

# Fortcoders Code Library

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# Contents

<b>Intro</b>	<b>2</b>
Main template . . . . .	2
Fast IO . . . . .	2
Pragmas (lol) . . . . .	2
<b>Data Structures</b>	<b>2</b>
Segment Tree . . . . .	2
Recursive . . . . .	2
Iterating . . . . .	3
Union Find . . . . .	4
Fenwick Tree . . . . .	4
Fenwick2D Tree . . . . .	4
PBDS . . . . .	5
Treap . . . . .	5
Implicit treap . . . . .	6
Persistent implicit treap . . . . .	6
2D Sparse Table . . . . .	6
K-D Tree . . . . .	7
Link/Cut Tree . . . . .	7
<b>Geometry</b>	<b>8</b>
Basic stuff . . . . .	8
Transformation . . . . .	8
Relation . . . . .	9
Area . . . . .	10
Convex . . . . .	10
Basic 3D . . . . .	11
Miscellaneous . . . . .	12
<b>Graph Theory</b>	<b>12</b>
Max Flow . . . . .	12
PushRelabel Max-Flow (faster) . . . . .	12
Min-Cost Max-Flow . . . . .	13
Max Cost Feasible Flow . . . . .	13
Heavy-Light Decomposition . . . . .	14
General Unweight Graph Matching . . . . .	14
Maximum Bipartite Matching . . . . .	15
2-SAT and Strongly Connected Components . . . . .	15
Enumerating Triangles . . . . .	15
Tarjan . . . . .	15
Kruskal reconstruct tree . . . . .	16
<b>Math</b>	<b>16</b>
Inverse . . . . .	16
Mod Class . . . . .	16
<b>Cancer mod class</b>	<b>17</b>
NTT, FFT, FWT . . . . .	17
Polynomial Class . . . . .	18
Sieve . . . . .	19
Gaussian Elimination . . . . .	20
is_prime . . . . .	20
Radix Sort . . . . .	20
lucas . . . . .	21
parity of n choose m . . . . .	21
sosdp . . . . .	21
<b>String</b>	<b>21</b>
AC Automaton . . . . .	21
KMP . . . . .	22

Z function . . . . .	22
General Suffix Automaton . . . . .	22
Manacher . . . . .	22
Lyndon . . . . .	22
minimal representation . . . . .	23

# Intro

## Main template

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define FOR(x,n) for(int x=0;x<n;x++)
5 #define forn(i, n) for (int i = 0; i < int(n); i++)
6 #define all(v) v.begin(),v.end()
7 using ll = long long;
8 using ld = long double;
9 using pii = pair<int, int>;
10 const char nl = '\n';
11
12 int main() {
13     cin.tie(nullptr)->sync_with_stdio(false);
14     cout << fixed << setprecision(20);
15     // mt19937
16     ↪ rng(chrono::steady_clock::now().time_since_epoch().count());
17 }
```

## Fast IO

```
1 namespace io {
2 constexpr int SIZE = 1 << 16;
3 char buf[SIZE], *head, *tail;
4 char get_char() {
5     if (head == tail) tail = (head = buf) + fread(buf, 1, SIZE,
6     ↪ stdin);
7     return *head++;
8 }
9 ll read() {
10     ll x = 0, f = 1;
11     char c = get_char();
12     for (; !isdigit(c); c = get_char()) (c == '-') && (f = -1);
13     for (; isdigit(c); c = get_char()) x = x * 10 + c - '0';
14     return x * f;
15 }
16 string read_s() {
17     string str;
18     char c = get_char();
19     while (c == ' ' || c == '\n' || c == '\r') c = get_char();
20     while (c != ' ' && c != '\n' && c != '\r') str += c, c =
21     ↪ get_char();
22     return str;
23 }
24 void print(int x) {
25     if (x > 9) print(x / 10);
26     putchar(x % 10 | '0');
27 }
28 void println(int x) { print(x), putchar('\n'); }
29 struct Read {
30     Read& operator>>(ll& x) { return x = read(), *this; }
31     Read& operator>>(long double& x) { return x =
32     ↪ stold(read_s()), *this; }
33 } in;
34 } // namespace io
```

## Pragmas (lol)

