Homework 9

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1. b) $\exists x \in \mathbb{Z}$, x is not prime AND not composite
   d) $\forall x \in \mathbb{Z}$, $5 | x$.
   f) $\forall x \in \mathbb{Z}$, $x^2 \geq 0$
   h) $\exists x, y \in \mathbb{Z}$, $x/y = 10$.
   j) $\forall x \in \mathbb{Z}$, $\exists y \in \mathbb{Z}$ s.t $y > x$.

2. a) FALSE because no animal is red.
   b) TRUE because dogs and cats are mammals.
   c) FALSE because 5 blue birds,
   d) TRUE because there are birds.
   e) FALSE because 5 blue birds.

3. FALSE : $x = 1$, $y = 1$; $x + y = 2$
   TRUE : $x = x$, $y = -x$; $x + y = 0$
   FALSE : $x = a$, $y = a + 1$; $x + y = 2a + 1$
   FALSE : $x = 1$, $y = 1$; $xy = 1$
   TRUE : $x = x$, $y = 0$; $xy = 0$

4. $\forall (x \in \mathbb{Z}; 9 | x), 3 | x$:

   By the laws of divisibility there is some integer c such that
   $$x = 9c = 3(3c).$$
   By the laws of integer multiplication there is some integer n such that $n = 3c$ and
   $$x = 3(3c) = 3n,$$
   which is to say $3 | x$ ◊

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4b) $\exists (x \in \mathbb{Z}, x \text{ is even}), 3 \mid x$.

Let $x = 6$. Observe:

$x = 2 \cdot 3$;

Therefore x is even and divisible by 3. ◊