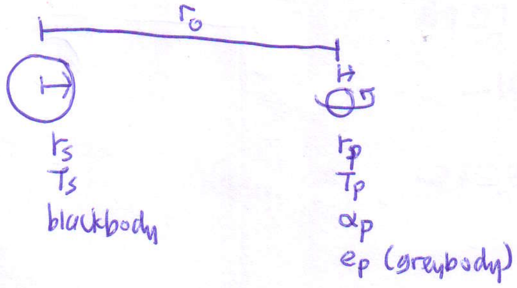
PC1142 2012-13

1.




Total power radiated by star, Stefan-Boltzman law,

$$P_{rad,s} = \sigma A_s e_s T_s^4 \quad \text{where } e_s = 1, \; A_s = 4\pi r_s^2$$

Power intercepted by planet,

$$P_{in,p} = \frac{\pi r_p^2}{4\pi r_0^2} \cdot P_{rad,s}$$

Power absorbed by planet

$$P_{ab,p} = (1-\alpha_p) \cdot P_{in,p}$$

Total power radiated by planet, Stefan-Boltzman law,

$$P_{rad,p} = \sigma A_p e_p T_p^4 \quad \text{where } A_p = 4\pi r_p^2$$

At steady state

$$P_{ab,p} = P_{rad,p} .$$

This gives $T_p = T_s \left( (1-\alpha_p) \frac{r_s^2}{4 r_0^2} \frac{e_s}{e_p} \right)^{1/4}$

where $e_s = 1$

2. H−C≡C−H

$C_V$ is contributed by translational KE and rotational KE terms. Acetylene has 3 degrees of translational freedom and two degrees of rotational freedom (due to linear molecule). Each of these degrees of freedom contributes $\frac{1}{2}R$ to $C_V$ (equipartition thm)

Hence $C_V = \frac{3}{2}R + \frac{2}{2}R = \frac{5}{2}R$ $(= 20.8 \text{ kJ mol}^{-1})$

3. $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)$ $\quad \Delta G^\circ_{rxn,298}$ ?

The max amt of electrical work that can be produced is given by $-\Delta G^\circ_{rxn,298}$

$$\Delta G_{rxn} = \Delta H_{rxn} - T\Delta S_{rxn}$$

$$\Delta H_{rxn} = \Delta H^\circ_{f\,298}(H_2O(l)) - \underbrace{\left(\Delta H^\circ_{f,298}(H_2(g)) + \tfrac{1}{2}\Delta H^\circ_{f,298}(O_2(g))\right)}_{\text{elements! in their std states } (=0)}$$

$$= -285.83 \text{ kJ mol}^{-1}$$

$$\Delta S_{rxn} = S^\circ_{298}(H_2O(l)) - \left( S^\circ_{298}(H_2(g)) + \tfrac{1}{2} S^\circ_{298}(O_2(g)) \right)$$

$$= 69.91 - \left(130.68 + \frac{205.14}{2}\right) \ J K^{-1} mol^{-1}$$

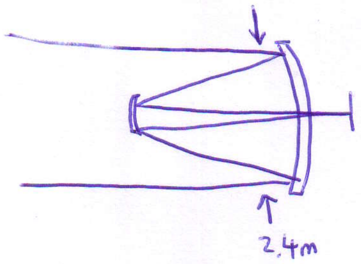
$$= -163.34 \ J K^{-1} mol^{-1}$$

$$\Delta G_{rxn} = -285.83 \times 10^3 - (298)(-163.34) \ J\, mol^{-1}$$

$$= -237.1 \ kJ\, mol^{-1}$$

Hence max amt of work produced is 237.1 kJ $mol^{-1}$ of $H_2O(l)$.

4.




Rayleigh's criteria,

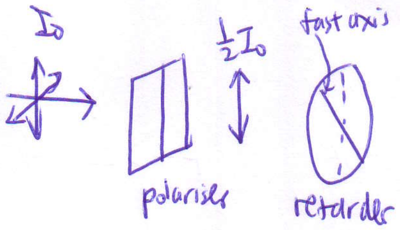
$$\sin \theta_{res} = \frac{1.22\lambda}{a}$$

For small angles, $\sin\theta \cong \theta$

green light

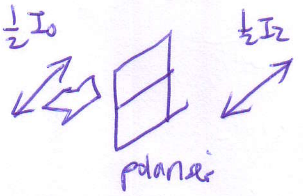
so, $$\theta_{res} = \frac{1.22\ (550nm)}{2.4m} = 2.8 \times 10^{-7} rad$$

5.




Resolve the vertically polarised light onto the fast axis and slow axis (perpendicular) of the retarder.

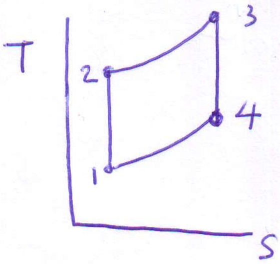

The retarder is known to retard the light polarised parallel to slow axis by $\pi$. This means the polarisation of the light is now horizontal.




The horizontally polarised light passes through the second polariser unchanged.