```
1 #pragma GCC optimize(2)
2 #pragma GCC optimize(3)
3 #pragma GCC optimize("Ofast")
4 #pragma GCC optimize("inline")
5 #pragma GCC optimize("-fgcse")
6 #pragma GCC optimize("-fgcse-lm")
7 #pragma GCC optimize("-fipa-sra")
8 #pragma GCC optimize("-ftree-pre")
9 #pragma GCC optimize("-ftree-vcv2")
10 #pragma GCC optimize("-fpeephole2")
11 #pragma GCC optimize("-ffast-math")
12 #pragma GCC optimize("-fsched-spec")
13 #pragma GCC optimize("unroll-loops")
14 #pragma GCC optimize("-falign-jumps")
15 #pragma GCC optimize("-falign-loops")
```

```
16 #pragma GCC optimize("-falign-labels")
17 #pragma GCC optimize("-fdevirtualize")
18 #pragma GCC optimize("-fcaller-saves")
19 #pragma GCC optimize("-fcrossjumping")
20 #pragma GCC optimize("-fthread-jumps")
21 #pragma GCC optimize("-funroll-loops")
22 #pragma GCC optimize("-fwhole-program")
23 #pragma GCC optimize("-freorder-blocks")
24 #pragma GCC optimize("-fschedule-insns")
25 #pragma GCC optimize("inline-functions")
26 #pragma GCC optimize("-ftree-tail-merge")
27 #pragma GCC optimize("-fschedule-insns2")
28 #pragma GCC optimize("-fstrict-aliasing")
29 #pragma GCC optimize("-fstrict-overflow")
30 #pragma GCC optimize("-falign-functions")
31 #pragma GCC optimize("-fcse-skip-blocks")
32 #pragma GCC optimize("-fcse-follow-jumps")
33 #pragma GCC optimize("-fsched-interblock")
34 #pragma GCC optimize("-fpartial-inlining")
35 #pragma GCC optimize("no-stack-protector")
36 #pragma GCC optimize("-freorder-functions")
37 #pragma GCC optimize("-findirect-inlining")
38 #pragma GCC optimize("-fhoist-adjacent-loads")
39 #pragma GCC optimize("-frerun-cse-after-loop")
40 #pragma GCC optimize("inline-small-functions")
41 #pragma GCC optimize("-finline-small-functions")
42 #pragma GCC optimize("-ftree-switch-conversion")
43 #pragma GCC optimize("-foptimize-sibling-calls")
44 #pragma GCC optimize("-fexpensive-optimizations")
45 #pragma GCC optimize("-funsafe-loop-optimizations")
46 #pragma GCC optimize("inline-functions-called-once")
47 #pragma GCC optimize("-fdelete-null-pointer-checks")
48 #pragma GCC
49     ↪ target("sse,sse2,sse3,ssse3,sse4.1,sse4.2,avx,avx2,popcnt,tune=na
```

## Data Structures

### Segment Tree

#### Recursive

- Implicit segment tree, range query + point update

```
1 struct Node {
2     int lc, rc, p;
3 };
4
5 struct SegTree {
6     vector<Node> t = {};
7     SegTree(int n) { t.reserve(n * 40); }
8     int modify(int p, int l, int r, int x, int v) {
9         int u = p;
10        if (p == 0) {
11            t.push_back(t[p]);
12            u = (int)t.size() - 1;
13        }
14        if (r - l == 1) {
15            t[u].p = t[p].p + v;
16        } else {
17            int m = (l + r) / 2;
18            if (x < m) {
19                t[u].lc = modify(t[p].lc, l, m, x, v);
20            } else {
21                t[u].rc = modify(t[p].rc, m, r, x, v);
22            }
23            t[u].p = t[t[u].lc].p + t[t[u].rc].p;
24        }
25        return u;
26    }
27    int query(int p, int l, int r, int x, int y) {
28        if (x <= l && r <= y) return t[p].p;
29        int m = (l + r) / 2, res = 0;
30        if (x < m) res += query(t[p].lc, l, m, x, y);
31        if (y > m) res += query(t[p].rc, m, r, x, y);
32        return res;
33    }
34 }
```

