MA 102 (Mathematics II)
IIT Guwahati

Tutorial Sheet No. 2 | Linear Algebra | January 24, 2019

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1. Let A be 4×3 matrix such that $\text{rank}(A) = 3$. Then show that there exists a 3×4 matrix B such that $BA = I_3$.

2. Find all the solutions of the linear system with the augmented matrix $[A|\mathbf{b}]$ as given below:

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 2 \\ 5 & 6 & 7 & 8 & 5 \\ 9 & 10 & 11 & 12 & 8 \end{array}\right]$$

   (a) Find $\mathbf{b}'$ such that $A\mathbf{x} = \mathbf{b}'$ does not have a solution.

   (b) By changing exactly one entry of A, find an A' such that $A'\mathbf{x} = \mathbf{b}$ will be consistent for all $\mathbf{b} \in \mathbb{R}^3$.

3. Let $A \in \mathcal{M}_5(\mathbb{R})$ be invertible with row sums 1. Show that the sum of all the elements of A^{-1} is 5.

4. True or False? Give justifications.

   (a) If for all $A \in \mathcal{M}_n(R)$, $AB = A$ then $B = I_n$.

   (b) If A and B are square matrices of order n with $AB = I_n$ then A and B are invertible and $BA = I_n$.
   Hint: If P is invertible then $\text{rank}(P) = n$. $AB = I$ implies there exists an invertible P such that $PAB = P$, where PA is in rref.

   (c) If A is an $m \times n$ matrix with at least one nonzero row (at least one entry of this row is nonzero) then A is row equivalent to a matrix B, with all nonzero rows.

   (d) If all the columns of an $n \times m$ nonzero matrix (it has at least one nonzero entry) A are equal then $rank(A) = 1$.

   (e) If A is an $m \times n$ matrix with a zero column (all entries of the column is zero) then the rref of A will again have a zero column.

   (f) If P is any invertible matrix such that PA is defined then, $Ax = b$ and $PAx = Pb$ are equivalent.

5. Using Gauss Jordan elimination prove that

$$\left\{\alpha \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} : \alpha \in \mathbb{R}\right\} + \left\{\alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} : \alpha \in \mathbb{R}\right\} + \left\{\alpha \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} : \alpha \in \mathbb{R}\right\} = \mathbb{R}^3.$$

6. If A is upper triangular and B is any matrix such that $AB = I$, then show that each diagonal entry of A is nonzero.

7. Show that $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \mid x_1 = 2x_3 + x_2 \right\}$ is a subspace of $\mathbb{R}^3$.

   (a) Find $\{\mathbf{u}, \mathbf{v}\}$ such that $span\{\mathbf{u}, \mathbf{v}\} = S$.

(b) Find a $\mathbf{v}'$ such that $span\{\mathbf{u}, \mathbf{v}'\} = span\{\mathbf{v}, \mathbf{v}'\} = S$.

(c) Find an $\mathbf{u}'$ such that $span\{\mathbf{u}', \mathbf{v}'\}$ is not a subspace of S. Geometrically what will be the picture of S and $span\{\mathbf{u}', \mathbf{v}'\}$?

8. By using Gauss Jordan elimination find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 9 & 10 & 12 \end{bmatrix}.$$

9. Using LU factorization of the matrix A solve the system of linear equations with the augmented matrix $[A|\mathbf{b}]$ as given below:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 10 \\ 1 & 2 & 3 & 4 & 30 \\ 1 & 4 & 8 & 15 & 93 \\ 1 & 3 & 6 & 10 & 65 \end{array}\right].$$

10. Show that $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \mid x_1 = 2x_3 - x_2,\ 2x_2 = x_3 \right\}$ is a subspace of $\mathbb{R}^3$.
Find an $\mathbf{u}$ such that $span\{\mathbf{u}\} = S$. Find an $\mathbf{u}'$ such that $span\{\mathbf{u}, \mathbf{u}'\}$ gives a plane in $\mathbb{R}^3$. Find a $\mathbf{v}$ such that $span\{\mathbf{v}\}$ is not a subspace of $span\{\mathbf{u}, \mathbf{u}'\}$. What will be the $span\{\mathbf{u}, \mathbf{u}', \mathbf{v}\}$?

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