Tutorial - 3 : Special Relativity

Q.1 **[Not to be discussed in tutorial but important to do at home]**

Show that these quantities are unchanged by Lorentz transformations [boost along x-direction only] (such quantities are called Lorentz invariants):

$$ds^2 \equiv c^2 dt^2 - dx^2 - dy^2 - dz^2; \quad E^2 - c^2 p^2; \quad x^2 - c^2 t^2 \tag{1}$$

A Lorentz 4-vector denoted by (v^0, v^1, v^2, v^3) is defined as any quantity that transforms like (ct, x, y, z) under Lorentz transformations (here the μ in the notation v^μ is just a label it does **NOT** mean "raised to the power"). Since we know that

$$ct^{'} = \gamma(ct - \frac{v}{c}x);\ x^{'} = \gamma(x - \frac{v}{c}ct);\ y^{'} = y;\ z^{'} = z \tag{2}$$

this means a Lorentz 4-vector would transform as,

$$v^{0'} = \gamma(v^0 - \frac{v}{c}v^1);\ v^{1'} = \gamma(v^1 - \frac{v}{c}v^0);\ v^{2'} = v^2;\ v^{3'} = v^3 \tag{3}$$

We have shown in the earlier tutorial that $(\frac{E}{c}, p_x, p_y, p_z) \equiv (p^0, p^1, p^2, p^3)$ is a Lorentz 4-vector. Show that (f^0, f^1, f^2, f^3) is also a Lorentz 4-vector where,

$$f^\mu \equiv \frac{dp^\mu}{d\tau};\ d\tau \equiv \frac{ds}{c} \tag{4}$$

here ds is the distance between events for an assumed time-like separation (ie. $ds^2 > 0$ where ds^2 is defined as $ds^2 \equiv c^2 dt^2 - dx^2 - dy^2 - dz^2$). Also $d\tau$ is called the **proper time** difference between the two events (ct, x, y, z) and $(ct + cdt, x + dx, y + dy, z + dz)$. The proper time difference is also the actual time difference between events that take place at the same point in space (ie. $d\tau \equiv dt$ if $dx = dy = dz = 0$). It is always possible to construct Lorentz invariants from 4-vectors. If (v^0, v^1, v^2, v^3) is a 4-vector, show that $(v^0)^2 - (v^1)^2 - (v^2)^2 - (v^3)^2$ is always a Lorentz invariant (also called a Lorentz scalar). This means the following is a Lorentz invariant.

$$|f|^2 \equiv (f^0)^2 - (f^1)^2 - (f^2)^2 - (f^3)^2 \tag{5}$$

Relate (f^0, f^1, f^2, f^3) to $(\frac{1}{c}\frac{dE}{dt}, \frac{dp_x}{dt}, \frac{dp_y}{dt}, \frac{dp_z}{dt})$ and $(\frac{E}{c}, p_x, p_y, p_z)$.

Q.2 Even in special relativity we may continue to define force as the rate of change of momentum vector.

$$\mathbf{f} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(\gamma_u \; m\mathbf{u}) \tag{6}$$

Show that the rate of change of energy of the particle may be written as,

$$\mathbf{f} \cdot \mathbf{u} = \frac{d}{dt}(\gamma_u \; mc^2) \equiv \frac{dE}{dt} \tag{7}$$

Ans. We showed in the lecture ppt that,

$$\mathbf{u} = \frac{c^2 \mathbf{p}}{E} \tag{8}$$

hence,

$$\mathbf{f} \cdot \mathbf{u} = \mathbf{u} \cdot \frac{d\mathbf{p}}{dt} = \frac{c^2 \mathbf{p}}{E} \cdot \frac{d\mathbf{p}}{dt} = \frac{1}{E} \frac{d}{dt} \frac{c^2 p^2}{2} \tag{9}$$

But $c^2 p^2 = E^2 - (mc^2)^2$. Hence,

$$\mathbf{f} \cdot \mathbf{u} = \frac{1}{E} \frac{d}{dt} \frac{c^2 p^2}{2} = \frac{1}{E} \frac{d}{dt} \frac{E^2 - (mc^2)^2}{2} = \frac{dE}{dt} \tag{10}$$

