CS528
Task Scheduling
(Part II)

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Parallel Machines

Ti	P1	P2	P3	P4
T1	10	10	10	10
T2	12	12	12	12
T3	16	16	16	16
T4	20	20	20	20

P: Identical

Ti	P1	P2	P3	P4
T1	10	15	20	25
T2	12	18	24	30
T3	16	24	32	40
T4	20	30	40	50

Q: Uniform : with speed difference (S_1=1, S_2=2/3, S_3=1/2, S_4=2/5

Ti	P1	P2	P3	P4
T1	10	8	12	2
T2	12	28	25	13
T3	16	4	32	14
T4	20	38	42	22

R: Unrelated : heterogeneous

Classification of Scheduling Problems

Classes of scheduling problems can be specified in terms of the three-field classification

$$\alpha \quad | \quad \beta \quad | \quad \gamma$$

where

- α specifies the **machine environment**,
- β specifies the **job characteristics**, and
- γ describes the **objective function(s).**

$P_m || C_{max}$

- n tasks, m processors
- ET: t_1, t_2, t_3,…., t_n
- **m-Subset Sum problem**
- **INDEP(m) Problem: NPC in strong sense**
- Divide the tasks in m sets such that
  - Difference of Sum of ETs of all the set is minimized**: does not exceed a value K**
  - Min (Max(Sum(Set_1), Sum(Set_2), …Sum(Set_m)))

$P_m | pmtn | C_{max}$

- n tasks, m processors, infinite pre-emption allowed
- ET: $t_1, t_2, t_3, \ldots, t_n$
- Divide all the work among all the cores equally
  - Avg=$(\Sigma\ t_i)/m$ work to each cores
  - If $max(t_i)>Avg$, $C_{max}=max(t_i)$, Task need to execute serially
- Handle the boundary cases

$Q_m | pmtn | C_{max}$

- n tasks, m uniform processors, infinite pre-emption allowed
- Longer task executed on high speed processor till it execution is long enough as compared to others
  - Sort the tasks based on LPT
  - Allocate long task to higher speed processors one by one
  - When execution time of longer task is no longer long as compared to other then co-execute

$Q_m \mid pmtn \mid C_{max}$

Algorithm level

```
t := 0;
WHILE there exist jobs with positive level {
        Assign(t);
    t1 := min{s > t | a job completes at time s};
    t2 := min{s > t | there are jobs i, j with pi(t) > pj(t)
                 and pi(s) = pj(s) at time s};
    t := min{t1, t2}
}
```

$t := 0;$

WHILE there exist jobs with positive level {

Assign(t);

$t_1 := \min\{s > t \mid \text{a job completes at time } s\};$

$t_2 := \min\{s > t \mid \text{there are jobs } i, j \text{ with } p_i(t) > p_j(t) \text{ and } p_i(s) = p_j(s) \text{ at time } s\};$

$t := \min\{t_1, t_2\}$

}

$Q_m \mid pmtn \mid C_{max}$

Assign (t)

$J := \{i \mid pi(t) > 0\}$;

$M := \{M1, . . .,Mm\}$;

WHILE $J = \emptyset$ and $M = \emptyset$ {

Find the set $I \subseteq J$ of jobs with highest level;

