Tutorial-1: Solutions:

1. The simplified Boolean expressions for the logic gate networks are as below:
    a. $Z = X\bar{Y} + \bar{X}Y$
    This cannot be simplified further. This is the XOR gate.
    b. $Z = (X + \bar{Y})(\bar{X} + Y) = XY + \bar{X}\bar{Y}$
    This cannot be simplified further. This is the XNOR gate.
    c. $Z = (X + \bar{X})(\bar{Y} + Y) = 1$
    d. $Z = X\bar{X} + Y\bar{Y} = 0$

2. The glow of the bulb will decrease.
3. This is an example of a linear function: where the plot describing the data set traces a straight line on a graph. From this line, and also from the numerical figures, you should be able to discern a constant ratio between voltage and current.
    Notes:
    The raw data figures were made intentionally "noisy" in this problem to simulate the types of measurement errors encountered in real life. One tool which helps overcome interpretational problems resulting from noise like this is graphing. Even with noise present, the linearity of the function is quite clearly revealed.
    The students should learn to make graphs as tools for their own understanding of data. When relationships between numbers are represented in graphical form, it lends another mode of expression to the data, helping people to apprehend patterns easier than by reviewing rows and columns of numbers.

4. The greater the resistance, the steeper the slope of the plotted line.
    The proper way to express the derivative of each of these plots is $[\frac{dv}{di}]$. The derivative of a linear function is a constant, and in each of these three cases that constant equals the resistor resistance in ohms. So, we could say that for simple resistor circuits, the instantaneous rate-of-change for a voltage/current function is the resistance of the circuit.

5. Unlike a resistor, which offers a relatively fixed (unchanging) amount of resistance to the motion of electrons over a wide range of operating conditions, the electrical resistance of light bulbs typically change dramatically over their respective operating ranges.
    From the graphs, determine where the resistance for each type of light bulb is at its maximum, and where the resistance is at its minimum.
    Notes:
    Many types of electrical and electronic components experience changes in electrical resistance over their operating ranges of current and voltage. Resistors, while simple to study, do not exhibit the behavior of most electronic components. It is important for students to understand that the real world of electricity and electronics is much more complex than what Ohm's Law might suggest (with an implicit assumption of fixed resistance). This is one concept that graphs really help to illustrate.

6. For the lamp L to be ON, S_1 and S_2 needs to be ON. Also, S_3 and S_4 cannot both be ON. Thus we have the truth table:

S_1	S_2	S_3	S_4	L
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

The standard SOP form is $L = S_1S_2\overline{S_3S_4} + S_1S_2S_3\overline{S_4} + S_1S_2\overline{S_3}S_4$.

7. The truth table describing the logic gate network is given below

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

The standard SOP form is $F = \bar{X}Y\bar{Z} + \bar{X}YZ + X\bar{Y}\bar{Z} + X\bar{Y}Z + XY\bar{Z} + XYZ$.
The standard POS form is $F = (X + Y + Z)(X + Y + \bar{Z})$.

8. The identities can be proven by considering all possible value the variable X_1 can take:
   e. Consider the identity

$$F(X_1, X_2, \cdots, X_n) = X_1.F(1, X_2, \cdots, X_n) + \overline{X_1}.F(0, X_2, \cdots, X_n).$$

Evaluating the LHS of the identity at $X_1 = 0$ we get

$$F(0, X_2, \cdots, X_n).$$

Evaluating the RHS of the identity at $X_1 = 0$ we get

$$0. F(1, X_2, \cdots, X_n) + 1. F(0, X_2, \cdots, X_n) = F(0, X_2, \cdots, X_n)$$

Thus LHS = RHS when $X_1 = 0$.

Evaluating the LHS of the identity at $X_1 = 1$ we get

$$F(1, X_2, \cdots, X_n).$$

Evaluating the RHS of the identity at $X_1 = 1$ we get

$$1. F(1, X_2, \cdots, X_n) + 0. F(0, X_2, \cdots, X_n) = F(1, X_2, \cdots, X_n)$$

Thus LHS = RHS when $X_1 = 1$.

This proves the identity for all values of $X_1, X_2, \cdots, X_n$.

f. Consider the identity

$$F(X_1, X_2, \cdots, X_n) = [X_1 + F(0, X_2, \cdots, X_n)]. [\overline{X_1} + F(1, X_2, \cdots, X_n)]$$

Evaluating the LHS of the identity at $X_1 = 0$ we get

$$F(0, X_2, \cdots, X_n).$$

Evaluating the RHS of the identity at $X_1 = 0$ we get

$$[0 + F(0, X_2, \cdots, X_n)]. [1 + F(1, X_2, \cdots, X_n)] = F(0, X_2, \cdots, X_n)$$

Thus LHS = RHS when $X_1 = 0$.

Evaluating the LHS of the identity at $X_1 = 1$ we get

$$F(1, X_2, \cdots, X_n).$$

Evaluating the RHS of the identity at $X_1 = 1$ we get

$$[1 + F(0, X_2, \cdots, X_n)]. [0 + F(1, X_2, \cdots, X_n)] = F(1, X_2, \cdots, X_n)$$

Thus LHS = RHS when $X_1 = 1$.

This prove the identity for all values of $X_1, X_2, \cdots, X_n$.