```
34 };
```

- Persistent implicit, range query + point update

```
1 struct Node {
2     int lc = 0, rc = 0, p = 0;
3 };
4
5 struct SegTree {
6     vector<Node> t = {{}}; // init all
7     SegTree() = default;
8     SegTree(int n) { t.reserve(n * 20); }
9     int modify(int p, int l, int r, int x, int v) {
10         // p: original node, update a[x] -> v
11         t.push_back(t[p]);
12         int u = (int)t.size() - 1;
13         if (r - l == 1) {
14             t[u].p = v;
15         } else {
16             int m = (l + r) / 2;
17             if (x < m) {
18                 t[u].lc = modify(t[p].lc, l, m, x, v);
19                 t[u].rc = t[p].rc;
20             } else {
21                 t[u].lc = t[p].lc;
22                 t[u].rc = modify(t[p].rc, m, r, x, v);
23             }
24             t[u].p = t[t[u].lc].p + t[t[u].rc].p;
25         }
26         return u;
27     }
28     int query(int p, int l, int r, int x, int y) {
29         // query sum a[x]...a[y-1] rooted at p
30         // t[p] holds the info of [l, r)
31         if (x <= l && r <= y) return t[p].p;
32         int m = (l + r) / 2, res = 0;
33         if (x < m) res += query(t[p].lc, l, m, x, y);
34         if (y > m) res += query(t[p].rc, m, r, x, y);
35         return res;
36     }
37 };
```

## Iterating

- Iterating, range query + point update

```
1 struct Node {
2     ll v = 0, init = 0;
3 };
4
5 Node pull(const Node &a, const Node &b) {
6     if (!a.init) return b;
7     if (!b.init) return a;
8     Node c;
9     return c;
10 }
11
12 struct SegTree {
13     ll n;
14     vector<Node> t;
15     SegTree(ll _n) : n(_n), t(2 * _n){};
16     void modify(ll p, const Node &v) {
17         t[p += n] = v;
18         for (p /= 2; p; p /= 2) t[p] = pull(t[p * 2], t[p * 2 +
19         1]);
20     }
21     Node query(ll l, ll r) {
22         Node left, right;
23         for (l += n, r += n; l < r; l /= 2, r /= 2) {
24             if (l & 1) left = pull(left, t[l++]);
25             if (r & 1) right = pull(t[--r], right);
26         }
27         return pull(left, right);
28 };
```

- Iterating, range query + range update

```
1 struct Node {
2     ll v = 0;
3 };
4 struct Tag {
5     ll v = 0;
6 };
7 Node pull(const Node& a, const Node& b) { return {max(a.v,
8     b.v)}; }
9 Tag pull(const Tag& a, const Tag& b) { return {a.v + b.v}; }
10 Node apply_tag(const Node& a, const Tag& b) { return {a.v +
11     b.v}; }
12
13 struct SegTree {
14     ll n, h;
15     vector<Node> t;
16     vector<Tag> lazy;
17     SegTree(ll _n) : n(_n), h((ll)log2(n)), t(2 * _n), lazy(2 *
18     _n) {}
19     void apply(ll x, const Tag& tag) {
20         t[x] = apply_tag(t[x], tag);
21         lazy[x] = pull(lazy[x], tag);
22     }
23     void build(ll l) {
24         for (l = (l + n) / 2; l > 0; l /= 2) {
25             if (!lazy[l].v) t[l] = pull(t[l * 2], t[2 * l + 1]);
26         }
27     }
28     void push(ll l) {
29         l += n;
30         for (ll s = h; s > 0; s--) {
31             ll i = l >> s;
32             if (lazy[i].v) {
33                 apply(2 * i, lazy[i]);
34                 apply(2 * i + 1, lazy[i]);
35             }
36             lazy[i] = Tag();
37         }
38     }
39     void modify(ll l, ll r, const Tag& v) {
40         push(l), push(r - 1);
41         ll l0 = l, r0 = r;
42         for (l += n, r += n; l < r; l /= 2, r /= 2) {
43             if (l & 1) apply(l++, v);
44             if (r & 1) apply(--r, v);
45         }
46         build(l0), build(r0 - 1);
47     }
48     Node query(ll l, ll r) {
49         push(l), push(r - 1);
50         Node left, right;
51         for (l += n, r += n; l < r; l /= 2, r /= 2) {
52             if (l & 1) left = pull(left, t[l++]);
53             if (r & 1) right = pull(t[--r], right);
54         }
55         return pull(left, right);
56     }
57 };
```

- AtCoder Segment Tree (recursive structure but iterative)

