Tutorial-9 Solutions

1. Fig. Q1(a) shows the connection of two wattmeters with the 3-phase load. The current I_A flows through the current coil of W_1 and V_{AC} is the voltage sensed by the voltage coil. Wattmeter W_1's reading will be the product of the voltage across its voltage coil (V_{AC}), the current through its current coil I_A and the cosine of the angle between V_{AC} and I_A.

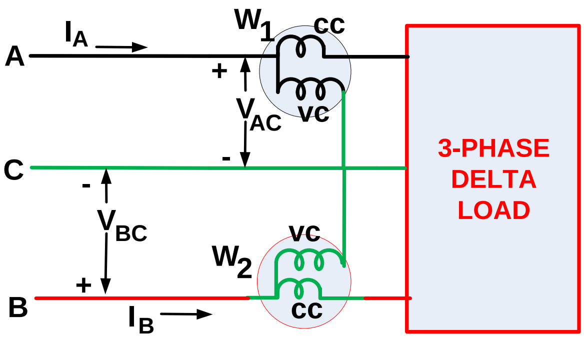

Fig. Q1(a)

From Fig. Q1(b), the reading of wattmeter, W_1, will be

$$W_1 = V_{AC} I_A \cos(\varphi)$$

where, φ is the angle between V_{AC} and I_A. V_{AB} is the reference voltage and the corresponding phase current I_{AB} lags the phase voltages by an angle θ. In case of a delta connection, the line current lags the phase current by an angle of 30^0. I_A lags I_{AB} by an angle of 30^0. From Fig. Q1(b), it is seen that the angle between V_{AC} ($-V_{CA}$) and V_{AB} is 60^0. Hence, the angle φ will be

$$\varphi = (60^0 - 30^0 - \theta)$$

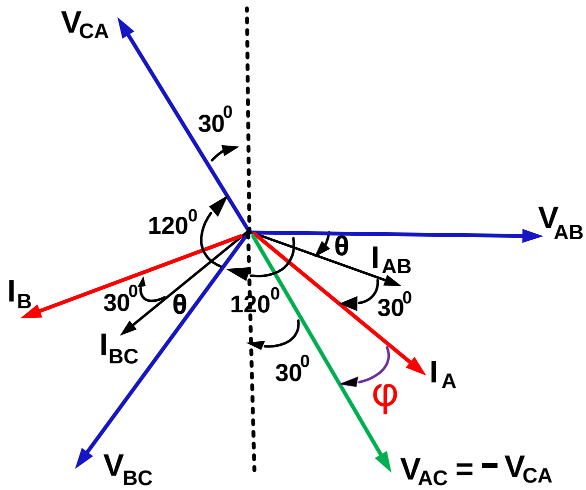

Fig. Q1(b)

$$W_1 = V_{AC} I_A \cos(\varphi) = V_{AC} I_A \cos(60^0 - 30^0 - \theta) = V_L I_L \cos(30^0 - \theta)$$

Similarly, I_B flows through the current coil of $\mathbf{W_2}$ and V_{BC} is the voltage across its voltage coil. The wattmeter reading $\boldsymbol{W_2}$ will be-

$$W_2 = V_{BC}I_B\cos(30^0 + \theta) = V_L I_L \cos(30^0 + \theta)$$

$$W_1 + W_2 = V_L I_L\{\cos(30^0 - \theta) + \cos(30^0 + \theta)\}$$
$$= V_L I_L\{2\cos 30^0 . \cos\theta\}$$
$$= V_L I_L\left\{2 \times \frac{\sqrt{3}}{2} \cdot \cos\theta\right\}$$
$$= \sqrt{3} V_L I_L \cos\theta = P = \ Total\ Power$$

$$W_1 - W_2 = V_L I_L sin\theta \qquad \tan\theta = \sqrt{3}(\frac{W_1 - W_2}{W_1 + W_2})$$

$$\textbf{Power Factor} = \cos[\tan^{-1}\{\sqrt{3}(\frac{W_1 - W_2}{W_1 + W_2})\}]$$

2. Real power consumed by the load = $(W_1 + W_2) = 20\ kW$

Reactive power consumed by the 3-phase load = $\sqrt{3} x V_L x I_L \sin\theta$
$$= \sqrt{3} \times (W_1 - W_2) = 17.32\ kVAR$$

(a) Conjugate of phase current = $I_{ph}^* = \frac{Complex\ Power}{3V_{ph}} = \frac{(20 + j17.32)\times 10^3}{3\times 440} = 20.04\angle 40.89^0$

Line current = $I_L = \sqrt{3} \times 20.04\angle(-40.89^0 - 30^0) = 34.71\angle - 70.89^0$

(b) Power factor = $\cos\left(\tan^{-1}\left(\frac{\sqrt{3}\,(W_1 - W_2)}{\mathrm{W1 + W2}}\right)\right) = 0.7559$ **lagging**

(c) Load resistance per phase = $R_{ph} = \frac{real\ power\ consumed}{|I_L|^2} = \frac{20\ kW}{34.71^2} = 16.6\ \Omega$

(d) Load reactance per phase = $X_{ph} = \frac{reactive\ power\ consumed}{|I_L|^2} = \frac{17.32\ kW}{34.71^2} = 14.376\ \Omega$

Note: In this question load is given as delta connected load. If the load would have been a star connected load, than **impedance** = $Z = \frac{power\ consumed}{3\times|I_L|^2}$

3. The equivalent circuit drawn as shown in Fig.Q3(b) using the approach described earlier.

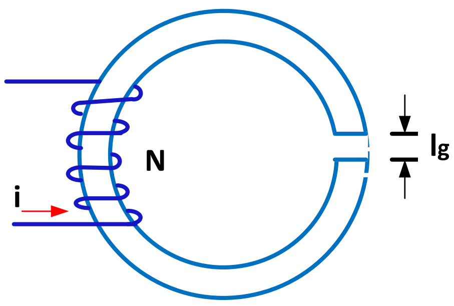

Fig. Q3(a)

