PH 102: Physics II

Lecture 19 (Spring 2019)

IIT Guwahati
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LECTURE PLAN (TENTATIVE) OF PH 102 (POST MID-SEM)

SN	Date	Topic	Griffith's section	Lectures	Division
Lec 15	13-3-2019	Lorentz Force, Biot-Savart law, Divergence & Curl of Magnetostatic Fields	5.1, 5.2, 5.3	1	I, II (4-4:55 pm)
Lec 15	13-3-2019	Lorentz Force, Biot-Savart law, Divergence & Curl of Magnetostatic Fields	5.1, 5.2, 5.3	1	III, IV (9-9:55 am)
Lec 16	14-3-2019	Applications of Ampere's law, Magnetic Vector Potential	5.3, 5.4	1	I, II (3-3:55 pm)
Lec 16	14-3-2019	Applications of Ampere's law, Magnetic Vector Potential	5.3, 5.4	1	III, IV (10-10:55 am)
Tut 8	19-3-2019	Lec 15, 16			
Lec 17	20-3-2019	Magnetic dipole, Force & torque on a magnetic dipole, Magnetic Materials, Magnetization	5.4, 6.1	1	I, II (4-4:55 pm)
Lec 17	20-3-2019	Magnetic dipole, Force & torque on a magnetic dipole, Magnetic Materials, Magnetization	5.4, 6.1	1	III, IV (9-9:55 am)
Lec 18	22-3-2019	Field of a magnetized object, Boundary Conditions	6.2, 6.3, 6.4	1	I, II (3-3:55 pm)
Lec 18	22-3-2019	Field of a magnetized object, Boundary Conditions	6.2, 6.3, 6.4	1	III, IV (10-10:55 am)
Tut 9	26-3-2019	Lec 17, 18			
Lec 19	27-3-2019	Ohm's law, motional emf, electromotive force	7.1	1	I, II (4-4:55 pm)
Lec 19	27-3-2019	Ohm's law, motional emf, electromotive force	7.1	1	III, IV (9-9:55 am)
Lec 20	28-3-2019	Faraday's law, Lenz's law, Self & Mutual Inductance, Energy Stored in Magnetic Field	7.2	1	I, II (3-3:55 pm)
Lec 20	28-3-2019	Faraday's law, Lenz's law, Self & Mutual Inductance, Energy Stored in Magnetic Field	7.2	1	III, IV (10-10:55 am)
Tut 10	2-4-2019	Lec 19, 20			
Lec 21	3-4-2019	Maxwell's equations	7.3	1	I, II (4-4:55 pm)
Lec 21	3-4-2019	Maxwell's equations	7.3	1	III, IV (9-9:55 am)
Lec 22	4-4-2019	Discussions, problem solving	7.3	1	I, II (3-3:55 pm)
Lec 22	4-4-2019	Discussions, problem solving	7.3	1	III, IV (10-10:55 am)
Tut 11	9-4-2019	Quiz II			

LECTURE PLAN (TENTATIVE) OF PH 102 (POST MID-SEM)

Lec 23	10-4-2019	Continuity equation, Poynting Theorem	8.1	1	I, II (4-4:55 pm)
Lec 23	10-4-2019	Continuity equation, Poynting Theorem	8.1	1	III, IV (9-9:55 am)
Lec 24	11-4-2019	Wave solution of Maxwell's equation, polarisation	9.1, 9.2	1	I, II (3-3:55 pm)
Lec 24	11-4-2019	Wave solution of Maxwell's equation, polarisation	9.1, 9.2	1	III, IV (10-10:55 am)
Tut 12	23-4-2019	Lec 23, 24			
Lec 25	24-4-2019	Electromagnetic waves in matter, Reflection and transmission: Normal incidence	9.3	1	I, II (4-4:55 pm)
Lec 25	24-4-2019	Electromagnetic waves in matter, Reflection and transmission: Normal incidence	9.3	1	III, IV (9-9:55 am)
Lec 26	25-4-2019	Reflection and transmission: Oblique Incidence	9.3, 9.4	1	I, II (3-3:55 pm)
Lec 26	25-4-2019	Reflection and transmission: Oblique Incidence	9.3, 9.4	1	III, IV (10-10:55 am)
Tut 13	30-4-2019	Lec 25, 26			
Lec 27	1-5-2019	Relativity and electromagnetism: Galilean and special relativity	12.1, 12.2, 12.3	1	I, II (4-4:55 pm)
Lec 27	1-5-2019	Relativity and electromagnetism: Galilean and special relativity	12.1, 12.2, 12.3	1	III, IV (9-9:55 am)
Lec 28	2-5-2019	Discussions, problem solving	12.1, 12.2, 12.3	1	I, II (3-3:55 pm)
Lec 28	2-5-2019	Discussions, problem solving	12.1, 12.2, 12.3	1	III, IV (10-10:55 am)

Ferromagnetism

Ferromagnets require no external fields to sustain the magnetisation.

In ferromagnet, each dipole likes to point in the same direction as its neighbours.

Such alignment occurs in small domains each of which contains billions of dipoles, all lined up.

But due to random orientations of domains, an object that contains enormous number of such domains whose magnetic fields cancel, is not magnetised as a whole (e.g. a nail is not a powerful magnet).

Ferromagnetism

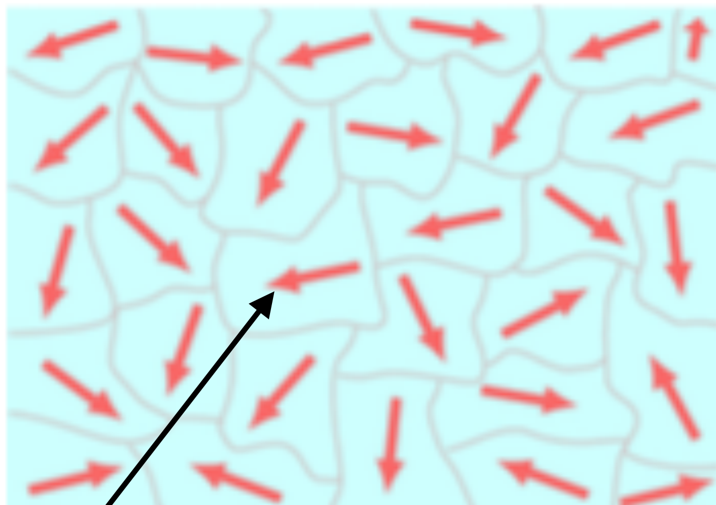
Under an applied magnetic field, domains parallel to the field grows and can take over the entire object, leading to saturation.

This process is not entirely reversible. After turning the field off, some randomly orientated domain configurations will be created again, but not completely. There remains some domains in the original direction: Permanent magnet.

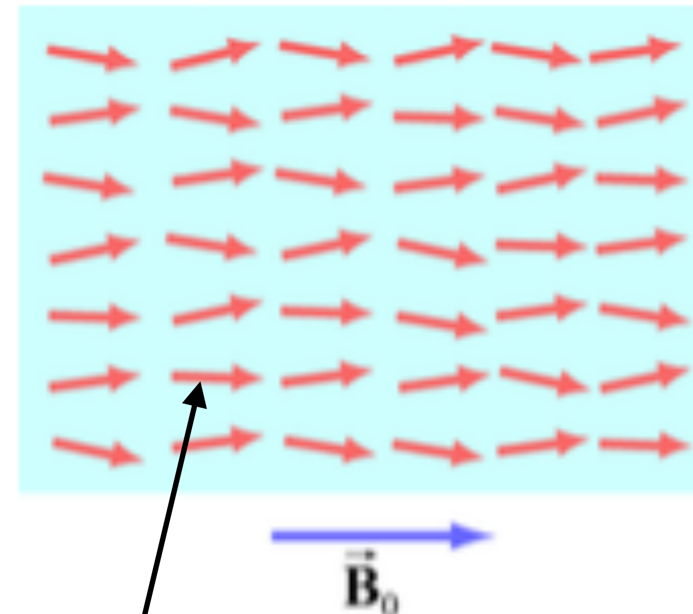
The irreversibility of magnetisation of ferromagnetic materials leads to the **hysteresis loop**.

They exhibit highly non-linear M versus H curve, unlike the linear relation in case of dia- and paramagnetic materials.

