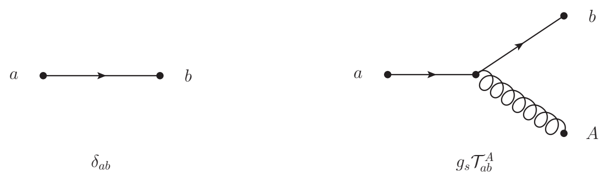

Fig. 2.5 Free quark-field and quark-gluon interaction term.

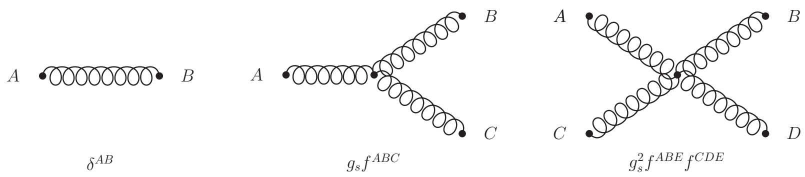

Fig. 2.6 Free gluon-field and cubic and quartic gluon self-interaction terms.

given by $C_F = 4/3$, $T_F = 1/2$, and $C_A = 3$. The ratio of a gluon emission by a gluon relative to the emission by a quark is therefore approximately $C_A/C_F = 9/4 = 2.25$, i.e. gluons radiate stronger than quarks by more than a factor of two. Similarly, gluons split into a gluon pair more often than into a quark-antiquark pair by roughly a factor of $C_A/T_F = 6$.

Of course, this classical QCD Lagrangian exhibits the property of local gauge invariance, i.e. invariance under a simultaneous redefinition of the quark and gluon fields. As a consequence of this internal symmetry, it is impossible to define the gluon field propagator without explicitly specifying a choice of gauge. A popular choice is given as a generalisation of the covariant Lorentz gauge $\partial^\mu \mathcal{A}^A_\mu = 0$ by the class of R_ξ gauges, imposed by adding the term

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{2\xi}\left(\partial^\mu \mathcal{A}^A_\mu\right)^2 \tag{2.10}$$

to the classical Lagrangian. Following L.D. Faddeev and V.N. Popov [53] this must be accompanied by the ghost term

$$\mathcal{L}_{\text{ghost}} = \partial_\mu \eta^{A\dagger}\left(\mathcal{D}^\mu_{AB}\eta^B\right) \tag{2.11}$$

because of the non-Abelian character of the QCD gauge group. The ghosts η^A, with conjugate-transpose $\eta^{A\dagger}$, represent complex scalar fields that nevertheless obey Fermi–Dirac statistics. They do not have a physical meaning, but should be considered as a mathematical trick to cancel nonphysical degrees of freedom otherwise present in calculations with covariant gauges.

This completes the Lagrangian for a consistent QFT of the strong interaction. Further invariant terms to add to the QCD Lagrangian are conceivable in principle. Renormalisability, however, forbids all additions that require coefficients with nega-