FRQ SEPT 7

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Satisfying the form

$$\frac{dy}{dx} + p(x)y = q(x)$$

for $p(x) = 4$ and $q(x) = x^2 e^{-4x}$

reveals that the ode is linear.

The integrating factor given by

$$\mu(x) = e^{\int p(x)dx} = e^{4x+b} = e^{4x} \Big|_{b=0}$$

can be used to solve the equation:

$$\mu(x)\left(\frac{dy}{dx} + 4y\right) = \mu(x)\left(x^2 e^{-4x}\right)$$

$$\boxed{e^{4x}\frac{dy}{dx} + 4e^{4x}y} = x^2$$

Checking the integrating factor

$$\frac{d}{dx}(\mu(x)y) = \frac{d}{dx}(e^{4x}y) = \boxed{e^{4x}\frac{dy}{dx} + 4e^{4x}y}$$

Substitute and solve:

$$\int \frac{d}{dx}(e^{4x}y)\,dx = \int x^2\,dx$$

$$e^{4x}y = \frac{1}{3}x^3 + C$$

FRQ SEPT 7 (continued)

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$e^{4x} y = \frac{1}{3}x^3 + C$; for $C \in \mathbb{R}$

$$y = \frac{\frac{1}{3}x^3 + C}{e^{4x}}.$$

The solution to the ode is

$y = \dfrac{\frac{1}{3}x^3 + C}{e^{4x}}$ for any value C.