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Chapter 14: Indexing

Database System Concepts, 7th Ed.

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Outline

- Basic Concepts
- Ordered Indices
- B⁺-Tree Index Files
- B-Tree Index Files
- Hashing
- Write-optimized indices
- Spatio-Temporal Indexing



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Basic Concepts

- Indexing mechanisms used to speed up access to desired data.
 - E.g., author catalog in library
- **Search Key** – a set of attributes used to look up records in a file. *1, 2, 3, 7, 8, 9*.
- An **index file** consists of records (called **index entries**) of the form



- Index files are typically much smaller than the original file
- Two basic kinds of indices:
 - **Ordered indices:** search keys are stored in sorted order
 - **Hash indices:** search keys are distributed uniformly across “buckets” using a “hash function”.

$$h(x) = \text{some} K \Rightarrow \text{search key } x$$

[hash]



Index Evaluation Metrics

index
Access time
Insertion time
Deletion time
Space overhead

- Access types supported efficiently. e.g.,
 - Records with a specified value in the attribute
 - Records with an attribute value falling in a specified range of values.

- Access time
- Insertion time
- Deletion time
- Space overhead

attribute l = index &
attribute l+1 ~ l+M

- ① $\Omega(\log n)$ for l <= i. insert, delete, $\Omega(n^2)$ for update
- ② storage space $\Omega(n^2)$
- ③ $\Omega(n^2)$ for index entries
- ④ $\Omega(n^2)$ for leaf pages



Ordered Indices

Q&A

- In an **ordered index**, index entries are stored sorted on the search key value.
- **Clustering index:** in a sequentially ordered file, the index whose search key specifies the sequential order of the file.
 - Also called **primary index**
 - The search key of a clustering index is usually but not necessarily the primary key.
- **Secondary index:** an index whose search key specifies an order different from the sequential order of the file. Also called **nonclustering index**.
- **Index-sequential file:** sequential file ordered on a search key, with a clustering index on the search key.

er.
Primary key is the key &
File is first sorted by primary key
Then it is clustered by clustering key
This is good for performance but
not for query processing

10101	Srinivasan	Comp. Sci.	65000		
12121	Wu	Finance	90000		
15151	Mozart	Music	40000		
22222	Einstein	Physics	95000		
32343	El Said	History	60000		
33456	Gold	Physics	87000		
45565	Katz	Comp. Sci.	75000		
58583	Califieri	History	62000		
76543	Singh	Finance	80000		
76766	Crick	Biology	72000		
83821	Brandt	Comp. Sci.	92000		
98345	Kim	Elec. Eng.	80000		

Figure 14.1 Sequential file for *instructor* records.



Dense Index Files

Search key is in index file.

- **Dense index** — Index record appears for every search-key value in the file.
- E.g. index on *ID* attribute of *instructor* relation

clustering index ID is primary key \Rightarrow there exists only one such record and the search is complete.

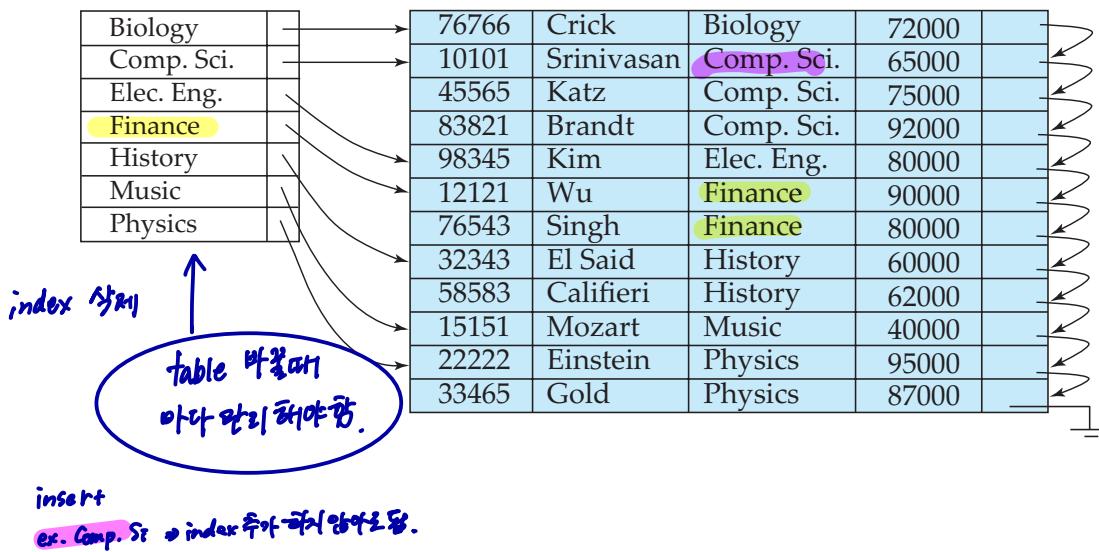
10101		10101	Srinivasan	Comp. Sci.	65000	
12121		12121	Wu	Finance	90000	
15151		15151	Mozart	Music	40000	
22222		22222	Einstein	Physics	95000	
32343		32343	El Said	History	60000	
33456		33456	Gold	Physics	87000	
45565		45565	Katz	Comp. Sci.	75000	
58583		58583	Califieri	History	62000	
76543		76543	Singh	Finance	80000	
76766		76766	Crick	Biology	72000	
83821		83821	Brandt	Comp. Sci.	92000	
98345		98345	Kim	Elec. Eng.	80000	



Dense Index Files (Cont.)

se

- Dense index on *dept_name*, with *instructor* file sorted on *dept_name*



Clustering vs Nonclustering Indices

- Indices offer substantial benefits when searching for records.
- BUT: indices imposes overhead on database modification
 - when a record is inserted or deleted, every index on the relation must be updated
 - When a record is updated, any index on an updated attribute must be updated
- Sequential scan (using clustering index) is efficient, but a sequential scan using a secondary (nonclustering) index is expensive on magnetic disk
 - Each record access may fetch a new block from disk
 - Each block fetch on magnetic disk requires about 5 to 10 milliseconds

↳ *Sequential scan → (ex. index & pointer access)*

클러스터링 인덱스

비클러스터링

인덱스



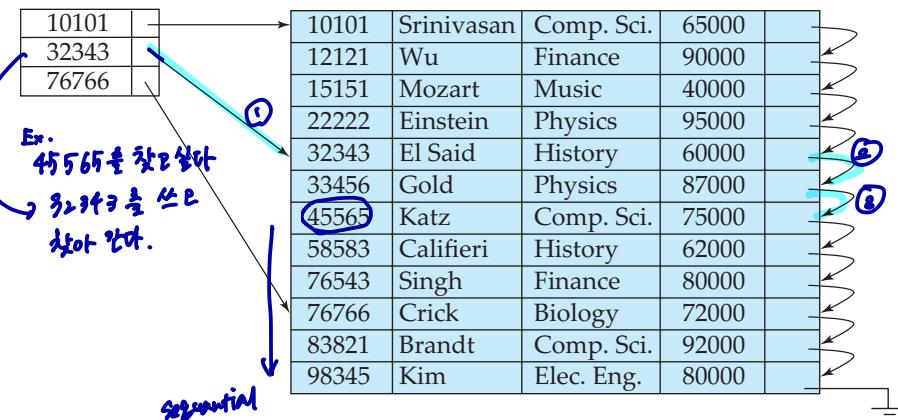
Search key - & index file.

Sparse Index Files

- **Sparse Index:** contains index records for only some search-key values.
 - Applicable when records are sequentially ordered on search-key
- To locate a record with search-key value K we:
 - Find index record with largest search-key value $\leq K$
 - Search file sequentially starting at the record to which the index record points

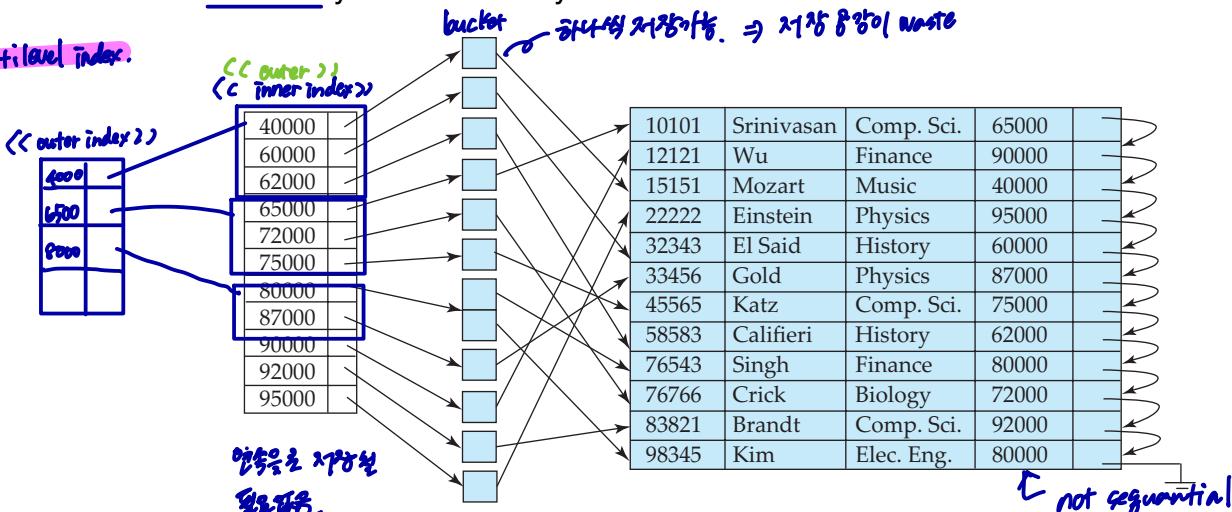
merit
 • memory 35%.
 • entry를 놓고나온다. 32343
 반복 필요 없어.
 ↗ index가 끝나면 끝나!

dangerit
 정부 시장이 정상.

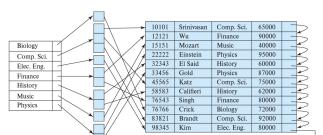


Secondary Indices Example

- P. 12
- Secondary index on salary field of instructor



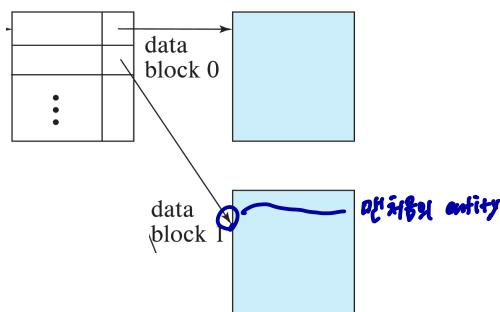
- Index record points to a bucket that contains pointers to all the actual records with that particular search-key value.
- Secondary indices have to be dense





Sparse Index Files (Cont.)

- Compared to dense indices:
 - + Less space and less maintenance overhead for insertions and deletions.
 - Generally slower than dense index for locating records.
- Good tradeoff:**
 - for clustered index: sparse index with an index entry for every block in file, corresponding to least search-key value in the block.



- For unclustered index: sparse index on top of dense index (multilevel index)

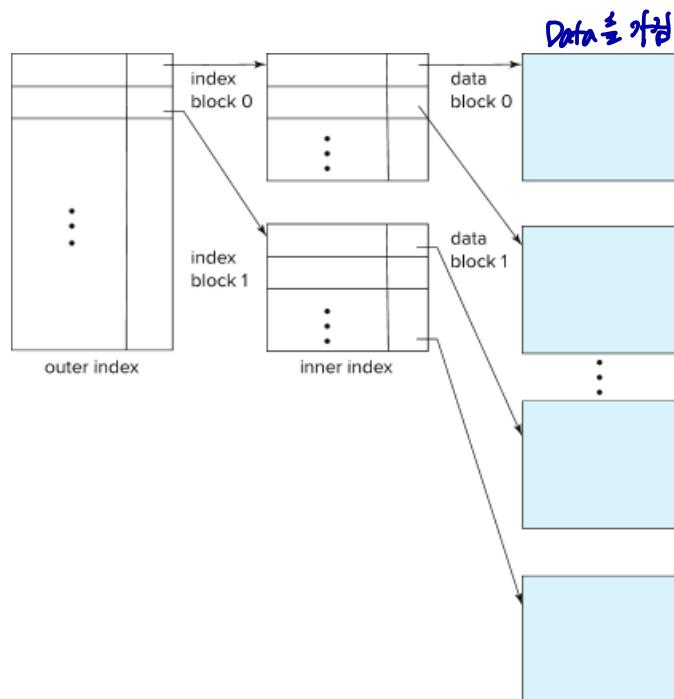


Multilevel Index

- (not all of it can be kept in memory.)*
- If index does not fit in memory access becomes expensive.
 - P. 104* Solution: treat index kept on disk as a sequential file and construct a sparse index on it. *index1 index2 ...*
 - + outer index – a sparse index of the basic index
 - + inner index – the basic index file
 - If even outer index is too large to fit in main memory, yet another level of index can be created, and so on. $\Rightarrow \dots$
 - Indices at all levels must be updated on insertion or deletion from the file.
↳ みんなで... . . .



