Tutorial- 11 (Solutions)

Q.1. For finding the equivalent inductance at ab terminals, let us assume a voltage $\boldsymbol{v}$ and currents $\boldsymbol{i}, \boldsymbol{i_1}$ and $\boldsymbol{i_2}$ as shown in Fig. S1. We can use KVL for loop1 and loop2:

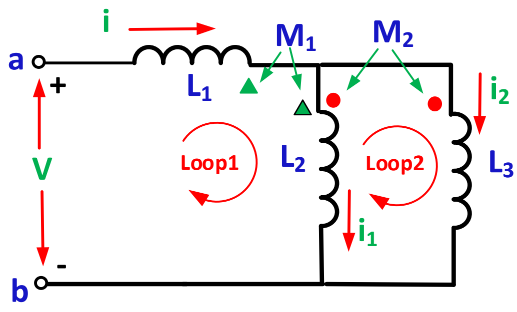


Fig. S1

$$\boldsymbol{i = i_1 + i_2}$$

For loop1, applying the KVL, $\boldsymbol{v = L_1\frac{di}{dt} + L_2\frac{di_1}{dt} - M_1\frac{di_1}{dt} - M_1\frac{di}{dt} + M_2\frac{di_2}{dt}}$

Similarly applying KVS in loop2, $\boldsymbol{L_2\frac{di_1}{dt} - L_3\frac{di_2}{dt} + M_2\frac{di_2}{dt} - M_1\frac{di}{dt} - M_2\frac{di_1}{dt} = 0}$

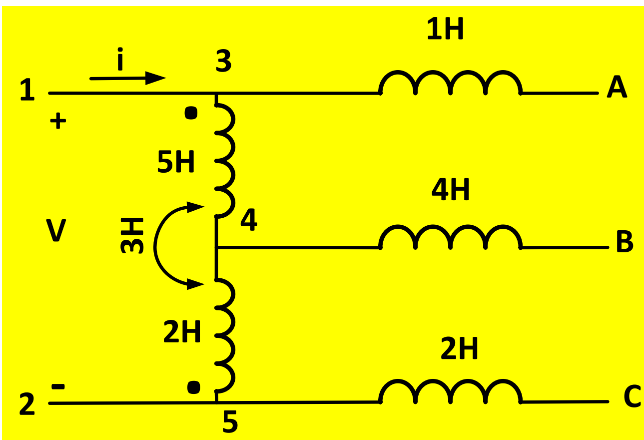
Solving the three equations and finding the relation between $\boldsymbol{v}$ and $\boldsymbol{i}$, one can find the equivalent inductance at ***ab*** terminal.

Q.2. Voltage induced in a coil due to a current in the second coil will have its +ve polarity at the dotted terminal, if the current enters into the dotted terminal at the second coil. Similarly, the induced voltage will have –ve polarity if the current leaves the dotted terminal.

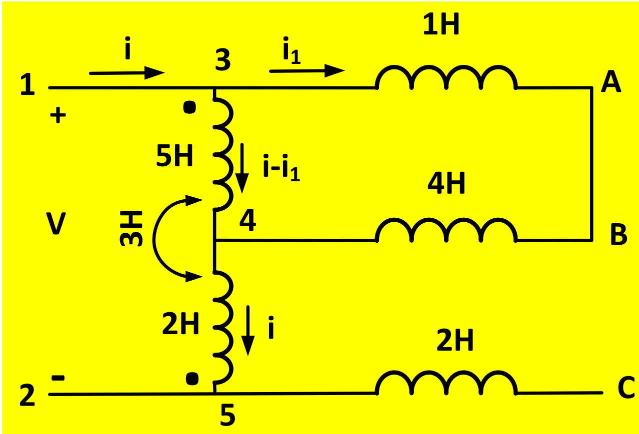
(a) Applying KVL in the loop 134521,

$\boldsymbol{v - 5\frac{di}{dt} + 3\frac{di}{dt} - 2\frac{di}{dt} + 3\frac{di}{dt} = 0 \Rightarrow v - \frac{di}{dt} = 0}$

