**NATIONAL UNIVERSITY OF SINGAPORE**

PC4130 QUANTUM MECHANICS III

(Semester I: AY 2009-10)

Time Allowed: 2 Hours

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## INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR** questions and comprises **THREE** printed pages including this page.
2. Answer ALL FOUR questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.

**Question I.**

A three-level quantum system is interacting with two laser fields. The Hamiltonian is given by

$$\begin{pmatrix} E_1 & \hbar\Omega_p \cos(\omega_p t) & 0 \\ \hbar\Omega_p^* \cos(\omega_p t) & E_2 & \hbar\Omega_s \cos(\omega_s t) \\ 0 & \hbar\Omega_s^* \cos(\omega_s t) & E_3 \end{pmatrix},$$

where $E_1 < E_2 < E_3$ are the energy eigenvalues of each level in the absence of the laser fields, $t$ is the time variable, and $\omega_p$ and $\omega_s$ are the two laser frequencies. We further assume that $\omega_p$ is close to the transition frequency $(E_2 - E_1)/\hbar$, and $\omega_s$ is close to the transition frequency $(E_3 - E_2)/\hbar$.

1. Use the rotating wave approximation to simplify the above Hamiltonian matrix.

2. Further using the "dressed-state" picture, derive an effective static Hamiltonian for this time-dependent problem.

**Question II.**

Consider the quantum scattering in a spherical square well potential. Specifically, the scattering potential $V(r)$ is rotationally symmetric: it equals $-V_0$ if $r \leq a$ and equals 0 if $r > a$, where $r$ is the radial distance from the origin.

1. Using the first Born approximation, calculate the scattering amplitude and the differential cross section.

2. Explain the "optical theorem" for quantum scattering.

3. If we apply the "optical theorem" using the scattering amplitude obtained in step 1, what is the value of the total cross section? Comment on your result.

**Question III.**

A hydrogen atom is initially in its ground state and then subject to a pulsed electric field $E(t) = E_0\delta(t)$ along the $z$ direction. In our considerations we neglect all fine-structure and hyperfine-structure corrections.

1. Given the selection rule for first-order dipole transitions, what are the final quantum numbers $l$ and $m$ in order to have nonzero transition amplitudes?

2. Using the first-order time-dependent perturbation theory, calculate the transition probability to an arbitrary excited state. You do not need to explicitly evaluate the matrix elements of the transition dipole moment.

**Question IV.**

Using the variational principle and a Gaussian trial wavefunction, estimate the energy of the ground state associated with the potential $V(x) = cx^4$, where $c > 0$. You may need the following integral:

$$\int_0^{+\infty} \exp(-ax^2)x^{2n}dx = \sqrt{\frac{\pi}{4a}}\left(\frac{1}{4a}\right)^n \frac{(2n)!}{n!}.$$

END OF PAPER, JG