2. Method of Variation of Parameters

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METHOD OF VARIATION OF PARAMETERS

- Used for second order LDEs only

Consider a second order LDE:

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = \phi(x)$$

Let the CF $= c_1\underbrace{y_1} + c_2\underbrace{y_2}$ → functions of x

Wronskian $= W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$$\boxed{\begin{aligned} &PI = A(x)y_1 + B(x)y_2 \\ &\text{where} \\ &A(x) = -\int \frac{y_2 \times \phi(x)}{W}\,dx \\ &B(x) = \int \frac{y_1 \times \phi(x)}{W}\,dx \end{aligned}}$$

NOTE: Bernoulli's method for integration by parts

APPLICATION BASED

Q. A body weighing 10 kg is hung from a spring. A pull of 20 kg will stretch the spring to 10 cm. The body is pulled down to 20 cm below the static equilibrium position and released. Find the displacement of the body from equilibrium position at a time 't' seconds, maximum velocity, period of oscillation.

The differential equation governing this principle:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

To find k:

$mg = k\delta$

$20 \times 9.8 = k(0.1)$

$k = 1960 \text{ N/m}$

$$\frac{d^2x}{dt^2} + \frac{1960}{10}x = 0$$

$x = c_1 \cos 14t + c_2 \sin 14t$

At $t = 0$, $x = 0.2$ m

$\frac{dx}{dt} = 0$

$\boxed{c_1 = 0.2}$ $\quad x = 0.2 \cos 14t$

$\boxed{c_2 = 0}$

Q A condenser of capacitance C discharges through a resistance R and inductance L in series, and the charge q at time t corresponds to the equation:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$$

$L = 0.25$ H

$C = 2 \times 10^{-6}$ F

$R = 250\ \Omega$

$t = 0$

$q = 0.002$ C

$\frac{dq}{dt} = 0$