PH 102 : Electromagnetism **Department of Physics**
Tutorial 10 **Indian Institute of Technology Guwahati**
02/04/19

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1. Two concentric metal spherical shells, of radius a and b, respectively, are separated by weakly conducting material of conductivity σ as shown in part (a) of figure 1.
(a) If they are maintained at a potential difference V, what current flows from one to the other?
(b) What is the resistance between the shells?
(c) Notice that if $b \gg a$ the outer radius b is irrelevant. How do you account for that? Exploit this observation to determine the current flowing between two metal spheres, each of radius a, immersed deep in the sea and held quite far apart (shown in part (b) of figure 1), if the potential difference between them is V. (This arrangement can be used to measure the conductivity of sea water.)

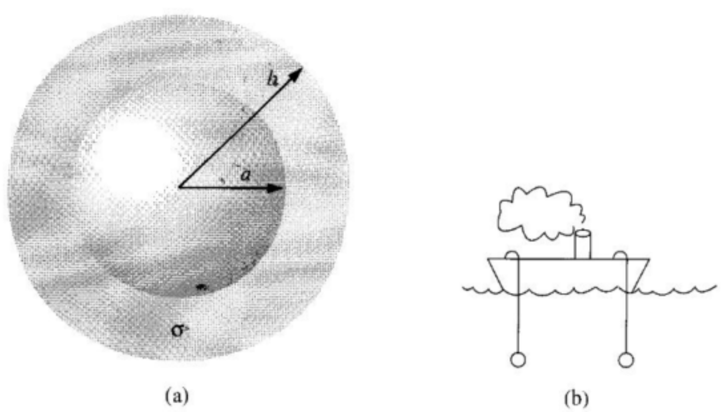

Figure 1: Figure for problem 1.

2. A capacitor C is charged upto a potential V and connected to an inductor L, as shown schematically in figure 2. At time $t = 0$ the switch S is closed. Find the current in the circuit as a function of time. How does your answer change if a resistor is included in series with C and L?

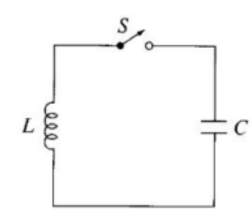

Figure 2: Figure for problem 2.

3. (a) Use the analogy between Faraday's law and Ampere's law, together with the Biot-Savart law, to show that

$$\vec{E}(\vec{r}, t) = -\frac{1}{4\pi}\frac{\partial}{\partial t}\int \frac{\vec{B}(\vec{r'}, t) \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$$

for Faraday-induced electric fields.
(b) Show that $\vec{E} = -\frac{\partial \vec{A}}{\partial t}$, where $\vec{A}$ is the vector potential. Check this result by taking the curl of both sides.
(c) A spherical shell of radius R carries a uniform charge σ. It spins about a fixed axis at an angular velocity $\omega(t)$ that changes slowly with time. Find the electric field inside and outside the sphere. [*Hint*: There are two contributions here: the Coulomb field due to the charge, and the Faraday field due to the changing $\vec{B}$.]

4. A rectangular closed loop of mass m and self inductance L is dropped with initial velocity $v_0\hat{i}_x$ between the pole faces of a magnet that has a concentrated uniform magnetic field $B_0\hat{i}_z$. Here $\hat{i}_n$ denotes unit vector along the n-axis, $(n \equiv x, y, z)$. Neglect the presence of gravity. The schematic diagram for the same is shown in figure 4 where s denotes the thickness of the field region whereas N, S denote north and south poles of the magnet respectively.
(a) What is the imposed flux through the loop as a function of the loop's position

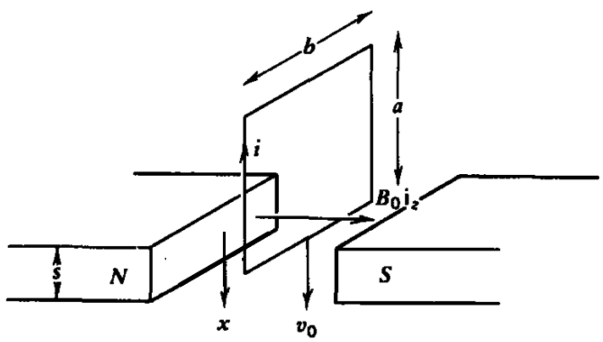

Figure 3: Figure for problem 4.

x $(0 < x < s)$ within the magnet?
(b) If the wire has conductivity σ and cross-sectional area A, what equation relates the induced current i in the loop and the loop's velocity?
(c) What is the force on the loop in terms of current i?
(d) Write down the second order differential equation for loop's velocity $v(t)$ in terms of $\omega_0^2 = \frac{B_0^2 b^2}{mL}, \alpha = \frac{2(a+b)}{\sigma AL}$.
(e) Find the loop's velocity at time $t = \frac{2\pi}{\beta}$ where $\beta = \sqrt{\omega_0^2 - (\alpha/2)^2}$ with ω_0, α are same as defined above. (*Hint*: This can be found by solving the second order differential equation for $v(t)$ in a way similar to solving for charge $q(t)$ in an LCR circuit without any emf source.)
(f) Find the induced current in the loop at time $t = \frac{2\pi}{\beta}$ where β is same as defined above.
(g) For $\sigma \to \infty$, what minimum initial velocity is necessary for the loop to pass through the magnetic field?

