Department of Electronics & Electrical Engineering

Lecture 4

RLC Networks

By

Dr. Praveen Kumar
Professor
Department of Electronics & Electrical Engineering

Operational Impedance

- When a time varying voltage ***e*** is applied to a simple resistor ***R***, the current is given by Ohm's law as

$$i = \frac{e}{R} \tag{1}$$

- If the same voltage is applied to a simple inductor, the relation between ***e*** and ***i*** is given by Faraday's law as

$$e = L\frac{di}{dt} \Rightarrow e = pLi$$

$$or\ i = \frac{e}{pL}$$

$$where \tag{2}$$

$$p = \text{the differential operator } d/dt$$

- The use of the differential operator permits the differential equation to be written as an algebraic expression, where ***pL*** plays a role for inductance that is the same as ***R*** for resistance. The inductor is said to posses an ***operational impedance*** of ***pL*.**

Operational Impedance

- When the voltage ***e*** is applied to a simple capacitor the pertinent relationship is

$$C\frac{de}{dt}=i \Rightarrow Cpe=i \Rightarrow i=\frac{e}{1/pC} \qquad (3)$$

- From the analogy of eq.1 and eq.3, the capacitor can be said to present to the forcing function ***e*** an operational impedance of ***1/pC***

- The term operational impedance can be applied to single elements or to several elements placed in series. Consider the network shown in Fig.1. The describing differential equation for the circuit is

$$e=Ri+L\frac{di}{dt}+\frac{1}{C}\int idt \qquad (4)$$

- In terms of the ***p*** operator the expression becomes

$$e=Ri+pLi+\frac{1}{pC}i \Rightarrow e=\left(R+pL+\frac{1}{pC}\right)i=Z(p)i \qquad (5)$$

where

$$Z(p)=R+pL+\frac{1}{pC}$$

Operational Impedance

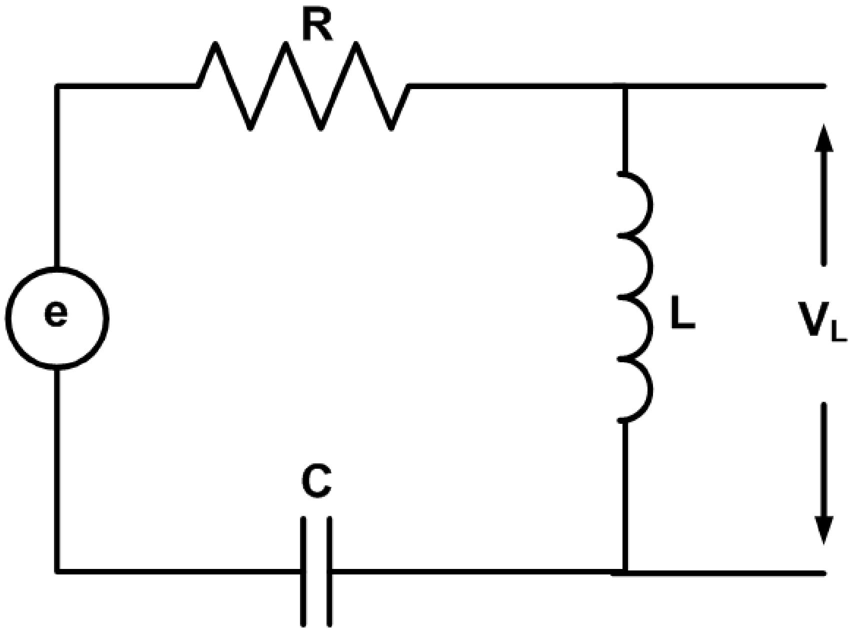

Fig.1: Circuit to illustrate equivalency between p operator algebraic equation and differential equation

Operational Impedance

- There are situations when it is required to relate an electrical quantity in one part of the circuit to an appropriate electrical quantity in another part of the circuit.
- In the network of Fig.1, it may be required to express the ratio of voltage appearing across the inductor to the source voltage.
- If vL is used to denote the voltage developed across the inductor, the desired result can then be written as

$$\frac{v_L}{e} = \frac{pL}{R + pL + 1/pC} \qquad (6)$$

- The eq.6 is commonly called the ***transfer function***. The transfer function relates two quantities that are identified at different terminal pairs.
- The transfer functions may be used to relate ratios of voltages, ratios of currents, ratios of current to voltage, ratios of voltage to current provided that the quantities are not identified at the same terminal pairs.

Example 1

- Write the expression for the operational impedance for each branch of the network shown in Fig.2 and determine the expression for the driving point impedance appearing at the terminal **a-d**.

