Assignment 11 Anthony J

1 a) $\begin{bmatrix} 3 & 8 & 8 \\ -7 & -7 & -7 \\ 3 & 6 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0\begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $\vec{u} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

---

b) $\det(N - \lambda I) = \lambda(3-\lambda)(\lambda+1) = 0$

$\lambda = 0, -1, 3;$

---

c) $\begin{bmatrix} 3 & 8 & 8 \\ -3 & -7 & -7 \\ 3 & 6 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -1\begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $\vec{v} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 3 & 8 & 8 \\ -3 & -7 & -7 \\ 3 & 6 & 6 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 3\begin{bmatrix} a \\ b \\ c \end{bmatrix}$; $\vec{w} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Let $S = \begin{bmatrix} \vec{u} & \vec{v} & \vec{w} \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

and $D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, s.t. $N = SDS^{-1}$.

Then $N^{2020} = SD^{2020}S^{-1}$.

$$N^{2020} = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & -1 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3^{2020} \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 3^{2020} \\ 0 & -1 & -3^{2020} \\ 0 & 0 & 3^{2020} \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3^{2020} & 2\cdot 3^{2020} - 2 & 2\cdot 3^{2020} - 2 \\ -3^{2020} & 1 - 2\cdot 3^{2020} & 1 - 2\cdot 3^{2020} \\ 3^{2020} & 2\cdot 3^{2020} & 2\cdot 3^{2020} \end{bmatrix}$$

$\det(N^{2020} - \lambda I) = (3^{2020} - \lambda)(\lambda+1)(\lambda-1) = 0$

$\lambda = \pm 1, 3^{2020}$.

Assignment 11 Anthony J

1 d) $P = S = [\vec{u}\ \vec{v}\ \vec{w}] = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix} =$

---

2 a) $\det(S - \lambda I) = (\lambda - 2)^2 (\lambda + 1)^2 = 0$

$\lambda = 2, -1;$

$\longrightarrow$ for $\lambda = 2$, RREF of $S - 2I$:

$\begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $a = 3b$, $c = 3d$

- solutions $\vec{u} = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$.

$\longrightarrow$ for $\lambda = -1$, RREF of $S + I$:

$\begin{bmatrix} 1 & -4 & 0 & 0 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $a = 4b$, $c = 4d$

- solutions $\vec{v} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 4 \\ 1 \end{bmatrix}$.

2 b) $P = [\vec{u}_1 \ \vec{u}_2 \ \vec{v}_1 \ \vec{v}_2] = \begin{bmatrix} 3 & 0 & 4 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ s.t. $S = PDP^{-1}$.

---

3) The characteristic polynomial of an identity matrix of size $n \times n$ will be $\det(I) = (1-\lambda)^n$.

- Looking at $n=2$, the only possibility

$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ s.t $\det(B) = \det(I)$ is:

$(a-\lambda)(d-\lambda) - (b)(c)$; $b = c = 0$
$a = d = 1$

which is the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

consider adding another dimension;

$\det(I) = (1-\lambda)^n$

$\det(B_{3\times3}) = (a-\lambda)\det(B_{2\times2}) - b\det(B_{2\times2}) \dots$

which must follow a similar pattern.