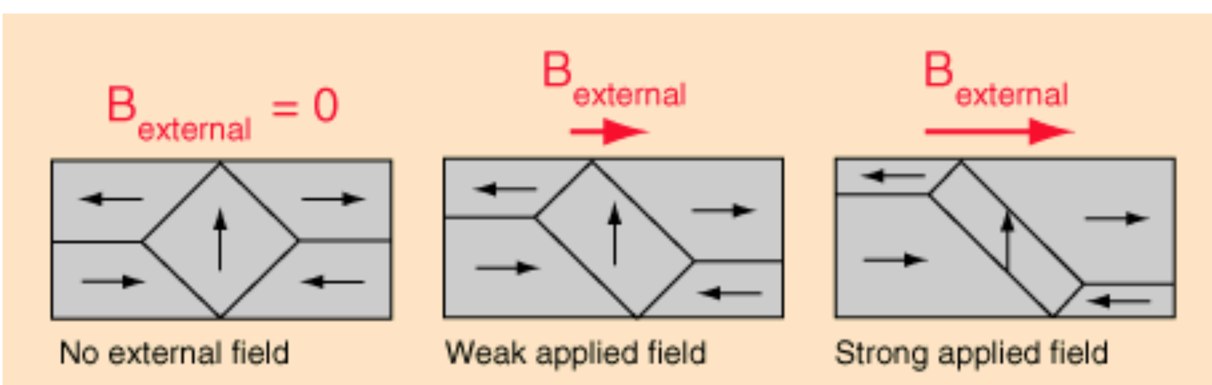
Ferromagnetism



Ferromagnetic domains



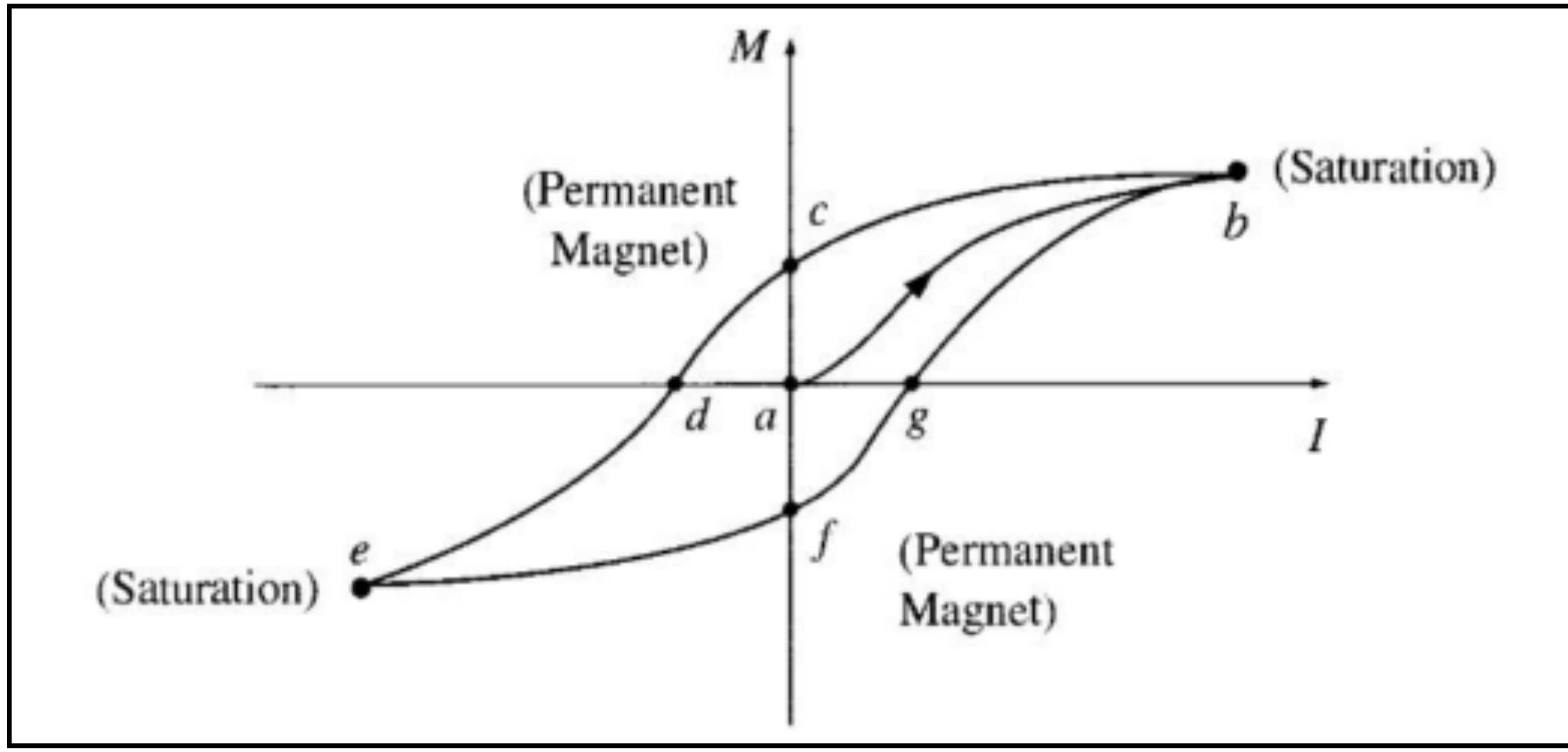
Alignment of magnetic dipoles along applied field



Growth of domains parallel to the applied field

Image credit: MIT, hyperphysics

Hysteresis Loop



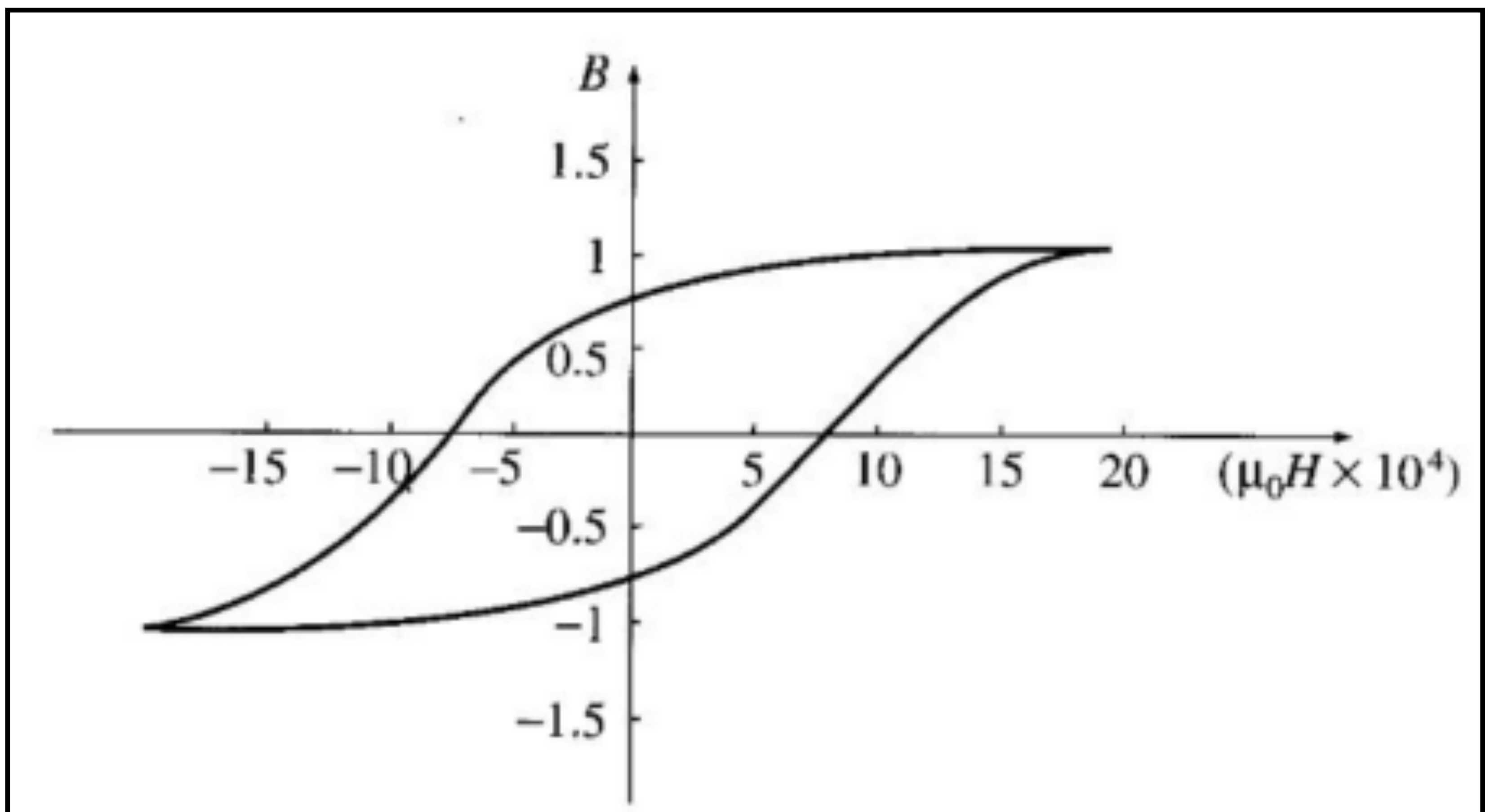


Figure 6.28, 6.29 Introduction to Electrodynamics, D. J. Griffiths

Ferromagnets

- Magnetisation depends not only on the applied field (or I) but also on its previous magnetic history.
- This property of magnetic memory is utilised to make magnetic storage of data, audio tap recorders etc.
- A solenoid coil will produce a field $\mu_0\vec{H}$ whereas $\vec{B}$ is the actual field one gets after introducing the iron.
- For iron, $\vec{B}$ is huge compared to $\mu_0\vec{H}$. Therefore, a little current goes a long way for ferromagnetic materials. That is why, a coil is wrapped around an iron core to make a powerful electromagnet. (A tiny applied field of 0.0002 T can produce 1 T field in iron!)

