3. t-test

30 September 2024 09:40

Testing Single Means (Small Samples)

Note: We can only use t-distribution when there are no outliers

Note: If σ is known, z-test regardless of sample size

(1) $\mu = 39.00$ mm

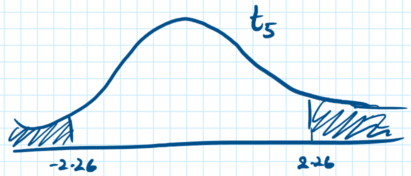
$\bar{x} = 39.011$ $\qquad s = 0.0119$

$n = 6$ } small sample $\qquad df = 5$

$H_0 : \mu = 39.00$ mm

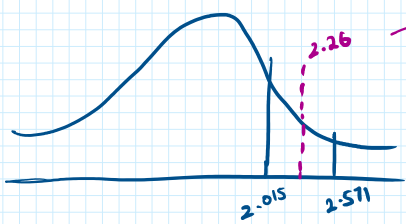
$H_1 : \mu \neq 39.00$ mm

$$t_{\text{test value}} = \frac{39.011 - 39}{\frac{0.0119}{\sqrt{6}}} = 2.26$$



P-value = $2(t(2.26))$

From t-distribution,



we can approximate this value as $\frac{t_5(2.015) + t_5(2.571)}{2} = \frac{0.05 + 0.025}{2} = 0.0375$

P-value $\approx 2 \times 0.0375 \approx 0.075$

$P > 0.05$

$\Rightarrow$ Enough evidence to accept H_0 as plausible

(2) $\mu = 7.0$

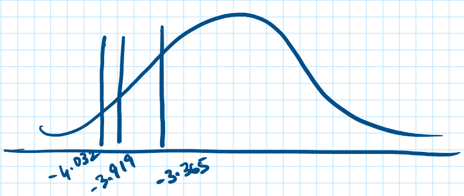
$\bar{x} = 6.68$ $\qquad s = 0.20$

$n = 6$ $\qquad df = 5$

$H_0 : \mu \geq 7.0$

$H_1 : \mu < 7.0$

$t_{\text{test val}} = -3.919$



From t-table, $t_5(-3.365) = 0.01$

$\Rightarrow t_5(-3.919) < 0.01$

$\Rightarrow$ P-value < 0.01

Since P-value is less than 0.01, there is enough evidence to reject H_0

TESTING POPULATION PROPORTION

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

① Believed to be 60% effective

$n = 100$

$\hat{p} = \frac{70}{100} = 0.7$

$p_0 = 0.6$, $q_0 = 0.4$

$H_0: p = 0.6$
$H_1: p > 0.6$

} why: we already have a drug 60% effective, so even if $p < 60$ for the new one it doesn't matter.
You can also do $H_0: p \leq 0.6$

Assuming H_0 is true

$$z = \frac{0.7 - 0.6}{\sqrt{\frac{(0.6)(0.4)}{100}}} = 2.04$$

$p(z > 2.04) = 0.0207 < 0.05$

Thus we reject H_0 and accept the plausibility of the new drug being superior

TESTING DIFFERENCE BETWEEN TWO MEANS (LARGE SAMPLE)

$$z = \frac{(\bar{x} - \bar{y}) - \Delta_0}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$$

σ_x, σ_y can be approximated with s_x, s_y

NOTE: Unless otherwise mentioned, $\Delta_0 \longrightarrow 0$

① $\mu_y\% = 72$, $\sigma_y = 8$, $n_y = 32$ } cured by physiotherapy
$\mu_x\% = 75$, $\sigma_x = 6$, $n_x = 36$ } cured by surgery

$H_0: \mu_x \leq \mu_y$ $(\mu_x - \mu_y \geq 0)$ $(\mu_y = \mu_x)$
$H_1: \mu_x > \mu_y$

Assume H_0 is true

$\Rightarrow z = \frac{(\mu_x - \mu_y) - \Delta_0}{} = 1.732$

$$\Rightarrow \quad z = \frac{(\mu_x - \mu_y) - \Delta_0}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} = 1.732$$

$$\text{P-value} = P(z > 1.732) = 1 - 0.9582 = 0.0418$$

Since p-value is smaller than 0.05, we reject H_0

Thus we conclude that surgery is not inferior to physiotherapy.

(2) Argon-CO_2 $\qquad$ CO_2 only

$n_x = 544$ $\qquad$ $n_y = 581$

$\bar{x} = 0.37\,\mu m$ $\quad$ $s_x = 0.25\,\mu m$ $\qquad$ $\bar{y} = 0.40\,\mu m$ $\quad$ $s_y = 0.26\,\mu m$

$H_0: \mu_x = \mu_y \quad (\mu_x - \mu_y = \Delta_0)$

$H_1: \mu_x \neq \mu_y$

Assuming H_0 is true,

$$z = \frac{(\bar{x} - \bar{y})}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}} = -1.97$$

$$\text{p-value} = P(z > 1.97) + P(z < -1.97)$$

$$= 2P(z < -1.97) = 2(0.0244) = 0.0488$$

P-value is lesser than 0.05, thus we reject H_0

Thus there is significant evidence to suggest that the two population means differ

(3) Same as above but:

Can you conclude that μ_y exceeds μ_x by more than $0.015\,\mu m$?

$\mu_y - \mu_x > 0.015\,\mu m$

$\Rightarrow \quad H_0: \mu_y - \mu_x \leq 0.015$

$H_1: \mu_y - \mu_x > 0.015$

$\Delta_0 = 0.015$

Assuming H_0 as true,

$$z = \frac{(0.40 - 0.37) - 0.015}{\sqrt{\frac{0.25^2}{544} + \frac{0.26}{581}}} = 0.986 \approx 0.99$$

$P(Z > 0.99) = 1 - 0.8389 = \underline{0.1611}$

Since p-value is greater than 0.05, we accept H_0

Thus we reject the plausibility of the fact that μ_y exceeds μ_x by more than 0.015 μm