1. Let $A \subset \mathbb{R}$ and $\mathbf{f} : A \to \mathbb{R}^n$ and $\mathbf{g} : A \to \mathbb{R}^n$ be two differentiable functions. Show that the function $h : A \to \mathbb{R}$ defined by $h(t) = (\mathbf{f}(t) \cdot \mathbf{g}(t))$ is differentiable for every $t \in A$. What is the derivative of h in terms of $\mathbf{f}$, $\mathbf{g}$ and their derivatives?

2. Suppose that $\mathbf{f} : A \subset \mathbb{R} \to \mathbb{R}^n$ is a differentiable function such that $\mathbf{f}'(t) = \mathbf{0} \in \mathbb{R}^n$ for every $t \in A$. Show that $\mathbf{f}$ is a constant function.

3. Suppose that c is positive real number and $\mathbf{f} : [a, b] \to \mathbb{R}^n$ is a twice-differentiable path in $\mathbb{R}^n$ such that $\|\mathbf{f}'(t)\| = c$ for every $t \in [a, b]$. Show that $\mathbf{f}'(t)$ and $\mathbf{f}''(t)$ are perpendicular for every $t \in [a, b]$.

4. Find the arc length of the given parametrized plane/space curves on the specified intervals:
   (a) $\mathbf{f}(t) = (t, t^2)$ on $[0, 1]$
   (b) $\mathbf{f}(t) = (\cos 3t, \sin 3t, t)$ on $[0, 3\pi]$
   (c) $\mathbf{f}(t) = (2(1 - \cos t) \cos t, 2(1 - \cos t) \sin t)$, on $[0, \pi]$

5. Show that the tangent lines to the regular parametrized curve $\mathbf{f}(t) = (3t, 3t^2, 2t^3)$ make a constant angle with the line $y = 0, z = x$ in $\mathbb{R}^3$.

6. Let $\mathbf{f} : [a, b] \to \mathbb{R}^3$ be a differentiable and regular parametrization of a curve with endpoints $\mathbf{p} := \mathbf{f}(a)$ and $\mathbf{q} := \mathbf{f}(b)$.
   (a) Show that for any constant vector $\mathbf{v} \in \mathbb{R}^3$ with $\|v\| = 1$,
   $$(\mathbf{q} - \mathbf{p}) \cdot \mathbf{v} = \int_a^b (\mathbf{f}'(t) \cdot \mathbf{v}) dt \leq \int_a^b \|\mathbf{f}'(t)\| dt$$
   (b) Using part (a) show that
   $$\|\mathbf{f}(b) - \mathbf{f}(a)\| \leq \int_a^b \|\mathbf{f}'(t)\| dt$$
   that is, the curve of shortest length from $\mathbf{f}(a)$ to $\mathbf{f}(b)$ is the straight line joining these points.

7. (a) Define $f(x, y) := xy$ for $(x, y) \in \mathbb{R}^2$. Draw the level curves of this curve corresponding to the levels: $2, 3, 0, -1$
   (b) Define $f(x, y) := (3 - x^2 - y^2)^2$ for $(x, y) \in \mathbb{R}^2$. Draw the level curves of this curve corresponding to the levels: $0, 2, 9, 16$.

8. Discuss the continuity of the following functions at the point $(0, 0)$:
   (a) $f(x, y) = \begin{cases} \frac{x^3 y}{x^6 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$
   (b) $f(x, y) = \begin{cases} xy\frac{x^2 - y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$

9. Let
   $$f(x, y) = \frac{x^2 y^2}{x^2 y^2 + (x - y)^2} \quad for (x, y) \neq (0, 0).$$
   Show that the iterated limits
   $$\lim_{x \to 0} \left[\lim_{y \to 0} f(x, y) \right] \quad and \quad \lim_{y \to 0} \left[\lim_{x \to 0} f(x, y) \right]$$
   exist and both are equal to 0, but $\lim_{(x,y) \to (0,0)} f(x, y)$ does not exist.