

PH 102: Physics II

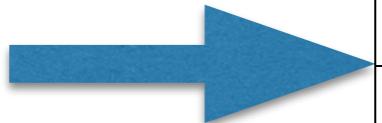
Lecture 22 (Spring 2019)

IIT Guwahati

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LECTURE PLAN (TENTATIVE) OF PH 102 (POST MID-SEM)

SN	Date	Topic	Griffith's section	Lectures	Division
Lec 15	13-3-2019	Lorentz Force, Biot-Savart law, Divergence & Curl of Magnetostatic Fields	5.1, 5.2, 5.3	1	I, II (4-4:55 pm)
Lec 15	13-3-2019	Lorentz Force, Biot-Savart law, Divergence & Curl of Magnetostatic Fields	5.1, 5.2, 5.3	1	III, IV (9-9:55 am)
Lec 16	14-3-2019	Applications of Ampere's law, Magnetic Vector Potential	5.3, 5.4	1	I, II (3-3:55 pm)
Lec 16	14-3-2019	Applications of Ampere's law, Magnetic Vector Potential	5.3, 5.4	1	III, IV (10-10:55 am)
Tut 8	19-3-2019	Lec 15, 16			
Lec 17	20-3-2019	Magnetic dipole, Force & torque on a magnetic dipole, Magnetic Materials, Magnetization	5.4, 6.1	1	I, II (4-4:55 pm)
Lec 17	20-3-2019	Magnetic dipole, Force & torque on a magnetic dipole, Magnetic Materials, Magnetization	5.4, 6.1	1	III, IV (9-9:55 am)
Lec 18	22-3-2019	Field of a magnetized object, Boundary Conditions	6.2, 6.3, 6.4	1	I, II (3-3:55 pm)
Lec 18	22-3-2019	Field of a magnetized object, Boundary Conditions	6.2, 6.3, 6.4	1	III, IV (10-10:55 am)
Tut 9	26-3-2019	Lec 17, 18			
Lec 19	27-3-2019	Ohm's law, motional emf, electromotive force	7.1	1	I, II (4-4:55 pm)
Lec 19	27-3-2019	Ohm's law, motional emf, electromotive force	7.1	1	III, IV (9-9:55 am)
Lec 20	28-3-2019	Faraday's law, Lenz's law, Self & Mutual Inductance, Energy Stored in Magnetic Field	7.2	1	I, II (3-3:55 pm)
Lec 20	28-3-2019	Faraday's law, Lenz's law, Self & Mutual Inductance, Energy Stored in Magnetic Field	7.2	1	III, IV (10-10:55 am)
Tut 10	2-4-2019	Lec 19, 20			
Lec 21	3-4-2019	Maxwell's equations	7.3	1	I, II (4-4:55 pm)
Lec 21	3-4-2019	Maxwell's equations	7.3	1	III, IV (9-9:55 am)
Lec 22	4-4-2019	Discussions, problem solving	7.3	1	I, II (3-3:55 pm)
Lec 22	4-4-2019	Discussions, problem solving	7.3	1	III, IV (10-10:55 am)
Tut 11	9-4-2019	Quiz II			



LECTURE PLAN (TENTATIVE) OF PH 102 (POST MID-SEM)

Lec 23	10-4-2019	Continuity equation, Poynting Theorem	8.1	1	I, II (4-4:55 pm)
Lec 23	10-4-2019	Continuity equation, Poynting Theorem	8.1	1	III, IV (9-9:55 am)
Lec 24	11-4-2019	Wave solution of Maxwell's equation, polarisation	9.1, 9.2	1	I, II (3-3:55 pm)
Lec 24	11-4-2019	Wave solution of Maxwell's equation, polarisation	9.1, 9.2	1	III, IV (10-10:55 am)
Tut 12	23-4-2019	Lec 23, 24			
Lec 25	24-4-2019	Electromagnetic waves in matter, Reflection and transmission: Normal incidence	9.3	1	I, II (4-4:55 pm)
Lec 25	24-4-2019	Electromagnetic waves in matter, Reflection and transmission: Normal incidence	9.3	1	III, IV (9-9:55 am)
Lec 26	25-4-2019	Reflection and transmission: Oblique Incidence	9.3, 9.4	1	I, II (3-3:55 pm)
Lec 26	25-4-2019	Reflection and transmission: Oblique Incidence	9.3, 9.4	1	III, IV (10-10:55 am)
Tut 13	30-4-2019	Lec 25, 26			
Lec 27	1-5-2019	Relativity and electromagnetism: Galilean and special relativity	12.1, 12.2, 12.3	1	I, II (4-4:55 pm)
Lec 27	1-5-2019	Relativity and electromagnetism: Galilean and special relativity	12.1, 12.2, 12.3	1	III, IV (9-9:55 am)
Lec 28	2-5-2019	Discussions, problem solving	12.1, 12.2, 12.3	1	I, II (3-3:55 pm)
Lec 28	2-5-2019	Discussions, problem solving	12.1, 12.2, 12.3	1	III, IV (10-10:55 am)

Maxwell's Equations

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Gauss's (Coulomb's) Law

$$\vec{\nabla} \cdot \vec{B} = 0$$

No name

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday's Law

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Ampere's Law with
Maxwell's Correction

Sample Problems

- (1) An alternating current $I = I_0 \cos(\omega t)$ flows down a long straight wire, and returns along a coaxial conducting tube of radius a . Assuming that the field goes to zero as $s \rightarrow \infty$, find $\mathbf{E}(s, t)$.

