Assignment 12 Anthony

1. $\vec{u} = (2, -1, -1)$
$\vec{v} = (0, -1, 1)$

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2. (a) $j = 0, -1$

(b) No j (the set is never orthogonal)

(c) $j = 0$

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3. Observe, that for a matrix A, where

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

the null space is defined using $A\vec{u} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, or in otherwords;

$$A\vec{u} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = u_1\begin{bmatrix} a \\ c \end{bmatrix} + u_2\begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This linear system finds the vector $\vec{u}$ that solves the equations

$$u_1 a + u_2 b = 0 \text{ and}$$
$$u_1 c + u_2 d = 0,$$

which is the same as finding the vector $\vec{u}$ which is orthogonal to the vectors (a, b) and (c, d). Observe that these two vectors fully express the space Row A, st

$$(\text{Row } A^{(2\times 2)})^{\perp} = \text{Nul } A^{(2\times 2)}$$

The same can be seen for all mxn matrices

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4. $\text{proj}_x(y) = \frac{y \cdot x}{x \cdot x} \vec{x}$

$$= \frac{(3-8+2)}{(9+16+4)}(3, 4, -2)$$

$$= -\frac{3}{29}(3, 4, -2)$$

$$\boxed{= \left(-\frac{9}{29}, -\frac{12}{29}, \frac{6}{29}\right)}$$

$\text{proj}_y(x) = \frac{x \cdot y}{y \cdot y} \vec{y}$

$$= \frac{(3-8+2)}{(1+4+1)}(1, -2, -1)$$

$$= -\frac{3}{6}(1, -2, -1)$$

$$\boxed{= \left(-\frac{1}{2}, 1, \frac{1}{2}\right)}$$