# Homework 7

1a) Let $x=2$. Observe that $x \in A$ when $a=0$, but that $x \notin B$ because for $2 = 10b-3$ to be true, $b$ must equal $\frac{1}{2}$ and $\frac{1}{2} \notin \mathbb{Z}$. Therefore $A \not\subseteq B$. ◊

1b) Let $x \in B$ and $a = 2b-1$ within $B$. Observe that, by the rules of integer arithmitic, $a \in \mathbb{Z}$, and:

$$\begin{aligned} x &= 10b-3 \\ &= \text{[illegible]} \\ &= 5(2b-1+1)-3 \\ &= 5(2b-1)+5-3 \\ &= 5a+2. \end{aligned}$$

This means that when $x \in B$, it also shows that $x \in A$. Therefore $B \subseteq A$. ◊

1c) Let $x \in B$ and $y \in C$ s.t. $c = b+1$ within $B$ and $C$. Observe that, using the rule of integer addition, $c \in \mathbb{Z}$. Also observe:

$$\begin{aligned} x &= 10b-3 \\ &= 10(b-1+1)-3 \\ &= 10c+10-3 \\ &= 10c+7 \\ &= y\text{, for any value } b. \end{aligned}$$

Meaning for any value $x \in B$; $x=y$, and $x \in C$. For any value $y \in C$; $y=x$, and $y \in B$. Therefore $B=C$.

Homework 7

2. (a) Let $x \in A$, where $A \subseteq B$ and $B \subseteq C$.

Since $A \subseteq B$, $x \in B$ by the rules of sets and subsets. Since $x \in B$ and $B \subseteq C$, $x \in C$ by the same rules.

Therefore any value for $x \in A$ is also $x \in C$, meaning: $A \subseteq C$. ◇

(b) $A = 1$
$B = \{1\}$
$C = \{1, 2, 3\}$
$1 \nsubseteq C$ because 1 is not a set.

(c) Let $A \in B$, where $B \subseteq C$.

Since $B \subseteq C$, $A \in C$ by the rules of sets and subsets.

Therefore any $A \in B$ also gives that $A \in C$. ◇

(d) $A = 1$
$B = \{1\}$
$C = \{\{1\}, \{1,2\}, \{1,2,3\}\}$
$A \notin C$