1. Determine the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ where a_n is given by

   (a) $\frac{n^n}{n!}$ (b) $(\ln n)^{-1}$ $n > 2$

2. Find the Taylor series of the functions $\sin x$, $\cos x$ and e^x, where $x \in \mathbb{R}$. Determine the radius of convergence of each of these series. Use the Taylor series of $\sin x$ to approximate $\sin 10$ correct to 3 decimal places.

3. Familiarize yourself with the geometry of $\mathbb{R}^3$. Understand the co-ordinate planes, the parametric equation of a line passing through a point (x_0, y_0, z_0) and parallel to a vector (a, b, c), the equation of a line passing through two given points, parametric equation of a plane passing through (x_0, y_0, z_0) and normal to the vector (a, b, c), the equation of a plane passing through three points. We will also assume the knowledge of various forms of equations of a line, plane, the foot of perpendicular of a point on a line, on a plane, distance of a point from a line/plane in $\mathbb{R}^3$ throughout the course. Consult Chapter 1 of your textbook (Basic Multivariable Calculus by Marsden, Tromba and Weinstein) for practice on these.

4. Define a Cauchy sequence in $\mathbb{R}^n$. Show that a sequence in $\mathbb{R}^n$ is convergent if and only if it is a Cauchy sequence.

5. Prove the Bolzano-Weierstrass theorem in $\mathbb{R}^n$: Suppose that A is a compact set in $\mathbb{R}^n$. Every sequence in A has a subsequence that converges to a point in A.

6. A subset S of $\mathbb{R}^n$ is said to be an open set if for every $\mathbf{x} \in S$, there is an $\epsilon > 0$ (depending on $\mathbf{x}$) such that $B_\epsilon(\mathbf{x}) \subset S$. Write the negation of this statement *i.e.* When is a subset of $\mathbb{R}^n$ not an open set?

7. Show that a subset of $\mathbb{R}^n$ is a closed set if and only if its complement is an open set.

8. Consider the following subsets of $\mathbb{R}^2$. Draw a picture of these subsets in the xy-plane. Choose all the adjectives among: open, closed, bounded, unbounded, compact that apply to these sets. Prove your assertions.
   (a) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$
   (b) $\{(x, y) \in \mathbb{R}^2 : -1 \leq y < 1\}$
   (c) $\{\mathbf{y} \in \mathbb{R}^2 : \|\mathbf{y}\| > 1\}$
   (d) $\{(x, y) \in \mathbb{R}^2 : x \neq y\}$
   (e) $\{(x, y) \in \mathbb{R}^2 : x^2 = y\}$
   (f) The graph of f where $f : \mathbb{R} \to \mathbb{R}$ is the function given by $f(x) = \cos x$.

9. Consider the following subsets of $\mathbb{R}^3$. Draw a picture of these subsets in the xyz-space. Choose all the adjectives among: open, closed, bounded unbounded, compact that apply to these sets. Prove your assertions.
   (a) $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 < 1\}$
   (b) $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1\}$
   (c) $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \geq 1\}$
   (d) $\{(x, y, z) \in \mathbb{R}^3 : x \neq 0, y \neq 0, z \neq 0\}$

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10. Find the volume of a regular tetrahedron in $\mathbb{R}^3$ of side a.

11. Let $A = \{(x, y, z) \in \mathbb{R}^3 : x, y, z \in \mathbb{Q}\}$
    (a) Is the set A a closed subset of $\mathbb{R}^3$? Is it an open subset of $\mathbb{R}^3$?
    (b) A has the property that, given any point in $\mathbb{R}^3$, there exists a sequence contained completely in this set which converges to the given point. Can you think of other subsets of $\mathbb{R}^3$, which have this property? Can you give infinitely many examples of such sets? Is it possible to describe all such sets?