Q.3 In presence of a uniform electric field, a charged particle experiences an electric force $q\vec{\mathcal{E}}$ where q is the charge and $\vec{\mathcal{E}}$ is the electric field. Also in presence of a uniform magnetic field, the magnetic force acting is $q\frac{\mathbf{u}}{c} \times \vec{\mathcal{B}}$ where $\vec{\mathcal{B}}$ is the magnetic field (in CGS units) and $\mathbf{u}$ is the velocity of the particle. This means Newton's Second Law becomes (called the Lorentz force equation):

$$\frac{d\mathbf{p}}{dt} = q\vec{\mathcal{E}} + q\frac{\mathbf{u}}{c} \times \vec{\mathcal{B}} \tag{11}$$

where $\mathbf{p} = \gamma_u \; m\mathbf{u}$ is the relativistic momentum of the particle. You know how $d\mathbf{p}$ transforms under Lorentz transformations (boost along x-direction only) and how dt transforms. Assume that the form of the above Lorentz force equation remains unchanged in reference frame $S^{'}$. This is expected since it is the incompatibility of electromagnetic theory with Galilean relativity that gave birth to Einstein's Special Relativity. This means,

$$\frac{d\mathbf{p}^{'}}{dt^{'}} = q\vec{\mathcal{E}}^{'} + q\frac{\mathbf{u}^{'}}{c} \times \vec{\mathcal{B}}^{'} \tag{12}$$

Combine Eq.(11) an Eq.(12) and the usual relations between $dt^{'}$ and dt and between $d\mathbf{p}^{'}$ and $d\mathbf{p}$ (and energy dE) to show that the relation between the electric and magnetic fields in the reference frame $S^{'}$ viz. $\vec{\mathcal{E}}^{'}$ and $\vec{\mathcal{B}}^{'}$ in terms of the original electric and magnetic fields in S viz. $\vec{\mathcal{E}}$ and $\vec{\mathcal{B}}$ and the boost velocity v is,

$$\mathcal{E}_x^{'} = \mathcal{E}_x, \quad \mathcal{E}_y^{'} = \gamma_v(\mathcal{E}_y - \frac{v}{c}\mathcal{B}_z), \quad \mathcal{E}_z^{'} = \gamma_v(\mathcal{E}_z + \frac{v}{c}\mathcal{B}_y) \tag{13}$$

$$\mathcal{B}_x^{'} = \mathcal{B}_x, \quad \mathcal{B}_y^{'} = \gamma_v(\mathcal{B}_y + \frac{v}{c}\mathcal{E}_z), \quad \mathcal{B}_z^{'} = \gamma_v(\mathcal{B}_z - \frac{v}{c}\mathcal{E}_y) \tag{14}$$

Show that $\vec{\mathcal{E}} \cdot \vec{\mathcal{B}}$ and $\vec{\mathcal{E}}^2 - \vec{\mathcal{B}}^2$ are invariant under Lorentz transformations.

A uniform electric field produces a force of 10^6 dynes in the y-direction on a charge momentarily at rest in reference frame S and there are no magnetic fields in reference frame S. What is the magnetic force on this charge as seen from reference frame $S^{'}$ moving with speed $v = \frac{2}{3}c$ relative to S in the x-direction?

Ans.

$$dp_x^{'} = \gamma_v(dp_x - \frac{v}{c^2}dE); \quad dp_y^{'} = dp_y; \quad dp_z^{'} = dp_z \tag{15}$$

$$dt^{'} = \gamma_v(dt - \frac{v}{c^2}dx) \tag{16}$$

Dividing one by the other,

$$\frac{dp_x^{'}}{dt^{'}} = \frac{(\frac{dp_x}{dt} - \frac{v}{c^2}\frac{dE}{dt})}{(1 - \frac{v}{c^2}u_x)}; \quad \frac{dp_y^{'}}{dt^{'}} = \frac{\frac{dp_y}{dt}}{\gamma_v(1 - \frac{v}{c^2}u_x)}; \quad \frac{dp_z^{'}}{dt^{'}} = \frac{\frac{dp_z}{dt}}{\gamma_v(1 - \frac{v}{c^2}u_x)} \tag{17}$$

From Q.2 we see that,

$$\frac{dE}{dt} = \frac{dp_x}{dt}u_x + \frac{dp_y}{dt}u_y + \frac{dp_z}{dt}u_z \tag{18}$$

