Tut 6

6.01. According to quantum mechanics, the electron cloud for a hydrogen atom in ground state has a charge density

$$\rho(r) = \frac{q}{\pi a^3} e^{-2r/a} , \tag{1}$$

where q is the charge of the electron and a is the Bohr radius. Find the atomic polarizability of such an atom.

Suppose $\vec{E}$ is the external field applied to the atom. Then nucleus will be shifted by d distance (say) from the center.

At equilibrium, the internal field $\vec{E}_e$ (say) of the atom, caused by the electron cloud, will balance $\vec{E}$.

Therefore our first aim will be to calculate $\vec{E}_e$ due to the electron cloud.

The electric field $\vec{E}_e$ at a distance r from the origin is computed from the Gauss's law:

$$\oint \vec{E}_e \cdot d\vec{s} = \frac{Q_{encl.}}{\epsilon_0}$$

$$\text{or,} \quad E_e = \frac{Q_{encl.}}{4\pi\epsilon_0 r^2}$$

Now,

$$Q_{encl.} = \iiint \rho(r) . r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= 4\pi \int_0^r \frac{q}{\pi a^3} e^{-\frac{2r}{a}} r^2 \, dr$$

$$= \frac{4q}{a^3} \left[-a/_2 \, e^{-\frac{2r}{a}} \left(r^2 + ar + \frac{a^2}{2} \right) \right]_0^r$$

$$\therefore \quad Q_{encl.} = q\left[1 - e^{-\frac{2r}{a}}\left(1 + \frac{2r}{a} + \frac{2r^2}{a^2}\right)\right].$$

[Note that the electron cloud extends to $r \to \infty$ and $Q_{encl.} = q$ for $r \to \infty$]

$\therefore$ The value of the external field $\vec{E}$ which causes this shift of nucleus by d distance is

$$E = |\vec{E}| = |\vec{E}_e(r = d)|$$
$$= \frac{q}{4\pi\epsilon_0 d^2}\left[1 - e^{-2d/a}\left(1 + \frac{2d}{a} + \frac{2d^2}{a^2}\right)\right].$$

Now, upto linear order of d, which makes the system as dipole, the dipole moment $\vec{p} = q\vec{d}$ is proportional to the applied field $\vec{E}$:

$$\underbrace{\vec{p} = \alpha \vec{E}}_{\text{atomic polarizability}}$$

So, let us now expand $\vec{E}$ and keep upto linear order in d:

$$E \simeq \frac{q}{4\pi\epsilon_0 d^2}\underbrace{\left[1 - \left(1 - \frac{2d}{a} + \frac{(2d/a)^2}{2!} - \frac{(2d/a)^3}{3!} + \cdots\right) \times \left(1 + \frac{2d}{a} + \frac{2d^2}{a^2}\right)\right]}_{\text{keep upto } d^3 \text{ order terms.}}$$

$$= \frac{q}{4\pi\epsilon_0 d^2}\left[1 - \left(1 + \frac{2d}{a} + \frac{2d^2}{a^2} - \frac{2d}{a} - \frac{4d^2}{a^2} - \frac{4d^3}{a^3} + \frac{4d^2}{2a^2} + \frac{8d^3}{2a^3} - \frac{8d^3}{6a^3}\right)\right]$$

$$= \frac{q}{4\pi\epsilon_0 d^2}\left(\frac{4d^3}{3a^3}\right)$$

$$= \frac{qd}{3\pi\epsilon_0 a^3} = \frac{p}{3\pi\epsilon_0 a^3} = \frac{p}{\alpha}$$

$$\therefore \boxed{\alpha = 3\pi\epsilon_0 a^3}$$

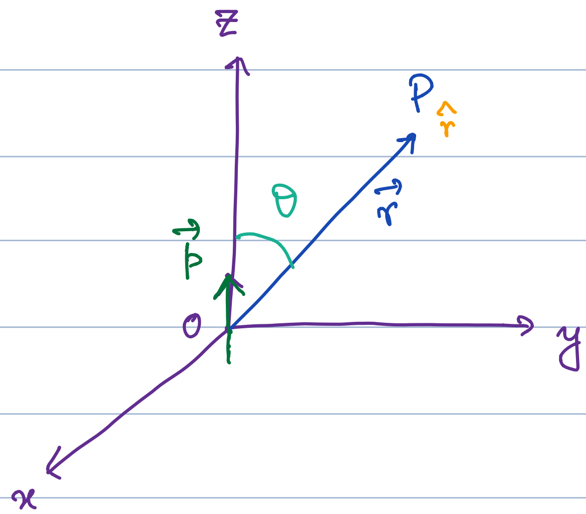
6.02. (a) Show that the electric field of a 'pure dipole' can be written as

