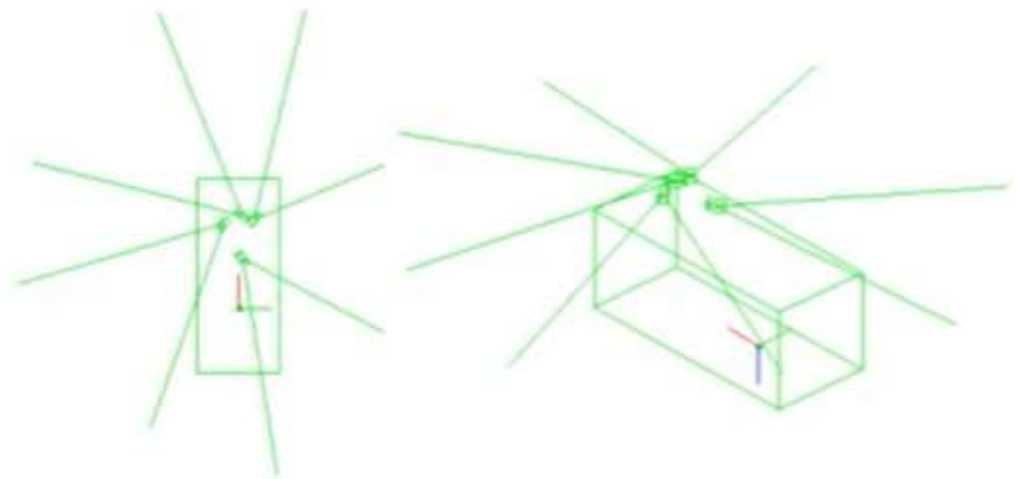


Bundle Adjustment

Some slides were adapted/taken from various sources, including 3D Computer Vision of Prof. Hee, NUS, Air Lab Summer School, The Robotic Institute, CMU, Computer Vision of Prof. Mubarak Shah, UCF, Computer Vision of Prof. William Hoff, Colorado School of Mines, Coursera Visual Odometry, Robotics: Perception, University of Pennsylvania and many more. We thankfully acknowledge them. Students are requested to use this material for their study only and **NOT** to distribute it.

Example Application for Bundle Adjustment

- A vehicle needs to map its environment that it is moving through, and estimate its self motion
- The main sensors are cameras mounted on the vehicle



Four camera configuration on vehicle

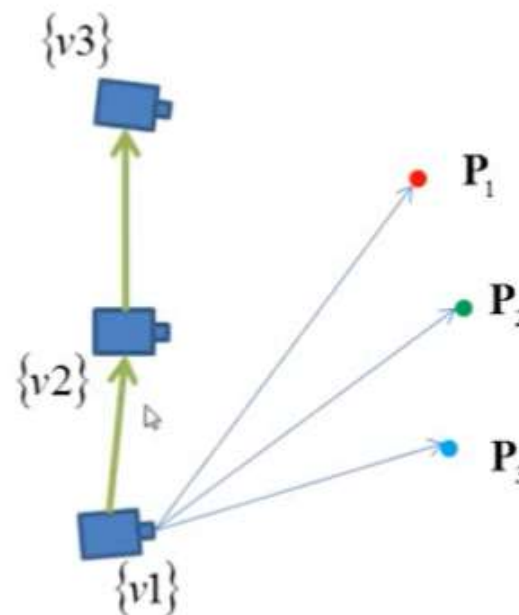
Four cameras are used for looking in the various diagonal directions.

Bundle adjustment

- The most accurate (and flexible) way to estimate structure and motion is to directly minimize the squared reprojection errors for the 2D points
- We can do this for N camera poses ($N \geq 2$)
- The unknowns are:
 - The (X,Y,Z) positions of the K 3D points
 - The relative poses of the camera in $N-1$ poses
 - The first pose is arbitrary and we can just set it to the origin
 - There are a lot of unknown parameters to find!
- We can use non-linear least squares
 - It will be slow ... probably not suitable for real-time
 - We will need a good starting guess

Structure from Motion

- Parameters to be estimated
 - $N-1$ vehicle poses w.r.t. 1st vehicle pose; each has 6 degrees of freedom
 - K 3D point locations w.r.t. 1st vehicle pose
- Measurements
 - Image point observations
$$\mathbf{p}_{ij} = (u \ v)^T, \quad i = 1..N, j = 1..K$$
 - Assumptions
 - Calibrated cameras
 - Point correspondences are known



Bundle adjustment

- We need a function that
 - Predicts the image locations of the K 3D points, in each of the N images
 - The inputs to the function are the current guess for the locations of the 3D points, and the $N-1$ poses
- As we did before, we can take the Jacobian of the function and iteratively add corrections to our guess
- Note - there are somewhat more efficient algorithms than the Newton's method that we used
 - One is called "Levenberg-Marquardt"
 - It adaptively switches between Newton's method and steepest descent
 - It is implemented in Matlab's "lsqnonlin" function
 - This is part of the optimization toolbox

Scale Factor

- Results have an unknown scale factor
 - All points, and all translations are scaled by the same unknown amount
- Possible ways to determine scale factor:
 - Take an picture of a known object in one of the images
 - Use stereo cameras
 - Use some other data source, such as odometry
- Odometry
 - Many vehicles can estimate self-motion from wheel encoders and heading sensors
 - It is fairly accurate over short distances, but can drift over longer distances

Structure from Motion

- Algorithm approach
 - Search for parameters to minimize residual errors between predictions and measurements

$$\text{minimize } E = \sum_i \left\| \underbrace{\begin{matrix} v_i \\ v_{i+1} \end{matrix} \boldsymbol{\beta}^{odom} - f\left(\begin{matrix} v_1 \\ v_{i-1} \end{matrix} \boldsymbol{\beta}, \begin{matrix} v_1 \\ v_i \end{matrix} \boldsymbol{\beta}\right)}_{\text{Predicts odometry measurement}} \right\|_{\Omega_i}^2 + \sum_{i,j} \left\| \underbrace{\mathbf{p}_{ij} - g(\mathbf{P}_j, \begin{matrix} v_1 \\ v_i \end{matrix} \boldsymbol{\beta})}_{\text{Projects point } i \text{ into image } j \text{ to predict image measurement}} \right\|_{\Xi_j}^2$$

– where

$$\|\mathbf{y}\|_{\Sigma}^2 = \mathbf{y}^T \Sigma^{-1} \mathbf{y}$$

- Nonlinear least squares problem
 - Levenberg Marquardt algorithm
 - “bundle adjust”

Minimization Algorithm

- Bundle adjust is computationally expensive
- Limit # poses, # points
- Use a sliding window of last N poses
 - Solutions are with respect to the first pose of this set
 - Effectively, mapping is done w.r.t. a moving local coordinate system



- Also, a good initial guess is needed
 - Pose guesses come from odometry
 - Point locations are initialized using triangulation

Mapping Program Control Flow

do

- Get next set of measurements (images and odometry)

- Predict pose of vehicle from odometry

- Track existing points

- Eliminate inactive points

- If predicted distance travelled is at least D meters

 - Define this pose as a “keyframe”

 - Try to acquire new points to track

 - Try to initialize 3D locations of 2D points using triangulation

 - Do bundle adjust algorithm over last 7 keyframes

while measurements are available

Thanks