AJ

Homework 20

1. Suppose R is an equivalence relation on a set A, and let $a, b \in A$.

Prove $(a \in [b]) \leftrightarrow (b \in [a])$:

($\rightarrow$) Suppose $a \in [b]$. This means that aRb, which implies bRa (through symmetry). Therefore $b \in [a]$.

($\leftarrow$) Suppose $b \in [a]$. This means that bRa, which implies aRb (through symmetry). Therefore $a \in [b]$.

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2. Let R and S be equivalence relations on a set A, and let $a, b \in A$.

Prove that $R = S$ if and only if the equivalence classes of R and S are the same:

($\rightarrow$) Suppose $R = S$. Observe, that if (a,b) is in R, then (a,b) is in S. Similarly, if (x,y) is in S, then (x,y) is in R.

Therefore $aRb \leftrightarrow aSb$.

Therefore, for the equivalence class of a:

$$[a] = \{x \in A : xRa\}$$
$$= \{x \in A : xSa\}.$$

Observe that the equivalence classes are the same.

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2. (cont.)

($\leftarrow$) Suppose the equivalence classes of R and S are the same, meaning for a:

$$[a] = \{x \in A : xRa\}$$
$$= \{y \in A : ySa\}.$$

1. Suppose bRa. This means that $b \in [a]$, which is to say that bSa. Therefore R is a subset of S.

2. Suppose bSa. This means that $b \in [a]$, which is to say that bRa. Therefore S is a subset of R.

Therefore $R = S$.

3. Let R be an equivalence relation on set A and let $a, x, y \in A$.

Prove that if $x, y \in [a]$, then xRy:

Suppose $x, y \in [a]$. This means that xRa and yRa. By symmetry, we see that therefore aRy. By transitivity, we have that xRy.

Therefore if $x, y \in [a]$, then xRy.

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4. Prove that if a relation R is symmetric and transitive, then R is also reflexive:

Suppose that R is a relation on set A which is both symmetric and transitive. Let $a, b \in A$.

This means $aRb \rightarrow bRa$ (symmetry).
This also means $(aRb \wedge bRa) \rightarrow aRa$ (transitive).

Therefore aRa, which is to say that the relation is reflexive.