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Free Response Question

Anthony
Jones

a) Solve the ode via separation of var. 8/27/20

$$\frac{dy}{dx} = e^{x-2y}$$

$$\frac{dy}{dx} = e^{x-2y}$$

$$dy = e^x e^{-2y} dx$$

$$e^{2y} dy = e^x dx \quad \checkmark$$

Integrate both sides:

$$\int e^{2y} dy = \int e^x dx$$

$$\frac{1}{2} e^{2y} = e^x + C_0 \quad \checkmark$$

Solve for y:

$$e^{2y} = 2e^x + C_0$$

$$2y = \ln(2e^x + C_0)$$

$$y = \frac{\ln(2e^x + C_0)}{2} \quad \checkmark$$

The solution to the ode is

$$y = \frac{\ln(2e^x + C_0)}{2} \quad \checkmark$$

Nice Job!

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b) Verify your solution.

$$\frac{dy}{dx} = e^{x-2y}, \quad y = \frac{\ln(2e^x + C_0)}{2}$$

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$$y = \frac{1}{2} \ln(2e^x + C_0)$$

$$\frac{dy}{dx} = \frac{1}{2} (2e^x + C_0)^{-1} (2e^x)$$

$$\frac{dy}{dx} = \frac{e^x}{2e^x + C_0}$$

.....

$$\frac{dy}{dx} = e^{x-2y}$$

Substitution for y:

$$\begin{aligned} \frac{dy}{dx} &= e^{x-2\left(\frac{\ln(2e^x + C_0)}{2}\right)} \\ &= e^x e^{-\ln(2e^x + C_0)} \end{aligned}$$

$$\frac{dy}{dx} = \frac{e^x}{2e^x + C_0}$$

Not the correct
way to verify

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The solution presented is a valid solution because the derivative expressed in terms of x equates to taking the derivative of our solution.