Multilevel Index (Cont.)



Index Update: Deletion

- If deleted record was the only record in the file with its particular search-key value, the search-key is deleted from the index also.
 - Single-level index entry deletion:**
 - Dense indices** – deletion of search-key is similar to file record deletion.
 - Sparse indices** –
 - if an entry for the search key exists in the index, it is deleted by replacing the entry in the index with the next search-key value in the file (in search-key order).
 - If the next search-key value already has an index entry, the entry is deleted instead of being replaced.
-
- A diagram illustrating index update during deletion. On the left, a small table shows index entries: 10101, 32343, and 76766. Above the table, a handwritten note says 'next search key 33456'. A red arrow labeled 'delete' points from the entry 32343 to a larger table on the right. The larger table contains 11 records with columns: ID, Name, Major, and Score. The record corresponding to index entry 32343 is highlighted with a blue underline. A red arrow labeled 'delete' points to this record. The table is followed by a series of right-pointing arrows.



Index Update: Insertion

- **Single-level index insertion:**
 - Perform a lookup using the search-key value of the record to be inserted.
 - **Dense indices** – if the search-key value does not appear in the index, insert it
 - Indices are maintained as sequential files
 - Need to create space for new entry, overflow blocks may be required
 - **Sparse indices** – if index stores an entry for each block of the file, no change needs to be made to the index unless a new block is created.
 - If a new block is created, the first search-key value appearing in the new block is inserted into the index.
- **Multilevel insertion and deletion:** algorithms are simple extensions of the single-level algorithms



Indices on Multiple Keys

- **Composite search key**
 - E.g., index on *instructor* relation on attributes (*name*, *ID*)
 - Values are sorted lexicographically → 與書順
 - E.g. (John, 12121) < (John, 13514) and (John, 13514) < (Peter, 11223)
 - Can query on just *name*, or on (*name*, *ID*)

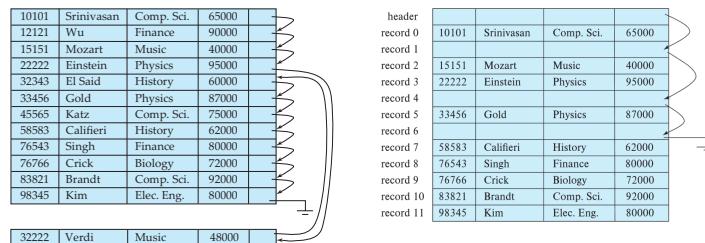
o name 本身 ⇒ ID 本身
o (name, ID) 本身
XX ID 本身 ⇒ name 本身 not prefix .



B+-Tree Index Files

- Disadvantage of indexed-sequential files

- Performance degrades as file grows, since many overflow blocks get created.
- Periodic reorganization of entire file is required.



+ Advantage of B+-tree index files: 자료 구조 안정화 및 성능 향상.

- Automatically reorganizes itself with small, local, changes, in the face of insertions and deletions. 자료 구조. 자체 재구조화에 있어 편리
- Reorganization of entire file is not required to maintain performance.

- (Minor) disadvantage of B+-trees:

- Extra insertion and deletion overhead, space overhead.

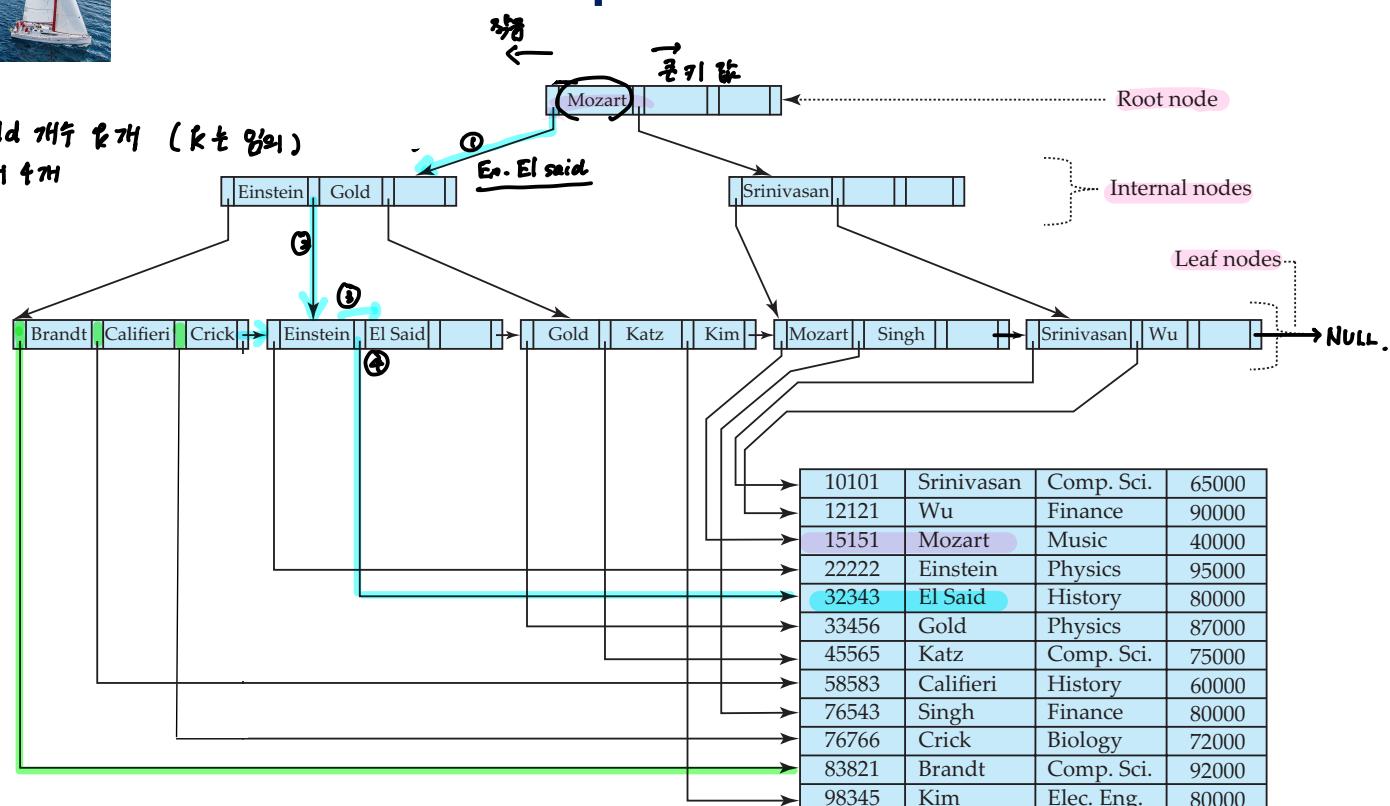
- Advantages of B+-trees outweigh disadvantages

- B+-trees are used extensively



node block & disk block & block.

Example of B+-Tree

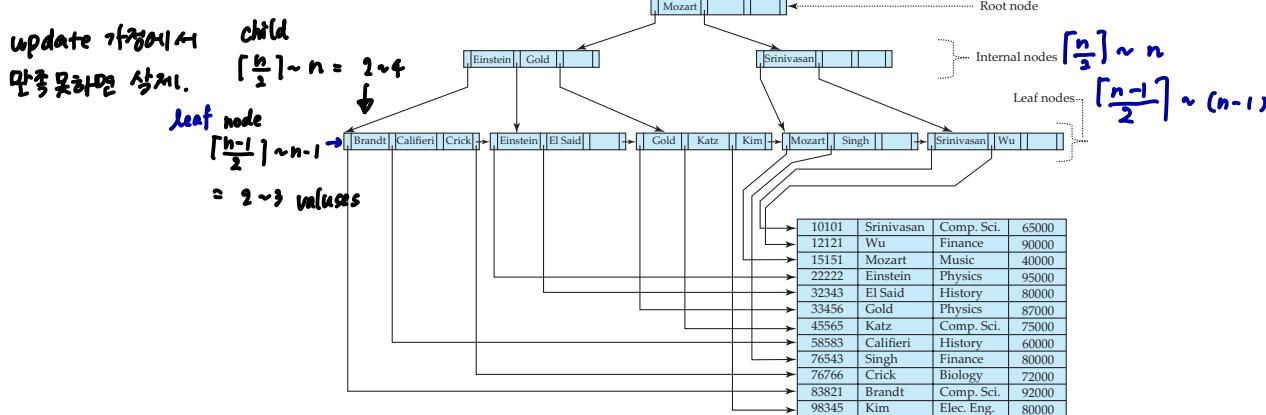




B+-Tree Index Files (Cont.)

- A B+-tree is a rooted tree satisfying the following properties:

- All paths from root to leaf are of the same length $\downarrow \text{depth} = 2$
- Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children.
- A leaf node has between $\lceil (n-1)/2 \rceil$ and $n-1$ values.
- Special cases:
 - If the root is not a leaf, it has at least 2 children.
 - If the root is a leaf (that is, there are no other nodes in the tree), it can have between 0 and $(n-1)$ values.



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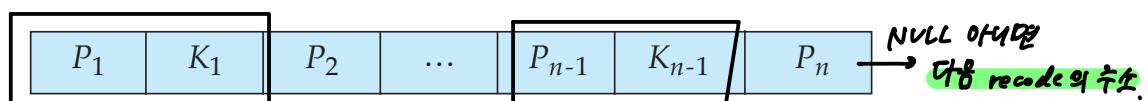
14.19

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B+-Tree Node Structure

- Typical node



- K_i are the search-key values
- P_i are pointers to children (for non-leaf nodes) or pointers to records or buckets of records (for leaf nodes).

- The search-keys in a node are ordered

$$K_1 < K_2 < K_3 < \dots < K_{n-1} \quad \text{ordering } \checkmark$$

(Initially assume no duplicate keys, address duplicates later)

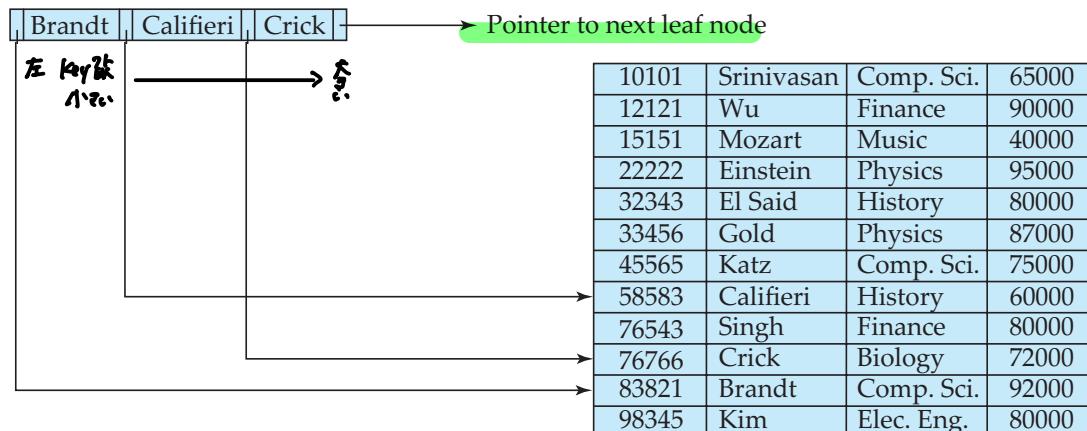
↑ 重複 허용.



