Chapter 3 Homework

Anthony Jones

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1 Exercises

3.6 We are provided that 3 out of the 4 drawn balls are white, meaning the last ball must be nonwhite. Since there are only 2 favorable outcomes, $\{W, W, W, B\}$ and $\{W, B, W, W\}$, and 4 total outcomes (one outcome for each position of the nonwhite ball), we have that

$$P = \frac{2}{4} = 0.5$$

regardless of replacement.

3.10 Normally the probability that a selected card is a spade is $\frac{13}{52}$; however, if we have already selected 2 spades, then there are 2 fewer total outcomes (since we are not replacing the cards) and 2 fewer favorable outcomes (since two of the spaces are already selected). Therefore

$$P = \frac{11}{50}$$

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3.14 For problem (a), we can consider drawing each ball a separate event. Therefore

$$P = \frac{7}{12} * \frac{9}{14} * \frac{5}{16} * \frac{7}{18} = \frac{35}{768}.$$

Now for problem (b), regardless of the order that they are drawn, we will draw 2 white balls and 2 black balls. Hence there will be 5 and then 7 favorable outcomes for selecting the first and second white ball, respectively; and 7 and then 9 favorable outcomes for selecting the first and second black ball. Since the total number of outcomes increases by two after each selection, we find that the answer is simply the same as above. Consider, for example, choosing the outcome $\{W, W, B, B\}$:

$$P = \frac{5}{12} * \frac{7}{14} * \frac{7}{16} * \frac{9}{18} = \frac{35}{768}.$$

Or, to illustrate, $\{B, W, W, B\}$:

$$P = \frac{7}{12} * \frac{5}{14} * \frac{7}{16} * \frac{9}{18} = \frac{35}{768}.$$

3.20 The probability of (a) is equal to the probability that they are both female and in computer science ($P(AB)$) over the probability that they are just in computer science ($P(B)$):

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.02}{0.05} = 0.4.$$

The probability of (b) is similar, but flipped:

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{0.02}{0.52} = \frac{1}{26}.$$

3.50 For the first question, we can simply add up the probabilities that each catagory of people have an accident:

$$P = 0.2 * (0.05) + 0.5 * (0.15) + 0.3 * (0.3) = 0.175$$

For the second question, we let A be the event that the policyholder is good or average risk, and let B be the event that they had no accidents in 1997:

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.2 * (1 - 0.05) + 0.5 * (1 - 0.15)}{1 - 0.175} \approx 0.745.$$

3.52 (a) $P = 0.6 * 0.15 + 0.4 * 0.05 = 0.11$

(b) $P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.6*0.2+0.4*0.1}{0.89} \approx 0.18$, since $P(AB) = P(A)$.

(c) We want to find the probability that she is accepted (event A) given that there is no mail through Wednesday (event B). Hence

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.25}{0.6 * 0.25 + 0.4 * 0.35} = \frac{0.25}{0.29} \approx 0.86.$$

(d) The probabilities of being accepted or not accepted for mail coming on Thursdays are the same, so $P = 0.5$.

(e) $P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.15}{0.6*0.15+0.4*0.4} = 0.25$

3.61 Since the sibling of nonalbino child was albino, we know that both of their parents are carriers. Therefore, the nonalbino child has only three possible gene pairs, $(A, a), (a, A)$, and (A, A), which are all equally likely to occur since the parents have pairs (A, a). Hence the probability that the nonalbino child is a carrier is $\frac{2}{3}$. Thus, the probability for (a) is:

$$\frac{2}{3} * 0.25 + \frac{1}{3} * 0 = \frac{1}{6}.$$

From above, we know that the probability of an offspring being albino is $\frac{1}{6}$. Thus, if A is the event that the first child is nonalbino, and B is the event that the second child is albino, then

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{\frac{2}{3} * 0.75 * 0.25 + \frac{1}{3} * 0 * 0}{5/6} = \frac{3}{20}.$$

3.76 Note that since E and F are mutually exclusive, they cannot both occur in the same trial; that is to say $P(E \cap F) = 0$. Next note that one of E, F, or G are certain to occur, where G is the event that neither E nor F occurs in a given trial. In other words, since $G = 1 - (E \cup F)$, we have

$$P(E) + P(F) + P(G) = 1.$$

We now consider the probability of EbF, the event where E occurs before F, by using the identity of EbF after conditioning on whether E, F, or G occurs:

$$P(EbF) = P(EbF|E)P(E) + P(EbF|F)P(F) + P(EbF|G)P(G).$$

If E is given, then $P(EbF) = 1$ since F will not occur in the trial, and so $P(EbF|E) = 1$; if F is given, then $P(EbF|F) = 0$ using a similar argument, so $P(EbF|F) = 0$; and finally, if G is given, then neither E nor F has yet occurred, so $P(EbF|G) = P(EbF)$. Thus

$$P(EbF) = P(E) + P(EbF) * P(G),$$

and hence, after rearranging:

$$P(EbF) = \frac{P(E)}{1 - P(G)} = \frac{P(E)}{P(E \cup F)} = \frac{P(E)}{P(E) + P(F) - P(E \cap F)} = \frac{P(E)}{P(E) + P(F)}.$$

3.77 Since outcome 3 is the last of the three outcomes to occur, we know that trial 1 cannot have outcome 3. Thus the first trial has an equal likelihood to either result in outcome 1 or 2, and so there is one favorable outcome out of two total outcomes. Hence $P = 0.5$.

2 Theoretical Exercises

3.4 Suppose boxes i and j are different ($i \neq j$). Let A be the event that the ball is in box j; B be the event that the ball is in box i; C be the event that the ball is uncovered after searching box i; and C^c be the event that the ball is not uncovered after searching box i. Note that if the ball is uncovered after searching through box i, because the two boxes are given to be different, this implies that the ball must not be in box j; in other words, $P(A|C) = 0$. We are given

$$P(A) = P_j,$$

$$P(B) = P_i,$$

$$P(C|B) = \alpha_i.$$

Observe that

$$\alpha_i = P(C|B) = \frac{P(C \cap B)}{P(B)} = \frac{P(B|C) * P(C)}{P(B)}.$$

Since the probability that the ball is in box i given that it was already uncovered after searching box i is certain, we have $P(B|C) = 1$; and therefore it follows that

$$P(C) = P(C|B) * P(B) = \alpha_i * P_i.$$

Thus the probability that the ball is not uncovered after searching box i is $P(C^c) = 1 - \alpha_i * P_i$. Now consider the identity of event A that is found by conditioning against the occurrence of event C:

$$P(A) = P(A|C)P(C) + P(A|C^c)P(C^c).$$

Since $P(A|C) = 0$, it follows that

$$P(A) = P(A|C^c)P(C^c),$$

and hence the conditional probability that the ball is in box j, given that a search of box i did not uncover it, is

$$P(A|C^c) = \frac{P(A)}{P(C^c)} = \frac{P_j}{1 - \alpha_i * P_i}.$$

Now suppose boxes i and j were the same $(i = j)$. Then the only probability that changes is $P(A|C)$, which becomes to equivalent to $P(B|C) = 1$ since events A and B are now the same. Thus, following from the same identity that's found by conditioning against the occurrence of event C:

$$P(A) = P(A|C)P(C) + P(A|C^c)P(C^c)$$

$$= \alpha_i * P_i + P(A|C^c)(1 - \alpha_i * P_i);$$

and hence, after rearranging:

$$P(A|C^c) = \frac{P_j - \alpha_i * P_i}{1 - \alpha_i * P_i} = \frac{(1 - \alpha_i)P_i}{1 - \alpha_i * P_i}.$$