Critical Temperature

- At very high temperatures, the random thermal motions overcome the dipole ordering and destroy it.
- This occurs at a precise temperature which is 770 degree C for iron.
- Below this temperature (known as the Curie point), iron is ferromagnetic.
- Above the Curie temperature, iron behaves like a paramagnet.
- Such abrupt changes in properties of a substance at sharply defined temperatures are called phase transitions (similar to water-vapour, water-ice transitions).

Summary: So far!

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$ Lorentz Force Law

$$\vec{F} = \int I(\vec{dl} \times \vec{B})$$ Force on current carrying wire

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$ Continuity Equation

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \hat{\mathfrak{r}}}{\mathfrak{r}^2} dl' = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l'} \times \hat{\mathfrak{r}}}{\mathfrak{r}^2}$$ Biot-Savart Law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{\text{enc}} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$ Ampere's Law

$$\vec{\nabla} \cdot \vec{B} = 0$$ Absence of monopole

Summary: So far!

Magnetic Vector Potential

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \cdot \vec{A} = 0 \qquad \nabla^2 \vec{A} = -\mu_0 \vec{J} \qquad \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r'})}{\mathfrak{r}} d\tau'$$

Magnetostatic Boundary Condition

$$\vec{B}_{\text{above}} - \vec{B}_{\text{below}} = \mu_0 (\vec{K} \times \hat{n})$$

Magnetic Dipole

$$\vec{m} = \frac{1}{2} \oint I(\vec{r} \times d\vec{l}) = \frac{1}{2} \int (\vec{r} \times \vec{J}) d\tau$$

$$\vec{A}_{\text{dipole}}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}, \qquad \vec{B}_{\text{dipole}}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^3} [3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m}]$$

$$\vec{N} = \vec{m} \times \vec{B} \qquad \vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B})$$

Force & Torque on Magnetic Dipole

Summary: So far!

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{M}(\vec{r'}) \times \hat{\mathfrak{r}}}{\mathfrak{r}^2} d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}_b(\vec{r'})}{\mathfrak{r}} d\tau' + \frac{\mu_0}{4\pi}\oint_S \frac{\vec{K}_b(\vec{r'})}{\mathfrak{r}} da'$$

$$\vec{J}_b = \vec{\nabla} \times \vec{M} \qquad \vec{K}_b = \vec{M} \times \hat{n}$$

$$\vec{\nabla} \times \left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right) = \vec{\nabla} \times \vec{H} = \vec{J}_f$$

$$H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}})$$

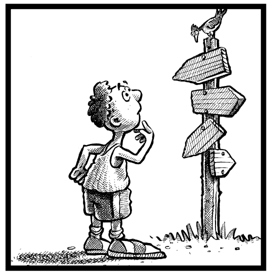
$$\vec{H}^{\parallel}_{\text{above}} - \vec{H}^{\parallel}_{\text{below}} = \vec{K}_f \times \hat{n}$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H}$$

$$\mu = \mu_0(1 + \chi_m) \qquad \vec{J}_b = \vec{\nabla} \times \vec{M} = \vec{\nabla} \times (\chi_m \vec{H}) = \chi_m \vec{J}_f$$

Magnetisation
Magnetic Materials
Field of Magnetised Object
Auxiliary Field
Boundary Conditions

Confusions?



- Magnetic field does no work on charged particles. Who does work when a magnetic dipole moves in non-uniform field?

- If a dipole moves in a non-uniform field, a net amount of work is done on the electrons, which also tries to change their flow!

- Concept of work is incomplete in magnetostatics: can't calculate the energy stored in magnetostatic field.

- While deriving the potential energy of a magnetic dipole $U = -\vec{\mu} \cdot \vec{B}$, we considered only the mechanical forces on the loop (See 15-2, 15-3, Feynman Lectures Vol II).

- Magnetic field does not do work on charged particle, but producing the field requires changing the field. Similarly, the motion of magnetic dipole leads to changing the field.

- And changing magnetic field produces electric field, which can work on charged particles! (Lecture 19-20 onwards!)

Ohm's Law

Current density $\vec{J}$ is proportional to the force per unit charge $\vec{f}$: $\vec{J} = \sigma \vec{f}$ where σ is the conductivity of the medium. It is related to the resistivity ρ as $\rho = 1/\sigma$

For perfect conductors, $\sigma \to \infty$

Material	Resistivity	Material	Resistivity
Conductors:		*Semiconductors:*	
Silver	1.59×10^{-8}	Salt water (saturated)	4.4×10^{-2}
Copper	1.68×10^{-8}	Germanium	4.6×10^{-1}
Gold	2.21×10^{-8}	Diamond	2.7
Aluminum	2.65×10^{-8}	Silicon	2.5×10^{3}
Iron	9.61×10^{-8}	*Insulators:*	
Mercury	9.58×10^{-7}	Water (pure)	2.5×10^{5}
Nichrome	1.00×10^{-6}	Wood	$10^{8} - 10^{11}$
Manganese	1.44×10^{-6}	Glass	$10^{10} - 10^{14}$
Graphite	1.4×10^{-5}	Quartz (fused)	$\sim 10^{16}$

Ohm's Law

If electromagnetic forces drive the charges then the current density is

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$$

Typically, the velocity of the charges is very small (not true for plasma) so that the second term can be ignored

$$\vec{J} = \sigma \vec{E}$$

Ohm's Law

This is consistent with the fact that for stationary charges ($\vec{J} = 0$), the field inside a conductor is zero ($\vec{E}$=0).

For a perfect conductor, $\vec{E} = \vec{J}/\sigma \to 0$ even though in this case $\vec{J} \neq 0$: Field required to drive current through a perfect conductor is negligible.

Resistors on the other hand, are made up of poor conductors resulting in non-zero electric field inside.

Example 7.1 (Introduction to Electrodynamics, D. J. Griffiths): A cylindrical resistor of cross-sectional area A and length L is made from material with conductivity σ. If the potential is constant over each end and the potential difference between the ends is V, what current flows?

$$I = JA = \sigma EA = \sigma \frac{V}{L} A$$

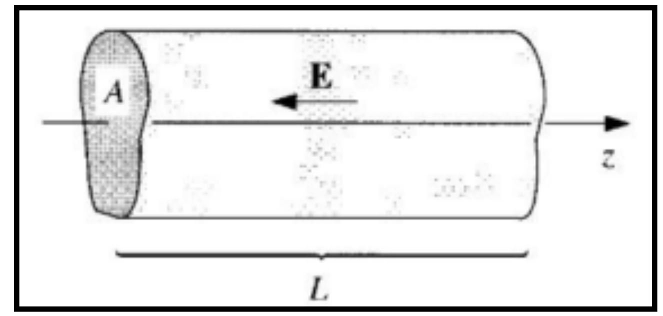


Figure 7.1, Introduction to Electrodynamics, D. J. Griffiths

Example 7.2 (Introduction to Electrodynamics, D. J. Griffiths): Two long cylinders (radii a and b) are separated by material of conductivity σ . If they are maintained at a potential difference V, what current flows from one to the other, in a length L?