From the Lorentz force equation Eq.(11) we see that,

$$\frac{dp_x}{dt} = q\mathcal{E}_x + \frac{q}{c}\left(u_y\mathcal{B}_z - u_z\mathcal{B}_y\right); \quad \frac{dp_y}{dt} = q\mathcal{E}_y + \frac{q}{c}\left(u_z\mathcal{B}_x - u_x\mathcal{B}_z\right); \quad \frac{dp_z}{dt} = q\mathcal{E}_z + \frac{q}{c}\left(u_x\mathcal{B}_y - u_y\mathcal{B}_x\right) \tag{19}$$

In reference frame $S^{'}$,

$$\frac{dp_x^{'}}{dt^{'}} = q\mathcal{E}_x^{'} + \frac{q}{c}\left(u_y^{'}\mathcal{B}_z^{'} - u_z^{'}\mathcal{B}_y^{'}\right); \quad \frac{dp_y^{'}}{dt^{'}} = q\mathcal{E}_y^{'} + \frac{q}{c}\left(u_z^{'}\mathcal{B}_x^{'} - u_x^{'}\mathcal{B}_z^{'}\right); \quad \frac{dp_z^{'}}{dt^{'}} = q\mathcal{E}_z^{'} + \frac{q}{c}\left(u_x^{'}\mathcal{B}_y^{'} - u_y^{'}\mathcal{B}_x^{'}\right) \tag{20}$$

Insert Eq.(19) and Eq.(20) into Eq.(18) and Eq.(17). Finally insert Eq.(18) into Eq.(17) and compare both sides to get,

$$q\mathcal{E}_x^{'} + \frac{q}{c}\left(u_y^{'}\mathcal{B}_z^{'} - u_z^{'}\mathcal{B}_y^{'}\right) = q\mathcal{E}_x + \frac{q}{c}\left(u_y\mathcal{B}_z - u_z\mathcal{B}_y\right) - \frac{v}{c^2}\frac{(q\mathcal{E}_y + \frac{q}{c}\left(u_z\mathcal{B}_x - u_x\mathcal{B}_z\right))u_y}{(1 - \frac{v}{c^2}u_x)} - \frac{v}{c^2}\frac{(q\mathcal{E}_z + \frac{q}{c}\left(u_x\mathcal{B}_y - u_y\mathcal{B}_x\right))u_z}{(1 - \frac{v}{c^2}u_x)} \tag{21}$$

$$q\mathcal{E}_y^{'} + \frac{q}{c}\left(u_z^{'}\mathcal{B}_x^{'} - u_x^{'}\mathcal{B}_z^{'}\right) = \frac{q\mathcal{E}_y + \frac{q}{c}\left(u_z\mathcal{B}_x - u_x\mathcal{B}_z\right)}{\gamma_v(1 - \frac{v}{c^2}u_x)} \tag{22}$$

$$q\mathcal{E}_z^{'} + \frac{q}{c}\left(u_x^{'}\mathcal{B}_y^{'} - u_y^{'}\mathcal{B}_x^{'}\right) = \frac{q\mathcal{E}_z + \frac{q}{c}\left(u_x\mathcal{B}_y - u_y\mathcal{B}_x\right)}{\gamma_v(1 - \frac{v}{c^2}u_x)} \tag{23}$$

Next we have to use the transformation laws for the velocity components of the particle.

$$u_x^{'} = \frac{u_x - v}{(1 - \frac{u_x\, v}{c^2})}; \quad u_y^{'} = \frac{u_y}{\gamma_v(1 - \frac{u_x\, v}{c^2})}; \quad u_z^{'} = \frac{u_z}{\gamma_v(1 - \frac{u_x\, v}{c^2})} \tag{24}$$