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\vec{p}.\hat{r})\hat{r} - \vec{p}]$$

Note that this form has the advantage of not committing to a particular coordinate system.

(b) Find the force and torque on a dipole in the field of a point charge. Let the charge q be at the origin and the dipole $\vec{p} = p_0(\sin\zeta_0\hat{x} + \cos\zeta_0\hat{z})$ be at the point $(0, 0, z_0)$. Also find the force on q due to the dipole and verify Newton's third law.

(a)



The electric field at P due to dipole is

$$\vec{E} = \frac{1}{4\pi\epsilon_0 r^3}\left(2p\cos\theta\,\hat{r} + p\sin\theta\,\hat{\theta}\right) \longrightarrow (1)$$

Now, from the above figure we can write:

$$\vec{p} = p\,\hat{z} \quad (\text{as dipole is along } z\text{-axis})$$

In spherical polar coordinates:

$$\hat{z} = (\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$$

$$\therefore\ \vec{p} = p\cos\theta\,\hat{r} - p\sin\theta\,\hat{\theta}$$

$$\Rightarrow\ p\cos\theta = \vec{p}\cdot\hat{r}$$

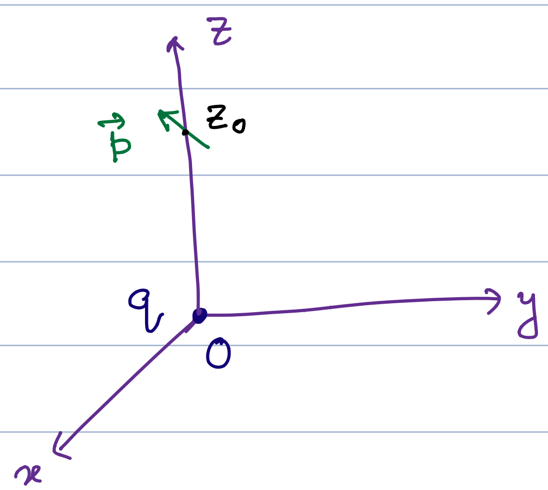
Rewriting ① as:

$$\vec{E} = \frac{1}{4\pi\epsilon_0 r^3}\left(3p\cos\theta\,\hat{r} - p\cos\theta\,\hat{r} + p\sin\theta\,\hat{\theta}\right)$$

we find

$$\vec{E} = \frac{1}{4\pi\epsilon_0 r^3}\left[3(\vec{p}\cdot\hat{r})\,\hat{r} - \vec{p}\right]$$

(b)



* <u>Force on the dipole:</u>

Force on a dipole is given by:

$$\vec{F} = (\vec{p}\cdot\vec{\nabla})\,\vec{E}_q$$

where $\vec{E}_q$ is the electric field due pt. charge q.

Here $\vec{p} = p_0(\sin\varphi_0\,\hat{x} + \cos\varphi_0\,\hat{z})$

and $\vec{E}_q$ at pt. (x, y, z) is

$$\vec{E}_q = \frac{q}{4\pi\epsilon_0}\left[\frac{x}{(x^2+y^2+z^2)^{3/2}}\hat{x} + \frac{y}{(x^2+y^2+z^2)^{3/2}}\hat{y} + \frac{z}{(x^2+y^2+z^2)^{3/2}}\hat{z}\right]$$

$$\therefore \vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E}_q \Big|_{(0,0,z_0)}$$

$$= \left(p_0 \sin\varphi_0 \frac{\partial}{\partial x} + p_0 \cos\varphi_0 \frac{\partial}{\partial z} \right) \vec{E}_q \Big|_{(0,0,z_0)}$$

$$= \frac{p_0 q}{4\pi\epsilon_0 z_0^3} \left[\sin\varphi_0 \, \hat{x} - 2\cos\varphi_0 \, \hat{z} \right]$$

