PH102: Tutorial Problem set

Tutorial 6

2019-2-15

6.01. According to quantum mechanics, the electron cloud for a hydrogen atom in ground state has a charge density

$$\rho(r) = \frac{q}{\pi a^3} e^{-2r/a} \ , \tag{1}$$

where q is the charge of the electron and a is the Bohr radius. Find the atomic polarizability of such an atom.

6.02. (a) Show that the electric field of a 'pure dipole' can be written as

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\vec{p}.\hat{r})\hat{r} - \vec{p}]$$

Note that this form has the advantage of not committing to a particular coordinate system.

(b) Find the force and torque on a dipole in the field of a point charge. Let the charge q be at the origin and the dipole $\vec{p} = p_0(\sin\zeta_0\hat{x} + \cos\zeta_0\hat{z})$ be at the point $(0, 0, z_0)$. Also find the force on q due to the dipole and verify Newton's third law.

6.03. Energy of a dipole:

(a) Show that the energy of a dipole with dipole moment $\vec{p}$ in an electric field $\vec{E}$

$$U = -\vec{p}.\vec{E} \ . \tag{2}$$

(b) Show that the interaction between two dipoles with dipole moments $\vec{p}_1$ and $\vec{p}_2$ separated by distance $\vec{r}$ is given by:

$$U = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[\vec{p}_1.\vec{p}_2 - 3(\vec{p}_1.\hat{r})(\vec{p}_2.\hat{r})] \ . \tag{3}$$

6.04. Two long coaxial cylindrical metal tubes (inner radius a and outer radius b) stand vertically in a tank of dielectric oil (susceptibility χ_e, mass density ρ). The inner one is maintained at a potential V and the outer one is grounded. To what height does the oil rise in the space between the tubes?

6.05. A spherical conductor of radius a, carries a charge Q. It is surrounded by linear dielectric material of susceptibility χ_e, out to radius b. Find the energy of this configuration.

6.06. A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a "frozen-in" polarization $\vec{P}(\vec{r}) = \frac{k}{r}\hat{r}$, where k is a constant and r is the distance from the center. There is no free charge in the problem. Find the electric field in all three regions by two different methods:

(a) Locate all the bound charge, and use Gauss's law to calculate the field it produces.

(b) Use $\oint \vec{D}.d\vec{S} = Q_{f_{encl}}$ to find $\vec{D}$ and then get $\vec{E}$ from $\vec{D} = \epsilon_0\vec{E} + \vec{P}$.

6.07. Given that $\vec{E}_1 = 2\hat{x} - 3\hat{y} + 5\hat{z}$ at the charge free dielectric interface of Figure 2. Find $\vec{D}_2$ and the angles θ_1 and θ_2.

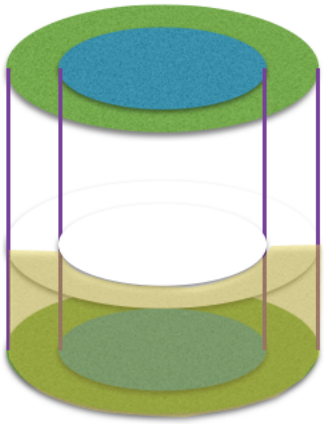

Figure 1: Problem 6.04

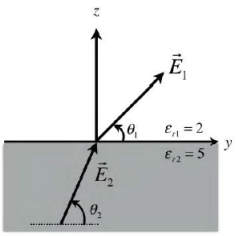

Figure 2: Problem 6.07