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}, \quad I = \int \vec{J}\cdot d\vec{a} = \sigma \int \vec{E}\cdot d\vec{a} = \sigma \frac{\lambda}{2\pi\epsilon_0 s}(2\pi s L) = \frac{\sigma}{\epsilon_0}\lambda L$$

$$V = -\int_b^a \vec{E}\cdot d\vec{l} = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$$

$$\implies I = \frac{2\pi\sigma L}{\ln(b/a)} V$$

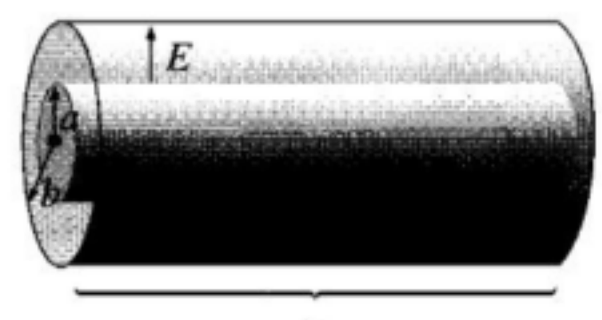


Figure 7.2, Introduction to Electrodynamics, D. J. Griffiths

Ohm's Law

In both the examples 7.1, 7.2, the current flowing is proportional to the potential difference between them V=IR which is the more familiar version of **Ohm's law**.

R is the resistance, a function of the geometry of the arrangement and the conductivity of the medium between the electrodes.

Resistance is measured in ohms (Ω): volt per ampere.

Proportionality between V and I is just a consequence of $\vec{J} = \sigma \vec{E}$: increasing charge increases potential, also increases electric field which then increases the current.

For steady current: $\vec{\nabla}\cdot\vec{E} = \frac{1}{\sigma}\vec{\nabla}\cdot\vec{J} = 0$ which implies zero charge density: Any unbalanced charge resides on the surface.

Although individual charges accelerate in an applied electric field, Ohm's law implies that constant electric field produces a constant current only.

Violating Newton's law?

Let λ be the mean free path between collisions of charged particles and v_{thermal} be the their thermal speed. Therefore, the time between successive collision is $t = \lambda/v_{\mathrm{thermal}}$

If an electric field is applied, then the charged particles are accelerated by $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$

Under the applied field, the charged particles drift through the conductor with an average speed given by

$$v_{\text{average}} = \frac{1}{2}at = \frac{a\lambda}{2v_{\text{thermal}}}$$

If there are n molecules per unit volume and f free electrons per molecule, each with charge q and mass m, the current density is

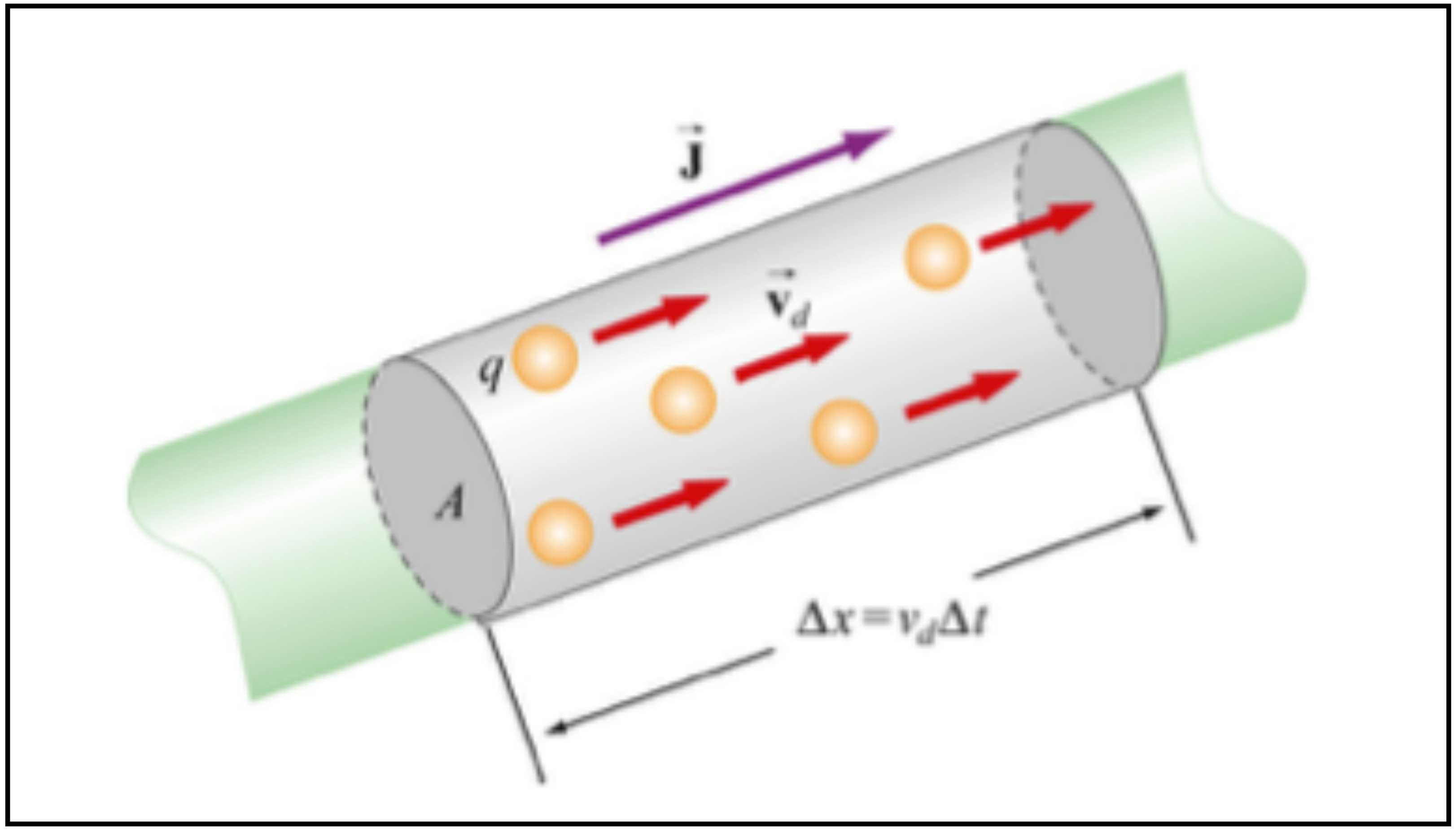
$$\vec{J} = nfq\vec{v}_{\text{average}} = \frac{nfq\lambda}{2v_{\text{thermal}}}\frac{\vec{F}}{m} = \left(\frac{nfq^2\lambda}{2mv_{\text{thermal}}}\right)\vec{E} \qquad \sigma$$

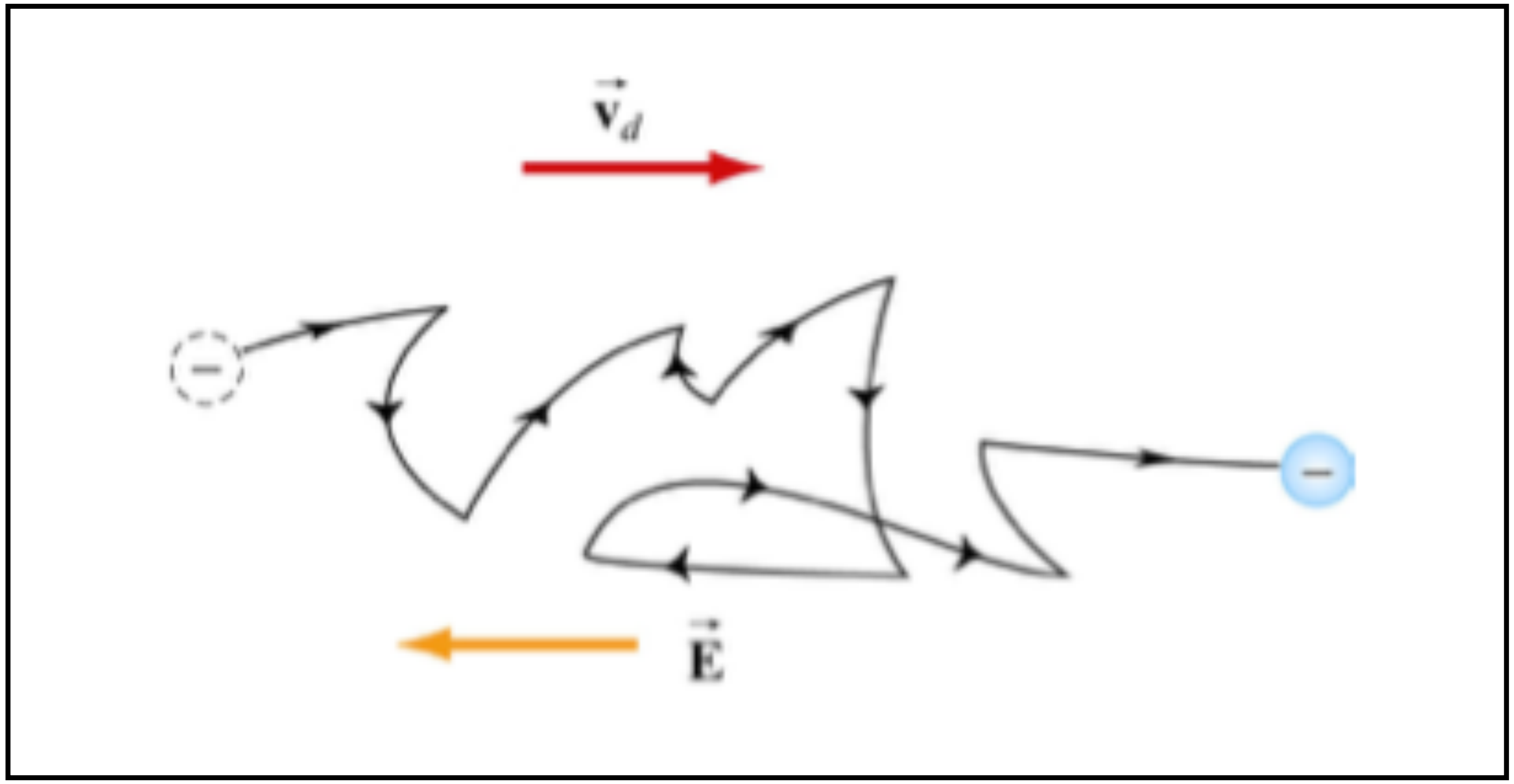
This approximate formula implies that **conductivity is directly proportional to the density of moving charges and decreases with increasing thermal velocities (or temperature)**.