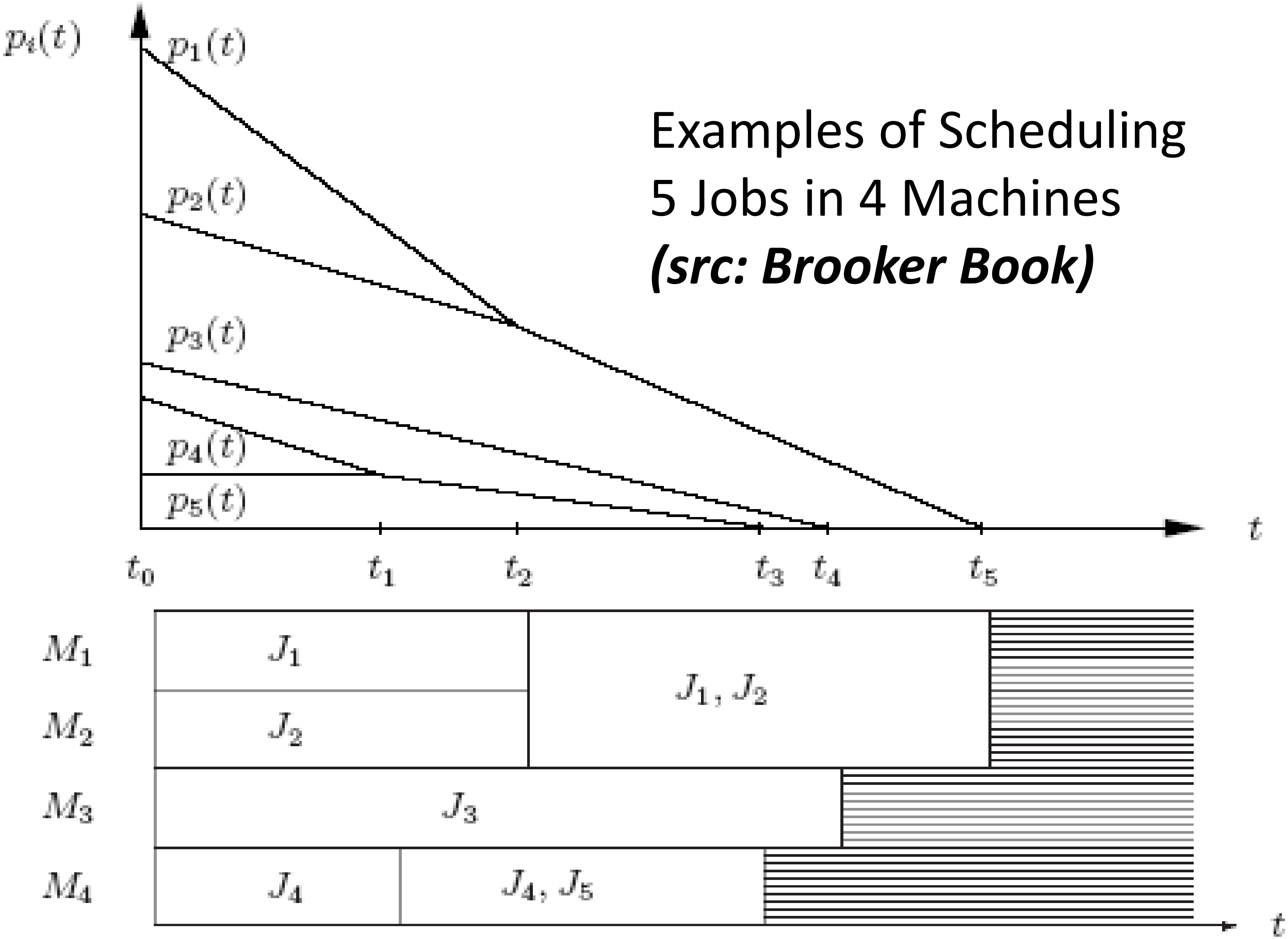
$r := \min\{|M|, |I|\}$;

Assign jobs in I to be processed jointly on the r fastest machines in M;

$J := J \setminus I$;

Eliminate the r fastest machines in M from M

$Q_m \mid pmtn \mid C_{max}$



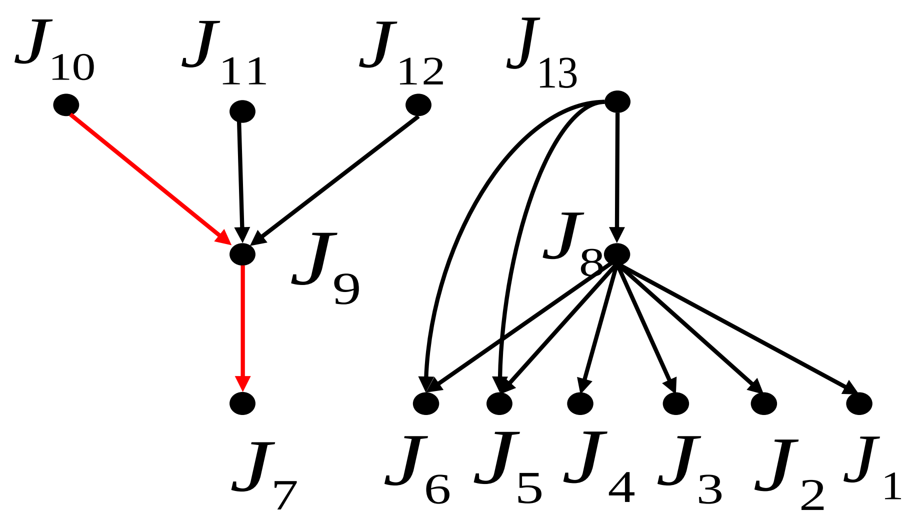
Examples of Scheduling
5 Jobs in 4 Machines
(src: Brooker Book)

Precedence constraints (*prec*)

Before certain jobs are allowed to start processing, one or more jobs first have to be completed.

