

COP-290: Design Practices

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1 Introduction

An engineering drawing, a type of technical drawing, is used to fully and clearly define requirements for engineered items. Engineering drawings specify requirements of a component or assembly which can be complicated. Standards provide rules for their specification and interpretation. Standardization also aids internationalization, because people from different countries who speak different languages can read the same engineering drawing, and interpret it the same way.

Drawings convey the following critical information:

- Geometry – the shape of the object; represented as views; how the object will look when it is viewed from various angles, such as front, top, side, etc.
- Dimensions – the size of the object is captured in accepted units.
- Tolerances – the allowable variations for each dimension.
- Material – represents what the item is made of.
- Finish – specifies the surface quality of the item, functional or cosmetic. For example, a mass-marketed product usually requires a much higher surface quality than, say, a component that goes inside industrial machinery.

1.1 Isometric Projection

An isometric projection shows the object from angles in which the scales along each axis of the object are equal. Isometric projection corresponds to rotation of the object by 45° about the vertical axis, followed by rotation of approximately 35.264° [$= \arcsin(\tan(30^\circ))$] about the horizontal axis starting from an orthographic projection view. "Isometric" comes from the Greek for "same measure". One of the things that makes isometric drawings so attractive is the ease with which 60° angles can be constructed with only a compass and straightedge.

Isometric projection is a type of axonometric projection.

2 Specifications

For this assignment we are limiting our range to **convex polyhedrons** and their derivatives. We will also not be considering objects with internal cavities.

3 Multiview Projection: 3D to 2D

3.1 Introduction

A multiview projection is a type of orthographic projection that shows the object as it looks from the front, right, left, top, bottom, or back (e.g. the primary views), and is typically positioned relative to each other according to the rules of either first-angle or third-angle projection. The origin and vector direction of the projectors (also called projection lines) differs, as explained below.

In first-angle projection, the parallel projectors originate as if radiated from behind the viewer and pass through the 3D object to project a 2D image onto the orthogonal plane behind it. The 3D object is projected into 2D "paper" space as if you were looking at a radiograph of the object: the top view is under the front view, the right view is at the left of the front view. First-angle projection is the ISO standard and is primarily used in Europe. In third-angle projection, the parallel projectors originate as if radiated from the far side of the object and pass through the 3D object to project a 2D image onto the orthogonal plane in front of it. The views of the 3D object are

like the panels of a box that envelopes the object, and the panels pivot as they open up flat into the plane of the drawing. Thus the left view is placed on the left and the top view on the top; and the features closest to the front of the 3D object will appear closest to the front view in the drawing. Third-angle projection is primarily used in the United States and Canada, where it is the default projection system according to ASME standard ASME Y14.3M. Until the late 19th century, first-angle projection was the norm in North America as well as Europe; but circa the 1890s, third-angle projection spread throughout the North American engineering and manufacturing communities to the point of becoming a widely followed convention, and it was an ASA standard by the 1950s. Circa World War I, British practice was frequently mixing the use of both projection methods.

As shown above, the determination of what surface constitutes the front, back, top, and bottom varies depending on the projection method used.

Not all views are necessarily used. Generally only as many views are used as are necessary to convey all needed information clearly and economically. The front, top, and right-side views are commonly considered the core group of views included by default, but any combination of views may be used depending on the needs of the particular design. In addition to the six principal views (front, back, top, bottom, right side, left side), any auxiliary views or sections may be included as serve the purposes of part definition and its communication. View lines or section lines (lines with arrows marked "A-A", "B-B", etc.) define the direction and location of viewing or sectioning. Sometimes a note tells the reader in which zone(s) of the drawing to find the view or section.

3.2 Input Specifications

We will be taking the input of the given 3D figure in the form of labelled **Cartesian co-ordinates**. Edges will be taken as pairs of the above mentioned labelled points. Faces of the body will be taken as set of edges. Since the input is in the form of Cartesian co-ordinates we do not consider the rotation of the figures with respect to the axes as that would not lead to any change in this case. In the sense that the final input will be Cartesian and the output will be according to that.

3.3 Mathematics and Proofs

We will first create a set of **3D co-ordinates** and also the **mid-points of all the edges** in the figure.

2D Recreation

3.3.1 Top View: First Angle Projection onto the X-Y Plane

For getting the top view we need to recreate a 2D set from the aforementioned 3D set by removing the z-coordinate from all the Cartesian points while maintaining the labels associated with them initially.

- Let's name the 3D set as **A** and assume an arbitrary point in it as **X**.
Now,

$$X = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$$

- To convert X to a 2D point in the top view we have to eliminate the z-coordinate and to do so we multiply it with the transition matrix $Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

$$X * Y = x = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$$

- We first retrieve the x-coordinates of all the points by multiplying each one with the matrix $\begin{bmatrix} 1 & 0 \end{bmatrix}$.
- Sort all the points based on their x-coordinates and retrieve the points with minimum and maximum value of the x. For multiple points having same maximum or minimum x, take any one. Let them be named *a* and *b* respectively. Add these points to a new set called **Out**. Add the line **AB** to another new set called **Perimeter**. In addition to these new sets define another set **In** of 2D points.

Theorem 1. A plane passing through (x_1, y_1, z_1) and (x_2, y_2, z_2) is $z - (ax + by + c) = 0$.

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$$

Lemma 1. A line passing through (x_1, y_1) and (x_2, y_2) is $lx + my + n = 0$, where $l = y_2 - y_1, m = x_1 - x_2$ and $n = x_2y_1 - x_1y_2$.