(a) Find the displacement current density \mathbf{J}_d .

(b) Integrate it to get the total displacement current, $I_d = \int \mathbf{J}_d \cdot d\mathbf{a}$.

Compare I_d and I . If the outer cylinder were, say, 2 mm in diameter, how high would the frequency have to be, for I_d to be 1% of I ?

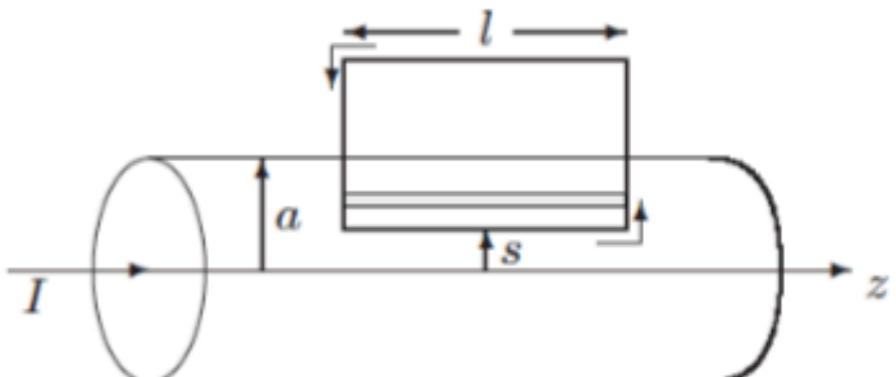
Solution: Using the “amperian loop” in the figure,

$$\oint \mathbf{E} \cdot d\mathbf{l} = El = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{a} = -\frac{d}{dt} \int_s^a \frac{\mu_0 I}{2\pi s'} l ds'$$

$$\therefore E = -\frac{\mu_0}{2\pi} \frac{dI}{dt} \ln\left(\frac{a}{s}\right). \quad \text{and,} \quad \frac{dI}{dt} = -I_0 \omega \sin \omega t,$$

Outside, $B = 0$,
so here $E = 0$
(like B outside
a solenoid).

$$\text{Hence, } \mathbf{E} = \frac{\mu_0 I_0 \omega}{2\pi} \sin(\omega t) \ln\left(\frac{a}{s}\right) \hat{\mathbf{z}}.$$



$$(a) \quad \mathbf{J}_d = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \epsilon_0 \frac{\mu_0 I_0 \omega^2}{2\pi} \cos(\omega t) \ln(a/s) \hat{\mathbf{z}} = \frac{\mu_0 \epsilon_0}{2\pi} \omega^2 I \ln(a/s) \hat{\mathbf{z}}.$$

What about \mathbf{E} along the vertical edges of the loop?

$$\begin{aligned}
 \text{(b)} \quad I_d &= \int \mathbf{J}_d \cdot d\mathbf{a} = \frac{\mu_0 \epsilon_0 \omega^2 I}{2\pi} \int_0^a \ln(a/s) (2\pi s ds) = \mu_0 \epsilon_0 \omega^2 I \int_0^a (s \ln a - s \ln s) ds \\
 &= \mu_0 \epsilon_0 \omega^2 I \left[(\ln a) \frac{s^2}{2} - \frac{s^2}{2} \ln s + \frac{s^2}{4} \right] \Big|_0^a = \mu_0 \epsilon_0 \omega^2 I \left[\cancel{\frac{a^2}{2} \ln a} - \cancel{\frac{a^2}{2} \ln a} + \frac{a^2}{4} \right] = \frac{\mu_0 \epsilon_0 \omega^2 I a^2}{4}.
 \end{aligned}$$

$$\text{(c)} \quad \frac{I_d}{I} = \frac{\mu_0 \epsilon_0 \omega^2 a^2}{4} \quad \text{where, } \mu_0 \epsilon_0 = 1/c^2, \quad I_d/I = (\omega a / 2c)^2.$$

Given, $a = 10^{-3}$ m, and $\frac{I_d}{I} = \frac{1}{100}$, implies $\frac{\omega a}{2c} = \frac{1}{10}$,

$$\omega = \frac{2c}{10a} = \frac{3 \times 10^8 \text{ m/s}}{5 \times 10^{-3} \text{ m}}, \text{ or } \omega = 0.6 \times 10^{11} / \text{s}$$

$$\nu = \frac{\omega}{2\pi} \approx 10^{10} \text{ Hz}, \text{ Microwave, way above the radio frequencies}$$

Thus the frequency required so that the displacement current is comparable ($\sim 1\%$) is very high @ microwave. Hence, in the early days of Faraday no experimental signal could appear giving clue for the displacement current!

$$(2) \text{ Consider the following fields, } \mathbf{E}(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \theta(vt - r) \hat{\mathbf{r}}; \quad \mathbf{B}(\mathbf{r}, t) = \mathbf{0}$$

Show that these fields satisfy all of Maxwell's equations, and determine ρ and \mathbf{J} . Describe the physical situation that gives rise to these fields.