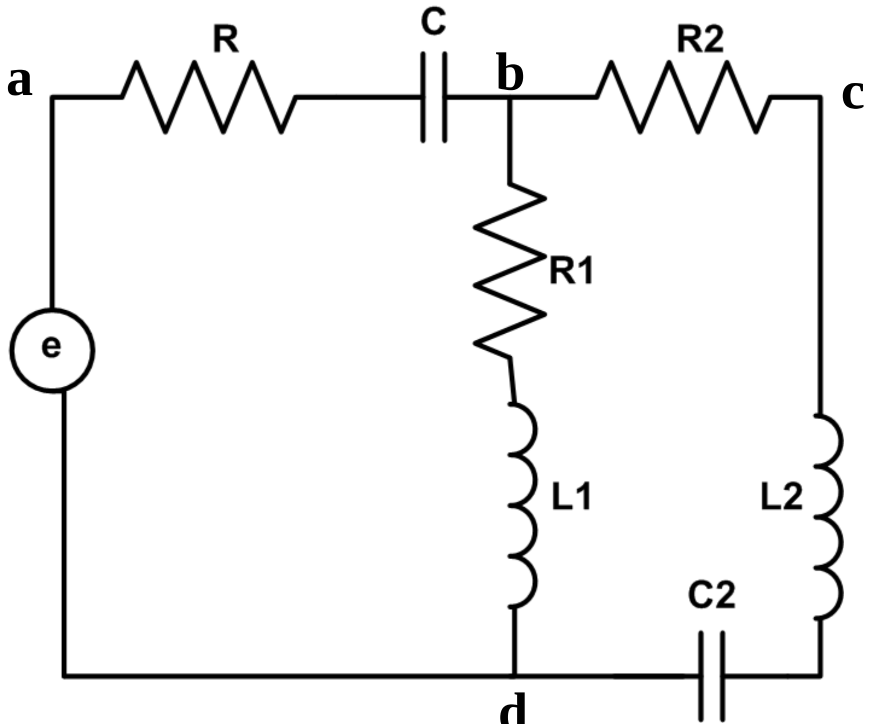

Fig.2: Circuit for the example 1

Solution 1

$$Z_{ab} = R + \frac{1}{pC}$$

$$Z_{bd} = R_1 + pL_1$$

$$Z_{bcd} = R_2 + pL_2 + \frac{1}{pC_2}$$

Driving point impedance

$$Z_{ad} = Z_{ab} + \frac{Z_{bd} Z_{bcd}}{Z_{bd} + Z_{bcd}}$$

The Forced or Particular Solution

- The general response equation of a network is

$$D(p)y(t) = N(p)f(t) \qquad (7)$$

- If ***f(t)*** is an applied source function, the ***y(t)*** is said to be a solution to the homogeneous linear differential equation.

- If ***f(t)*** is a sinusoidal function, then ***y(t)*** must also be sinusoidal. In general ***y(t)*** will differ from the source function in amplitude and phase and it will continue to exist in the circuit so long as the source function remains applied.

- This is the reason why ***y(t)*** is referred to as the ***forced solution***. It is also referred to as ***steady state solution*** because it remains in the circuit long after the transient terms disappear.

- Another commonly used description for this part of the solution is the term ***particular solution***. This name is due to the fact that the form of ***y(t)*** is particularized to the nature of the source function. When the source function is an exponential, the particular solution will likewise be exponential.

