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1 Basic

1.1 .vimrc

```
syn on
se ai nu ru cul mouse=a
se cin et ts=4 sw=4 sts=4
so $VIMRUNTIME/mswin.vim
colo desert
se gfn=Monospace\ 14
```

1.2 Misc

```
#include <random>
mt19937 rng(0x5EED);
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(rng); }

#define SECS (clock() / CLOCKS_PER_SEC)

struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    }
};
typedef unordered_map<Key,int,KeyHasher> map_t;
```

1.3 python-related

```
from fractions import Fraction
from decimal import Decimal, getcontext
getcontext().prec = 250 # set precision

itwo = Decimal(0.5)
two = Decimal(2)

N = 200
def angle(cosT):
    """given cos(theta) in decimal return theta"""
    for i in range(N):
        cosT = ((cosT + 1) / two) ** itwo
        sinT = (1 - cosT * cosT) ** itwo
        return sinT * (2 ** N)
pi = angle(Decimal(-1))
```

2 flow

2.1 ISAP

```
#define SZ(c) ((int)(c).size())
struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r):
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
    void init(int x) {
        tot = x+2;
        s = x+1, t = x+2;
        for(int i = 0; i <= tot; i++) {
            G[i].clear();
            iter[i] = d[i] = gap[i] = 0;
        }
    }
    void addEdge(int u, int v, int c) {
        G[u].push_back(Edge(v, c, SZ(G[v])));
        G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
    }
};
```

2.2 MinCostFlow

```

    }
}
if(mom[t] == -1) break ;
int df = INFF;
for(int u = t; u != s; u = mom[u])
    df = min(df, g[mom[u]][id[u]].cap);
for(int u = t; u != s; u = mom[u]){
    Edge &e = g[mom[u]][id[u]];
    e.cap -= df;
    g[e.v][e.rev].cap += df;
}
mxf += df;
mnc += df*d[t];
}
return mnc;
}
} flow;

```

2.3 Dinic

2.4 Kuhn Munkres

```
struct KM{
// Maximum Bipartite Weighted Matching (Perfect Match)
static const int MXN = 650;
static const int INF = 2147483647; // LL
int n,match[MXN],vx[MXN],vy[MXN];
```

```

int edge[MXN][MXN],lx[MXN],ly[MXN],slack[MXN];
// **** LL
void init(int _n){
    n = _n;
    for(int i=0; i<n; i++) for(int j=0; j<n; j++)
        edge[i][j] = 0;
}
void addEdge(int x, int y, int w) // LL
{ edge[x][y] = w; }
bool DFS(int x){
    vx[x] = 1;
    for (int y=0; y<n; y++){
        if (vy[y]) continue;
        if (lx[x]+ly[y] > edge[x][y]){
            slack[y]=min(slack[y], lx[x]+ly[y]-edge[x][y]);
        } else {
            vy[y] = 1;
            if (match[y] == -1 || DFS(match[y]))
                { match[y] = x; return true; }
        }
    }
    return false;
}
int solve(){
    fill(match,match+n,-1);
    fill(lx,lx+n,-INF); fill(ly,ly+n,0);
    for (int i=0; i<n; i++)
        for (int j=0; j<n; j++)
            lx[i] = max(lx[i], edge[i][j]);
    for (int i=0; i<n; i++){
        fill(slack,slack+n,INF);
        while (true){
            fill(vx,vx+n,0); fill(vy,vy+n,0);
            if ( DFS(i) ) break;
            int d = INF; // long long
            for (int j=0; j<n; j++)
                if (!vy[j]) d = min(d, slack[j]);
            for (int j=0; j<n; j++){
                if (vx[j]) lx[j] -= d;
                if (vy[j]) ly[j] += d;
                else slack[j] -= d;
            }
        }
    }
    int res=0;
    for (int i=0; i<n; i++)
        res += edge[match[i]][i];
    return res;
}
}graph;

```

2.5 DMST

```

/*
 * Edmond's algorithm for Directed MST
 * runs in O(VE)
 */
const int MAXV = 10010;
const int MAXE = 10010;
const int INF = 2147483647;
struct Edge{
    int u, v, c;
    Edge(int x=0, int y=0, int z=0) : u(x), v(y), c(z){}
};
int V, E, root;
Edge edges[MAXE];
inline int newV(){ return ++ V; }
inline void addEdge(int u, int v, int c)
{ edges[++E] = Edge(u, v, c); }
bool con[MAXV];
int mnInW[MAXV], prv[MAXV], cyc[MAXV], vis[MAXV];
inline int DMST(){
    fill(con, con+V+1, 0);
    int r1 = 0, r2 = 0;
    while(1){
        fill(mnInW, mnInW+V+1, INF);
        fill(prv, prv+V+1, -1);
        REP(i, 1, E){
            int u=edges[i].u, v=edges[i].v, c=edges[i].c;
            if(u != v && v != root && c < mnInW[v])

```

```

                mnInW[v] = c, prv[v] = u;
            }
        }
        fill(vis, vis+V+1, -1);
        fill(cyc, cyc+V+1, -1);
        r1 = 0;
        bool jf = 0;
        REP(i, 1, V){
            if(con[i]) continue;
            if(prv[i] == -1 && i != root) return -1;
            if(prv[i] > 0) r1 += mnInW[i];
            int s;
            for(s = i; s != -1 && vis[s] == -1; s = prv[s])
                vis[s] = i;
            if(s > 0 && vis[s] == i){
                // get a cycle
                jf = 1; int v = s;
                do{
                    cyc[v] = s, con[v] = 1;
                    r2 += mnInW[v]; v = prv[v];
                }while(v != s);
                con[s] = 0;
            }
        }
        if(!jf) break;
        REP(i, 1, E){
            int &u = edges[i].u;
            int &v = edges[i].v;
            if(cyc[v] > 0) edges[i].c -= mnInW[edges[i].v];
            if(cyc[u] > 0) edges[i].u = cyc[edges[i].u];
            if(cyc[v] > 0) edges[i].v = cyc[edges[i].v];
            if(u == v) edges[i--] = edges[E--];
        }
        return r1+r2;
    }
}

```

2.6 SW min-cut

```

// global min cut
struct SW{ // O(V^3)
    static const int MXN = 514;
    int n,vst[MXN],del[MXN];
    int edge[MXN][MXN],wei[MXN];
    void init(int _n){
        n = _n; FZ(edge); FZ(del);
    }
    void addEdge(int u, int v, int w){
        edge[u][v] += w; edge[v][u] += w;
    }
    void search(int &s, int &t){
        FZ(vst); FZ(wei);
        s = t = -1;
        while (true){
            int mx=-1, cur=0;
            for (int i=0; i<n; i++)
                if (!del[i] && !vst[i] && mx<wei[i])
                    cur = i, mx = wei[i];
            if (mx == -1) break;
            vst[cur] = 1;
            s = t; t = cur;
            for (int i=0; i<n; i++)
                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    int solve(){
        int res = 2147483647;
        for (int i=0,x,y; i<n-1; i++){
            search(x,y);
            res = min(res,wei[y]);
            del[y] = 1;
            for (int j=0; j<n; j++)
                edge[x][j] = (edge[j][x] += edge[y][j]);
        }
        return res;
    }
}graph;

```

2.7 Max Cost Circulation

```

struct MaxCostCirc {
    static const int MAXN = 33;
    int n , m;
    struct Edge { int v , w , c , r; };
    vector<Edge> g[ MAXN ];
    int dis[ MAXN ] , prv[ MAXN ] , prve[ MAXN ];
    bool vis[ MAXN ];
    int ans;
    void init( int _n , int _m ) : n(_n), m(_m) {}
    void adde( int u , int v , int w , int c ) {
        g[ u ].push_back( { v , w , c , SZ( g[ v ] ) } );
        g[ v ].push_back( { u , -w , 0 , SZ( g[ u ] )-1 } );
    }
    bool poscyc() {
        fill( dis , dis+n+1 , 0 );
        fill( prv , prv+n+1 , 0 );
        fill( vis , vis+n+1 , 0 );
        int tmp = -1;
        FOR( t , n+1 ) {
            REP( i , 1 , n ) {
                FOR( j , SZ( g[ i ] ) ) {
                    Edge& e = g[ i ][ j ];
                    if( e.c && dis[ e.v ] < dis[ i ]+e.w ) {
                        dis[ e.v ] = dis[ i ]+e.w;
                        prv[ e.v ] = i;
                        prve[ e.v ] = j;
                        if( t == n ) {
                            tmp = i;
                            break;
                        }
                    }
                }
            }
        }
        if( tmp == -1 ) return 0;
        int cur = tmp;
        while( !vis[ cur ] ) {
            vis[ cur ] = 1;
            cur = prv[ cur ];
        }
        int now = cur , cost = 0 , df = 100000;
        do{
            Edge &e = g[ prv[ now ] ][ prve[ now ] ];
            df = min( df , e.c );
            cost += e.w;
            now = prv[ now ];
        }while( now != cur );
        ans += df*cost; now = cur;
        do{
            Edge &e = g[ prv[ now ] ][ prve[ now ] ];
            Edge &re = g[ now ][ e.r ];
            e.c -= df;
            re.c += df;
            now = prv[ now ];
        }while( now != cur );
        return 1;
    }
} circ;

```

2.8 Gusfield

```

#define SOURCE 0
#define SINK 1
const unsigned int inf=4000000000u;
int n,m,deg[MAXN],adj[MAXN][MAXN];
unsigned int res[MAXN][MAXN],cap[MAXN][MAXN];
int nei[MAXN],gdeg[MAXN],gadj[MAXN][MAXN];
unsigned int gres[MAXN][MAXN];
unsigned int cut[MAXN][MAXN];
unsigned int cutarr[MAXN*MAXN];
int cutn,ql,qr,que[MAXN],pred[MAXN];
unsigned int aug[MAXN];
bool cutset[MAXN];
int visited[MAXN],visid=0;
inline void augment(int src,int sink) {
    int v=sink; unsigned a=aug[sink];
    while(v!=src) {
        res[pred[v]][v]-=a;
        res[v][pred[v]]+=a;
        v=pred[v];
    }
}
inline bool bfs(int src,int sink) {

```

```

    int i,v,u; ++visid;
    ql=qr=0; que[qr++]=src;
    visited[src]=visid; aug[src]=inf;
    while(ql<qr) {
        v=que[ql++];
        for(i=0;i<deg[v];i++) {
            u=adj[v][i];
            if(visited[u]==visid||res[v][u]==0) continue;
            visited[u]=visid; pred[u]=v;
            aug[u]=min(aug[v],res[v][u]);
            que[qr++]=u;
            if(u==sink) return 1;
        }
    }
    return 0;
}
void dfs_src(int v) {
    int i,u;
    visited[v]=visid;
    cutset[v]=SOURCE;
    for(i=0;i<deg[v];i++) {
        u=adj[v][i];
        if(visited[u]<visid&&res[v][u]) dfs_src(u);
    }
}
inline unsigned int maxflow(int src,int sink) {
    int i,j;
    unsigned int f=0;
    for(i=0;i<n;i++) {
        for(j=0;j<deg[i];j++) res[i][adj[i][j]]=cap[i][adj[i][j]];
        cutset[i]=SINK;
    }
    while(bfs(src,sink)) {
        augment(src,sink);
        f+=aug[sink];
    }
    ++visid;
    dfs_src(src);
    return f;
}
inline void gusfield() {
    int i,j;
    unsigned int f;
    for(i=0;i<n;i++) { nei[i]=0; gdeg[i]=0; }
    for(i=1;i<n;i++) {
        f=maxflow(i,nei[i]);
        gres[i][nei[i]]=gres[nei[i]][i]=f;
        gadj[i][gdeg[i]++]=nei[i];
        gadj[nei[i]][gdeg[nei[i]]++]=i;
        for(j=i+1;j<n;j++)
            if(nei[j]==nei[i]&&cutset[j]==SOURCE) nei[j]=i;
    }
}
void dfs(int v,int pred,int src,unsigned int cur) {
    int i,u;
    cut[src][v]=cur;
    for(i=0;i<gdeg[v];i++) {
        u=gadj[v][i];
        if(u==pred) continue;
        dfs(u,v,src,min(cur,gres[v][u]));
    }
}
inline void find_all_cuts() {
    int i;
    cutn=0; gusfield();
    for(i=0;i<n;i++) dfs(i,-1,i,inf);
}

```

2.9 Max flow with lower/upper bound

```

// Max flow with lower/upper bound on edges
// source = 1 , sink = n
int in[ N ] , out[ N ];
int l[ M ] , r[ M ] , a[ M ] , b[ M ];
int solve() {
    flow.init( n );
    for( int i = 0 ; i < m ; i ++ ) {
        in[ r[ i ] ] += a[ i ];
        out[ l[ i ] ] += a[ i ];
    }
}

```

```

    flow.addEdge( l[ i ] , r[ i ] , b[ i ] - a[ i ] );
    // flow from l[i] to r[i] must in [a[i], b[i]]
}
int nd = 0;
for( int i = 1 ; i <= n ; i ++ ){
    if( in[ i ] < out[ i ] ){
        flow.addEdge( i , flow.t , out[ i ] - in[ i ] );
        nd += out[ i ] - in[ i ];
    }
    if( out[ i ] < in[ i ] )
        flow.addEdge( flow.s , i , in[ i ] - out[ i ] );
}
// original sink to source
flow.addEdge( n , 1 , INF );
if( flow.maxflow() != nd )
    // no solution
    return -1;
int ans = flow.G[ 1 ].back().c; // source to sink
flow.G[ 1 ].back().c = flow.G[ n ].back().c = 0;
// take out super source and super sink
for( size_t i = 0 ; i < flow.G[ flow.s ].size() ; i
    ++ ){
    flow.G[ flow.s ][ i ].c = 0;
    Edge &e = flow.G[ flow.s ][ i ];
    flow.G[ e.v ][ e.r ].c = 0;
}
for( size_t i = 0 ; i < flow.G[ flow.t ].size() ; i
    ++ ){
    flow.G[ flow.t ][ i ].c = 0;
    Edge &e = flow.G[ flow.t ][ i ];
    flow.G[ e.v ][ e.r ].c = 0;
}
flow.addEdge( flow.s , 1 , INF );
flow.addEdge( n , flow.t , INF );
flow.reset();
return ans + flow.maxflow();
}

```

2.10 Flow Method

Maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$;
 with the corresponding symmetric dual problem,
 Minimize $b^T y$ subject to $A^T y \geq c$, $y \geq 0$.

