MA 102 (Mathematics II)

Tutorial Sheet No. 7 **Ordinary Differential Equations** **March 28, 2019**

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1. Determine the *order* and *degree* of the following differential equations. Also, state whether they are *linear* or *nonlinear*.
   $(a)\ \frac{d^4y}{dx^4} + 19\left(\frac{dy}{dx}\right)^2 = 11y$; $(b)\ \frac{d^2y}{dx^2} + x\sin y = 0$; $(c)\ \frac{d^2y}{dx^2} + y\sin x = 0$; $(d)\ (1+\frac{dy}{dx})^{\frac{1}{2}} = x\frac{d^2y}{dx^2}$;
   $(e)\ \frac{d^6y}{dx^6} + \left(\frac{d^4y}{dx^4}\right)\left(\frac{d^3y}{dx^3}\right) + y = x$; $(f)\ x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} + y = e^x$.

2. Eliminating the arbitrary constants c_1, c_2, obtain the differential equation satisfied by the following functions.
   $(a)\ y = c_1e^{-x} + c_2e^{2x}$; $(b)\ x^2 + c_1y^2 = 1$; $(c)\ y = c_1x - c_1^3$.

3. Consider the equation $y'(x) = cy(x)$, $0 < x < \infty$, where c is a real constant. Then
   (a) Show that if ϕ is any solution and $\psi(x) = \phi(x)e^{-cx}$ then $\psi(x)$ is a constant.
   (b) If $c < 0$, show that every solution tends to zero as $x \to \infty$.
   (c) If $c > 0$, prove that the magnitude of every non-trivial solution tends to ∞ as $x \to \infty$.
   (d) When $c = 0$, what can be said about the magnitude of the solution?

4. Find all real valued C^1 solutions $y(x)$ of the differential equation $xy'(x) + y(x) = x$, $x \in (-1, 1)$.

5. Discuss the existence and uniqueness of a solution of the following initial value problems (IVP) in the region $R: |x| \leq 1\ |y| \leq 1$.
   $(a)\ \dfrac{dy}{dx} = 3y^{2/3},\ \ y(0) = 0$; $(b)\ \dfrac{dy}{dx} = \sqrt{|y|},\ \ y(0) = 0$;
   $(c)\ \dfrac{dy}{dx} = x^2 + y^2,\ \ y(0) = 0$.

6. Show that the equation $|y'(x)| + |y(x)| + 1 = 0$ has no real solutions.

7. Find the particular solution of
   $(a)\ \ xy' + 3y = \frac{\sin x}{x^2},\ x \neq 0,\ \ y(\pi/2) = 1.$
   $(b)\ \ y' + y = f(x),\ \ y(0) = 0$, where $f(x) = \begin{cases} 2, & 0 \leq x < 1, \\ 0, & x \geq 1. \end{cases}$
   $(c)\ \ x^2y' + xy = \frac{y^3}{x},\ \ y(1) = 1,\ \ x \neq 0.$

8. Under what conditions, the following differential equations are exact?
   $(a)\ (ax + by)dx + (kx + ly)dy = 0$; $(b)\ [f(x) + g(y)]dx + [h(x) + l(y)]dy = 0$;
   $(c)\ (x^3 + xy^2)dx + (ax^2y + bxy^2)dy = 0$.

9. Are the following equations exact? If exact, obtain the general solution.
   $(a)\ (2xy - \sec^2 x)dx + (x^2 + 2y)dy = 0$. $(b)\ (x - 2xy + e^y)dx + (y - x^2 + xe^y)dy = 0$.

10. In each case find an integrating factor and solve:
    $(a)\ y' - (2/x)y = x^2\cos x$, $(b)\ ydx + (x^2y - x)dy = 0$, $(c)\ y(2x^2y^3 + 3)dx + x(x^2y^3 - 1)dy = 0$

11. Show that if $(N_x - M_y)/(xM - yM) = g(xy)$ then the equation $M(x, y)dx + N(x, y)dy = 0$ has an integrating factor of the form $\mu(xy)$, where $\mu(u) = \exp\left(\int g(u)du\right)$.

12. Given that $y_1(x) = x$ is a solution of $\dfrac{dy}{dx} = -y^2 + xy + 1$, obtain the general solution.

13. Find the value of n such that the curves $x^n + y^n = c_1$ are the orthogonal trajectories of the family $y = \dfrac{x}{1 - c_2 x}$, where c_1 and c_2 are arbitrary constants.

14. A point P is dragged along the xy plane by a string PT of length a. If T starts at the origin and moves along the positive y axis, and if P starts at $(a, 0)$, what is the path of P?

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