Exam 2:

~~a) A permutation in S_n is even when~~

g) The subgroup is characteristic in G.

i) A group is soluable if the commuter subgroups of G diverge to order 0.

h) G must be soluable.

a) A permutation is even if the factor group generated by that permutation is isomorphic to $\mathbb{Z}$.

c) $orb(a) = \{ g \mid ga = ag \in G \}$

d) An action is transitive if the orbit of that action is $ga = ag$ for all $g \in G$.

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k) An element $x \in R$ of a ring is nilpotent if for some $n \in \mathbb{N}$, $x^n = 0$.

L) An integral domain is a ring where, for $a, b \in R$, $ab = 0 \Rightarrow a = 0$ or $b = 0$.

m) The ideal of a ring is a subgroup I that, for all $r \in R$, either $rI \subseteq R$ or $Ir \subseteq R$ (left and right ideals).

i) The invariant factor decomposition of a finite group A is given by the direct products of subgroups of $\mathbb{Z}$ where, for $\mathbb{Z}_n$, the right products of the decomposition are all factors of every factor to its left: for a, b, c, etc...
$\mathbb{Z}_a \oplus \mathbb{Z}_b \oplus \mathbb{Z}_c \ldots$ has that c is a factor of b, which is a factor of a.

f) Given a group with order $|G| = m \cdot p^a$ for $(m, p) = 1$:

There exists at least 1 sylow p-group,
The sylow p-groups are all communicative,
and $n \equiv 1 \bmod (p)$ describes how many n sylow-p-groups can exist for G.

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2) Let R be a Ring, $I \subseteq R$ an ideal, $S \subseteq R$ a subring. Let A be a subgroup of S:

A subgroup of S is an ideal if, for all $s \in S$, $sA \subseteq S$ or $As \subseteq S$.

Consider $I \cap S$. This intersection is a subgroup of S as $S \subseteq R$, and I is already a subgroup of R. Thus, as S and R share the same operations $I \cap S \subseteq S$.

Consider $s(I \cap S)$. If $a \in I \cap S$, then $sa \in R$ because $a \in I$ and I is an ideal of R. Furthermore, because $a \in S$ and S is a subring, $sa \in S$ as well as it contains the same operations as R. Thus $sa \in S$ and $s(I \cap S) \subseteq S$.

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3) Z_{22} : $22 = 2 \cdot \cancel{11}$

Z_{25} : $25 = \cancel{5} \cdot \cancel{5}$

Z_{24} : $2 \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{3}$

Z_{27} : $\cancel{3} \cdot 3 \cdot \cancel{3}$

a) The elementery divisor decomposition of G:

$$(\mathbb{Z}_{2^3} \oplus \mathbb{Z}_2) \oplus (\mathbb{Z}_{3^3} \oplus \mathbb{Z}_3) \oplus (\mathbb{Z}_{5^2}) \oplus \mathbb{Z}_{11}$$

b) The invariant factor decomposition of G:

$$(\mathbb{Z}_{2^3} \oplus \mathbb{Z}_{3^3} \oplus \mathbb{Z}_{5^2} \oplus \mathbb{Z}_{11}) \oplus (\mathbb{Z}_2 \oplus \mathbb{Z}_3)$$

4) Let G be a group of order 12.

By the Sylow theorems, the possible sylow p-subgroups are given by $12 = 2 \cdot 2 \cdot 3$.

Therefore there are $n \equiv 1 \bmod (3)$ either 1 or 4, and $m \equiv 1 \bmod (2)$ exactly 3 sylow 2-subgroups of G, or

Observe that for G to be simple, there must exist one p-subgroup unique to that p (for it to be characteristic.

If the 2-subgroup was unique, however, then there would be $2^2 = 4$ elements in the 2-subgroups and $4 \cdot 3 = 12$ elements in the 3-subgroups, which is more than the 12 available.