PH 102 : Electromagnetism **Department of Physics**
Tutorial 9 **Indian Institute of Technology Guwahati**
26/03/19

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1. A thin uniform donut, carrying charge Q and mass M, rotates about its axis as shown in the figure 1.
(a) Find the gyromagnetic ratio (g), i.e. the ratio of its magnetic dipole moment to its angular momentum.
(b) What is the gyromagnetic ratio a uniform spinning sphere of total charge Q and mass M?
(c) According to quantum mechanics, the angular momentum of a spinning electron is $\frac{\hbar}{2}$, where $\hbar$ is Planck's constant. What, then, is the electron's magnetic dipole moment (in units of $A.m^2$)?

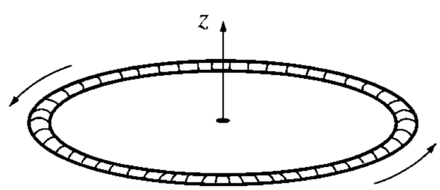

Figure 1: Figure for problem 1.

2. Find the magnetic dipole moment of a spherical shell, of radius R, carrying a uniform surface charge σ which is set to spin at angular velocity $\vec{\omega}$. Show that for points $r > R$, the vector potential is same as that of a perfect dipole. Hint: The vector potential for $r > R$ is:
$$\vec{A}(\vec{r}) = \frac{\mu_0 R^4 \sigma}{3r^3}(\vec{\omega} \times \vec{r})$$

3. A uniform current density $\vec{J} = J_0\hat{z}$ fills a slab straddling the yz plane as shown in figure 2, from $x = -a$ to $x = +a$. A magnetic dipole $\vec{m} = m_0\hat{x}$ is situated at the origin.
(a) Find the force on the dipole.
(b) Do the same for a dipole pointing in the y direction: $\vec{m} = m_0\hat{y}$.
(c) In the *electrostatic case*, the expressions $\vec{F} = \vec{\nabla}(\vec{p}.\vec{E})$ and $\vec{F} = (\vec{p}.\vec{\nabla})\vec{E}$ are equivalent (prove it), but this is not the case for the magnetic analogues (explain why). As an example, calculate $(\vec{m}.\vec{\nabla})\vec{B}$ for the configurations in (a) and (b).

4. An iron rod of length L and square cross section (side a), is given a uniform longitudinal magnetisation $\vec{M}$, and then bent around into a circle with a narrow gap (width w), as shown in figure 3. Find the magnetic field at the centre of the gap, assuming $w\langle a\langle L$.

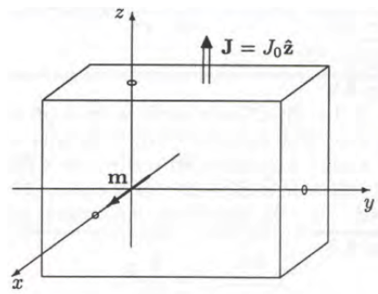

Figure 2: Figure for problem 3.

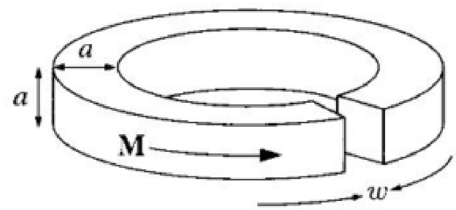

Figure 3: Figure for problem 4.

5. Consider the following similarities between electrostatics and magnetostatics (in matter):

$$\vec{\nabla}\cdot\vec{D}=0,\ \ \vec{\nabla}\times\vec{E}=0,\ \ \epsilon_0\vec{E}=\vec{D}-\vec{P}$$

$$\vec{\nabla}\cdot\vec{B}=0,\ \ \vec{\nabla}\times\vec{H}=0,\ \ \mu_0\vec{H}=\vec{B}-\mu_0\vec{M}$$

Thus, the transcription $\vec{D}\to\vec{B},\vec{E}\to\vec{H},\vec{P}\to\mu_0\vec{M},\epsilon_0\to\mu_0$ turns an electrostatic problem into an analogous magnetostatic one. Use this observation, together with the analogous electrostatic results (namely, (i) electric field inside a uniformly polarised sphere $\vec{E}=-\frac{\vec{P}}{3\epsilon_0}$, (ii) electric field inside a sphere of linear dielectric in an otherwise uniform electric field E_0 is $\vec{E}=\frac{3}{\epsilon_r+2}\vec{E}_0=\frac{1}{1+\frac{\chi_e}{3}}\vec{E}_0$) to rederive

(a) the magnetic field inside a uniformly magnetised sphere.