```
1 template <class T> struct PointSegmentTree {
2     int size = 1;
3     vector<T> tree;
4     PointSegmentTree(int n) : PointSegmentTree(vector<T>(n)) {}
5     PointSegmentTree(vector<T>& arr) {
6         while(size < (int)arr.size())
7             size <<= 1;
8         tree = vector<T>(size << 1);
9         for(int i = size + arr.size() - 1; i >= 1; i--)
10             if(i >= size) tree[i] = arr[i - size];
11             else consume(i);
12     }
13     void set(int i, T val) {
14         tree[i += size] = val;
15         for(i >>= 1; i >= 1; i >>= 1)
16             consume(i);
17     }
18     T get(int i) { return tree[i + size]; }
```

```

19 T query(int l, int r) {
20     T resl, resr;
21     for(l += size, r += size + 1; l < r; l >>= 1, r >>= 1) {
22         if(l & 1) resl = resl * tree[l++];
23         if(r & 1) resr = tree[--r] * resr;
24     }
25     return resl * resr;
26 }
27 T query_all() { return tree[1]; }
28 void consume(int i) { tree[i] = tree[i << 1] * tree[i << 1 |
↪ 1]; }
29 };
30
31
32 struct SegInfo {
33     ll v;
34     SegInfo() : SegInfo(0) {}
35     SegInfo(ll val) : v(val) {}
36     SegInfo operator*(SegInfo b) {
37         return SegInfo(v + b.v);
38     }
39 };

```

## Union Find

```

1 vector<int> p(n);
2 iota(p.begin(), p.end(), 0);
3 function<int(int)> find = [&](int x) { return p[x] == x ? x :
↪ (p[x] = find(p[x])); };
4 auto merge = [&](int x, int y) { p[find(x)] = find(y); };

```

- Persistent version

```

1 struct Node {
2     int lc, rc, p;
3 };
4
5 struct SegTree {
6     vector<Node> t = {{0, 0, -1}}; // init all
7     SegTree() = default;
8     SegTree(int n) { t.reserve(n * 20); }
9     int modify(int p, int l, int r, int x, int v) {
10         // p: original node, update a[x] -> v
11         t.push_back(t[p]);
12         int u = (int)t.size() - 1;
13         if (r - l == 1) {
14             t[u].p = v;
15         } else {
16             int m = (l + r) / 2;
17             if (x < m) {
18                 t[u].lc = modify(t[p].lc, l, m, x, v);
19                 t[u].rc = t[p].rc;
20             } else {
21                 t[u].lc = t[p].lc;
22                 t[u].rc = modify(t[p].rc, m, r, x, v);
23             }
24             t[u].p = t[t[u].lc].p + t[t[u].rc].p;
25         }
26         return u;
27     }
28     int query(int p, int l, int r, int x, int y) {
29         // query sum a[x]...a[y-1] rooted at p
30         // t[p] holds the info of [l, r)
31         if (x <= l && r <= y) return t[p].p;
32         int m = (l + r) / 2, res = 0;
33         if (x < m) res += query(t[p].lc, l, m, x, y);
34         if (y > m) res += query(t[p].rc, m, r, x, y);
35         return res;
36     }
37 };
38
39 struct DSU {
40     int n;
41     SegTree seg;
42     DSU(int _n) : n(_n), seg(n) {}
43     int get(int p, int x) { return seg.query(p, 0, n, x, x + 1);
↪ }

```

