Three-phase System with Delta Load

For a Y-connected load the line voltage is √3 times the phase voltage and line current is same as the phase current. In a delta connected system, the loads are connected back to back as shown in Fig. 1.

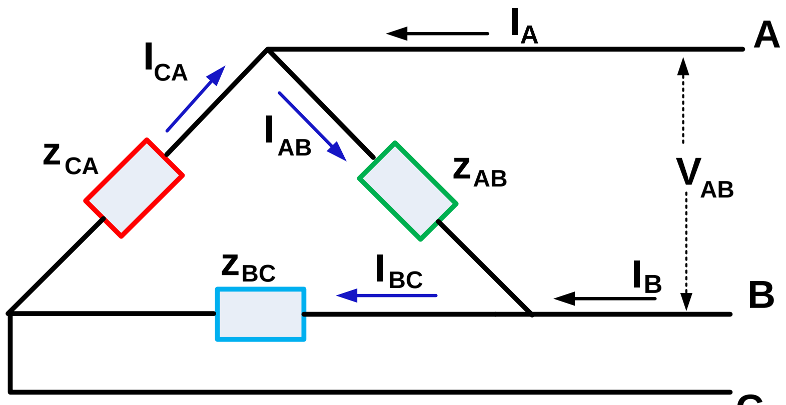

Fig. 1

The line voltages and the phase voltages are same. They are not distinguishable. The relation between the line and the phase voltages is

$$|V_{AB}| = |V_{BC}| = |V_{CA}| = |V_L| = |V_p|$$

The line voltages or the phase voltages in the delta connection are equal in magnitude and they differ from each other by an angle of 120^0. If we consider a balanced three phase delta load, i.e., $Z_{AB} = Z_{BC} = Z_{CA} = Z_P$ and V_{AB} as the reference voltage ($V_{AB} = V_P\angle 0^0$), the phase currents will be

$$I_{AB} = \frac{V_{AB}}{Z_P} = \frac{V_P\angle 0^0}{Z_P} = I_P\angle 0^0,\ I_{BC} = I_P\angle -120^0 \text{ and } I_{CA} = I_P\angle -240^0$$

The line currents can be found as follows:

$$I_A = I_{AB} - I_{CA}$$

$$= I_p\angle 0^o - I_p\angle -240^\circ$$

$$= I_p\sin\omega t - I_p\sin(\omega t - 240^0)$$

$$= I_p[\sin\omega t - (\sin\omega t.cos240^0 - \cos\omega t \cdot sin240^0)$$

$$= I_p\left[\sin\omega t + \frac{1}{2}\sin\omega t - \frac{\sqrt{3}}{2}\cos\omega t\right]$$

$$= \sqrt{3}I_P\left[\frac{\sqrt{3}}{2}\sin\omega t - \frac{1}{2}cos\omega t\right] = \sqrt{3}I_p sin(\omega t - 30^0)$$

Hence, $\boldsymbol{I_A = \sqrt{3}I_P\angle -30^0}$, $\boldsymbol{I_B = \sqrt{3}I_P\angle -150^0}$ and $\boldsymbol{I_C = \sqrt{3}I_P\angle -270^0}$

In delta connection, the phase voltage and the line voltage are same. The magnitude of the line current is √3 times the magnitude of the phase current. One can convert a delta connected load to a star connected load by using star-delta transformation.

Example: A balanced three-phase three-wire system has a line voltage of 500 V. Two balanced Y-connected loads are present. One is a capacitive load with 7-j2 per phase and the other is an inductive load of 4+j2 a per phase. Find (a) the phase voltage, (b) the line current, (c) the total power drawn by the load. (d) the power factor at which the source is operating.