The response to constant sources

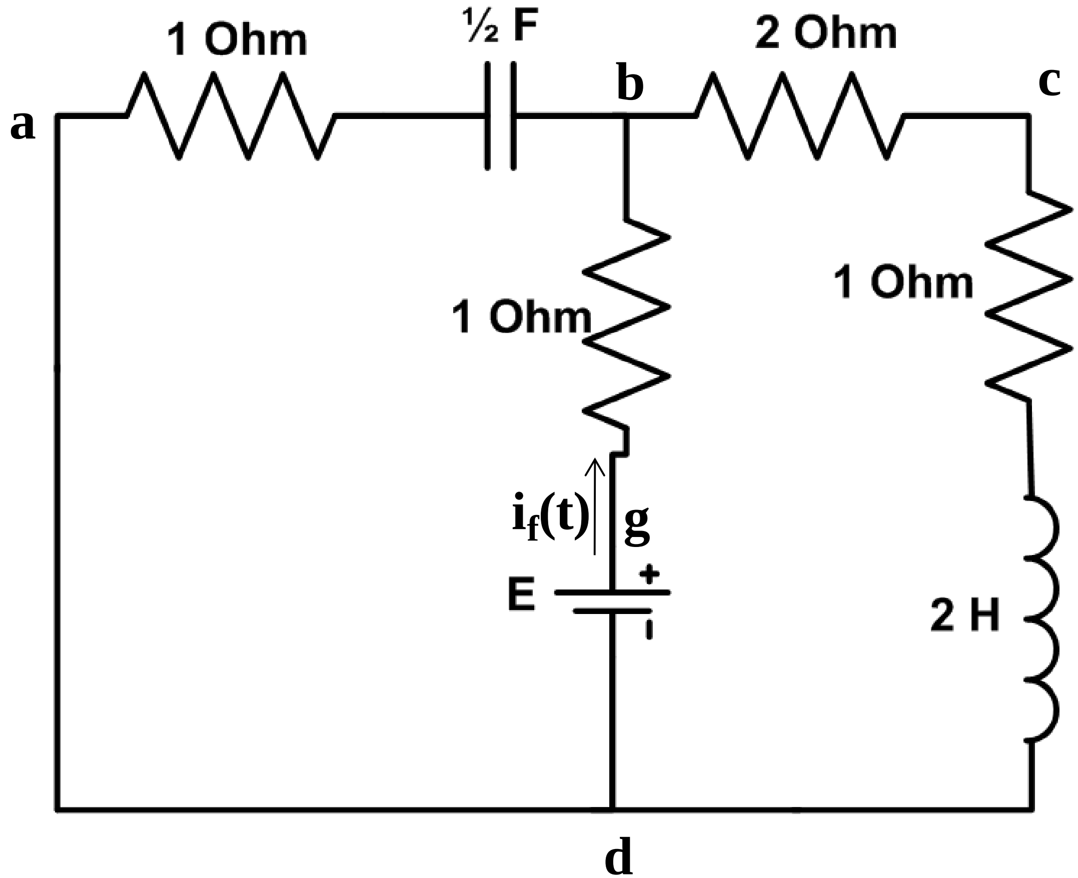

Fig.3: The network 1

The response to constant sources

- The response to a constant source is developed for the network shown in Fig.3. It is required to find the battery current $\mathbf{i_f(t)}$ due to the source voltage **E**.

- The general procedure to obtain the solution is:
  - Obtain the equilibrium equation by applying circuit theory to the configuration of circuit elements described, in terms of their operational impedances.
  - Put the resulting expression in the form of eq.7. Then examine the right side of this equation to determine the exact form needed by the forced solution to qualify as a solution to the equilibrium equation.
  - Finally, solve for the unknown quantities.

- To find the expression for the current $\mathbf{i_f(t)}$ in the circuit shown in Fig.3, it is required to find the impedance **Z**.

The response to constant sources

- The operation impedance across the terminals **g-d** is:

$$Z(p) = Z_{bg} + \frac{Z_{ab}Z_{bcd}}{Z_{ab}+Z_{bcd}} = 1 + \frac{\frac{p+2}{p}(3+2p)}{\frac{p+2}{p}+3+2p} = \frac{4p^2+11p+8}{2p^2+4p+2} \qquad (8)$$

- The current $i_f(t)$ is given by

$$i_f(t) = \frac{E}{Z_{gd}} = \frac{2p^2+4p+2}{4p^2+11p+8}E \qquad (9)$$

$$\Rightarrow (4p^2+11p+8)i_f(t) = (2p^2+4p+2)E$$

- The corresponding differential equation is

$$4\frac{d^2 i_f}{dt^2} + 11\frac{di_f}{dt} + 8i_f = 2E \qquad (10)$$

The response to constant sources

- Examination of the equilibrium equation (eq.10) reveals that because there are no derivative terms on the right side of the equation, the derivative terms of the response function must be identically equal to zero.
- Hence, the solution for $\mathbf{i_f(t)}$ is

$$i_f(t) = I_0 \tag{11}$$

- Substituting the solution in eq.11 into eq.10 gives

$$I_0 = \frac{E}{4} \tag{12}$$

This the forced or steady stae solution

- The examination of the network given in Fig.3 reveals that this result is entirely expected. The current $\mathbf{i_f(t)}$ finds a closed path only in loop ***bcd***. The left hand part of the loop is open circuit in the steady state due to presence of the capacitor.

- The voltage across the 2H inductor is zero in steady state. Hence, the current is just a d-c current of magnitude **E/4**.

