Lecture 02

PH101: Physics 1

Module 2: Special Theory of Relativity - Basics

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RECAP

Within Galilean Relativity

While space coordinates change when seen from different inertial frames,
Time is absolute, and is the same in all inertial frames.

Galilean relativity fails to accommodate the constancy of speed of light
(in all inertial frames).

Special Theory of Relativity suggests

Speed of light is constant in all frames (as required by Maxwell's EM waves).

(Consequently) Time is not absolute, as in the classical Galilean relativity.

New ideas to compare things in different frames.
(in place of constancy of spatial distance)

Consider Two frames, S and S'.

S' is moving with speed v along x-axis

Consider Two events: Event 1: Light pulse emitted Event 2: This light pulse detected.

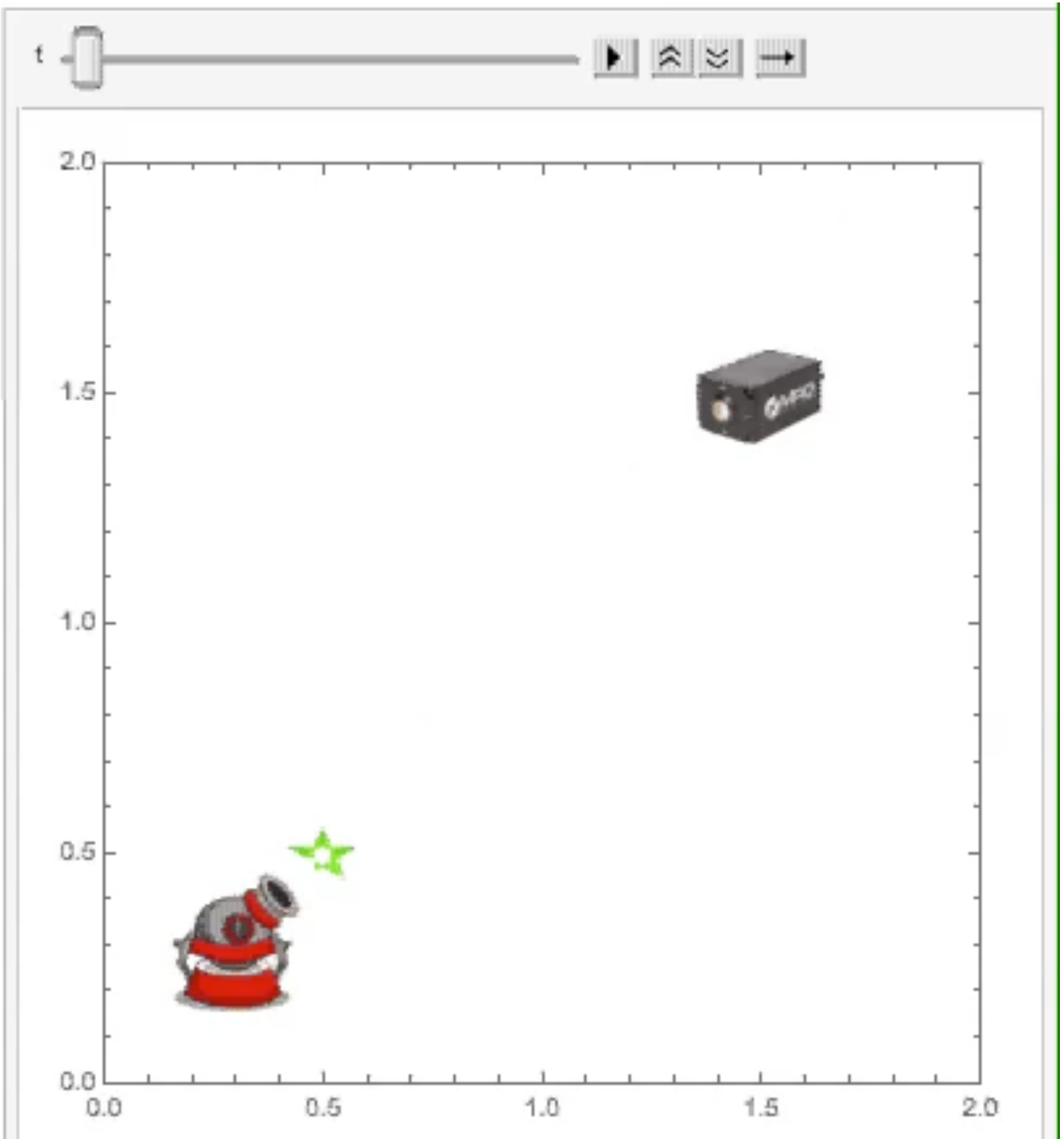
In frame S, let the coordinates of these events are: $(0, 0,\ 0,\ 0)$ and $(t, x, y,\ z)$

The same events as recorded in S': $(0, 0,\ 0,\ 0)$ $(t', x', y',\ z')$

The two frames are synchronised at the first event.