Torque on the dipole:

$$\vec{N}(0,0,z_0) = (\vec{p}\times\vec{E}_q)\Big|_{(0,0,z_0)} = -\frac{p_0 q \sin\varphi_0}{4\pi\epsilon_0 z_0^2}\hat{y}$$

* Force on the charge:

$\vec{F}_q$ = force on the charge due to dipole

$$= q\vec{E}(-z_0\hat{z})$$

where $\vec{E} = \frac{1}{4\pi\epsilon_0 r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right]$ with $\hat{r} = -\hat{z}$ (as according to our convension $\vec{r}$ is directed from dipole to field pt.).

$$\therefore \vec{E}(-z_0\hat{z}) = \frac{1}{4\pi\epsilon_0 z_0^3}\left[3(\vec{p}\cdot\hat{z})\hat{z} - \vec{p}\right]$$

$$= \frac{1}{4\pi\epsilon_0 z_0^3}\left[3p_0\cos\varphi_0\,\hat{z} - p_0\sin\varphi_0\,\hat{x} - p_0\cos\varphi_0\,\hat{z}\right]$$

$$= \frac{p_0 q}{4\pi\epsilon_0 z_0^3}\left[-\sin\varphi_0\,\hat{x} + 2\cos\varphi_0\,\hat{z}\right]$$

$$\therefore \vec{F_q} = \frac{q p_0}{4\pi\epsilon_0 z_0^3}\left(-\sin\varphi_0\,\hat{x} + 2\cos\varphi_0\,\hat{z}\right) = -\vec{F}$$

This is as expected from Newton's 3rd law.

6.03. Energy of a dipole:

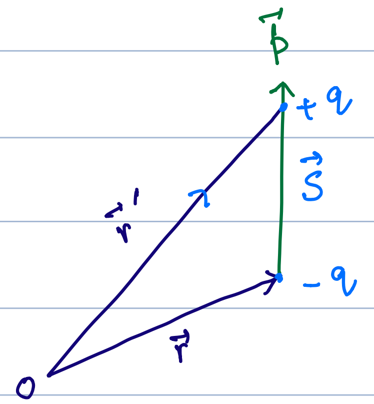
(a) Show that the energy of a dipole with dipole moment $\vec{p}$ in an electric field $\vec{E}$

$$U = -\vec{p}.\vec{E} \ . \tag{2}$$

(b) Show that the interaction between two dipoles with dipole moments $\vec{p}_1$ and $\vec{p}_2$ separated by distance $\vec{r}$ is given by:

$$U = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[\vec{p}_1.\vec{p}_2 - 3(\vec{p}_1.\hat{r})(\vec{p}_2.\hat{r})] \ . \tag{3}$$

(a)



$\vec{r}\,' = \vec{r} + \vec{s}$

Here we shall calculate the work done on a dipole by bringing it from infinity to a position in electric field $\vec{E}$.

This is equivalent to bringing $+q$ charge at $\vec{r}\,'$ (say) and $-q$ charge at $\vec{r}$ (say) with $\vec{s}$ is the separation vector from $-q$ to $+q$. Remember that here s is very small i.e. $\frac{s}{r} << 1$

In this configuration, work done is

$$U = -q\,V(r) + qV(r+s)$$

where V is the potential due to electric field $\vec{E}$;

i.e. $\vec{E} = -\vec{\nabla} V$.

Since $s \ll r$; the Taylor expansion of $V(r+s)$ around $s=0$ is :

$$V(r+s) \simeq V(r) + \vec{s} \cdot \vec{\nabla} V(r)$$

(Keeping only upto linear order in s)

$$= V(r) - \vec{s} \cdot \vec{E}$$

$$\therefore U = -qV(r) + q\left[V(r) - \vec{s} \cdot \vec{E}\right]$$

$$= -q\vec{s} \cdot \vec{E} = -\vec{p} \cdot \vec{E}$$

where $\vec{p} = q\vec{s}$ = dipole moment.