Definition

- Successor
- Predecessor
- Immediate successor
- Immediate predecessor
- Transitive Reduction



$$p(J_i) = 1$$

Precedence constraints (*prec*)

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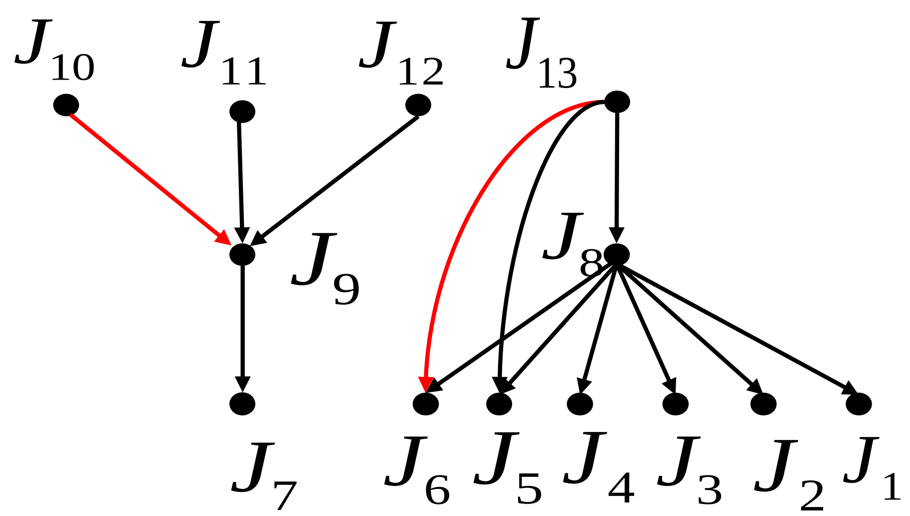


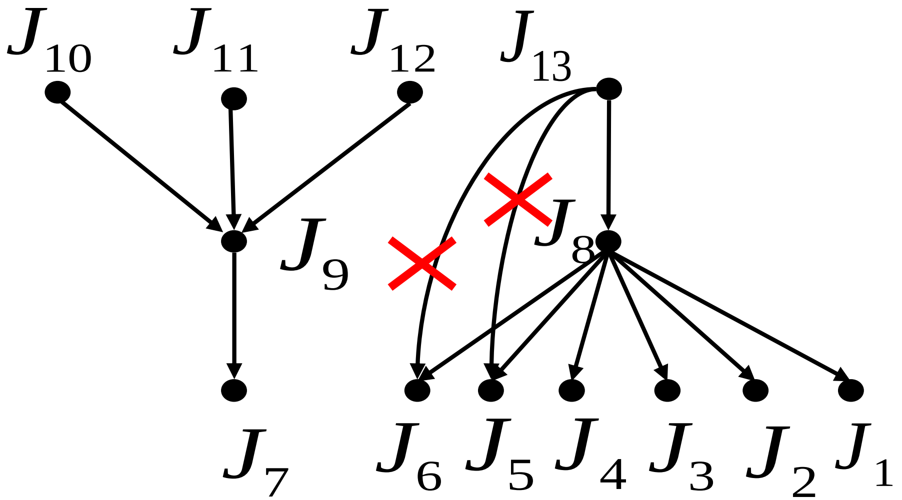
$$p(J_i) = 1$$

Precedence constraints (*prec*)

One or more job have to be completed before another job is allowed to start processing. *Prec : Arbitrary acyclic graph*

Definition

- Successor
- Predecessor
- Immediate successor
- Immediate predecessor
- Transitive Reduction

