Star-Delta Transformation

Three-phase loads can be either star (Y) connected or delta (Δ) connected. In delta connection, the impedances are connected back-to-back as shown in Fig. 1 (a). There is no neutral point in delta connection. Fig. 1(b) shows a star connected load. Depending on the application, either of the two connections is used. In case of balanced three phase circuits, the impedances of all the phases are equal. For circuit analysis, the star connected loads can be represented using the delta connected loads and vice-versa. The equivalent star connected loads in terms of delta connected loads are derived as follows.

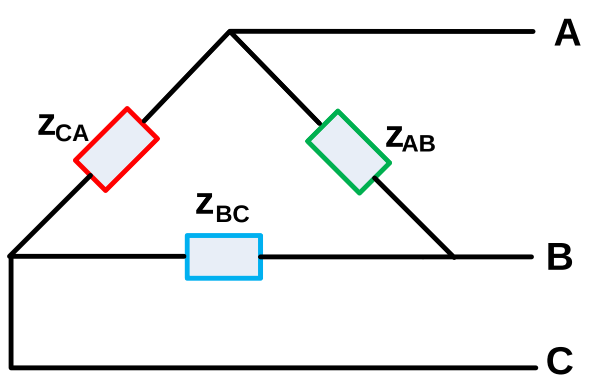

Fig. 1(a)

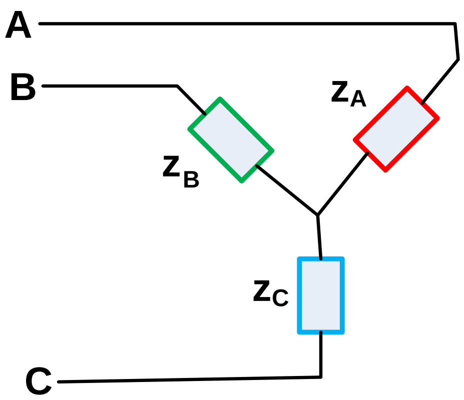

Fig. 1(b)

The impedance seen between the lines A and B in the star connected load is $\mathbf{Z_A + Z_B}$ (series combination of $\mathbf{Z_A}$ and $\mathbf{Z_B}$). In the delta connected load, the impedance seen between A and B is $\mathbf{Z_{AB}}$ in parallel with the series combination of $\mathbf{Z_{CA}}$ and $\mathbf{Z_{BC}}$.

$$\boldsymbol{Z_A + Z_B = Z_{AB} \,||\, (Z_{BC} + Z_{CA}) = \frac{Z_{AB}(Z_{BC} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}}}$$

Similarly,

$$\boldsymbol{Z_B + Z_c = \frac{Z_{BC}(Z_{AB} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}}}$$

$$\boldsymbol{Z_c + Z_A = \frac{Z_{CA}(Z_{AB} + Z_{BC})}{Z_{AB} + Z_{BC} + Z_{CA}}}$$

Solving these three equations, the star connected impedances can be represented with equivalent delta connected impedances as

$$Z_A = \frac{Z_{AB}Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$
$$Z_B = \frac{Z_{AB}Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}$$
$$Z_c = \frac{Z_{BC}Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

Similarly, delta connected loads (**Z_{AB}, Z_{BC}** and **Z_{CA}**) can be derived in terms of star connected loads (**Z_A, Z_B** and **Z_C**). Students can attempt this as an exercise.

Balanced Three-Phase System

A balanced three-phase four-wire system has a balanced three phase source and a balanced three-phase load. The loads in all the three phases are same. The fourth wire is the neutral wire. Fig. 2 shows a balanced three-phase system with a Y-connected load.

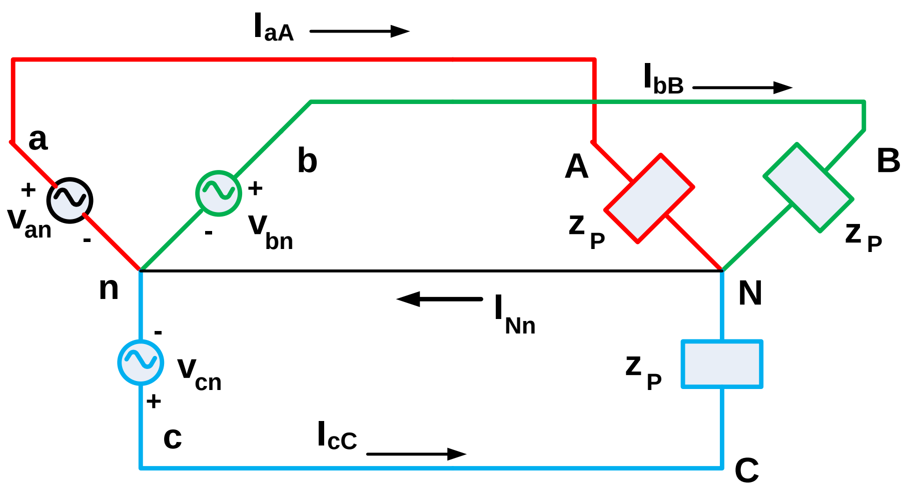

Fig. 2

The three line currents are $I_{aA,} I_{bB}$ and I_{cC} . In case of an Y-connected system the line currents are equal the phase currents (they are not distinguishable). Using KVL in the three single phases separately

$$I_{aA} = \frac{V_{an}}{z_P}, \quad I_{bB} = \frac{V_{bn}}{z_P} = \frac{V_{an}\angle -120^0}{z_P} = I_{aA}\angle - 120^0,$$

Similarly $I_{cC} = I_{aA}\angle -240^0$

The three line currents have equal magnitude and they differ from each other by a phase angle of 120 degree. The current in the neutral wire is

$$\boldsymbol{I_{Nn} = I_{aA} + I_{bB} + I_{cC} = 0}$$

The zero current in the neutral wire can be visualized as having an infinite impedance or it is an open circuit condition. There will be no effect on the line currents if the neutral wire is taken out from the circuit. This can save the cost of the transmission line.