Insert Eq.(24) into Eq.(21), Eq.(22) and Eq.(23). Note that the electric and magnetic fields in reference frame $S^{'}$ depend on the electric and magnetic fields in S and the boost velocity v but they **CANNOT** depend on the velocity $\mathbf{u}$ of the particle. The fields influence the particle motion but the particles do not influence the external fields! This means we may equate the coefficients of u_x on both sides of Eq.(21), Eq.(22) and Eq.(23), similarly for u_y and u_z to get,

$$\mathcal{B}_x^{'} = \mathcal{B}_x, \quad \mathcal{B}_y^{'} = \gamma_v(\mathcal{B}_y + \frac{v}{c}\mathcal{E}_z), \quad \mathcal{B}_z^{'} = \gamma_v(\mathcal{B}_z - \frac{v}{c}\mathcal{E}_y) \tag{25}$$

$$\mathcal{E}_x^{'} = \mathcal{E}_x, \quad \mathcal{E}_y^{'} = \gamma_v(\mathcal{E}_y - \frac{v}{c}\mathcal{B}_z), \quad \mathcal{E}_z^{'} = \gamma_v(\mathcal{E}_z + \frac{v}{c}\mathcal{B}_y) \tag{26}$$

From this it is clear that $\vec{\mathcal{E}} \cdot \vec{\mathcal{B}} \equiv \mathcal{E}_x\mathcal{B}_x + \mathcal{E}_y\mathcal{B}_y + \mathcal{E}_z\mathcal{B}_z$ and $\vec{\mathcal{E}}^2 - \vec{\mathcal{B}}^2 \equiv (\mathcal{E}_x^2 - \mathcal{B}_x^2) + (\mathcal{E}_y^2 - \mathcal{B}_y^2) + (\mathcal{E}_z^2 - \mathcal{B}_z^2)$ are Lorentz invariants since,

$$\vec{\mathcal{E}}' \cdot \vec{\mathcal{B}}' = \vec{\mathcal{E}} \cdot \vec{\mathcal{B}} \quad ; \qquad \vec{\mathcal{E}'}^2 - \vec{\mathcal{B}'}^2 = \vec{\mathcal{E}}^2 - \vec{\mathcal{B}}^2 \tag{27}$$

Lastly, in the reference frame S there is only an electric field in the y-direction ie. $q\mathcal{E}_x = 0, q\mathcal{E}_y = 10^6 \ dynes, q\mathcal{E}_z = 0$ The magnetic field seen in reference frame $S^{'}$ is $(\frac{v}{c} = \frac{2}{3}, \gamma_v = \frac{3}{\sqrt{5}})$

$$q\mathcal{B}_x^{'} = 0, \quad q\mathcal{B}_y^{'} = 0, \quad q\mathcal{B}_z^{'} = \frac{2}{\sqrt{5}} \times 10^6 \ dynes \tag{28}$$

It is given that the particle is momentarily at rest in reference frame S. This means $u_x = u_y = u_z = 0$. This means $u_x^{'} = -v, u_y^{'} = u_z^{'} = 0$. This means the magnetic force acting on the particle in reference frame $S^{'}$ is,

$$\mathbf{f}_{mag}^{'} = \frac{\mathbf{u}^{'}}{c} \times q\vec{\mathcal{B}}' \tag{29}$$

This means $f_{x,mag}^{'} = f_{z,mag}^{'} = 0$ and

$$f_{y,mag}^{'} = \frac{v}{c}q\mathcal{B}_z^{'} = \frac{2}{3} \times \frac{2}{\sqrt{5}} \times 10^6 \ dynes = 5.96285 \times 10^5 dynes \tag{30}$$

Q.4 A photon (quantum of light) in vacuum is a particle whose rest mass is equal to zero. In Newtonian mechanics a particle with zero mass cannot have kinetic energy but in special relativity it has an energy proportional to its momentum since in general, $E = \sqrt{c^2p^2 + (mc^2)^2}$ but when $m = 0$,

$$E = c\ p \tag{31}$$

where $p = |\mathbf{p}|$. But we also know that the velocity of a particle in relativity is related to the momentum through the (fully general) relation [show this],

$$\mathbf{u} = c\frac{c\mathbf{p}}{E} \tag{32}$$

For a photon in vacuum,

$$\mathbf{u} = c\frac{c\mathbf{p}}{E} = c\frac{c\mathbf{p}}{cp} = c\ \hat{p} \tag{33}$$

This means a massless particle can exist in nature provided it moves with a speed c. Now consider a photon in a medium like glass. We know that light in glass travels slower than c. The velocity $\mathbf{u}$ in Eq.(32) is the velocity of the particle aspect of light. We will show in the next part of this course that there is a dual description which says that every particle has a dual wave description and vice versa.