```

44 int set(int p, int x, int v) { return seg.modify(p, 0, n, x,
↪ v); }
45 int find(int p, int x) {
46     int parent = get(p, x);
47     if (parent < 0) return x;
48     return find(p, parent);
49 }
50 int is_same(int p, int x, int y) { return find(p, x) ==
↪ find(p, y); }
51 int merge(int p, int x, int y) {
52     int rx = find(p, x), ry = find(p, y);
53     if (rx == ry) return -1;
54     int rank_x = -get(p, rx), rank_y = -get(p, ry);
55     if (rank_x < rank_y) {
56         p = set(p, rx, ry);
57     } else if (rank_x > rank_y) {
58         p = set(p, ry, rx);
59     } else {
60         p = set(p, ry, rx);
61         p = set(p, rx, -rx - 1);
62     }
63     return p;
64 }
65 };

```

## Fenwick Tree

```

1 template <typename T> struct FenwickTree {
2     int size = 1, high_bit = 1;
3     vector<T> tree;
4     FenwickTree(int _size) : size(_size) {
5         tree.resize(size + 1);
6         while((high_bit << 1) <= size) high_bit <<= 1;
7     }
8     FenwickTree(vector<T>& arr) : FenwickTree(arr.size()) {
9         for(int i = 0; i < size; i++) update(i, arr[i]);
10    }
11    int lower_bound(T x) {
12        int res = 0; T cur = 0;
13        for(int bit = high_bit; bit > 0; bit >>= 1) {
14            if((res|bit) <= size && cur + tree[res|bit] < x) {
15                res |= bit; cur += tree[res];
16            }
17        }
18        return res;
19    }
20    T prefix_sum(int i) {
21        T ret = 0;
22        for(i++; i > 0; i -= (i & -i)) ret += tree[i];
23        return ret;
24    }
25    T range_sum(int l, int r) { return (l > r) ? 0 :
↪ prefix_sum(r) - prefix_sum(l - 1); }
26    void update(int i, T delta) { for(i++; i <= size; i += (i &
↪ -i)) tree[i] += delta; }
27 };

```

## Fenwick2D Tree

```

1 struct Fenwick2D {
2     ll n, m;
3     vector<vector<ll>> a;
4     Fenwick2D(ll _n, ll _m) : n(_n), m(_m), a(n, vector<ll>(m))
↪ {}
5     void add(ll x, ll y, ll v) {
6         for (int i = x + 1; i <= n; i += i & -i) {
7             for (int j = y + 1; j <= m; j += j & -j) {
8                 (a[i - 1][j - 1] += v) %= MOD;
9             }
10        }
11    }
12    void add(ll x1, ll x2, ll y1, ll y2, ll v) {
13        // [(x1, y1), (x2, y2))
14        add(x1, y1, v);
15        add(x1, y2, MOD - v), add(x2, y1, MOD - v);
16        add(x2, y2, v);

```

```

17     }
18     ll sum(ll x, ll y) { // [(0, 0), (x, y))
19         ll ans = 0;
20         for (int i = x; i > 0; i -= i & -i) {
21             for (int j = y; j > 0; j -= j & -j) {
22                 (ans += a[i - 1][j - 1]) %= MOD;
23             }
24         }
25         return ans;
26     }
27 };

```

## PBDS

```

1  #include <bits/stdc++.h>
2  #include <ext/pb_ds/assoc_container.hpp>
3  using namespace std;
4  using namespace __gnu_pbds;
5  template<typename T>
6  using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
7  ↪ tree_order_statistics_node_update>;
8  template<typename T, typename X>
9  using ordered_map = tree<T, X, less<T>, rb_tree_tag,
10 ↪ tree_order_statistics_node_update>;
11 template<typename T, typename X>
12 using fast_map = cc_hash_table<T, X>;
13 template<typename T, typename X>
14 using ht = gp_hash_table<T, X>;
15 mt19937_64
16 ↪ rng(chrono::steady_clock::now().time_since_epoch().count());
17
18 struct splitmix64 {
19     size_t operator()(size_t x) const {
20         static const size_t fixed =
21         ↪ chrono::steady_clock::now().time_since_epoch().count();
22         x += 0x9e3779b97f4a7c15 + fixed;
23         x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
24         x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
25         return x ^ (x >> 31);
26     }
27 };

```

## Treap

- (No rotation version)

