Homework 2*

Data Structures
Fall 2019 CS203@IITG

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(1) (a) By giving the specific input that caused the non-asymptotic worst-case behaviour, determine the worst-case non-asymptotic number of comparisons in bubble, insertion, and selection sorts.

(b) Show that the worst-case asymptotic number of exchanges in selection sort is an order (in n) lesser to the worst-case asymptotic number of exchanges in bubble sort.

(c) Between bubble, insertion, and selection sort algorithms, determine which ones are efficiently adaptable when the input is given in a singly linked list.

(2) (a) Analyze the average-case time complexity of selection sort algorithm.

(b) In the randomized selection sort algorithm, the input is randomly permuted and the resultatnt permutation is fed to the (deterministic) selection sort algorithm presented in class. Analyze the expected time complexity of randomized selection sort.

(3) In the threaded binary tree for efficient (without stack) inorder traversal, analyze the asymptotic worst-case time complexities of inorder traversal as well as the node insertion to the current tree.

In the threaded binary tree for efficient (without stack) inorder traversal, is it possible to do node deletion so that the asymptotic worst-case deletion time is upper bounded by the asymptotic worst-case node insertion time?

Devise a threaded binary tree for efficient preorder traversal. Analyze its worst-case time complexity for node insertion and the preorder traversal.

(4) Determine whether the mergesort can be (slightly) modified so that it becomes adaptive? Prove the correctness and analyze the new algorithm.

(5) For every n, give one concrete example with n positive integers which when given as input to the quicksort algorithm results in a balanced partitioning at every internal node of the corresponding recursion tree.

(6) Using the right proof technique, prove the correctness of counting sort algorithm.

(7) Determine the asymptotic worst-case number of comparisons done in each of the following algorithms: bubble sort, selection sort, mergesort, and quicksort.

In which of these, the asymptotic worst-case time complexity is same as the asymptotic worst-case number of comparisons?

Further, draw the decision tree for each of these algorithms when the number of input elements is equal to 3.

(8) Generalize the argument to give the asymptotic worst-case lower bound via decision trees for the comparison sort algorithm when each internal node of the tree has at most k nodes, for any integer $k > 2$.

Discuss whether it is possible for the decision tree of any comparison sort to have more than two children.

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