Due to collisions, the work done by the electrical force gets converted into heat in the resistor. Since work done per unit charge is V and the charge per unit time is I, the power delivered is

$$P = VI = I^2R$$

Joule heating law





Credit: MIT

Electron drift velocity in Cu

1 gm of Cu has $(6.022 \times 10^{23})/63.54 = 9.477 \times 10^{21}$ number of atoms. Mass density of Cu is 9.0 gm per cc.

Therefore, number density of Cu is 8.52×10^{22} which is same as the free electron density.

Each Cu atom provides
1 free electron

For 1 A current flowing through a Cu wire of 1 mm square cross section, the drift velocity of electron is

$$v_{\text{average}} = \frac{\frac{1\ \text{A}}{10^{-6}\ \text{m}^2}}{(8.52 \times 10^{22} \times 10^{6}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})} = 7.33 \times 10^{-5} \text{ms}^{-1} \qquad v_{\text{average}} = \frac{J}{nfq}$$

which is tiny!

Although e's get accelerated by E, due to frequent collisions, they stop and start all over again, keeping average speed constant: No violation of Newton's law!

Electromotive Force (emf)

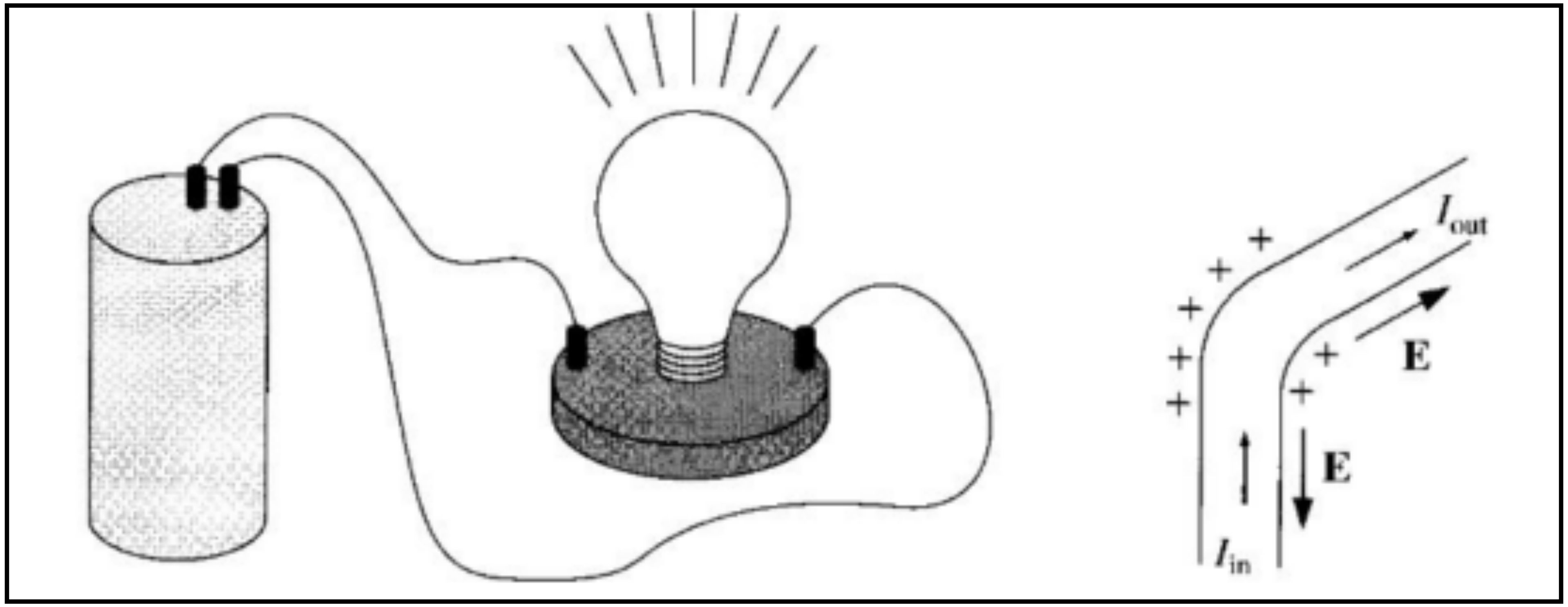


Figure 7.7, 7.8, Introduction to Electrodynamics, D. J. Griffiths

The current I through the circuit is same all the way around the loop.

If I is not same everywhere ($I_{\text{in}} > I_{\text{out}}$), charge accumulates and hence a field **E** is produced. This field then slows the incoming current and supports the outflow of current.

This self-correction mechanism helps to maintain uniformity of current throughout.

Electromotive Force (emf)

Force per unit charge driving current around a circuit:

$$\vec{f} = \vec{f}_s + \vec{E}$$

$\vec{f}_s$ is the force due to the source (battery), $\vec{E}$ is the electrostatic force that maintains uniform current throughout.

The line integral of the force per unit charge:

$$\mathcal{E} \equiv \oint \vec{f} \cdot d\vec{l} = \oint \vec{f}_s \cdot d\vec{l}$$

Electromotive force (emf)

where the conservative nature of electrostatic field is utilised that is $\oint \vec{E} \cdot d\vec{l} = 0$

Electromotive Force (emf)

For an ideal source of emf (zero internal resistance), the net force on the charges is zero $(\sigma \to \infty)$. This implies

$$\vec{E} = -\vec{f}_s$$

Therefore, the potential difference between the two terminals is

$\vec{f}_s = 0$ outside the source

$$V = -\int_a^b \vec{E} \cdot d\vec{l} = \int_a^b \vec{f}_s \cdot d\vec{l} = \oint \vec{f}_s \cdot d\vec{l} = \mathcal{E}$$

For a real battery with internal resistance r, the potential difference is $\mathcal{E} - Ir$ where ***I*** is the current flowing.

Electromotive Force (emf)

- Electromotive force can, therefore, be defined as the amount of electric energy per Coulomb of positive charge as the charge passes through the source from low potential to high potential. **It is not a force**.
- Typical sources of emf: batteries, solar cells, fuel cells, electric generators etc.
- However, emf can be more general and not restricted to such sources, as we will see in next few slides.

Motional emf

Motional emf arises when a wire is moved through a magnetic field.

