PH 102 : Electromagnetism
Tutorial 12
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1. Consider the propagation of an electromagnetic wave in a medium characterised by complex dielectric constant $\epsilon = \epsilon_r - i\epsilon_i$. An x-polarised electromagnetic wave propagating in the z-direction is given by $\vec{E} = E_0 e^{i(\omega t - kz)}\hat{x} = E_0 e^{-\gamma z} e^{i(\omega t - \beta z)}\hat{x}$. Calculate the magnetic field and the time averaged Poynting vector.

2. Light of angular frequency ω passes from medium 1. through a slab (thickness d) of medium 2, and into medium 3 (for instance, from water through glass into air, as shown in figure 1). Show that the transmission coefficient for normal incidence is given by
$$T^{-1} = \frac{1}{4n_1n_3}\left[(n_1+n_3)^2 + \frac{1}{n_2^2}(n_1^2 - n_2^2)(n_3^2 - n_2^2)\sin^2\left(\frac{n_2\omega d}{c}\right)\right].$$

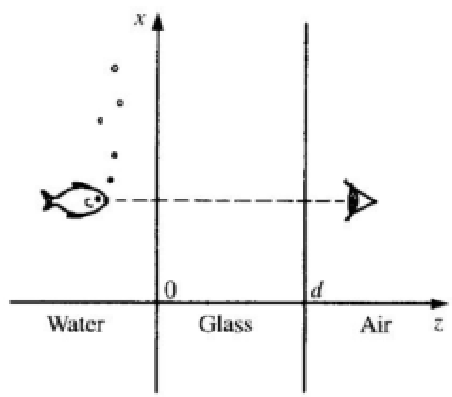

Figure 1: Figure for problem 2, 4.

Hint: To the left, there is an incident wave and a reflected wave; to the right, there is a transmitted wave; inside the slab there is a wave going to the right and a wave going to the left. Express each of these in terms of its complex amplitude, and relate the amplitudes by imposing suitable boundary conditions at the two interfaces. All three media are linear and homogeneous; assume $\mu_1 = \mu_2 = \mu_3 = \mu_0$.

3. A microwave antenna is radiating at 10 GHz is to be protected from the environment by a plastic shield of dielectric constant 2.5. What is the minimum thickness of the shielding that will allow perfect transmission (assuming normal incidence)? Hint: Use the result for transmission coefficient in problem 2.

4. Light from an aquarium (figure 1) goes from water ($n = 4/3$) through a plane of glass ($n = 3/2$) into air ($n = 1$). Assuming it is a monochromatic plane wave and that it

strikes the glass at normal incidence, find the minimum and maximum transmission coefficients. You can see the fish clearly, how well can it see you? Hint: Use the result for transmission coefficient in problem 2.

5. Consider the incidence of s plane electromagnetic wave (with its electric field perpendicular to the plane of incidence) at the interface of two dielectrics with $n_2 < n_1$. Assume the angle of incidence θ_1 to be greater than the critical angle θ_c. Calculate time averaged Poynting vector $\langle S_{2x}\rangle, \langle S_{2z}\rangle$ (where $\vec{S}_2$ is the Poynting vector associated with the transmitted wave) and interpret the results physically.

1 Take Home Problem

1. Consider the propagation of en electromagnetic wave in a conducting medium where $\vec{J} = \sigma\vec{E}$ with σ representing the conductivity of the medium.
(a) Show that $\vec{E}$ would now satisfy the equation

$$\nabla^2\vec{E} - \mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

(b) Assume a plane wave solution of the above equation of the form $\vec{E} = \vec{E}_0 E^{i(\omega t - kz)}$. Write $k = \beta - i\gamma$ and obtain expression for β, γ.
(c) For a good conductor $\frac{\sigma}{\omega\epsilon}$ becomes very large. Write the limiting forms of β and γ.

2. Consider a plane electromagnetic wave of frequency ω, travelling in the z direction, polarised in the x direction, incident at the interface ($z = 0$) separating medium 1 and medium 2. In the derivation for reflected and transmitted amplitudes discussed in the class, the reflected and transmitted waves were assumed to have the same polarisation as the incident wave (along x direction). Prove that this has to be true always. Hint: Let the polarisation vectors of the transmitted and reflected waves be $\hat{n}_T = \cos\theta_T\hat{x} + \sin\theta_T\hat{y}, \hat{n}_R = \cos\theta_R\hat{x} + \sin\theta_R\hat{y}$, and then prove from the electromagnetic boundary conditions that $\theta_T = \theta_R = 0$.

3. Consider a plane wave (with its electric field to be along the y-axis) incident normally on a dielectric film of thickness d. Calculate the reflectivity of the film.