Leaf Nodes in B+-Trees

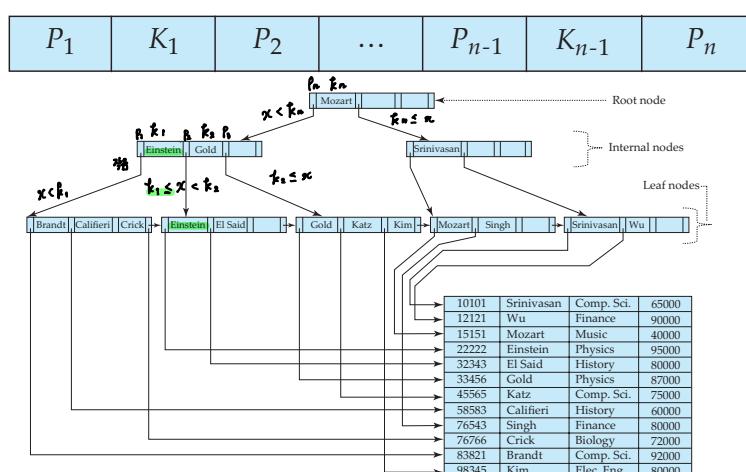
Properties of a leaf node:

- For $i = 1, 2, \dots, n-1$, pointer P_i points to a file record with search-key value K_i ,
- If L_i, L_j are leaf nodes and $i < j$, L_i 's search-key values are less than or equal to L_j 's search-key values
- P_n points to next leaf node in search-key order



Non-Leaf Nodes in B+-Trees

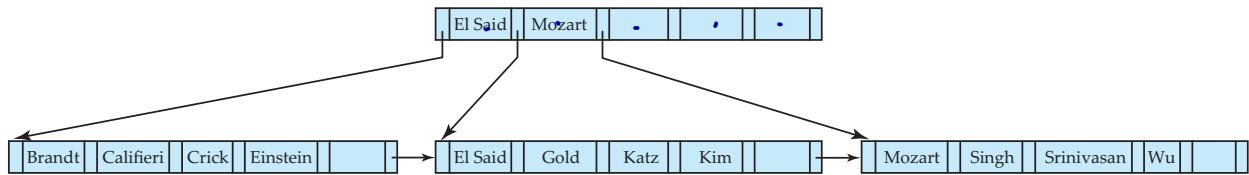
- Non leaf nodes form a multi-level sparse index on the leaf nodes. For a non-leaf node with m pointers:
 - All the search-keys in the subtree to which P_1 points are less than K_1
 - For $2 \leq i \leq n-1$, all the search-keys in the subtree to which P_i points have values greater than or equal to K_{i-1} and less than K_i
 - All the search-keys in the subtree to which P_n points have values greater than or equal to K_{n-1}
 - General structure





Example of B+-tree

- B+-tree for *instructor* file ($n = 6$)



- Leaf nodes must have between $\lceil(n-1)/2\rceil$ and $n-1$, with $n=6$. $\lceil\frac{6-1}{2}\rceil = \lceil\frac{5}{2}\rceil = 3$
- Non-leaf nodes other than root must have between $\lceil(n/2)\rceil$ and n with $n=6$.
- Root must have at least 2 children.

Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children.

A leaf node has between $\lceil(n-1)/2\rceil$ and $n-1$ values

Special cases:

- If the root is not a leaf, it has **at least 2 children**.
- If **the root is a leaf** (that is, there are no other nodes in the tree), it can have between **0 and $(n-1)$ values**.



Observations about B+-trees

- Since the inter-node connections are done by pointers, “logically” close blocks need not be “physically” close. → ~~data physically & logically don't match~~
- The non-leaf levels of the B+-tree form a hierarchy of sparse indices.
- The B+-tree contains a relatively small number of levels
 - Level below root has at least $2 * \lceil n/2 \rceil$ values
 - Next level has at least $2 * \lceil n/2 \rceil * \lceil n/2 \rceil$ values
 - .. etc.
- If there are K search-key values in the file, the tree height is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$
- thus searches can be conducted efficiently.
- Insertions and deletions to the main file can be handled efficiently, as the index can be restructured in logarithmic time (as we shall see).

Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children.

A leaf node has between $\lceil(n-1)/2\rceil$ and $n-1$ values

Special cases:

- If the root is not a leaf, it has **at least 2 children**.
- If **the root is a leaf** (that is, there are no other nodes in the tree), it can have between **0 and $(n-1)$ values**.

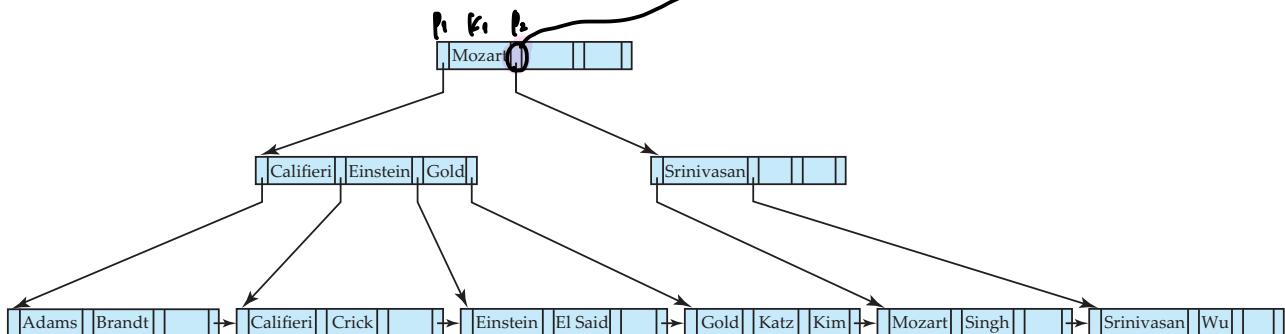


Queries on B⁺-Trees

key value read & fnd.

function *find(v)*

1. *C=root*
2. **while** (*C* is not a leaf node)
 1. Let *i* be least number s.t. $v \leq K_i$. *C* *ki* *vi* *value*.
 2. **if** there is no such number *i* then $K_i < v$ *v* *not* *in C* *key* *exists*.
 3. Set *C* = *last non-null pointer in C*
 4. **else if** ($v = C.K_i$) Set *C* = *P_{i+1}* ↳ ex. *Mozart* *not* *in C*
 5. **else set** *C* = *C.P_i*
3. **if** for some *i*, $K_i = v$ **then return** *C.P_i*
4. **else return null** /* no record with search-key value *v* exists. */



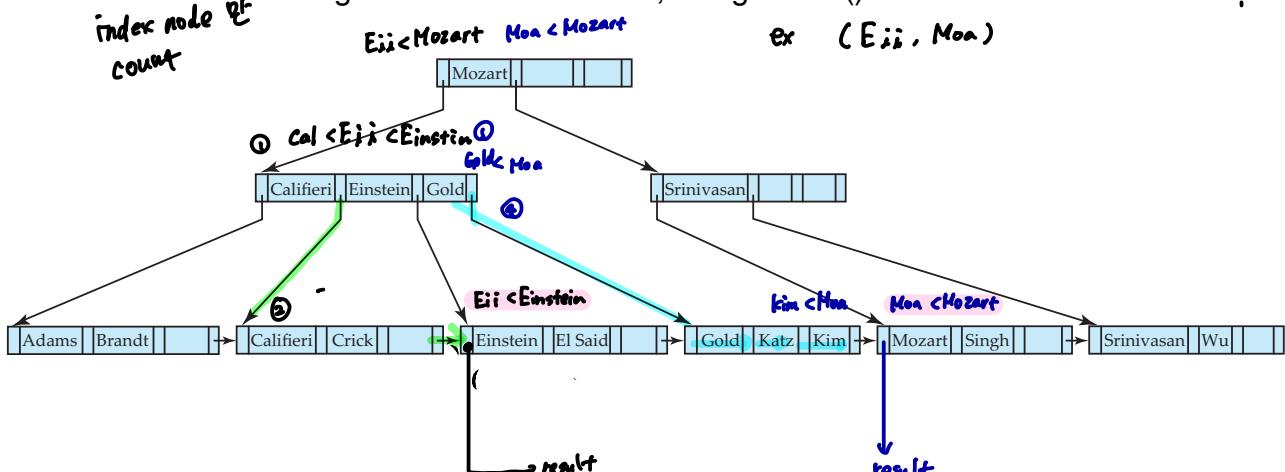
Queries on B⁺-Trees (Cont.)

부호

- **Range queries** find all records with search key values in a given range
 - See book for details of function *findRange(lb, ub)* which returns set of all such records >
 - Real implementations usually provide an iterator interface to fetch matching records one at a time, using a *next()* function

ex

index node
count





Queries on B+-Trees (Cont.)



tree search, data retrieval, insertion, deletion, update, ... \rightarrow n data $\in 10^3$

とてもゆっくり増加
安全性確保できる

- If there are K search-key values in the file, the height of the tree is no more than $\lceil \log_{n/2}(K) \rceil$.
- A node is generally the same size as a disk block, typically 4 kilobytes and n is typically around 100 $n = 100 \text{ bytes}$
 - $(4 \times 10^3) / 100 = 40 \text{ bytes per index entry}$.
- With 1 million search key values and $n = 100$ $\lceil \log_{\frac{n}{2}}(1,000,000) \rceil = 4$ nodes are accessed in a lookup traversal from root to leaf.
- Contrast this with a balanced binary tree with 1 million search key values — around 20 nodes are accessed in a lookup $\text{binary tree} \approx 20 \text{ nodes}$
 - above difference is significant since every node access may need a disk I/O, costing around 20 milliseconds 20 ms per cost

@

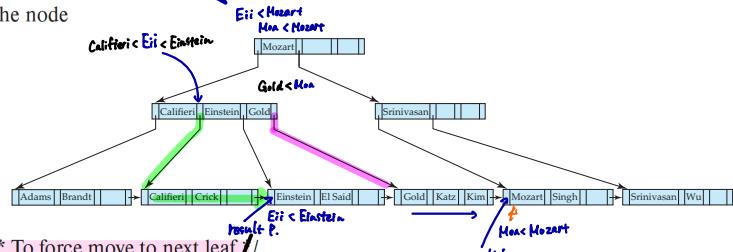


Range Query on a B+-tree

```
function findRange(lb, ub)
/* Returns all records with search key value V such that lb ≤ V ≤ ub. */
Set resultSet = {};
Set C = root node
while (C is not a leaf node) begin
    Let i = smallest number such that lb ≤ C.Ki
    if there is no such number i then begin
        Let Pm = last non-null pointer in the node
        Set C = C.Pm
    end
    else if (lb = C.Ki) then Set C = C.Pi+1
    else Set C = C.Pi /* lb < C.Ki */
end
/* C is a leaf node */
Let i be the least value such that Ki ≥ lb
if there is no such i
    then Set i = 1 + number of keys in C; /* To force move to next leaf */
Set done = false;
while (not done) begin
    Let n = number of keys in C.
    if (i ≤ n and C.Ki ≤ ub) then begin
        Add C.Pi to resultSet
        Set i = i + 1
    end
    else if (i ≤ n and C.Ki > ub)
        then Set done = true;
    else if (i > n and C.Pn+1 is not null)
        then Set C = C.Pn+1, and i = 1 /* Move to next leaf */
    else Set done = true; /* No more leaves to the right */
end
return resultSet;
```

p. 26

Ex. (Eii, Moa)





Non-Unique Keys

- If a search key a_i is not unique, create instead an index on a composite key (a_i, A_p) , which is unique
 - A_p could be a primary key, record ID, or any other attribute that guarantees uniqueness
- Search for $a_i = v$ can be implemented by a range search on composite key, with range $(v, -\infty)$ to $(v, +\infty)$ ex $(Computer, -\infty) \dots (Computer, +\infty) \rightarrow$ ex ID.
- But more I/O operations are needed to fetch the actual records
 - If the index is clustering, all accesses are sequential
 - If the index is non-clustering, each record access may need an I/O operation



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Updates on B+-Trees: Insertion



Assume record already added to the file. Let

- pr be pointer to the record, and let
- v be the search key value of the record

Find leaf node $\xrightarrow{\text{of } v}$ (v, pr)

- Find the leaf node in which the search-key value would appear
 - If there is room in the leaf node, insert (v, pr) pair in the leaf node
 - Otherwise, split the node (along with the new (v, pr) entry) as discussed in the next slide, and propagate updates to parent nodes.

leaf node 미 풀이가 가능한 상태.

해결 / 풀이

 $\lceil \frac{n}{2} \rceil$

Updates on B+-Trees: Insertion (Cont.)

Degree of $\frac{1}{2}$

- Splitting a leaf node:

- take the n (search-key value, pointer) pairs (including the one being inserted) in sorted order. Place the first $\lceil \frac{n}{2} \rceil$ in the original node, and the rest in a new node.

* $\lceil \frac{n}{2} \rceil = \lceil \frac{n-1}{2} + 1 \rceil$ $\lceil \frac{n}{2} \rceil$ is a new node

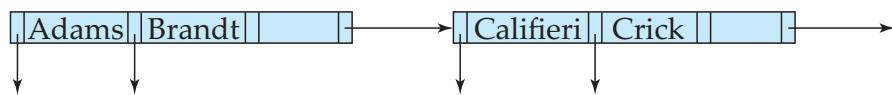
first key & pointer is
the node's $\frac{1}{2}$ nodes

- let the new node be p , and let k be the least key value in p . Insert (k, p) in the parent of the node being split.

