1. Consider the rectangular co-ordinates in terms of spherical coordinates as $x = r\cos\theta\sin\phi, y = r\sin\theta\sin\phi, z = r\cos\phi$. Consider the function of the rectangular co-ordinates, $u = x^2 + y^2 + z^2$. Find $\frac{\partial u}{\partial r}$, $\frac{\partial u}{\partial \theta}$ and $\frac{\partial u}{\partial \phi}$.

2. For $t \in \mathbb{R}$, let $g(t) = e^t$ and $h(t) = e^{-t}$. For $(x, y) \in \mathbb{R}^2$, define $f(x, y) = \frac{x}{y}$. Find the tangent vector to the curve $\mathbf{c}(t) = (g(t), h(t), f(g(t), h(t)))$ on the surface $z = f(x, y)$ at the point $t = 1$.

3. Suppose that $x = t^2 - s^2, y = ts$; where $t, s \in \mathbb{R}$. Let $u = x^2 + y^2, v = -xy$.
   (a) Compute the derivative matrices for u, v, x and y.
   (b) Express the ordered pair (u, v) in terms of t and s.
   (c) Let $f(t, s) = (u(t, s), v(t, s))$. Calculate Df at a point (t, s) and verify that the chain rule holds.

4. Suppose that the temperature around a point (x, y, z) in space is $T(x, y, z) = x^2 + y^2 + z^2$. Let a particle follow the right circular helix $\sigma(t) = (\cos t, \sin t, t)$ and let $T(t)$ be its temperature at time t.
   (a) What is $T'(t)$?
   (b) Find an approximate value for the temperature at $t = \frac{\pi}{2} + 0.01$.
   (c) Compute the composition function and verify the chain rule for the composition.

5. Find the gradient vector to $f(x, y) = \frac{x^2}{8} - \frac{y^2}{12}$ at the point $(\pi, 2\pi)$.

6. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by:

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & \text{if} \quad (x, y) \neq (0, 0) \\ 0, & \text{if} \quad (x, y) = (0, 0) \end{cases}$$

   (a) Show that the directional derivatives of f at $(0, 0)$ exist along every direction.
   (b) Show that f is not continuous at $(0, 0)$.

7. Let $f : A \subset \mathbb{R}^2 \to \mathbb{R}$ be differentiable at $(a_1, a_2) \in A$. Let $f(a_1, a_2) = c$. Further let $(\alpha(t), \beta(t))$ for $t \in \mathbb{R}$ be a regular parametrization of the level curve $f(x, y) = c$. Assuming that $\nabla f(a_1, a_2) \neq (0, 0)$, show that the gradient of f is normal to the level curve $f(x, y) = c$ at the point $(x, y) = (a_1, a_2)$.

8. Let $f(x, y, z) = x^2 + y^2 + z^2$ for $(x, y, z) \in \mathbb{R}^3$. Show that the gradient of f at $(3, 1, 5)$ is normal to the tangent plane to the level surface $f(x, y, z) = 35$.