<u>Another method</u> :

We can also evaluate the energy of the dipole by the work done in rotating it :

$$U = \int_{\pi/2}^{\theta} N \, d\theta' = \int_{\pi/2}^{\theta} pE \sin\theta' \, d\theta'$$

$$= -pE\cos\theta = -\vec{p} \cdot \vec{E}.$$

Here we have judiciously chosen the reference pt. to be at $\theta = \pi/2$.

(b) The energy stored in a system where two dipoles are interating can be calculated by determining the energy when one dipole is kept in the electric field due to the other dipole.

Here let us first calculate the electric field at a position $\vec{r}$ w.r.t. 2nd dipole due itself:

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0 r^3}\left[3(\vec{p}_2\cdot\hat{r})\,\hat{r} - \vec{p}_2\right].$$

Now, when 1st dipole is brought at $\vec{r}$, the energy of the composite system will be:

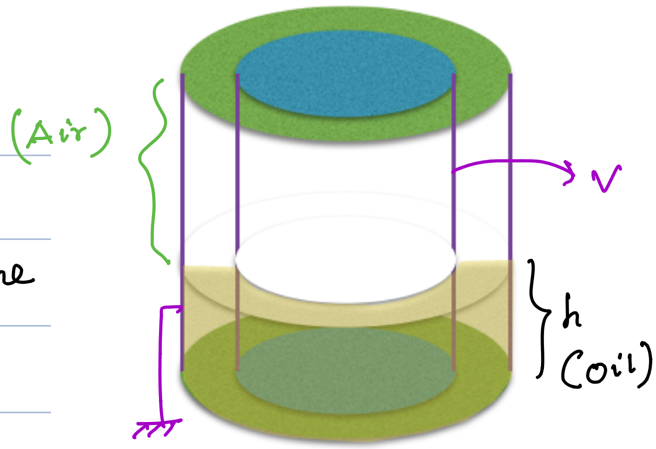
$$U = -\vec{p}_1\cdot\vec{E}_2$$

$$= -\frac{1}{4\pi\epsilon_0 r^3}\,\vec{p}_1\cdot\left[3(\vec{p}_2\cdot\hat{r})\,\hat{r} - \vec{p}_2\right]$$

$$= \frac{1}{4\pi\epsilon_0 r^3}\left[-3(\vec{p}_1\cdot\hat{r})(\vec{p}_2\cdot\hat{r}) + \vec{p}_1\cdot\vec{p}_2\right]$$

Note: The above expression is symmetric under exchange of 1 and 2; which is as expected.

6.04. Two long coaxial cylindrical metal tubes (inner radius a and outer radius b) stand vertically in a tank of dielectric oil (susceptibility χ_e, mass density ρ). The inner one is maintained at a potential V and the outer one is grounded. To what height does the oil rise in the space between the tubes?



Air part

Suppose,
σ = surface charge density on the inner cylinder in the air part.

So the electric field between two cylinders in the air part will satisfy

$$\oint \vec{E}\cdot d\vec{s} = \frac{Q_{encl.}}{\epsilon_0}$$

$$\Rightarrow E \cdot 2\pi s l = \frac{1}{\epsilon_0}\left(\sigma \cdot 2\pi a l\right)$$

$$\text{or, } E = \frac{\sigma a}{\epsilon_0 s} \Rightarrow \vec{E} = \frac{\sigma a}{\epsilon_0 s}\hat{s}$$

h = height of oil, raised in the space between the tubes.

$\therefore$ The potential at the surface of the inner cylinder w.r.t. outer cylinder is

$$V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{r}$$

$$= -\int_b^a \frac{\sigma a}{\epsilon_0 s}\, ds$$

$$= -\frac{\sigma a}{\epsilon_0}\int_b^a \frac{ds}{s}$$

$$= -\frac{\sigma a}{\epsilon_0} \int_b^a \frac{ds/a}{s/a} = -\frac{\sigma a}{\epsilon_0} \left[\ln \; s/a \right]_b^a$$

$$= \frac{\sigma a}{\epsilon_0} \ln (b/a)$$

Since $V_b = 0$ and $V_a = V$ (given) : we find

$$\boxed{V = \frac{\sigma a}{\epsilon_0} \ln (b/a)} \longrightarrow (1)$$

Oil part

For the oil part, suppose σ' is the free surface charge density on the inner surface.