5. Consider a solid cylindrical wire of radius R_1 surrounded by a thin long cylindrical

coaxial shell of radius R_2. In the inner cylindrical solid wire, current I is distributed uniformly. In the outer cylindrical shell the same current flows, but in the opposite direction. Find the
(a) Magnetic energy stored in the cable per unit length of the cable.
(b) Self inductance per unit length of the cable.

1 Take Home Problems

1. A copper rod of length L is made to rotate in the xy plane at angular velocity ω where there is a uniform time invariant magnetic field $\vec{B} = B_0\hat{x}$. Find the induced emf between the two ends of the rod.
2. A rectangular loop of wire is situated so that one end (height h) is between the plates of a parallel plate capacitor (shown in figure 4), oriented parallel to the field $\vec{E}$. The other end is way outside, where the field is essentially zero. What is the emf in this loop? If the total resistance is R, what current flows? Explain.

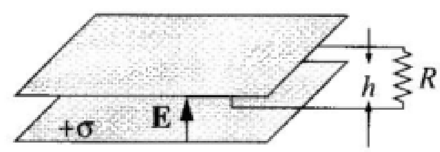

Figure 4: Figure for take home problem 2.

3. A square loop is cut out of a thick sheet of aluminium. It is then placed so that the top portion is in a uniform magnetic field $\vec{B}$, and allowed to fall under gravity (shown in figure 5 where the shading indicates the field region and $\vec{B}$ points into the page). If the magnetic field is 1 T, find the terminal velocity of the loop (in m/s). Find the velocity of the loop as a function of time. How long does it take (in seconds) to reach, say, 90% of the terminal velocity? What would happen if you cut a tiny slit in the ring, breaking the circuit? Write your final answer in numbers by using acceleration due to gravity $g = 9.8$ m/s^2, mass density of aluminium $\eta = 2.7 \times 10^3$ kg/m^3, resistivity of aluminium $\rho = 2.8 \times 10^{-8}$ Ωm.

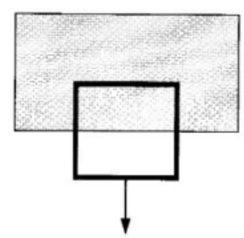

Figure 5: Figure for take home problem 3.

4. A square loop, side a, resistance R, lies a distance s from an infinite straight wire that carries current I (as shown in figure 6). Now someone cuts the wire, so that I drops to zero. In what direction does the induced current in the square loop flow, and what total charge passes a given point in the loop during the time this current flows? If you don't like the scissors model, turn the current down gradually:

$$I(t) = \begin{cases} (1 - \alpha t)I, & \text{for } 0 \le t \le 1/\alpha, \\ 0, & \text{for } t > 1/\alpha. \end{cases}$$

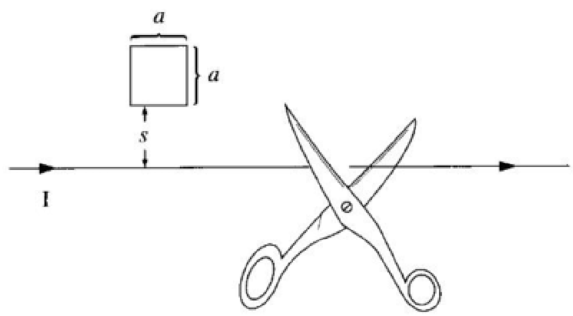

Figure 6: Figure for take home problem 4.

5. Find the energy stored in a section of length l of a long solenoid (radius R, current I, n turns per unit length) using the following formulas discussed in the class:
(a) $W = \frac{1}{2}LI^2$ where L is the inductance.
(b) $W = \frac{1}{2}\oint(\vec{A}\cdot\vec{I})dl$ where $\vec{A}$ is the magnetic vector potential.
(c) $W = \frac{1}{2\mu_0}\int_{\text{all space}} B^2 d\tau$.
(d) $W = \frac{1}{2\mu_0}\left[\int_V B^2 d\tau - \oint_S(\vec{A}\times\vec{B})\cdot d\vec{a}\right]$, where S is the surface bounding the volume V. Take as your volume the cylindrical tube from radius $a < R$ out to radius $b > R$.

6. Consider a magnetic field given by

$$\begin{aligned}\vec{B} &= B_0(t)\hat{z} \qquad s < a\\ &= 0 \qquad\quad s > a\end{aligned}$$

Calculate the induced electric field.