Defuzzification via Center of Area

André Siviero

August 24, 2012

Abstract

Mathematical detailment which serves as a support to Matlab code that computes the defuzzified value to an Intuitionistic Fuzzy Number (IFN) via Center of Area

1 Center of Area Method

As seen on Angelov, the Center of Area Method applied to IFNs is defined as:

$$x_{IF\_COA} = \frac{\sum_a^d (\mu(x_i) - v(x_i))x_i}{\sum_{i=1}^N (\mu(x_i) - v(x_i))}$$

However, in continuous fuzzy sets, this sum is turned into an integral:

$$x_{IF\_COA} = \frac{\int_a^d (\mu(x) - v(x))xdx}{\int_a^d (\mu(x) - v(x))dx}$$

1.1 Numerator

It can ben seen in Chen & Li that the definitions of pertinence and non-pertinence functions would lead to the numerator to be divided in 3 integrals:

$$\int_a^d (\mu(x) - v(x))xdx = NA_a^b + NB_b^c + NC_c^d$$

Each of them is gonna be calculated separately and each of these factors corresponds to 2 lines of code in the I4FN_defuzzificationCOA implemmentation

$$NA_a^b = \int_a^b x \left[\frac{\tilde{\mu_a}(x-a)}{b-a} - \frac{b - x + \tilde{v_a}(x-a)}{b-a} \right] dx =$$

$$\int_a^b \frac{x}{b-a} \left[x(\tilde{\mu_a} + 1 - \tilde{v_a}) + (a\tilde{v_a} - b - a\tilde{\mu_a}) \right] dx =$$

$$\left[\frac{x^3}{3} \left(\frac{\tilde{\mu_a} + 1 - \tilde{v_a}}{b-a} \right) + \frac{x^2}{2} \left(\frac{a\tilde{v_a} - b - a\tilde{\mu_a}}{b-a} \right) \right]_a^b =$$

$$\frac{b^3 - a^3}{3} \left(\frac{\tilde{\mu_a} + 1 - \tilde{v_a}}{b-a} \right) + \frac{b^2 - a^2}{2} \left(\frac{a\tilde{v_a} - b - a\tilde{\mu_a}}{b-a} \right)$$

$$NB_b^c = \int_b^c (\tilde{\mu_a} - \tilde{v_a}) x dx = \left[\frac{x^2}{2}(\tilde{\mu_a} - \tilde{v_a})\right]_b^c = \frac{c^2 - b^2}{2}(\tilde{\mu_a} - \tilde{v_a})$$

$$NC_c^d = \int_c^d x \left[\frac{\tilde{\mu_a}(d-x)}{d-c} - \frac{x - c + \tilde{v_a}(d-x)}{d-c}\right] dx =$$

$$\int_c^d \frac{x}{d-c} \left[x(-\tilde{\mu_a} - 1 + \tilde{v_a}) + (d\tilde{\mu_a} + c - d\tilde{v_a})\right] dx =$$

$$\left[\frac{x^3}{3}\left(\frac{-\tilde{\mu_a} - 1 + \tilde{v_a}}{d-c}\right) + \frac{x^2}{2}\left(\frac{d\tilde{\mu_a} + c - d\tilde{v_a}}{d-c}\right)\right]_c^d =$$

$$\frac{d^3 - c^3}{3}\left(\frac{-\tilde{\mu_a} - 1 + \tilde{v_a}}{d-c}\right) + \frac{d^2 - c^2}{2}\left(\frac{d\tilde{\mu_a} + c - d\tilde{v_a}}{b-a}\right)$$

By summing these factors, the numerator is obtained. Next the denominator is obtained, in a very similar calculation:

1.2 Denominator

$$\int_a^d (\mu(x) - v(x)) dx = DA_a^b + DB_b^c + DC_c^d$$

$$DA_a^b = \int_a^b \left[\frac{\tilde{\mu_a}(x-a)}{b-a} - \frac{b - x + \tilde{v_a}(x-a)}{b-a}\right] dx =$$

$$\int_a^b \frac{1}{b-a} \left[x(\tilde{\mu_a} + 1 - \tilde{v_a}) + (a\tilde{v_a} - b - a\tilde{\mu_a})\right] dx =$$

$$\left[\frac{x^2}{2}\left(\frac{\tilde{\mu_a} + 1 - \tilde{v_a}}{b-a}\right) + x\left(\frac{a\tilde{v_a} - b - a\tilde{\mu_a}}{b-a}\right)\right]_a^b =$$

$$\frac{b^2 - a^2}{2}\left(\frac{\tilde{\mu_a} + 1 - \tilde{v_a}}{b-a}\right) + (a\tilde{v_a} - b - a\tilde{\mu_a})$$

$$DB_b^c = \int_b^c (\tilde{\mu_a} - \tilde{v_a}) dx = [x(\tilde{\mu_a} - \tilde{v_a})]_b^c = (c-b)(\tilde{\mu_a} - \tilde{v_a})$$

$$DC_c^d = \int_c^d \left[\frac{\tilde{\mu_a}(d-x)}{d-c} - \frac{x - c + \tilde{v_a}(d-x)}{d-c}\right] dx =$$

$$\int_c^d \frac{1}{d-c} \left[x(-\tilde{\mu_a} - 1 + \tilde{v_a}) + (d\tilde{\mu_a} + c - d\tilde{v_a})\right] dx =$$

$$\left[\frac{x^2}{2}\left(\frac{-\tilde{\mu_a} - 1 + \tilde{v_a}}{d-c}\right) + x\left(\frac{d\tilde{\mu_a} + c - d\tilde{v_a}}{d-c}\right)\right]_c^d =$$

$$\frac{d^2 - c^2}{2}\left(\frac{-\tilde{\mu_a} - 1 + \tilde{v_a}}{d-c}\right) + (d\tilde{\mu_a} + c - d\tilde{v_a})$$

1.3 Defuzzified Number

Finally, the Defuzzified number x_{IF_COA} is:

$$x_{IF\_COA} = \frac{N}{D} = \frac{NA_a^b + NB_b^c + NC_c^d}{DA_a^b + DB_b^c + DC_c^d}$$