Maximize $c^T x$ subject to $Ax \leq b$;
 with the corresponding asymmetric dual problem,
 Minimize $b^T y$ subject to $A^T y = c$, $y \geq 0$.

General Graph:

IMax Ind. SetI + IMin Vertex CoverI = IVI

IMax Ind. Edge SetI + IMin Edge CoverI = IVI

Bipartite Graph:

IMax Ind. SetI = IMin Edge CoverI

IMax Ind. Edge SetI = IMin Vertex CoverI

To reconstruct the minimum vertex cover, dfs from each unmatched vertex on the left side **and** with unused edges only. Equivalently, dfs from source with unused edges only **and** without visiting sink. Then, a vertex is chosen iff. it is on the left side **and** without visited **or** on the right side **and** visited through dfs.

Maximum density subgraph ($\sum W_e + \sum W_v$) / IVI

Binary search on answer:

For a fixed D, construct a Max flow model as follow:

Let S be Sum of all weight(**or** inf)

1. from source to each node with cap = S

2. For each (u,v,w) in E, (u->v, cap=w), (v->u, cap=w)

3. For each node v, from v to sink with cap = S + 2 * D
 - deg[v] - 2 * (W of v)

where deg[v] = \sum weight of edge associated with v

If maxflow < S * IVI, D is an answer.

Requiring subgraph: all vertex can be reached from source with edge whose cap > 0.

3 Math

3.1 FFT

```

// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
    for(int i=0; i<=MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN)
                : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^= k); k >= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if(inv) for (i = 0; i < n; i++) a[i] /= n;
}

```

3.2 NTT

```

typedef long long LL;
// Remember coefficient are mod P
// p=a*2^n+1
n    2^n    p    a    root
16   65536   65537   1    3
20   1048576 7340033   7    3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >= 1, bs = (bs * bs) % P)
            if(b&1) res=(res*bs)%P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if(a==1)return 1;
        return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
    }
}
LL omega[MAXN+1];
NTT() {
    omega[0] = 1;
    LL r = bigmod(root, (P-1)/MAXN);
    for (int i=1; i<=MAXN; i++)
        omega[i] = (omega[i-1]*r)%P;
}
// n must be 2^k
void tran(int n, LL a[], bool inv_ntt=false){
    int basic = MAXN / n, theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            LL w = omega[i*theta%MAXN];
            for (int j = i; j < n; j += m) {

```

```

    int k = j + mh;
    LL x = a[j] - a[k];
    if (x < 0) x += P;
    a[j] += a[k];
    if (a[j] > P) a[j] -= P;
    a[k] = (w * x) % P;
}
}
theta = (theta * 2) % MAXN;
}
int i = 0;
for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^ k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
}
if (inv_ntt) {
    LL ni = inv(n,P);
    reverse(a+1, a+n);
    for (i = 0; i < n; i++)
        a[i] = (a[i] * ni) % P;
}
}
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;

```

3.3 Fast Walsh Transform

```

/* xor convolution:
 * x = (x0,x1) , y = (y0,y1)
 * z = ( x0y0 + x1y1 , x0y1 + x1y0 )
 * =>
 * x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
 * z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
 * z = (1/2) * z'
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
typedef long long LL;
const int MAXN = (1<<20)+10;
const LL MOD = 1e9+7;
inline LL pw( LL x , LL k ) {
    LL res = 1;
    for( LL bs = x ; k ; k >>= 1, bs = (bs * bs)%MOD )
        if( k&1 ) res = ( res * bs ) % MOD;
    return res;
}
inline LL inv( LL x ) {
    return pw( x , MOD-2 );
}
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
    for( int d = 1 ; d < N ; d <= 1 ) {
        int d2 = d<<1;
        for( int s = 0 ; s < N ; s += d2 )
            for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
                LL ta = x[ i ] , tb = x[ j ];
                x[ i ] = ta+tb;
                x[ j ] = ta-tb;
                if( x[ i ] >= MOD ) x[ i ] -= MOD;
                if( x[ j ] < 0 ) x[ j ] += MOD;
            }
    }
    if( inv )
        for( int i = 0 ; i < N ; i++ ) {
            x[ i ] *= inv( N );
            x[ i ] %= MOD;
        }
}

```

3.4 Poly operator

```

struct PolyOp {
#define FOR(i, c) for (int i = 0; i < (c); ++i)
    NTT<P, root, MAXN> ntt;
    static int nxt2k(int x) {
        int i = 1; for (; i < x; i <= 1); return i;
    }
};

```

```

}
void Mul(int n, LL a[], int m, LL b[], LL c[]) {
    static LL aa[MAXN], bb[MAXN];
    int N = nxt2k(n+m);
    copy(a, a+n, aa); fill(aa+n, aa+N, 0);
    copy(b, b+m, bb); fill(bb+m, bb+N, 0);
    ntt(N, aa); ntt(N, bb);
    FOR(i, N) c[i] = aa[i] * bb[i] % P;
    ntt(N, c, 1);
}
void Inv(int n, LL a[], LL b[]) {
    // ab = aa^-1 = 1 mod x^(n/2)
    // (b - a^-1)^2 = 0 mod x^n
    // bb - a^-2 + 2 ba^-1 = 0
    // bba - a^-1 + 2b = 0
    // bba + 2b = a^-1
    static LL tmp[MAXN];
    if (n == 1) {b[0] = ntt.inv(a[0], P); return;}
    Inv((n+1)/2, a, b);
    int N = nxt2k(n*2);
    copy(a, a+n, tmp);
    fill(tmp+n, tmp+N, 0);
    fill(b+n, b+N, 0);
    ntt(N, tmp); ntt(N, b);
    FOR(i, N) {
        LL t1 = (2 - b[i] * tmp[i]) % P;
        if (t1 < 0) t1 += P;
        b[i] = b[i] * t1 % P;
    }
    ntt(N, b, 1);
    fill(b+n, b+N, 0);
}
void Div(int n, LL a[], int m, LL b[], LL d[], LL r
    []) {
    // Ra = Rb * Rd mod x^(n-m+1)
    // Rd = Ra * Rb^-1 mod
    static LL aa[MAXN], bb[MAXN], ta[MAXN], tb[MAXN];
    if (n < m) {copy(a, a+n, r); fill(r+n, r+m, 0);
        return;}
    // d: n-1 - (m-1) = n-m (n-m+1 terms)
    copy(a, a+n, aa); copy(b, b+m, bb);
    reverse(aa, aa+n); reverse(bb, bb+m);
    Inv(n-m+1, bb, tb);
    Mul(n-m+1, ta, n-m+1, tb, d);
    fill(d+n-m+1, d+n, 0); reverse(d, d+n-m+1);
    // r: m-1 - 1 = m-2 (m-1 terms)
    Mul(m, b, n-m+1, d, ta);
    FOR(i, n) { r[i] = a[i] - ta[i]; if (r[i] < 0) r[i]
        += P; }
}
void dx(int n, LL a[], LL b[]) { REP(i, 1, n-1) b[i]
    -1] = i * a[i] % P; }
void Sx(int n, LL a[], LL b[]) {
    b[0] = 0;
    FOR(i, n) b[i+1] = a[i] * ntt.iv[i+1] % P;
}
void Ln(int n, LL a[], LL b[]) {
    // Integral a' a^-1 dx
    static LL a1[MAXN], a2[MAXN], b1[MAXN];
    int N = nxt2k(n*2);
    dx(n, a, a1); Inv(n, a, a2);
    Mul(n-1, a1, n, a2, b1);
    Sx(n+n-1-1, b1, b);
    fill(b+n, b+N, 0);
}
void Exp(int n, LL a[], LL b[]) {
    // Newton method to solve g(a(x)) = ln b(x) - a(x)
    // = 0
    // b' = b - g(b(x)) / g'(b(x))
    // b' = b (1 - lnb + a)
    static LL lnb[MAXN], c[MAXN], tmp[MAXN];
    assert(a[0] == 0); // dont know exp(a[0]) mod P
    if (n == 1) {b[0] = 1; return;}
    Exp((n+1)/2, a, b);
    fill(b+(n+1)/2, b+n, 0);
    Ln(n, b, lnb);
    fill(c, c+n, 0); c[0] = 1;
    FOR(i, n) {
        c[i] += a[i] - lnb[i];
        if (c[i] < 0) c[i] += P;
        if (c[i] >= P) c[i] -= P;
    }
}

```



```

    Mul(n, b, n, c, tmp);
    copy(tmp, tmp+n, b);
}
} polyop;

```

3.5 Miller Rabin

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
bool witness(LL a, LL n, LL u, int t){
    if(!a) return 0;
    LL x=mypow(a,u,n);
    for(int i=0; i<t; i++){
        LL nx=mul(x,x,n);
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x=nx;
    }
    return x!=1;
}
bool miller_rabin(LL n, int s=100) {
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if(n<2) return 0;
    if(!(n&1)) return n == 2;
    LL u=n-1; int t=0;
    // n-1 = u*2^t
    while(!(u&1)) u>>=1, t++;
    while(s--){
        LL a=randll()%(n-1)+1;
        if(witness(a,n,u,t)) return 0;
    }
    return 1;
}

```

3.6 Simplex

```

const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXN], d[MAXN][MAXM];
double x[MAXN];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
               double c[MAXM], int n, int m){
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
    }
}

```

```

r = -1; s = -1;
for (int j = 0; j < m; ++j)
    if (s < 0 || ix[s] > ix[j]) {
        if (d[n + 1][j] > eps ||
            (d[n + 1][j] > -eps && d[n][j] > eps))
            s = j;
    }
if (s < 0) break;
for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
    if (r < 0 ||
        (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps ||
        (dd < eps && ix[r + m] > ix[i + m]))
        r = i;
}
if (r < 0) return -1; // not bounded
}
if (d[n + 1][m] < -eps) return -1; // not executable
double ans = 0;
for (int i = 0; i < m; i++) x[i] = 0;
for (int i = m; i < n + m; ++i) { // the missing
    enumerated x[i] = 0
    if (ix[i] < m - 1) {
        ans += d[i - m][m] * c[ix[i]];
        x[ix[i]] = d[i - m][m];
    }
}
return ans;
}

```

3.7 Faulhaber

```

/* faulhaber 's formula -
 * cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2]; // coefficient of x^j when p=i
inline int getinv(int x) {
    int a=x, b=mod, a0=1, a1=0, b0=0, b1=1;
    while(b) {
        int q,t;
        q=a/b; t=b; b=a-b*q; a=t;
        t=b0; b0=a0-b0*q; a0=t;
        t=b1; b1=a1-b1*q; a1=t;
    }
    return a0<0?a0+mod:a0;
}
inline void pre() {
    /* combinational */
    for (int i=0; i<=MAXK; i++) {
        cm[i][0]=cm[i][i]=1;
        for (int j=1; j<i; j++)
            cm[i][j]=add(cm[i-1][j-1], cm[i-1][j]);
    }
    /* inverse */
    for (int i=1; i<=MAXK; i++) inv[i]=getinv(i);
    /* bernoulli */
    b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
    for (int i=2; i<=MAXK; i++) {
        if (i&1) { b[i]=0; continue; }
        b[i]=1;
        for (int j=0; j<i; j++)
            b[i]=sub(b[i], mul(cm[i][j], mul(b[j], inv[i-j+1])));
    }
    /* faulhaber */
    // sigma_x=1~n {x^p} =
    // 1/(p+1) * sigma_j=0~p {C(p+1,j)*B_j*n^(p-j+1)}
    for (int i=1; i<=MAXK; i++) {
        co[i][0]=0;
        for (int j=0; j<=i; j++)
            co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]));
    }
}
/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n, int p) {
}

```

```

int sol=0,m=n;
for(int i=1;i<=p+1;i++) {
    sol=add(sol,mul(co[p][i],m));
    m = mul(m, n);
}
return sol;
}

```

3.8 Chinese Remainder

