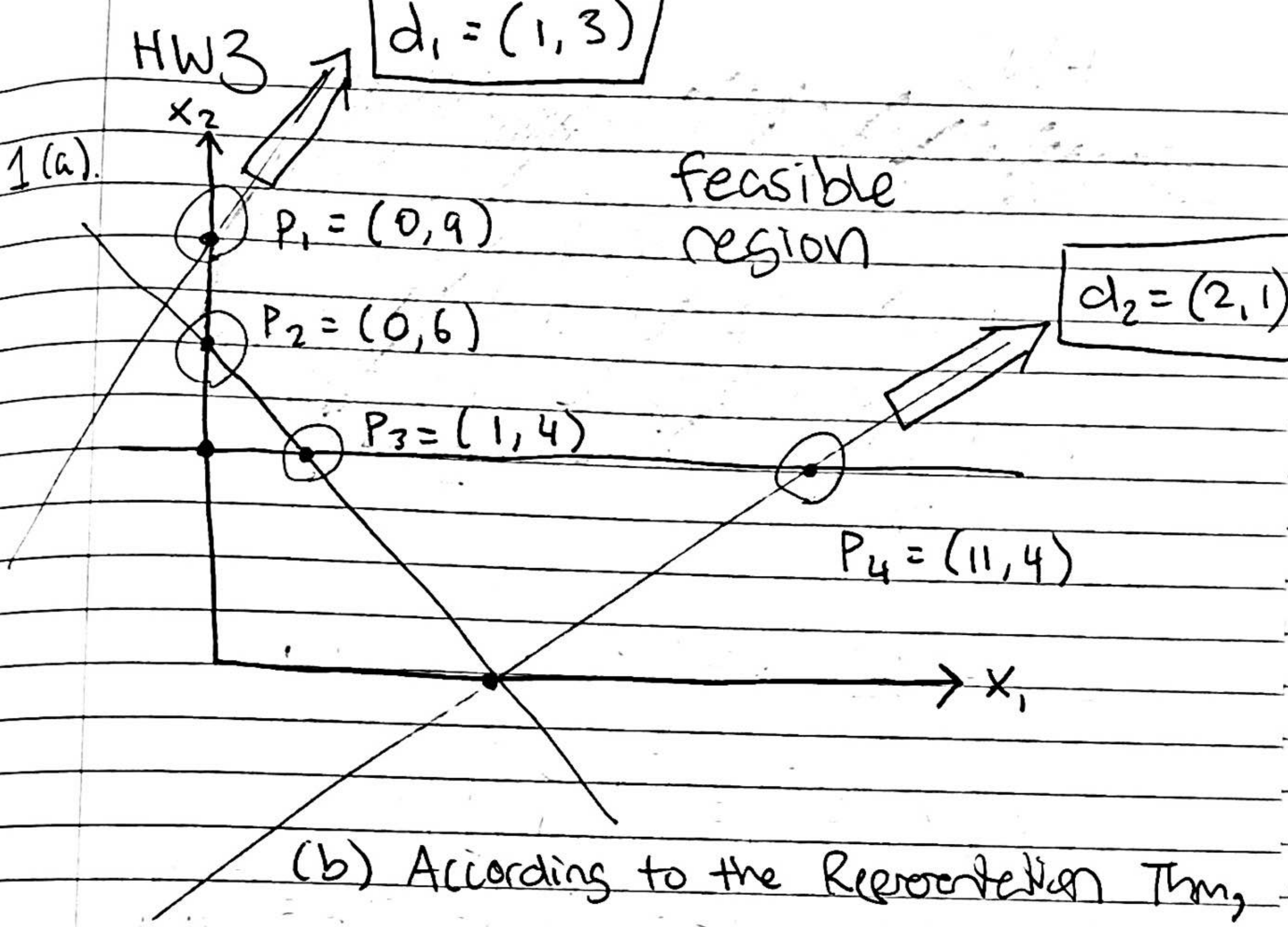
HW3

1 (a).



(b) According to the Representation Thm,

$$\vec{x} = \alpha_1 \vec{p}_1 + \alpha_2 \vec{p}_2 + \alpha_3 \vec{p}_3 + \alpha_4 \vec{p}_4 + \lambda_1 \vec{d}_1 + \lambda_2 \vec{d}_2,$$

where $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$, and

$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \lambda_1, \lambda_2 \geq 0$.