The second event correspond to detecting the light pulse => $x^2 + y^2 + z^2 = c^2 t^2$

$$x'^2 + y'^2 + z'^2 = c'^2 t'^2 = c^2 t'^2$$


t
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0

Observation:

y and z coordinates (of each event) are the same in both the frames, but x coordinate would be different.

From the point of view of S', the frame S is moving with speed v in the -ve x direction.

The relation between x and x' should be invertible, and should look similar. **Should be a linear relation**

$$x' = a_1 x + a_2\, ct$$

$$ct' = b_1 ct + b_2 x$$

The origin of S' is moving with speed v => $x' = 0 \Rightarrow x = vt$

$$\Rightarrow 0 = a_1 vt + a_2\, ct; \quad a_2 = -\frac{v}{c} a_1$$

$$x'^2 - c^2 t'^2 = x^2 - c^2 t^2$$

(Remember the case of light pulse)

$$\Rightarrow a_1 = b_1 = \sqrt{\frac{c^2}{c^2 - v^2}} = \gamma \quad a_2 = b_2 = -\frac{v}{c}\sqrt{\frac{c^2}{c^2 - v^2}} = -\gamma\beta$$

Lorentz transformation: relating the coordinates and time in two different coordinates.

$$x' = \gamma\left(x - \beta x_0\right); \qquad y' = y; \quad z' = z$$

We introduce $x_0 = ct$

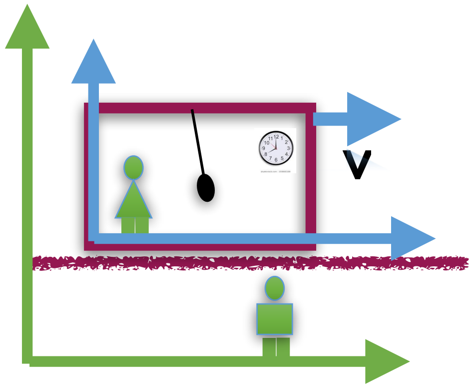
$$x'_0 = \gamma\left(x_0 - \beta x\right)$$

Inverting the relation:

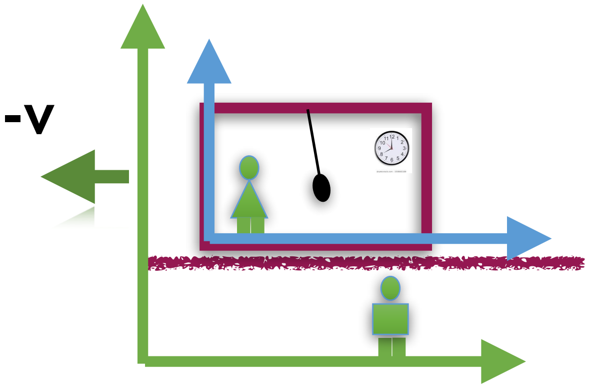
$$x = \gamma\left(x' + \beta x'_0\right); \qquad y' = y; \quad z' = z$$

$$x_0 = \gamma\left(x'_0 + \beta x'\right)$$

Exercise



$$\equiv$$



Illustrations:

Consider frame S and another frame S'.

Their origins coincide at $t = t' = 0$. Their axes are parallel to each other. The origin of S' moves with a constant speed $v = 10\,km/s$ along the x-axis.

An event occurs at $x = 1m, \quad y = 2m, \quad z = 10\,m$ at $t = 8\,s$ in S.

What are its coordinates in S'?

Illustrations:

Consider frame S and another frame S’.

Their origins coincide at $t = t' = 0.$ Their axes are parallel to each other. The origin of S’ moves with a constant speed $v = 0.2c$ along the x-axis.

An event occurs at $x = 1m, \quad y = 2m, \quad z = 10\,m$ at $t = 8\,s$ in S.

What are its coordinates in S’?

Illustrations:

Consider frame S and another frame S'.