This part acts as a dielectric. So the electric displacement vector between the two cylinders for this part satisfies:

$$\oint \vec{D} \cdot d\vec{s} = Q_{free}.$$

$$\Rightarrow \quad D \cdot 2\pi s l = \sigma' \cdot 2\pi a l$$

$$\Rightarrow \quad \vec{D} = \frac{\sigma' a}{s} \hat{s}$$

$\therefore$ Electric field here will be $\vec{E}' = \frac{\vec{D}}{\epsilon} = \frac{\sigma' a}{\epsilon s} \hat{s}$,

where ϵ is the permittivity of oil.

$\therefore$ The potential difference between two cylinders for this oil part is

$$V_a - V_b = -\int_b^a \vec{E}' \cdot d\vec{r}$$

$$= \frac{\sigma' a}{\epsilon} \ln(b/a)$$

Again, since $V_b = 0$ and $V_a = V$ (given), we have:

$$\boxed{V = \frac{\sigma' a}{\epsilon} \ln(b/a)} \longrightarrow (2)$$

Equating (1) and (2) one finds:

$$\sigma' = \frac{\epsilon}{\epsilon_0}\sigma = \epsilon_r \sigma \; ; \; \epsilon_r = \epsilon/\epsilon_0 = 1 + \chi_e$$

Now, if L is the total length of the cylinders, then total charge on the inner cylinder is

$$Q = \sigma' \, 2\pi a h + \sigma \cdot 2\pi a (L-h)$$

$$= 2\pi a \left[\sigma' h + \sigma (L-h)\right]$$

$$= 2\pi a \left[\epsilon_r \sigma h + \sigma (L-h)\right]$$

$$= 2\pi a\sigma \left[\underbrace{(\epsilon_r - 1)}_{\chi_e \text{ = electric susceptibility of oil.}} h + L \right]$$

or $$Q = 2\pi a\sigma (\chi_e h + L).$$

Hence the capacitance of the system is

$$C = \frac{Q}{V} = \frac{2\pi a\sigma(\chi_e h + L)}{\frac{\sigma a}{\epsilon_0} \ln(b/a)} \quad \left[\text{Using } ①\right]$$

$$= 2\pi\epsilon_0 \frac{\chi_e h + L}{\ln(b/a)}.$$

Now, due to this charge accumulation, the oil experiences an upward force which is given by

$$F\uparrow = \frac{1}{2} V^2 \frac{dC}{dh}$$

Here $$\frac{dC}{dh} = 2\pi\epsilon_0 \frac{\chi_e}{\ln(b/a)}$$

$$\therefore \quad F\uparrow = \frac{1}{2} V^2 \cdot 2\pi\epsilon_0 \frac{\chi_e}{\ln(b/a)} = \frac{\pi\epsilon_0 \chi_e V^2}{\ln(b/a)}.$$

This force will try to raise the oil level. Whereas the downward gravitational force will try to pull it down. At equilibrium, they will balance each other.

The gravitation force is given by

$$F_g \downarrow = mg = \rho \pi (b^2 - a^2) h g.$$

At equilibrium we must have $F \uparrow = F_g \downarrow$.

$$\Rightarrow \quad h = \frac{\epsilon_0 \chi_e V^2}{\rho (b^2 - a^2) g \ln(b/a)}.$$

6.05. A spherical conductor of radius a, carries a charge Q. It is surrounded by linear dielectric material of susceptibility χ_e, out to radius b. Find the energy of this configuration.

Inside the conductor ($r < a$), $\vec{D} = 0$. Let us calculate electric displacement for $r > a$ using Gauss's formula:

$$\oint \vec{D}.d\vec{a} = Q_{fenc} \implies D4\pi r^2 = Q \implies \vec{D} = \frac{Q}{4\pi r^2}\hat{r}$$

Therefore electric field is

$$\vec{E} = \begin{cases} 0 & \text{for} \quad r < a \\ \frac{Q}{4\pi \epsilon r^2}\hat{r} & \text{for} \quad a < r < b \\ \frac{Q}{4\pi \epsilon_0 r^2}\hat{r} & \text{for} \quad r > b \end{cases}$$

