PH102: Tutorial Problem set

Tutorial 1

2018-10-24

1.01. Show that $\vec{\nabla} f(r) = \frac{f'(r)\vec{r}}{r}$, where $\vec{r}$ is the position vector and $r = |\vec{r}|$ while $f(r)$ is an arbitrary, regular function of r.

1.02. The equation for the surface of an ellipsoid is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = \text{constant} \ ,$$

where a, b, c are constants. Find the unit normal to each point of the above surface.

1.03. The height of a certain hill is given by

$$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12) \ ,$$

where y is the distance north, and x is the distance east of South Hadley. All distances are measured in some arbitrary units.

(a) Where is the top of the hill located?

(b) How high is the hill?

(c) How steep is the slope at a point one unit north and one unit east of South Hadley? In which direction is the slope steepest at that point?

1.04. The figure 1 shows an ellipse with foci at points A and B. Let P be a point on the ellipse. Show that lines AP and BP make equal angles with the tangent to the ellipse at P. [Hint: Use the fact that $R_1 + R_2 =$ constant].

1.05. Prove the identities:

$$(a)\ \vec{\nabla} \times (f\vec{A}) = f(\vec{\nabla} \times \vec{A}) - \vec{A} \times (\vec{\nabla} f) \ .$$
$$(b)\ \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} \ .$$

1.06. Show that the work done for a particle moving under force field $\vec{F} = (2xy + z^3)\hat{x} + x^2\hat{y} + 3xz^2\hat{z}$ from point $a = (1, 1, 0)$ to $b = (2, 2, 0)$ as shown in Figure 2 following path 1 and path 2 is equal. Show that curl of the force field $\vec{F}$ vanishes. Calculate the corresponding scalar potential.

1.07. Find the work done in moving a particle once around a circle in xy plane, if the circle centre at the origin and radius 3 and if the force is given by

$$\vec{F} = (2x - y + z)\hat{x} + (x + y - z^2)\hat{y} + (3x - 2y + 4z)\hat{z} \ .$$

Evaluting $\vec{\nabla} \times \vec{F}$, infer the nature of vector field $\vec{F}$.

1.08. Evaluate

$$\int\int_S \vec{A} \cdot \hat{n} dS \ ,$$

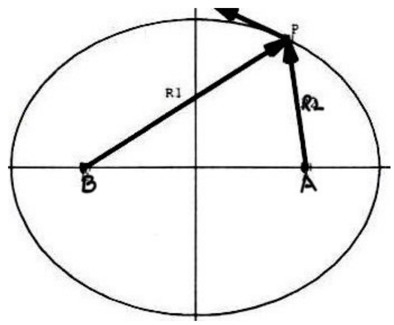

Figure 1: Problem 1.04

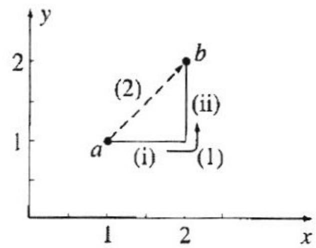

Figure 2: Problem 1.06

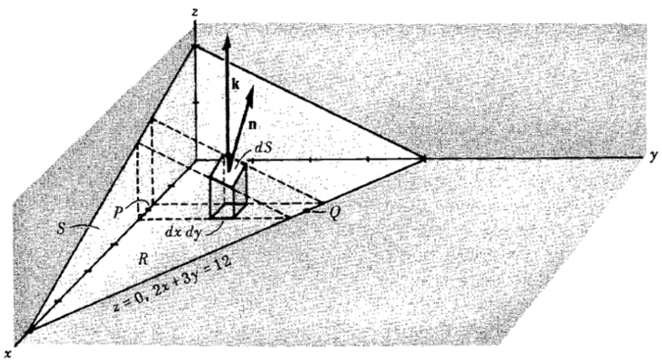

Figure 3: Problem 1.08

where $\vec{A} = 18z\hat{x} - 12\hat{y} + 3y\hat{z}$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant (see Figure 3).

Take home problems

H1.01. Find the gradients of the following functions:

$$(a)\ f(x,y,z) = x^2 + y^3 + z^4\ .$$
$$(b)\ f(x,y,z) = x^2y^2z^4\ .$$
$$(c)\ f(x,y,z) = e^x \sin y \ln z\ .$$
$$(d)\ f(x,y,z) = r^n\ .$$

H1.02. Prove the following identities:

$$(a)\ \vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f\ .$$
$$(b)\ \vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})\ .$$

H1.03 Evalute $\vec{\nabla}\times(\vec{\nabla}\phi)$.