Solution:

Physically, this is the field of a point charge $+q$ at the origin, out to an expanding spherical shell of radius vt . Outside the shell the field is zero. Evidently the shell carries the opposite charge, $-q$

$$\text{Using } \nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f) \quad \text{and} \quad \nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2} \right) = 4\pi \delta^3(\mathbf{r}).$$

$$\nabla \cdot \mathbf{E} = \theta(vt - r) \nabla \cdot \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}} \right) + \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}} \cdot \nabla [\theta(vt - r)] = \frac{q}{\epsilon_0} \delta^3(\mathbf{r}) \theta(vt - r) + \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} (\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}) \frac{\partial}{\partial r} \theta(vt - r).$$

However,

$$\delta^3(\mathbf{r}) \theta(vt - r) = \delta^3(\mathbf{r}) \theta(t), \text{ and } \frac{\partial}{\partial r} \theta(vt - r) = -\delta(vt - r)$$

Chapter 1
D J Griffiths

so,

$$\rho = \epsilon_0 \nabla \cdot \mathbf{E} = q \delta^3(\mathbf{r}) \theta(t) - \frac{q}{4\pi r^2} \delta(vt - r).$$

For $t < 0$ the field and the charge density are zero everywhere.

$$\nabla \cdot \mathbf{B} = 0, \text{ and } \nabla \times \mathbf{E} = \mathbf{0}$$

(since \mathbf{E} has only an r component, and it is independent of θ and ϕ)

$$\nabla \times \mathbf{B} = \mathbf{0} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \partial \mathbf{E} / \partial t.$$

$$\mathbf{J} = -\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = -\epsilon_0 \left\{ \frac{q}{4\pi\epsilon_0 r^2} \frac{\partial}{\partial t} [\theta(vt - r)] \right\} \hat{\mathbf{r}} = -\frac{q}{4\pi r^2} v \delta(vt - r) \hat{\mathbf{r}}.$$

The stationary charge at the origin doesn't contribute to \mathbf{J} ,
of course; for the expanding shell we have $\mathbf{J} = \rho \mathbf{v}$.

Practice similar problems where Maxwell's equations are used
to find source terms from the fields and vice versa!

(3) A long straight solenoid having N turns per unit length carries a current $I = I_0 \sin \omega t$. Calculate the displacement current density as a function of s within the solenoid.

Solution: Displacement current is $\vec{J} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Due to the change in current and hence the magnetic field inside the solenoid, there will be induced electric field which can be found out by using Faraday's law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$

$$\implies E(2\pi s) = -\pi s^2 \frac{dB}{dt} = -\pi s^2 \mu_0 N \frac{dI}{dt}$$

$$\implies E_\phi = -\frac{s}{2} \mu_0 N I_0 \omega \cos \omega t$$

Consider a circular loop of radius s coaxial with the solenoid axis

Inside a solenoid, magnetic field is axial, given by $B = \mu_0 NI$

4. (a)

Show that Maxwell's equations with magnetic charge are invariant under the duality transformation,

$$\begin{aligned}\mathbf{E}' &= \mathbf{E} \cos \alpha + c\mathbf{B} \sin \alpha, \\ c\mathbf{B}' &= c\mathbf{B} \cos \alpha - \mathbf{E} \sin \alpha, \\ cq'_e &= cq_e \cos \alpha + q_m \sin \alpha, \\ q'_m &= q_m \cos \alpha - cq_e \sin \alpha,\end{aligned}$$

where $c = 1/\sqrt{\epsilon_0 \mu_0}$ and α is an arbitrary rotation angle in "E/B-space." Charge and current densities transform in the same way as q_e and q_m .

4. (b)

Show that the force law, $\mathbf{F} = q_e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + q_m \left(\mathbf{B} - \frac{1}{c^2} \mathbf{v} \times \mathbf{E} \right)$, is also invariant under the duality transformation.

Solution:

$$\begin{aligned}4. (a) \quad \nabla \cdot \mathbf{E}' &= (\nabla \cdot \mathbf{E}) \cos \alpha + c(\nabla \cdot \mathbf{B}) \sin \alpha = \frac{1}{\epsilon_0} \rho_e \cos \alpha + c\mu_0 \rho_m \sin \alpha \\ &= \frac{1}{\epsilon_0} (\rho_e \cos \alpha + c\mu_0 \epsilon_0 \rho_m \sin \alpha) = \frac{1}{\epsilon_0} (\rho_e \cos \alpha + \frac{1}{c} \rho_m \sin \alpha) = \frac{1}{\epsilon_0} \rho'_e.\end{aligned}$$