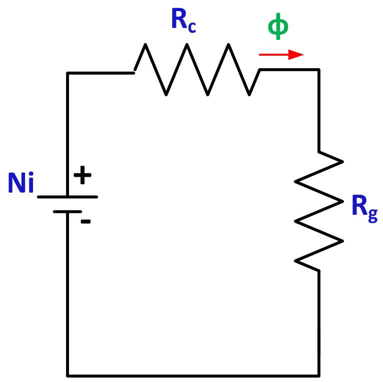

Fig. Q3(b)

Cross-sectional area of the core and the air-gap are-

$$A_c = A_g = 2 \times 2\ cm^2 = 4 \times 10^{-4}\ m^2$$

The mean radius $\mathbf{r} = (10 + 12)/2 = 11\ \mathrm{cm}$

The length of the core $l_c = 2\pi\mathrm{r} - 1 = 68.12\ \mathrm{cm}$

The length of the air-gap $l_g = 1\ cm.$ The reluctances of the core and the air-gap are

$$\Re_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{0.6812}{1200 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 1.129 \times 10^6\ H^{-1}$$

$$\Re_g = \frac{l_g}{\mu_0 A_g} = \frac{0.01}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 19.894 \times 10^6 H^{-1}$$

$$\Re_{eq} = \Re_g + \Re_c = 21.023 \times 10^6\ H^{-1}$$

The flux = $\mathbf{\Phi} = \frac{1300 \times 5}{21.023 \times 10^6} = 0.309\ \mathrm{mWb}$

The flux density = $B = \frac{0.309 \times 10^{-3}}{4 \times 10-4} = 0.7725\ \mathrm{T}$

4.

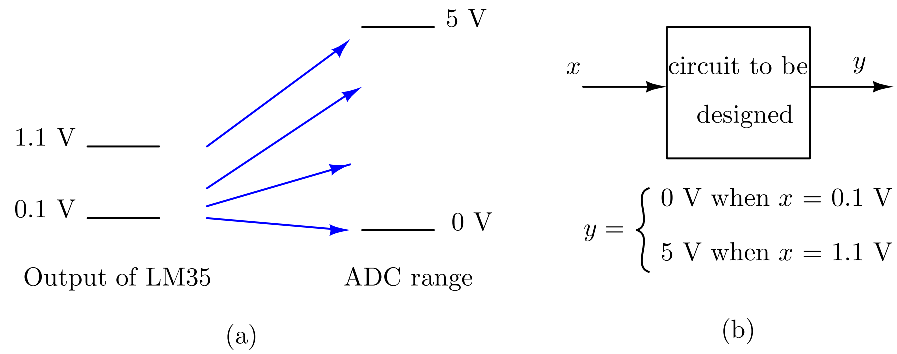

Figure 1: (a) Voltage mapping, and (b) the circuit transfer function.

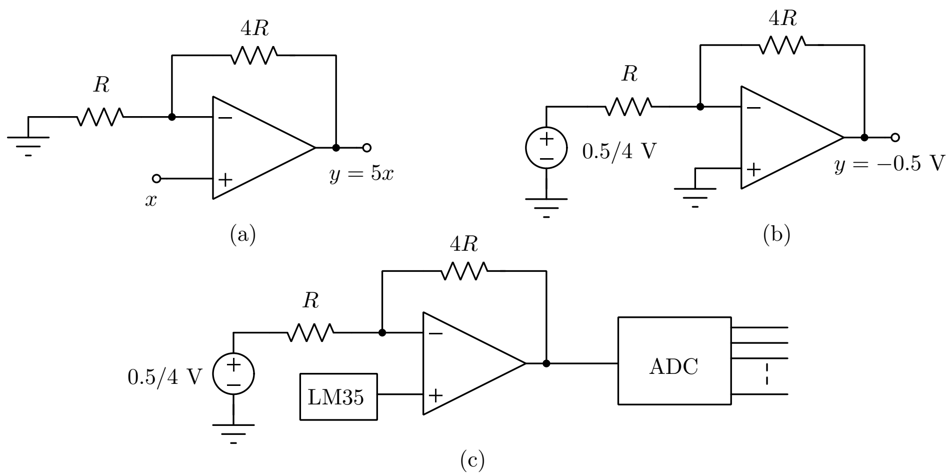

Figure 2: (a) A non-inverting amplifier with a gain 5, (b) a DC offset introducer, and (c) the circuit with the required transfer characteristics.

Design steps:

1. Fig. 1(a) shows the voltage mapping that needs to be realized with a circuit. Fig. 1(b) shows the circuit to be designed. Let the input to the circuit be x and the output be y.

2. Even though any one-to-one mapping function is a valid solution, a mapping function $y = mx + c$ is easy to realize.

3. By solving the below equations, we get $m = 5$ and $c = -0.5$.

$$0 = 0.1m + c \text{ and } 5 = 1.1m + c$$

4. Understand the transfer function: We need to amplify the input x by $m = +5$ and subtract a fixed offset voltage $c = 0.5$ V from the amplified output.

5. A non-inverting amplifier with the component values shown in Fig. 2(a) can be used to implement $y = 5x$. Any resistance with a practically available value can be used for R.

6. The constant value -0.5 V can be implemented by the same circuit and is shown in Fig. 2(b). Here we have assumed the availability of a constant voltage $\frac{0.5}{4}$ V. In reality, one should realize this with the given battery voltage.

7. Final circuit with the transfer characteristics $y = 5x - 0.5$ is shown in Fig. 2(c).