Homework 12

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1: Suppose there is an integer x s.t. x is even and $x+1$ is even.
Therefore we know $2|x$ and $2|x+1$;
that is, there is an integer a and b s.t.

$$x = 2a \quad \text{and}$$
$$x+1 = 2b.$$

Rearranging and substituting gives:

$$2a+1 = 2b$$
$$a + \tfrac{1}{2} = b,$$

which is false because a and b are both integers. $\Rightarrow\Leftarrow$

2: Suppose n is an integer divisible by 4.
Then there is an integer a s.t.

$$n = 4a.$$

Assume $n+2$ is divisible by 4.
Then there is an integer b s.t.

$$n+2 = 4b.$$

Substituting and rearranging gives:

$$4a+2 = 4b$$
$$a + \tfrac{1}{2} = b,$$

which is false because a and b are integers. $\Rightarrow\Leftarrow$

3: Assume there is a smallest element $x \in E$. If $x|2$ then $(x-2)|2$, because

$$x = 2a \rightarrow (x-2) = 2b \rightarrow x = 2b+2$$
$$\rightarrow x = 2(b+1).$$

$x-2 < x$. $\Rightarrow\Leftarrow$

4: IDK ☹