Homework 4

Anthony Jones

1. Let $a, b, c, d \in \mathbb{Z}$ s.t.

$$b = a + 1,$$
$$c = a^2, \text{ and}$$
$$d = b^2.$$

Observe that c and d are consecutive perfect squares. Their difference:

$$c - d = a^2 - b^2 = a^2 - (a+1)^2,$$

can be reduced as follows:

$$c - d = a^2 - (a+1)^2$$
$$c - d = a^2 - (a^2 + 2a + 1)$$
$$c - d = -2a - 1.$$

By the rules of integers multiplication and addition, there exists an integer n s.t.

$$n = -a - 1 \text{ and}$$
$$a = -n - 1.$$

Substituting for a gives the new equation

$$c - d = -2(-n-1) - 1$$
$$c - d = 2n + 2 - 1$$
$$c - d = 2n + 1,$$

which is to say the difference of two consecutive perfect squares are odd, $\Diamond$
By the definition of odd.

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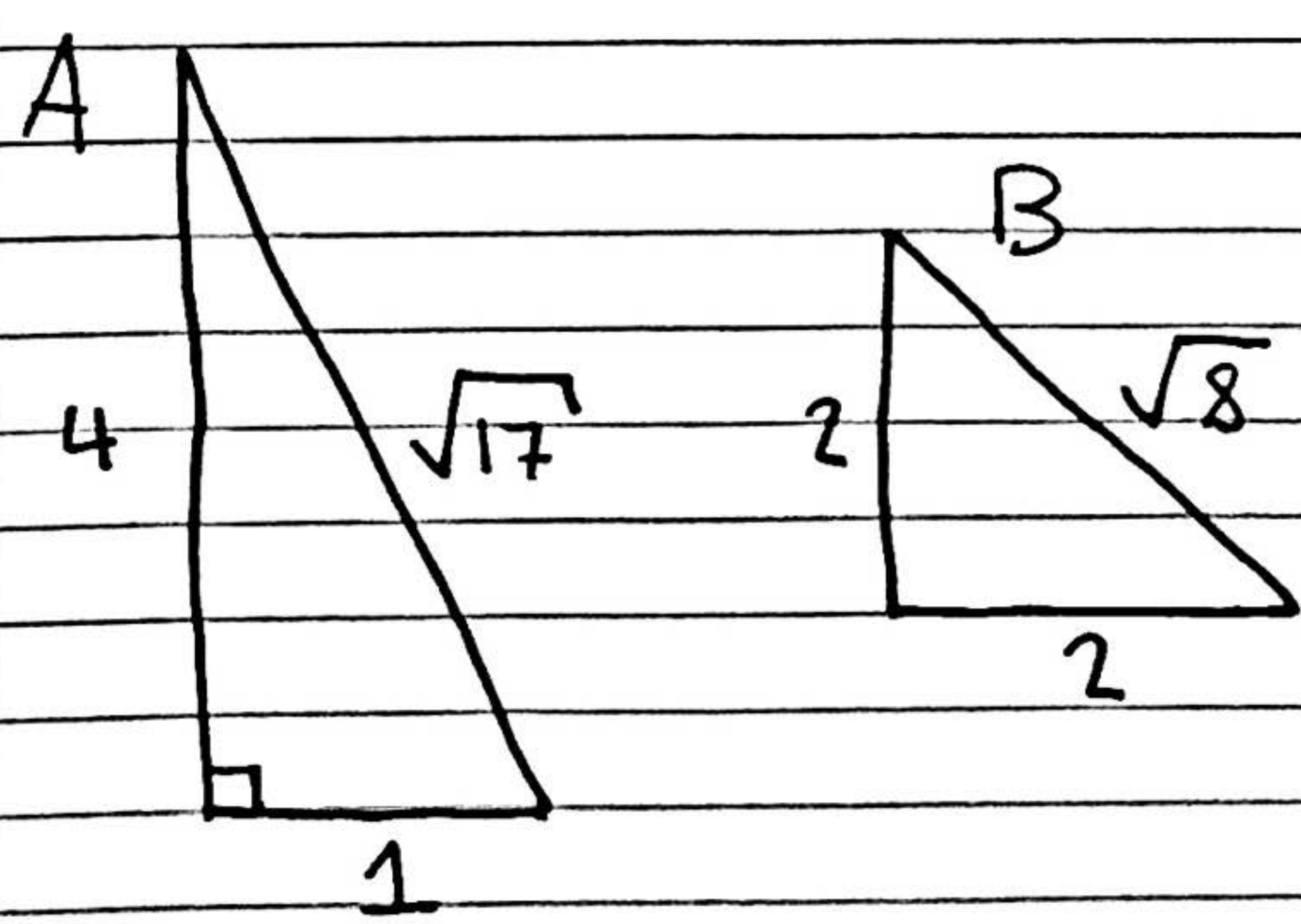
2. Let $a = 2$; $b = 1$; $c = 3$.
Observe:

$$a^{(b^c)} = 2^{(2^3)} = 2 \neq (a^b)^c = (2^1)^3 = 8.$$

3. Let $x = 0$; $x + 1 = 1$ is a positive integer, but $x = 0$ is not.

4. Consider the triangles



Their areas $A = \frac{1}{2}(4 \cdot 1) = 2$ and $B = \frac{1}{2}(2 \cdot 2) = 2$ are the same, but their hypothenuses are not.