Homework 6

Anthony Jones

1. (a) $\{-2, 2\}$
   (b) $\{\} = \emptyset$
   (c) $\{1, 2, 3, 4, 5, 6\}$
   (d) $\{-1, 0, 1, 2, 3\}$
   (e) $\{5, 10, 15, 20, 25\}$

2. (a) $\{x \in \mathbb{N} : 2 \mid x+1\}$
   (b) $\{x \in \mathbb{N} : x = c^2,\ c \in \mathbb{N} : 0 < c \leq 10\}$
   (c) $\{x \in \mathbb{Z} : 2 \mid x+1\}$
   (d) $\{x \in \mathbb{N} : x = 2^c,\ c \in \mathbb{N}\}$
   (e) $\{x \in \mathbb{N} : 6 < x < 13\}$

3. Let $A = \{x \in \mathbb{Z} : x = 4k+1 \text{ for some } k \in \mathbb{Z}\}$
   and $B = \{y \in \mathbb{Z} : y = 4k-3 \text{ for some } k \in \mathbb{Z}\}$

   • Suppose $x \in A$
   > Then there is some $k \in \mathbb{Z}$ s.t.
   $x = 4k+1$.
   > Using the rules of integer addition, there
   is some $q \in \mathbb{Z}$ s.t.
   $k = q-1$.
   > Substituting gives
   $x = 4(q-1)+1$
   $= 4q-4+1$
   $= 4q-3$
   > Thus $x = 4q-3$ for some $q \in \mathbb{Z}$, meaning
   > $x \in B$ ◊

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4. • Suppose $y \in B$
7 Then there is some $K \in \mathbb{Z}$ s.t.
$$y = 4K - 3.$$
7 Using the rules of integer addition, there is some $q \in \mathbb{Z}$ s.t.
$$K = q + 1.$$
7 substituting gives
$$\begin{aligned} y &= 4(q+1) - 3 \\ &= 4q + 4 - 3 \\ &= 4q + 1 \end{aligned}$$
7 Thus ~~Hence~~. $y = 4q+1$ for some $q \in \mathbb{Z}$, meaning 7 $y \in A$.

E.C. We say that $A = B$.