At exit, polarisation is horizontal, intensity is half of the incident intensity

6(i)




$1 \to 2$ : adiabatic reversible, $\Delta S = 0$

$2 \to 3$ :

Isobaric process, $dS = \frac{dq_{rev}}{T} = \frac{nC_p\,dT}{T}$

$$\Delta S = nC_p \ln\left(\frac{T_2}{T_1}\right)$$

(ii) $1 \to 2$ : adiabatic process. Eqn of adiabat is $T_2^{\gamma} P_2^{1-\gamma} = T_1^{\gamma} P_1^{1-\gamma}$ -- Eq(1)

$3 \to 4$ : also adiabatic process. $T_3^{\gamma} P_3^{1-\gamma} = T_4^{\gamma} P_4^{1-\gamma}$ -- Eq(2)

since $P_2 = P_3$ and $P_1 = P_4$, we divide Eq(2) by Eq(1)

we have,

$$\left(\frac{T_3}{T_2}\right)^{\gamma} = \left(\frac{T_4}{T_1}\right)^{\gamma}$$

which rearranges to

$$\frac{T_3}{T_4} = \frac{T_2}{T_1}$$

Thermal efficiency of Brayton cycle

n number of moles of gas

$$e_{th} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{nC_p(T_4 - T_1)}{nC_p(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

$$= 1 - \frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)}$$

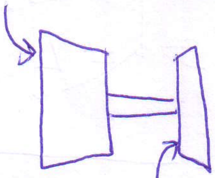
$\frac{T_3}{T_1}$

$$= 1 - \frac{T_1}{T_2}$$

From Eq(1) $\frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}} = (r_p)^{\frac{1-\gamma}{\gamma}}$

Hence $e_{th} = 1 - (r_p)^{\frac{1-\gamma}{\gamma}}$

(iii) $T_1 = 30°C = 303K$



$T_3 = 1200°C = 1473K$

Thermal efficiency

$$e_{th} = 1 - (r_p)^{\frac{1-\gamma}{\gamma}}$$

$r_p = 6.0$

$\gamma = 1.4$, so $\frac{1-\gamma}{\gamma} = -0.286$

and $e_{th} = 0.40$

Power input

$$P_{in} = \left(\frac{dm}{dt}\right) \cdot C_p (T_3 - T_2)$$

$$T_2 = \frac{T_1}{\left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}}} = \frac{303K}{\quad}$$

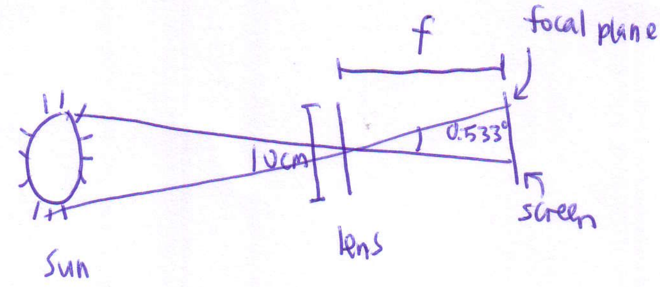
$$= (0.2\, kgs^{-1})(1.005\, kJ\, kg^{-1}K^{-1})(1473K - 506K)$$

$$= 194\, kW$$

Power output

$$P_{out} = P_{in} \cdot e_{th} = 78\, kW$$

7.




(i) The screen has to be placed on the focal plane of the glass.

Thin-lens equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For plano-convex lens, one face is flat and the other is convex,
Let us take

$R_1 = 33\text{ cm}$ and $R_2 = \infty$

$$\frac{1}{f} = (1.55-1)\left(\frac{1}{33\text{ cm}} - \frac{1}{\infty}\right)$$

$f = 60\text{ cm}$

(ii) Size of sun $d' = f \cdot \theta = (60\text{ cm})\left(0.533° \frac{\pi}{180°}\right) = 0.56\text{ cm}$

(iii) Image Intensity $I_{im} = \dfrac{I_0 \cdot \pi r_{lens}^2}{\pi \left(\frac{d'}{2}\right)^2} = \dfrac{4 I_0 r_{lens}^2}{(d')^2} = \dfrac{4(1\text{ kW m}^{-2})(0.10\text{ m})^2}{(0.56 \times 10^{-2}\text{ m})^2}$

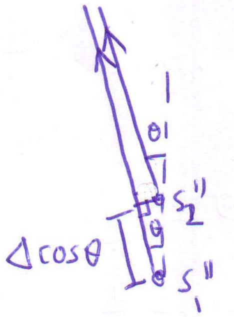
$= 320\text{ kW m}^{-2}$

(iv) If convex surface is facing the sun, the image has less spherical aberration.

8. (i) $\Delta = 2(d_1 - d_2)$

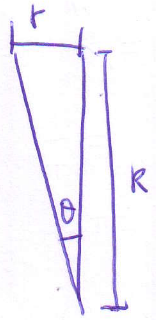
(ii) For $R \gg \Delta$, the rays from $S_2''$ and $S_1''$ are parallel.

Optical path difference is $\Delta\cos\theta$. Constructive interference occurs when




$$\Delta\cos\theta = m\lambda \text{ where } m \text{ is integer}$$

(iii)




$r = R\sin\theta$.

For small $\theta$, $r = R\theta$

Also since

$$\Delta\cos\theta = m\lambda$$

For small angles

$$\Delta\left(1 - \frac{1}{2}\theta^2\right) = m\lambda$$

$$\theta = \left[2\left(1 - \frac{m\lambda}{\Delta}\right)\right]^{1/2} \qquad \frac{m\lambda}{\Delta} < 1$$

So $R\theta = R\left[2\left(\frac{\Delta - m\lambda}{\Delta}\right)\right]^{1/2}$

If $\Delta = m\lambda$,

$r = 0$, central region is bright