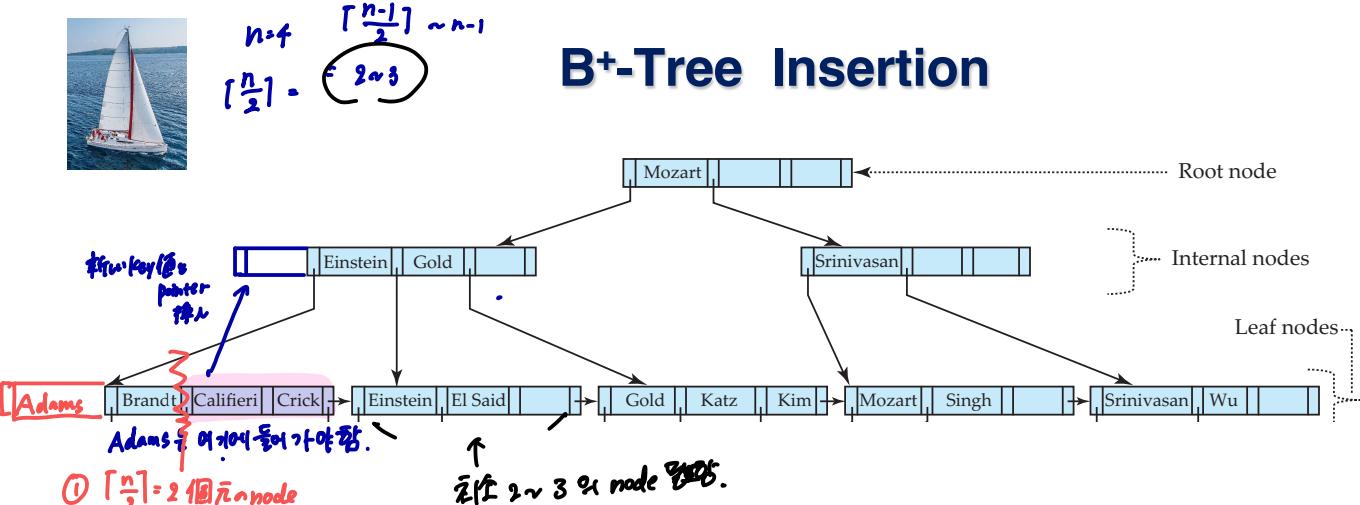
- If the parent is full, split it and propagate the split further up.

- Splitting of nodes proceeds upwards till a node that is not full is found.

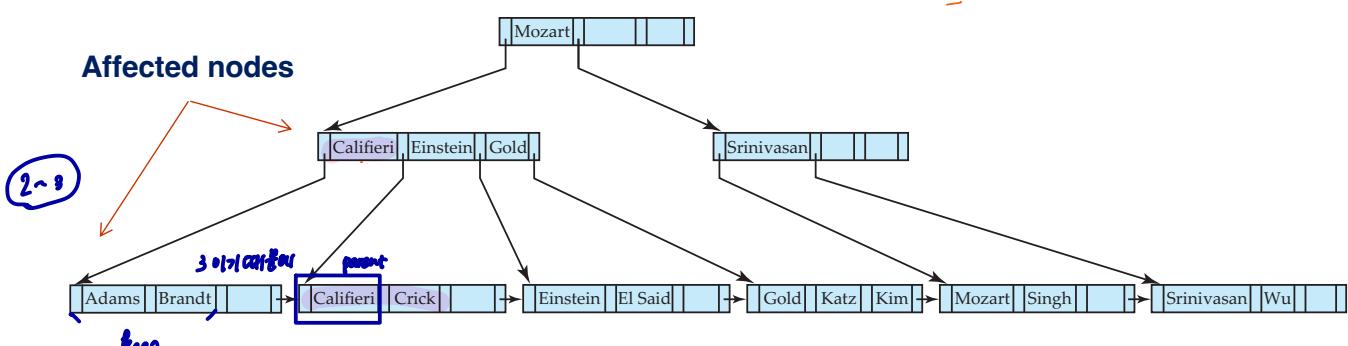
- In the worst case the root node may be split increasing the height of the tree by 1.



Result of splitting node containing Brandt, Califieri and Crick on inserting Adams
Next step: insert entry with (Califieri, pointer-to-new-node) into parent

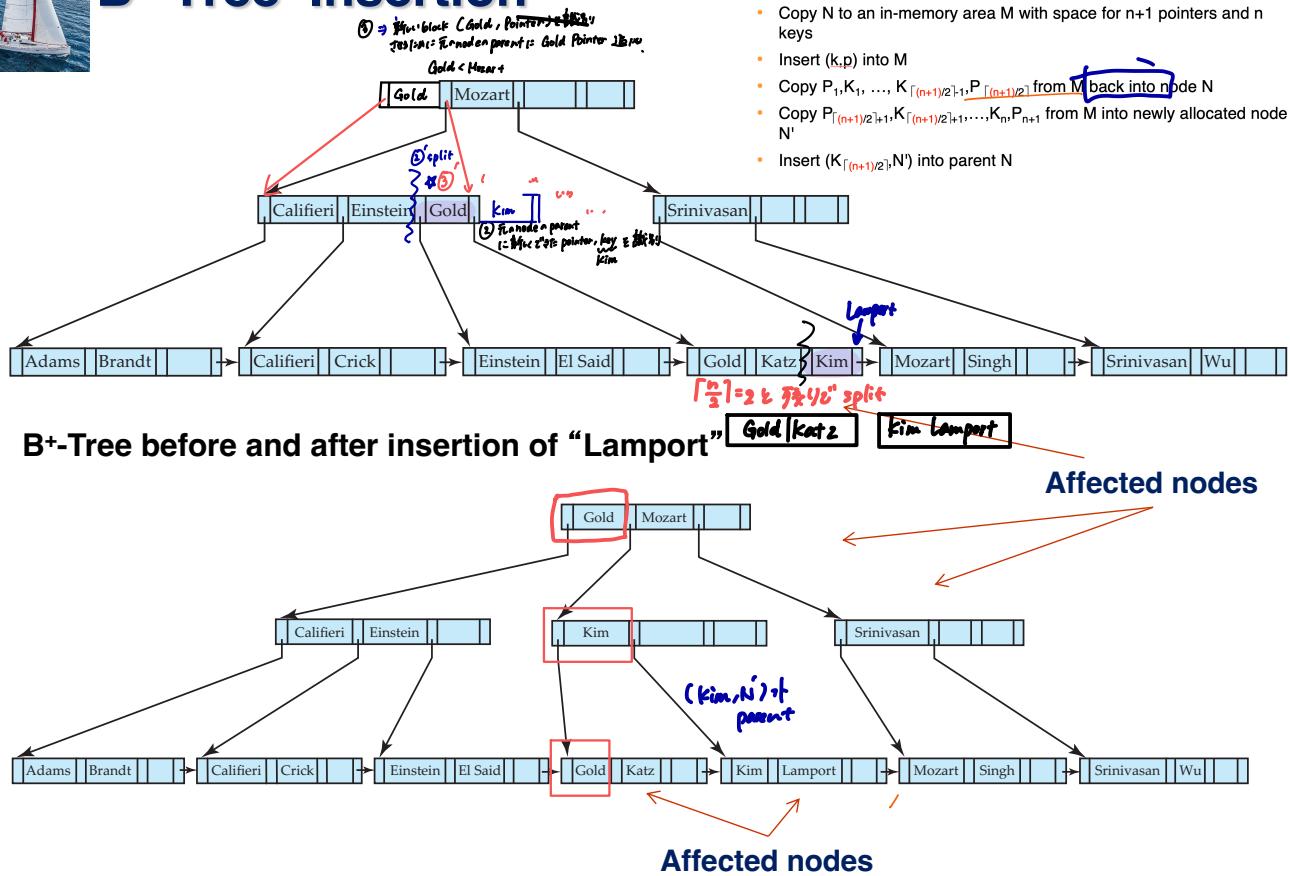


Affected nodes





B+ -Tree Insertion



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14.33

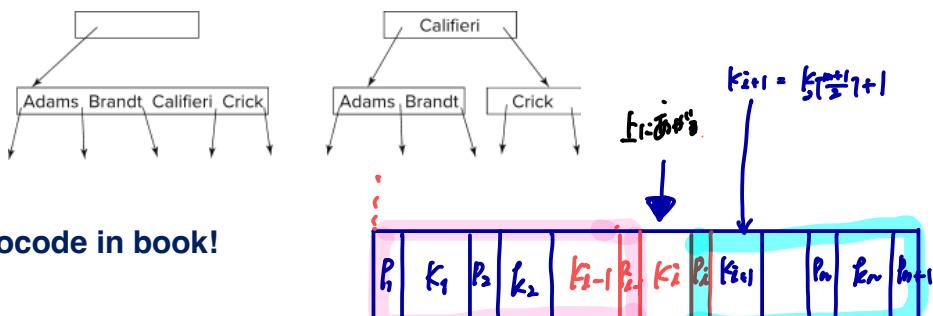
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Insertion in B+ -Trees (Cont.)

- Splitting a non-leaf node: when inserting (k,p) into an already full internal node N
 - Copy N to an in-memory area M with space for n+1 pointers and n keys
 - Insert (k,p) into M
 - Copy $P_1, K_1, \dots, K_{\lceil \frac{n+1}{2} \rceil - 1}, P_{\lceil \frac{n+1}{2} \rceil}$ from M back into node N
 - Copy $P_{\lceil \frac{n+1}{2} \rceil + 1}, K_{\lceil \frac{n+1}{2} \rceil + 1}, \dots, K_n, P_{n+1}$ from M into newly allocated node N'
 - Insert $(K_{\lceil \frac{n+1}{2} \rceil}, N')$ into parent N

Example



Read pseudocode in book!

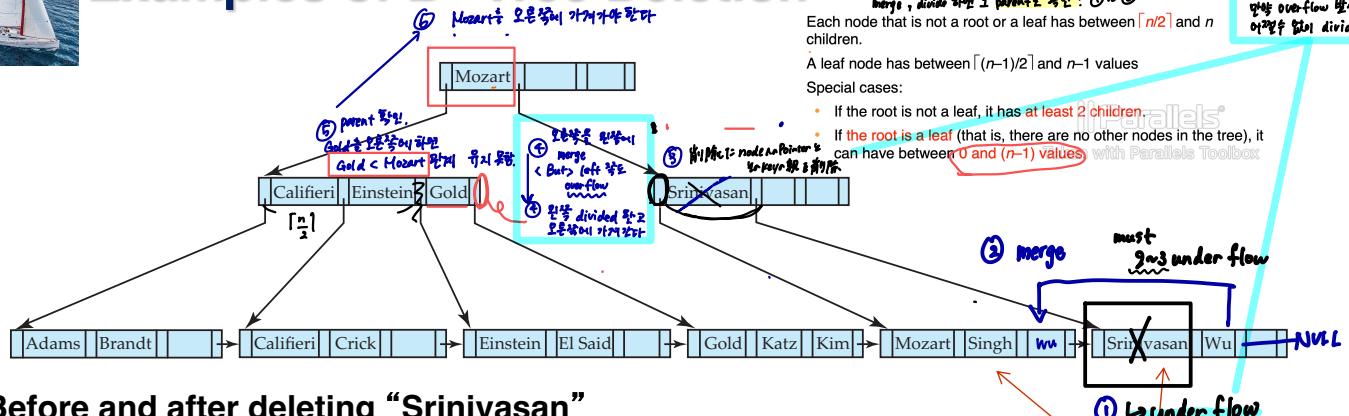
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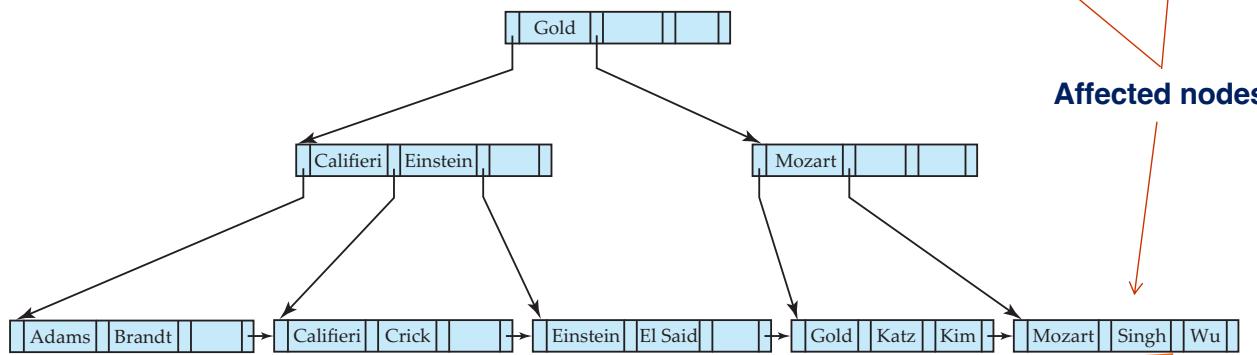
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Examples of B+-Tree Deletion



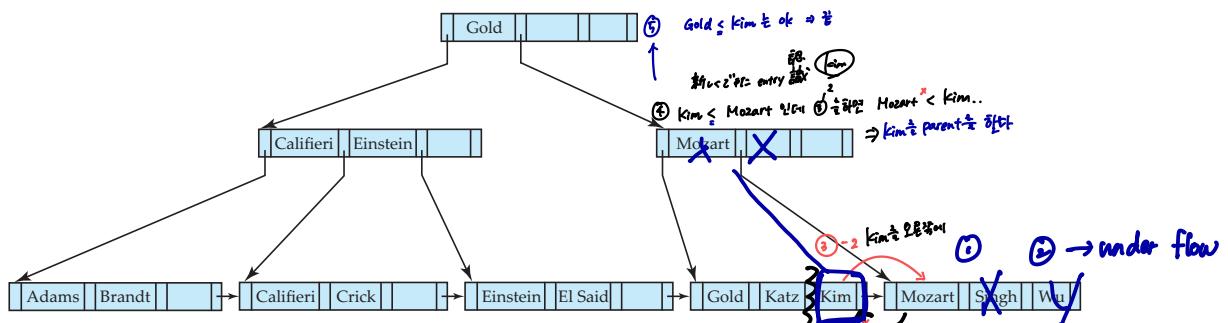
Before and after deleting “Srinivasan”