$$\begin{aligned}\nabla \cdot \mathbf{B}' &= (\nabla \cdot \mathbf{B}) \cos \alpha - \frac{1}{c} (\nabla \cdot \mathbf{E}) \sin \alpha = \mu_0 \rho_m \cos \alpha - \frac{1}{c \epsilon_0} \rho_e \sin \alpha \\ &= \mu_0 (\rho_m \cos \alpha - \frac{1}{c \mu_0 \epsilon_0} \rho_e \sin \alpha) = \mu_0 (\rho_m \cos \alpha - c \rho_e \sin \alpha) = \mu_0 \rho'_m.\end{aligned}$$

$$\begin{aligned}\nabla \times \mathbf{E}' &= (\nabla \times \mathbf{E}) \cos \alpha + c(\nabla \times \mathbf{B}) \sin \alpha = \left(-\mu_0 \mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t} \right) \cos \alpha + c \left(\mu_0 \mathbf{J}_e + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \sin \alpha \\ &= -\mu_0 (\mathbf{J}_m \cos \alpha - c \mathbf{J}_e \sin \alpha) - \frac{\partial}{\partial t} \left(\mathbf{B} \cos \alpha - \frac{1}{c} \mathbf{E} \sin \alpha \right) = -\mu_0 \mathbf{J}'_m - \frac{\partial \mathbf{B}'}{\partial t}.\end{aligned}$$

$$\begin{aligned}\nabla \times \mathbf{B}' &= (\nabla \times \mathbf{B}) \cos \alpha - \frac{1}{c} (\nabla \times \mathbf{E}) \sin \alpha = \left(\mu_0 \mathbf{J}_e + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \cos \alpha - \frac{1}{c} \left(-\mu_0 \mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t} \right) \sin \alpha \\ &= \mu_0 (\mathbf{J}_e \cos \alpha + \frac{1}{c} \mathbf{J}_m \sin \alpha) + \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\mathbf{E} \cos \alpha + c \mathbf{B} \sin \alpha) = \mu_0 \mathbf{J}'_e + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}'}{\partial t}.\end{aligned}$$

$$\begin{aligned}4. \text{ (b)} \quad \mathbf{F}' &= q'_e (\mathbf{E}' + \mathbf{v} \times \mathbf{B}') + q'_m (\mathbf{B}' - \frac{1}{c^2} \mathbf{v} \times \mathbf{E}') \\ &= \left(q_e \cos \alpha + \frac{1}{c} q_m \sin \alpha \right) \left[(\mathbf{E} \cos \alpha + c \mathbf{B} \sin \alpha) + \mathbf{v} \times \left(\mathbf{B} \cos \alpha - \frac{1}{c} \mathbf{E} \sin \alpha \right) \right] \\ &\quad + (q_m \cos \alpha - c q_e \sin \alpha) \left[\left(\mathbf{B} \cos \alpha - \frac{1}{c} \mathbf{E} \sin \alpha \right) - \frac{1}{c^2} \mathbf{v} \times (\mathbf{E} \cos \alpha + c \mathbf{B} \sin \alpha) \right] \\ &= q_e \left[(\mathbf{E} \cos^2 \alpha + c \mathbf{B} \sin \alpha \cos \alpha - c \mathbf{B} \sin \alpha \cos \alpha + \mathbf{E} \sin^2 \alpha) \right. \\ &\quad \left. + \mathbf{v} \times \left(\mathbf{B} \cos^2 \alpha - \frac{1}{c} \mathbf{E} \sin \alpha \cos \alpha + \frac{1}{c} \mathbf{E} \sin \alpha \cos \alpha + \mathbf{B} \sin^2 \alpha \right) \right] \\ &\quad + q_m \left[\left(\frac{1}{c} \mathbf{E} \sin \alpha \cos \alpha + \mathbf{B} \sin^2 \alpha + \mathbf{B} \cos^2 \alpha - \frac{1}{c} \mathbf{E} \sin \alpha \cos \alpha \right) \right. \\ &\quad \left. + \mathbf{v} \times \left(\frac{1}{c} \mathbf{B} \sin \alpha \cos \alpha - \frac{1}{c^2} \mathbf{E} \sin^2 \alpha - \frac{1}{c^2} \mathbf{E} \cos^2 \alpha - \frac{1}{c} \mathbf{B} \sin \alpha \cos \alpha \right) \right] \\ &= q_e (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + q_m \left(\mathbf{B} - \frac{1}{c^2} \mathbf{v} \times \mathbf{E} \right) = \mathbf{F}.\end{aligned}$$

(5) Consider a parallel plate capacitor of circular cross section of radius a . If the current to the plates is time dependent, obtain the induced magnetic field in between the plates of the capacitor.

Solution: Using Ampere's law (corrected one) in terms of conduction as well as displacement current

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(I + I_D)$$

In between the plates, the conduction current is obviously zero. However, the displacement current is given by

$$I_D = \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a}$$

Considering a circular loop of radius $s < a$ (inside the capacitor), we can find the magnetic field as

$$B(2\pi s) = \mu_0 \epsilon_0 \pi s^2 \frac{\partial E}{\partial t}$$

$$\implies \vec{B} = \frac{\mu_0 \epsilon_0}{2} s \frac{\partial E}{\partial t} \hat{\phi}$$

Constant E in between the plates is assumed ignoring the edge effects!

