Homework 17

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1 Proof: We will prove this by cases. Let n be an integer.

Case 1: n is even.

i) By definition of even, there exists an integer s.t. $2a = n$.

ii) $(2a)^2 + (2a) + 3 = 2(2a^2 + a + 1) + 1$

ii) By definition of odd, if n is even then $n^2 + n + 3$ is odd.

Case 2: n is odd.

i) By definition of odd, there exists an integer s.t. $2a+1 = n$

ii) $(2a+1)^2 + (2a+1) + 3 = 4a^2 + 4a + 1 + 2a + 1 + 3$
$= 4a^2 + 6a + 5$
$= 2(2a^2 + 3a + 2) + 1$

iii) By definition of multiple addition and multiplication, there exists an integer $b = 2a^2 + 3a + 2$

iv) By definition of odd, if n is odd then $n^2 + n + 3$ is odd.

Therefore we have shown that where n is an integer, the equation is odd.

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2 Proof: We will show proof using cases:
Let n be an integer.

Case 1: $n = 3k$, for $k \in \mathbb{Z}$.

i) $(3k)^3 - (3k) = 3(9k^2 - k)$

ii) By integer arithmetic, $9k^2 - k \in \mathbb{Z}$.

iii) By rules of divisibility, if $n = 3k$, then $n^3 - n$ is divisible by 3.

Case 2: $n = 3k+1$, for $k \in \mathbb{Z}$.

i) $(3k+1)^3 - (3k+1) = 27k^3 + 27k^2 + 9k + 1 - 3k - 1$
$= 3(9k^3 + 9k^2 + 2k)$

ii) By integer arithmetic, $9k^3 + 9k^2 + 2k \in \mathbb{Z}$.

iii) By rules of divisibility, if $n = 3k+1$, then $n^3 - n$ is divisible by 3.

Case 3: $n = 3k+2$, for $k \in \mathbb{Z}$.

i) $(3k+2)^3 - (3k+2) = 27k^3 + 54k^2 + 36k + 8 - 3k - 2$
$= 3(9k^3 + 18k^2 + 11k + 2)$

ii) By integer arithmetic, $9k^3 + 18k^2 + 11k + 2 \in \mathbb{Z}$.

iii) By rules of divisibility, if $n = 3k+2$, then $n^3 - n$ is divisible by 3.

Therefore we have shown where $n \in \mathbb{Z}$, $n^3 - n$ is divisible by 3.

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3 Proof: We will prove this by cases: Let n be a positive real number.

Case 1: $\sqrt[3]{n} > 0$.

i) $(\sqrt[3]{n})^3 = n$.

ii) By positive multiplication, if $\sqrt[3]{n} > 0$ then $n > 0$.

Case 2: $\sqrt[3]{n} \leq 0$.

i) $(\sqrt[3]{n})^3 = n$; $(0)^3 = 0$.

ii) By negative multiplication, if $\sqrt[3]{n} < 0$ then $n < 0$ and if $\sqrt[3]{n} = 0$ then $n = 0$.
Thus by ~~contrapositive if $\sqrt[3]{n} > 0$ then $n > 0$,~~ contradiction $\sqrt[3]{n}$ can only be less than or equal to 0 if $n \leq 0$.

Therefore for $n \in \mathbb{R}^+$, $\sqrt[3]{n} \in \mathbb{R}^+$.

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4 Proof: We will prove using cases.

Let $8 \mid (x^2-1)$.

Case 1: $8 \mid (x+1)$

i) By rules of divisibity $2 \mid (x+1)$, which means x is odd.

ii) Therefore x is odd.

Case 2: $8 \mid (x-1)$

i) By rules of integer arithmetic,

$c + 2 = x$.

$8 \mid (c+1)$

ii) By rules of divisibility, $2 \mid (c+1)$ which means c is odd.

iii) By definition of oddity, if c is odd then $c+2$ is odd, so

iv) x is odd.

Therefor if $8 \mid (x^2-1)$ then x is odd, so if x is even then $8 \nmid (x^2-1)$.