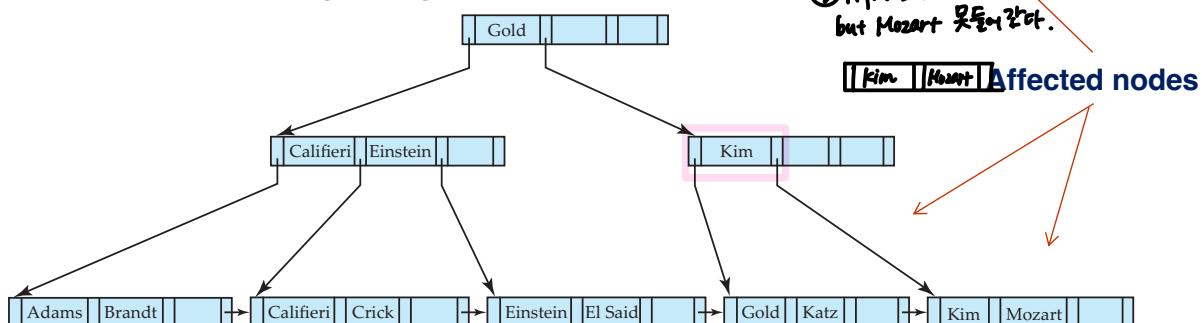
- Deleting “Srinivasan” causes merging of under-full leaves



Examples of B+-Tree Deletion (Cont.)



Before and after deleting “Singh” and “Wu”

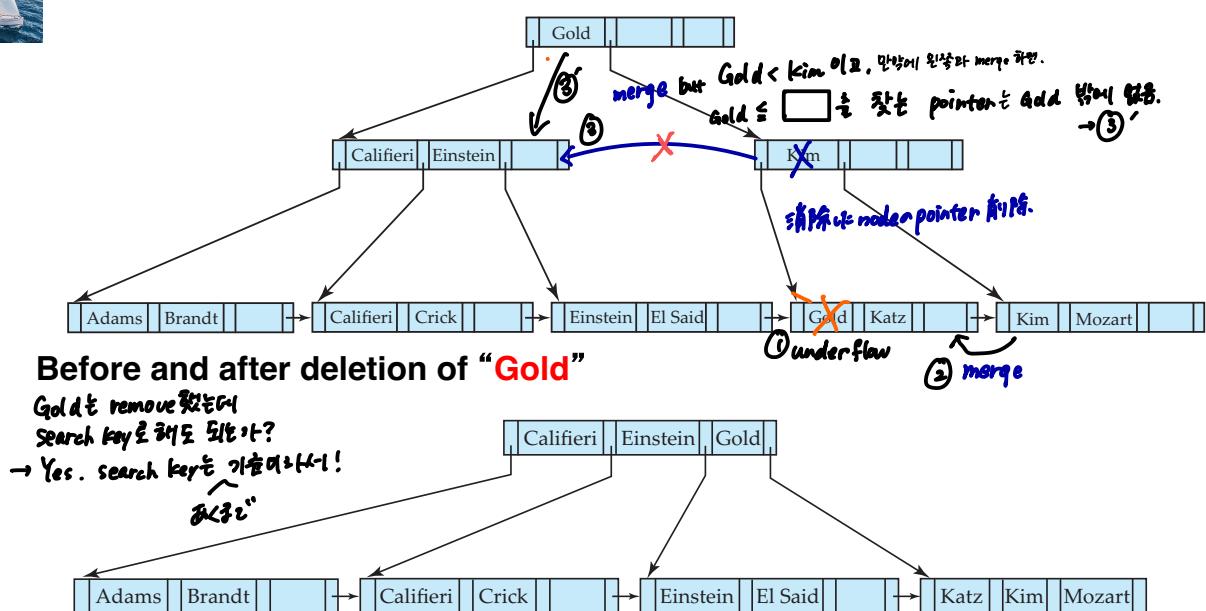


- Leaf containing 'Singh' and 'Wu' became underfull, and borrowed a value from its left sibling 'Kim'
- Search-key value in the parent changes as a result



rootnode $\lceil \frac{n-1}{2} \rceil \approx n-1 \Rightarrow 2 \text{ or } 3$

Example of B⁺-tree Deletion (Cont.)



- Node with Gold and Katz became underfull, and was merged with its sibling
- Parent node becomes underfull, and is merged with its sibling
 - Value separating two nodes (at the parent) is pulled down when merging
- Root node then has only one child, and is deleted



Updates on B⁺-Trees: Deletion

Assume record already deleted from file. Let V be the ~~delete~~ search key value of the record, and P_r be the pointer to the record.

- Remove (P_r, V) from the leaf node
- If the node has too few entries due to the removal, and the entries in the node and a sibling fit into a single node, then **merge siblings**:
 - Insert all the search-key values in the two nodes into a single node (the one on the left), and delete the other node.
 - Delete the pair (K_{i-1}, P_i) , where P_i is the pointer to the deleted node, from its parent, recursively using the above procedure.

~~1 2 3 4 5 6 7 8~~ ← 5 6 7 8 & 1 2 3 4 are merged.
2 ~ 3 keys of 5 6 7 8 → under flow ↑



Updates on B+-Trees: Deletion

- Otherwise, if the node has too few entries due to the removal, but the entries in the node and a sibling do not fit into a single node, then **redistribute pointers**:
 - Redistribute the pointers between the node and a sibling such that both have more than the minimum number of entries.
 - Update the corresponding search-key value in the parent of the node.
- The node deletions may cascade upwards till a node which has $\lceil n/2 \rceil$ or more pointers is found.
- If the root node has only one pointer after deletion, it is deleted and the sole child becomes the root.



Complexity of Updates

- Cost (in terms of number of I/O operations) of insertion and deletion of a single entry proportional to height of the tree
 - With K entries and maximum fanout of n , worst case complexity of insert/delete of an entry is $O(\log_{\lceil n/2 \rceil}(K))$ *tree depth*
- In practice, number of I/O operations is less:
 - Internal nodes tend to be in buffer
 - Splits/merges are rare, most insert/delete operations only affect a leaf node *大部分插入删除操作只影响叶节点*
- Average node occupancy depends on insertion order
 - $\frac{2}{3}$ with random, $\frac{1}{2}$ with insertion in sorted order



Non-Unique Search Keys

- Alternatives to scheme described earlier
 - Buckets on separate block (bad idea)
 - List of tuple pointers with each key
 - Extra code to handle long lists
 - Deletion of a tuple can be expensive if there are many duplicates on search key (why?)
 - Worst case complexity may be linear!
 - Low space overhead, no extra cost for queries
 - Make search key unique by adding a record-identifier
 - Extra storage overhead for keys
 - Simpler code for insertion/deletion
 - Widely used



B+-Tree File Organization

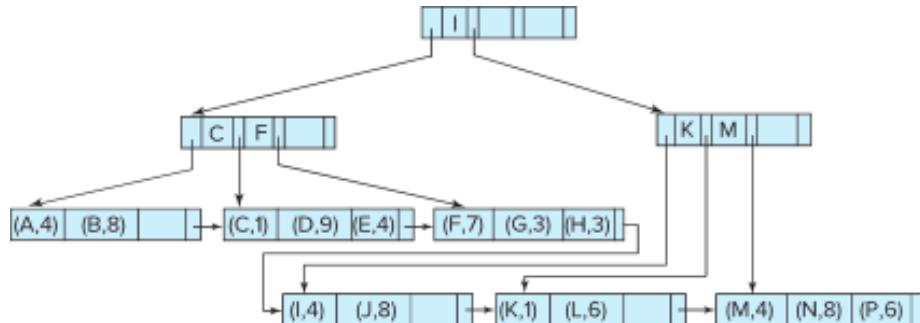
- B+-Tree File Organization:
 - Leaf nodes in a B+-tree file organization store records, instead of pointers
 - Helps keep data records clustered even when there are insertions/deletions/updates
- Leaf nodes are still required to be half full
 - Since records are larger than pointers, the maximum number of records that can be stored in a leaf node is less than the number of pointers in a nonleaf node.
- Insertion and deletion are handled in the same way as insertion and deletion of entries in a B+-tree index.



B+-Tree File Organization (Cont.)

7 u

- Example of B+-tree File Organization



- Good space utilization important since records use more space than pointers.
- To improve space utilization, involve more sibling nodes in redistribution during splits and merges
 - Involving 2 siblings in redistribution (to avoid split / merge where possible) results in each node having at least $\lfloor 2n/3 \rfloor$ entries



Other Issues in Indexing

- Record relocation and secondary indices**
 - If a record moves, all secondary indices that store record pointers have to be updated
 - Node splits in B+-tree file organizations become very expensive
 - Solution:* use search key of B+-tree file organization instead of record pointer in secondary index *for ex. name, 주소가 아닌 search key*
 - Add record-id if B+-tree file organization search key is non-unique
 - Extra traversal of file organization to locate record
- Higher cost for queries, but node splits are cheap



Indexing Strings

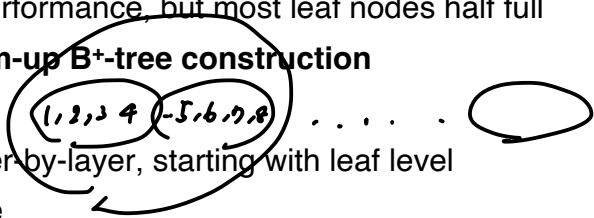
- Variable length strings as keys
 - Variable fanout
 - Use space utilization as criterion for splitting, not number of pointers
- **Prefix compression**
 - Key values at internal nodes can be prefixes of full key
 - Keep enough characters to distinguish entries in the subtrees separated by the key value
 - E.g., "Silas" and "Silberschatz" can be separated by "Silb"
 - Keys in leaf node can be compressed by sharing common prefixes



Bulk Loading and Bottom-Up Build

1
2
3
4
5
bottom up
from 3
to 2 & 3
then 1 & 2 ...
etc.

- Inserting entries one-at-a-time into a B+-tree requires ≥ 1 IO per entry
 - assuming leaf level does not fit in memory
 - can be very inefficient for loading a large number of entries at a time (**bulk loading**)
- Efficient alternative 1: *先排序后插入。即先将所有数据排序，然后按序插入。*
 - sort entries first (using efficient external-memory sort algorithms discussed later in Section 12.4)
 - insert in sorted order
 - insertion will go to existing page (or cause a split)
 - much improved IO performance, but most leaf nodes half full
- Efficient alternative 2: **Bottom-up B+-tree construction**
 - As before sort entries
 - And then create tree layer-by-layer, starting with leaf level
 - details as an exercise
 - Implemented as part of bulk-load utility by most database systems





↳ B⁺tree
B-tree 데는 ??

↳ B⁺tree < 5단계

B-Tree Index Files

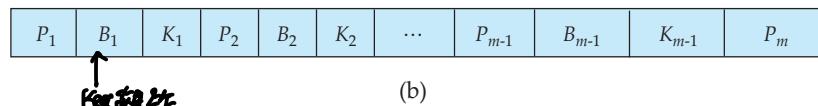
만약

binary search tree 라면

- Similar to B⁺-tree, but B-tree allows search-key values to appear only once; eliminates redundant storage of search keys.
- Search keys in nonleaf nodes appear nowhere else in the B-tree; an **additional pointer field for each search key** in a nonleaf node must be included. (b)
- Generalized B-tree leaf node (a)



(a)



↑
Key
값

(b)

- Nonleaf node – pointers B_i are the bucket or file record pointers.

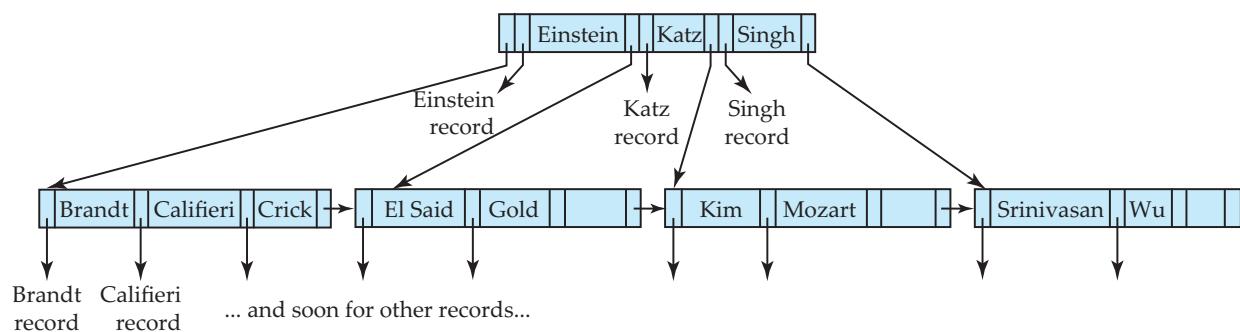


B-Tree Index Files (Cont.)