For points outside the capacitor plates $s > a$, the magnetic field is

$$B(2\pi s) = \mu_0 \epsilon_0 \pi a^2 \frac{\partial E}{\partial t}$$

$$\implies \vec{B} = \frac{\mu_0 \epsilon_0}{2s} a^2 \frac{\partial E}{\partial t} \hat{\phi}$$

E outside the plates is taken to be zero!

Ampere's Law for Finite Current Carrying Wire

- It was argued not to use Ampere's law (in magnetostatics) to find magnetic field of a finite current carrying wire. Biot-Savart law was used instead to find the field.
- But why did Ampere's law give the wrong result?
- Will we get a correct result if we use Ampere's law with Maxwell's correction?

Finite Current Carrying Wire

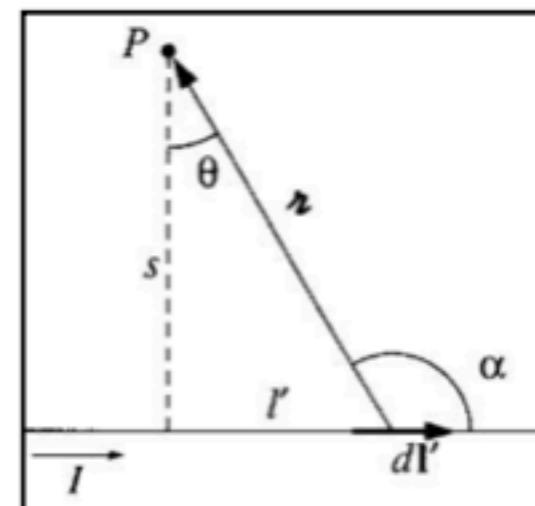
Recall the discussion from Lecture 15

Example 5.5 (Introduction to Electrodynamics, D. J. Griffiths):
Find the magnetic field at a distance s from a long straight wire carrying a steady current I .

Using Biot-Savart law: $B(\vec{s}) = \frac{\mu_0}{4\pi} I \int \frac{|d\vec{l}' \times \hat{\tau}|}{r^2}, \hat{\tau} = \vec{s} - \vec{l}'$

Using $|d\vec{l}' \times \hat{\tau}| = dl' \sin \alpha = dl' \cos \theta$

$$l' = s \tan \theta \implies dl' = \frac{s}{\cos^2 \theta} d\theta \quad s = r \cos \theta \implies \frac{1}{r^2} = \frac{\cos^2 \theta}{s^2}$$



we can write,

$$B(\vec{s}) = \frac{\mu_0}{4\pi} I \int_{\theta_1}^{\theta_2} \left(\frac{s}{\cos^2 \theta} \cos \theta d\theta \right) \left(\frac{\cos^2 \theta}{s^2} \right)$$

$$\implies B(\vec{s}) = \frac{\mu_0 I}{4\pi s} \int_{\theta_1}^{\theta_2} \cos \theta d\theta$$

$$\implies B(\vec{s}) = \frac{\mu_0 I}{4\pi s} (\sin \theta_2 - \sin \theta_1)$$

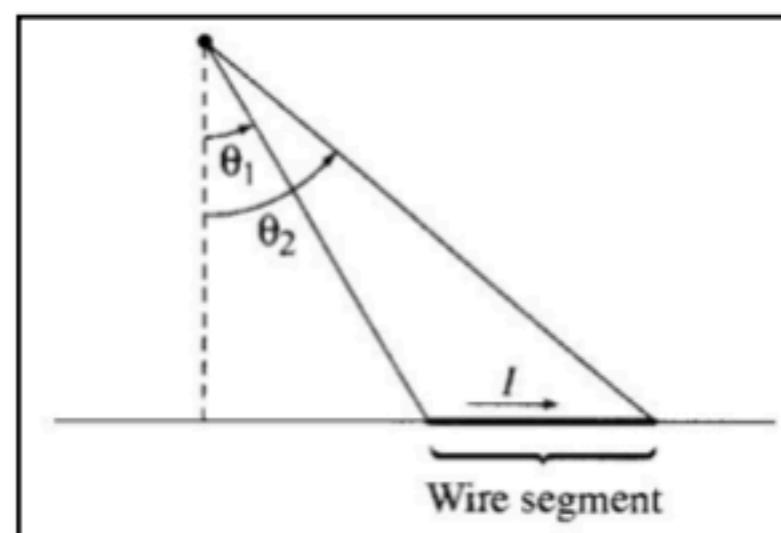


Fig. 5.18, 5.19 (Introduction to Electrodynamics, D. J. Griffiths)

Finite Current Carrying Wire

- If we find the field at a point in a plane half way between the end points of the wire, we will have

$$\theta_1 = -\theta_2 = -\theta \implies B(s) = \frac{\mu_0 I}{2\pi s} \sin \theta$$

