PH 102 : Electromagnetism **Department of Physics**
Tutorial 11 **Indian Institute of Technology Guwahati**
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1. A flat wire of radius a carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w\langle a$ forms a parallel plate capacitor as shown in figure 1.
(a) Find the electric and magnetic fields in the gap as functions of the distance s from the axis and the time t.

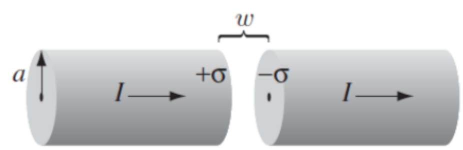

Figure 1: Figure for problem 1.

(b) Find the energy density u_{em} and the Poynting vector $\vec{S}$ in the gap. Note specially the direction of $\vec{S}$. Check that $\partial u_{\text{em}}/\partial t = -\vec{\nabla}\cdot\vec{S}$ is satisfied.
(c) Determine the total energy in the gap, as a function of time. Calculate the total power flowing into the gap, by integrating the Poynting vector over the appropriate surface. Check that the power input is equal to the rate of increase of energy in the gap (Poynting's Theorem-in this case $W = 0$, because there is no charge in the gap).

2. A parallel plate capacitor is made up of two circular discs of diameter d each, spaced a distance h apart. A potential difference of V is applied between the plates and the spacing between the plates is increased at a uniform rate to $2h$ in one second. Find the induced magnetic field and the Poynting vector at the edge of the capacitor plates as the plates are separated. Use the Poynting's theorem to correlate the change of stored energy to the energy loss as the plates are being separated. (Ignore the effects due to edges of the plates)

3. The linearly polarised wave is denoted by $\vec{\tilde{f}}(z,t) = \tilde{A}e^{i(kz-\omega t)}\hat{n}$. Linear polarisation results from the combination of horizontally and vertically polarised waves of the same phase. If the two components are of equal amplitude, but out of phase by $\pi/2$ (say, $\delta_v = 0, \delta_h = \pi/2$), the result is a circularly polarised wave. In that case:
(a) At a fixed point z, show that the string moves in a circle about the z axis. Does it go clockwise (right circular polarised) or counterclockwise (left circular polarised), as you look down the axis toward the origin? How would you construct a wave circling the other way?
(b) Sketch the string at time $t = 0$.
(c) How would you shake the string in order to produce a circularly polarised wave?

4. The simplest possible spherical wave can be represented by

$$E(r,\theta,\phi,t) = A\frac{\sin\theta}{r}\left[\cos\left(kr-\omega t\right) - \frac{1}{kr}\sin\left(kr-\omega t\right)\right]\hat{\phi}, \quad \frac{\omega}{k} = c.$$

(a) Show that $\vec{E}$ obeys all four of Maxwell's equations, in vacuum, and find the associated magnetic field.
(b) Calculate the Poynting vector. Average $\vec{S}$ over a full cycle to get the intensity vector $\vec{I}$. Does it point in the expected direction? Does it fall off like r^{-2}, as it should?
(c) Integrate $\vec{I}\cdot d\vec{a}$ over a spherical surface to determine the total power radiated.

1 Take Home Problems

1. In complex notation, we use the Complex wave function which, as discussed in the class, is given by $\tilde{f}(z,t) = \tilde{A}e^{i(kz-\omega t)}$ with $\tilde{A} = Ae^{i\delta}$ being the complex amplitude. Use the method of separation of variables to solve the wave equation and to show that any wave can be expressed as a linear combination of sinusoidal waves:

$$\tilde{f}(z,t) = \int_{-\infty}^{\infty} \tilde{A}(k)e^{i(kz-\omega t)}dk.$$

2. Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and phase angle zero that is (a) travelling in the negative x direction and polarised in the z direction; (b) travelling in the direction from the origin to the point $(1,1,1)$, with polarisation parallel to the $x-z$ plane. In each case, sketch the wave, and give the explicit Cartesian components of $\vec{k}$ and $\hat{n}$.