The response to exponential sources

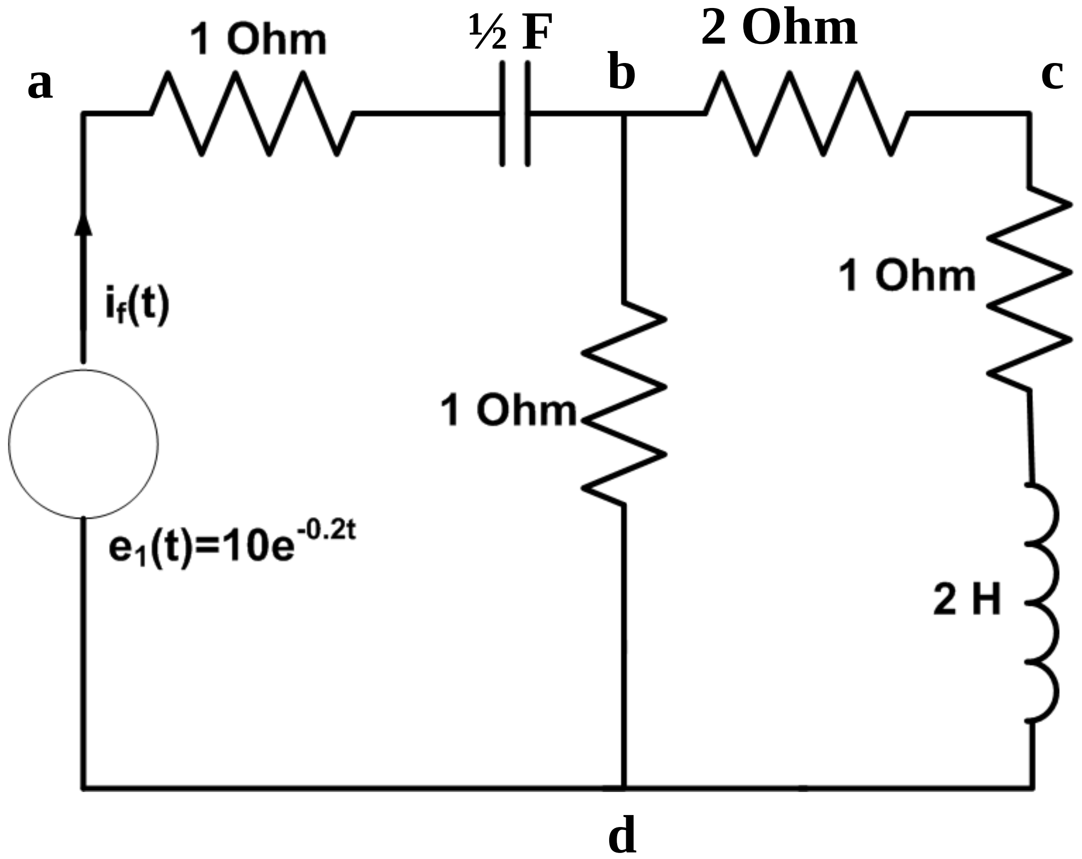

Fig.4: The network 2

The response to exponential sources

- The exponential function is unique in the sense that it is the only function which preserves its form after being subjected to differentiation and integration.

- It is due to this property that it has presence in many mathematical situations and most particularly in the solutions to homogeneous linear differential equation.

- Let the excitation function **f(t)** in the Fig.4 be exponential

$$f(t) = 10e^{-0.2t} \tag{13}$$

- The operational impedance $\mathbf{Z_{ad}}$ is

$$Z_{ad} = \frac{4p^2 + 11p + 8}{2p^2 + 4p} \tag{14}$$

- The current $\mathbf{i_f(t)}$ is given by

$$i_f(t) = \frac{2p^2 + 4p}{4p^2 + 11p + 8} 10e^{-02.t} \tag{15}$$

The response to exponential sources

- Rearranging the eq.8,

$$(4p^2+11p+8)i_f(t)=(2p^2+4p)10e^{-0.2t} \qquad (16)$$

- The entire right side of the eq.16 can be expressed as

$$(2p^2+4p)10e^{-0.2t}=20p^2e^{-0.2t}+40pe^{0.2t}$$

$$=20\frac{d^2}{dt^2}e^{-0.2t}+40\frac{d}{dt}e^{-0.2t}=-7.2e^{-0.2t} \qquad (17)$$

- Hence, eq.17 becomes

$$(4p^2+11p+8)i_f(t)=-7.2e^{-0.2t} \qquad (18)$$

- The right hand side of eq.18 contains just an exponential function, it follows that to satisfy this equation it is necessary for the forced solution to be of the form

$$i_f(t)=Ae^{-0.2t} \qquad (19)$$

The response to exponential sources

- From eq.18 and eq.19, the magnitude of ***A*** is obtained as

$$A = -\frac{7.2}{5.96} = -1.21 \qquad (20)$$

- The final expression for the forced solution becomes

$$i_f(t) = -1.21e^{-0.2t} \qquad (21)$$