The phenomenon of refraction is related to this dual wave description of light rather than the photon description. The speed with which energy is transported by light waves (not particles) is $v_g = \nu\lambda$. We will see later when we discuss quantum mechanics that the wave description of light which has a property like frequency ν is related to the energy E of particle description through $E = h\nu$ where h is Planck's constant. This formula is due to Einstein himself. Another physicist called de Broglie said that the other wave property viz. wavelength λ has to be related to the momentum $p \equiv |\mathbf{p}|$ of the particle description through $\lambda = \frac{h}{p}$. This is called de Broglie relation. From this we may conclude that the speed with which energy is transported by light waves in a medium is,

$$v_g = \nu\lambda = \frac{E}{p} \tag{34}$$

The refractive index of a medium in which this light wave is propagating is defined as,

$$n = \frac{c}{v_g} = \frac{cp}{E} \tag{35}$$

Consider two media with refractive indices n_1 and n_2. Imagine a light ray propagates from medium 1 to medium 2 as shown. Snell's Law states that (in a reference frame S),

$$n_1 \; sin(\theta_1) = n_2 \; sin(\theta_2) \tag{36}$$

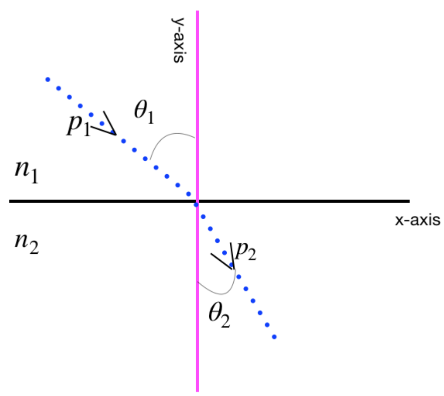

Figure 1: Snell's Law of Refraction

Show that in a reference frame $S^{'}$ moving with a constant speed v parallel to the (flat) interface between the two media, the form of Snell's Law remains unchanged. In other words,

$$n_1^{'} \; sin(\theta_1^{'}) = n_2^{'} \; sin(\theta_2^{'}) \tag{37}$$

where $n^{'} = \frac{c}{v_g^{'}}$. Note that the frequency of light is not affected by refraction, only the wavelength and the speed of the light waves are affected.

Ans. Substituting the formula for the refractive index in terms of momentum and energy we get,

$$\frac{cp_1}{E_1}\ sin(\theta_1) = \frac{cp_2}{E_2}\ sin(\theta_2) \tag{38}$$

The energy $E_1 = h\nu_1$ and $E_2 = h\nu_2$ but the frequency of light in both the media are the same $\nu_1 = \nu_2$ so that Snell's law takes the form,

$$p_1\ sin(\theta_1) = p_2\ sin(\theta_2) \tag{39}$$

But $p\ sin(\theta) \equiv p_x$. This means Snell's Law becomes,

$$p_{1,x} = p_{2,x} \tag{40}$$

Now we know that in reference frame $S^{'}$,

$$p^{'}_{j,x} = \gamma(p_{j,x} - \frac{v}{c^2}E_j)\ ;\quad E^{'}_j = \gamma(E_j - vp_{j,x}) \tag{41}$$

Call $\Delta p_x \equiv p_{1,x} - p_{2,x}$ and $\Delta E \equiv E_1 - E_2$. Both these are zero in reference frame S: $\Delta p_x = 0$ and $\Delta E = 0$. The question is, are they still zero in the Lorentz transformed frame?

$$\Delta p^{'}_x = \gamma(\Delta p_x - \frac{v}{c^2}\Delta E)\ ;\quad \Delta E^{'} = \gamma(\Delta E - v\Delta p_x) \tag{42}$$

Hence $\Delta p^{'}_x = 0$ and $\Delta E^{'} = 0$. This means,

$$p^{'}_{1,x} = p^{'}_{2,x}\ ;\quad E^{'}_1 = E^{'}_2 \tag{43}$$

This means the frequency of light in the two media are the same even in the Lorentz transformed frame. But more importantly,

$$p^{'}_1\ sin(\theta^{'}_1) = p^{'}_2\ sin(\theta^{'}_2)\ ;\quad E^{'}_1 = E^{'}_2 \tag{44}$$

