HW 21

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1: a)
- Yes
- dom $f = \{1, 3\}$ im $f = \{2, 4\}$
- Yes
- $g = f^{-1} = \{(2,1), (4,3)\}$

b)
- Yes
- dom $f = \mathbb{Z}$ im f = even integers
- Yes
- $g = f^{-1} = \{(x,y) : x, y \in \mathbb{Z},\ 2y = x\}$

c)
- Yes
- dom $f = \mathbb{Z}$ im $f = \mathbb{Z}$
- Yes
- $g = f^{-1} = \{(x,y) : x, y \in \mathbb{Z},\ y + x = 0\} = f$

d)
- No, because multiple values of y exist for $x = 0$.

e)
- Yes
- dom $f = \mathbb{Z}$ im f = positive squares
- No, because the negatives all map to the same value as the positives: $2^2 = (-2)^2 = 4$.

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2:

a) The function is ~~both~~ one-to-one ~~onto~~ only:

The dom $f = \mathbb{Z}$ but the im $f \neq \mathbb{Z}$; therefore the function is not onto.

Suppose $f(a) = f(b)$.
Then $2a = 2b$,
which follows that $a = b$.
Therefore the function is one-to-one.

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b) The function is both:

The dom $f = \mathbb{Z}$ and the im $f =$ $\mathbb{Z}$, because the input and output can be any integer. Therefore the function is onto.

Suppose $f(a) = f(b)$
Then $10 + a = 10 + b$,
which follows that $a = b$.
Therefore f is one to one,

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c) The function is both:

The dom $f = \mathbb{N}$ and im $f = \mathbb{N}$.
Therefore the function is onto.
(similar to problem b)

The same proof for c.

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3. $\boxed{f \text{ is one-to-one:}}$

Suppose $f(a) = f(b)$.

Then $5a = 5b$, which follows that $a = b$. Therefore f is one-to-one.

$\boxed{f \text{ is NOT onto:}}$

The dom f is $\mathbb{Z}$, as shown in the function's definition.
The im f is not $\mathbb{Z}$, however, as $c = 4$ shows that $c \in \mathbb{Z}$ but no $d \in \mathbb{Z}$ shows $c = 5 \cdot d$.
Therefore the function is not onto.

4. $\boxed{f \text{ is onto:}}$

The dom $f = \mathbb{Z}$, as shown in the function's definition.
The im f is defined as an integer, which given that the inputs are $[1, \infty)$, ~~the~~ will be $a \in \mathbb{Z}$, s.t. $0 \leq a \leq \infty$, which is $\mathbb{N}_0$. Therefore the function is onto.