1. Double Integration

AREA

$$\int_a^b f(x)\, dx \qquad\qquad \int_c^d \int_a^b dx\, dy$$

$$\int_c^d \int_a^b \underbrace{f(x,y)}_{\text{height}}\ \underbrace{dx\, dy}_{\text{Area}} \xrightarrow{\ \ } \text{Volume}$$
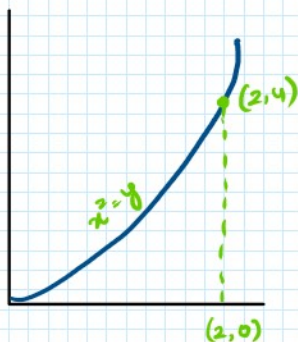

$g(x)$

$f(x)$
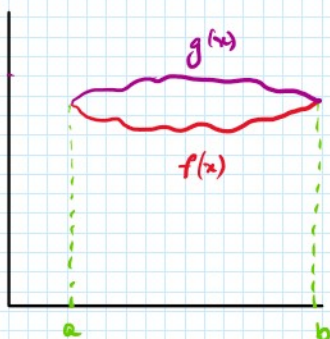
$x: a \longrightarrow b$
$y: f(x) \longrightarrow g(x)$

NOTE: Double Integration Rules

- Inner integration first, then outer; inner operator corresponds to inner integral
- Constant limits $\longrightarrow$ outer integral
  Variable limits $\longrightarrow$ inner integral
- You cannot have both limits to be variable limits.

EXAMPLES

$(2,4)$

$x^2 = y$

$(2,0)$

$y: 0 \longrightarrow 4$
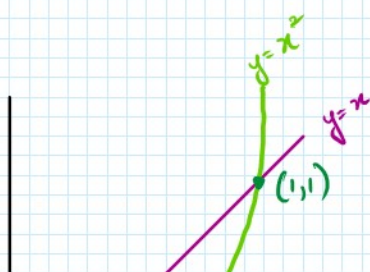$x: \sqrt{y} \longrightarrow 2$

$$\text{Area under curve} = \int_0^2 \int_0^{x^2} dy\, dx$$

[OR]

$x: 0 \longrightarrow 2$
$y: 0 \longrightarrow x^2$
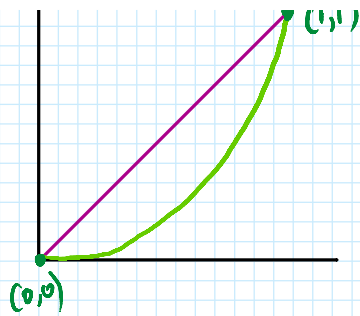
$$\text{Area under the curve} = \int_0^2 \int_0^{x^2} dy\, dx$$

$y = x^2$

$y = x$

$(1,1)$

x w/ constant limits   y w/ constant limits

$x: 0 \longrightarrow 1$ $y: 0 \longrightarrow 1$

(1,1)

(0,0)

x: 0 ⟶ 1   y = 0 ⟶ 1

y = x² ⟶ x   x = y ⟶ √y