Uniform magnetic field **B** in shaded region, pointing into the plane

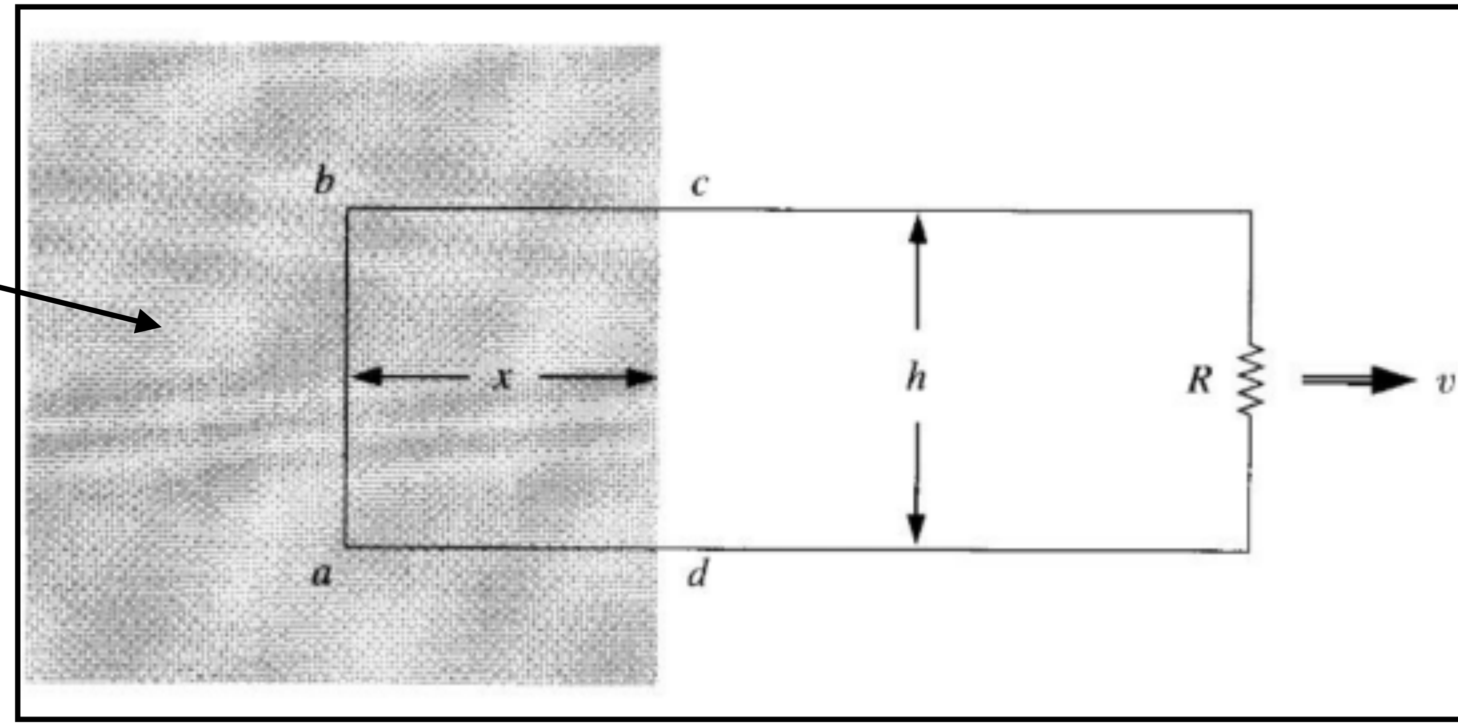


Figure 7.10, Introduction to Electrodynamics, D. J. Griffiths

Due to the motion towards right, the charges in the portion ab of the loop experience magnetic force qvB in the vertical direction which drives a current in the loop.

The emf generated is

$$\mathcal{E} = \oint \vec{f}_{\mathrm{mag}} \cdot d\vec{l} = vBh$$

Width of the loop

Motional emf

u: drift speed

Let the charges in the vertical portion of the loop (previous page) have speed u, in addition to the horizontal velocity v. This leads to a force quB on the charged particles towards left.

To balance the magnetic force towards left, the person pulling the wire must exert a force per unit charge uB towards right.

The resultant motion of the charged particle is along the resultant of vertical (u) and horizontal (v) velocities, covering an actual distance $h/\cos\theta$

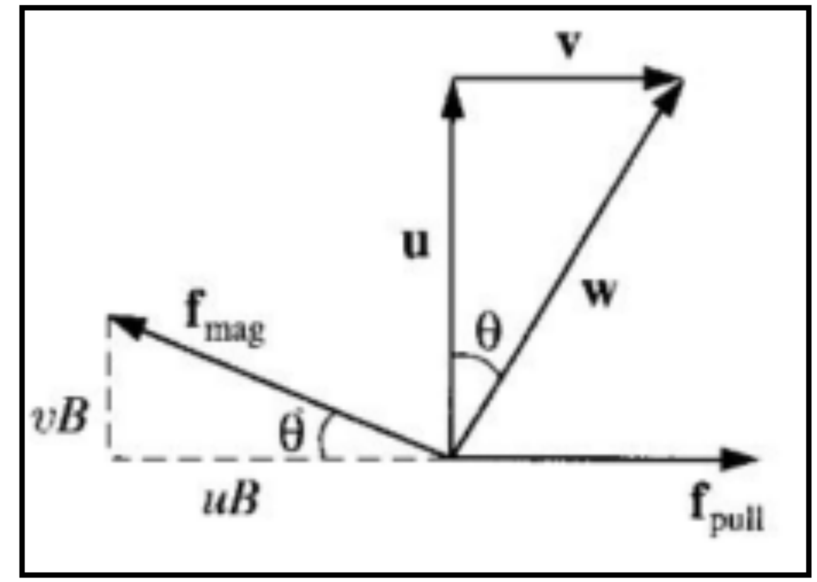


Figure 7.11, Introduction to Electrodynamics, D. J. Griffiths

The work done per unit charge is given by

$$\int \vec{f}_{\text{pull}} \cdot d\vec{l} = (uB)\left(\frac{h}{\cos\theta}\right)\cos\left(\frac{\pi}{2}-\theta\right)$$
$$= vBh = \mathcal{E}$$

Also see Example 5.3, Introduction to Electrodynamics, D J Griffiths

Motional emf

The work done per unit charge (by the external agent) is exactly equal to the motional emf generated. This agrees with the fact that **magnetic forces do not do any work.**

Emf can also be expressed as $\mathcal{E} = -\frac{d\Phi}{dt}$ where Φ is the flux of magnetic field $\Phi = \int \vec{B} \cdot d\vec{a}$

For the rectangular loop, $\Phi = Bhx$ and its rate of change is given by $\frac{d\Phi}{dt} = Bh\frac{dx}{dt} = -Bhv$

