NATIONAL UNIVERSITY OF SINGAPORE

PC4130 Quantum Mechanics III

(Semester I: AY 2010-11)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR** questions and comprises **THREE** printed pages.
2. Answer **ALL FOUR** questions.
3. Answers to the questions are to be written in the answer books.
4. Each question carries 25 marks.
5. This is a CLOSED BOOK examination.
6. One Help Sheet (A4 size, both sides) is allowed for this examination.

Question 1.

Consider a particle of mass m moving in the one-dimensional potential $V(x) = \alpha|x|$, where $\alpha > 0$.

(a) Using the variational principle with a Gaussian trial wavefunction, estimate the ground state energy. You may need this integral: $\int_0^{+\infty} \exp(-ax^2)x^2 dx = \sqrt{\frac{\pi}{16a^3}}$.

(b) Using the WKB quantization rule, predict how the transition frequency between two neighboring energy levels scales with energy if the system is in a highly excited state.

Question 2.

The Hamiltonian describing a spin-$\frac{1}{2}$ particle interacting with an external magnetic field is given by

$$H = -\frac{\gamma B \hbar}{2} \begin{pmatrix} \cos(\theta) & \sin(\theta)e^{-i\phi} \\ \sin(\theta)e^{i\phi} & -\cos(\theta) \end{pmatrix}, \tag{1}$$

where γ is the gyro-magnetic ratio, B is the field strength, (θ, ϕ) are the two angles in a spherical coordinate system describing the direction of the magnetic field.

(a) Find the spinor wavefunction that describes a "spin-down" state along the field direction.

(b) If θ is fixed and ϕ is adiabatically changed from 0 to 2π over a total duration T, calculate the dynamical phase and the Berry phase for the "spin-down" state obtained in (a).

Question 3.

Consider the electric dipole induced spontaneous emission of the hydrogen (or deuterium) atom modeled solely by a Coulomb potential. The bare eigenstates are characterized by the principle quantum number n, the orbital angular momentum quantum number l and the magnetic quantum number m.

(a) Suppose the initial state is given by $n = 4, l = 1, m = 1$. What are the possible final states after the atom spontaneously emits one photon?

(b) If the system does not reach the ground state after emitting one photon, what are the possible deexcitation routes through which the atom can further decay to the ground state via spontaneous emission?

(c) Referring to the spontaneous emission rate of a state characterized by a fixed set of quantum numbers, which of the two, namely, the hydrogen atom or the deuterium atom, has a slightly larger rate?

Question 4.

Briefly explain the following items.

(a) Lippmann-Schwinger equation

(b) "Optical theorem" in quantum scattering theory

(c) Interaction picture

(d) Fine-structure correction to hydrogen spectrum

(e) Fermi's golden rule for quantum transition rate to a continuum

END OF PAPER (JG)