#37 of 114/MA101/Quiz-3/2018f Roll... 180101037
L1/Oct. 28/p. 1 of 10/#230 Name: Kotkar Anket Sanjay

INSTRUCTIONS: The maximum possible score on this Quiz is **15 marks**. Show all steps. Answer all questions in designated space or over the extra pages towards the end of the booklet. Use supplementary sheets ONLY for rough work.

1. Suppose that $f : [0, 10] \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} x & \text{if } x \text{ is an integer,} \\ 1 & \text{otherwise.} \end{cases}$$

Show that f is Riemann integrable on $[0, 10]$ and that $\int_0^{10} f(x)dx = 10$. [3 marks]

→ Divide the interval $[0,10]$ in $[0,1) \cup [1,2) \cup [2,3) \cup \ldots \cup [9,10]$

We partition the interval like above.

Now let ε be given, $\varepsilon > 0$, $\varepsilon \in \mathbb{R}$.

Let $S(f, \dot{P})$ be the reimann sum with tagged partition $\dot{P}$.

~~Note~~ $\dot{P}$ is a partition of $[0,10]$.

Let $\dot{P}_1$ be tagged partition of $[0,1)$, $\dot{P}_2$ be tagged partition of $[1,2)$, ...

$\therefore \dot{P} = \dot{P}_1 + \dot{P}_2 + \ldots + \dot{P}_{10}$

& $S(f, \dot{P}) = S(f, \dot{P}_1) + S(f, \dot{P}_2) + \cdots + S(f, \dot{P}_{10})$

Let $\delta_1 = \|\dot{P}_1\|$, $\delta_2 = \|\dot{P}_2\|$ ..., i.e. δ_1 is norm of partition $\dot{P}_1$

Let $\delta' = \max\{\delta_1, \delta_2, \cdots, \delta_{10}\}$

Take $\delta' < \frac{\varepsilon}{55}$ arrange all the δ_i, $1 \le i \le 10$ less than $\frac{\varepsilon}{55}$.

Now

$S(f, \dot{P}_1) = f(t_1)(x_1 - 0) + f(t_2)(x_2 - x_1) + \cdots + f(t_n)(x_n - x_{n-1})$

Now $x_{n-1} \le 1$, $x_n < 1 + \delta'$, $t_n < 1$

$\therefore S(f, \dot{P}_1) = 1(x_1 - 0) + 1(x_2 - x_1) + \cdots + 1(x_n - x_{n-1})$

$= 1(1 + \delta')$ $(\because f(t) = 1 \ \forall \ t \in (0,1)$

$\therefore S(f, \dot{P}_1) = 1 + \delta'$

PageID: 180101037-01

MarkingScheme:

PageMark:

Incomplete justification.

PageID: 180101037-01

MarkingScheme:

PageMark:

PageID: 180101037-02
MarkingScheme: 1+.5+.5+1
PageMark: 0+.5+0+0

Now

Similarly $S(f, \dot{P}_2) = 1+\delta'$

Now for $\dot{P}_3$, $[2,3)$,

$S(f, \dot{P}_3) = f(t_1)(x_1-2) + f(t_2)(x_2-x_1) + \dots + f(t_n)(x_n - x_{n-1})$

$\leq 2(x_1-2) + f(t_2)(x_2-x_1) + \dots + f(t_n)(x_n - x_{n-1})$

$(\because$ if $t_1 = 2$, then $f(t_1) = 2$ & $2 \leq t_n < 3$, otherwise $f(t_1) = 1)$

$= (x_1-2) + (x_1-2) + 1(x_2-x_1) + \dots + 1(x_n - x_{n-1})$

$\leq \delta' + 1 + \delta' = 1 + 2\delta''$

↳ remaining sum

Similarly $S(f, \dot{P}_4) = 1 + 3\delta''$

$\vdots$

$S(f, \dot{P}_{10}) = 1 + 9\delta'$

$\therefore S(f, \dot{P}) \leq 10 + 46\delta'$

$\therefore |S(f, \dot{P}) - 10| = 46\delta' \leq 46 \times \frac{\varepsilon}{55} < \varepsilon$