```

LL solve(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL,LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}

```

3.9 Pollard Rho

```

// does not work when n is prime
LL f(LL x, LL mod){ return add(mul(x,x,mod),1,mod); }
LL pollard_rho(LL n) {
    if(!(n&1)) return 2;
    while(true){
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2) {
            for(int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}

```

3.10 ax+by=gcd

```

PII gcd(LL a, LL b){
    if(b == 0) return {1, 0};
    PII q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}

```

3.11 Discrete sqrt

```

void calcH(int &t, int &h, const int p) {
    int tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
}
// solve equation x^2 mod p = a
bool solve(int a, int p, int &x, int &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
    if ((p + 1) % 4 == 0) {
        x=mypow(a,(p+1)/4,p); y=p-x; return true;
    } else {
        int t, h, b, pb; calcH(t, h, p);
        if (t >= 2) {
            do {b = rand() % (p - 2) + 2;
                } while (mypow(b, p / 2, p) != p - 1);
            pb = mypow(b, h, p);
        } int s = mypow(a, h / 2, p);
        for (int step = 2; step <= t; step++) {
            int ss = (((LL)(s * s) % p) * a) % p;
            for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
            if (ss + 1 == p) s = (s * pb) % p;
            pb = ((LL)pb * pb) % p;
        } x = ((LL)s * a) % p; y = p - x;
    } return true;
}

```

3.12 Romberg

```

// Estimates the definite integral of
// \int_a^b f(x) dx
template<class T>
double romberg( T& f, double a, double b, double eps=1e-8){
    vector<double>t; double h=b-a,last,curr; int k=1,i=1;
    t.push_back(h*(f(a)+f(b))/2);
    do{ last=t.back(); curr=0; double x=a+h/2;
        for(int j=0;j<k;j++) curr+=f(x), x+=h;
        curr=(t[0] + h*curr)/2; double k1=4.0/3.0,k2=1.0/3.0;
        for(int j=0;j<i;j++){ double temp=k1*curr-k2*t[j];
            t[j]=curr; curr=temp; k2/=4*k1-k2; k1=k2+1;
        } t.push_back(curr); k*=2; h/=2; i++;
    }while( fabs(last-curr) > eps);
    return t.back();
}

```

3.13 Prefix Inverse

```

void solve( int m ){
    inv[ 1 ] = 1;
    for( int i = 2 ; i < m ; i ++ )
        inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;
}

```

3.14 Roots of Polynomial

```

const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ];
int n;
int sign( double x ){return (x < -eps)?(-1):(x>eps);}
double f(double a[], int n, double x){
    double tmp=1,sum=0;
    for(int i=0;i<n;i++){
        sum=sum+a[i]*tmp; tmp=tmp*x;
    }
    return sum;
}
double binary(double l,double r,double a[],int n){
    int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
    if(sl==0) return l; if(sr==0) return r;
    if(sl*sr>0) return inf;
    while(r-l>eps){
        double mid=(l+r)/2;
        int ss=sign(f(a,n,mid));
        if(ss==0) return mid;
        if(ss*sl>0) l=mid; else r=mid;
    }
    return l;
}
void solve(int n,double a[],double x[],int &nx){
    if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
    double da[10], dx[10]; int ndx;
    for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
    solve(n-1,da,dx,ndx);
    nx=0;
    if(ndx==0){
        double tmp=binary(-inf,inf,a,n);
        if (tmp<inf) x[++nx]=tmp;
        return;
    }
    double tmp;
    tmp=binary(-inf,dx[1],a,n);
    if(tmp<inf) x[++nx]=tmp;
    for(int i=1;i<=ndx-1;i++){
        tmp=binary(dx[i],dx[i+1],a,n);
        if(tmp<inf) x[++nx]=tmp;
    }
    tmp=binary(dx[ndx],inf,a,n);
    if(tmp<inf) x[++nx]=tmp;
}
int main() {
    scanf("%d",&n);
    for(int i=n;i>=0;i--) scanf("%lf",&a[i]);
}

```



```
int nx;
solve(n,a,x,nx);
for(int i=1;i<=nx;i++) printf("%.6f\n",x[i]);
}
```

3.15 Result

- Lucas' Theorem :
For $n, m \in \mathbb{Z}^*$ and prime P , $C(m, n) \bmod P = \prod (C(m_i, n_i))$ where m_i is the i -th digit of m in base P .
- Stirling Numbers(permutation $|P| = n$ with k cycles):
 $S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x + i)$
- Stirling Numbers(Partition n elements into k non-empty set):
$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$
- Pick's Theorem : $A = i + b/2 - 1$
- Kirchhoff's theorem :
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$, Deleting any one row, one column, and cal the $\det(A)$
- Burnside Lemma: $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- Polya theorem: $|Y^x/G| = \frac{1}{|G|} \sum_{g \in G} m^{c(g)}$
 $m = |Y|$: num of colors, $c(g)$: num of cycle

4 Geometry

4.1 Intersection of 2 lines

```
Pt LLIntersect(Line a, Line b) {
    Pt p1 = a.s, p2 = a.e, q1 = b.s, q2 = b.e;
    double f1 = (p2-p1)^(q1-p1), f2 = (p2-p1)^(p1-q2);
    double f = f1+f2;
    if(dcmp(f) == 0) return Pt(nan(""), nan(""));
    return q1*(f2/f) + q2*(f1/f);
}
```

4.2 halfPlaneIntersection

```
bool onleft(Line L, Pt p) {
    return dcmp(L.v^(p-L.s)) > 0;
}
// assume that Lines intersect
vector<Pt> HPI(vector<Line>& L) {
    sort(L.begin(), L.end());
    int n = L.size(), fir, las;
    Pt *p = new Pt[n];
    Line *q = new Line[n];
    q[fir=las=0] = L[0];
    for(int i = 1; i < n; i++) {
        while(fir < las && !onleft(L[i], p[las-1])) las--;
        while(fir < las && !onleft(L[i], p[fir])) fir++;
        q[++las] = L[i];
        if(dcmp(q[las].v^q[las-1].v) == 0) {
            las--;
            if(onleft(q[las], L[i].s)) q[las] = L[i];
        }
        if(fir < las) p[las-1] = LLIntersect(q[las-1], q[las]);
    }
    while(fir < las && !onleft(q[fir], p[las-1])) las--;
    if(las-fir <= 1) return {};
    p[las] = LLIntersect(q[las], q[fir]);
    int m = 0;
    vector<Pt> ans(las-fir+1);
    for(int i = fir; i <= las; i++) ans[m++] = p[i];
    return ans;
}
```

4.3 Intersection of 2 segments

```
bool onseg(Pt p, Line L) {
    Pt x = L.s-p, y = L.e-p;
    return dcmp(x^y) == 0 && dcmp(x*y) <= 0; // inseg:
    dcmp(x*y) < 0
}
Pt SSIntersect(Line a, Line b) {
    Pt p1 = a.s, p2 = a.e, q1 = b.s, q2 = b.e;
    Pt p = LLIntersect(a, b);
    if(!isnan(p.x) && onseg(p, a) && onseg(p, b)) return
        q;
    return Pt(nan(""), nan(""));
}
```

4.4 Intersection of circle and segment

```
bool Inter(const Pt& p1, const Pt& p2, Circle& cc) {
    Pt dp = p2 - p1;
    double a = dp * dp;
    double b = 2 * (dp * (p1 - cc.o));
    double c = cc.o * cc.o + p1 * p1 - 2 * (cc.o * p1)
        - cc.R * cc.R;
    double bb4ac = b * b - 4 * a * c;
    return !(fabs(a) < eps || bb4ac < 0);
}
```

4.5 Intersection of polygon and circle

```
Pt ORI, info[N];
D r; int n;
// Divides into multiple triangle, and sum up
// oriented area
D area2(Pt pa, Pt pb) {
    if(norm(pa) < norm(pb)) swap(pa, pb);
    if(norm(pb) < eps) return 0;
    D S, h, theta;
    D a = norm(pb), b = norm(pa), c = norm(pb - pa);
    D cosB = (pb * (pb - pa)) / a / c, B = acos(cosB);
    D cosC = (pa * pb) / a / b, C = acos(cosC);
    if(a > r) {
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if(h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
    } else if(b > r) {
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    } else S = .5*sin(C)*a*b;
    return S;
}
D area() {
    D S = 0;
    for(int i = 0; i < n; ++i)
        S += abs(area2(info[i], info[i+1])) * sign(det(
            info[i], info[i+1]));
    return fabs(S);
}
```

4.6 Intersection of 2 circles

4.7 Circle cover

```
#define N 1021
struct CircleCover {
    int C; Circ c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    D Area[N];
    void init(int _C) { C = _C; }
    bool CCinter(Circ& a, Circ& b, Pt& p1, Pt& p2) {
        Pt o1 = a.o, o2 = b.o;
        D r1 = a.R, r2 = b.R;
        if(norm(o1 - o2) > r1 + r2) return {};
    }
}
```

```

    if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
        return {};
    D d2 = ( o1 - o2 ) * ( o1 - o2 );
    D d = sqrt(d2);
    if( d > r1 + r2 ) return false;
    Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
    D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
    Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
    p1 = u + v; p2 = u - v;
    return true;
}
struct Teve {
    Pt p; D ang; int add;
    Teve() {}
    Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
    bool operator<(const Teve &a)const
    {return ang < a.ang;}
}eve[ N * 2 ];
// strict: x = 0, otherwise x = -1
bool disjunct( Circ& a, Circ &b, int x )
{return sign( norm( a.o - b.o ) - a.R - b.R ) > x;}
bool contain( Circ& a, Circ &b, int x )
{return sign( a.R - b.R - norm( a.o - b.o ) ) > x;}
bool contain(int i, int j){
    /* c[j] is non-strictly in c[i]. */
    return (sign(c[i].R - c[j].R) > 0 ||
            (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
            contain(c[i], c[j], -1);
}
void solve(){
    for( int i = 0 ; i <= C + 1 ; i ++ )
        Area[ i ] = 0;
    for( int i = 0 ; i < C ; i ++ )
        for( int j = 0 ; j < C ; j ++ )
            overlap[i][j] = contain(i, j);
    for( int i = 0 ; i < C ; i ++ )
        for( int j = 0 ; j < C ; j ++ )
            g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                        disjunct(c[i], c[j], -1));
    for( int i = 0 ; i < C ; i ++ ){
        int E = 0, cnt = 1;
        for( int j = 0 ; j < C ; j ++ )
            if( j != i && overlap[j][i] )
                cnt ++;
        for( int j = 0 ; j < C ; j ++ )
            if( i != j && g[i][j] ){
                Pt aa, bb;
                CCinter(c[i], c[j], aa, bb);
                D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
                D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
                eve[E ++] = Teve(bb, B, 1);
                eve[E ++] = Teve(aa, A, -1);
                if(B > A) cnt ++;
            }
        if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
        else{
            sort( eve , eve + E );
            eve[E] = eve[0];
            for( int j = 0 ; j < E ; j ++ ){
                cnt += eve[j].add;
                Area[cnt] += (eve[j].p ^ eve[j + 1].p) * .5;
                D theta = eve[j + 1].ang - eve[j].ang;
                if (theta < 0) theta += 2. * pi;
                Area[cnt] +=
                    (theta - sin(theta)) * c[i].R*c[i].R * .5;
            }
        }
    }
}
};

```

4.8 Convex Hull trick

```

/* Given a convexhull, answer queries in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
    int n;

```

```

    vector<Pt> a;
    vector<Pt> upper, lower;
    Conv(vector<Pt> _a) : a(_a){
        n = a.size();
        int ptr = 0;
        for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
        for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
        for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
        upper.push_back(a[0]);
    }
    int sign( LL x ){ // fixed when changed to double
        return x < 0 ? -1 : x > 0; }
    pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
        int l = 0, r = (int)conv.size() - 2;
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
            else l = mid;
        }
        return max(make_pair(det(vec, conv[r]), r),
                   make_pair(det(vec, conv[0]), 0));
    }
    void upd_tang(const Pt &p, int id, int &i0, int &i1){
        if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
        if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
    }
    void bi_search(int l, int r, Pt p, int &i0, int &i1){
        if(l == r) return;
        upd_tang(p, l % n, i0, i1);
        int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
            if (smid == sl) l = mid;
            else r = mid;
        }
        upd_tang(p, r % n, i0, i1);
    }
    int bi_search(Pt u, Pt v, int l, int r) {
        int sl = sign(det(v - u, a[l % n] - u));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            int smid = sign(det(v - u, a[mid % n] - u));
            if (smid == sl) l = mid;
            else r = mid;
        }
        return l % n;
    }
    // 1. whether a given point is inside the CH
    bool contain(Pt p) {
        if (p.X < lower[0].X || p.X > lower.back().X)
            return 0;
        int id = lower_bound(lower.begin(), lower.end(), Pt(
            p.X, -INF)) - lower.begin();
        if (lower[id].X == p.X) {
            if (lower[id].Y > p.Y) return 0;
        }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
        id = lower_bound(upper.begin(), upper.end(), Pt(p.X
            , INF), greater<Pt>()) - upper.begin();
        if (upper[id].X == p.X) {
            if (upper[id].Y < p.Y) return 0;
        }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
        return 1;
    }
    // 2. Find 2 tang pts on CH of a given outside point
    // return true with i0, i1 as index of tangent points
    // return false if inside CH
    bool get_tang(Pt p, int &i0, int &i1) {
        if (contain(p)) return false;
        i0 = i1 = 0;
        int id = lower_bound(lower.begin(), lower.end(), p)
            - lower.begin();
        bi_search(0, id, p, i0, i1);
        bi_search(id, (int)lower.size(), p, i0, i1);
        id = lower_bound(upper.begin(), upper.end(), p,
            greater<Pt>()) - upper.begin();
        bi_search((int)lower.size() - 1, (int)lower.size()
            - 1 + id, p, i0, i1);
        bi_search((int)lower.size() - 1 + id, (int)lower.
            size() - 1 + (int)upper.size(), p, i0, i1);
        return true;
    }
}

```

