Magnetic Circuits

Electric & magnetic fields are complementary to each other. Electric current is associated with a magnetic field. All electro-mechanical energy conversion devices are associated with magnetic fields. Isolation of electrical supply from sensitive electrical or electronic equipment such as medical equipment requires magnetic field at the interface between the two circuits. Electromagnetic ideas are exploited for designing sophisticated medical imaging devices such as magnetic resonance imaging (MRI) equipment. Electromagnetic principles are used in induction cookers.

Following are some basic parameters used in connection with the magnetic circuits,

μ - Permeability (henry / meter)

μ_0 - Permeability of the free space ($4\pi \times 10^{-7}$ henry / meter)

ϕ - Flux (Weber, Wb)

B -Flux density (Wb/m^2, weber per square meter)

H – Magnetic Field intensity (A/m, ampere / meter)

F - Mmf., Magneto motive force (AT, ampere-turns)

R – Reluctance

ρ – Permeance

Ampere's Circuit Law:

Line integral of the magnetic field intensity around a closed path is equal to the total current linked by the contour. Ampere's circuit law relates the current with the magnetic field intensity. Applying Ampere's circuit law in Fig. 1(a), the magnetic field intensity will be related to the total current linked with the contour as

$$\oint \boldsymbol{H} \cdot \mathbf{dl} = \sum \boldsymbol{i} = \boldsymbol{i_1} + \boldsymbol{i_2} - \boldsymbol{i_3}$$

$$\oint \boldsymbol{H}\mathbf{dlcos}\boldsymbol{\theta} = \sum \boldsymbol{i}$$

Fig. 1(b) shows a current carrying conductor with current **i** through a circular magnetic material. The magnetic field intensity at a distance ***r*** from the conductor can be evaluated using Ampere's circuit law. The angle between the length element ***dl*** and the magnetic field intensity ***H*** is zero degree.

$$\oint \boldsymbol{H} \cdot \mathbf{dl} = \boldsymbol{i} \Rightarrow \boldsymbol{H}\mathbf{2}\boldsymbol{\pi r} = \boldsymbol{i}$$
$$\Rightarrow \boldsymbol{H} = \frac{\boldsymbol{i}}{\mathbf{2}\boldsymbol{\pi R}}$$

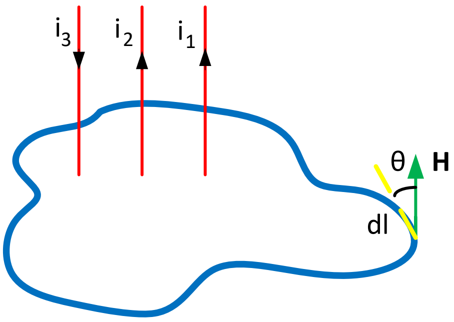

Fig. 1 (a)

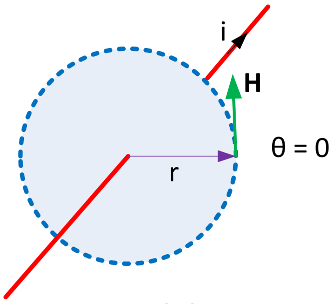

Fig. 1 (b)

Toroid is a ring shaped magnetic cone over which the current carrying coil is wound. Fig. 2 shows a toroid. Magnetic flux is confined to the core. Flux outside the toroid is called leakage flux. Neglecting the leakage flux, the field intensity at the mean radius (***r***) of core can be found using the Ampere's circuit law. Most of the practical circuits are some form of modifications or extensions of this configuration. The effective core length can be considered as l_c.

$$\oint \boldsymbol{H} \cdot \mathbf{dl} = \boldsymbol{Ni} =\rangle \boldsymbol{H}\mathbf{2}\boldsymbol{\pi}\mathbf{r} = \boldsymbol{Ni}$$
$$\Rightarrow \boldsymbol{H} = \frac{\boldsymbol{Ni}}{\mathbf{2}\boldsymbol{\pi r}} = \frac{\boldsymbol{Ni}}{\boldsymbol{l_c}}$$