Since $c^T = [-7, 2]$, the problem can be reformulated as

$$\max \ (c^T p_1)\alpha_1 + (c^T p_2)\alpha_2 + (c^T p_3)\alpha_2 + (c^T p_4)\alpha_4$$
$$\text{s.t } \ \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1,$$
$$\alpha_1, \alpha_2, \alpha_3, \alpha_4 \geq 0$$

as $c^T d_1 = -1 \leq 0$ and $c^T d_2 = -12 \leq 0$. Thus the solution is bounded and equals the maximal product of an extreme point and the constraint.

HW3

1(c) $c^T p_1 = [-7, 2] \begin{bmatrix} 0 \\ 9 \end{bmatrix} = 18$

$c^T p_2 = [-7, 2] \begin{bmatrix} 0 \\ 6 \end{bmatrix} = 12$

$c^T p_3 = [-7, 2] \begin{bmatrix} 1 \\ 4 \end{bmatrix} = 1$

$c^T p_4 = [-7, 2] \begin{bmatrix} 11 \\ 4 \end{bmatrix} = -69$

Thus the solution to the resulting problem is $\alpha_1 = 1$ and $\alpha_2 = \alpha_3 = \alpha_4 = 0$; the solution to the original problem is $\vec{x} = (0, 9)$.

(d) When we change the objective to max of $z = 4x_1 - x_2$, the new $c^T = [4, -1]$. However

$$c^T d_1 = [4, -1] \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 1 > 0, \text{ and}$$
$$c^T d_2 = [4, -1] \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 7 > 0$$

Thus the problem is unbounded and can be arbitrarily large for values λ_1 and λ_2. Thus the linear problem is also unbounded, and there is no one maximum solution.

HW3

2. $A\vec{x} = \vec{b}$, so

$$A = \begin{bmatrix} 1 & 3 & -1 & 1 \\ 2 & 1 & 2 & 1 \end{bmatrix},$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \vec{b} = \begin{bmatrix} 30 \\ 15 \end{bmatrix}$$

Solution 1: $\boxed{x = (3, 9, 0, 0)}$

$$B = [A_1 \; A_2] = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix},$$

basic variables $x_B = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = B^{-1}b = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$,

nonbasic variables $x_N = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$(3) + 3(9) - 1(0) + (0) = 3 + 27 = 30$$
$$2(3) + (9) + 2(0) + (0) = 6 + 9 = 15$$
✓

Solution 2: $\boxed{x = (0, 15/2, 0, 15/2)}$

$$B = [A_2 \; A_4] = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix},$$

basic variables $x_B = \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = B^{-1}b = \begin{bmatrix} 15/2 \\ 15/2 \end{bmatrix}$,

nonbasic variables $x_N = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

✓
$$(0) + 3(15/2) - 1(0) + (15/2) = 45/2 + 15/2 = 30$$
$$2(0) + (15/2) + 2(0) + (15/2) = 15/2 + 15/2 = 15$$

HW3

3. (a) The basic solution for B is

$\vec{x} = (3, 0, 1, -1, 0)$:

$$B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} A_1 & A_3 & A_4 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} = B^{-1}b = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$

$$x_N = \begin{bmatrix} x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} = 3\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\checkmark \quad = \begin{bmatrix} 6+1+0 \\ 3+0-1 \\ 3+0+0 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 3 \end{bmatrix}$$

(b) This is not feasible, because $\vec{x} < 0$ as $x_4 = -1$.

HW3

4. (a) $s(4,10)$ lies in the intersection of $x_5 = 0$ and $x_4 = 0$, so they are the nonbasic variables. Thus

$$B = \begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

(b) We will try to find the basic solution following where $x_3 = 0$ intersects $x_1 = 0$, which is clearly somewhere when $x_2 < 0$:

$$B = \begin{bmatrix} A_2 & A_4 & A_5 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Thus the basic variables are

$$x_B = \begin{bmatrix} x_2 \\ x_4 \\ x_5 \end{bmatrix} = B^{-1}b = \begin{bmatrix} -2 \\ 20 \\ 12 \end{bmatrix},$$ and

the nonbasic variables are

$$x_N = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ (the intersection I mentioned).

$$\checkmark \begin{bmatrix} -2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 20 \\ 12 \end{bmatrix} = \begin{bmatrix} 4+0+0 \\ -2+20+0 \\ -2+0+12 \end{bmatrix} = \begin{bmatrix} 4 \\ 18 \\ 10 \end{bmatrix}$$