```
#define eps 1e-8
class PY{ public:
    int n;
    Pt pt[5];
    Pt& operator[](const int x){ return pt[x]; }
    void input(){
        int i; n=4;
        for(i=0;i<n;i++) scanf("%lf%lf",&pt[i].x,&pt[i].y);
    }
    double getArea(){
        int i; double s=pt[n-1]^pt[0];
        for(i=0;i<n-1;i++) s+=pt[i]^pt[i+1];
        return s/2;
    }
};
PY py[500];
pair<double,int> c[5000];
inline double segP(Pt &p,Pt &p1,Pt &p2){
    if(SG(p1.x-p2.x)==0) return (p.y-p1.y)/(p2.y-p1.y);
    return (p.x-p1.x)/(p2.x-p1.x);
}
double polyUnion(int n){
    int i,j,ii,jj,ta,tb,r,d;
    double z,w,s,sum,tc,td;
    for(i=0;i<n;i++) py[i][py[i].n]=py[i][0];
    sum=0;
    for(i=0;i<n;i++){
        for(ii=0;ii<py[i].n;ii++){
            r=0;
            c[r++]=make_pair(0.0,0);
            c[r++]=make_pair(1.0,0);
            for(j=0;j<n;j++){
                if(i==j) continue;
                for(jj=0;jj<py[j].n;jj++){
                    ta=SG(tri(py[i][ii],py[i][ii+1],py[j][jj]));
                    tb=SG(tri(py[i][ii],py[i][ii+1],py[j][jj+1]));
                    ;
                    if(ta==0 && tb==0){
                        if((py[j][jj+1]-py[j][jj])*(py[i][ii+1]-py[i][ii])>0 && j<i){
                            c[r++]=make_pair(segP(py[j][jj],py[i][ii],py[i][ii+1]),1);
                            c[r++]=make_pair(segP(py[j][jj+1],py[i][ii+1],py[i][ii]),-1);
                        }
                    }
                }
            }
            if(ta>=0 && tb<0){
```

```

        tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
        td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
        c[r++]=make_pair(tc/(tc-td),1);
    }else if(ta<0 && tb>=0){
        tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
        td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
        c[r++]=make_pair(tc/(tc-td),-1);
    }
    }
    sort(c,c+r);
    z=min(max(c[0].first,0.0),1.0);
    d=c[0].second; s=0;
    for(j=1;j<r;j++){
        w=min(max(c[j].first,0.0),1.0);
        if(!d) s+=w-z;
        d+=c[j].second; z=w;
    }
    sum+=(py[i][ii]^py[i][ii+1])*s;
}
return sum/2;
}
int main(){
    int n,i,j,k;
    double sum,ds;
    scanf("%d",&n); sum=0;
    for(i=0;i<n;i++){
        py[i].input();
        ds=py[i].getArea();
        if(ds<0){
            for(j=0,k=py[i].n-1;j<k;j++,k--) swap(py[i][j],
                py[i][k]);
            ds=-ds;
        } sum+=ds;
    } printf("%.9f\n",sum/polyUnion(n));
}

```

4.12 Lower Concave Hull

```

/****
    maintain a "concave hull" that support the following
    1. insertion of a line
    2. query of height(y) on specific x on the hull
****/
/* set as needed */
typedef long double LD;
const LD eps=1e-9;
const LD inf=1e19;
class Seg {
public:
    LD m,c,x1,x2; // y=mx+c
    bool flag;
    Seg(
        LD _m,LD _c,LD _x1=-inf,LD _x2=inf,bool _flag=0)
        :m(_m),c(_c),x1(_x1),x2(_x2),flag(_flag) {}
    LD evaly(LD x) const { return m*x+c; }
    const bool operator<(LD x) const{return x2-eps<x;}
    const bool operator<(const Seg &b) const {
        if(flag!b.flag) return *this<b.x1;
        return m+eps<b.m;
    }
};
class LowerConcaveHull { // maintain a hull like: \_/_/
public:
    set<Seg> hull;
    /* functions */
    LD xintersection(Seg a,Seg b)
    { return (a.c-b.c)/(b.m-a.m); }
    inline set<Seg>::iterator replace(set<Seg> &
        hull,set<Seg>::iterator it,Seg s) {
        hull.erase(it);
        return hull.insert(s).first;
    }
    void insert(Seg s) {
        // insert a line and update hull
        set<Seg>::iterator it=hull.find(s);
        // check for same slope
        if(it!=hull.end()) {
            if(it->c+eps>=s.c) return;

```

```

        hull.erase(it);
    }
    // check if below whole hull
    it=hull.lower_bound(s);
    if(it!=hull.end()&&
        s.evaly(it->x1)<=it->evaly(it->x1)+eps) return;
    // update right hull
    while(it!=hull.end()) {
        LD x=xintersection(s,*it);
        if(x>=it->x2-eps) hull.erase(it++);
        else {
            s.x2=x;
            it=replace(hull,it,Seg(it->m,it->c,x,it->x2));
            break;
        }
    }
    // update left hull
    while(it!=hull.begin()) {
        LD x=xintersection(s,*(--it));
        if(x<=it->x1+eps) hull.erase(it--);
        else {
            s.x1=x;
            it=replace(hull,it,Seg(it->m,it->c,it->x1,x));
            break;
        }
    }
    // insert s
    hull.insert(s);
}
void insert(LD m,LD c) { insert(Seg(m,c)); }
LD query(LD x) { // return y @ given x
    set<Seg>::iterator it =
        hull.lower_bound(Seg(0.0,0.0,x,x,1));
    return it->evaly(x);
}
};

```

4.13 Delaunay Triangulation

```

/* Delaunay Triangulation:
    Given a sets of points in 2D plane, find a
    triangulation such that no points will strictly
    inside circumcircle of any triangle.

    find : return a triangle contain given point
    add_point : add a point into triangulation

    A Triangle is in triangulation iff. its has_chd is 0.
    Region of triangle u: iterate each u.edge[i].tri,
    each points are u.p[(i+1)%3], u.p[(i+2)%3]

    calculation involves  $O(|V|^6)$  */
const int N = 100000 + 5;
const type inf = 2e3;
type eps = 1e-6; // 0 when integer
type sqr(type x) { return x*x; }
// return p4 is in circumcircle of tri(p1,p2,p3)
bool in_cc(const Pt& p1, const Pt& p2, const Pt& p3,
    const Pt& p4){
    type u11 = p1.X - p4.X; type u12 = p1.Y - p4.Y;
    type u21 = p2.X - p4.X; type u22 = p2.Y - p4.Y;
    type u31 = p3.X - p4.X; type u32 = p3.Y - p4.Y;
    type u13 = sqr(p1.X)-sqr(p4.X)+sqr(p1.Y)-sqr(p4.Y);
    type u23 = sqr(p2.X)-sqr(p4.X)+sqr(p2.Y)-sqr(p4.Y);
    type u33 = sqr(p3.X)-sqr(p4.X)+sqr(p3.Y)-sqr(p4.Y);
    type det = -u13*u22*u31 + u12*u23*u31 + u13*u21*u32
        -u11*u23*u32 - u12*u21*u33 + u11*u22*u33;
    return det > eps;
}
type side(const Pt& a, const Pt& b, const Pt& p)
{ return (b - a) ^ (p - a); }
typedef int SdRef;
struct Tri;
typedef Tri* TriRef;
struct Edge {
    TriRef tri; SdRef side;
    Edge():tri(0), side(0){}
    Edge(TriRef _tri, SdRef _side):tri(_tri), side(_side)
    {}
};

```

```

struct Tri {
    Pt p[3];
    Edge edge[3];
    TriRef chd[3];
    Tri() {}
    Tri(const Pt& p0, const Pt& p1, const Pt& p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return chd[0] == 0 ? 0
            : chd[1] == 0 ? 1
            : chd[2] == 0 ? 2 : 3;
    }
    bool contains(Pt const& q) const {
        for( int i = 0 ; i < 3 ; i ++ )
            if( side(p[i], p[(i + 1) % 3] , q) < -eps )
                return false;
        return true;
    }
} pool[ N * 10 ], *tris;
void edge( Edge a, Edge b ){
    if(a.tri) a.tri->edge[a.side] = b;
    if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig(){
        the_root = // Tri should at least contain all
                    points
        new(tris++)Tri(Pt(-inf,-inf),Pt(+inf+inf,-inf),Pt
        (-inf,+inf+inf));
    }
    TriRef find(Pt p)const{ return find(the_root,p); }
    void add_point(const Pt& p){ add_point(find(the_root,
        p),p); }
    TriRef the_root;
    static TriRef find(TriRef root, const Pt& p) {
        while( true ){
            if( !root->has_chd() )
                return root;
            for( int i = 0; i < 3 && root->chd[i] ; ++i )
                if (root->chd[i]->contains(p)) {
                    root = root->chd[i];
                    break;
                }
        }
        assert( false ); // "point not found"
    }
    void add_point(TriRef root, Pt const& p) {
        TriRef tab,tbc,tca;
        /* split it into three triangles */
        tab=new(tris++) Tri(root->p[0],root->p[1],p);
        tbc=new(tris++) Tri(root->p[1],root->p[2],p);
        tca=new(tris++) Tri(root->p[2],root->p[0],p);
        edge(Edge(tab,0), Edge(tbc,1));
        edge(Edge(tbc,0), Edge(tca,1));
        edge(Edge(tca,0), Edge(tab,1));
        edge(Edge(tab,2), root->edge[2]);
        edge(Edge(tbc,2), root->edge[0]);
        edge(Edge(tca,2), root->edge[1]);
        root->chd[0] = tab;
        root->chd[1] = tbc;
        root->chd[2] = tca;
        flip(tab,2);
        flip(tbc,2);
        flip(tca,2);
    }
    void flip(TriRef tri, SdRef pi) {
        TriRef trj = tri->edge[pi].tri;
        int pj = tri->edge[pi].side;
        if (!trj) return;
        if (!in_cc(tri->p[0],tri->p[1],tri->p[2],trj->p[pj]
            )) return;
        /* flip edge between tri,trj */
        TriRef trk = new(tris++) Tri(tri->p[(pi+1)%3], trj
            ->p[pj], tri->p[pi]);
        TriRef trl = new(tris++) Tri(trj->p[(pj+1)%3], tri
            ->p[pi], trj->p[pj]);
        edge(Edge(trk,0), Edge(trl,0));
        edge(Edge(trk,1), tri->edge[(pi+2)%3]);
        edge(Edge(trk,2), trj->edge[(pj+1)%3]);

```

```

        edge(Edge(trl,1), trj->edge[(pj+2)%3]);
        edge(Edge(trl,2), tri->edge[(pi+1)%3]);
        tri->chd[0]=trk; tri->chd[1]=trl; tri->chd[2]=0;
        trj->chd[0]=trk; trj->chd[1]=trl; trj->chd[2]=0;
        flip(trk,1); flip(trk,2);
        flip(trl,1); flip(trl,2);
    }
};
vector<TriRef> triang;
set<TriRef> vst;
void go( TriRef now ){
    if( vst.find( now ) != vst.end() )
        return;
    vst.insert( now );
    if( !now->has_chd() ){
        triang.push_back( now );
        return;
    }
    for( int i = 0 ; i < now->num_chd() ; i ++ )
        go( now->chd[ i ] );
}
void build( int n , Pt* ps ){
    tris = pool;
    random_shuffle(ps, ps + n);
    Trig tri;
    for(int i = 0; i < n; ++ i)
        tri.add_point(ps[i]);
    go( tri.the_root );
}

```

4.14 Min Enclosing Circle

```

struct Mec{
    // return pair of center and r
    static const int N = 101010;
    int n;
    Pt p[ N ], cen;
    double r2;
    void init( int _n , Pt _p[] ){
        n = _n;
        memcpy( p , _p , sizeof(Pt) * n );
    }
    double sqr(double a){ return a*a; }
    Pt center(Pt p0, Pt p1, Pt p2) {
        Pt a = p1-p0;
        Pt b = p2-p0;
        double c1=norm2( a ) * 0.5;
        double c2=norm2( b ) * 0.5;
        double d = a ^ b;
        double x = p0.X + (c1 * b.Y - c2 * a.Y) / d;
        double y = p0.Y + (a.X * c2 - b.X * c1) / d;
        return Pt(x,y);
    }
    pair<Pt,double> solve(){
        random_shuffle(p,p+n);
        r2=0;
        for (int i=0; i<n; i++){
            if (norm2(cen-p[i]) <= r2) continue;
            cen = p[i];
            r2 = 0;
            for (int j=0; j<i; j++){
                if (norm2(cen-p[j]) <= r2) continue;
                cen=Pt((p[i].X+p[j].X)/2,(p[i].Y+p[j].Y)/2);
                r2 = norm2(cen-p[j]);
                for (int k=0; k<j; k++){
                    if (norm2(cen-p[k]) <= r2) continue;
                    cen = center(p[i],p[j],p[k]);
                    r2 = norm2(cen-p[k]);
                }
            }
        }
        return {cen,sqrt(r2)};
    }
} mec;

```

4.15 Minkowski sum