Dividing one by the other,

$$\frac{c\ p^{'}_1}{E^{'}_1}\ sin(\theta^{'}_1) = \frac{c\ p^{'}_2}{E^{'}_2}\ sin(\theta^{'}_2) \tag{45}$$

This means,

$$n^{'}_1\ sin(\theta^{'}_1) = n^{'}_2\ sin(\theta^{'}_2) \tag{46}$$

Q.5 A bullet of mass $m = 100$ grams fired at a speed of 0.5c horizontally hits and get stuck on to a ball of mass $M = 200$ grams sitting at rest with respect to the earth. What is the speed of the combined system after the collision as per special theory of relativity (momentum defined as $\vec{p} = \gamma_u m\vec{u}$ is conserved)? (Neglect all other effects including that of air drag and gravity.)

Ans.

Consider the direction of velocity of the bullet as $\hat{x}$.

Initial momentum of the bullet: $\vec{p}_1 = \frac{1}{\sqrt{1-0.5^2}}\ 0.1 \times 0.5c\ \hat{x} = 1.1547 \times 0.05c\ \hat{x}$ (SI units)

Initial momentum of the ball: 0

Total initial momentum: $\vec{p}_i = 0.057735c\ \hat{x}$

Final momentum: $\vec{p}_f = \gamma_{u_f}(M+m)\vec{u}_f$. Momentum conservation $=> \vec{p}_f = \vec{p}_i = 0.057735c\ \hat{x}$.

Inverting the relation (discussed in the class), we have $\vec{u}_f = \frac{c\vec{p}_f}{\sqrt{|\vec{p}_f|^2+(M+m)^2c^2}} = \frac{0.057735c\ \hat{x}}{\sqrt{0.057735^2+0.3^2}} = 0.189c\ \hat{x}$

Q.6 Consider a shell of total mass $M = 1$ kg at rest with respect an observer on earth. At time $t = 0$ it explodes into three equal parts, each of which fly away with the same speed $u = 0.2c$, making angle 120^o between each other. The velocities may be considered as $\vec{u}_1 = 0.2c\ \hat{x}$, $\vec{u}_2 = 0.2c\left(-\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right)$, $\vec{u}_3 = 0.2c\left(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\right)$. For the observer on the earth, what is the energy and what is momentum of each of the pieces, (let us denote these by $E_1,\ E_2,\ E_3$ and $\vec{p}_1,\ \vec{p}_2,\ \vec{p}_3$, respectively)?

Denoting the energy-momentum four-vectors as $p_1,\ p_2,\ p_3$ with $p_1 \equiv \left(\frac{E_1}{c},\ \vec{p}_1\right)$, etc., find $(p_1+p_2+p_3)\cdot(p_1+p_2+p_3)$. (In general, $p\cdot p = \frac{E^2}{c^2} - \vec{p}\cdot\vec{p} = m^2c^2$. please refer to the lectures.)

Consider another observer moving with velocity $\vec{v} = 0.5c\ \hat{x}$ with respect to the observer on the earth. Find $E_1',\ E_2',\ E_3'$ and $\vec{p}_1{}',\ \vec{p}_2{}',\ \vec{p}_3{}'$.

What are the masses of each of these parts as seen from the moving frame?

Find the magnitude of the sum of the energy-momentum four-vector in this frame $((p_1' + p_2' + p_3')\cdot(p_1' + p_2' + p_3'))$.

Ans.

For an object moving with velocity, $\vec{u}$, we have energy, $E = \gamma_u mc^2$ and $\vec{p} = \gamma_u m\vec{u}$.

Before the explosion:

Total energy available: $E_i = mc^2 = 9\times10^{16}$ J. Initial momentum, $\vec{p}_i = 0$.

All the three pieces have the same mass (equal parts), and move with the same speed. Therefore, they have the same energy.