Hence, energy of the configuration

$$\begin{aligned} W &= \frac{1}{2}\int \vec{D}.\vec{E}d\tau = \frac{1}{2}\int \vec{D}.\vec{E}4\pi r^2 dr \\ &= \frac{1}{2}\frac{Q^2}{(4\pi)^2}4\pi\left[\frac{1}{\epsilon}\int_a^b \frac{1}{r^2}\frac{1}{r^2}r^2 dr + \frac{1}{\epsilon_0}\int_b^\infty \frac{1}{r^2}\frac{1}{r^2}r^2 dr\right] \\ &= \frac{Q^2}{8\pi\epsilon_0(1+\chi_e)}\left(\frac{1}{a} + \frac{\chi_e}{b}\right). \end{aligned}$$

6.06. A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a "frozen-in" polarization $\vec{P}(\vec{r}) = \frac{k}{r}\hat{r}$, where k is a constant and r is the distance from the center. There is no free charge in the problem. Find the electric field in all three regions by two different methods:

(a) Locate all the bound charge, and use Gauss's law to calculate the field it produces.

(b) Use $\oint \vec{D}.d\vec{S} = Q_{fencl}$ to find $\vec{D}$ and then get $\vec{E}$ from $\vec{D} = \epsilon_0\vec{E} + \vec{P}$.

(a) Volume bound charge density

$$\rho_b = -\vec{\nabla}.\vec{P}$$
$$= -\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{k}{r}\right) = -\frac{k}{r^2}.$$

Surface bound charge density:

$$\sigma_b = \vec{P}.\hat{n} = \begin{cases} \vec{P}.(-\hat{r})\Big|_{r=a} = -\frac{k}{a} & \text{at } r=a \\ \vec{P}.\hat{r}\Big|_{r=b} = \frac{k}{b} & \text{at } r=b. \end{cases}$$

Now use Gauss's to find fields at three regions:

$$\oint_S \vec{E}.d\vec{S} = \frac{Q_{encl.}}{\epsilon_0}$$

$\underline{\underline{r<a}}$:

$$Q_{encl.} = 0$$
$$\Rightarrow \boxed{E_{(r<a)} = 0}$$

$\underline{\underline{r > b}}$:

$$Q_{encl.} = \int \sigma_b \, da + \int \rho_b \, dv$$

$$= \int \left(-\frac{k}{a}\right) \cdot a^2 \sin\theta \, d\theta \, d\phi + \int \frac{k}{b} \cdot b^2 \sin\theta \, d\theta \, d\phi$$

$$+ \int \left(-\frac{k}{r^2}\right) r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= -ak \cdot 4\pi + bk \cdot 4\pi - k \cdot 4\pi \int_a^b dr$$

$$= 4\pi k (b-a) - 4\pi k (b-a) = 0$$

$$\therefore \boxed{E_{(r>b)} = 0}$$

$\underline{\underline{a < r < b}}$

$$Q_{encl.} = \int \sigma_b \, da + \int \rho_b \, dv$$

$$= -\frac{k}{a} \cdot a^2 \cdot 4\pi - k \int \frac{1}{r^2} r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= -4\pi ak - 4\pi k \int_a^r dr$$

$$= -4\pi ak - 4\pi k (r-a) = -4\pi kr$$

$\therefore$ By Gauss's theorem : $E \cdot 4\pi r^2 = -\dfrac{4\pi k r}{\epsilon_0}$

$$\Rightarrow \boxed{\vec{E}_{(a<r<b)} = -\frac{k}{\epsilon_0 r} \hat{r}}$$

(b)

Here $Q_{free} = 0$ everywhere, and $\vec{\nabla} \times \vec{P} = \vec{0}$, we may use Gauss's law to determine $\vec{D}$:

$$\therefore \oint_S \vec{D} \cdot d\vec{S} = 0$$

$$\Rightarrow \vec{D} = 0 \text{ everywhere.}$$

i.e. $\epsilon_0 \vec{E} + \vec{P} = 0$ everywhere.

