Assignment 10

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1a) U has a domain $\mathbb{R}^2 = \begin{bmatrix} a \\ b \end{bmatrix}$, which has a basis of $\{(1,1)\}$ and a dimension of 2.

1b) U has a kernal = span $\{(0,0)\}$, which has a basis of $\{(0,0)\}$ and a dimension of 2.

1c) U has a range $\mathbb{P}^2 = at^2 + (a+b)$ which has a basis of $\{(t^2+1, 1)\}$ and a dimension of 2.

2a) $\vec{x} = 5\begin{bmatrix} 2 \\ 3 \end{bmatrix} - 2\begin{bmatrix} -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 18 \\ 13 \end{bmatrix}$

2b) $\vec{y} = a\begin{bmatrix} 2 \\ 3 \end{bmatrix} + b\begin{bmatrix} -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 16 \end{bmatrix}$

$a = 5$, $b = 1$, $[\vec{y}]_B = (5, 1)$

2c) $\vec{z} = a\begin{bmatrix} 2 \\ 3 \end{bmatrix} + b\begin{bmatrix} -4 \\ 1 \end{bmatrix} = 2\begin{bmatrix} a \\ b \end{bmatrix}$?

$\vec{z} = 0\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 0\begin{bmatrix} -4 \\ 1 \end{bmatrix} = 2\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

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3) $M = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$

$$M - \lambda I = \begin{bmatrix} 3-\lambda & 0 & 0 \\ 1 & 2-\lambda & 1 \\ 2 & 0 & 1-\lambda \end{bmatrix}$$

$$\det(M-\lambda I) = (3-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix}$$

$$= (3-\lambda)\left((2-\lambda)(1-\lambda)\right)$$

$$= (3-\lambda)(2-3\lambda+\lambda^2)$$

$$0 = 6-11\lambda+6\lambda^2-\lambda^3$$

EIGENVALUE: $\boxed{\lambda = 3, 2, 1}$

$\lambda = 3$: $\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 2 & 0 & -2 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 2 & -4 & 0 \end{array}\right]$

$$= \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right]$$

$x_1 - x_3 = 0$

$x_2 - 2x_3 = 0$

$\vec{x} = x_3 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

4) $C = \begin{bmatrix} 2 & -3 & -5 \\ -12 & -3 & -15 \\ 8 & 6 & 16 \end{bmatrix}$

$$\det(C-\lambda I) = (2-\lambda)\begin{vmatrix} -3-\lambda & -15 \\ 6 & 16-\lambda \end{vmatrix}$$

$$-(-3)\begin{vmatrix} -12 & -15 \\ 8 & 16-\lambda \end{vmatrix}$$

$$+(-5)\begin{vmatrix} -12 & -3-\lambda \\ 8 & 6 \end{vmatrix}$$

$$= (2-\lambda)\big((-3-\lambda)(16-\lambda) + 99\big) + 3(-12(16-\lambda)+120)$$