1. Jacobian, Double Integration

Thursday, February 8, 2024 9:17 PM

JACOBIAN

$u = xyz$, $v = y^2$, $w = x+z$. Evaluate J.

values of u,v,w

```
>>> syms x y z
>>> J = jacobian([x*y*z, y^2, x+z], [x,y,z])
```

independent variables

```
>>> d = det(J)
```

$u = x^2-2y$, $v = x+y+z$, $w = x-2y+3z$. Evaluate J.

```
>>> syms x y z
>>> J = jacobian([x^2.-2y, x+y+z, x-2*y+3*z], [x,y,z])
>>> d = det (J)
```

$u = x^2-2y^2$, $v = 2x^2-y^2$ where $x = r\cos\theta$, $y = r\sin\theta$. Evaluate J.

```
>>> syms x y r theta
>>> x = r*cos(theta)
>>> y = r* sin(theta)
>>> J = jacobian ([x^2-2*y^2, 2*x^2-y^2], [r, theta])
>>> d = det(J)
>>> simplify (d)
```

$x = u(1-v)$, $y = uv$. Prove $JJ' = 1$

```
>>> syms u v
>>> J = jacobian([u*(1-v), u*v], [u,v])
```

```
>>> J = jacobian([u*(1-v), u*v], [u,v])
>>> J* inv(J)
```

INTEGRATION

int(func, lower_limit, upper_limit)] → analytically solvable

integral(func, lower_limit, upper_limit)] → not analytically solvable; gives numerical answer

integral2(func, $\underbrace{\text{ll1, ul1}}_{\text{outer limit}}$, ll2, ul2)] → double integration

> **NOTE: Function handle**
> Integral only takes a function handle as an argument for func.
> This is defined as follows:
> y = @(x) x.^2

Evaluate $\int_0^1\int_0^{1-x} \frac{1}{(\sqrt{x+y})(1+x+y)^2}\,dy\,dx$

```
>>> func = @(x,y) 1./(sqrt(x+y).*(1+x+y).^2);
>>> ymax = @(x) 1-x;
>>> q = integral2(func, 0,1,0, ymax)
```

Evaluate $\int_{-\pi}^{2\pi}\int_0^{\pi} [y\sin(x) + x\cos(y)]\,dy\,dx$

```
>>> func = @(x,y) y.*sin(x) + x.*cos(y);
>>> q = integral2(func, -pi, 2*pi, 0, pi)
```

Evaluate $\int_0^5\int_0^{x^2} x(x^2+y^2)\,dy\,dx$

Evaluate $\int_0 \int_0 x(x^2+y^2)\,dy\,dx$

```
>>> func = @(x,y) x.*(x.^2 + y.^2);
>>> ymax = @(x) x.^2;
>>> q = integral2(func, 0, 5, 0, ymax)
```

Evaluate $$\int_0^{\frac{\pi}{2}} \int_0^{1/(\sin\theta+\cos\theta)} \frac{r}{\sqrt{r\cos\theta + r\sin\theta}\left(1 + r\cos\theta + r\sin\theta\right)}\,dr\,d\theta$$

```
>> rmax = @(theta)1./(sin(theta)+cos(theta))

rmax =

  function_handle with value:

    @(theta)1./(sin(theta)+cos(theta))
```

variables in the order of their limits in integration

```
>> func=@(theta,r)r./(sqrt(r.*cos(theta)+r.*sin(theta)).*(1+r
   .*cos(theta)+r.*sin(theta)).^2)

func =

  function_handle with value:

    @(theta,r)r./(sqrt(r.*cos(theta)+r.*sin(theta)).*(1+r.*cos(theta)+r
.*sin(theta)).^2)


>> q=integral2(func,0,pi./2,0,rmax)

q =

    0.2854
```