```

vector<Pt> minkowski(vector<Pt> p, vector<Pt> q){
    int n = p.size(), m = q.size();
    Pt c = Pt(0, 0);
    for( int i = 0; i < m; i++) c = c + q[i];
    c = c / m;
    for( int i = 0; i < m; i++) q[i] = q[i] - c;
    int cur = -1;
    for( int i = 0; i < m; i++)
        if( (q[i] ^ (p[0] - p[n-1])) > -eps)
            if( cur == -1 || (q[i] ^ (p[0] - p[n-1])) >
                (q[cur] ^ (p[0] - p[n-1])) )
                cur = i;
    vector<Pt> h;
    p.push_back(p[0]);
    for( int i = 0; i < n; i++)
        while( true ){
            h.push_back(p[i] + q[cur]);
            int nxt = (cur + 1 == m ? 0 : cur + 1);
            if((q[cur] ^ (p[i+1] - p[i])) < -eps) cur = nxt;
            else if( (q[nxt] ^ (p[i+1] - p[i])) >
                (q[cur] ^ (p[i+1] - p[i])) ) cur = nxt;
            else break;
        }
    for(auto &i : h) i = i + c;
    return convex_hull(h);
}

```

4.16 Min dist on Cuboid

```

typedef LL T;
T r;
void turn(T i, T j, T x, T y, T z,
          T x0, T y0, T L, T W, T H) {
    if (z==0) { T R = x*y; if (R<r) r=R; return; }
    if(i>=0 && i< 2) turn(i+1, j, x0+L+z, y, x0+L-x,
                          x0+L, y0, H, W, L);
    if(j>=0 && j< 2) turn(i, j+1, x, y0+W+z, y0+W-y,
                          x0, y0+W, L, H, W);
    if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0,
                          x0-H, y0, H, W, L);
    if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0,
                          x0, y0-H, L, H, W);
}
T solve(T L, T W, T H,
        T x1, T y1, T z1, T x2, T y2, T z2){
    if( z1!=0 && z1!=H ){
        if( y1==0 || y1==W )
            swap(y1,z1), swap(y2,z2), swap(W,H);
    }else swap(x1,z1), swap(x2,z2), swap(L,H);
    if (z1==H) z1=0, z2=H-z2;
    r=INF; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
    return r;
}

```

4.17 Heart of Triangle

```

Pt inCenter( Pt &A, Pt &B, Pt &C) { // 內心
    double a = norm(B-C), b = norm(C-A), c = norm(A-B);
    return (A * a + B * b + C * c) / (a + b + c);
}
Pt circumCenter( Pt &a, Pt &b, Pt &c) { // 外心
    Pt bb = b - a, cc = c - a;
    double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc);
    return a-Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d;
}
Pt orthoCenter( Pt &a, Pt &b, Pt &c) { // 垂心
    Pt ba = b - a, ca = c - a, bc = b - c;
    double Y = ba.Y * ca.Y * bc.Y,
           A = ca.X * ba.Y - ba.X * ca.Y,
           x0 = (Y+ca.X*ba.Y*bc.X-ba.X*ca.Y*c.X) / A,
           y0 = -ba.X * (x0 - c.X) / ba.Y + ca.Y;
    return Pt(x0, y0);
}

```

5 Graph

5.1 HeavyLightDecomp

```

#define REP(i, s, e) for(int i = (s); i <= (e); i++)
#define REPD(i, s, e) for(int i = (s); i >= (e); i--)
const int MAXN = 100010;
const int LOG = 19;
struct HLD{
    int n;
    vector<int> g[MAXN];
    int sz[MAXN], dep[MAXN];
    int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
    // ts : timestamp , useless after yutruli
    // tid[ u ] : pos. of node u in the seq.
    // tdi[ i ] : node at pos i of the seq.
    // tl , tr[ u ] : subtree interval in the seq. of
    // node u
    int prt[MAXN][LOG], head[MAXN];
    // head[ u ] : head of the chain contains u
    void dfsz(int u, int p){
        dep[u] = dep[p] + 1;
        prt[u][0] = p; sz[u] = 1; head[u] = u;
        for(int& v:g[u]) if(v != p){
            dep[v] = dep[u] + 1;
            dfsz(v, u);
            sz[u] += sz[v];
        }
    }
    void dfshl(int u){
        ts++;
        tid[u] = tl[u] = tr[u] = ts;
        tdi[tid[u]] = u;
        sort(ALL(g[u]),
            [&](int a, int b){return sz[a] > sz[b];});
        bool flag = 1;
        for(int& v:g[u]) if(v != prt[u][0]){
            if(flag) head[v] = head[u], flag = 0;
            dfshl(v);
            tr[u] = tr[v];
        }
    }
    inline int lca(int a, int b){
        if(dep[a] > dep[b]) swap(a, b);
        int diff = dep[b] - dep[a];
        REPD(k, LOG-1, 0) if(diff & (1<<k)){
            b = prt[b][k];
        }
        if(a == b) return a;
        REPD(k, LOG-1, 0) if(prt[a][k] != prt[b][k]){
            a = prt[a][k]; b = prt[b][k];
        }
        return prt[a][0];
    }
    void init( int _n ){
        n = _n; REP( i , 1 , n ) g[ i ].clear();
    }
    void addEdge( int u , int v ){
        g[ u ].push_back( v );
        g[ v ].push_back( u );
    }
    void yutruli(){
        dfsz(1, 0);
        ts = 0;
        dfshl(1);
        REP(k, 1, LOG-1) REP(i, 1, n)
            prt[i][k] = prt[prt[i][k-1]][k-1];
    }
    vector< PII > getPath( int u , int v ){
        vector< PII > res;
        while( tid[ u ] < tid[ head[ v ] ] ){
            res.push_back( PII(tid[ head[ v ] ] , tid[ v ] ) );
            v = prt[ head[ v ] ][ 0 ];
        }
        res.push_back( PII( tid[ u ] , tid[ v ] ) );
        reverse( ALL( res ) );
        return res;
    }
    /* res : list of intervals from u to v
    * u must be ancestor of v

```



```

* usage :
* vector< PII >& path = tree.getPath( u , v )
* for( PII tp : path ) {
*     int l , r; tie( l , r ) = tp;
*     upd( l , r );
*     uu = tree.tdi[ l ] , vv = tree.tdi[ r ];
*     uu ~> vv is a heavy path on tree
* }
*/
}
} tree;

```

5.2 DominatorTree

```

const int MAXN = 100010;
struct DominatorTree{
#define REP(i,s,e) for(int i=(s);i<=(e);i++)
#define REPD(i,s,e) for(int i=(s);i>=(e);i--)
    int n , m , s;
    vector< int > g[ MAXN ] , pred[ MAXN ];
    vector< int > cov[ MAXN ];
    int dfn[ MAXN ] , nfd[ MAXN ] , ts;
    int par[ MAXN ];
    int sdom[ MAXN ] , idom[ MAXN ];
    int mom[ MAXN ] , mn[ MAXN ];
    inline bool cmp( int u , int v )
    { return dfn[ u ] < dfn[ v ]; }
    int eval( int u ){
        if( mom[ u ] == u ) return u;
        int res = eval( mom[ u ] );
        if( cmp( sdom[ mn[ mom[ u ] ] ] , sdom[ mn[ u ] ] ) )
            mn[ u ] = mn[ mom[ u ] ];
        return mom[ u ] = res;
    }
    void init( int _n , int _m , int _s ){
        ts = 0; n = _n; m = _m; s = _s;
        REP( i , 1 , n ) g[ i ].clear(); pred[ i ].clear();
    }
    void addEdge( int u , int v ){
        g[ u ].push_back( v );
        pred[ v ].push_back( u );
    }
    void dfs( int u ){
        ts++;
        dfn[ u ] = ts;
        nfd[ ts ] = u;
        for( int v : g[ u ] ) if( dfn[ v ] == 0 ){
            par[ v ] = u;
            dfs( v );
        }
    }
    void build(){
        REP( i , 1 , n ){
            dfn[ i ] = nfd[ i ] = 0;
            cov[ i ].clear();
            mom[ i ] = mn[ i ] = sdom[ i ] = i;
        }
        dfs( s );
        REPD( i , n , 2 ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
            for( int v : pred[ u ] ) if( dfn[ v ] ){
                eval( v );
                if( cmp( sdom[ mn[ v ] ] , sdom[ u ] ) )
                    sdom[ u ] = sdom[ mn[ v ] ];
            }
            cov[ sdom[ u ] ].push_back( u );
            mom[ u ] = par[ u ];
            for( int w : cov[ par[ u ] ] ){
                eval( w );
                if( cmp( sdom[ mn[ w ] ] , par[ u ] ) )
                    idom[ w ] = mn[ w ];
                else idom[ w ] = par[ u ];
            }
            cov[ par[ u ] ].clear();
        }
        REP( i , 2 , n ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
            if( idom[ u ] != sdom[ u ] )

```

```

        idom[ u ] = idom[ idom[ u ] ];
    }
} domT;

```

5.3 MaxClique

```

#define N 111
struct MaxClique{ // 0-base
    typedef bitset< N > Int;
    Int linkto[ N ] , v[ N ];
    int n;
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            linkto[ i ].reset();
            v[ i ].reset();
        }
    }
    void addEdge( int a , int b ){
        v[ a ][ b ] = v[ b ][ a ] = 1;
    }
    int popcount( const Int& val )
    { return val.count(); }
    int lowbit( const Int& val )
    { return val._Find_first(); }
    int ans , stk[ N ];
    int id[ N ] , di[ N ] , deg[ N ];
    Int cans;
    void maxclique( int elem_num , Int candi ){
        if( elem_num > ans ){
            ans = elem_num;
            cans.reset();
            for( int i = 0 ; i < elem_num ; i ++ )
                cans[ id[ stk[ i ] ] ] = 1;
        }
        int potential = elem_num + popcount( candi );
        if( potential <= ans ) return;
        int pivot = lowbit( candi );
        Int smaller_candi = candi & (~linkto[pivot]);
        while( smaller_candi.count() && potential > ans ){
            int next = lowbit( smaller_candi );
            candi[next] = !candi[next];
            smaller_candi[ next ] = !smaller_candi[ next ];
            potential --;
            if( next == pivot || (smaller_candi & linkto[next]
                ).count() ){
                stk[elem_num] = next;
                maxclique( elem_num + 1 , candi & linkto[next] );
            }
        }
    }
    int solve(){
        for( int i = 0 ; i < n ; i ++ ){
            id[ i ] = i;
            deg[ i ] = v[ i ].count();
        }
        sort( id , id + n , [&]( int id1 , int id2 ){
            return deg[id1] > deg[id2]; } );
        for( int i = 0 ; i < n ; i ++ )
            di[ id[ i ] ] = i;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                if( v[ i ][ j ] )
                    linkto[ di[ i ] ][ di[ j ] ] = 1;
        Int cand; cand.reset();
        for( int i = 0 ; i < n ; i ++ )
            cand[ i ] = 1;
        ans = 1;
        cans.reset(); cans[ 0 ] = 1;
        maxclique( 0 , cand );
        return ans;
    }
} solver;

```

5.4 Strongly Connected Component

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++)
            E[i].clear(), rE[i].clear();
    }
    void addEdge(int u, int v){
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(),vec.end());
        FZ(vst);
        for (auto v : vec)
            if (!vst[v]){
                rDFS(v); nScc++;
            }
    }
};

```

5.5 Dynamic MST

```

/* Dynamic MST O( Q lg^2 Q )
(qx[i], qy[i]) -> chg weight of edge No.qx[i] to qy[i]
delete an edge: (i, \infy)
add an edge: change from \infy to specific value
*/
const int SZ=M+3*MXQ;
int a[N],*tz;
int find(int xx){
    int root=xx; while(a[root]) root=a[root];
    int next; while((next=a[xx])){a[xx]=root; xx=next; }
    return root;
}
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,
            int *z,int m1,long long ans){
    if(Q==1){
        for(int i=1;i<=n;i++) a[i]=0;
        z[ qx[0] ]=qy[0]; tz = z;
        for(int i=0;i<m1;i++) id[i]=i;
        sort(id,id+m1,cmp); int ri,rj;
        for(int i=0;i<m1;i++){
            ri=find(x[id[i]]); rj=find(y[id[i]]);
            if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
        }
        printf("%lld\n",ans);
        return;
    }
    int ri,rj;
    //contract
    kt=0;
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<Q;i++){
        ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[ri]=rj;
    }
    int tm=0;
    for(int i=0;i<m1;i++) extra[i]=true;
    for(int i=0;i<Q;i++) extra[ qx[i] ]=false;
    for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;
    tz=z; sort(id,id+tm,cmp);
    for(int i=0;i<tm;i++){