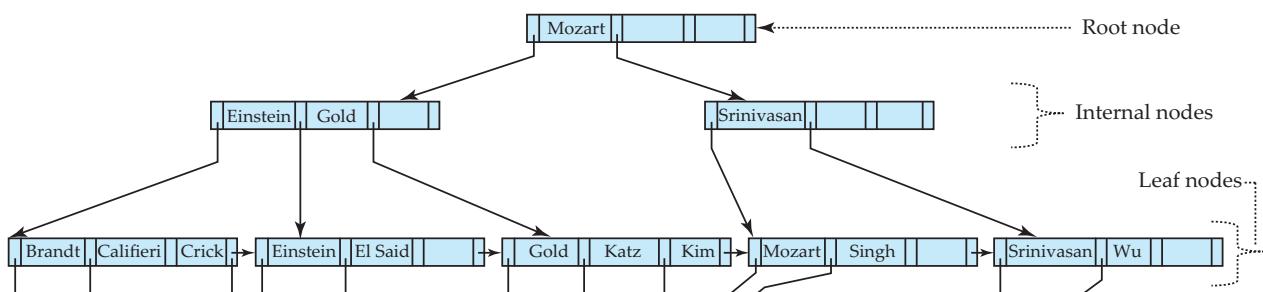
- Advantages of B-Tree indices:
 - May use less tree nodes than a corresponding B⁺-Tree.
 - Sometimes possible to find search-key value before reaching leaf node. ↗ 한 번에 찾을 수 있다.
- Disadvantages of B-Tree indices:
 - Only small fraction of all search-key values are found early
 - Non-leaf nodes are larger, so fan-out is reduced. Thus, B-Trees typically have greater depth than corresponding B⁺-Tree
 - Insertion and deletion more complicated than in B⁺-Trees
 - Implementation is harder than B⁺-Trees.
- **Typically, advantages of B-Trees do not outweigh disadvantages.**



B-Tree Index File Example



B-tree (above) and B⁺-tree (below) on same data



Indexing on Flash

- Random I/O cost much lower on flash
 - 20 to 100 microseconds for read/write
- Writes are not in-place, and (eventually) require a more expensive erase
- Optimum page size therefore much smaller
- Bulk-loading still useful since it minimizes page erases
- Write-optimized tree structures (discussed later) have been adapted to minimize page writes for flash-optimized search trees



Indexing in Main Memory

- Random access in memory
 - Much cheaper than on disk/flash
 - But still expensive compared to cache read
 - Data structures that make best use of cache preferable
 - **Binary search for a key value within a large B⁺-tree node** results in many cache misses
- B⁺- trees with small nodes that fit in cache line are preferable to reduce cache misses
- Key idea: use large node size to optimize disk access, but structure data within a node using a tree with small node size, instead of using an array.



Hashing



Static Hashing

- A **bucket** is a unit of storage containing one or more entries (a bucket is typically a disk block).
 - we obtain the bucket of an entry from its search-key value using a **hash function**
- Hash function h is a function from the set of all search-key values K to the set of all bucket addresses B .
- Hash function is used to locate entries for access, insertion as well as deletion.
- Entries with different search-key values may be mapped to the same bucket; thus entire bucket has to be searched sequentially to locate an entry.
- In a **hash index**, buckets store entries with pointers to records
- In a **hash file-organization** buckets store records



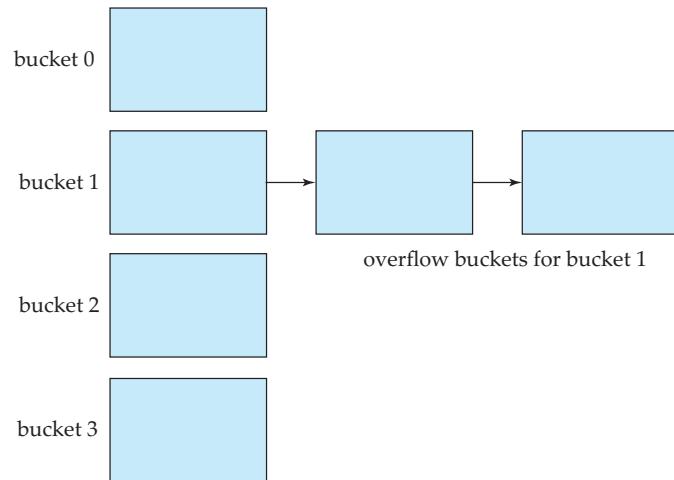
Handling of Bucket Overflows

- Bucket overflow can occur because of
 - Insufficient buckets
 - Skew in distribution of records. This can occur due to two reasons:
 - multiple records have same search-key value
 - chosen hash function produces non-uniform distribution of key values
- Although the probability of bucket overflow can be reduced, it cannot be eliminated; it is handled by using **overflow buckets**.



Handling of Bucket Overflows (Cont.)

- **Overflow chaining** – the overflow buckets of a given bucket are chained together in a linked list.
- Above scheme is called **closed addressing** (also called **closed hashing** or **open hashing** depending on the book you use)
 - An alternative, called **open addressing** (also called **open hashing** or **closed hashing** depending on the book you use) which does not use overflow buckets, is not suitable for database applications.



Example of Hash File Organization

Hash file organization of *instructor* file, using *dept_name* as key
(See figure in next slide.)

- There are 10 buckets,
- The binary representation of the *i*th character is assumed to be the integer *i*.
- The hash function returns the sum of the binary representations of the characters modulo 10
 - E.g. $h(\text{Music}) = 1 \quad h(\text{History}) = 2$
 $h(\text{Physics}) = 3 \quad h(\text{Elec. Eng.}) = 3$



Example of Hash File Organization

Hash file organization of *instructor* file, using *dept_name* as key.

bucket 0			

bucket 1			
15151	Mozart	Music	40000

bucket 2			
32343	El Said	History	80000
58583	Califieri	History	60000

bucket 3			
22222	Einstein	Physics	95000
33456	Gold	Physics	87000
98345	Kim	Elec. Eng.	80000

bucket 4			
12121	Wu	Finance	90000
76543	Singh	Finance	80000

bucket 5			
76766	Crick	Biology	72000

bucket 6			
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000

bucket 7			



Deficiencies of Static Hashing

- In static hashing, function h maps search-key values to a fixed set of B of bucket addresses. Databases grow or shrink with time.
 - If initial number of buckets is too small, and file grows, performance will degrade due to too much overflows.
 - If space is allocated for anticipated growth, a significant amount of space will be wasted initially (and buckets will be underfull).
 - If database shrinks, again space will be wasted.
- One solution: periodic re-organization of the file with a new hash function
 - Expensive, disrupts normal operations
- Better solution: allow the number of buckets to be modified dynamically.



Dynamic Hashing

- Periodic rehashing
 - If number of entries in a hash table becomes (say) 1.5 times size of hash table,
 - create new hash table of size (say) 2 times the size of the previous hash table
 - Rehash all entries to new table
- Linear Hashing
 - Do rehashing in an incremental manner
- Extendable Hashing
 - Tailored to disk based hashing, with buckets shared by multiple hash values
 - Doubling of # of entries in hash table, without doubling # of buckets



Comparison of Ordered Indexing and Hashing

- Cost of periodic re-organization
- Relative frequency of insertions and deletions
- Is it desirable to optimize average access time at the expense of worst-case access time?
- Expected type of queries:
 - Hashing is generally better at retrieving records having a specified value of the key.
 - If range queries are common, ordered indices are to be preferred
- In practice:
 - PostgreSQL supports hash indices, but discourages use due to poor performance
 - Oracle supports static hash organization, but not hash indices
 - SQLServer supports only B⁺-trees



Multiple-Key Access

- Use multiple indices for certain types of queries.
- Example:

```
select ID
      from instructor
     where dept_name = "Finance" and salary = 80000
```
- Possible strategies for processing query using indices on single attributes:
 1. Use index on *dept_name* to find instructors with department name Finance; test *salary* = 80000
 2. Use index on *salary* to find instructors with a salary of \$80000; test *dept_name* = "Finance".
 3. Use *dept_name* index to find pointers to all records pertaining to the "Finance" department. Similarly use index on *salary*. Take intersection of both sets of pointers obtained.



Indices on Multiple Keys

- **Composite search keys** are search keys containing more than one attribute
 - E.g., (*dept_name*, *salary*)
- Lexicographic ordering: $(a_1, a_2) < (b_1, b_2)$ if either
 - $a_1 < b_1$, or
 - $a_1=b_1$ and $a_2 < b_2$



Indices on Multiple Attributes

Suppose we have an index on combined search-key
(*dept_name*, *salary*).

- With the **where** clause
 - where *dept_name* = “Finance” **and** *salary* = 80000
the index on (*dept_name*, *salary*) can be used to fetch only records that satisfy both conditions.
 - Using separate indices is less efficient — we may fetch many records (or pointers) that satisfy only one of the conditions.
- Can also efficiently handle
 - where *dept_name* = “Finance” **and** *salary* < 80000
- But cannot efficiently handle
 - where *dept_name* < “Finance” **and** *balance* = 80000
 - May fetch many records that satisfy the first but not the second condition



Other Features

- **Covering indices**
 - Add extra attributes to index so (some) queries can avoid fetching the actual records
 - Store extra attributes only at leaf
 - Why?
- Particularly useful for secondary indices
 - Why?



Creation of Indices

- Example
 - **create index** *takes_pk* **on** *takes* (*ID, course_ID, year, semester, section*)
drop index *takes_pk*
- Most database systems allow specification of type of index, and clustering
- Indices on primary key created automatically by all databases
 - Why?
 - Whenever a tuple is inserted into the relation, the index can be used to check that the primary-key constraint is not violated (i.e., there are no duplicates on the primary-key value)
 - Without the index on the primary key, whenever a tuple is inserted, the entire relation has to be scanned to ensure that the primary-key constraint is satisfied.
- Some database also create indices on foreign key attributes
 - Why might such an index be useful for this query:
 - *takes* $\bowtie \sigma_{name='Shankar'}$ (*student*)
- Indices can greatly speed up lookups, but impose cost on updates
 - Index tuning assistants/wizards supported on several databases to help choose indices, based on query and update workload



Index Definition in SQL

- Create an index
 - **create index** <index-name> **on** <relation-name>
<attribute-list>
 - E.g.,: **create index** *b-index* **on** *branch*(*branch_name*)
- Use **create unique index** to indirectly specify and enforce the condition that the search key is a candidate key
- Not really required if SQL **unique** integrity constraint is supported
- To drop an index
 - **drop index** <index-name>
- Most database systems allow specification of type of index, and clustering.



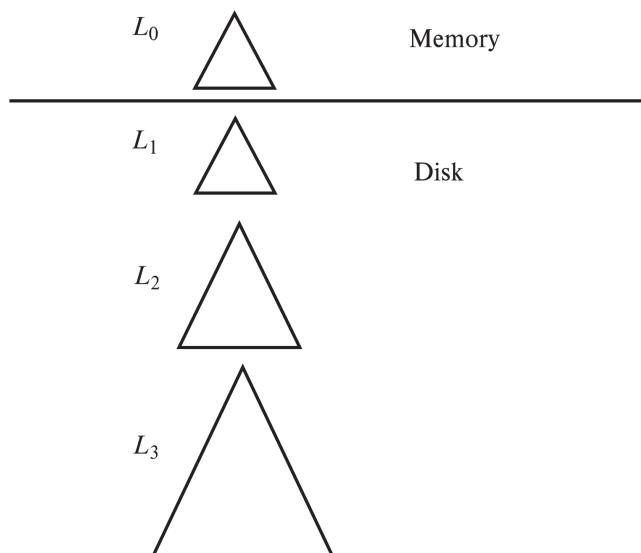
Write Optimized Indices

- Performance of B⁺-trees can be poor for write-intensive workloads
 - One I/O per leaf, assuming all internal nodes are in memory
 - With magnetic disks, < 100 inserts per second per disk
 - With flash memory, one page overwrite per insert
- Two approaches to reducing cost of writes
 - **Log-structured merge tree**
 - **Buffer tree**



Log Structured Merge (LSM) Tree

- Consider only inserts/queries for now
- Records inserted first into in-memory tree (L_0 tree)
- When in-memory tree is full, records moved to disk (L_1 tree)
 - B⁺-tree constructed using bottom-up build by merging existing L_1 tree with records from L_0 tree
- When L_1 tree exceeds some threshold, merge into L_2 tree
 - And so on for more levels
 - Size threshold for L_{i+1} tree is k times size threshold for L_i tree





LSM Tree (Cont.)

