PH102: Tutorial Problem set

Tutorial 4

2018-10-24

4.01. A rod placed along the x axis has charge density $\lambda(x) = \frac{\lambda_0 x}{L}$ in the interval $-L < x < L$. Find the electric field at a point $x = x_0 > L$ along the x axis. Examine this result for $x_0 \to \infty$ and show that it falls off like a dipole field $\vec{E} = \hat{x}\frac{\lambda_0 L^2}{3\pi\epsilon_0 x_0^3}$ and find the associated dipole moment. Hint: Expand in a Taylor series to an order that yields a nonzero result.

4.02. A long coaxial cable (see Fig. 1) carries a uniform volume charge density ρ on the inner cylinder (radius a), and a uniform surface charge density on the outer cylindrical shell (radius b). This surface charge is negative and of just the right magnitute so that the cable as a whole is electrically neutral. Find the electric field in each of the three regions:

(a) inside the inner cylinder ($s < a$);
(b) between the cylinders ($a < s < b$);
(c) outside the cable ($s > b$).
Plot $|\vec{E}|$ as a function of s for these three regions.

4.03. Two spheres, each of radius R and carrying uniform charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap (see Fig. 2). Call the vector from the positive center to the negative center $\vec{d}$. Show that the field in the region of overlap is constant, and find its value.

4.04. A spherical surface of radius R and center at the origin carries a surface charge $\sigma(\theta, \phi) = \sigma_0 \cos\theta$. Find the electric field at z on z-axis. Treat the case $z < R$ (inside) as well as $z > R$ (outside).
[Hint: Be sure to take the positive square root : $\sqrt{R^2 + z^2 - 2Rz} = (R - z)$ if $R > z$; but it is $(z - R)$ if $R < z$.]

4.05. A sphere of radius R carries a charge density $\rho(r) = kr$ (where k is a constant). Find the energy of the configuration. Check your answer by calculating it in at least two different ways.

4.06. Two spherical cavities, of radii a and b, are hollowed out from the interior of a neutral conducting sphere of radius R (see Fig. 3). At the center of each cavity a point charge is placed, call these charges q_a and q_b respectively.

(a) Find the surface charges σ_a, σ_b and σ_R.
(b) What is the field outside the conductor?
(c) What is the field within each cavity?
(d) What is the force on q_a and q_b?
(e) Which of these answers would change if a third charge, say q_c, were brought near the conductor?

Take home problems

H4.01. Which of these is an electrostatic field?

$$(a)\ \vec{E}_1 = k(xy\hat{x} + 2yz\hat{y} + 3xz\hat{z})\ ;$$

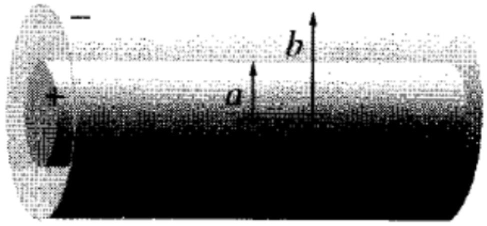

Figure 1: Problem 4.02

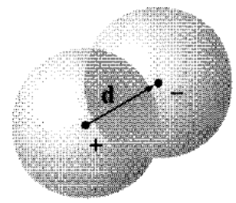

Figure 2: Problem 4.03

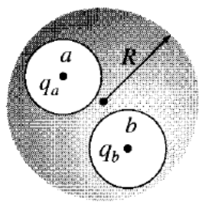

Figure 3: Problem 4.06

$$(b)\ \vec{E}_2 = k(y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z})\ .$$
$$(c)\ \vec{E}_3 = 2r\sin\theta\cos\phi\hat{r} + r\cos\theta\cos\phi\hat{\theta} - r\sin\phi\hat{\phi}\ .$$

Here k is a constant. For the possible field, find the electrostatic potential V at (x_0, y_0, z_0) using the origin as your reference point. Check your answer by computing $\vec{\nabla}V$.

H4.02. Suppose the electric field in some region is found to be $\vec{E} = 2r\sin\theta\cos\phi\hat{r} + r\cos\theta\cos\phi\hat{\theta} - r\sin\theta\hat{\phi}$ in spherical polar coordinates (k is some constant).

(a) Find the charge density ρ.

(b) Find the total charge contained in the sphere of radius R, centered at the origin. (Do it two different ways.)

H4.03. If the potential $V(s, \phi, z) = s^2 z\sin\phi$ in cylindrical coordinates, calculate the energy within the region defined by $1 < s < 4$, $-2 < z < 2$ and $0 < \phi < \pi/3$.