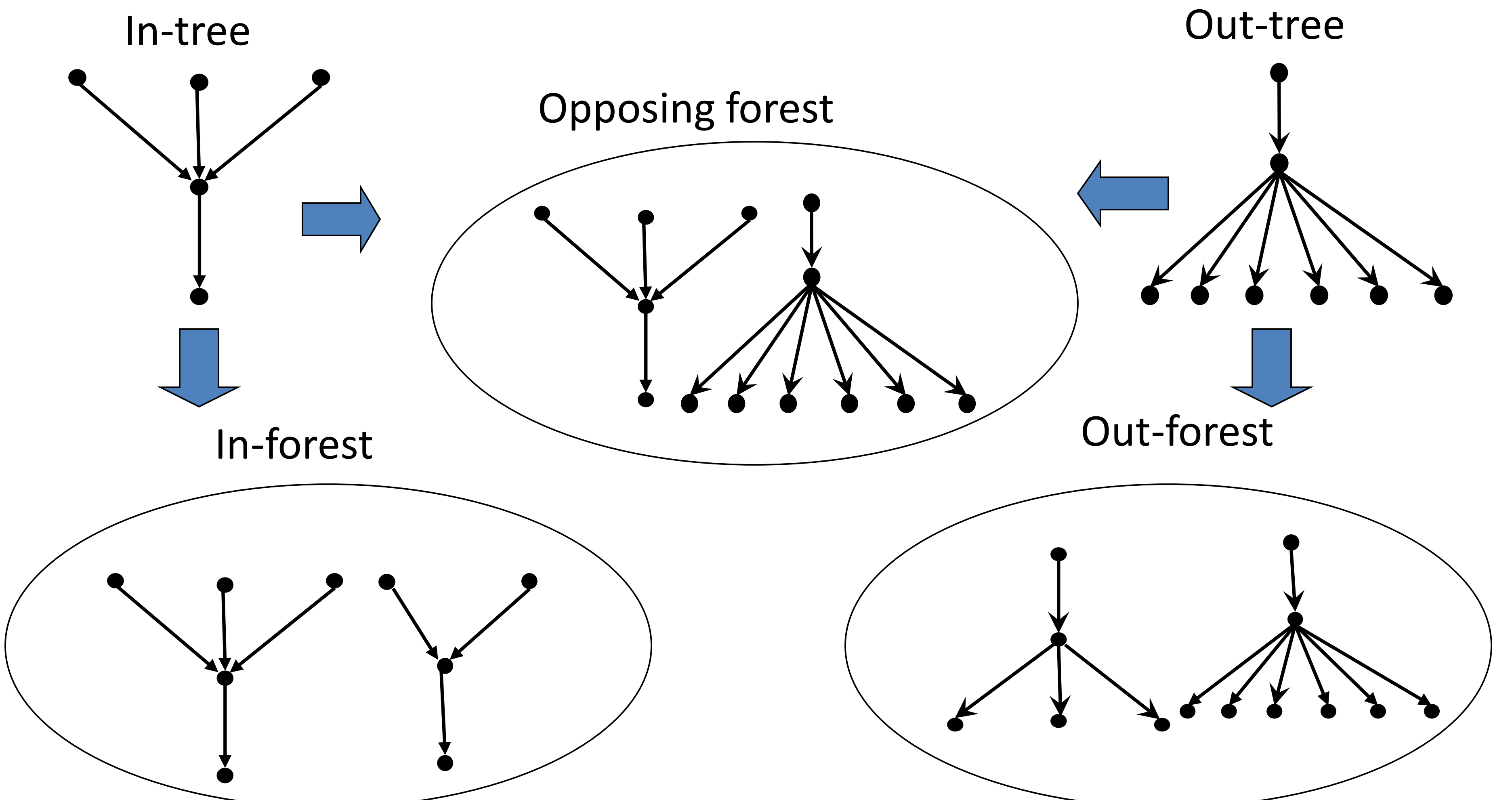


$$p(J_i) = 1$$

Special precedence constraints

- In-tree (Out-tree)
- In-forest (Out-forest)
- Opposing forest
- ***Interval orders***
- ***Series-parallel orders***
- ***Level orders***

Special precedence constraints



A Sahu

P_m | prec, $p_j = 1$ | C_{max} ($m \geq 1$)

- Processor Environment
  - m identical processors are in the system.
- Job characteristics
  - Precedence constraints are given by a precedence graph;
  - Preemption is not allowed;
  - The release time of all the jobs is 0.
- Objective function
  - C_{max} : the time the last job finishes execution.
  - If c_j denotes the finishing time of J_j in a schedule S,

$$C_{max} = max_{1 \leq j \leq n} c_j$$

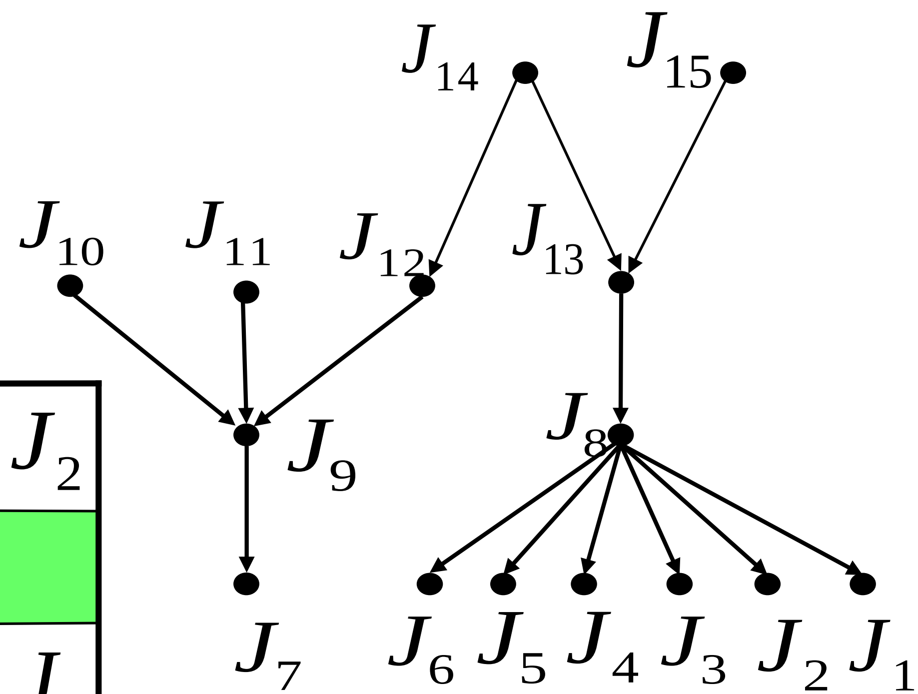
Gantt Chart

A Gantt chart indicates the time each job spends in execution, as well as the processor on which it executes ~~of some Schedule~~