```

```

        ri=find(x[id[i]]); rj=find(y[id[i]]);
        if(ri!=rj){
            a[ri]=rj; ans += z[id[i]];
            kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
        }
    }
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);
    int n2=0;
    for(int i=1;i<=n;i++) if(a[i]==0)
        vd[i]=++n2;
    for(int i=1;i<=n;i++) if(a[i])
        vd[i]=vd[find(i)];
    int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
    for(int i=0;i<m1;i++) app[i]=-1;
    for(int i=0;i<Q;i++){ if(app[qx[i]]==-1){
        Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ];
        Nz[m2]=z[ qx[i] ];
        app[qx[i]]=m2; m2++;
    }
    }
    for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[i]]; }
    for(int i=1;i<=n2;i++) a[i]=0;
    for(int i=0;i<tm;i++){
        ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
        if(ri!=rj){
            a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
            Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
        }
    }
    int mid=Q/2;
    solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
    solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
}
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
    scanf("%d",&n,&m);
    for(int i=0;i<m;i++) scanf("%d%d",x+i,y+i,z+i);
    scanf("%d",&Q);
    for(int i=0;i<Q;i++){ scanf("%d",qx+i,qy+i); qx[i]--; }
}
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
int main(){init(); work(); }

```

5.6 Maximum General graph Matching

```

const int N = 514, E = (2e5) * 2;
struct Graph{
    int to[E],bro[E],head[N],e;
    int lnk[N],vis[N],stp,n;
    void init( int _n ){
        stp = 0; e = 1; n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            lnk[i] = vis[i] = 0;
    }
    void add_edge(int u,int v){
        to[e]=v,bro[e]=head[u],head[u]=e++;
        to[e]=u,bro[e]=head[v],head[v]=e++;
    }
    bool dfs(int x){
        vis[x]=stp;
        for(int i=head[x];i;i=bro[i]){
            int v=to[i];
            if(!lnk[v]){
                lnk[x]=v,lnk[v]=x;
                return true;
            }else if(vis[lnk[v]]<stp){
                int w=lnk[v];
                lnk[x]=v,lnk[v]=x,lnk[w]=0;
                if(dfs(w)){
                    return true;
                }
                lnk[w]=v,lnk[v]=w,lnk[x]=0;
            }
        }
        return false;
    }
    int solve(){
        int ans = 0;

```

```

    for(int i=1;i<=n;i++)
        if(!lnk[i]){
            stp++; ans += dfs(i);
        }
    return ans;
}
} graph;

```

5.7 Minimum General Weighted Matching

```

struct Graph {
    // Minimum General Weighted Matching (Perfect Match)
    static const int MXN = 105;
    int n, edge[MXN][MXN];
    int match[MXN], dis[MXN], onstk[MXN];
    vector<int> stk;
    void init(int _n) {
        n = _n;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                edge[ i ][ j ] = 0;
    }
    void add_edge(int u, int v, int w)
    { edge[u][v] = edge[v][u] = w; }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.PB(u);
        onstk[u] = 1;
        for (int v=0; v<n; v++){
            if (u != v && match[u] != v && !onstk[v]){
                int m = match[v];
                if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1;
                    stk.PB(v);
                    if (SPFA(m)) return true;
                    stk.pop_back();
                    onstk[v] = 0;
                }
            }
        }
        onstk[u] = 0;
        stk.pop_back();
        return false;
    }
    int solve() {
        // find a match
        for (int i=0; i<n; i+=2){
            match[i] = i+1;
            match[i+1] = i;
        }
        while (true){
            int found = 0;
            for( int i = 0 ; i < n ; i ++ )
                onstk[ i ] = dis[ i ] = 0;
            for (int i=0; i<n; i++){
                stk.clear();
                if (!onstk[i] && SPFA(i)){
                    found = 1;
                    while (SZ(stk)>=2){
                        int u = stk.back(); stk.pop_back();
                        int v = stk.back(); stk.pop_back();
                        match[u] = v;
                        match[v] = u;
                    }
                }
            }
            if (!found) break;
        }
        int ret = 0;
        for (int i=0; i<n; i++)
            ret += edge[i][match[i]];
        ret /= 2;
        return ret;
    }
} graph;

```

5.8 Maximum General Weighted Matching

```

struct WeightGraph {
    static const int INF = INT_MAX;
    static const int N = 514;
    struct edge{
        int u,v,w; edge(){}
        edge(int ui,int vi,int wi)
            :u(ui),v(vi),w(wi){}
    };
    int n,n_x;
    edge g[N*2][N*2];
    int lab[N*2];
    int match[N*2],slack[N*2],st[N*2],pa[N*2];
    int flo_from[N*2][N+1],S[N*2],vis[N*2];
    vector<int> flo[N*2];
    queue<int> q;
    int e_delta(const edge &e){
        return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
    }
    void update_slack(int u,int x){
        if(!slack[x]||e_delta(g[u][x])<e_delta(g[slack[x]][x]))
            slack[x]=u;
    }
    void set_slack(int x){
        slack[x]=0;
        for(int u=1;u<=n;++u)
            if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
                update_slack(u,x);
    }
    void q_push(int x){
        if(x<=n)q.push(x);
        else for(size_t i=0;i<flo[x].size();i++)
            q_push(flo[x][i]);
    }
    void set_st(int x,int b){
        st[x]=b;
        if(x>n)for(size_t i=0;i<flo[x].size();++i)
            set_st(flo[x][i],b);
    }
    int get_pr(int b,int xr){
        int pr=find(flo[b].begin(),flo[b].end(),xr)-flo[b].begin();
        if(pr%2==1){
            reverse(flo[b].begin()+1,flo[b].end());
            return (int)flo[b].size()-pr;
        }else return pr;
    }
    void set_match(int u,int v){
        match[u]=g[u][v].v;
        if(u<=n) return;
        edge e=g[u][v];
        int xr=flo_from[u][e.u],pr=get_pr(u,xr);
        for(int i=0;i<pr;++i)set_match(flo[u][i],flo[u][i+1]);
        set_match(xr,v);
        rotate(flo[u].begin(),flo[u].begin()+pr,flo[u].end());
    }
    void augment(int u,int v){
        for(;;){
            int xnv=st[match[u]];
            set_match(u,v);
            if(!xnv)return;
            set_match(xnv,st[pa[xnv]]);
            u=st[pa[xnv]],v=xnv;
        }
    }
    int get_lca(int u,int v){
        static int t=0;
        for(++t;u!=v;swap(u,v)){
            if(u==0)continue;
            if(vis[u]==t)return u;
            vis[u]=t;
            u=st[match[u]];
            if(u)u=st[pa[u]];
        }
        return 0;
    }
    void add_blossom(int u,int lca,int v){
        int b=n+1;
        while(b<=n_x&&st[b])++b;
        if(b>n_x)++n_x;
        lab[b]=0,S[b]=0;
    }

```

```

match[b]=match[lca];
flo[b].clear();
flo[b].push_back(lca);
for(int x=u,y;x!=lca;x=st[pa[y]])
    flo[b].push_back(x),flo[b].push_back(y=st[match[x]]),q.push(y);
reverse(flo[b].begin()+1,flo[b].end());
for(int x=v,y;x!=lca;x=st[pa[y]])
    flo[b].push_back(x),flo[b].push_back(y=st[match[x]]),q.push(y);
set_st(b,b);
for(int x=1;x<=n_x;++x)g[b][x].w=g[x][b].w=0;
for(int x=1;x<=n;++x)flo_from[b][x]=0;
for(size_t i=0;i<flo[b].size();++i){
    int xs=flo[b][i];
    for(int x=1;x<=n_x;++x)
        if(g[b][x].w==0||e_delta(g[xs][x])<e_delta(g[b][x]))
            g[b][x]=g[xs][x],g[x][b]=g[x][xs];
    for(int x=1;x<=n;++x)
        if(flo_from[xs][x])flo_from[b][x]=xs;
}
set_slack(b);
}
void expand_blossom(int b){
    for(size_t i=0;i<flo[b].size();++i)
        set_st(flo[b][i],flo[b][i]);
    int xr=flo_from[b][g[b][pa[b]].u],pr=get_pr(b,xr);
    for(int i=0;i<pr;i+=2){
        int xs=flo[b][i],xns=flo[b][i+1];
        pa[xs]=g[xns][xs].u;
        S[xs]=1,S[xns]=0;
        slack[xs]=0,set_slack(xns);
        q.push(xns);
    }
    S[xr]=1,pa[xr]=pa[b];
    for(size_t i=pr+1;i<flo[b].size();++i){
        int xs=flo[b][i];
        S[xs]=-1,set_slack(xs);
    }
    st[b]=0;
}
bool on_found_edge(const edge &e){
    int u=st[e.u],v=st[e.v];
    if(S[v]==-1){
        pa[v]=e.u,S[v]=1;
        int nu=st[match[v]];
        slack[v]=slack[nu]=0;
        S[nu]=0,q.push(nu);
    }else if(S[v]==0){
        int lca=get_lca(u,v);
        if(!lca)return augment(u,v),augment(v,u),true;
        else add_blossom(u,lca,v);
    }
    return false;
}
bool matching(){
    memset(S+1,-1,sizeof(int)*n_x);
    memset(slack+1,0,sizeof(int)*n_x);
    q=queue<int>();
    for(int x=1;x<=n_x;++x)
        if(st[x]==x&&!match[x])pa[x]=0,S[x]=0,q.push(x);
    if(q.empty())return false;
    for(;;){
        while(q.size()){
            int u=q.front();q.pop();
            if(S[st[u]]==1)continue;
            for(int v=1;v<=n;++v)
                if(g[u][v].w>0&&st[u]!=st[v]){
                    if(e_delta(g[u][v])==0){
                        if(on_found_edge(g[u][v]))return true;
                    }else update_slack(u,st[v]);
                }
        }
        int d=INF;
        for(int b=n+1;b<=n_x;++b)
            if(st[b]==b&&S[b]==1)d=min(d,lab[b]/2);
        for(int x=1;x<=n_x;++x)
            if(st[x]==x&&slack[x]){
                if(S[x]==-1)d=min(d,e_delta(g[slack[x]][x]));
                else if(S[x]==0)d=min(d,e_delta(g[slack[x]][x])/2);
            }
    }
}

```

```

}
for(int u=1;u<=n;++u){
    if(S[st[u]]==0){
        if(lab[u]<=d)return 0;
        lab[u]-=d;
    }else if(S[st[u]]==1)lab[u]+=d;
}
for(int b=n+1;b<=n_x;++b)
    if(st[b]==b){
        if(S[st[b]]==0)lab[b]+=d*2;
        else if(S[st[b]]==1)lab[b]-=d*2;
    }
q=queue<int>();
for(int x=1;x<=n_x;++x)
    if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta(g[slack[x]][x])==0)
        if(on_found_edge(g[slack[x]][x]))return true;
for(int b=n+1;b<=n_x;++b)
    if(st[b]==b&&S[b]==1&&lab[b]==0)expand_blossom(b);
}
return false;
}
pair<long long,int> solve(){
    memset(match+1,0,sizeof(int)*n);
    n_x=n;
    int n_matches=0;
    long long tot_weight=0;
    for(int u=0;u<=n;++u)st[u]=u,flo[u].clear();
    int w_max=0;
    for(int u=1;u<=n;++u)
        for(int v=1;v<=n;++v){
            flo_from[u][v]=(u==v?0);
            w_max=max(w_max,g[u][v].w);
        }
    for(int u=1;u<=n;++u)lab[u]=w_max;
    while(matching())n_matches++;
    for(int u=1;u<=n;++u)
        if(match[u]&&match[u]<u)
            tot_weight+=g[u][match[u]].w;
    return make_pair(tot_weight,n_matches);
}
void add_edge( int ui , int vi , int wi ){
    g[ui][vi].w = g[vi][ui].w = wi;
}
void init( int _n ){
    n = _n;
    for(int u=1;u<=n;++u)
        for(int v=1;v<=n;++v)
            g[u][v]=edge(u,v,0);
}
} graph;

```

5.9 Minimum Steiner Tree

```

// Minimum Steiner Tree
//  $O(V^3 T + V^2 2^T)$ 
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
    int n , dst[V][V] , dp[1<<T][V] , tdst[V];
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            for( int j = 0 ; j < n ; j ++ )
                dst[ i ][ j ] = INF;
            dst[ i ][ i ] = 0;
        }
    }
    void add_edge( int ui , int vi , int wi ){
        dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
        dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
    }
    void shortest_path(){
        for( int k = 0 ; k < n ; k ++ )
            for( int i = 0 ; i < n ; i ++ )
                for( int j = 0 ; j < n ; j ++ )
                    dst[ i ][ j ] = min( dst[ i ][ j ] ,
                        dst[ i ][ k ] + dst[ k ][ j ] );
    }
}