Solution:

(a) As this is a Y-connected system, the phase voltage will be

$$\boldsymbol{v_p = \frac{V_L}{\sqrt{3}} = \frac{500}{\sqrt{3}}V = 288.67V}$$

(b) Two loads are connected in parallel. The per phase load $\mathbf{Z_P}$ can be estimated as

$$\boldsymbol{Z_1 = 7 - j2 \qquad Z_2 = 4 + j2}$$

$$\boldsymbol{Z_P = \frac{Z_1Z_2}{Z_1 + Z_2} = \frac{32 + j6}{11}}$$

$$\boldsymbol{I_P = I_L = \frac{V_P}{Z_P} = 97.53L - 10 \cdot 6^0}$$

(c) Total power is

$$\boldsymbol{P = 3V_pI_p\cos\theta = 3 \times 288 \cdot 67 \times 97 \cdot 53 \cos(-10.6^0)}$$

$$\mathbf{= 83\ KW}$$

(d) The source power factor is

$$\boldsymbol{P \cdot F. = cos\theta = cos(-10 \cdot 6^0)}$$

$$\mathbf{= 0 \cdot 983\ lagging}$$

In part (a) and part (b) of the above question, the magnitudes of voltage and current were asked. This question can be modified by asking the phasor voltage, $\boldsymbol{V_{BN}}$ and the phasor current $\boldsymbol{I_B}$. Given line voltage is $\boldsymbol{V_{AB}}$ is equal to 500 v.

Solution: The magnitude of the voltages and currents will be same as found in the previous case. The difference will be the extra information which is the phase angle of the voltage and the current. In this question $\boldsymbol{V_{AB}}$ is the reference phasor as its angle is zero. The phase voltage V_{BN} will lag the line voltage by an angle of 150 degree. Similarly, the line current I_B will lag the line voltage V_{AB} by an angle of 160.6 degree. Hence

$$V_{BN} = 288 \cdot 67\angle -150^0 V$$
$$I_B = 97 \cdot 53\angle -160.6^0 A$$

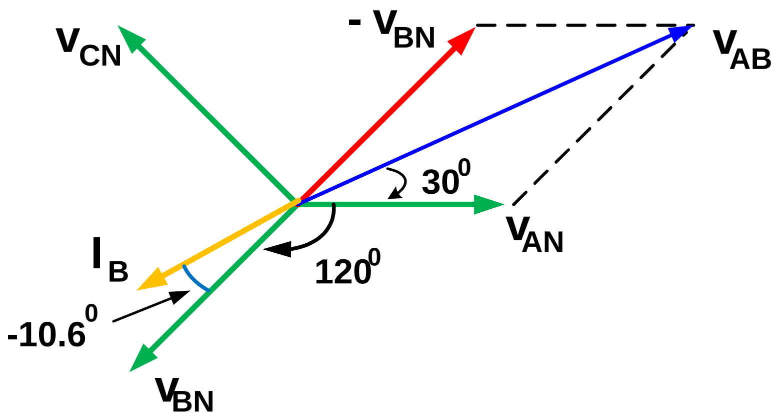

Fig. 2

Three-Phase Power Measurement

Wattmeter is the instrument used for power measurement. Fig. 3 shows a wattmeter (W) and its connection with a single-phase load. It has two coils, one is called as current coil (cc) and the other is the voltage coil (vc). The current coil is connected in series and the voltage coil is c onnected across the load or supply whose power is intended to be measured.

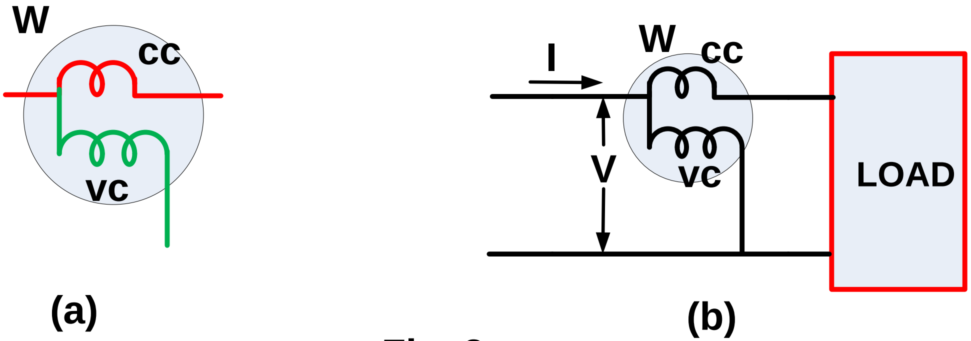

Fig. 3

For a three-phase system three wattmeters are required. Three-phase power can also be measured using two wattmeters. In addition to real power, the power factor and the reactive power can be estimated from the readings of the two wattmeters. Two wattmeter method for measurement of three-phase power is depicted in Fig. 4.

