1) The linear system represented by the matrix is:

$$\begin{aligned} 3x_1 - 2x_2 + x_3 &= 35 \\ -2x_2 + x_3 &= 5 \\ x_3 &= 3 \end{aligned} \qquad \begin{aligned} x_1 &= (35-5)/3 \\ x_2 &= (5-3)/-2 \\ x_3 &= 3 \end{aligned}$$

Solving with substitution yields:

$\boxed{x_1 = 10}$; $\boxed{x_2 = -1}$; $\boxed{x_3 = 3}$

---

2a) Applying the operations yield the following matrices:

$$M_1 = M\Big|_{\substack{R'_1 = R_3 \\ R'_3 = R_1}} = \begin{bmatrix} 3 & 6 & 2 & 0 & 4 \\ 1 & 4 & 5 & -2 & 2 \\ 3 & -2 & 6 & -1 & 3 \end{bmatrix}$$

$$M_2 = M_1\Big|_{R'_2 = 4R_2} = \begin{bmatrix} 3 & 6 & 2 & 0 & 4 \\ 4 & 16 & 20 & -8 & 8 \\ 3 & -2 & 6 & -1 & 3 \end{bmatrix}$$

$$M_3 = M_2\Big|_{R'_2 = R_2 - 2R_3} = \begin{bmatrix} 3 & 6 & 2 & 0 & 4 \\ (4-6) & (16+4) & (20-12) & (-8+2) & (8-6) \\ 3 & -2 & 6 & -1 & 3 \end{bmatrix}$$

$$N = M_3 = \begin{bmatrix} 3 & 6 & 2 & 0 & 4 \\ -2 & 20 & 8 & -6 & 2 \\ 3 & -2 & 6 & -1 & 3 \end{bmatrix}$$

The final result after applying the operations is:

$$N = \begin{bmatrix} 3 & 6 & 2 & 0 & 4 \\ -2 & 20 & 8 & -6 & 2 \\ 3 & -2 & 6 & -1 & 3 \end{bmatrix}$$

---

2b) To transform matrix N back into matrix M: First add twice row 3 to row 2; then scale row 2 by 1/4; and then interchange rows 1 and 3.

3) The linear system is represented by the following augmented matrix:

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ \downarrow \\ R_{24} \\ R_{25} \\ R_{26} \end{array} \left[\begin{array}{ccccccc|c} 1 & -1 & 1 & \rightarrow & -1 & 1 & -1 & 1 \\ 0 & 1 & -1 & \rightarrow & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & \rightarrow & -1 & 1 & -1 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0 & 0 & 0 & \rightarrow & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & \rightarrow & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & \rightarrow & 0 & 0 & 1 & 1 \end{array}\right]$$

Apply Row Operations...

$R'1 = R1 + R2$

$R'2 = R2 + R3$

$R'3 = R3 + R4$

$\downarrow \quad \downarrow \quad \downarrow$

$R'24 = R24 + R25$

$R'25 = R25 + R26$

Applying the row operations $R'n = Rn + Rn{+}1$ yields:

$$\begin{array}{c} R'_1 \\ R'_2 \\ R'_3 \\ \downarrow \\ R'_{24} \\ R'_{25} \\ R'_{26} \end{array} \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & \rightarrow & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & \rightarrow & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & \rightarrow & 0 & 0 & 0 & 2 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0 & 0 & 0 & \rightarrow & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & \rightarrow & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & \rightarrow & 0 & 0 & 1 & 1 \end{array}\right]$$

Solution set...

$a = 2$

$b = 2$

$c = 2$

$\downarrow \quad \downarrow$

$x = 2$

$y = 2$

$z = 1$

... which preserves the solution set, where $z = 1$ and every other variable equals 2.

---

4) Reducing the system to echelon form yields;

$$\left[\begin{array}{cc|c} 1 & \beta & 1 \\ 0 & (1-\beta) & (\beta-1) \end{array}\right]$$

which is consistent for all values for $\beta$, as a linear system is only inconsistent where one of its rows are all zero and the augmented column is not zero in echelon form. When $\beta = 1$ in this system, the augmented column is also 0.