1NF

RHS atomic: no multivalued

2NF

} attributes depend on whole key

Given any candidate key, non-prime attributes are fully functionally dependent on it + 1NF
all keys.

Only needed for multi-attribute keys

3NF

} attr. depend on nothing but key

No non-prime attribute transitively dependent on superkeys + 2NF

If $X \to A$:

$X \to$ superkey

[OR]

$A \to$ prime attribute

violations → prime attr. → non-prime } 2NF violation

→ non-prime attr → non-prime } 3NF violation
transitive dependency

BCNF

Stricter 3NF.

If $X \to A$:
$X \to$ candidate key

Decomposition

critical ✓ Lossless join: Joining should not create spurious tuples

Dependency preservation: All FDs should be preserved post decomposition
can be relaxed

Non-additive join test for binary decompositions (NJB)

$D = \{R_1, R_2\}$ on R is lossless iff:

$$[R_1 \cap R_2] \longrightarrow [R_1 - R_2] \in F^+$$

[OR]

$$[R_1 \cap R_2] \longrightarrow [R_2 - R_1] \in F^+$$

Algorithm to achieve BCNF:

If R not in BCNF, $X \to A$ is BCNF violation

$$R \to [R-A], \quad R \to [X \cup A]$$

and repeat if either $(R-A)$ or $(X \cup A)$ are not in BCNF

Multivalued Dependencies (MVD)

One attribute determines a set of values in the other

$$X \twoheadrightarrow Y$$

$$Z = R - (X \cup Y)$$

Now for t_1, t_2 with same X, there should be t_3, t_4 s.t.:

$t_3 : (X, Y_1, Z_2)$ $\quad t_4 : (X, Y_2, Z_1)$

for $t_1 = (X, Y_1, Z_1)$ $\quad t_2 = (X, Y_2, Z_2)$

Basically, all combinations should be there

Trivial MVD

① $Y \subseteq X$

"X is independently determining values of X"

② $X \cup Y = R$

"X determines all possible combinations of R"
↓
stupid because all combinations in R = all rows
= no constraint

4NF

For every non-trivial MVD $X \twoheadrightarrow A \in F^+$,
X = superkey (R)

Join dependencies

$JD(R_1, R_2, \ldots, R_n)$

Decomposition of R s.t. joins of individual R_i = R for every valid state of R (r)

MVD = JD with $n = 2$

$$*(\pi_{R_1}(r), \pi_{R_2}(r), \ldots, \pi_{R_n}(r)) = r$$

Trivial join dependency

One $R_i = R$
"join of R with any subset of R will give back R"

5NF / Project-Join Normal Form (PJNF)

For every non-trivial JD in F^+:
R_i = superkey (R)

① $PQ \to R$
$S \to T$
$PQS \to RS$

$PQS^+ = \{P, Q, S, R, T\}$

R dependent on PQ, not full functional dependency → not 2NF
Illy S dependent on T

Decompose:
$R1(P, Q, R)$, $R_2(S, T)$, $R_3(P, Q, S)$ → for the key

② $R(X, Y, Z)$

$X \to Y$
$Y \to Z$

Not in 3NF
$X \to Y \to Z$

Y not candidate key
Z not prime attr

Decompose:
$R - A$, $X \cup A$

$R_1(X, Y)$, $R_2(Y, Z)$

③ $R(A, B, C, D, E)$
$A \to BC$, $C \to DE$

$A \to B$
$A \to C$
$C \to D$
$C \to E$

1NF ✓

$A^+ = \{A, B, C, D, E\}$
$C^+ = \{C, D, E\}$

No compound key; 2NF ✓

3NF:
prime = A
non-prime = B, C, D, E

$A \to B$: A superkey
$A \to C$: ✓
$C \to D$: D not in superkey, C not CK ✗ → not in 3NF/BCNF

$R_1 = R - A = (A, B, C)$ $\quad R_2 = X \cup A = (C, D, E)$

Now in BCNF

④ Minimal cover

$X \to W$
$WZ \to X$
$WZ \to Y$
$Y \to W$
$Y \to X$
$Y \to Z$

① Redundancy

(i) $X \to W$
$X^+ = \{X\}$

(ii) $WZ \to X$
$WZ^+ = \{W, Z, Y, X \ldots\}$
Redundant

$FD1 = \{X \to W, WZ \to Y, Y \to W, Y \to X, Y \to Z\}$

(iii) $WZ \to Y$
$(WZ)^+ = \{W, Z\}$

(iv) $Y \to W$
$Y^+ = \{Y, X, Z, W \ldots\}$
redundant

$FD2 = \{X \to W, WZ \to Y, Y \to X, Y \to Z\}$

(v) $Y \to X$
$Y^+ = \{Y, Z\}$

(vi) $Y \to Z$
$Y^+ = \{Y, X, W\}$

② Extraneous

Check $WZ \to Y$.
$(WZ)^+ = \{W, Z, Y, X\}$

Remove Z
$W^+ = \{W, Y, X, Z\}$

Z = extraneous

$FD3 = \{X \to W, W \to Y, Y \to X, Y \to Z\}$