

## INF

RHS atomic: no multivalued

**2NF** } attributes depend on whole key

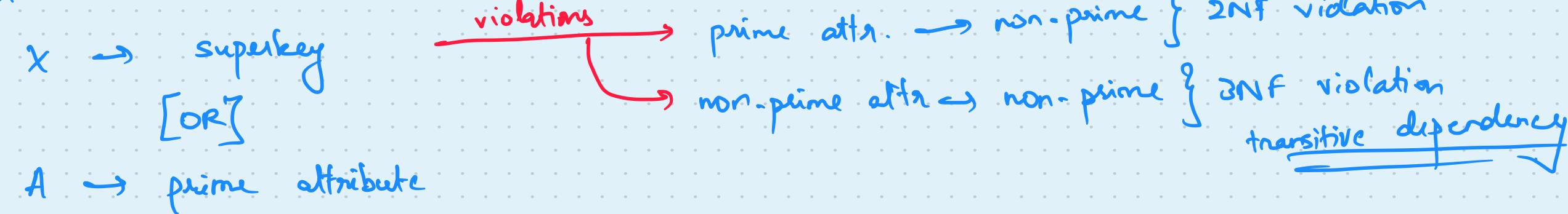
given any candidate key, non-prime attributes are fully functionally dependent on it + INF

Only needed for multi-attribute keys all keys.

**3NF** } attr. depend on nothing but key

No non-prime attribute transitively dependent on superkeys + 2NF

If  $X \rightarrow A$ :



## BCNF

Stricter 3NF.

If  $X \rightarrow A$ :

$X \rightarrow$  candidate key

## Decomposition

Critical lossless join: Joining should not create spurious tuples  
Dependency preservation: All FDs should be preserved post-decomposition  
 can be relaxed

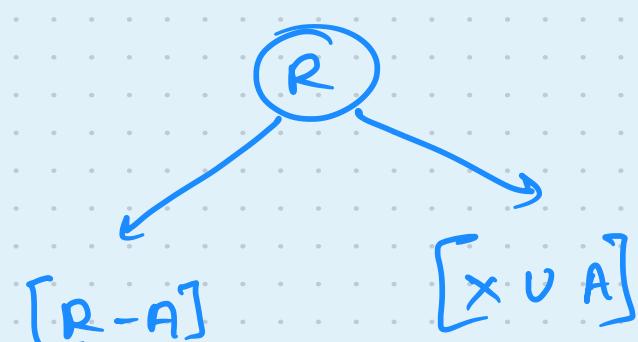
Non-additive join test for binary decompositions (NTB)

$D = \{R_1, R_2\}$  on  $R$  is lossless iff:

$$\begin{aligned} [R_1 \cap R_2] &\rightarrow [R_1 - R_2] \in F^+ \\ [\text{OR}] \\ [R_1 \cap R_2] &\rightarrow [R_2 - R_1] \in F^+ \end{aligned}$$

## Algorithm to achieve BCNF:

If  $R$  not in BCNF,  $X \rightarrow A$  is BCNF violation



and repeat if either  $(R-A)$  or  $(X \cup A)$  are not in BCNF

## Multivalued Dependencies (MVD)

One attribute determines a set of values in the other

$$X \rightarrow Y$$

$$Z = R - (X \cup Y)$$

Now for  $t_1, t_2$  with same  $X$ , there should be  $t_3, t_4$  etc:

$$t_3 = (x, y_1, z_2) \quad t_4 = (x, y_2, z_1)$$

$$\text{for } t_1 = (x, y_1, z_1) \quad t_2 = (x, y_2, z_2)$$

Basically, all combinations should be there

## 4NF

For every non-trivial MVD  $X \rightarrow A \in F^+$ ,

$X$  = superkey ( $R$ )

## Join dependencies

$$JD(R_1, R_2, \dots, R_n)$$

Decomposition of  $R$  st. joins of individual  $R_i = R$  for every valid state of  $R$  ( $n$ )