$E_1 = E_2 = E_3 = \frac{E_i}{3} = 3\times10^{16}$ J

Magnitude of the momentum of each piece, $|\vec{p}_1| = E_1\frac{|\vec{u}_1|}{c^2}$ kg m/s $= 3\times10^{16}\ \frac{0.2}{c} = 2\times10^7$ kg m/s $= |\vec{p}_2| = |\vec{p}_3|$

That means, $\vec{p}_1 = 2\times10^7\ \hat{x}$, $\vec{p}_2 = 2\times10^7\ \left(-\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right)$, $\vec{p}_3 = 2\times10^7\ \left(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\right)$. The rest mass of each piece is $m_j = \frac{2\sqrt{\frac{2}{3}}}{5}kg$. The apparent mass is E_j/c^2 (this interpretation of energy of a moving object divided by c^2 as an apparent or relativistic mass is sometimes given but we prefer to reserve the term mass to denote rest mass). Now $\gamma_v = \frac{2}{\sqrt{3}}$. Hence,

$$p_x^{'} = \gamma_v(p_x - \frac{v}{c^2}E);\ p_y^{'} = p_y;\ p_z^{'} = p_z\ ;\ E^{'} = \gamma_v(E - vp_x)$$

For mass m_1,

$$p^{'}_{1,x} = \frac{2}{\sqrt{3}}(2\times 10^7 - \frac{0.5}{3\times 10^8}\times 3\times 10^{16});\ p^{'}_{1,y} = 0;\ p^{'}_{1,z} = 0\ ;\ E^{'}_1 = \frac{2}{\sqrt{3}}(3\times 10^{16} - 0.5\times 3\times 10^8\times 2\times 10^7)$$

For mass m_2,

$$p^{'}_{2,x} = \frac{2}{\sqrt{3}}(-\frac{1}{2}\times 2\times 10^7 - \frac{0.5}{3\times 10^8}\times 3\times 10^{16});\ p^{'}_{2,y} = \frac{\sqrt{3}}{2}\times 2\times 10^7;\ p^{'}_{2,z} = 0\ ;\ E^{'}_2 = \frac{2}{\sqrt{3}}(3\times 10^{16} + \frac{1}{2}\times 0.5\times 3\times 10^8\times 2\times 10^7)$$

For mass m_3,

$$p^{'}_{3,x} = \frac{2}{\sqrt{3}}(-\frac{1}{2}\times 2\times 10^7 - \frac{0.5}{3\times 10^8}\times 3\times 10^{16});\ p^{'}_{3,y} = -\frac{\sqrt{3}}{2}\times 2\times 10^7;\ p^{'}_{2,z} = 0\ ;\ E^{'}_3 = \frac{2}{\sqrt{3}}(3\times 10^{16} + \frac{1}{2}\times 0.5\times 3\times 10^8\times 2\times 10^7)$$

OR

For mass m_1,

$$p^{'}_{1,x} = -2\sqrt{3}\times 10^7\ kg.m/s;\ p^{'}_{1,y} = 0;\ p^{'}_{1,z} = 0\ ;\ E^{'}_1 = 18\sqrt{3}\times 10^{15}\ J$$

For mass m_2,

$$p^{'}_{2,x} = 4\sqrt{3}\times 10^7\ kg.m/s;\ p^{'}_{2,y} = \sqrt{3}\times 10^7\ kg.m/s;\ p^{'}_{2,z} = 0\ ;\ E^{'}_2 = 21\sqrt{3}\times 10^{15}\ J$$

For mass m_3,

$$p^{'}_{3,x} = -4\sqrt{3}\times 10^7\ kg.m/s;\ p^{'}_{3,y} = -\sqrt{3}\times 10^7\ kg.m/s;\ p^{'}_{2,z} = 0\ ;\ E^{'}_3 = 21\sqrt{3}\times 10^{15}\ J$$

Define the 4-vector $p^{'} = \sum_{j=1,2,3}(\frac{E^{'}_j}{c}, p^{'}_{j,x}, p^{'}_{j,y}, p^{'}_{j,z})$. Hence,

$$p^{'} = 10^8\ (2\sqrt{3}, -\sqrt{3}, 0, 0)$$

The length (squared) of the 4-vector is,

$$|p^{'}|^2 = 10^{16}(12-3) = 9\times 10^{16}\ (kg.m/s)^2$$

But we know that $|p^{'}| = M_i c = 1kg\times 3\times 10^8 = 3\times 10^8\ (kg.m/s)$. Hence these two are consistent.