(b) the magnetic field inside a sphere of linear magnetic material in an otherwise uniform magnetic field.

(c) the average magnetic field over a sphere, due to steady currents within the sphere.

6. At the interface between one linear magnetic material and another, the magnetic field lines bend as shown in figure 4. Assuming there is no free current at the boundary, show that $\tan\theta_2/\tan\theta_1=\mu_2/\mu_1$.

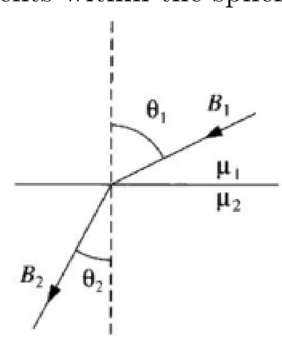

Figure 4: Figure for problem 6.

1 Take home problems

1. A uniformly charged solid sphere of radius R carries a total charge Q, and is set spinning with angular velocity ω about the z axis.
(a) What is the magnetic dipole moment of the sphere?
(b) Find the magnetic field at a point (r, θ) inside the sphere.
(c) Using the results of (b) find the average magnetic field within the sphere. Hint: Average magnetic field is defined as

$$\vec{B}_{\text{avg}} = \frac{1}{\frac{4}{3}\pi R^3} \int \vec{B} d\tau$$

Compare this result with the result of (a) and show that the average magnetic field is related to the magnetic dipole moment as

$$\vec{B}_{\text{avg}} = \frac{\mu_0}{4\pi} \frac{2\vec{m}}{R^3}$$

2. A thin glass rod of radius R and length L carries a uniform charge σ. It is spinning about its axis, at an angular velocity ω. Find the magnetic field at a distance $s \gg R$ from the center of the rod (see figure 5).

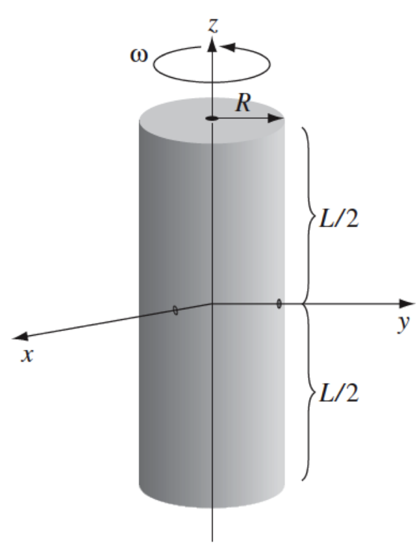

Figure 5: Figure for take home problem 2.

3. Suppose the field inside a large piece of magnetic material is $\vec{B}_0$, so that $\vec{H}_0 = \vec{B}_0/\mu_0 - \vec{M}$.
(a) Now a small spherical cavity is hollowed out of the material (as shown in figure 6).

Find the field at the centre of the cavity, in terms of $\vec{B_0}, \vec{M}$. Also find $\vec{H}$ at the centre of the cavity in terms of $\vec{H_0}, \vec{M}$.
(b) Do the same for a long needle-shaped cavity running parallel to $\vec{M}$.
(c) Do the same for a thin wafer-shaped cavity perpendicular to $\vec{M}$.

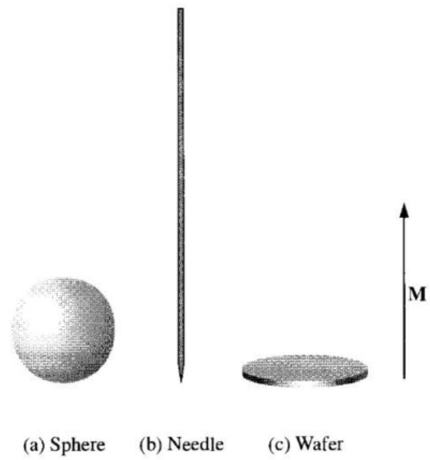

Figure 6: Figure for take home problem 3.

4. Given that $\vec{H_1} = -2\hat{i} + 6\hat{j} + 4\hat{k}$ A/m in the region $y - x - 2 \leq 0$, where $\mu_1 = 5\mu_0$. Calculate
(a) $\vec{M_1}$ and $\vec{B_1}$.
(b) $\vec{M_2}$ and $\vec{B_2}$ in the region $y - x - 2 \geq 0$, where $\mu_2 = 2\mu_0$.

5. A short circular cylinder of radius a and length L carries a "frozen-in" uniform magnetisation $\vec{M}$ parallel to its axis. Find the bound current and sketch the magnetic field of the cylinder: one for $L \gg a$, one for $L\langle a$ and one for $L \approx a$.