```

```

}
int solve( const vector<int>& ter ){
    int t = (int)ter.size();
    for( int i = 0 ; i < ( 1 << t ) ; i ++ )
        for( int j = 0 ; j < n ; j ++ )
            dp[ i ][ j ] = INF;
    for( int i = 0 ; i < n ; i ++ )
        dp[ 0 ][ i ] = 0;
    for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
        if( msk == ( msk & (-msk) ) ){
            int who = __lg( msk );
            for( int i = 0 ; i < n ; i ++ )
                dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
            continue;
        }
        for( int i = 0 ; i < n ; i ++ )
            for( int submsk = ( msk - 1 ) & msk ; submsk ;
                submsk = ( submsk - 1 ) & msk )
                dp[ msk ][ i ] = min( dp[ msk ][ i ],
                    dp[ submsk ][ i ] +
                    dp[ msk ^ submsk ][ i ] );
        for( int i = 0 ; i < n ; i ++ ){
            tdst[ i ] = INF;
            for( int j = 0 ; j < n ; j ++ )
                tdst[ i ] = min( tdst[ i ],
                    dp[ msk ][ j ] + dst[ j ][ i ] );
        }
        for( int i = 0 ; i < n ; i ++ )
            dp[ msk ][ i ] = tdst[ i ];
    }
    int ans = INF;
    for( int i = 0 ; i < n ; i ++ )
        ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
    return ans;
}
} solver;

```

5.10 BCC based on vertex

```

struct BccVertex {
    int n,nScc,step,dfn[MXN],low[MXN];
    vector<int> E[MXN],sccv[MXN];
    int top,stk[MXN];
    void init(int _n) {
        n = _n; nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v)
    { E[u].PB(v); E[v].PB(u); }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v:E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v,u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z;
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            } else
                low[u] = min(low[u], dfn[v]);
        }
    }
    vector<vector<int>> solve() {
        vector<vector<int>> res;
        for (int i=0; i<n; i++)
            dfn[i] = low[i] = -1;
        for (int i=0; i<n; i++)
            if (dfn[i] == -1) {
                top = 0;
                DFS(i,i);
            }
        REP(i,nScc)res.PB(sccv[i]);
    }
}

```

```

    return res;
}
} graph;

```

5.11 Min Mean Cycle

```

/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n )
    { n = _n; m = 0; }
    // WARNING: TYPE matters
    void addEdge( int vi , int ui , double ci )
    { e[ m ++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
                    prv[i+1][u] = v;
                    prve[i+1][u] = j;
                }
            }
        }
    }
    double solve(){
        // returns inf if no cycle, mmc otherwise
        double mmc=inf;
        int st = -1;
        bellman_ford();
        for(int i=0; i<n; i++) {
            double avg=-inf;
            for(int k=0; k<n; k++) {
                if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
                else avg=max(avg,inf);
            }
            if (avg < mmc) tie(mmc, st) = tie(avg, i);
        }
        FZ(vst); edgeID.clear(); cycle.clear(); rho.clear();
        for (int i=n; !vst[st]; st=prv[i--][st]) {
            vst[st]++;
            edgeID.PB(prve[i][st]);
            rho.PB(st);
        }
        while (vst[st] != 2) {
            int v = rho.back(); rho.pop_back();
            cycle.PB(v);
            vst[v]++;
        }
        reverse(ALL(edgeID));
        edgeID.resize(SZ(cycle));
        return mmc;
    }
} mmc;

```

5.12 Directed Graph Min Cost Cycle

```

// works in O(N M)
#define INF 100000000000000LL
#define N 5010
#define M 200010
struct edge{
    int to; LL w;
    edge(int a=0, LL b=0): to(a), w(b){}
};

```

```

struct node{
    LL d; int u, next;
    node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
}b[M];
struct DirectedGraphMinCycle{
    vector<edge> g[N], grev[N];
    LL dp[N][N], p[N], d[N], mu;
    bool inq[N];
    int n, bn, bsz, hd[N];
    void b_insert(LL d, int u){
        int i = d/mu;
        if(i >= bn) return;
        b[++bsz] = node(d, u, hd[i]);
        hd[i] = bsz;
    }
    void init( int _n ){
        n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            g[ i ].clear();
    }
    void addEdge( int ai , int bi , LL ci )
    { g[ai].push_back(edge(bi,ci)); }
    LL solve(){
        fill(dp[0], dp[0]+n+1, 0);
        for(int i=1; i<=n; i++){
            fill(dp[i]+1, dp[i]+n+1, INF);
            for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
                for(int k=0; k<(int)g[j].size(); k++){
                    dp[i][g[j][k].to] = min(dp[i][g[j][k].to],
                                            dp[i-1][j]+g[j][k].w);
                }
            }
        }
        mu=INF; LL bunbo=1;
        for(int i=1; i<=n; i++) if(dp[n][i] < INF){
            LL a=-INF, b=1;
            for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
                if(a*(n-j) < b*(dp[n][i]-dp[j][i])){
                    a = dp[n][i]-dp[j][i];
                    b = n-j;
                }
            }
            if(mu*b > bunbo*a)
                mu = a, bunbo = b;
        }
        if(mu < 0) return -1; // negative cycle
        if(mu == INF) return INF; // no cycle
        if(mu == 0) return 0;
        for(int i=1; i<=n; i++){
            for(int j=0; j<(int)g[i].size(); j++){
                g[i][j].w *= bunbo;
            }
            memset(p, 0, sizeof(p));
            queue<int> q;
            for(int i=1; i<=n; i++){
                q.push(i);
                inq[i] = true;
            }
            while(!q.empty()){
                int i=q.front(); q.pop(); inq[i]=false;
                for(int j=0; j<(int)g[i].size(); j++){
                    if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
                        p[g[i][j].to] = p[i]+g[i][j].w-mu;
                        if(!inq[g[i][j].to]){
                            q.push(g[i][j].to);
                            inq[g[i][j].to] = true;
                        }
                    }
                }
            }
        }
        for(int i=1; i<=n; i++) grev[i].clear();
        for(int i=1; i<=n; i++){
            for(int j=0; j<(int)g[i].size(); j++){
                g[i][j].w += p[i]-p[g[i][j].to];
                grev[g[i][j].to].push_back(edge(i, g[i][j].w));
            }
        }
        LL mldc = n*mu;
        for(int i=1; i<=n; i++){
            bn=mldc/mu, bsz=0;
            memset(hd, 0, sizeof(hd));
            fill(d+1, d+n+1, INF);
            b_insert(d[i]=0, i);
            for(int j=0; j<=bn-1; j++) for(int k=hd[j]; k; k=
                b[k].next){

```

```

                int u = b[k].u;
                LL du = b[k].d;
                if(du > d[u]) continue;
                for(int l=0; l<(int)g[u].size(); l++) if(g[u][l]
                    ].to > i){
                    if(d[g[u][l].to] > du + g[u][l].w){
                        d[g[u][l].to] = du + g[u][l].w;
                        b_insert(d[g[u][l].to], g[u][l].to);
                    }
                }
            }
            for(int j=0; j<(int)grev[i].size(); j++) if(grev[
                i][j].to > i)
                mldc=min(mldc,d[grev[i][j].to] + grev[i][j].w);
        }
        return mldc / bunbo;
    }
} graph;

```

5.13 K-th Shortest Path

```

// time: O(|E| \lg |E| + |V| \lg |V| + K)
// memory: O(|E| \lg |E| + |V|)
struct KSP{ // 1-base
    struct nd{
        int u, v, d;
        nd(int ui = 0, int vi = 0, int di = INF)
        { u = ui; v = vi; d = di; }
    };
    struct heap{
        nd* edge; int dep; heap* chd[4];
    };
    static int cmp(heap* a, heap* b)
    { return a->edge->d > b->edge->d; }
    struct node{
        int v; LL d; heap* H; nd* E;
        node(){
            node(LL _d, int _v, nd* _E)
            { d = _d; v = _v; E = _E; }
            node(heap* _H, LL _d)
            { H = _H; d = _d; }
        friend bool operator<(node a, node b)
        { return a.d > b.d; }
    };
    int n, k, s, t, dst[ N ];
    nd *nxt[ N ];
    vector<nd*> g[ N ], rg[ N ];
    heap *nullNd, *head[ N ];
    void init( int _n , int _k , int _s , int _t ){
        n = _n; k = _k; s = _s; t = _t;
        for( int i = 1 ; i <= n ; i ++ ){
            g[ i ].clear(); rg[ i ].clear();
            nxt[ i ] = head[ i ] = NULL;
            dst[ i ] = -1;
        }
    }
    void addEdge( int ui , int vi , int di ){
        nd* e = new nd(ui, vi, di);
        g[ ui ].push_back( e );
        rg[ vi ].push_back( e );
    }
    queue<int> dfsQ;
    void dijkstra(){
        while(dfsQ.size()) dfsQ.pop();
        priority_queue<node> Q;
        Q.push(node(0, t, NULL));
        while (!Q.empty()){
            node p = Q.top(); Q.pop();
            if(dst[p.v] != -1) continue;
            dst[ p.v ] = p.d;
            nxt[ p.v ] = p.E;
            dfsQ.push( p.v );
            for(auto e: rg[ p.v ])
                Q.push(node(p.d + e->d, e->u, e));
        }
    }
    heap* merge(heap* curNd, heap* newNd){
        if(curNd == nullNd) return newNd;
        heap* root = new heap;
        memcpy(root, curNd, sizeof(heap));
    }

```



```

if(newNd->edge->d < curNd->edge->d){
    root->edge = newNd->edge;
    root->chd[2] = newNd->chd[2];
    root->chd[3] = newNd->chd[3];
    newNd->edge = curNd->edge;
    newNd->chd[2] = curNd->chd[2];
    newNd->chd[3] = curNd->chd[3];
}
if(root->chd[0]->dep < root->chd[1]->dep)
    root->chd[0] = merge(root->chd[0],newNd);
else
    root->chd[1] = merge(root->chd[1],newNd);
root->dep = max(root->chd[0]->dep, root->chd[1]->
    dep) + 1;
return root;
}
vector<heap*> V;
void build(){
    nullNd = new heap;
    nullNd->dep = 0;
    nullNd->edge = new nd;
    fill(nullNd->chd, nullNd->chd+4, nullNd);
    while(not dfsQ.empty()){
        int u = dfsQ.front(); dfsQ.pop();
        if(!nxt[ u ]) head[ u ] = nullNd;
        else head[ u ] = head[nxt[ u ]->v];
        V.clear();
        for( auto&& e : g[ u ] ){
            int v = e->v;
            if( dst[ v ] == -1 ) continue;
            e->d += dst[ v ] - dst[ u ];
            if( nxt[ u ] != e ){
                heap* p = new heap;
                fill(p->chd, p->chd+4, nullNd);
                p->dep = 1;
                p->edge = e;
                V.push_back(p);
            }
        }
        if(V.empty()) continue;
        make_heap(V.begin(), V.end(), cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
        for( size_t i = 0 ; i < V.size() ; i ++ ){
            if(L(i) < V.size()) V[i]->chd[2] = V[L(i)];
            else V[i]->chd[2]=nullNd;
            if(R(i) < V.size()) V[i]->chd[3] = V[R(i)];
            else V[i]->chd[3]=nullNd;
        }
        head[u] = merge(head[u], V.front());
    }
}
vector<LL> ans;
void first_K(){
    ans.clear();
    priority_queue<node> Q;
    if( dst[ s ] == -1 ) return;
    ans.push_back( dst[ s ] );
    if( head[s] != nullNd )
        Q.push(node(head[s], dst[s]+head[s]->edge->d));
    for( int _ = 1 ; _ < k and not Q.empty() ; _ ++ ){
        node p = Q.top(); q; Q.pop();
        ans.push_back( p.d );
        if(head[ p.H->edge->v ] != nullNd){
            q.H = head[ p.H->edge->v ];
            q.d = p.d + q.H->edge->d;
            Q.push(q);
        }
        for( int i = 0 ; i < 4 ; i ++ )
            if( p.H->chd[ i ] != nullNd ){
                q.H = p.H->chd[ i ];
                q.d = p.d - p.H->edge->d + p.H->chd[ i ]->
                    edge->d;
                Q.push( q );
            }
    }
}
void solve(){
    dijkstra();
    build();
    first_K();
}

```

```

} solver;

```

6 String

6.1 PalTree

```

/*
 * sfail: compressed fail links with same diff
 * O(lgn): length of sfail link path
 */
const int MAXN = 1e6+10;
struct PalT{
    int tot,lst;
    int nxt[MAXN][26], len[MAXN];
    int fail[MAXN], diff[MAXN], sfail[MAXN];
    char* s;
    int newNode(int l, int _fail) {
        int res = ++tot;
        fill(nxt[res], nxt[res]+26, 0);
        len[res] = l, fail[res] = _fail;
        diff[res] = l - len[_fail];
        if (diff[res] == diff[_fail])
            sfail[res] = sfail[_fail];
        else
            sfail[res] = _fail;
        return res;
    }
    void push(int p) {
        int np = lst;
        int c = s[p]-'a';
        while (p-len[np]-1 < 0 || s[p] != s[p-len[np]-1])
            np = fail[np];
        if ((lst=nxt[np][c])) return;
        int nq_f = 0;
        if (len[np]+2 == 1) nq_f = 2;
        else {
            int tf = fail[np];
            while (p-len[tf]-1 < 0 || s[p] != s[p-len[tf]-1])
                tf = fail[tf];
            nq_f = nxt[tf][c];
        }
        int nq = newNode(len[np]+2, nq_f);
        nxt[np][c] = nq;
        lst=nq;
    }
    void init(char* _s){
        s = _s;
        tot = 0;
        newNode(-1, 1);
        newNode(0, 1);
        diff[2] = 0;
        lst = 2;
    }
} palt;

