1. Suppose that $\mathbf{x}$ and $\mathbf{y}$ are in $\mathbf{R}^n$ and $\mathbf{x} \neq \mathbf{y}$. Show that there is a continuous function $f : \mathbb{R}^n \to \mathbb{R}$ with $f(\mathbf{x}) = 1$, $f(\mathbf{y}) = 0$ and $0 \leq f(\mathbf{z}) \leq 1$ for every $\mathbf{z} \in \mathbf{R}^n$.

2. Consider $f : \mathbb{R}^2 \to \mathbb{R}$ given by
$$f(x,y) = \begin{cases} x \sin\left(\frac{1}{x}\right) + y \sin\left(\frac{1}{y}\right), & \text{if } x \neq 0, y \neq 0 \\ x \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0, y = 0 \\ y \sin\left(\frac{1}{y}\right), & \text{if } x = 0, y \neq 0 \\ 0 & \text{if } x = 0, y = 0. \end{cases}$$
   (a) Show that f is continuous at $(0,0)$.
   (b) Show that none of the partial derivatives of f exist at $(0,0)$.

3. In each of the following case, determine whether the function f is differentiable at $(0,0)$:
   (a) $f(x,y) = \begin{cases} e^{-\frac{1}{x^2+y^2}}, & \text{if } x^2+y^2 \neq 0 \\ 0, & \text{if } x = y = 0 \end{cases}$
   (b) $f(x,y) = \sqrt{|xy|}$

4. Show that the function $f(r,\theta) = \frac{1}{2} r \sin 2\theta, r > 0$ is differentiable at every point in its domain. Determine whether this function is of class C^1.

5. Use linear approximation to calculate:
   (a) $\sin 29° \cdot \tan 46°$
   (b) $\frac{1.03^2}{\sqrt[3]{0.98\sqrt[4]{1.05^3}}}$

6. Let
$$f(x,y) = \begin{cases} (x^2+y^2) \sin \frac{1}{x^2+y^2}, & \text{if } x^2+y^2 \neq 0 \\ 0, & \text{if } x = y = 0 \end{cases}$$
   Prove that f is differentiable at $(0,0)$ but its partial derivatives are not continuous at $(0,0)$.

7. Let a, b be two real numbers. Show that the function $f(x,y) = ax + by, (x,y) \in \mathbb{R}^2$ is differentiable at every point in its domain and that the vector (a,b) is its derivative. Hence show that the tangent plane to the graph of f at any point on the graph co-incides with the graph of f.

8. Consider the surface $S : z = x^2 + 3y^2$.
   (a) Find the slope of the tangent line to the curve of intersection of the surface S and the plane $y = 1$ at the point $(1,1,4)$.
   (b) Find a parametric equation for the tangent line whose slope you computed in part (a).
   (c) Find the slope of the tangent line to the curve of intersection of the surface S and the plane $x = 1$ at the point $(1,1,4)$.
   (d) Find a parametric equation for the tangent line whose slope you computed in part (b).
   (e) Find an equation of the tangent plane to the surface S at the point $(1,1,4)$.

9. Find the equation of the tangent plane to the graph $z = \cos x \cos y$ at the point $(0, \frac{\pi}{2}, 0)$.

10. Find the linear approximation of $f(x,y) = (xe^y + \cos y, x, x + e^y)$ at the point $(1,0)$.