Tutorial - 2 : Quantum Mechanics

To be discussed on 12 November 2018

Q1. Show that three operators, A, B and C satisfy the relation, $[AB,\ C] = A[B,C] + [A,C]B$.
Q2. Consider the position and momentum operators, X and P, and the Hamiltonian operator of a free particle, H. Find the commutators, (a) $[X,P^2]$, (b) $[H,P]$
Q3. Consider a particle in a one-dimensional box of length a, defined by the potential,

$$\begin{aligned} V(x) &= 0, \quad \text{for} \ \ (0 \leq x \leq a) \\ &= \infty, \quad \text{for} \ \ (0 > x, \quad x > a) \end{aligned}$$

The energy eigenvalues of the particle are given by $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$, where $n = 1,\ 2,\ 3,\ \cdots$.
Is $\psi_n(x)$ also eigenstate of momentum? (Compare with the case of unbound free particle.)
Obtain the momentum space wave function, $\phi_1(k)$ corresponding to the ground state ($\psi_1(x)$).

Q4. The inner product (dot product) of two functions is defined as,

$$\langle\psi|\phi\rangle = \int \psi^*(x)\phi(x)\ dx$$

The length of a function is defined as

$$||\psi|| = \sqrt{\langle\psi|\psi\rangle}$$

Prove Cauchy-Scwartz inequality, $|\langle\psi|\phi\rangle| \leq ||\psi||\ ||\phi||$.