- Use of Ampere's law will give same answer as for an infinite wire

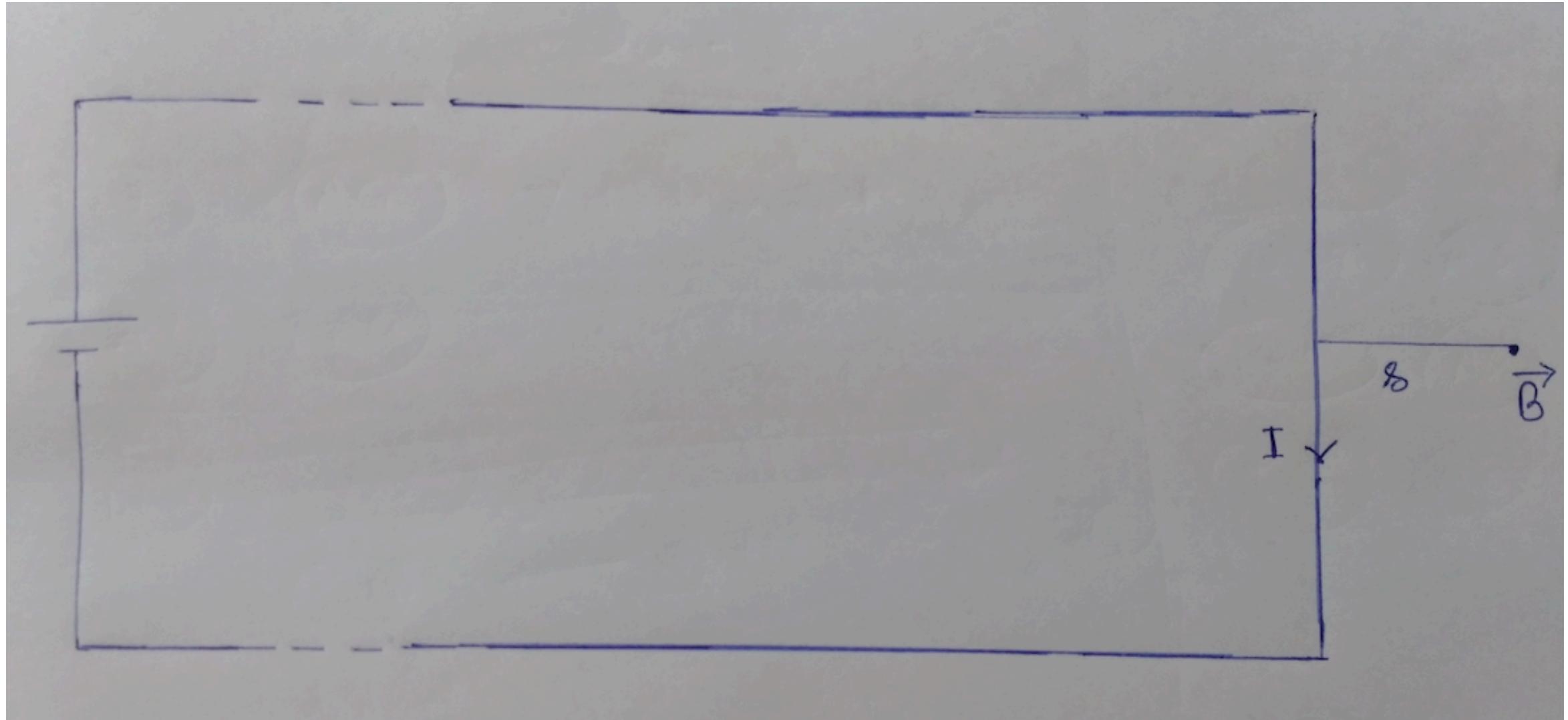
$$B(s) = \frac{\mu_0 I}{2\pi s}$$

which is independent of the position of the plane where the point in question is located.

- Thus, Ampere's law and Biot-Savart law give contradictory results: both can not be true!

Finite Current Carrying Wire

- Finite current is an idealised concept. If it is steady current, then it has to be part of a loop:



- But even if we consider the other end of the loop to be infinitely away, yet we lose the azimuthal symmetry, to use Ampere's law.