```

6.2 SAIS

```

const int N = 300010;
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;

```

```

    hei[r[i]] = ans;
}
}
void sais(int *s, int *sa, int *p, int *q, bool *t,
          int *c, int n, int z){
    bool uniq = t[n-1] = true, neq;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
        lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
    XD; \
    memcpy(x + 1, c, sizeof(int) * (z - 1)); \
    REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]
        ]-1]]++ = sa[i]-1; \
    memcpy(x, c, sizeof(int) * z); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]
        ]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
    MS0(c, z);
    REP(i,n) uniq &= ++c[s[i]] < 2;
    REP(i,z-1) c[i+1] += c[i];
    if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
    for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
        +1] ? t[i+1] : s[i]<s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i]
        ]]=p[q[i]=nn++]=i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
        neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
            [i])*sizeof(int));
        ns[q[lst=sa[i]]]=nmzx+=neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx
        + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s[p[
        nsa[i]]]] = p[nsa[i]]);
}
}
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}

```

6.3 SuffixAutomata

```

const int MAXM = 1000010;
struct SAM{
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode(){
        int res = ++tot;
        fill(nxt[res], nxt[res]+33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init(){
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c){
        int p = lst;
        int np = newNode();
        mx[np] = mx[p]+1;
        for(; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if(p == 0) mom[np] = root;
        else{
            int q = nxt[p][c];
            if(mx[p]+1 == mx[q]) mom[np] = q;
            else{

```

```

                int nq = newNode();
                mx[nq] = mx[p]+1;
                for(int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for(; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void push(char *str){
        for(int i = 0; str[i]; i++)
            push(str[i]-'a'+1);
    }
} sam;

```

6.4 Aho-Corasick

```

struct ACautomata{
    struct Node{
        int cnt,dp;
        Node *go[26], *fail;
        Node (){
            cnt = 0; dp = -1; fail = 0;
            memset(go,0,sizeof(go));
        }
    };
    Node *root, pool[1048576];
    int nMem;
    Node* new_Node(){
        pool[nMem] = Node();
        return &pool[nMem++];
    }
    void init(){
        { nMem = 0; root = new_Node(); }
    }
    void add(const string &str)
    { insert(root,str,0); }
    void insert(Node *cur, const string &str, int pos){
        if (pos >= (int)str.size())
            { cur->cnt++; return; }
        int c = str[pos]-'a';
        if (cur->go[c] == 0)
            cur->go[c] = new_Node();
        insert(cur->go[c],str,pos+1);
    }
    void make_fail(){
        queue<Node*> que;
        que.push(root);
        while (!que.empty()){
            Node* fr=que.front();
            que.pop();
            for (int i=0; i<26; i++){
                if (fr->go[i]){
                    Node *ptr = fr->fail;
                    while (ptr && !ptr->go[i]) ptr = ptr->fail;
                    if (!ptr) fr->go[i]->fail = root;
                    else fr->go[i]->fail = ptr->go[i];
                    que.push(fr->go[i]);
                }
            }
        }
    }
};

```

6.5 Z Value

```

char s[MAXN];
int len,z[MAXN];
void Z_value() {
    int i,j,left,right;
    left=right=0; z[0]=len;
    for(i=1;i<len;i++) {
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
    }
}

```

```

    if(i+z[i]>right) {
        right=i+z[i];
        left=i;
    }
}
}

```

6.6 BWT

```

struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
    vector<int> v[ SIGMA ];
    void BWT(char* ori, char* res){
        // make ori -> ori + ori
        // then build suffix array
    }
    void iBWT(char* ori, char* res){
        for( int i = 0 ; i < SIGMA ; i ++ )
            v[ i ].clear();
        int len = strlen( ori );
        for( int i = 0 ; i < len ; i ++ )
            v[ ori[i] - BASE ].push_back( i );
        vector<int> a;
        for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
            for( auto j : v[ i ] ){
                a.push_back( j );
                ori[ ptr ++ ] = BASE + i;
            }
        for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
            res[ i ] = ori[ a[ ptr ] ];
            ptr = a[ ptr ];
        }
        res[ len ] = 0;
    }
} bwt;

```

6.7 ZValue Palindrome

```

int len, zv[MAX*2];
char ip[MAX], op[MAX*2];
int main(){
    cin >> ip; len = strlen(ip);
    int l2 = len*2 - 1;
    for(int i=0; i<l2; i++)
        if(i&1) op[i] = '@';
        else op[i] = ip[i/2];
    int l=0, r=0; zv[0] = 1;
    for(int i=1; i<l2; i++){
        if( i > r ){
            l = r = i;
            while( l>0 && r<l2-1 && op[l-1] == op[r+1] )
                l --, r ++;
            zv[i] = (r-l+1);
        }else{
            int md = (l+r)/2, j = md + md - i;
            zv[i] = zv[j];
            int q = zv[i] / 2, nr = i + q;
            if( nr == r ){
                l = i + i - r;
                while( l>0 && r<l2-1 && op[l-1] == op[r+1] )
                    l --, r ++;
                zv[i] = r - l + 1;
            }else if( nr > r )
                zv[i] = (r - i) * 2 + 1;
        }
    }
}

```

6.8 Smallest Rotation

```

string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){

```

```

        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

6.9 Cyclic LCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
    int i=r+al,j=bl,l=0;
    while(i>r) {
        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];
        j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al&&j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl&&pred[i+1][j+1]==LU) {
            i++;
            j++;
            pred[i][j]=L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if(al>bl) {
        swap(al,bl);
        strcpy(tmp,a);
        strcpy(a,b);
        strcpy(b,tmp);
    }
    strcpy(tmp,a);
    strcat(a,tmp);
    // basic lcs
    for(int i=0;i<=2*al;i++) {
        dp[i][0]=0;
        pred[i][0]=U;
    }
    for(int j=0;j<=bl;j++) {
        dp[0][j]=0;
        pred[0][j]=L;
    }
    for(int i=1;i<=2*al;i++) {
        for(int j=1;j<=bl;j++) {
            if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
            else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
            if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
            else if(a[i-1]==b[j-1]) pred[i][j]=LU;
            else pred[i][j]=U;
        }
    }
    // do cyclic lcs
}

```

```

int clcs=0;
for(int i=0;i<al;i++) {
    clcs=max(clcs,lcs_length(i));
    reroot(i+1);
}
// recover a
a[al]='\0';
return clcs;
}

```

7 Data Structure

7.1 Link-Cut Tree

```

const int MEM = 100005;
struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay (int _val=-1) : val(_val), rev(0), size(1)
    { f = ch[0] = ch[1] = &nil; }
    bool isr()
    { return f->ch[0] != this && f->ch[1] != this; }
    int dir()
    { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d){
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push(){
        if( !rev ) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev=0;
    }
    void pull(){
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[d], d);
    x->setCh(p, !d);
}
vector<Splay*> splayVec;
void splay(Splay *x){
    splayVec.clear();
    for (Splay *q=x;; q=q->f){
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
    Splay *q = nil;
    for (;x!=nil;x=x->f){
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
    return q;
}

```

```

}
void chroot(Splay *x){
    access(x);
    splay(x);
    x->rev ^= 1;
}
void link(Splay *x, Splay *y){
    chroot(y);
    y->f=x;
}
void cut_p(Splay *y) {
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
    chroot(x);
    cut_p(y);
}
Splay* get_root(Splay *x) {
    x=access(x);
    for(; x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    x = get_root(x), y = get_root(y);
    return x == y;
}
Splay* lca(Splay *x, Splay *y) {
    access(x);
    return access(y);
}
/*
query(Splay *x,Splay *y){
    setroot(y);
    x=access(x);
    return x->tag;
}
query(Splay *x,Splay *y){
    Splay *p=lca(x,y);
    return p->tag+p->ch[1]->tag+(x!=p?x->tag:0);
}
*/

```

7.2 Black Magic

```

#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int,int> umap_t;
typedef priority_queue<int> heap;
#include<ext/rope>
using namespace __gnu_cxx;
int main(){
    // Insert some entries into s.
    set_t s; s.insert(12); s.insert(505);
    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);
    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);
    // Erase an entry.
    s.erase(12);
    // The order of the keys should be: 505.
    assert(*s.find_by_order(0) == 505);
    // The order of the keys should be: 505.
    assert(s.order_of_key(505) == 0);

    heap h1 , h2; h1.join( h2 );

    rope<char> r[ 2 ];
    r[ 1 ] = r[ 0 ]; // persistenet
    string t = "abc";
    r[ 1 ].insert( 0 , t.c_str() );
}

```

```

r[ 1 ].erase( 1 , 1 );
cout << r[ 1 ].substr( 0 , 2 );
}

```

8 Others

8.1 Find max tangent(x,y is increasing)

```

typedef long long LL;
const int MAXN = 100010;
struct Coord{
    LL x, y;
    Coord operator - (Coord ag) const{
        Coord res;
        res.x = x - ag.x;
        res.y = y - ag.y;
        return res;
    }
}sum[MAXN], pnt[MAXN], ans, calc;

inline bool cross(Coord a, Coord b, Coord c){
    return (c.y-a.y)*(c.x-b.x) > (c.x-a.x)*(c.y-b.y);
}

int main(){
    int n, l, np, st, ed, now;
    scanf("%d %d\n", &n, &l);
    sum[0].x = sum[0].y = np = st = ed = 0;
    for (int i = 1, v; i <= n; i++){
        scanf("%d", &v);
        sum[i].y = sum[i-1].y + v;
        sum[i].x = i;
    }
    ans.x = now = 1;
    ans.y = -1;
    for (int i = 0; i <= n - l; i++){
        while (np > 1 &&
            cross(pnt[np-2], pnt[np-1], sum[i]))
            np--;
        if (np < now && np != 0) now = np;
        pnt[np++] = sum[i];
        while (now < np &&
            !cross(pnt[now-1], pnt[now], sum[i+l]))
            now++;
        calc = sum[i+l] - pnt[now-1];
        if (ans.y * calc.x < ans.x * calc.y){
            ans = calc;
            st = pnt[now-1].x;
            ed = i+l;
        }
    }
    double res = (sum[ed].y - sum[st].y) /
        (sum[ed].x - sum[st].x);
    printf("%f\n", res);
    return 0;
}

```

8.2 Exact Cover Set

```

// given n*m 0-1 matrix
// find a set of rows s.t.
// for each column, there's exactly one 1
#include <stdio.h>
#include <string.h>
#define N 1024 //row
#define M 1024 //column
#define NM ((N+2)*(M+2))
char A[N][M]; //n*m 0-1 matrix
int used[N]; //answer: the row used
int id[N][M];
int L[NM], R[NM], D[NM], U[NM], C[NM], S[NM], ROW[NM];
void remove(int c){
    L[R[c]] = L[c]; R[L[c]] = R[c];
    for (int i=D[c]; i!=c; i=D[i]){
        for (int j=R[i]; j!=i; j=R[j]){
            U[D[j]] = U[j]; D[U[j]] = D[j]; S[C[j]]--;

```

```

        }
    }
}

void resume(int c){
    for (int i=D[c]; i!=c; i=D[i]){
        for (int j=L[i]; j!=i; j=L[j]){
            U[D[j]] = D[U[j]] = j; S[C[j]]++;
        }
        L[R[c]] = R[L[c]] = c;
    }
}

int dfs(){
    if(R[0]==0) return 1;
    int md=100000000,c;
    for (int i=R[0]; i!=0; i=R[i]){
        if(S[i]<md){ md=S[i]; c=i; }
        if(md==0) return 0;
        remove(c);
        for (int i=D[c]; i!=c; i=D[i]){
            used[ROW[i]] = 1;
            for (int j=R[i]; j!=i; j=R[j]) remove(C[j]);
            if(dfs()) return 1;
            for (int j=L[i]; j!=i; j=L[j]) resume(C[j]);
            used[ROW[i]] = 0;
        }
        resume(c);
        return 0;
    }
}

int exact_cover(int n, int m){
    for (int i=0; i<=m; i++){
        R[i] = i+1; L[i] = i-1; U[i] = D[i] = i;
        S[i] = 0; C[i] = i;
    }
    R[m] = 0; L[0] = m;
    int t = m+1;
    for (int i=0; i<n; i++){
        int k = -1;
        for (int j=0; j<m; j++){
            if(!A[i][j]) continue;
            if(k == -1) L[t] = R[t] = t;
            else{ L[t] = k; R[t] = R[k]; }
            k = t; D[t] = j+1; U[t] = U[j+1];
            L[R[t]] = R[L[t]] = U[D[t]] = D[U[t]] = t;
            C[t] = j+1; S[C[t]]++; ROW[t] = i; id[i][j] = t++;
        }
    }
    for (int i=0; i<n; i++) used[i] = 0;
    return dfs();
}

```