MATH 3190 NOTES

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$A \cup B = \{x : x \in A \text{ or } x \in B\}$

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

$A - B = \{x : x \in A \text{ and } x \notin B\}$

$A \Delta B = (A \cup B) - (A \cap B)$

$A - (B \cup C) = (A - B) \cap (A - C)$

$A - (B \cap C) = (A - B) \cup (A - C)$

$A \times B = \{(a, b) : a \in A, b \in B\}$

$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

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1. ~~i) Let A, B, and C be sets~~

ii)

x	y	z	$y \wedge z$	$x \to y$	"A" $x \to (y \wedge z)$	"B" $z \to (x \to y)$	$A \to B$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	T	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

$[x \to (y \wedge z)] \to [z \to (x \to y)]$ is a tautology because for any value, true or false, for $x, y,$ or z the statement is true, as shown above.

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iii) a) ~~$\{x : x \in \mathbb{Z}^+, x = 10^a ; a : a \in \mathbb{Z}, \ldots$~~

$\{x : x \in \mathbb{Z} \text{ and } x = 10^a, a : a \in \mathbb{Z} \text{ and } 0 \leq a \leq 6\}$

b) $\{x : x \in \mathbb{R} \text{ and } x = \frac{1}{a}, a : a \in \mathbb{Z} \text{ and } a \neq 0\}$

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2. i) Let $a|n^2$ and $a \leq n$.

By the rules of divisiblity, there is some integer x s.t.

$ax = n^2$.

Observe that $a \leq n$ also implies $ax \leq nx$, which we know implies $n^2 \leq nx$.

If $n^2 = nx$ then $x = n$ and $a = n$, which shows $a|n$ because $a \cdot 1 = n$.

If $n^2 < nx$ then $n < x$ and we already know $x = n^2/a$.

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iii) a) TRUE because 2 is an element of $\{1,2,3\}$.

b) FALSE because 2 is not a set and all elements shown are themselves sets.

c) FALSE because the set on the right does not contain the element "3".

d) TRUE because the set on the right has $\{1,2,3\}$ as one element.

e) TRUE because both sets contain $\{\emptyset\}$ and the set on the left doesnt contain anything else.

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2i) Let $a | n^2$ and $a \leq n$.

Observe that when n is prime, n^2 only has factors 1 and n, so a must either be 1 or n, which by laws of identity are both dividers of n.

When n is composite then

$$n = q \cdot p \,, \quad q, p \in \mathbb{Z} \quad \text{so}$$

$$n^2 = q^2 \cdot p^2$$

which has at least factors q and p.

Observe that when you take the root of both sides, $n = q \cdot p$.

$a | n^2 \rightarrow a | (q^2 \cdot p^2) \rightarrow a | (q \cdot p)$ as a is an element of the set of possible factors $\{1, q, p, n\}$.

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3) ii) Let $a, b, c \in \mathbb{Z}$ s.t.

$b = a+1$ and $c = b+1$.

Observe $a+b+c = a + (a+1) + ((a+1)+1)$

$= 3a + 3$

$= 3(a+1)$

$= 3b$

Because $b \in \mathbb{Z}$ ~~[illegible]~~, $3 \mid (a+b+c)$. ◊

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iii) Let X and Y be sets where ~~[illegible]~~.

Observe $(X \cap Y) \cup (Y - X)$ reduces twice to

$P = \{a : a \in X \text{ and } a \in Y\} \cup \{b : b \in Y \text{ and } b \notin X\}$ by the laws of set operations. The reduces further:

$Q = \{c : (c \in X \text{ and } c \in Y) \text{ or } (c \in Y \text{ and } c \notin X)\}$ by the laws of set operations. Observe that when $c \in X$, $c \in Q$ iff $c \in Y$ and that when $c \notin X$, $c \in Q$ iff $c \in Y$. This is to say that when $c \in Y$, $c \in Q$ and when $c \in Q$, $c \in Y$; meaning $Q = Y$ ◊

4)

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~~Let A and B be sets.~~

~~$A \triangle B = \{x :$~~

i) Let A, B be sets, where $A \subseteq B$.

That means $x \in B$ for all $x \in A$.

Observe that for any element $x \in A$, there is a new set Q s.t

$$Q_A = \{y : y = 2^x\} = 2^A.$$

This also means that for the same elements $x \in B$, $y \in Q_A \rightarrow y \in Q_B$ where

$$Q_B = \{y : y = 2^c, c \in B\}.$$

For any element $y \in Q_A$, there is the same element y for $y \in Q_B$, meaning

$$Q_A = 2^A \subseteq Q_B = 2^B.$$

Let A, B be sets where $2^A \subseteq 2^B$.

That means $x \in 2^B$ for all $x \in 2^A$.

Observe:

$$Q_A = \{y : y = \log_2 x, x \in A\} = A \quad \text{and}$$

$$Q_B = \{y : y = \log_2 x, x \in B\} = B.$$

For any element $y \in Q_A$, there is an element y for $y \in Q_B$, meaning $Q_A = A \subseteq Q_B = B$. $\diamond$

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4 iii) ~~Let~~ $\cancel{a, b \in \mathbb{Z} \text{ s.t.}}$

$\cancel{a = n^2, \; n \in \mathbb{Z}^+}$ ~~and~~

$\cancel{b = (n+1)^2 = n^2 + 2n + 1}$

~~Observe that~~

$\cancel{b - a = n^2 + 2n + 1 - n^2 = 2n + 1}$

Let $a, b, n \in \mathbb{Z}$ s.t.

$a = n^2$, and

$b = (n+c)^2$, where $|n| \neq 1$

Observe

$$b - a = (n+c)^2 - n^2$$
$$= n^2 + 2cn + c^2 - n^2$$
$$= 2cn + c^2$$
$$= c(2n + c).$$

By the rules of integer addition there is some integer a s.t.

$a = 2n + c$,

which means $b - a = c(a)$ which by the diffinition of divisibity means $c \mid a$, which is to say $b - a$ is composite $\square$.