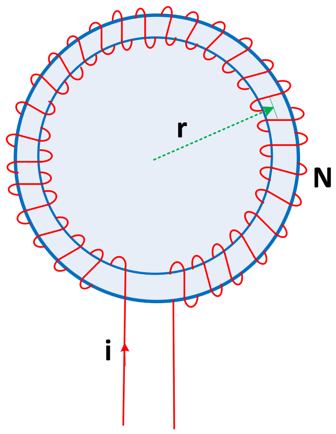

Fig. 2

The quantity ***Ni*** is called magneto motive force (mmf) which is denoted as ***F***. Its unit is ampere turns (AT). Magnetic field intensity is related to magnetic flux density.

$$B = \mu H = \frac{Ni}{lc/\mu}$$
$$\phi = BA = \frac{Ni}{l_c/\mu A}$$

If $R = \frac{l_c}{\mu A}$, where R is the reluctance of a magnetic circuit. The flux in terms of mmf and the reluctance can be written as

$$\phi = \frac{Ni}{R} = \frac{\text{mmf}}{Reluctance}$$

To represent a magnetic circuit as an equivalent electrical circuit, mmf is considered as equivalent to emf (voltage), reluctance is equivalent to resistance and flux is equivalent to current. Ohm's law can be used for the relation between the mmf, the reluctance and the flux in a magnetic circuit.

$$\text{mmf} \leftrightarrow \text{emf}$$
$$\phi \leftrightarrow I$$
$$R \leftrightarrow \mathbb{R}$$

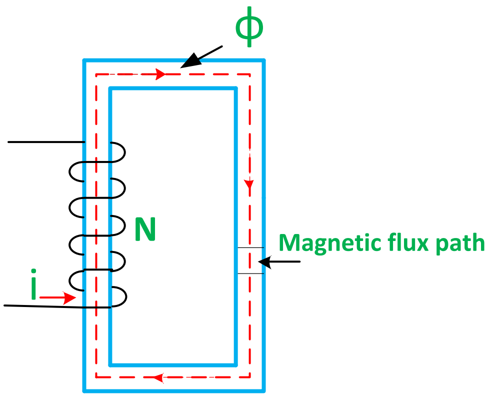

Fig. 3(a)

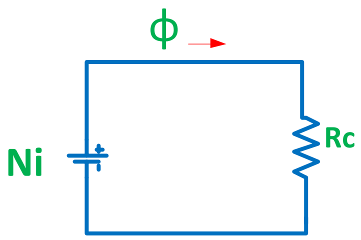

Fig. 3(b)

Fig. 3(a) shows a coil with N number of turns which carries a current i. This coil will produce flux Φ in the magnetic core. If the core has an effective length l_C, cross section area A and its permeability is μ, then its reluctance is $R = l_C/\mu A$. The electrical equivalent circuit (Fig. 3(b)) will consist of an equivalent voltage source which represents the mmf (***F=Ni***) and an equivalent resistance equal to the reluctance R of the magnetic circuit. The circuit shows flux (Φ) as equivalent to the current in the circuit.

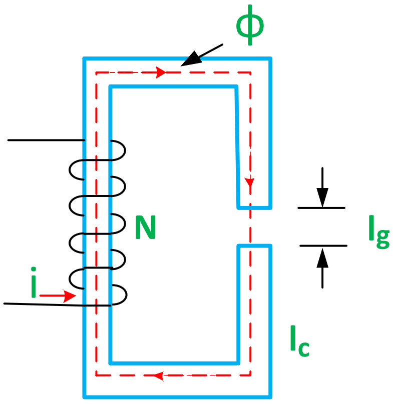

Fig. 4(a)

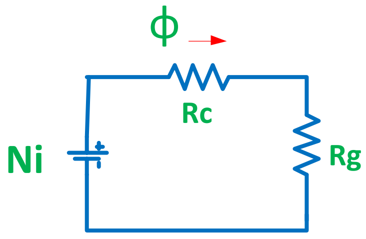

Fig. 4(b)

Fig. 4(a) shows a coil with ***N*** number of turns wound around a core of effective length l_C. There is an air gap of length l_g. The current in the coil will produce flux **Φ** in the magnetic core. This flux will link the core through the air gap. If the cross section area is ***A***, the permeability of the core material is μ, then its reluctance is $\boldsymbol{R_C = l_C/\mu A}$. The reluctance of the air gap is $\boldsymbol{R_g = l_g/\mu_g A}$. The electrical equivalent circuit (Fig. 4(b)) will consist of an equivalent voltage source which represents the mmf (***F=Ni***) and two equivalent resistances equal to two reluctances R_c and R_g in series. The circuit shows flux (**Φ**) as equivalent to the current in the circuit.