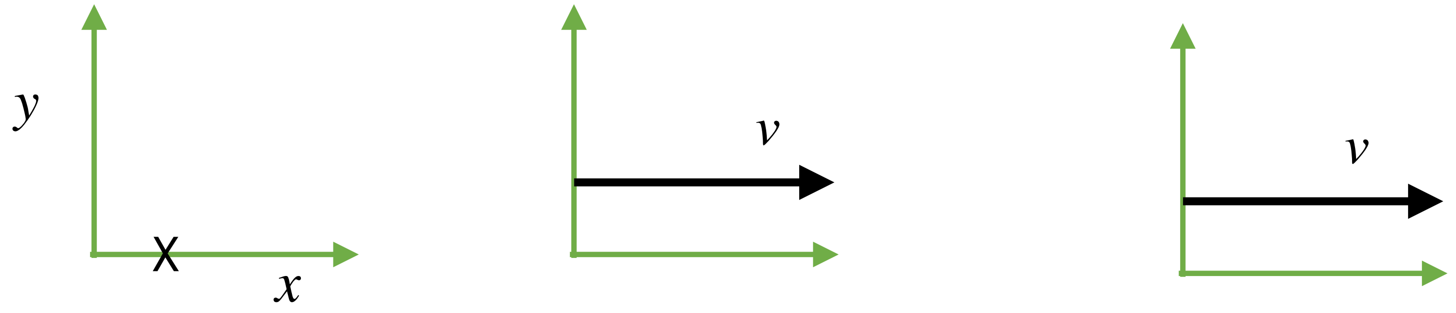
Their origins coincide at $t = t' = 0$. Their axes are parallel to each other. The origin of S' moves with a constant speed $v = 0.7c$ along the x-axis.

An event occurs at $x = 1m, \quad y = 2m, \quad z = 10\,m$ at $t = 8\,s$ in S.

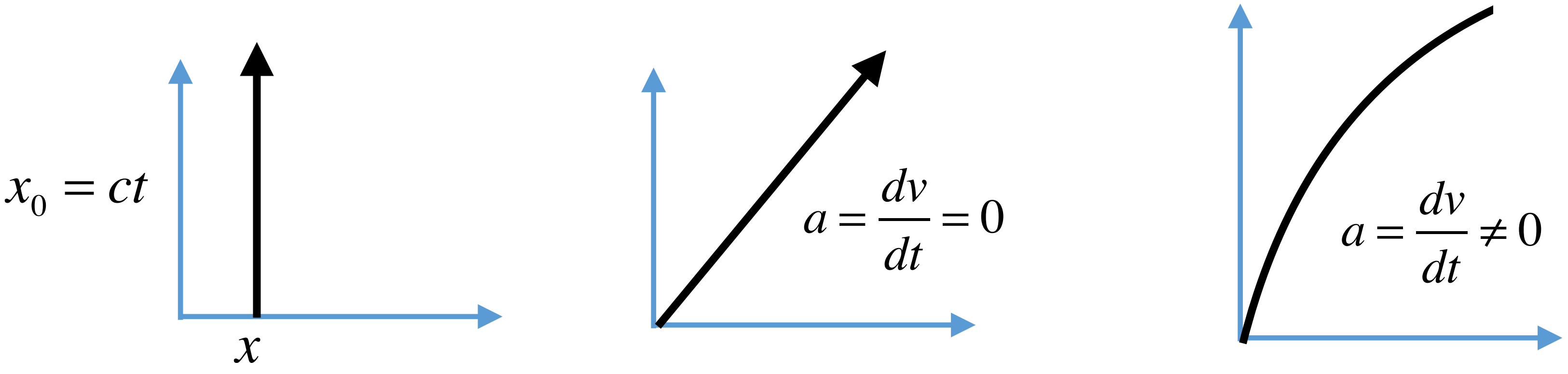
What are its coordinates in S'?

WORLD LINE

Meaning of trajectory in the usual sense is lost in STR, as the space and time are interlinked.
The concept of **world line** is introduced instead.



Path of particles
in the usual sense.



World lines in STR

Object at rest

Moving with const
speed along x

Accelerating
object

Consider the invariant interval (between two events, E1 and E2): $ds^2 = c^2\,dt^2 - dx^2 - dy^2 - dz^2$

Three possibilities:

$$ds^2 = 0$$

Events are called light-like events.

For example, E1 is the flashing of light at $(0, 0, 0, 0)$
and E2 is detecting it at (t, x, y, z)

Distance covered by the light pulse in time $t = ct$

This is equal to the spatial distance between the points.

$$ds^2 > 0$$

Events are called time-like events.