Slot 1 Slot 2

P1			J_{15}	J_{13}	J_9	J_5	J_2
P2		J_{14}	J_{12}	J_8	J_7	J_4	
P3	J_{10}	J_{11}			J_6	J_3	J_1

Time axis



$$p(J_i) = 1$$

P_m | prec, $p_j = 1$ | C_{max}

Theorem 1

Pm| prec, $p_j = 1$ | C_{max} is NP-complete.

1. Ullman (1976)

   3SAT ≤ Pm | prec, $p_j = 1$ | C_{max}

2. Lenstra and Rinooy Kan (1978)

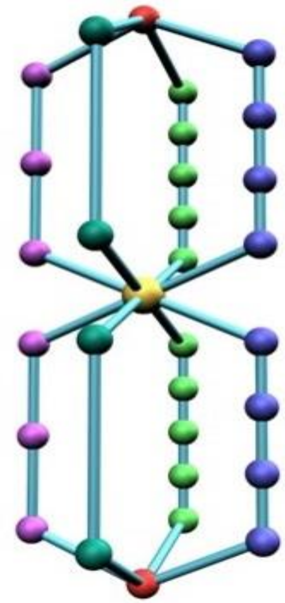
   k-clique ≤ Pm| prec, $p_j = 1$ | C_{max}

P_m| prec, pj = 1 | C_{max} is NP-complete.

Proof: out of Syllabus

P_m | prec, $p_j = 1$ | C_{max}

Mayr (1985)



- **Theorem 2**

$Pm \mid p_j = 1, SP \mid C_{max}$ is NP-complete.

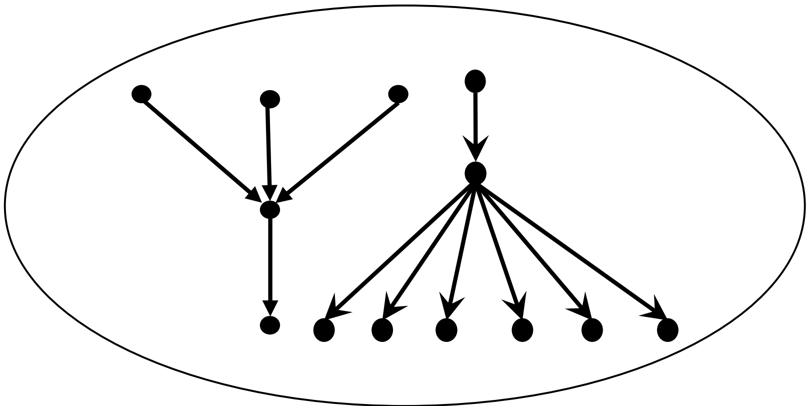
SP: Series - parallel

- **Theorem 3**

$Pm \mid p_j = 1, OF \mid C_{max}$ is NP-complete.

