Tuesday, February 20, 2024 12:01 AM

MARKOV'S INEQUALITY

$X \rightarrow$ non-negative random variable with finite mean μ

$$\boxed{P(X \geq c) \leq \frac{\mu}{c}}$$

CHEBYSHEV'S INEQUALITY

$X \rightarrow$ random variable w/ finite μ and σ^2 with any distribution

does not need to be non-negative

$$\boxed{P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}}$$

Markov's inequality applied to $(X-\mu)^2$

describes the percentage of values within a certain $k\sigma$ of the mean; gives bounds for $P(X)$ where X is outside $k\sigma$ limits from the mean.

Standard units: No. of standard deviations that a particular value of a random variable is away from the mean

- Determined by $\frac{X-\mu}{\sigma}$
- $-k\sigma \leq X-\mu \leq k\sigma$ for some small k
- $\mu - k\sigma \leq X \leq \mu + k\sigma$

Other forms

① $P(|X-\mu| \geq c) \leq \frac{\sigma^2}{c^2}$

$k = \frac{c}{\sigma}$

② $P((X-\mu)^2 \geq k^2\sigma^2) \leq \frac{1}{k^2}$

③ $P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$ } complement of original statement

④ $P(X \geq \mu + k\sigma) + P(X \leq \mu - k\sigma) \leq \frac{1}{k^2}$ } $|X-\mu| \geq k\sigma \xrightarrow{\text{2 cases}}$ $X \geq \mu + k\sigma$, $X \leq \mu - k\sigma$

- $P(X \geq \mu + k\sigma) \leq \frac{1}{k^2}$
- $P(X \leq \mu - k\sigma) \leq \frac{1}{k^2}$

PROBLEMS

① $\mu = 50$ mm $\sigma = 0.45$ mm

Largest value for probability that the length of the rivet is outside the interval 49.1 - 50.9 mm?

$$P(|X-\mu| \geq k\sigma) = \frac{1}{k^2}$$

$$P(|X-50| \geq 0.9) \Rightarrow 0.9 = k\sigma, \quad k = \frac{0.9}{0.45} = 2$$

Substituting k as 2,

$$P(X \leq 49.1 \text{ or } X \geq 50.9) \leq \frac{1}{k^2}$$

$$P(X \leq 49.1 \text{ or } X \geq 50.9) \leq \frac{1}{4}$$

② Assume the pdf for X above is:

$$f_X(x) = \begin{cases} \dfrac{477 - 471(x-50)^2}{640} & 49 \le x \le 51 \\ 0 & \text{otherwise} \end{cases}$$

$\mu_X = 50$, $\sigma = 0.45$

Compute $P(X)$ where X is outside 49.1 - 50.9 mm. How close is this probability to Chebyshev's bound?

Chebyshev bound

$P(|X-50| \ge 0.9) \le 0.25$

Actual probability

$$P(|X-50| \ge 0.9) = 1 - P(|X-50| \le 0.9)$$

$$= 1 - \int_{49.1}^{50.9} f(x)\,dx = 1 - 0.9838$$

$$= 0.016$$

③ $\mu = 3$ years $= 36$ months

$\sigma = 2$ months

What percent of computers last b/w 31 months to 41 months?

$$P(|X - \mu| \le k\sigma) \ge 1 - \frac{1}{k^2}$$

where $k\sigma$ is 5 months

$\Rightarrow k = \frac{5}{2} = 2.25$

Putting $k = 2.25$

$$P(|X-36| \le 10) \ge 1 - \frac{1}{(2.25)^2}$$

$$P(|X-36| \le 10) \ge 0.84$$

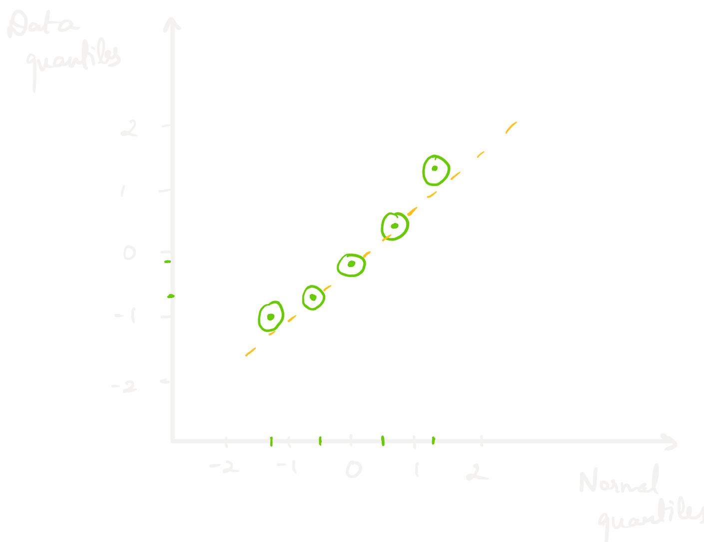
$\Rightarrow$ At least 84% of computers last between 31 months and 41 months

= QUANTILE - QUANTILE PLOTS =

Checking whether given data values came from a normally distributed population

Check whether 3.01, 3.35, 4.79, 5.96, 7.89 are from a normal distribution

Data values	Index (i)	Quantiles $\left(\frac{i-0.5}{n}\right)$	Quantiles for normal distribution	Quantiles for data $z = \frac{x-\mu}{\sigma}$	Theoretical values (expected) $z = \frac{x-\mu}{\sigma}$, $x = \mu + z\sigma$
3.01	1	0.1	-1.28	-0.995	$5+(-1.28)2 = 2.44$
3.35	2	0.3	-0.52	-0.825	$5+(-0.52)2 = 3.96$
4.79	3	0.5	0	-0.105	$5+(0)2 = 5$
5.96	4	0.7	0.52	0.48	$5+(0.52)2 = 6.04$
7.89	5	0.9	1.28	1.45	$5+(1.28)2 = 7.56$



Conclusion:

Q-Q plot is somewhat linear

$\Downarrow$

Data taken from normal distribution

Check whether 43, 23, 33, 25, 28, 38 came from a normal distribution

23, 25, 28, 33, 38, 43 ; $\mu = 31.667$ $\sigma = \sqrt{E(x^2) - [E(x)]^2} = \sqrt{1053.333 - 1002.778} = 7.11$

Data values	Index (i)	Quantiles Hazen: $\left(\frac{i-0.5}{n}\right)$	Quantiles for normal distribution	Quantiles for data $z = \frac{x-\mu}{\sigma}$	Theoretical values (expected) $z = \frac{x-\mu}{\sigma}$, $x = \mu + z\sigma$
23	1				

23	1
25	2
28	3
33	4
38	5
43	6

Figure out how this works