For example, E1 is firing a bullet at $(0, 0, 0, 0)$
and E2 is it hitting a target at (t, x, y, z)

Distance covered by the bullet in time $t = vt < ct$

The spatial distance between the points $vt = \sqrt{x^2 + y^2 + z^2}$

$$ds^2 < 0$$

Events are called space-like

This cannot be the case with normal events.
Possible for particles moving faster than c

Such particles with $v > c$ are called Tachyons

Consider two time-like events E1: (t_1, x_1, y_1, z_1) E2: (t_2, x_2, y_2, z_2)

with $dt = t_2 - t_1, \quad dx = x_2 - x_1, \quad dy = y_2 - y_1, \quad dz = z_2 - z_1$

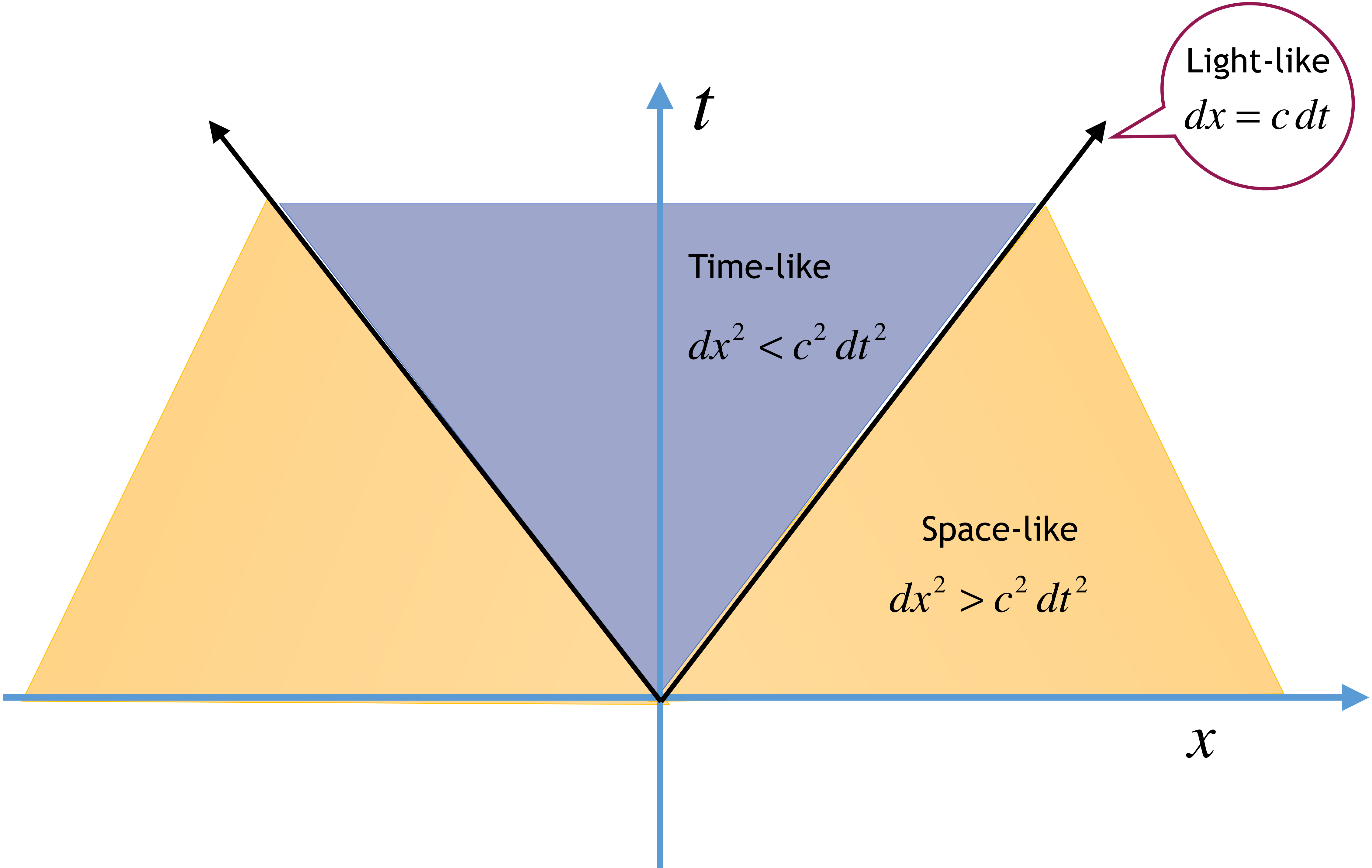
Since the interval is the same
when seen from different inertial frames,

$$ds^2 = c^2\, dt^2 - dx^2 - dy^2 - dz^2$$
$$= c^2\, dt'^2 - dx'^2 - dy'^2 - dz'^2 = ds'^2$$

The events will remain time-like in all inertial frames.

Similarly, light-like and space-like events also would remain so when seen from any inertial frame.

Similarly, light-like and space-like events also would remain so when seen from any inertial frame.


Light-like
$dx = c\,dt$
t
Time-like
$dx^2 < c^2\,dt^2$
Space-like
$dx^2 > c^2\,dt^2$
x

We shall continue the discussion in the next class