, 0.1\rangle$
$w(t_3)$	0.3869	0.31983	0.29327

Table 7: Final decision matrix, $\tilde{R}\{\tilde{\xi}_{ik}\}_{m\times k}$.

	t_1	t_2	t_3
A_1	$\langle(0.5628, 0.702, 0.8351); 0.7271, 0.2018\rangle$	$\langle(0.6382, 0.8023, 0.9411); 0.7249, 0.1901\rangle$	$\langle(0.6156, 0.7896, 0.9881); 0.7019, 0.1699\rangle$
A_2	$\langle(0.5289, 0.6761, 0.82); 0.7096, 0.2039\rangle$	$\langle(0.662, 0.8394, 0.992); 0.7545, 0.1672\rangle$	$\langle(0.6028, 0.7646, 0.9138); 0.7446, 0.1731\rangle$
A_3	$\langle(0.6504, 0.8346, 0.9934); 0.8214, 0.138\rangle$	$\langle(0.6131, 0.7742, 0.9757); 0.7798, 0.1806\rangle$	$\langle(0.5943, 0.7449, 0.8425); 0.7827, 0.1662\rangle$
A_4	$\langle(0.5866, 0.742, 0.9111); 0.7015, 0.1941\rangle$	$\langle(0.5696, 0.7153, 0.872); 0.7126, 0.2116\rangle$	$\langle(0.5749, 0.7191, 0.8361); 0.7714, 0.1612\rangle$
A_5	$\langle(0.5655, 0.7033, 0.8151); 0.717, 0.1953\rangle$	$\langle(0.6523, 0.8133, 0.8761); 0.7423, 0.1771\rangle$	$\langle(0.6049, 0.7568, 0.827); 0.7988, 0.1287\rangle$
$\lambda(t_1)$	0.45507	0.28064	0.26429

Table 8: Distances, closeness coefficient and ranking order of five alternatives.

Alternatives	$D(A_i, \gamma^+)$	$D(A_i, \gamma^-)$	CC_i	Rank
A_1	1.1526	1.6994	0.59586	4
A_2	1.1446	1.7031	0.59805	3
A_3	0.93287	1.8046	0.65922	1
A_4	1.1918	1.6835	0.58551	5
A_5	1.1168	1.7151	0.60563	2

Table 9: Final ranking results obtained by the two DMADM models.

	The DMADM Model with TFNs		The DMADM Model with TIFNs	
	CC_i	Ranking	CC_i	Ranking
A_1	0.51454	2	0.59586	4
A_2	0.50153	3	0.59805	3
A_3	0.56709	1	0.65922	1
A_4	0.49184	4	0.58551	5
A_5	0.49123	5	0.60563	2

As shown in Figures 2–4, the ranking order of all alternatives will remain constant when the variation values of the membership and non-membership degrees, with respect to the three attributes, vary in the range from −0.06 to 0.04. But over the range, the ranking order of the two alternatives A_1 and A_2 will change with the membership and non-membership degrees, with respect to the three attributes. It demonstrates that the alternatives A_1 and A_2 are more sensitive to membership and non-membership degrees than the other three alternatives.

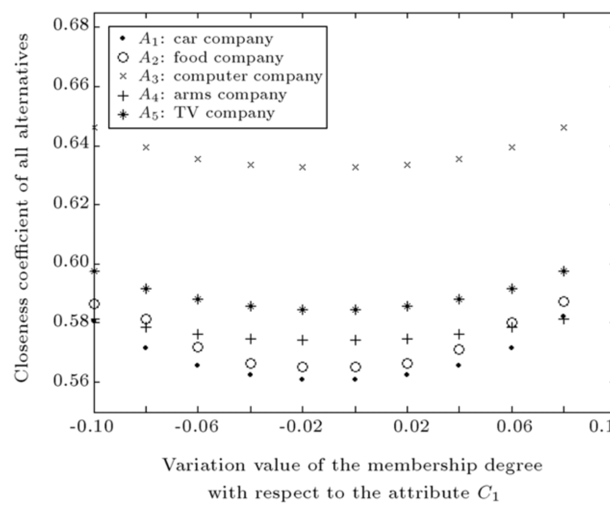

Figure 2: Ranking order sensitivity to the membership and non-membership degrees with respect to the first attribute, C_1.

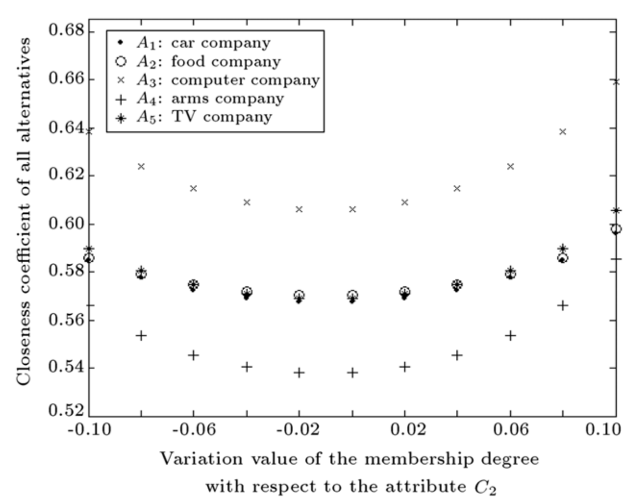

Figure 3: Ranking order sensitivity to the membership and non-membership degrees with respect to the second attribute, C_2.

6. Conclusions

In this paper, a DMADM model with triangular intuitionistic fuzzy numbers is presented to deal with vagueness and uncertain information. Using an aggregation operator (TIFN-WAA), a procedure for the DMADM model with TIFNs, based on the priority attribute, is developed to solve the DMADM problem, where all attribute values are expressed in triangular intuitionistic fuzzy numbers. In addition, a new distance measure between two TIFNs is proposed to determine the entropy weights in the DMADM model.

Another DMADM model with TFNs is established to compare with the DMADM model with TIFNs. The comparison results demonstrate that the DMADM model with TIFNs can provide a more reliable ranking order than the DMADM model with TFNs.

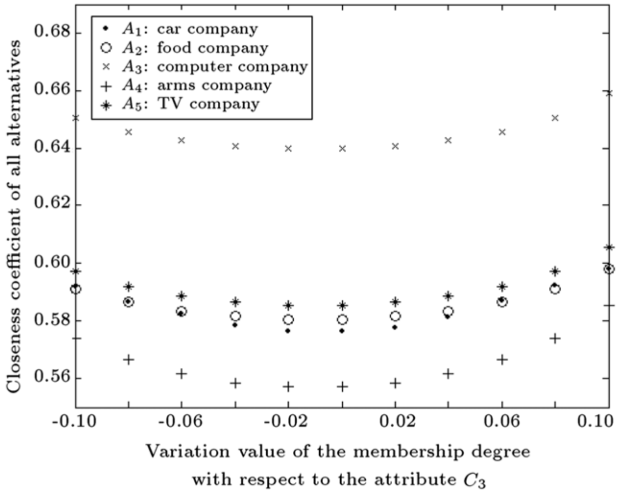

Figure 4: Ranking order sensitivity to the membership and non-membership degrees with respect to the third attribute, C_3.

Although the example provided here is for selecting a desirable investment company, the proposed model can be applied to many different fields.

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