Leq = 1 H



(b)



Applying KVL in the loop 134521,

$$V - 5\,\frac{d(i-i_1)}{dt} + 3\,\frac{di}{dt} - 2\,\frac{di}{dt} + 3\,\frac{d(i-i_1)}{dt} = 0 \;\Rightarrow V - \frac{di}{dt} + 2\,\frac{di_1}{dt} = 0 \quad (1)$$

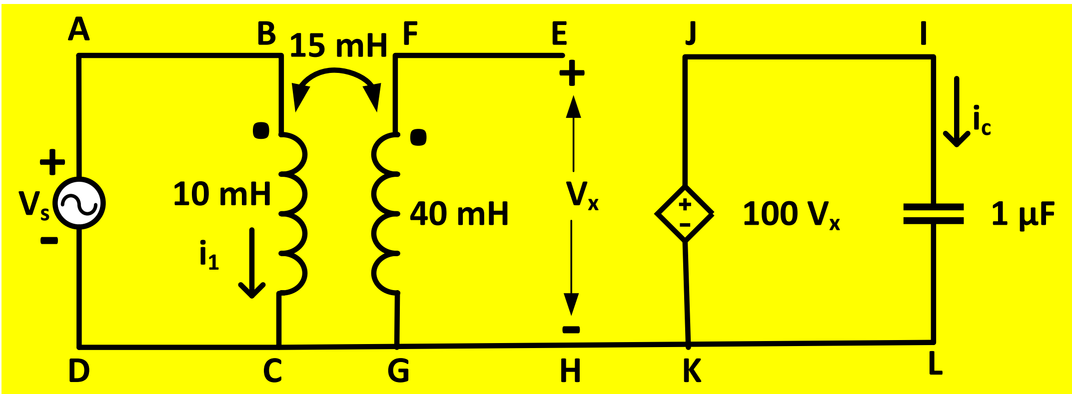
Applying KVL in the loop 3AB43,

$$-\frac{di_1}{dt} - 4\,\frac{di_1}{dt} + 5\,\frac{d(i-i_1)}{dt} - 3\,\frac{di}{dt} = 0 \;\Rightarrow \frac{di_1}{dt} = \frac{1}{5}\frac{di}{dt} \quad \text{---------------------}(2)$$

Replacing the value of $\frac{di_1}{dt}$ from (2) in equation (1)

$$V - \frac{di}{dt}\left(1 - \frac{2}{5}\right) = 0 \;\Rightarrow L_{eq} = \frac{3}{5}\,H$$

Q.3.



Applying KVL in the loop ABCDA

$$\frac{10t^2}{t^2 + 0.01} = 10 \times 10^{-3}\,\frac{di_1}{dt}$$

$$\Rightarrow \frac{di_1}{dt} = \frac{1000t^2}{t^2 + 0.01}$$

There will be an induced voltage in 40 mH coil due to the current $\boldsymbol{i_1}$ in 10 mH coil. Applying KVL in the loop EFGHE

$$\mathbf{15 \times 10^{-3} \frac{d i_1}{dt} = V_x}$$

$$\Rightarrow \boldsymbol{V_x = \frac{15t^2}{t^2 + 0.01}}$$

Applying KVL in the loop LKJIL

$$\mathbf{100 V_x - \frac{\int i_c dt}{C} = 0}$$

$$\Rightarrow \boldsymbol{i_c = 100C \times \frac{dV_x}{dt} = \frac{0.03t}{(t^2 + 0.01)^2}\ mA}$$

EE101: Basic Electronics

Theme: Micro-electronics

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Tutorial Problems

Solutions

Q4.

1. The diode equation can be rewritten in the following form: $V_D = V_T \ln\left(\frac{I_D}{I_s}\right)$.

2. From Fig. 1 and using the above equation, we can write the following expressions for output voltages.

$$V_1 = V_T \ln\left(\frac{nI_0}{I_s}\right) \quad (1)$$

$$V_2 = V_T \ln\left(\frac{I_0}{I_s}\right) \quad (2)$$

$$V_{OUT} = V_1 - V_2 = V_T \ln(n) \quad (3)$$

3. Recall that $V_T = \frac{kT}{q}$ is the thermal voltage. Here k is the Boltzmann constant, T is the absolute temperature in Kelvin and q is the magnitude of the electron charge.

