PH 102 : Electromagnetism **Department of Physics**
Tutorial 8 **Indian Institute of Technology Guwahati**
19/03/19

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1. Consider the motion of a charged particle in the simultaneous presence of magnetic field $\vec{B}$ (in x-direction) and electric field $\vec{E}$ (in z-direction). Find and sketch the trajectory of the particle if it starts at the origin with velocity (a) $\vec{v}(0) = (E/B)\hat{y}$, (b) $\vec{v}(0) = (E/2B)\hat{y}$, (c) $\vec{v}(0) = (E/B)(\hat{y} + \hat{z})$.

2. Consider two infinite straight line charges λ, a distance d apart, moving along at a constant speed v as shown in figure 1. How large would v have to be in order for the magnetic attraction to balance the electrical repulsion?

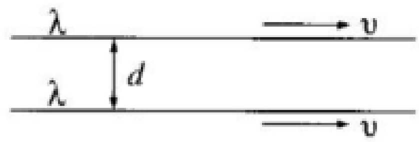

Figure 1: Figure for tutorial problem 2.

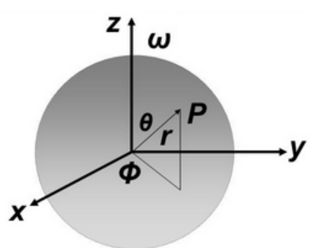

Figure 2: Figure for tutorial problem 3.

3. Calculate the magnetic force of attraction between the northern and southern hemispheres of a spinning charged spherical shell, shown in figure 2. The magnetic force on a surface current $\vec{K}$ is given by

$$\vec{F} = \int (\vec{K} \times \vec{B}_{\text{avg}}) da, \quad \vec{B}_{\text{avg}} = \frac{1}{2}(\vec{B}_{\text{inside}} + \vec{B}_{\text{outside}})$$

4. Find the magnetic field due to a current I in a coaxial cable whose inner conductor has radius a and the outer conductor has the radii $b, c (b < c)$. Also, express the magnetic field as a vector in terms of the current density.

5. A long hollow coaxial wire has inner radius a and outer radius b. Uniform current I flows along its inner surface and return through the outer surface as shown in figure 3. Find vector potential at a distance s from its axis.

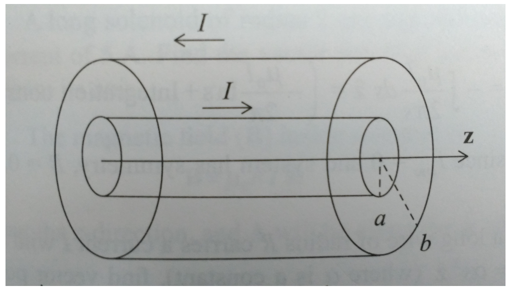

Figure 3: Figure for tutorial problem 5.

6. Show that for uniform magnetic field $\vec{B}$, the magnetic vector potential can be written as $\vec{A} = -\frac{1}{2}(\vec{r} \times \vec{B})$. Is this result unique, or are there other functions with the same properties?

1 Take Home Problems

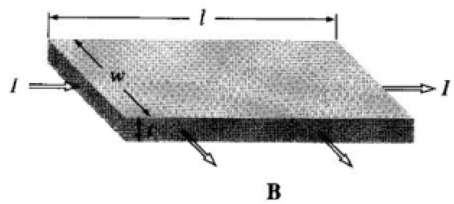

Figure 4: Figure for take home problem 1.

1. A current I flows to the right through a rectangular bar of conducting material, in the presence of a uniform magnetic field $\vec{B}$ pointing out of the page, as shown in figure 4.

   (a) If the moving charges are positive, in which direction are they deflected by the magnetic field? This deflection results in an accumulation of charge on the upper and lower surfaces of the bar, which in turn produces an electric force to counteract the magnetic one. Equilibrium occurs when the two exactly cancel each other. (A phenomenon known as the Hall effect, to be studied as a part of an experiment in PH 110).

   (b) Find the resulting potential difference (the Hall voltage) between the top and bottom of the bar, in terms of B, v (the speed of the charges), and the relevant dimensions of the bar.

   (c) How would your analysis change if the moving charges were negative? (The Hall effect is the classic way of determining the sign of the mobile charge carriers in a material, which is also one of the objectives of the Hall effect related experiment in PH 110).

2. (a) A rotating disk (angular velocity ω) carries a uniform density of "static electricity" σ. Find the surface current density K at a distance r from the center.

(b) Consider a uniformly charged solid sphere of radius R and total charge Q, centered at the origin and spinning at a constant angular velocity ω about the z axis. Find the current density $\vec{J}$ at any point (r, θ, ϕ)

3. Find the magnetic field at a point $z > R$ on the axis of (a) the rotating disk and (b) the rotating sphere, in problem 2.

4. A semicircular wire carries a steady current $\vec{I}$. Find the magnetic field at a point P on the other semicircle (see figure 5). The semicircular wire must be connected to some other wire to complete the circuit. Neglect this wire needed to complete the circuit.

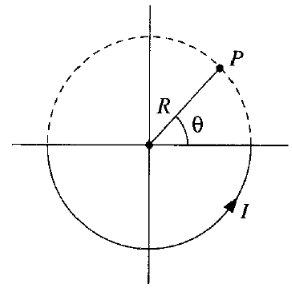

Figure 5: The path

5. Two long coaxial solenoids each carry current I, but in opposite directions, as shown in figure 6. The inner solenoid (radius a) has n_1 turns per unit length, and the outer one (radius b) has n_2. Find $\vec{B}$ in each of the three regions: (i) inside the inner solenoid, (ii) between them, and (iii) outside both.

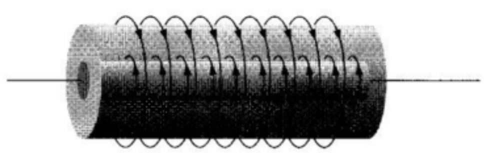

Figure 6: Figure for take home problem 5.

6. Just as $\vec{\nabla} \cdot \vec{B} = 0$ allows us to express $\vec{B}$ as the curl of a vector potential ($\vec{B} = \vec{\nabla} \times \vec{A}$), so $\vec{\nabla} \cdot \vec{A} = 0$ permits us to write $\vec{A}$ itself as the curl of a higher potential: $\vec{A} = \vec{\nabla} \times \vec{W}$.
(a) Find the general formula for $\vec{W}$ (as an integral over $\vec{B}$), which holds when $\vec{B} \to 0$ at ∞.
(b) Determine $\vec{W}$ for the case of a uniform magnetic field $\vec{B}$.
(c) Find $\vec{W}$ inside and outside an infinite solenoid.