OF: Opposing - forest

Opposing forest

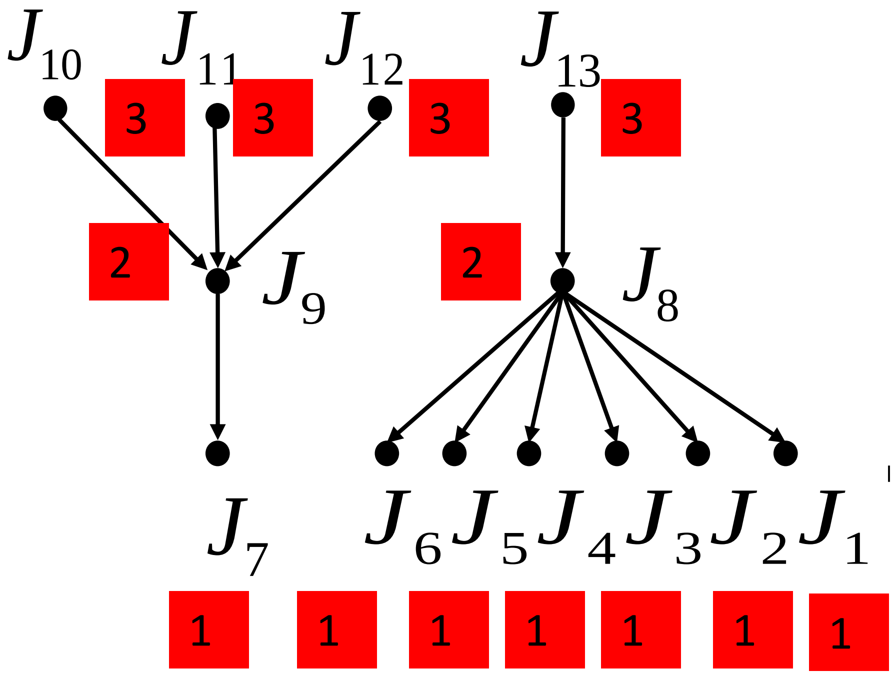


Proof: out of Syllabus

Hu's HLF/CP Algorithm

- T. C. Hu (1961), **Critical Path/Highest Level First**
- Assign a level h to each job.
  - **If job has no successors, h(j) equals 1.**
  - **Otherwise, h(j) equals one plus the maximum level of its immediate successors.**
- Set up a priority list L by nonincreasing order of the jobs' levels.
- Execute the list scheduling policy on this level based priority list L.

HLF/CP algorithm : Example



M2	J_{10}	J_{13}	J_8	J_6	J_3
M2	J_{11}	J_9	J_7	J_5	J_2
M1	J_{12}			J_4	J_1

$$L = (\overbrace{J_{10}, J_{11}, J_{12}, J_{13}}^{\text{Level 3}}, \overbrace{J_9, J_8}^{\text{Level 2}}, \overbrace{J_7, J_6, J_5, J_4, J_3, J_2, J_1}^{\text{Level 1}})$$

HLF/CP algorithm

- **Time complexity**

  O(|V|+|E|)   (|V| is the number of jobs and |E| is the number of edges in the precedence graph)

- **Theorem (Hu, 1961) : HLF/CP for Tree**
  - **The HLF algorithm is optimal for $P_m \mid p_j = 1$, in-tree (out-tree) $\mid C_{max}$.**
  - **The HLF algorithm is optimal for $P_m \mid p_j = 1$, in-forest (out-forest) $\mid C_{max}$.**

HLF/CP algorithm

- N.F. Chen & C.L. Liu (1975)

The approximation ratio of HLF algorithm for the problem with general precedence constraints:

If $m = 2$, $\delta_{HLF} \leq 4/3$.

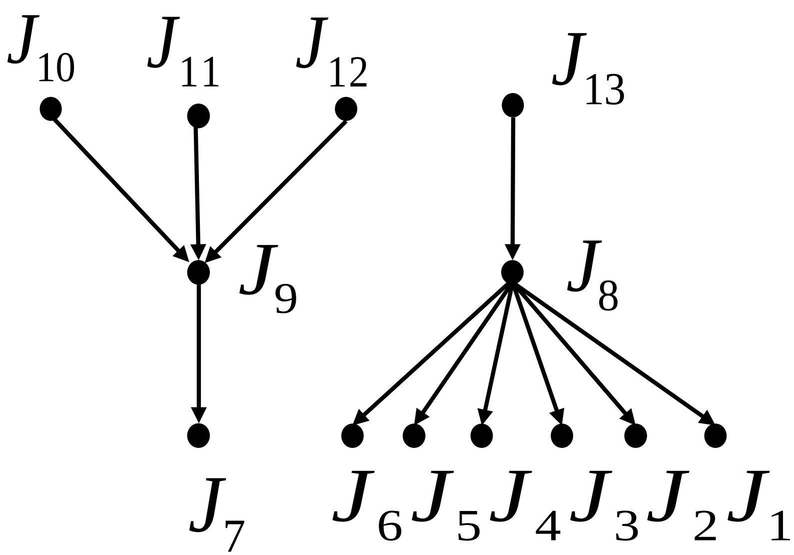
If $m \geq 3$, $\delta_{HLF} \leq 2 - 1/(m-1)$.

PTAS Algorithms: Pm | prec, $p_j = 1$ | C_{max}

- PTAS : Polynomial Time Approximation Scheme
- Approximation List scheduling policies
  - Graham's list algorithm/Greedy List
    - **Discussed in Cilk Lectures : $T \leq 2T^*$, Also proved**
    - **CLR Book Chapter 27, Multi-threaded Algorithm**
  - HLF algorithm
  - MSF algorithm

List scheduling policies

- Set up a priority list L of jobs.
- When a processor is idle, assign the first ready job to the processor and remove it from the list L.



