

## Equivalent Inductance

Equivalent inductance for more than one inductors in **series** is the sum of their inductance values. Figure 13(a) shows two inductors in series. The equivalent inductance seen from the terminals **ab** is the sum of two inductance values ( $L_{eq} = L_1 + L_2$ ). For finding this equivalent inductance, it is required to find the voltage current relationship. For the circuit shown in Fig. 13 (a),

$$\begin{aligned}
 v - L_1 \frac{di}{dt} - L_2 \frac{di}{dt} &= 0 \\
 \Rightarrow v &= (L_1 + L_2) \frac{di}{dt} \\
 v = L_{eq} \frac{di}{dt} &= L_{eq} = L_1 + L_2
 \end{aligned}$$

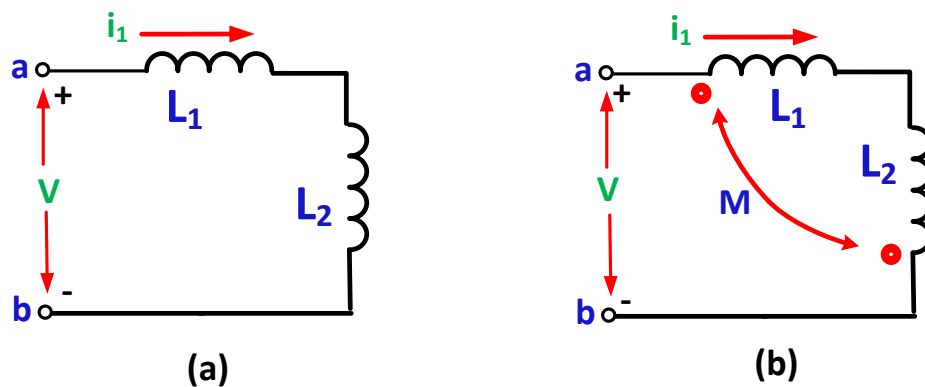


Fig. 13

In Fig. 13 (b), two series inductors are magnetically coupled. The mutual inductance is  $M$ . In this case, there will be two induced voltages. One will be induced in coil  $L_1$  due to current in  $L_2$  and the other induced voltage will be in coil  $L_2$  due to current in  $L_1$ . As the current in  $L_2$  leaves the dotted terminal, its induced voltage in  $L_1$  will have its negative polarity at its dotted terminal. Similarly the current enters the dotted terminal in  $L_1$ . Hence its induced voltage in  $L_2$  will have its positive polarity at its dotted terminal. Applying KVL

$$\begin{aligned}
 v - L_1 \frac{di}{dt} - L_2 \frac{di}{dt} + M \frac{di}{dt} + M \frac{di}{dt} &= 0 \\
 \Rightarrow v &= (L_1 + L_2 - 2M) \frac{di}{dt} \\
 L_{eq} &= L_1 + L_2 - 2M
 \end{aligned}$$

If the circuit consists of more than two inductors as mutually coupled inductors, then we can evaluate the equivalent inductance by finding the voltage and current relationship.

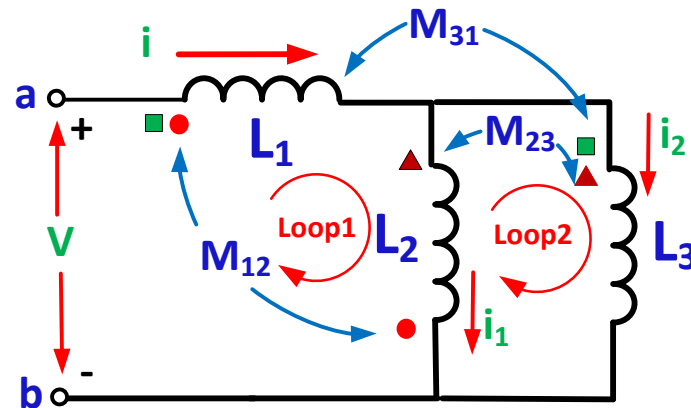


Fig. 14

Fig. 14 shows a circuit with three magnetically coupled inductors. The self inductances of the three coils are  $L_1$ ,  $L_2$  and  $L_3$ . The mutual inductance between the coils  $L_1$  and  $L_2$  is  $M_{12}$ . Circular dots are used for this case. Triangular dots are used for mutual inductance  $M_{23}$  between the coils  $L_2$  and  $L_3$ . Square dots are used for mutual inductance ( $M_{31}$ ) between the coils  $L_3$  and  $L_1$ . for finding the equivalent inductance at  $ab$  terminals, we can use KVL for loop1 and loop2

$$\begin{aligned}
 i &= i_1 + i_2 \\
 v &= L_1 \frac{di}{dt} + L_2 \frac{di_1}{dt} - M_{12} \frac{di_1}{dt} - M_{12} \frac{di}{dt} \\
 &\quad + M_{31} \frac{di_2}{dt} + M_{23} \frac{di_2}{dt} \\
 L_2 \frac{di_1}{dt} - L_3 \frac{di_2}{dt} + M_{23} \frac{di_2}{dt} - M_{12} \frac{di}{dt} \\
 - M_{31} \frac{di}{dt} - M_{23} \frac{di_1}{dt} &= 0
 \end{aligned}$$

Solving the three equations and finding the relation between  $v$  and  $i$ , we can determine the equivalent inductance at  $ab$  terminal.

