Assignment for practice

1.Suppose $\{X_n, n \geq 0\}$ is a discrete-time Markov chain (DTMC)with $V = \{0,1,2\}$. Given

$$\mathbf{P} = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.5 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{bmatrix} \text{ and } \mathbf{p}^{(0)} = \begin{bmatrix} 0.5 & 0.4 & 0.1 \end{bmatrix}.$$

(a) Determine (i) $P(X_1 = 2 / X_0 = 2)$ (ii) $P(X_1 = 2, X_2 = 1 / X_0 = 0)$ and (iii) $P(X_2 = 1)$

(b) If the steady state probabilities of the chain exist, find $\lim_{n\to\infty} \mathbf{p}^{(n)}$ and $\lim_{n\to\infty} \mathbf{P}^{(n)}$

2.Consider the Markov chain represented by the state transition matrix. Answer the following by inspection

$$\mathbf{P} = \begin{bmatrix} 0 & 0.3 & 0.7 & 0 \\ 0.4 & 0.5 & 0 & 0.1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(a) Draw the state transition diagram for the chain

(b) Partition the state-space into communicating classes.

(c) Find the closed communicating class of the chain.

3. Suppose $\{X_n, n \geq 0\}$ is a discrete-time Markov chain (DTMC)with $V = \{0,1,2,3\}$ and the state transition matrix

$$\mathbf{P} = \begin{bmatrix} 0.7 & 0.3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix}$$

(a) Find the first return probability to state 3 and comment if this state is recurrent.

(b) Find the mean first return time μ_{00} to and the steady state transition probability $\lim_{n\to\infty} p_{00}^{(n)}$.

(c) Examine if state 2 is periodic.

4. Let $N(t)$ be a Poisson process with intensity λ=2, and let $T1$, $T2$, ⋯ be the corresponding interarrival times.

(a)Find the probability that the first arrival occurs after t=0.5

(b) Given that we have had no arrivals before t=1, find $P(T1>3)$
(c) Given that the third arrival occurred at time t=2, find the probability that the fourth arrival occurs after t=4
(d) You start watching the process at time t=10. Let T be the time of the first arrival that you see. (i) Find ET and Var(T). (ii) Find the conditional expectation and the conditional variance of T given you are informed that the last arrival occurred at time t=9
5.(a)Consider a finite-state CTMC {X(t)}, with the generator matrix **Q**. If the steady-state probability vector $\boldsymbol{\pi}$ exists, use with the forward Kolmogorov equation to show that
$\boldsymbol{\pi}\mathbf{Q} = \mathbf{0}$
(b) Consider a CTMC with $V = \{0,1,2\}$ and the transition probability matrix of the embedded MC as

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 1 \end{bmatrix}$$

(i) Obtain the generator matrix