J_{11}	J_9	J_8	J_6	J_3
J_{10}	J_{13}	J_7	J_5	J_2
J_{12}			J_4	J_1

First job of the list may not be ready

$$L=(J_9,J_8,J_7,J_6,J_5,J_{11},J_{10},J_{12},J_{13},J_4,J_3,J_2,J_1)$$

Graham's list algorithm

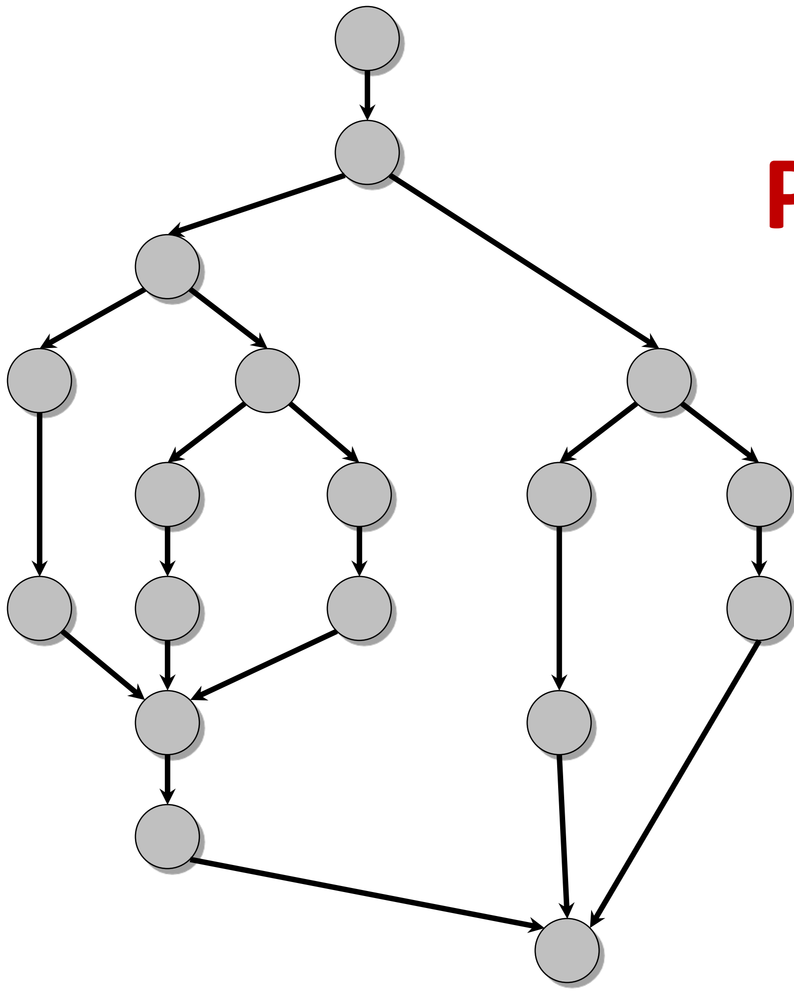
- Graham first analyzed the performance of the simplest list scheduling algorithm.
- List scheduling algorithm with an arbitrary job list is called Graham's list algorithm.
- Approximation ratio for $Pm \mid prec, p_j = 1 \mid C_{max}$

  δ = 2-1/m. (Tight bound!)

  - Approximation ratio is δ if for each input instance, the makespan produced by the algorithm is at most δ times of the optimal makespan.

