Homework 3

Anthony Jones

1. Let $a, b, c \in \mathbb{Z}$ s.t. $a|b$ and $a|c$.
By the definition of divisibility, there are two integers u and v s.t.

$$b = a \cdot u \text{ and}$$
$$c = a \cdot v.$$

Substituting for b and c, observe that

$$b + c = a \cdot u + a \cdot v = a(u+v).$$

By the rules of integer addition, there is some integer n s.t.

$$b + c = a \cdot n,$$

which is to say $a|(b+c)$. ◊

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2. Let $a, b, d, x, y \in \mathbb{Z}$ s.t. $d|a$ and $d|b$.
Also let $u = ax$ and $v = by$.
By the definition of divisibility, there are two integers q and p s.t.

$$a = d \cdot q \text{ and}$$
$$b = d \cdot p, \text{ so that now}$$
$$u = d \cdot q \cdot x \text{ and}$$
$$v = d \cdot p \cdot y.$$

By the rules of integer multiplication, there are now integers f and g s.t.

$$u = d \cdot f \text{ and}$$
$$v = d \cdot g,$$

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2. (cont)

Let $a, b, d, x, y, u, v \in \mathbb{Z}$ s.t.

$d | a$,
$d | b$,
$u = ax$, and
$v = by$.

By the definition of divisibility, there are two integers p and q s.t.

$a = d \cdot p$ and
$b = d \cdot q$, so that now
$u = d \cdot p \cdot x$ and
$v = d \cdot q \cdot y$.

By the rules of integer multiplication, there are two integers f and g s.t.

$u = d \cdot f$ and
$v = d \cdot g$,

which is to say $d | u$ and $d | v$.

Given the proof to question (1), we know that $d | (u + v)$, or after substituting: $d | (ax + by)$. ◇

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3. Let $x, b, c \in \mathbb{Z}$ s.t.

$$x = 2b - 1 \text{ and } b = c + 1.$$

Observe that substituting for b gives

$$x = 2(c+1) - 1 = 2c + 1$$

which by the definition of odd implies that x is odd.

$\Updownarrow$

Let $x, \in \mathbb{Z}$ s.t. x is odd.

By the defi of odd, there is an integer c s.t.

$$x = 2c + 1.$$

Arranging the terms gives the equation:

$$x = 2(c+1) - 1$$

Let b be and Integer s.t.

$$b = c + 1.$$

~~Observe that~~ Thus when x is odd, there is some integer b that satisfies

$$x = 2b - 1. \quad \Diamond$$

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4: The err in the proof was dividing both sides by $(a-b)$, given that $a=b$. Here you are dividing by zero, changing the statement

"$0 = 0$"

into

"$a+b = b$".