dx/dt=-v as x decreases with time

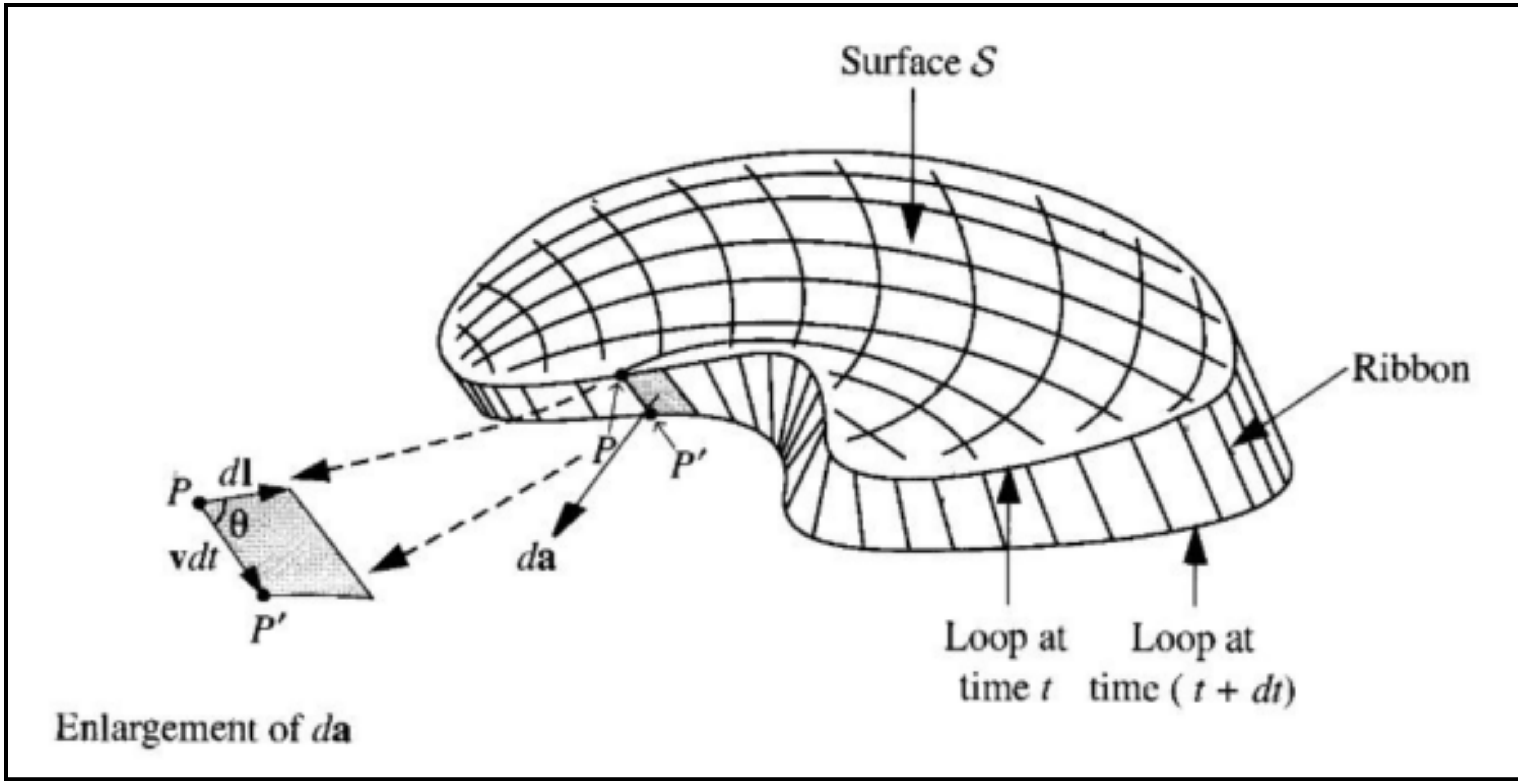
Therefore,

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

Flux rule for motional emf

Flux rule: a general proof

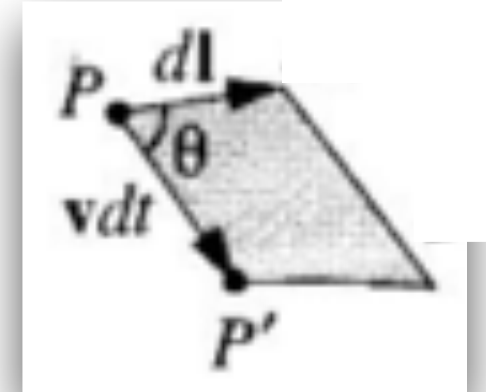
Consider a loop of wire in the presence of a magnetic field. Figure below shows the wire at time t and t+dt.



Difference between flux through the loop at two different times is the flux leaking through the edge, denoted by Ribbon

Figure 7.13, Introduction to Electrodynamics, D. J. Griffiths

Flux rule: a general proof



Change in flux through the loop during time interval dt is $$d\Phi = \Phi(t+dt) - \Phi(t) = \Phi_{\text{ribbon}} = \int_{\text{ribbon}} \vec{B} \cdot d\vec{a}$$

As shown in the figure, v is the speed of the wire and u is the speed of a charge. The infinitesimal element of the ribbon has an area $d\vec{a} = (\vec{v} \times d\vec{l})dt$

Therefore, $$\frac{d\Phi}{dt} = \oint \vec{B} \cdot (\vec{v} \times d\vec{l}) = \oint \vec{B} \cdot (\vec{w} \times d\vec{l})$$

$\vec{w} = \vec{v} + \vec{u}$ is the resultant velocity of the charge

and $\vec{u} \parallel d\vec{l} \implies \vec{u} \times d\vec{l} = 0$

Flux rule: a general proof

Using scalar triple product

$$\vec{B} \cdot (\vec{w} \times d\vec{l}) = (\vec{B} \times \vec{w}) \cdot d\vec{l} = -(\vec{w} \times \vec{B}) \cdot d\vec{l}$$

Therefore,
$$\frac{d\Phi}{dt} = -\oint (\vec{w} \times \vec{B}) \cdot d\vec{l}$$

Since $\vec{w} \times \vec{B}$ is the magnetic force per unit charge, we can write
$$\frac{d\Phi}{dt} = -\oint \vec{f}_{\mathrm{mag}} \cdot d\vec{l}$$

Since the line integral of force per unit charge is the emf, we can write
$$\mathcal{E} = -\frac{d\Phi}{dt}$$

Flux rule for motional emf

Problem 7.7 (Introduction to Electrodynamics, D. J. Griffiths) A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart. A resistor R is connected across the rails and a uniform magnetic field B, pointing into the page, fills the entire region.

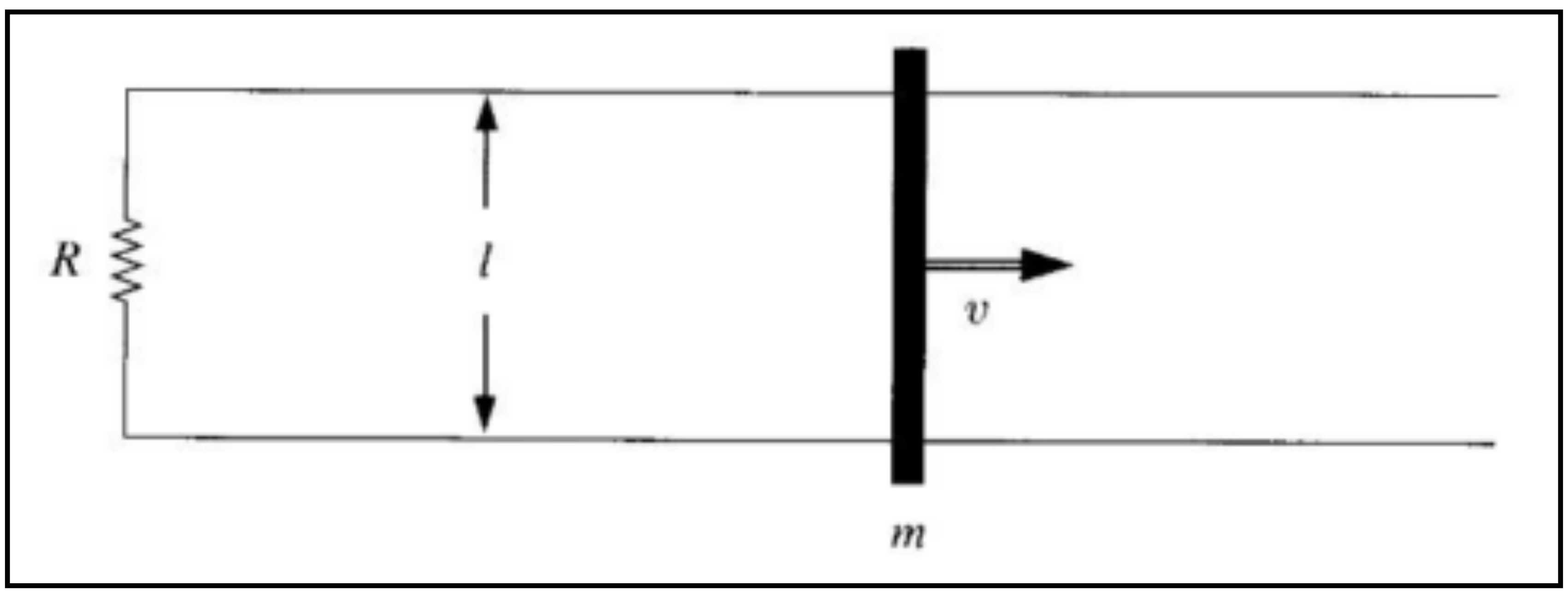


Figure 7.16, Introduction to Electrodynamics, D. J. Griffiths

(a) If the bar moves to the right at speed v, what is the current in the resistor? In what direction does it flow?

$$\mathcal{E} = -\frac{d\Phi}{dt} = Bl\frac{dx}{dt} = Blv = IR \implies I = \frac{Blv}{R}$$

which flows in counterclockwise direction evident from the direction of $\vec{v} \times \vec{B}$

(b) What is the magnetic force on the bar? In what direction?

$$|\vec{F}| = |I\vec{l} \times \vec{B}| = IlB = \frac{B^2 l^2 v}{R}$$

The direction of this force is towards left.

(c) If the bar starts out with speed v_0 at time t=0, and is left to slide, what is its speed at a later time t?

Since the force on the bar is opposite to the direction of motion, we can write

$$F = ma = m\frac{dv}{dt} = -\frac{B^2l^2}{R}v \implies \frac{dv}{dt} = -\left(\frac{B^2l^2}{Rm}\right)v$$

$$\implies v(t) = v_0 \exp\left[-\left(\frac{B^2l^2}{Rm}\right)t\right]$$

(d) Show that the energy delivered to the resistor is same as the initial kinetic energy of the bar.

$$P = \frac{dW}{dt} = I^2R = \frac{B^2l^2v^2}{R^2}R = \frac{B^2l^2}{R}v_0^2e^{-2\beta t}, \quad \beta = \frac{B^2l^2}{mR}$$

$$\implies W = \beta m v_0^2 \int_0^\infty e^{-2\beta t}dt = \beta m v_0^2 \frac{1}{2\beta} = \frac{1}{2}mv_0^2$$

No work by magnetic force!