CP Algo: CLR Book Page 779-783

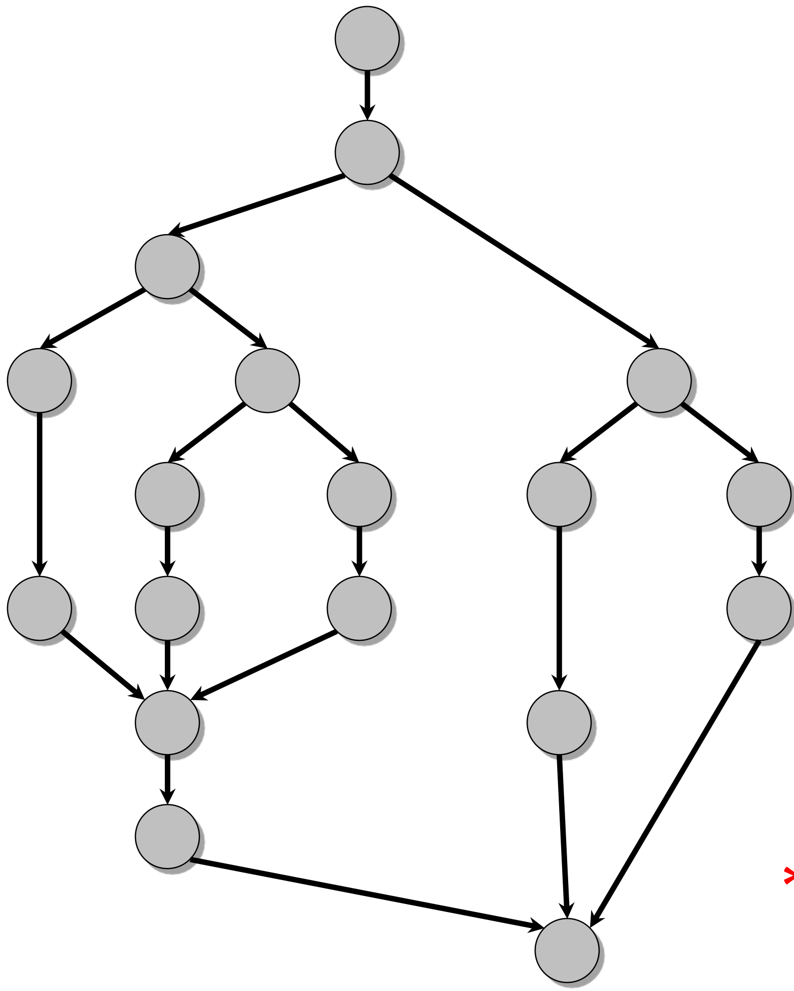
T_P = execution time on P processors



$Pm \mid p_j = 1 \text{ , prec} \mid C_{max}$

CP Algorithms

T_P = execution time on P processors



T_1 = ***work***

T_∞ = ***span****

LOWER BOUNDS

- $T_P \geq T_1/P$
- $T_P \geq T_\infty$

*Also called ***critical-path length*** or ***computational depth***.

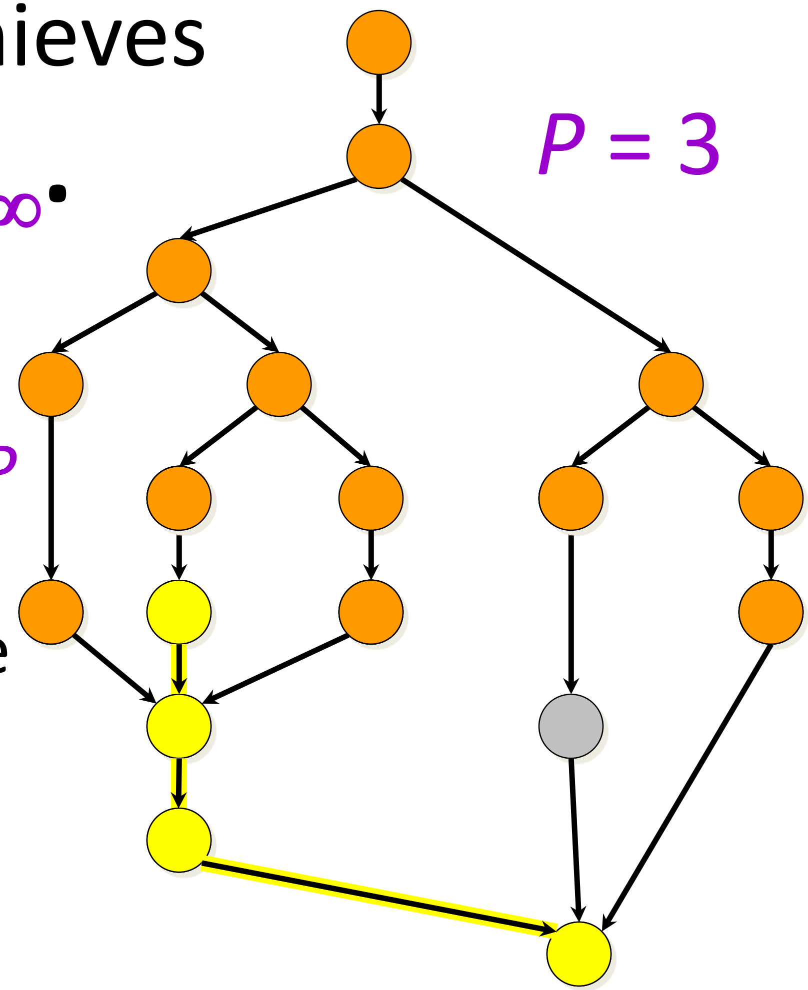
CP: Greedy-Scheduling Theorem

Theorem [Graham '68 & Brent '75].

Any greedy scheduler achieves

$$T_P \leq T_1/P + T_\infty.$$

$P = 3$



Proof.

- # complete steps $\leq T_1/P$, since each complete step performs P work.
- # incomplete steps $\leq T_\infty$, since each incomplete step reduces the span of the unexecuted dag by 1. ■

CP: Optimality of Greedy

Corollary. Any greedy scheduler achieves within a factor of 2 of optimal.

Proof. Let T_P^* be the execution time produced by the optimal scheduler. Since $T_P^* \geq \max\{T_1/P, T_\infty\}$ (lower bounds), we have

$$
\begin{aligned}
T_P &\leq T_1/P + T_\infty \\
&\leq 2 \cdot \max\{T_1/P, T_\infty\} \\
&\leq 2T_P^* . \quad \blacksquare
\end{aligned}
$$