Example: Fig. 5 shows a rectangular magnetic core with an air gap. Find the exciting current needed to cause a flux density of $B_g = 1.25\,T$ in the air gap. Given $N = 500\ turns$ and $\mu_r(iron) = 4000$.

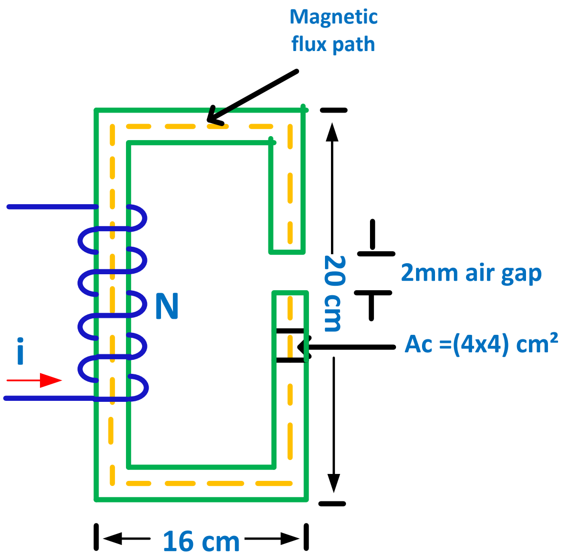

Fig. 5

Solution: It is a simple series magnetic circuit with its analog shown in Fig. 6.

Core length = $\boldsymbol{l_c = 2[(20-4)+(16-4)]-0.2 = 55.8cm}$

Cross-sectional area of core $\boldsymbol{A_c = 16cm^2}$

Core reluctance $\boldsymbol{\mathfrak{R}_c = \frac{l_c}{\mu_r\mu_0 A_c} = \frac{55.8\times10^{-2}}{4000\times4\pi\times10^{-7}\times16\times10^{-4}} = 0.694\times10^5}$

Air gap length $\boldsymbol{l_g = 0.2\ cm}$

Area of air gap $\boldsymbol{A_g = 16cm^2}$

Air gap reluctance $\boldsymbol{\mathfrak{R}_g = \frac{l_g}{\mu_0 A_g} = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times16\times10^{-4}} = 9.95x10^5}$

Total $\boldsymbol{\mathfrak{R} = \mathfrak{R}_c + \mathfrak{R}_g = 10.64\times10^5 AT/Wb}$

Flux in the magnetic circuit, $\boldsymbol{\Phi = BA = 1.25\times16\times10^{-4} = 2.0\ mWb}$

Now $$\boldsymbol{Ni = \Phi(\mathfrak{R}_c + \mathfrak{R}_g) = \Phi\mathfrak{R}}$$

$$\boldsymbol{= 2.0\times10^{-3}\times10.64\times10^5 = 2128\ AT}$$

So, the exciting current $\boldsymbol{i = \frac{\Phi\mathfrak{R}}{N} = \frac{2128}{500} = 4.26\ A}$

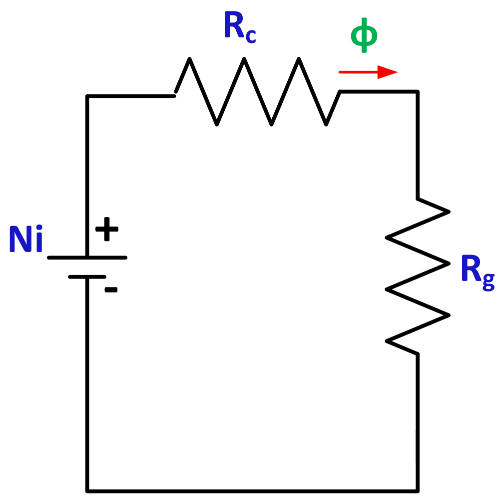

Fig. 6