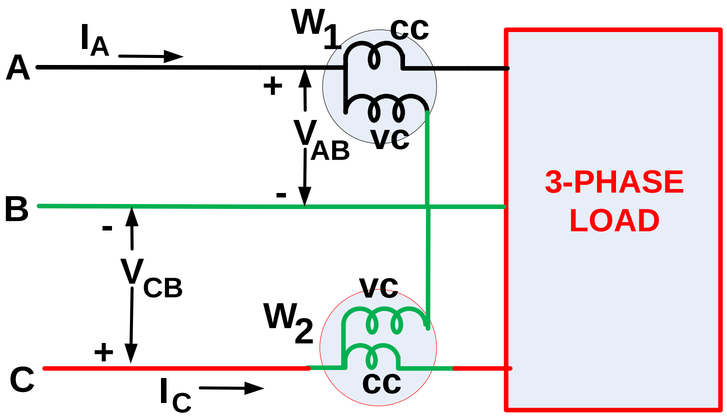

Fig. 4

The current I_A flows through the current coil of W_1 and V_{AB} is the voltage sensed by the voltage coil of W_1. Similarly, I_C flows through the current coil of W_2 and V_{CB} is the voltage across its voltage coil. From Fig.12, one can see

$$W_1 = V_{AB}I_A\cos(30^0+\theta) = V_LI_L\cos(30^0+\theta)$$
$$W_2 = V_{CB}I_c\cos(30^0-\theta) = V_LI_L\cos(30^0-\theta)$$
$$W_1+W_2 = V_LI_L\{\cos(30^0+\theta)+\cos(30^0-\theta)\}$$
$$= V_LI_L\{2\cos 30^0.\cos\theta\}$$
$$= V_LI_L\left\{2\times\frac{\sqrt{3}}{2}\cdot\cos\theta\right\}$$
$$= \sqrt{3}V_LI_L\cos\theta = P = \mathit{Total\ Power}$$

$$W_2 - W_1 = V_LI_L sin\theta \qquad \tan\theta = \sqrt{3}\left(\frac{W_2-W_1}{W_1+W_2}\right)$$

Power Factor = $\cos\left[\tan^{-1}\left\{\sqrt{3}\left(\frac{W_2-W_1}{W_1+W_2}\right)\right\}\right]$

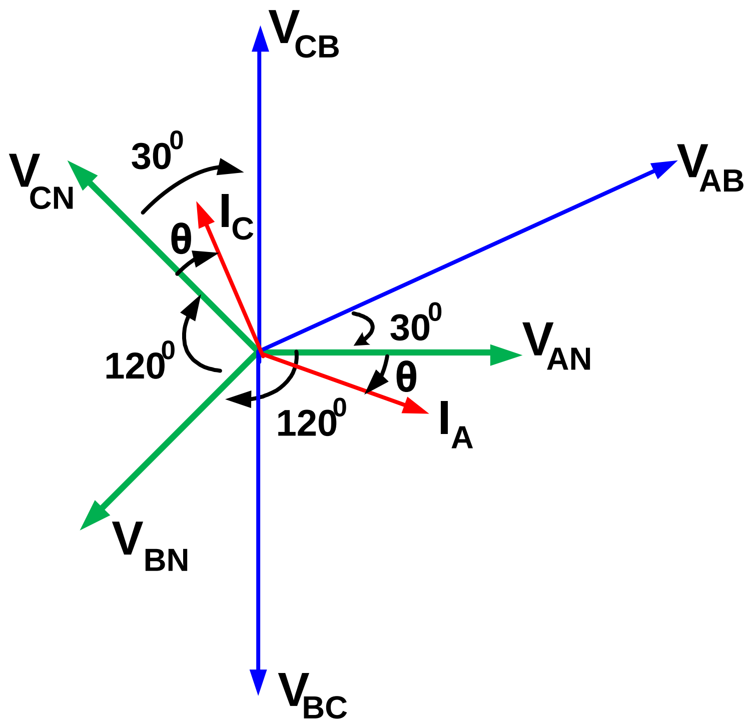

Fig. 5