$\therefore$ For $\|\dot{P}\| < \frac{\varepsilon}{55}$,

$|S(f, \dot{P}) - 10| < \varepsilon$

$\therefore$ According to def^n of remainn integral,

$S(f, \dot{P}) = 10$

i.e. $\int_0^{10} f(x)dx = 10$

Def^n of remainn integral: $\int_a^b f(x)dx = S(f, \dot{P}) = L$

if $\forall \varepsilon > 0, \varepsilon \in \mathbb{R}$, $\exists \delta$ s.t. when $\|\dot{P}\| < \delta$,

$|S(f, \dot{P}) - L| < \varepsilon$

Roll... 180101037
Name: Kotkar Anket Sanjay

2. Is the set $\{(x, y, z) \in \mathbb{R}^3 : 3x - y + 2z = 0\}$ a closed subset of $\mathbb{R}^3$? If so, give a proof and if not, justify. [3 marks]

→ Let (x_n) be a convergent sub. of x
(y_n) be of y
& (z_n) be of z.

Let $\lim x_n \to a$
$\lim y_n \to b$
& $\lim z_n \to c$

Now $\exists\, \delta$ s.t. , $\delta = \frac{\varepsilon}{4}$

$a - \delta < x_n < a + \delta$
$b - \delta < y_n < b + \delta$
$c - \delta < z_n < c + \delta$

$\therefore 3a - b + 2c - 4\delta < 3x_n - y_n + 2z_n < 3a - b + 2c + 4\delta$

$\therefore 3x_n - y_n + 2z_n = 0$

$\therefore |(3a - b + 2c) - 0| < 4\delta$

Let $\delta = \frac{\varepsilon}{4}$.

$|(3a - b + 2c) - 0| < 4 \times \frac{\varepsilon}{4} = \varepsilon$ ✓

$\therefore$ It is closed.

PageID: 180101037-03
MarkingScheme:
PageMark:

PageID: 180101037-03
MarkingScheme:
PageMark:

PageID: 180101037-04
MarkingScheme: 1+1+1
PageMark: 1+1+1

BlankPage

PageID: 180101037-04
MarkingScheme: 1+1+1
PageMark: 1+1+1

BlankPage

Roll... 180101037
Name: Kotkar Anket Sanjay

3. Consider the function $f(x,y) = 4x^3 + 3\sqrt{3}y$ for $(x,y) \in \mathbb{R}^2$ and $S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$. Let g be the restriction of f to the set S.

   (a) Show that g has a maximum and a minimum value on the set S. [2 marks]

   (b) Find the maximum and minimum values of g on the set S. [2 marks]

PageID: 180101037-05

MarkingScheme:

PageMark:

BlankPage

PageID: 180101037-05

MarkingScheme:

PageMark:

BlankPage

Roll... 180101037
Name: Kotkar Anket Sanjay

#37 of 114/MA101/Quiz-3/2...
L1/Oct. 28/p. 6 of 10/#2...

PageID: **180101037-06**

MarkingScheme: **1+1+1+1**

PageMark: **0+0+0+0**

BlankPage

PageID: **180101037-06**

MarkingScheme: **1+1+1+1**

PageMark: **0+0+0+0**

BlankPage

#37 of 114/MA101/Quiz-3/2018f
L1/Oct. 28/p. 7 of 10/#230

Roll... 180101037
Name: Kotkar Anket Sanjay

PageID: 180101037-07

MarkingScheme:

PageMark:

PageID: 180101037-07

MarkingScheme:

PageMark:

4. Let $\mathbb{Q}$ be the set of rational numbers. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as follows :

$$f(x,y) = \begin{cases} x^2 - y^2 & \text{if } x \in \mathbb{Q}, y \in \mathbb{R}, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that f is not continuous at the point $(0,1)$. [2 marks]