- Benefits of LSM approach
 - Inserts are done using only sequential I/O operations
 - Leaves are full, avoiding space wastage
 - Reduced number of I/O operations per record inserted as compared to normal B⁺-tree (up to some size)
- Drawback of LSM approach
 - Queries have to search multiple trees
 - Entire content of each level copied multiple times
- Stepped-merge index
 - Variant of LSM tree with multiple trees at each level
 - Reduces write cost compared to LSM tree
 - But queries are even more expensive
 - Bloom filters to avoid lookups in most trees
- Details are covered in Chapter 24



LSM Trees (Cont.)

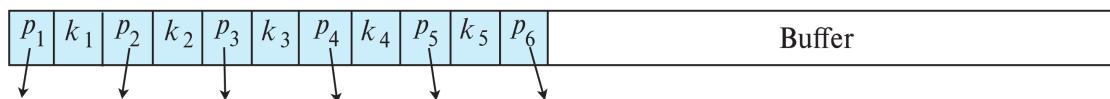
- Deletion handled by adding special “delete” entries
 - Lookups will find both original entry and the delete entry, and must return only those entries that do not have matching delete entry
 - When trees are merged, if we find a delete entry matching an original entry, both are dropped.
- Update handled using insert+delete
- LSM trees were introduced for disk-based indices
 - But useful to minimize erases with flash-based indices
 - The stepped-merge variant of LSM trees is used in many BigData storage systems
 - Google BigTable, Apache Cassandra, MongoDB
 - And more recently in SQLite4, LevelDB, and MyRocks storage engine of MySQL



Buffer Tree

- Alternative to LSM tree
- Key idea: each internal node of B⁺-tree has a buffer to store inserts
 - Inserts are moved to lower levels when buffer is full
 - With a large buffer, many records are moved to lower level each time
 - Per record I/O decreases correspondingly
- Benefits
 - Less overhead on queries
 - Can be used with any tree index structure
 - Used in PostgreSQL Generalized Search Tree (GiST) indices
- Drawback: more random I/O than LSM tree

Internal node



Bitmap Indices

- Bitmap indices are a special type of index designed for efficient querying on multiple keys
- Records in a relation are assumed to be numbered sequentially from, say, 0
 - Given a number n it must be easy to retrieve record n
 - Particularly easy if records are of fixed size
- Applicable on attributes that take on a relatively small number of distinct values
 - E.g., gender, country, state, ...
 - E.g., income-level (income broken up into a small number of levels such as 0-9999, 10000-19999, 20000-50000, 50000- infinity)
- A bitmap is simply an array of bits



Bitmap Indices (Cont.)

- In its simplest form a bitmap index on an attribute has a bitmap for each value of the attribute
 - Bitmap has as many bits as records
 - In a bitmap for value v, the bit for a record is 1 if the record has the value v for the attribute, and is 0 otherwise
- Example

record number	ID	gender	income_level
0	76766	m	L1
1	22222	f	L2
2	12121	f	L1
3	15151	m	L4
4	58583	f	L3

Bitmaps for gender		Bitmaps for income_level	
m	10010	L1	10100
f	01101	L2	01000
		L3	00001
		L4	00010
		L5	00000



Bitmap Indices (Cont.)

- Bitmap indices are useful for queries on multiple attributes
 - not particularly useful for single attribute queries
- Queries are answered using bitmap operations
 - Intersection (and)
 - Union (or)
- Each operation takes two bitmaps of the same size and applies the operation on corresponding bits to get the result bitmap
 - E.g., $100110 \text{ AND } 110011 = 100010$
 $100110 \text{ OR } 110011 = 110111$
 $\text{NOT } 100110 = 011001$
 - Males with income level L1: $10010 \text{ AND } 10100 = 10000$
 - Can then retrieve required tuples.
 - Counting number of matching tuples is even faster



Bitmap Indices (Cont.)

- Bitmap indices generally very small compared with relation size
 - E.g., if record is 100 bytes, space for a single bitmap is 1/800 of space used by relation.
 - If number of distinct attribute values is 8, bitmap is only 1% of relation size



Efficient Implementation of Bitmap Operations

- Bitmaps are packed into words; a single word and (a basic CPU instruction) computes and of 32 or 64 bits at once
 - E.g., 1-million-bit maps can be and-ed with just 31,250 instruction
- Counting number of 1s can be done fast by a trick:
 - Use each byte to index into a precomputed array of 256 elements each storing the count of 1s in the binary representation
 - Can use pairs of bytes to speed up further at a higher memory cost
 - Add up the retrieved counts
- Bitmaps can be used instead of Tuple-ID lists at leaf levels of B⁺-trees, for values that have a large number of matching records
 - Worthwhile if > 1/64 of the records have that value, assuming a tuple-id is 64 bits
 - Above technique merges benefits of bitmap and B⁺-tree indices



Spatial and Temporal Indices



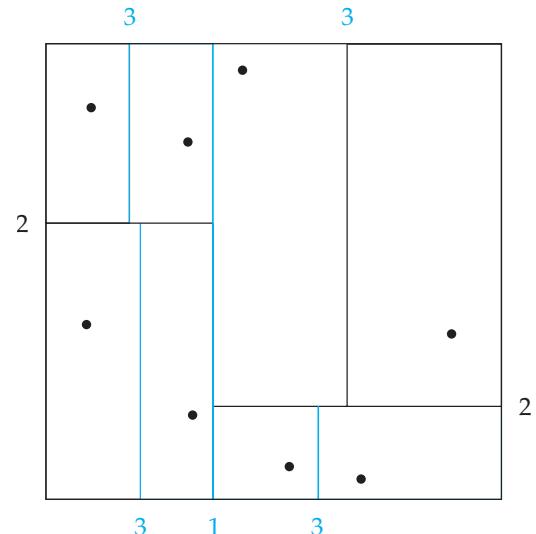
Spatial Data

- Databases can store data types such as lines, polygons, in addition to raster images
 - allows relational databases to store and retrieve spatial information
 - Queries can use spatial conditions (e.g. contains or overlaps).
 - queries can mix spatial and nonspatial conditions
- **Nearest neighbor queries**, given a point or an object, find the nearest object that satisfies given conditions.
- **Range queries** deal with spatial regions. e.g., ask for objects that lie partially or fully inside a specified region.
- Queries that compute intersections or **unions** of regions.
- **Spatial join** of two spatial relations with the location playing the role of join attribute.



Indexing of Spatial Data

- **k-d tree** - early structure used for indexing in multiple dimensions.
- Each level of a *k-d* tree partitions the space into two.
 - Choose one dimension for partitioning at the root level of the tree.
 - Choose another dimensions for partitioning in nodes at the next level and so on, cycling through the dimensions.
- In each node, approximately half of the points stored in the sub-tree fall on one side and half on the other.
- Partitioning stops when a node has less than a given number of points.

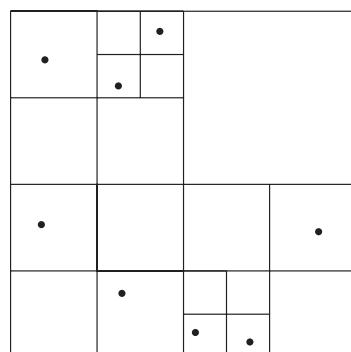


- The **k-d-B tree** extends the *k-d* tree to allow multiple child nodes for each internal node; well-suited for secondary storage.



Division of Space by Quadtrees

- Each node of a quadtree is associated with a rectangular region of space; the top node is associated with the entire target space.
- Each non-leaf nodes divides its region into four equal sized quadrants
 - correspondingly each such node has four child nodes corresponding to the four quadrants and so on
- Leaf nodes have between zero and some fixed maximum number of points (set to 1 in example).





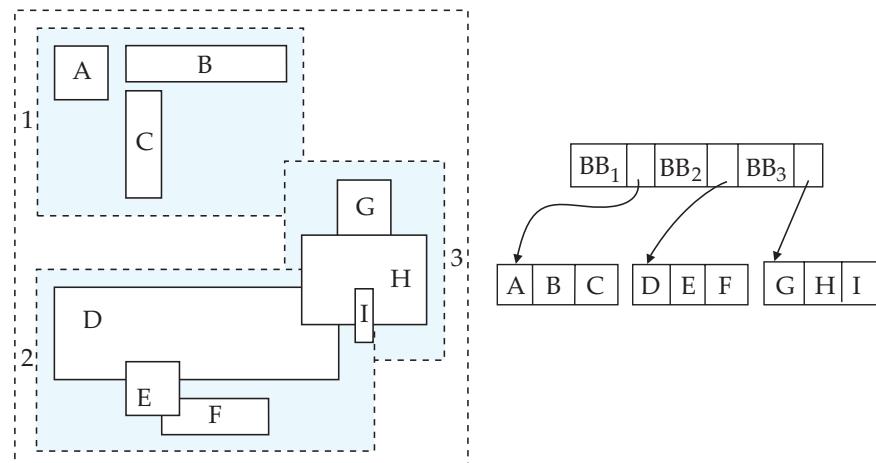
R-Trees

- **R-trees** are a N-dimensional extension of B+-trees, useful for indexing sets of rectangles and other polygons.
- Supported in many modern database systems, along with variants like R⁺-trees and R^{*}-trees.
- Basic idea: generalize the notion of a one-dimensional interval associated with each B+ -tree node to an N-dimensional interval, that is, an N-dimensional rectangle.
- Will consider only the two-dimensional case ($N = 2$)
 - generalization for $N > 2$ is straightforward, although R-trees work well only for relatively small N
- The **bounding box** of a node is a minimum sized rectangle that contains all the rectangles/polygons associated with the node
 - *Bounding boxes of children of a node are allowed to overlap*



Example R-Tree

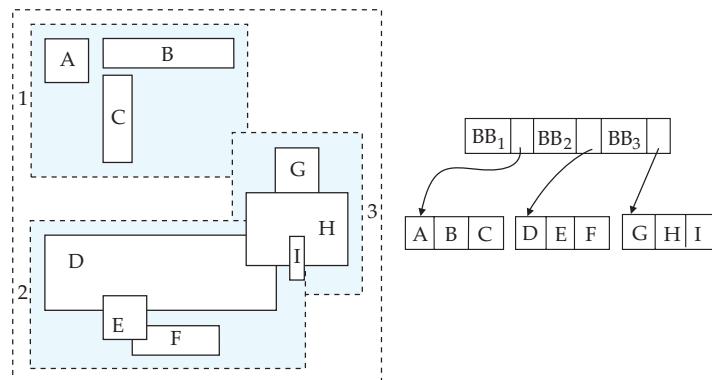
- A set of rectangles (solid line) and the bounding boxes (dashed line) of the nodes of an R-tree for the rectangles.
- The R-tree is shown on the right.





Search in R-Trees

- To find data items intersecting a given query point/region, do the following, starting from the root node:
 - If the node is a leaf node, output the data items whose keys intersect the given query point/region.
 - Else, for each child of the current node whose bounding box intersects the query point/region, recursively search the child
- Can be very inefficient in worst case since multiple paths may need to be searched, but works acceptably in practice.



Indexing Temporal Data

- Temporal data refers to data that has an associated time period (interval)
 - Example: a temporal version of the *course* relation
- Time interval has a start and end time
 - End time set to infinity (or large date such as 9999-12-31) if a tuple is currently valid and its validity end time is not currently known
- Query may ask for all tuples that are valid at a point in time or during a time interval
 - Index on valid time period speeds up this task

course_id	title	dept_name	credits	start	end
BIO-101	Intro. to Biology	Biology	4	1985-01-01	9999-12-31
CS-201	Intro. to C	Comp. Sci.	4	1985-01-01	1999-01-01
CS-201	Intro. to Java	Comp. Sci.	4	1999-01-01	2010-01-01
CS-201	Intro. to Python	Comp. Sci.	4	2010-01-01	9999-12-31



Indexing Temporal Data (Cont.)