MVD = JD with  $n=2$

$$*(\Pi_{R_1}(n), \Pi_{R_2}(n), \dots, \Pi_{R_n}(n)) = n$$

## Trivial join dependency

One  $R_i = R$

"join of  $R$  with any subset of  $R$  will give back  $R$ "

## Trivial MVD

①  $Y \subseteq X$

" $X$  is independently determining values of  $Y$ "

②  $X \cup Y = R$

" $X$  determines all possible combinations of  $R$ "

↓  
stupid because all combinations in  $R$  = all rows  
= no constraint

①  $PQ \rightarrow R$

$S \rightarrow T$

$PQS \rightarrow RS$

$$PQS^+ = \{P, Q, S, R, T\}$$

$R$  dependent on  $PQ$ , not full functional dependency  $\rightarrow$  not 2NF  
 Only  $S$  dependent on  $T$

Decompose:

$$R_1(P, Q, R), R_2(S, T), \underline{R_3(P, Q, S)} \text{ for the key}$$

②  $R(X, Y, Z)$

$$\begin{aligned} X \rightarrow Y \\ Y \rightarrow Z \\ X \rightarrow Y \rightarrow Z \end{aligned}$$

$Y$  not candidate key  
 $\geq$  not prime attr

Decompose:

$$R_1(X, Y), R_2(Y, Z)$$

③  $R(A, B, C, D, E)$

$$A \rightarrow BC, C \rightarrow DE$$

$$\begin{aligned} A \rightarrow B \\ A \rightarrow C \\ C \rightarrow D \\ C \rightarrow E \\ A^+ = \{A, B, C, D, E\} \\ C^+ = \{C, D, E\} \end{aligned}$$

No compound key: 2NF ✓

3NF:

$$\begin{aligned} \text{prime} = A \\ \text{non-prime} = B, C, D, E \end{aligned}$$

$$A \rightarrow B: A \text{ superkey}$$

$A \rightarrow C: \text{ } \times$  → not in 3NF/BCNF

$$C \rightarrow D: D \text{ not in superkey}, C \text{ not CK } \times$$

$$\begin{aligned} R_1 = R - A \\ R_2 = X \cup A \\ = (A, B, C) \\ = (C, D, E) \end{aligned}$$

Now in BCNF

④ Minimal cover

① Redundancy

$$\begin{aligned} X \rightarrow W \\ WZ \rightarrow X \\ WZ \rightarrow Y \\ Y \rightarrow W \\ Y \rightarrow X \\ Y \rightarrow Z \\ (WZ)^+ = \{W, Z\} \\ (WZ)^+ = \{W, Z, Y, X, \dots\} \\ \text{Redundant.} \end{aligned}$$

$$FD1 = \{X \rightarrow W, WZ \rightarrow Y, Y \rightarrow W, Y \rightarrow X, Y \rightarrow Z\}$$

$$(iii) WZ \rightarrow Y$$

$$(WZ)^+ = \{W, Z\}$$

$$(iv) Y \rightarrow W$$

$$Y^+ = \{Y, X, Z, W, \dots\}$$

Redundant.

$$FD2 = \{X \rightarrow W, WZ \rightarrow Y, Y \rightarrow X, Y \rightarrow Z\}$$

$$(v) Y \rightarrow X$$

$$Y^+ = \{Y, Z\}$$

$$(vi) Y \rightarrow Z$$

$$Y^+ = \{Y, X, W\}$$

② Extraneous

Check  $WZ \rightarrow Y$ .

$$(WZ)^+ = \{W, Z, Y, X\}$$

Remark Z

$$W^+ = \{W, Y, X, Z\}$$

Z = extraneous.

$$FD3 = \{X \rightarrow W, W \rightarrow Y, Y \rightarrow X, Y \rightarrow Z\}$$

## 5NF / Project-join Normal Form (PJNF)

For every non-trivial JD in  $F^+$ :

$$R_i = \text{superkey } (R)$$