Proof. Using the general line form $y = mx + c$, where m is the slope of the line. \square

- Construct a set called **Plane** consisting of all the plane equations calculated by taking the set of edges in the input for all such sets.
- Let the line in Perimeter be $L : lx + my + n = 0$, find the distance of all the points in the 2D plane from **L** using the formula

$$d = lx_a + my_a + n \quad (1)$$

where \mathbf{d} is the distance from the line and (x_a, y_a) is the point whose distance we are finding out.

Remark. We are not considering the $\sqrt{l^2 + m^2}$ because we only need a comparison and not the absolute value.

- Sort all the points wrt the distance from L and find the minimum and maximum points. Let them be **C** and **D** respectively. Now we analyze the different cases that will arise in this situation:

Case1: ($d_{min} > 0$ and $d_{max} > 0$)

Add C to the set In and D to the set Out. Remove line AB from the Perimeter set and add lines AD and DB to it.

Case2: ($d_{min} = 0$ and $d_{max} > 0$)

Add C and D to the set Out. Remove line AB from the Perimeter set

and add lines AD and DB to it.

Case3:($d_{min} = 0$ and $d_{max} = 0$)

Not possible for a 3D figure.

Case4:($d_{min} < 0$ and $d_{max} = 0$)

Add C and D to the set Out. Remove line AB from the Perimeter set and add lines AC and BC to it.

Case5:($d_{min} < 0$ and $d_{max} < 0$)

Add D to the set In and C to the set Out. Remove line AB from the Perimeter set and add lines AC and BC to it.

Case6:($d_{min} < 0$ and $d_{max} > 0$)

Add C and D to the set Out. Remove line AB from the Perimeter set and add lines AD,DB,AC and BC to it.

- Now take a line $X \in Perimeter$ and m and n be the end points of X. Let $l \in Out$ such that $l \neq m$ and $l \neq n$. Calculate the distance of l from the line X using the equation 1. Again we will look at the different cases:

Case1:($distance = 0$)

Then take another point in place of l from Out because distance = 0 implies that the point is on the line.

Case2:($distance > 0$)

From the set containing all the points in the 2D projection, find the point which is closest (signed) to the line X and name it p. If the distance of p is < 0 , then add p into Out, add edges p-m and p-n from Perimeter and remove X from Perimeter.

If distance of p = 0, then add p into Out and remove X from Perimeter.

If distance of p > 0 , then add p into In and remove X from Perimeter.

Case3:($distance < 0$)

From the set containing all the points in the 2D projection, find the

point which is farthest(signed) to the line X and name it p. If the distance of p is > 0 , then add p into Out, add edges p-m and p-n from Perimeter and remove X from Perimeter.

If distance of p = 0, then add p into Out and remove X from Perimeter.

If distance of p < 0 , then add p into In and remove X from Perimeter.

- Repeat this process for each edge in Perimeter until the whole set is empty. Now all the points in the set Out are visible and they form the boundary of the 2D representation.
- Now for each $x \in \text{In}$, calculate the distance from each plane in Plane. If the distance is positive for all planes then the point is marked visible and we add it to a new set **Vis**. If it is negative for even one plane then we mark it hidden and add it to another set **Hid**. Calculate the distance as $z_1 - ax_1 - by_1 - c$, where $z - (ax + by + c) = 0$ is the plane equation and (x_1, y_1, z_1) is the point.

Theorem 2. *Edge X: $(x_1, y_1) - (x_2, y_2)$ is hidden if either one of the end points or the mid-point of the edge is in Hid and the rest are in Vis.*

Proof. The orthographic projection basically is the shadow formed by an object when a light from a source at infinity falls on it. Now we claim that our procedure recreates this "shadow". We start our procedure by finding both x_{min} and x_{max} , since they are farthest points in the x-direction they are definitely visible and will be the left-most and right-most corners of the shadow. In the next step we find the farthest points from the line joining $x_{min} - x_{max}$ in the perpendicular direction. Now we will have a closed figure, either a triangle or a quadrilateral, using the cases mentioned above.

After repeating the process for each edge we get a k-sided polygon figure ($k \geq 3$) and for each edge of the polygon we know that all points lie to one side of it (inside wrt to the polygon) because if this was not the case then the edge wouldn't be removed from the set Perimeter.

Now a point is hidden from the 2D projection if the "light" falls on a face of the object first and not on the point. So we find the signed distance (d) of the point from all the planes. If $d \forall \text{ planes} \in \text{Plane} \geq 0$

then it is visible. If $d < 0$ for even one plane \in Plane then it is hidden. An edge is hidden if either of its ends or the mid-point of the edge is in Hid. Hence a hidden edge is not visible. \square

3.4 Front View: First Angle Projection onto the X-Z Plane

- Similarly for front view ignore the y-coordinate and change the transition matrix to $Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and follow similar steps to form the front view diagram or the "shadow" we were referring to.

3.5 Side View: First Angle Projection onto the Y-Z Plane

- Similarly for side view ignore the x-coordinate and change the transition matrix to $Y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and follow similar steps to form the side view diagram or the "shadow" we were referring to.

3D Recreation

3.6 Isometric Projection

Since the points which form edges will be given in the input as mentioned in the specifications 3D recreation is just finding the x,y,z coordinates of a point from two views.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (x \ y) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + (y \ z) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where (x, y) are the coordinates of the point in top view and (y, z) are its coordinates in the side view.

Remark. We can use any two views, it's not necessary to use only the above mentioned two views.

4 Ending Remarks

The present documentation is subject to minor changes which can be caused due to implementation difficulties in the software testing part of the assignment.