PH102: Tutorial Problem set

Tutorial 2

2018-10-24

2.01. Find the volume of the region common to the intersecting cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ (See Figure 1)

2.02. Verify the divergence theorem for $\vec{A} = 4x\hat{x} - 2y^2\hat{y} + z^2\hat{z}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$ (see Figure 2).

2.03. Prove $\int\int\int_V \vec{\nabla}\phi \; dV = \int\int_S \phi \; \hat{n} dS$.

2.04. Prove $\int\int\int_V \vec{\nabla} \times \vec{B} \; dV = \int\int_S \hat{n} \times \vec{B} \; dS$.

2.05. (a) Verify Stoke's theorem by calculating the line integral of $\vec{F} = 2z\hat{x} + x\hat{y} + y\hat{z}$ over a circle of radius R in the xy plane centered at the origin, where the open surface is the hemisphere in $z > 0$ (see Fig. 3).

(b) Calculate the same line integral using Divergence theorem imagining the hemispherical surface as well as the disc on the $x - y$ plane to form a closed surface.

2.06. Prove $\oint d\vec{r} \times \vec{B} = \int\int_S (\hat{n} \times \vec{\nabla}) \times \vec{B} \; dS$.

Take home problems

H2.01. Prove $\int\int\int_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right) = \int\int_S \left(\phi\vec{\nabla}\psi - \psi\vec{\nabla}\phi\right) \cdot d\vec{S}$.

H2.02. Prove the identity $\int\int_S \vec{\nabla}\phi \times d\vec{S} = -\oint_C \phi d\vec{r}$.

H2.03. Evaluate $\oint_C \vec{r} \times d\vec{r}$ by using the identity in Problem (2.06) where the loop is on the $x - y$ plane. Check that if the magnitude is twice the area enclosed by the loop C.

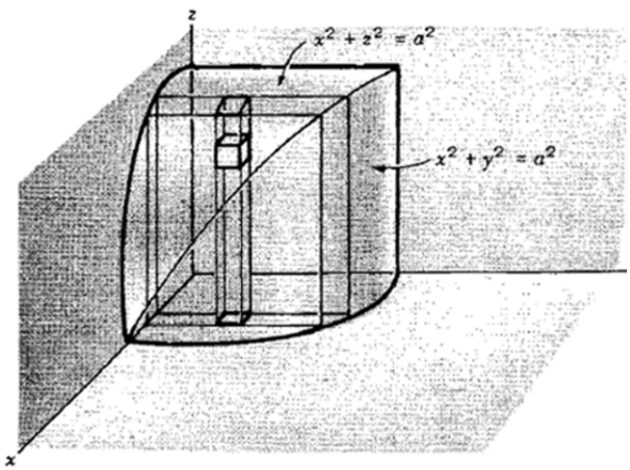

Figure 1: Problem 2.01

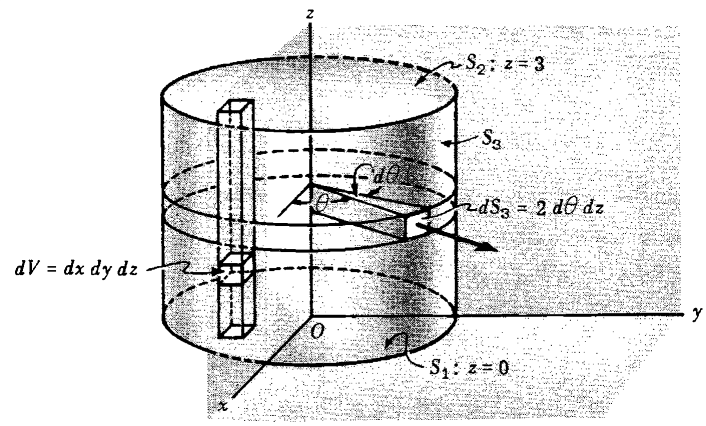

Figure 2: Problem 2.02

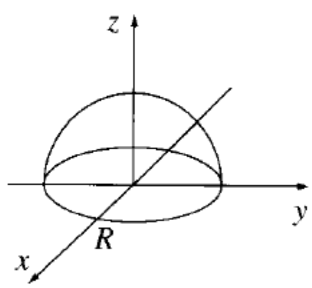

Figure 3: Problem 2.05