Tutorial 2 on Relativity

Q.1 In the file MichelsonMorley.pdf, the authors discuss an analysis of the expected results of the experiment if Galilean relativity was valid - which means the speed of light depends on the reference frame. They find that the total change in the time taken between the rotated and non-rotated apparatus for the beam of light to reach the eye is non-zero. Re-analyze this experiment in a similar way except now assume that the speed of light is independent of the reference frame (special relativity). You should find that the above total change in the time taken is zero.

Q.2 On earth, every person that is born at some place at some time, finally dies at some other (or rarely, the same) place at a later time. Show that there is no reference frame in which it is going to appear that the person (called P) died before she was born. This is a non-issue in Galilean relativity since time is absolute, but requires a proof in Special Relativity.

Q.3 In 1962, the famous Indian Physicist E.C.G. Sudarshan (1931-2018) along with his collaborators Bilaniuk and Deshpande proposed that there may be particles that travel faster than light which do not violate Einstein's special relativity [American Journal of Physics 30, 718 (1962); doi: 10.1119/1.1941773]. These particles were later called "tachyons". In fact, the following classification is standard. With respect to some reference frame S,

a) 'Bradyons' are particles with speed $|u| < c$ ('bradys' in Greek means 'slow')

b) 'Luxons' are particles that move with the speed of light $|u| = c$ (eg. photons, gluons and gravitons) ('lux' means light in Latin)

c) 'Tachyons' are particles that move with the speed $|u| > c$ ('tachus' means 'speedy' in Greek)

In Special Relativity, only reference frames have to move with (constant) velocities less than c, particles that live in them can do whatever they want.

With respect to another frame S' moving with speed v, $|v| < c$ relative to S in the x-direction, show that
A bradyon in S will also be a bradyon in S', a luxon in S will also be a luxon in S' and finally a tachyon in S (if you can find one) will also be a tachyon in S', no matter how small or large $|v| < c$ is.

Note that the boost velocity v has to be less than the speed of light – you are not allowed to sit on a tachyon and observe the world. This is because Lorentz transformations don't make sense if $|v| > c$ since $\gamma = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$ will become imaginary and $x^{'} = \gamma(x - vt)$ etc. will not make sense etc.

Q.4 *Throwing: The International Cricket Council (ICC) declares a bowling action illegal when a player is throwing rather than bowling the ball. This is defined by the ICC as being when the arm's angle at the elbow is less than 165 degrees at time of release of ball (180 degrees would be when the arm is perfectly straight).*

Two brothers who are umpires are watching an India vs Pakistan match where the Pakistani bowler Saeed Ajmal is seen bowling to Sachin Tendulkar. One of the umpires, Gary Gaia is the TV umpire. His brother, John Ouranos is an astronaut in a spaceship moving with speed $0.95c$ parallel to the cricket pitch in the direction of the ball being bowled. Gary notices that the Ajmal has bowled illegally since at the time of the release of the ball, the upper half of his arm was horizontal but the lower half was making an angle of 20^0 with the horizontal. Gary awards a no-ball to India for throwing. Does John who is watching the match from his spaceship agree or disagree with his brother? What angle does John measure? Answer these questions if John's spaceship is moving perpendicular (instead of parallel) to the pitch with the same speed ie. $0.95c$.

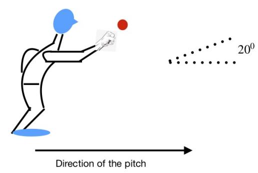

Figure 1: Saeed Ajmal

Q.5 A particle with (rest) mass m moving with velocity $\mathbf{u}$ in a reference frame S is defined to have a (relativistic) momentum $\mathbf{p}$ and (relativistic) energy E given by

$$\mathbf{p} = \frac{m\mathbf{u}}{\sqrt{1-\frac{u^2}{c^2}}}; \quad E = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} \tag{1}$$

These unusual definitions ensure that in collisions, if momentum is conserved before and after collision in one reference frame, it is also seen to be conserved in a reference frame moving relative to the earlier one. The usual non-relativistic definitions $\mathbf{p} = m\mathbf{u}$ and $E = \frac{1}{2}mu^2$ do not have this property [under Lorentz transformations, but under Galilean transformation these simple older formulas are sufficient to ensure conservation laws are applicable in all inertial reference frames]. If $|\mathbf{u}| \ll c$ show by binomial expansion that,

$$\mathbf{p} = m\mathbf{u} + m\mathbf{u}\frac{u^2}{2c^2} + m\mathbf{u}\frac{3u^4}{8c^4} + ... \tag{2}$$

$$E = mc^2 + \frac{1}{2}mu^2 + \frac{1}{2}mu^2\ \frac{3u^2}{4c^2} + ... \tag{3}$$

Show that in a reference frame S' moving with a speed v in the x-direction, the energy and momentum of the particle becomes,

$$E^{'} = \gamma_v(E - vp_x); \quad p^{'}_x = \gamma_v(p_x - \frac{v}{c^2}E); \quad p^{'}_y = p_y; \quad p^{'}_z = p_z; \quad where\ \gamma_v = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{4}$$

Using this, show that in a two-body collision, if the (relativistic) energy and (relativistic) momentum are conserved in one reference frame S they are also conserved in reference frame S'.

Not to be discussed in tutorial but important to do at home: Show that energy and momentum as defined by the usual formulas $E = \frac{1}{2}mu^2$, $\mathbf{p} = m\mathbf{u}$ ARE NOT CONSERVED in a two body collision if you assume *Lorentz transformations* are correct. Also show that energy and momentum as defined by the usual formulas $E = \frac{1}{2}mu^2$, $\mathbf{p} = m\mathbf{u}$ ARE CONSERVED in a two body collision if you assume *Galilean transformations* are correct.

Q.6 Imagine a particle moving in the x-y plane in reference frame S. Now S' is a spaceship that moves in the x direction with speed v relative to S. If u_x, u_y, u_z are the instantaneous velocity components of the particle and a_x, a_y, a_z are the components of acceleration, find the corresponding components as seen from S'.