- To create a temporal index on attribute a :
 - Use spatial index, such as R-tree, with attribute a as one dimension, and time as another dimension
 - Valid time forms an interval in the time dimension
 - Tuples that are currently valid cause problems, since value is infinite or very large
 - Solution: store all current tuples (with end time as infinity) in a separate index, indexed on $(a, start-time)$
 - To find tuples valid at a point in time t in the current tuple index, search for tuples in the range $(a, 0)$ to (a, t)
- Temporal index on primary key can help enforce temporal primary key constraint

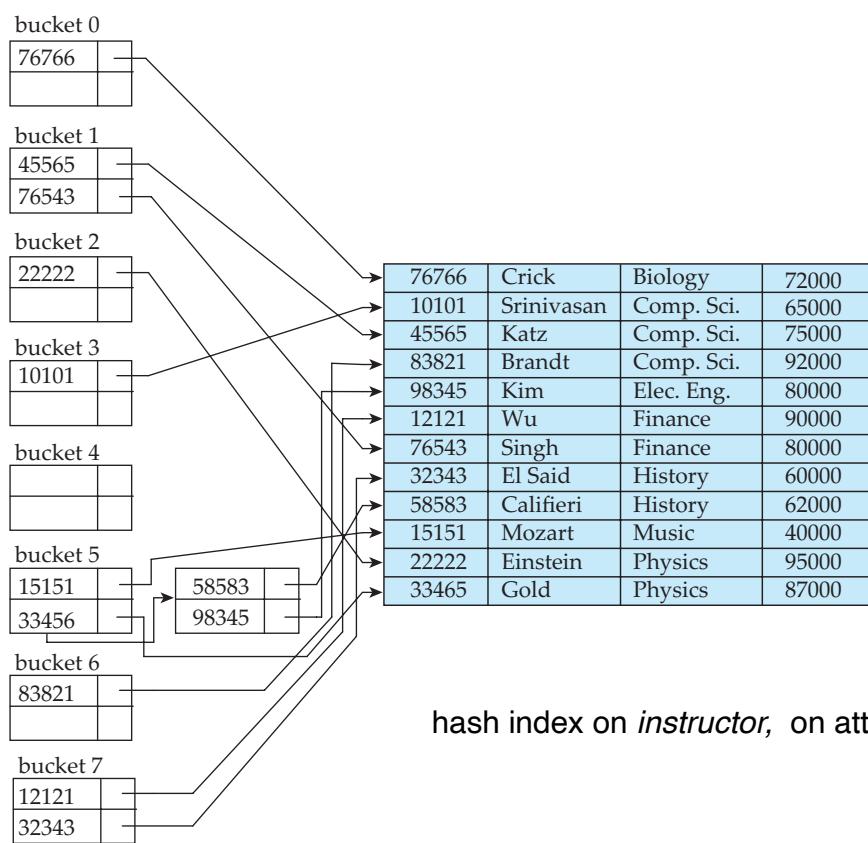
course_id	title	dept_name	credits	start	end
BIO-101	Intro. to Biology	Biology	4	1985-01-01	9999-12-31
CS-201	Intro. to C	Comp. Sci.	4	1985-01-01	1999-01-01
CS-201	Intro. to Java	Comp. Sci.	4	1999-01-01	2010-01-01
CS-201	Intro. to Python	Comp. Sci.	4	2010-01-01	9999-12-31



End of Chapter 14



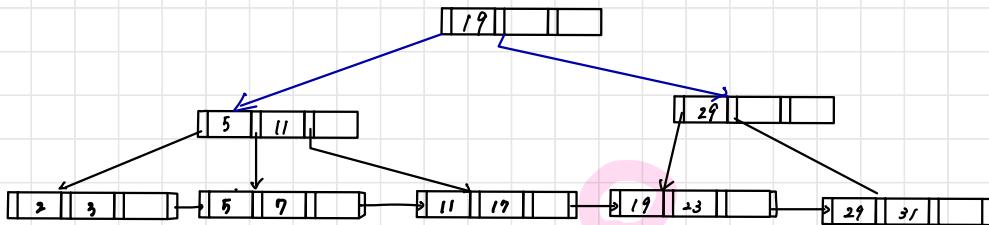
Example of Hash Index



14-3. (2, 3, 5, 7, 11, 17, 19, 23, 29, 31)

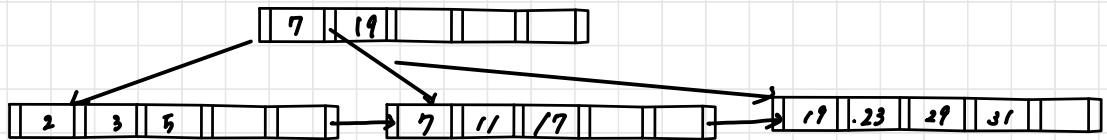
a: node 4 $\Rightarrow \lceil \frac{n}{2} \rceil \sim n = 2 \sim 4$ child

$$\lceil \frac{n-1}{2} \rceil \sim (n-1) = 2 \sim 3 \text{ leaf}$$

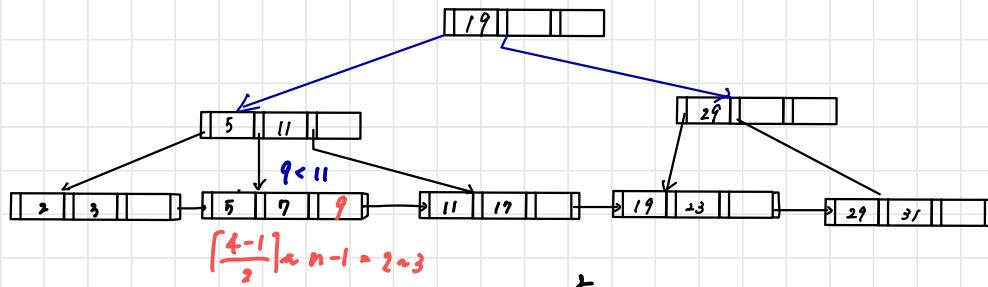


b: node 6 child $\lceil \frac{n}{2} \rceil \sim n = 3 \sim 6$ $6 \div 2 = 2 \sim 2$

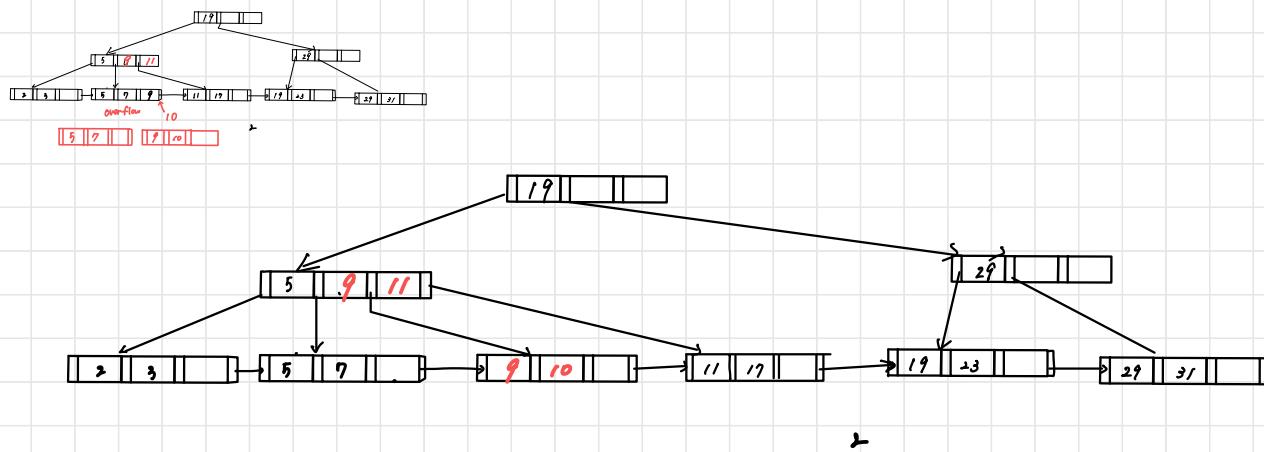
$$r \quad \text{node } \lceil \frac{n-1}{2} \rceil \sim n-1 = \lceil \frac{6-1}{2} \rceil \sim 5 = 3 \sim 5$$



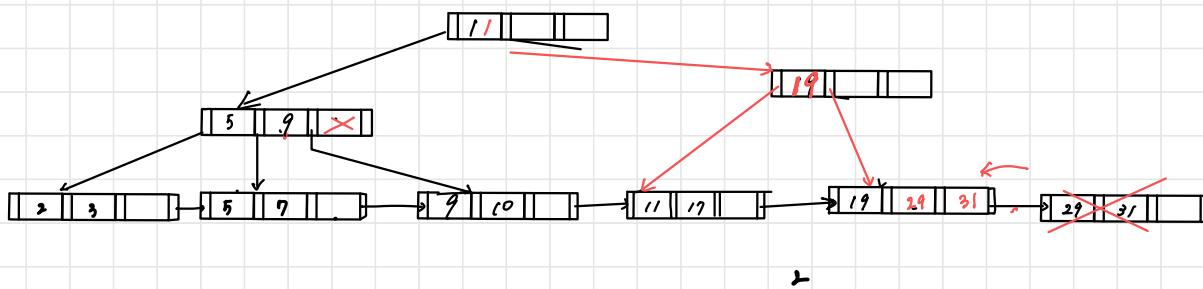
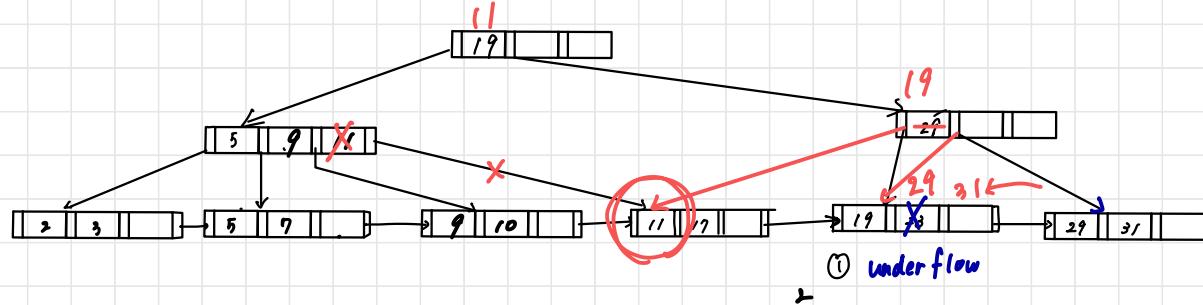
14-4 insert 9

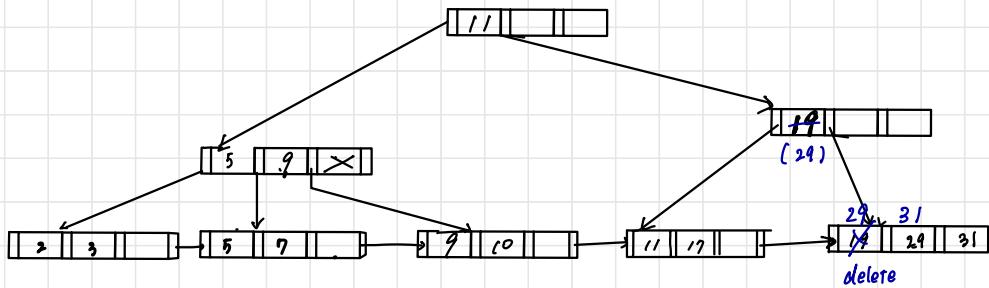


b. insert 10

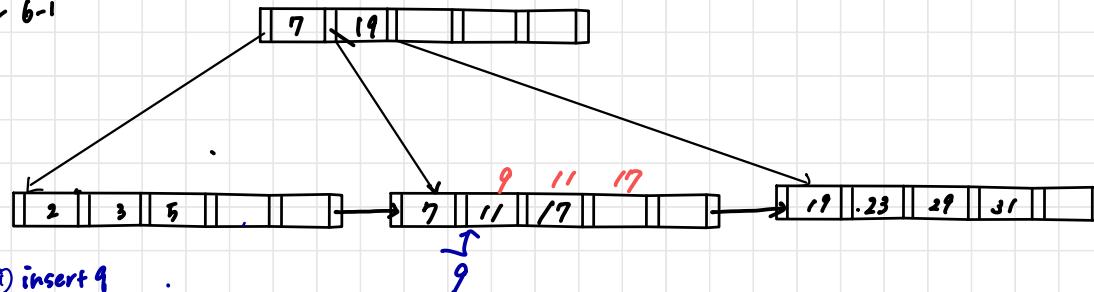


delete 23





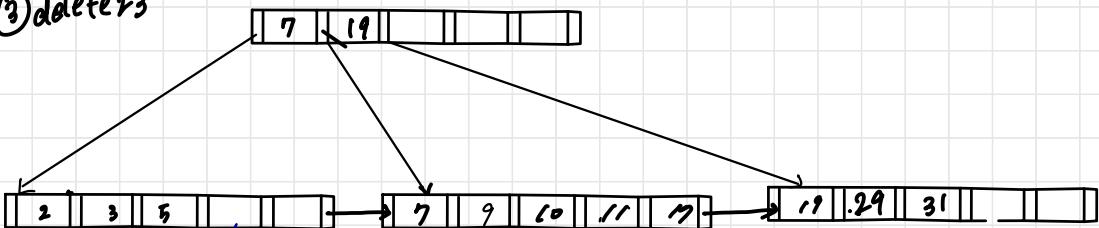
$$\begin{aligned}
 \text{node } 6 \\
 \left\lceil \frac{n-1}{2} \right\rceil &\sim 6-1 \\
 &= 3 \sim 5
 \end{aligned}$$



① insert 9

② insert 10

③ delete 23



④ delete 19