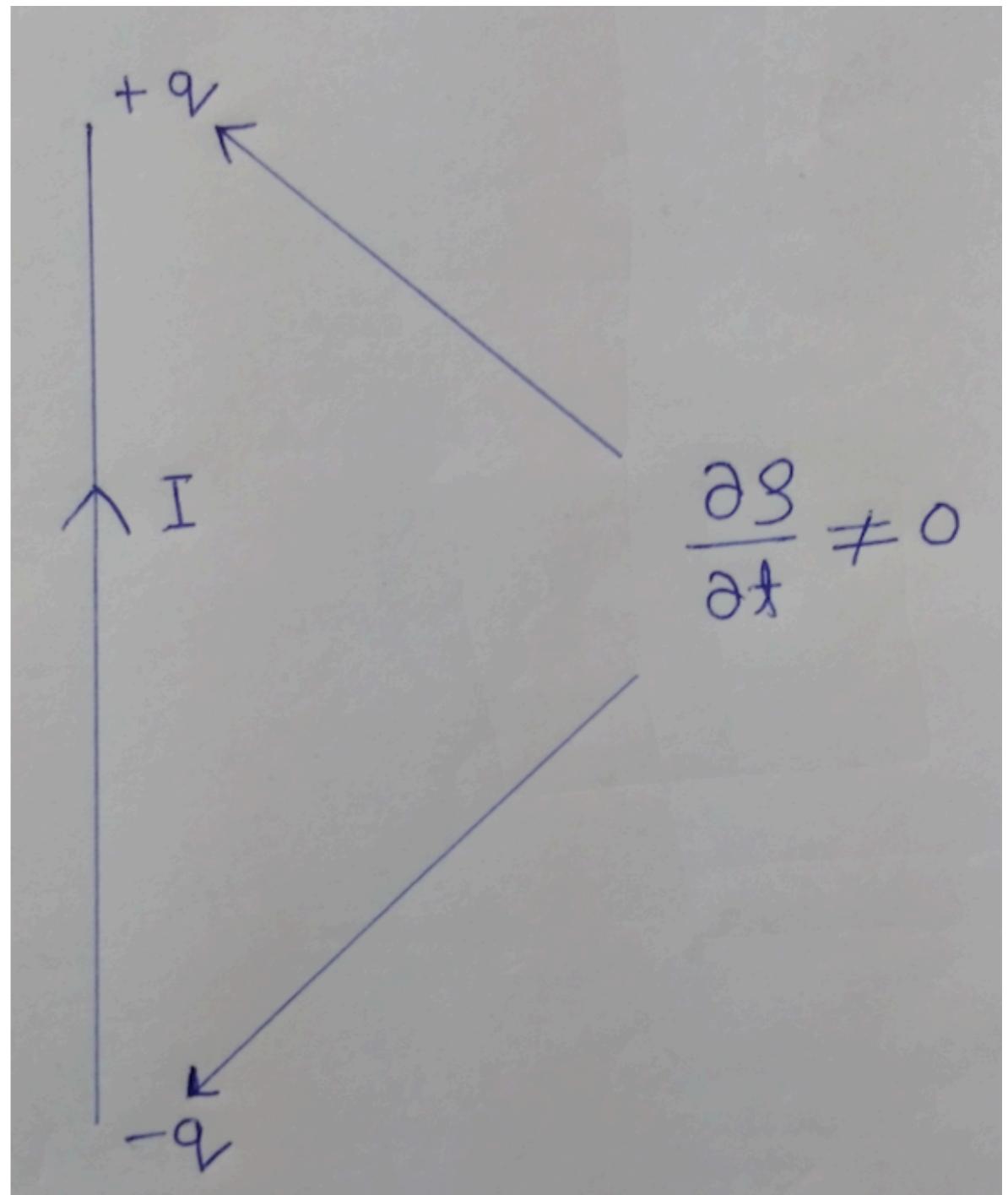
Finite Current Carrying Wire

1. Let us now consider a symmetric situation: the finite wire is not part of a loop and hence has an azimuthal symmetry.

2. But, in accordance with continuity equation,

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

there should be change of charge density at two ends of the wire due to flow of current.



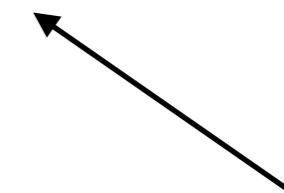
Finite Current Carrying Wire

- Since the current is non-steady, we need to use the corrected version of Ampere's law which is consistent with the continuity equation.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\int (\vec{\nabla} \times \vec{B}) \cdot d\vec{a} = \mu_0 \int \vec{J} \cdot d\vec{a} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{a}$$