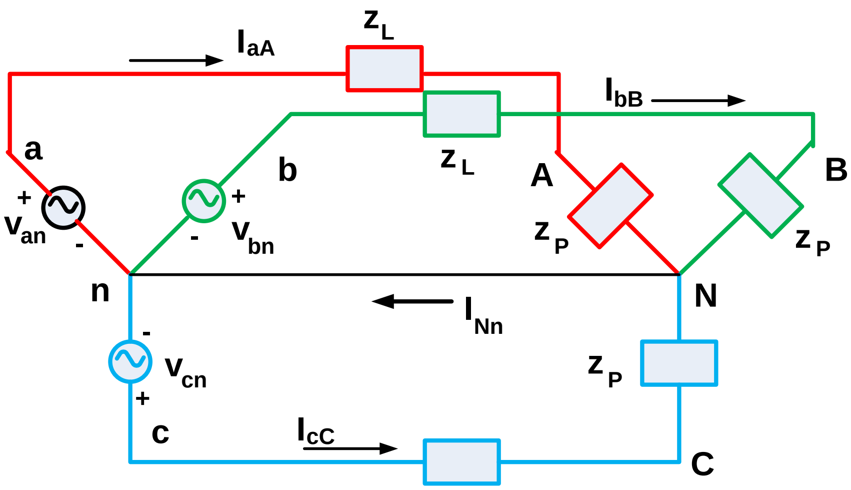

Fig. 3

If we introduce a line impedance (Fig.3), the currents will be

$$\boldsymbol{I_{aA} = \frac{V_{an}}{Z_L + Z_p}}$$

$$\boldsymbol{I_{bB} = \frac{V_{bn}}{Z_L + z_p} = I_{aA}L - 120^0}$$

$$\boldsymbol{I_{cC} = I_{aA}L - 240^0}$$

$$\boldsymbol{I_{Nn} = I_{aA} + I_{bB} + I_{cC} = 0}$$

The three line currents have the same relationship as seen earlier. Now considering an impedance in the neutral wire (Fig. 4), the circuit can be analyzed using Kirchoff's current law at node N. The voltage at node n is zero as this is the common reference point for the three voltage sources.

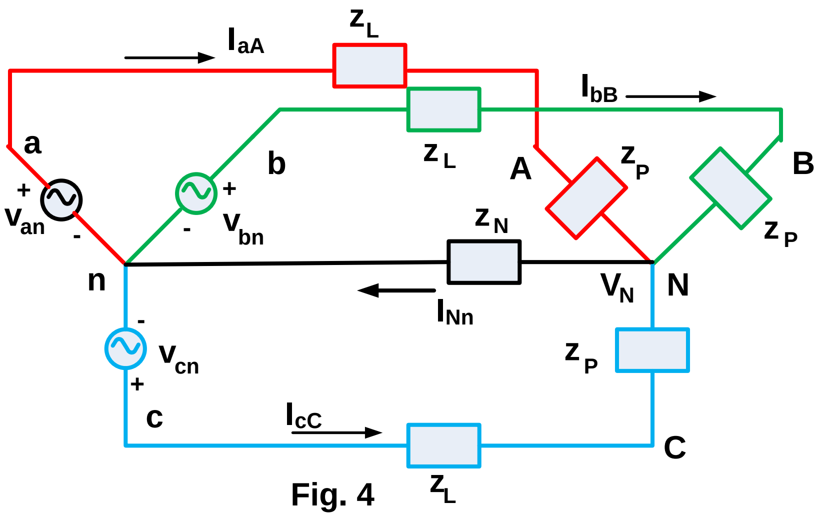

Fig. 4

$$\frac{V_N}{Z_N} = \frac{V_{an} - V_N}{Z_L + Z_P} + \frac{V_{bn} - V_N}{Z_L + Z_p} + \frac{V_{cn} - V_N}{Z_L + Z_p}$$

$$\Rightarrow V_N\left[\frac{1}{Z_N} + \frac{3}{Z_L + Z_p}\right] = \frac{V_{an} + V_{bn} + V_{cn}}{Z_L + Z_p}$$

$$V_{an} + V_{bn} + V_{cn} = 0 \Rightarrow V_N = 0$$

The voltage of node N is zero. The current in the neutral wire will be zero in this case.

These analysis show that the current in the neutral wire is always zero for a balanced three phase system. This is true for any impedance in the neutral wire even for short circuit (zero impedance) or open circuit (infinite impedance) case. A short circuit or zero impedance neutral wire enables us to analyse the balanced polyphase circuit on a per phase basis. The phase-B (B) equivalent circuit for analysis is shown in Fig. 5.

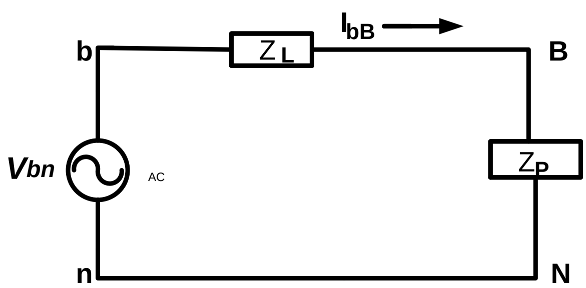

Fig. 5

In a star connected network the line and phase currents are same, $I_L = I_P$. In fact, they are not distinguishable. In a Y-connected system, the line voltage is $\sqrt{3}$ times the phase voltage ($V_L = \sqrt{3}\, V_P$).