### Energy in Mutually Coupled Circuits

For the circuit shown in Fig. 15, the right hand side is open circuited. The current  $i_1$  is increased from zero to some constant value  $I_1$  during the time interval  $t = 0$  to  $t_1$ . Power delivered to  $L_1$  from left is

$$v_1 i_1 = L_1 \frac{di_1}{dt} i_1$$

Power entering from right side circuit is

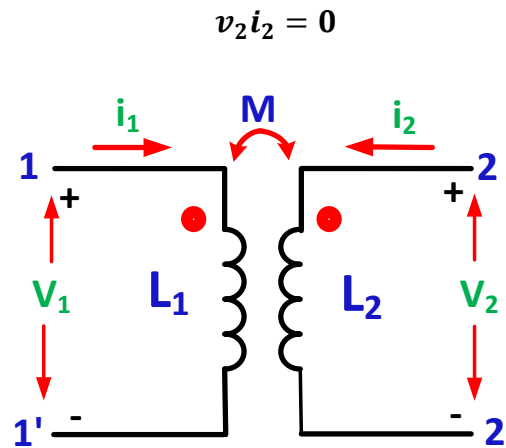


Fig. 15

Energy stored in  $L_1$  at  $t = t_1$  is

$$\int_0^{t_1} v_1 i_1 dt = \int_0^{I_1} L_1 \frac{di_1}{dt} i_1 dt = \int_0^{I_1} L_1 i_1 di_1 = \frac{1}{2} L_1 I_1^2$$

Then  $i_1$  is kept constant at a value  $I_1$  during the time interval  $t = t_1 \rightarrow t_2$  and allowing  $i_2$  to change from 0 at  $t = t_1$  to a constant value  $I_2$  at  $t = t_2$ . The energy delivered to  $L_2$  from the right side source will be

$$\int_{t_1}^{t_2} v_2 i_2 dt = \int_0^{I_2} L_2 i_2 di_2 = \frac{1}{2} L_2 I_2^2$$

During the time interval  $t_1$  to  $t_2$ ,  $i_1$  remains constant at  $I_1$ . It will not produce any induced voltage in  $L_2$  as this is not a time varying current. But the current  $i_2$  varies from 0 to  $I_2$  in the interval  $t_1$  to  $t_2$ . Hence,  $i_2$  will produce an induced voltage in  $L_1$  which can be given as  $V_1'$

$$v_1' = M \frac{di_2}{dt}$$

$I_1$  is the current through  $v_1'$ . As the current is entering into  $v_1'$ , energy will be delivered to  $L_1$  from the right hand source. This energy will be given as

$$\int_{t_1}^{t_2} v_1' I_1 dt = \int_{t_1}^{t_2} M \frac{di_2}{dt} I_1 dt = M I_1 \int_0^{I_2} di_2 = M I_1 I_2$$

Total energy delivered by the right hand side source in the interval  $t_1$  to  $t_2$  will be

$$\frac{1}{2} L_2 I_2^2 + M I_1 I_2$$

Total energy delivered when  $i_1$  and  $i_2$  have reached constant values during the interval  $t = 0$  to  $t_2$  will be

$$W_{Total} = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2$$

If one current enters a dotted terminal while the other leaves a dotted terminal, the sign of the mutual energy term will be negative.

$$W_{Total} = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 - M I_1 I_2$$

Energy stored expression can be valid for instantaneous values of  $i_1$  and  $i_2$ .

$$W(t) = \frac{1}{2}L_1 [(i_1(t))]^2 + \frac{1}{2}L_2 [(i_2(t))]^2 \pm M [i_1(t)][i_2(t)]$$

Since  $W(t)$  represents the energy stored within a passive network, it cannot be negative for any value of  $i_1$ ,  $i_2$ ,  $L_1$ ,  $L_2$  or  $M$ . An upper bound for the value of  $M$  can be found from the value of  $W(t)$ .

$$\begin{aligned} W &= \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2 \\ &= \frac{1}{2}(\sqrt{L_1} i_1 - \sqrt{L_2} i_2)^2 + \sqrt{L_1 L_2} i_1 i_2 - M i_1 i_2 \end{aligned}$$

The first term is positive as it is a square term. Its minimum value is zero. With this condition the energy can be nonnegative if

$$\begin{aligned} \sqrt{L_1 L_2} &\geq M \\ \Rightarrow M &\leq \sqrt{L_1 L_2} \end{aligned}$$

This is the upper limit for the magnitude of the mutual inductance. The degree to which  $M$  approaches its maximum value is described by the coefficient of coupling given as

$$\begin{aligned} K = \text{Coeff. of Coupling} &= \frac{M}{\sqrt{L_1 L_2}} \\ 0 &\leq k \leq 1 \end{aligned}$$