$$\Rightarrow \vec{E} = -\vec{P}/\epsilon_0 \text{ everywhere.}$$

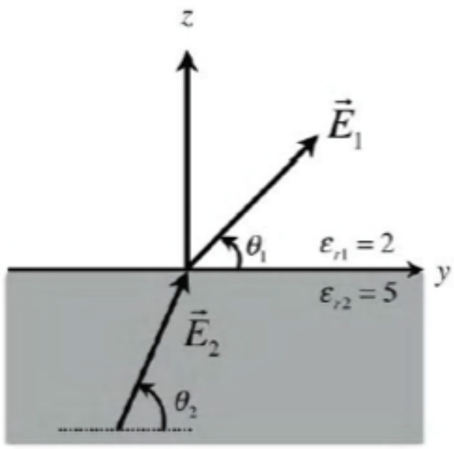
For regions $r < a$ and $r > b$, we have $\vec{P} = 0$.

$$\Rightarrow \boxed{\vec{E}_{(r<a;\, r>b)} = 0}$$

For $a < r < b$, we have $\vec{P} = \frac{k}{r}\hat{r}$.

$$\therefore \boxed{\vec{E}(a < r < b) = -\frac{k}{\epsilon_0 r}\hat{r}}$$

6.07. Given that $\vec{E}_1 = 2\hat{x} - 3\hat{y} + 5\hat{z}$ at the charge free dielectric interface of Figure 2. Find $\vec{D}_2$ and the angles θ_1 and θ_2.



Given $\vec{E}_1 = 2\hat{x} - 3\hat{y} + 5\hat{z}$.

$\therefore$ From the figure it is evident that

$\vec{E}_1^{\perp} = 5\hat{z}$

and $\vec{E}_1^{\parallel} = 2\hat{x} - 3\hat{y}$.

Now, $\vec{D}_1 = \epsilon_0 \epsilon_{r_1} \vec{E}_1$

$\therefore \vec{D}_1^{\perp} = \epsilon_0 \epsilon_{r_1} \vec{E}_1^{\perp} = 2\epsilon_0 \times 5\hat{z} = 10\,\epsilon_0 \hat{z}$

and $\vec{D}_1^{\parallel} = \epsilon_0 \epsilon_{r_1} \vec{E}_1^{\parallel} = 2\epsilon_0 (2\hat{x} - 3\hat{y}) = 4\epsilon_0\hat{x} - 6\epsilon_0\hat{y}$.

Recall the boundary conditions:

$D_1^{\perp} - D_2^{\perp} = \sigma_f = 0$ (since here there is no free charge)

and $E_1^{\parallel} = E_2^{\parallel}$.

$$\therefore \vec{D}_2^{\perp} = \vec{D}_1^{\perp} = 10\,\epsilon_0\,\hat{z}$$

$$\text{and } \vec{E}_2^{\parallel} = \vec{E}_1^{\parallel} = 2\hat{x} - 3\hat{y}$$

$$\therefore \vec{D}_2^{\parallel} = \epsilon_0\,\epsilon_{r2}\,\vec{E}_2^{\parallel} = 5\epsilon_0\,(2\hat{x} - 3\hat{y}) = 10\,\epsilon_0\,\hat{x} - 15\,\epsilon_0\,\hat{y}.$$

$$\therefore \vec{D}_2 = \vec{D}_2^{\perp} + \vec{D}_2^{\parallel}$$

$$= \epsilon_0\,(10\,\hat{x} - 15\,\hat{y} + 10\,\hat{z})$$

$$\Rightarrow \vec{E}_2 = \frac{\vec{D}_2}{\epsilon_0\,\epsilon_{r2}} = 2\hat{x} - 3\hat{y} + 2\hat{z}$$

Now, from the figure we have

$$|\vec{E}_1| \cos(90^\circ - \theta_1) = \vec{E}_1 \cdot \hat{z} = 5$$

$$\Rightarrow \sin\theta_1 = \frac{5}{|\vec{E}_1|} = \frac{5}{\sqrt{38}}$$

$$\Rightarrow \theta_1 = 54.2^\circ.$$

$$\text{Also, } |\vec{E}_2| \cos(90^\circ - \theta_2) = \vec{E}_2 \cdot \hat{z}$$

$$\therefore \cos(90^\circ - \theta_2) = \frac{2}{|\vec{E}_2|} = \frac{2}{\sqrt{17}}$$

$$\Rightarrow \sin\theta_2 = \frac{2}{\sqrt{17}}$$

$$\therefore \theta_2 = 29^\circ.$$