Example 7.4 (Introduction to Electrodynamics, D J Griffiths): A metal disk of radius a rotates with angular velocity o about the vertical axis, through a uniform field pointing up. Find the current in the resistor shown in figure.

Solution: The force per unit charge is $\vec{f} = \vec{v} \times \vec{B} = \omega s B \hat{s}$

The emf and current are therefore,

$$\mathcal{E} = \int_0^a f ds = \frac{\omega B a^2}{2}$$
$$I = \frac{\mathcal{E}}{R} = \frac{\omega B a^2}{2R}$$

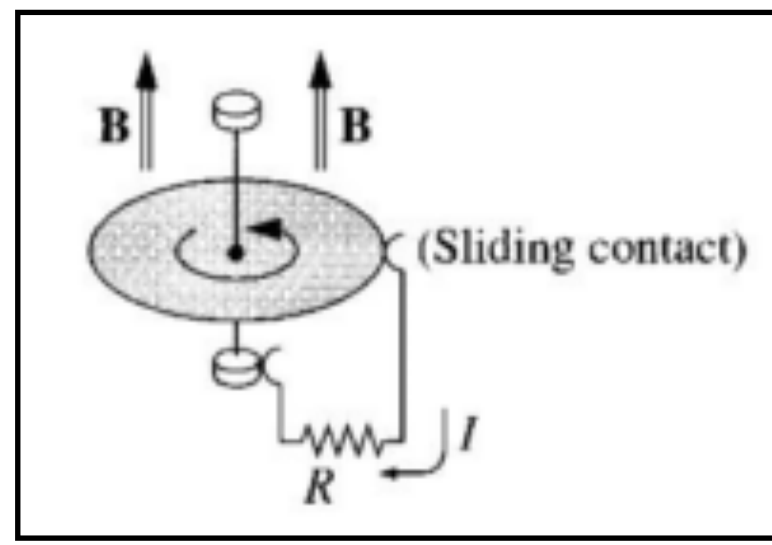


Figure 7.14, Introduction to Electrodynamics, D J Griffiths

Eddy Currents!

Currents are induced in the metallic disc when it swings across the region of magnetic field: Eddy currents!

Eddy currents slow it down when it enters the field region (Why?).

Using a disk with many cuts prevents large scale Eddy currents and the it swings freely in the field region without any drag.

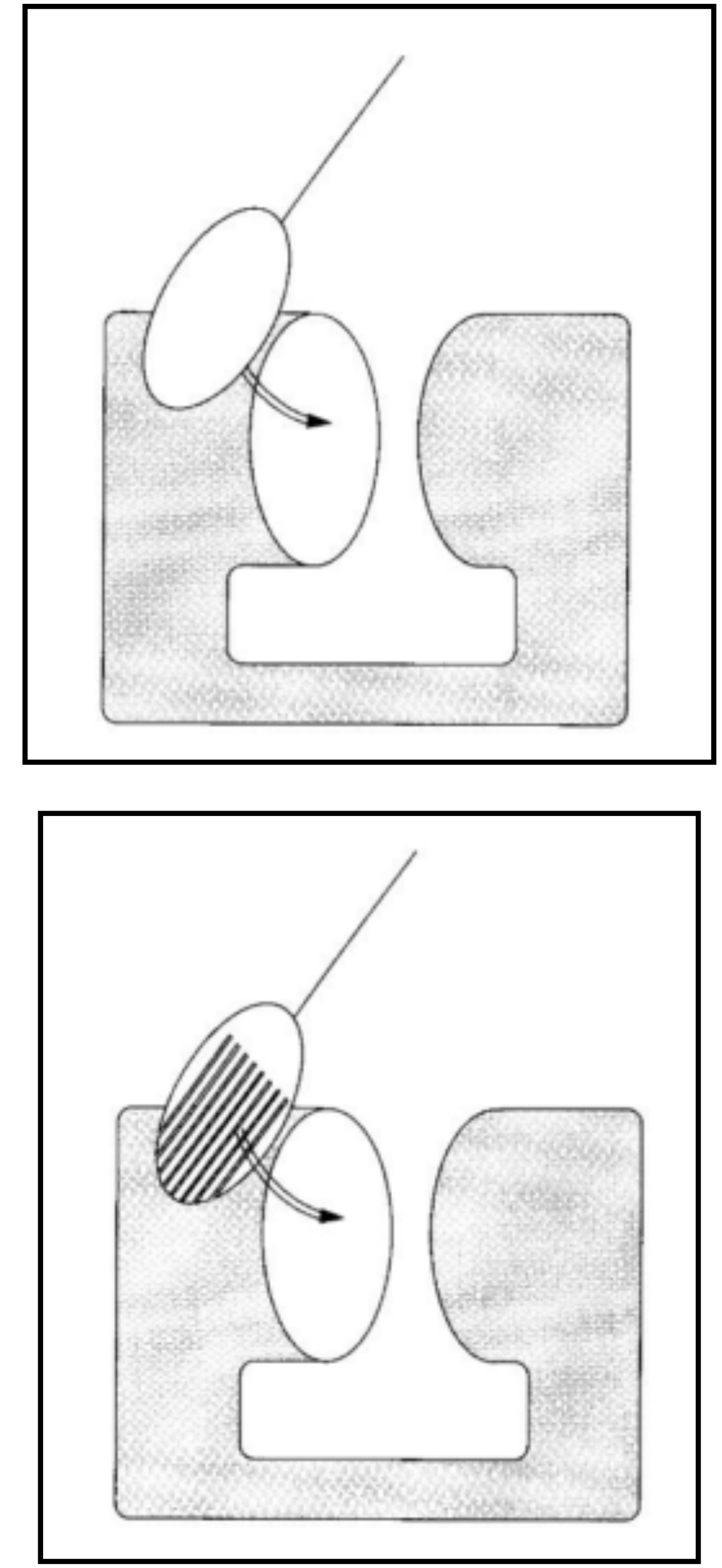
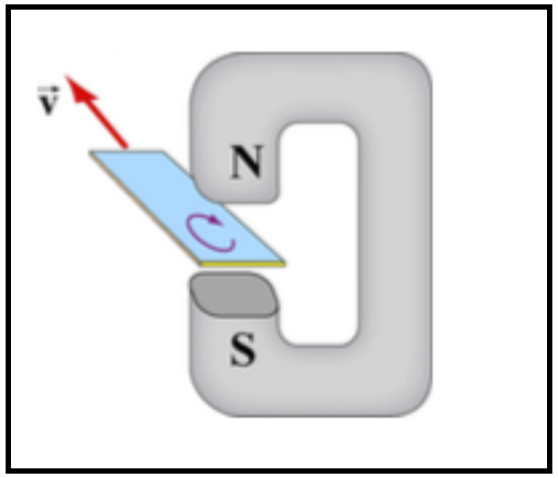
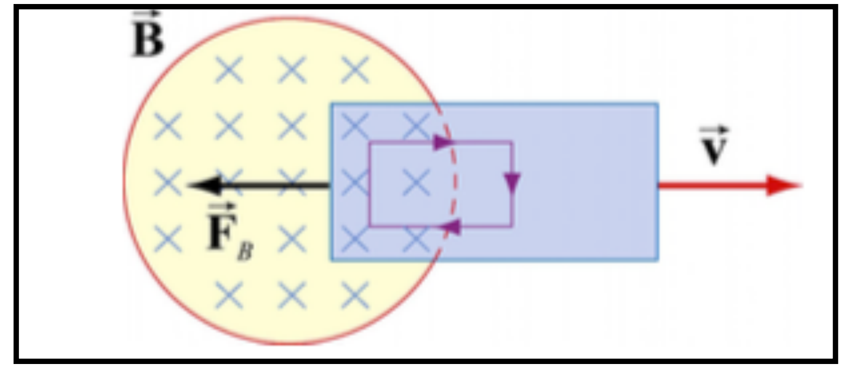


Figure 7.14, Introduction to Electrodynamics, D J Griffiths

Eddy Currents!

How does it slow down the swinging conducting plate?





Laminations/cuts decreases the power loss ($P = V^2/R$) due to Joule heating by increasing the resistance R

Applications of Eddy currents:

1. To suppress unwanted mechanical oscillations.

2. Magnetic brakes.

Image credit: MIT