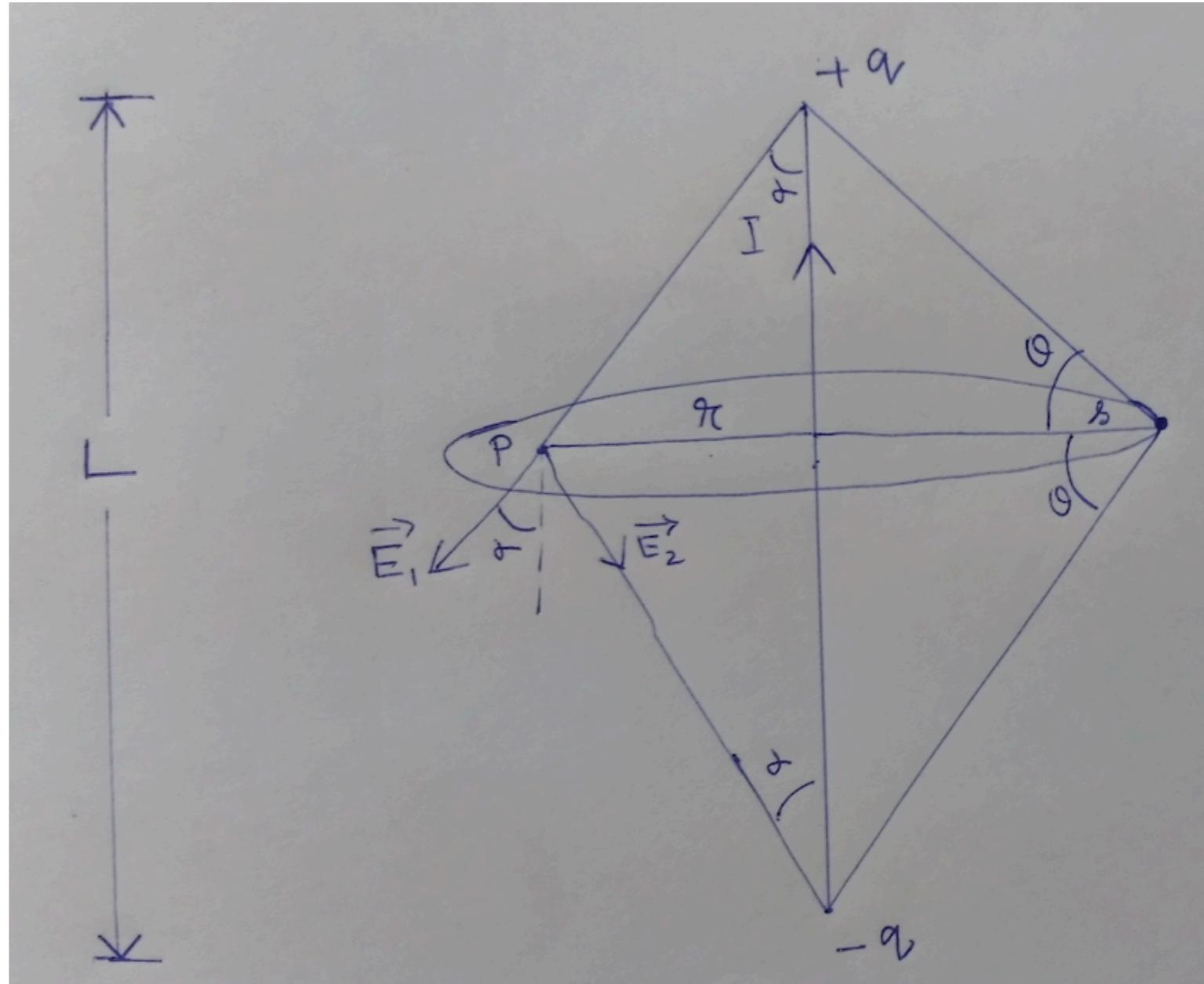
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t}, \quad \Phi_E = \int \vec{E} \cdot d\vec{a}$$



Electric flux through
the surface enclosed
by the Amperian loop

Finite Current Carrying Wire

- One has to find the electric flux passing through the surface enclosed by the Amperian loop (See figure on right).
- Let us consider the flux passing through an elemental area da at point P on the surface.



$$d\Phi_E = -\frac{2q}{4\pi\epsilon_0} \frac{\cos\alpha}{(L/2)^2 + r^2} da$$

$$da = r dr d\phi$$

$$r = \frac{L}{2} \tan\alpha$$

Finite Current Carrying Wire

$$\Phi_E = \int d\Phi_E = -\frac{2q}{4\pi\epsilon_0} \frac{4}{L^2} \int \frac{\cos \alpha}{\sec^2 \alpha} r dr d\phi$$

$$\Phi_E = -\frac{2q}{\pi\epsilon_0 L^2} \int \frac{\cos \alpha}{\sec^2 \alpha} \frac{L}{2} \tan \alpha \frac{L}{2} \sec^2 \alpha d\alpha d\phi$$

$$\Phi_E = -\frac{2q}{4\pi\epsilon_0} \int_0^{\alpha_1} \sin \alpha d\alpha \int_0^{2\pi} d\phi$$

$$\Phi_E = -\frac{q}{\epsilon_0} (1 - \cos \alpha_1) = -\frac{q}{\epsilon_0} (1 - \sin \theta)$$

Finite Current Carrying Wire

- Using the expression for electric flux, we can find magnetic field from Ampere's law (with Maxwell's correction) as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(-\frac{q}{\epsilon_0} (1 - \sin \theta) \right)$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I - \mu_0 I + \mu_0 I \sin \theta$$

$$B(2\pi s) = \mu_0 I \sin \theta$$

$$B(s) = \frac{\mu_0 I}{2\pi s} \sin \theta$$

- Which is same as the answer we got using Biot-Savart Law.

Reminder: Quiz 2 (April 9, 2019)

Syllabus: Lecture 15-20

- Lorentz Force, Biot-Savart law, Divergence & Curl of Magnetostatic Fields.
- Applications of Ampere's law, Magnetic Vector Potential.
- Magnetic dipole, Force & torque on a magnetic dipole, Magnetic Materials, Magnetization.
- Field of a magnetized object, Boundary Conditions.
- Ohm's law, motional emf, electromotive force.
- Faraday's law, Lenz's law, Self & Mutual Inductance, Energy Stored in Magnetic Field.

Solutions to take home problems (as many as possible!) will be uploaded in Moodle by tomorrow.