(b) Show that f is continuous at the point $(1,1)$. [3 marks]

(a) First path Let $x \in \mathbb{Q}$, $x \to 0$, & $y \to 1$, $y \in \mathbb{R}$

$\therefore \lim_{(x\to 0, y\to 1)} f(x,y) = x^2 - y^2$

$= 0^2 - 1^2 = -1$

Also let another path be $x \in \mathbb{Q}'$, $x \to 0$, $y \to 1$, $y \in \mathbb{R}$

$\therefore \lim_{(x\to 0, y\to 1)} f(x,y) = 0$ ($\because \mathbb{Q}'$ is set of irrational numbers)

Along 2 diff. paths, value of lim of $f(x,y)_{(x\to 0, y\to 1)}$ is diff.

$\therefore f(x,y)$ is discontinuous at $(0,1)$

(b) Let P be any path of $(x \to 1, y \to 1)$

Now $x \in \mathbb{Q}$, $y \in \mathbb{R}$

$f(x,y) = x^2 - y^2$

& when $x \in \mathbb{Q}'$, $y \in \mathbb{R}$

$f(x,y) = 0$

We can divide the path P in 2 path union P_1 & P_2.

where $P_1 = (x \to 1, x \in \mathbb{Q}, y \to 1, y \in \mathbb{R})$

& $P_2 = (x \to 1, x \in \mathbb{Q}', y \to 1, y \in \mathbb{R})$

PageID: 180101037-08
MarkingScheme: 2+1+2
PageMark: 2+0+0

$\therefore$ Now

$P_1 \cap P_2 = 0$ & $P_1 \cup P_2 = P$

$\therefore$ Along P_1,

$$\lim_{(x\to 1, y\to 1)} f(x,y) = x^2 - y^2 = 1 - 1 = 0$$

& along P_2,

$$\lim_{(x\to 1, y\to 1)} f(x,y) = 0$$

Or

let

$x \in$ δ neighbourhood of 1 & $y \in$ δ neighbhood of 1.

Also $f(1,1) = 0$

Then,

if $x \in \mathbb{Q}$, $y \in \mathbb{R}$, $1-\delta < x < 1+\delta$

then $f(x,y) = x^2 - y^2$

$\therefore \lim f(x,y) = x^2 - y^2$

$\therefore |f(x,y) - f(1,1)| \leq |(1+\delta)^2 - 1| = |2\delta + \delta^2|$

let $\delta < 2$.

$\therefore |f(x,y) - f(1,1)| < |2\delta + 2\delta|$

$= 4\delta$

Take $\delta = \frac{\varepsilon}{4}$ or 2 whichever is minimum.

$\therefore |f(x,y) - f(1,1)| < 4\delta \leq 4 \times \frac{\varepsilon}{4} = \varepsilon$

$\therefore |f(x,y) - f(1,1)| < \varepsilon$

Incorrect proof

Roll... 180101037
Name: Kotkar Anket Sanjay

Xtra space for answers, NOT rough work. Clearly MENTION Question Number.

(4) continued

if $x \in \mathbb{Q}'$, $y \in \mathbb{R}$,

then $f(x,y) = 0$

$\therefore |f(x,y) - f(1,1)| = 0 < \varepsilon$

$\therefore$ for $\delta = \min\left(\frac{\varepsilon}{4}, 2\right)$,

$|f(x,y) - f(1,1)| < \varepsilon$

$\therefore f(x,y)$ is continuous at $(1,1)$

PageID: 180101037-09

MarkingScheme:

PageMark:

PageID: 180101037-09

MarkingScheme:

PageMark:

MA101:2018:Monsoon:Quiz-3

Mathematics 1

Graded Answer Script of 180101037:Kotkar Anket Sanjay

Question	YourScore	ClassAverage	ClassMax	ClassMin
1	.5	.08	3.0	0
2	3	.27	3	0
3	0	.39	4	0
4	2	.64	5	0
TOTAL	5.5	1.40	11.5	0