1. Jacobian, Double Integration

Thursday, February 8, 2024 9:17 PM

JACOBIAN

$u = xyz$, $v = y^2$, $w = x+z$. Evaluate J.

values of u,v,w

```
>>> syms x y z
>>> J = jacobian([x*y*z, y^2, x+z], [x,y,z])
>>> d = det(J)
```

independent variables

$u = x^2 - 2y$, $v = x+y+z$, $w = x-2y+3z$. Evaluate J.

```
>>> syms x y z
>>> J = jacobian([x^2.-2y, x+y+z, x-2*y+3*z], [x,y,z])
>>> d = det (J)
```

$u = x^2 - 2y^2$, $v = 2x^2 - y^2$ where $x = r\cos\theta$, $y = r\sin\theta$. Evaluate J.

```
>>> syms x y r theta
>>> x = r*cos(theta)
>>> y = r* sin(theta)
>>> J = jacobian ([x^2-2*y^2, 2*x^2-y^2], [r, theta])
>>> d = det(J)
>>> simplify (d)
```

$x = u(1-v)$, $y = uv$. Prove $JJ' = 1$

```
>>> syms u v
>>> J = jacobian([u*(1-v), u*v], [u,v])
```

```
>>> J = jacobian([u*(1-v), u*v], [u,v])
>>> J* inv(J)
```

INTEGRATION

int (func, lower_limit, upper_limit) → analytically solvable

integral (func, lower_limit, upper_limit) → not analytically solvable; gives numerical answer

integral2(func, $\underbrace{\text{ll1, ul1}}_{\text{outer limit}}$, ll2, ul2) → double integration

NOTE: Function handle

Integral only takes a function handle as an argument for func.
This is defined as follows:

```
y = @(x) x.^2
```

Evaluate $\int_0^1\int_0^{1-x} \frac{1}{(\sqrt{x+y})(1+x+y)^2}\,dy\,dx$

```
>>> func = @(x,y) 1./(sqrt(x+y).*(1+x+y).^2);
>>> ymax = @(x) 1-x;
>>> q = integral2(func, 0,1,0, ymax)
```

Evaluate $\int_{-\pi}^{2\pi}\int_0^{\pi} [y\sin(x) + x\cos(y)]\,dy\,dx$

```
>>> func = @(x,y) y.*sin(x) + x.*cos(y);
>>> q = integral2(func, -pi, 2*pi, 0, pi)
```

Evaluate $\int_0^5\int_0^{x^2} x(x^2+y^2)\,dy\,dx$

Evaluate $\int_0 \int_0 x(x^2+y^2)\,dy\,dx$

```
>> func = @(x,y) x.*(x.^2 + y.^2);
>> ymax = @(x) x.^2;
>> q = integral2(func, 0, 5, 0, ymax)
```

Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^{1/(\sin\theta+\cos\theta)} \frac{r}{\sqrt{r\cos\theta + r\sin\theta}\,(1 + r\cos\theta + r\sin\theta)^2}\, dr\, d\theta$

```
>> rmax = @(theta)1./(sin(theta)+cos(theta))


rmax =

  function_handle with value:

    @(theta)1./(sin(theta)+cos(theta))
```

variables in the order of their limits in integration

```
>> func=@(theta,r)r./(sqrt(r.*cos(theta)+r.*sin(theta)).*(1+r
   .*cos(theta)+r.*sin(theta)).^2)


func =

  function_handle with value:

    @(theta,r)r./(sqrt(r.*cos(theta)+r.*sin(theta)).*(1+r.*cos(theta)+r
.*sin(theta)).^2)



>> q=integral2(func,0,pi./2,0,rmax)


q =

    0.2854
```

Evaluate $\int_0^3 \int_0^2 \int_0^1 (x+y+z)\,dz\,dx\,dy$

```
>> fun = @(x,y,z) x+y+z
>> q = integral3(fun, 0,3,0,2,0,1, 'Method', 'tiled')
```

$q = \text{integral3}(\text{fun}, 0, 3, 0, 2, 0, 1, \underbrace{\text{Method}, \text{tiled}}_{\text{for definite limits}})$

> **NOTE: Infinite limits**
>
> inf $\longrightarrow$ infinity
>
> integral . . . . . 'Method', 'iterative'

Evaluate

$$\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\,dx\,dy\,dz$$

```
fun = @(x,y,z) exp(x+y+z)
xmin = 0
xmax = log(2)
ymin = 0
ymax = @(x) x
zmin = 0
zmax = @(x,y) x + log(y)
q = integral3(fun, xmin, xmax, ymin, ymax, zmin, zmax)
```