4. $\frac{\partial V_{OUT}}{\partial T} = \frac{k}{q}\ln(n)$ is a positive constant!

Inference: Even though the individual voltages V_1 and V_2 have negative temperature coefficients (-2mV/°C), the differential voltage $(V_1 - V_2)$ has a positive constant temperature coefficient. Moreover, the voltage V_{OUT} is proportional to the absolute temperature. This circuit can be used as a temperature sensor.

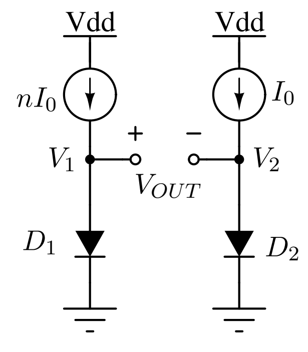

Figure 1: Diode circuits

Q5.

1. V_1 (or V_2) has a negative temperature coefficient and V_{OUT} has a positive temperature coefficient.

2. If we can generate a voltage $V_{OUT,0T} = A_1 V_1 + A_2 V_{OUT}$, then

$$\frac{\partial V_{OUT,0T}}{\partial T} = A_1(-2mV/°C) + A_2(\frac{k}{q}\ln(n)) \approx [-2A_1 + 0.087 \times A_2 \ln(n)] \ \ \mathrm{mV/°C}.$$

3. If we choose $0.087 \times A_2 \ln(n) = 2A_1$, then $\frac{\partial V_{OUT,0T}}{\partial T} = 0$ and the voltage $V_{OUT,0T}$ becomes independent of temperature.

4. $V_{OUT,0T} = A_1 V_1 + A_2 V_{OUT} = (A_1 + A_2)V_1 - A_2 V_2$.

5. Notes on realization:

   - We need to sense the voltages V_1 and V_2 without drawing any current from the diode circuit.
   - Since V_1 and V_2 require two different scaling factors $(A_1 + A_2)$ and A_2 respectively, the 3-opamp realization shown in Fig. 2 can be used[1].

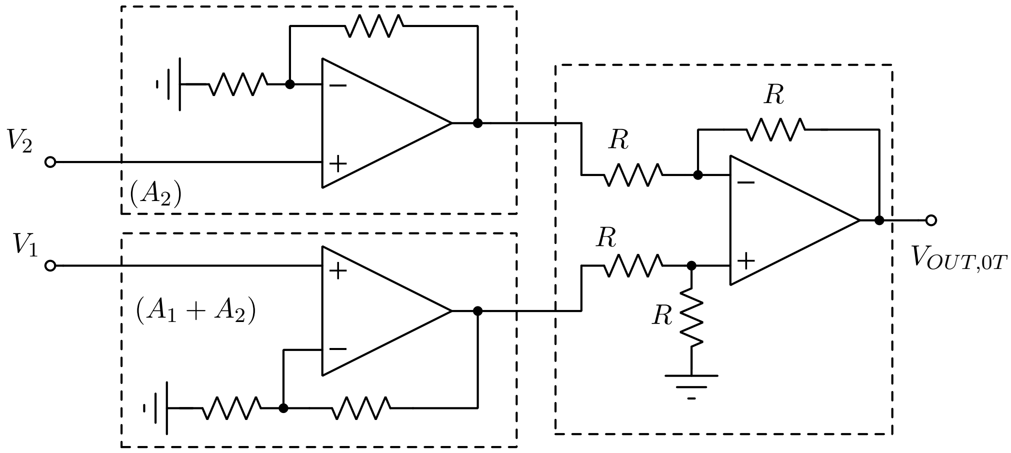

Figure 2:

[1]Important: This realization will not work in practice as the characteristics of opamps, resistors, and current sources change with temperature.