```

1  struct Node {
2      Node *l, *r;
3      int s, sz;
4      // int t = 0, a = 0, g = 0; // for lazy propagation
5      ll w;
6
7      Node(int _s) : l(nullptr), r(nullptr), s(_s), sz(1),
8      ↪ w(rng()) {}
9      void apply(int vt, int vg) {
10         // for lazy propagation
11         // s -= vt;
12         // t += vt, a += vg, g += vg;
13     }
14     void push() {
15         // for lazy propagation
16         // if (l != nullptr) l->apply(t, g);
17         // if (r != nullptr) r->apply(t, g);
18         // t = g = 0;
19     }
20     void pull() { sz = 1 + (l ? l->sz : 0) + (r ? r->sz : 0); }
21 };
22
23 std::pair<Node *, Node *> split(Node *t, int v) {
24     if (t == nullptr) return {nullptr, nullptr};
25     t->push();
26     if (t->s < v) {
27         auto [x, y] = split(t->r, v);
28         t->r = x;
29         t->pull();
30         return {t, y};
31     }
32 }

```

```

30     } else {
31         auto [x, y] = split(t->l, v);
32         t->l = y;
33         t->pull();
34         return {x, t};
35     }
36 }
37
38 Node *merge(Node *p, Node *q) {
39     if (p == nullptr) return q;
40     if (q == nullptr) return p;
41     if (p->w < q->w) swap(p, q);
42     auto [x, y] = split(q, p->s + rng() % 2);
43     p->push();
44     p->l = merge(p->l, x);
45     p->r = merge(p->r, y);
46     p->pull();
47     return p;
48 }
49
50 Node *insert(Node *t, int v) {
51     auto [x, y] = split(t, v);
52     return merge(merge(x, new Node(v)), y);
53 }
54
55 Node *erase(Node *t, int v) {
56     auto [x, y] = split(t, v);
57     auto [p, q] = split(y, v + 1);
58     return merge(merge(x, merge(p->l, p->r)), q);
59 }
60
61 int get_rank(Node *t, int v) {
62     auto [x, y] = split(t, v);
63     int res = (x ? x->sz : 0) + 1;
64     t = merge(x, y);
65     return res;
66 }
67
68 Node *kth(Node *t, int k) {
69     k--;
70     while (true) {
71         int left_sz = t->l ? t->l->sz : 0;
72         if (k < left_sz) {
73             t = t->l;
74         } else if (k == left_sz) {
75             return t;
76         } else {
77             k -= left_sz + 1, t = t->r;
78         }
79     }
80 }
81
82 Node *get_prev(Node *t, int v) {
83     auto [x, y] = split(t, v);
84     Node *res = kth(x, x->sz);
85     t = merge(x, y);
86     return res;
87 }
88
89 Node *get_next(Node *t, int v) {
90     auto [x, y] = split(t, v + 1);
91     Node *res = kth(y, 1);
92     t = merge(x, y);
93     return res;
94 }

```

### • USAGE

```

1  int main() {
2      cin.tie(nullptr)->sync_with_stdio(false);
3      int n;
4      cin >> n;
5      Node *t = nullptr;
6      for (int op, x; n--;) {
7          cin >> op >> x;
8          if (op == 1) {
9              t = insert(t, x);
10             } else if (op == 2) {

```

```

11     t = erase(t, x);
12 } else if (op == 3) {
13     cout << get_rank(t, x) << "\n";
14 } else if (op == 4) {
15     cout << kth(t, x)->s << "\n";
16 } else if (op == 5) {
17     cout << get_prev(t, x)->s << "\n";
18 } else {
19     cout << get_next(t, x)->s << "\n";
20 }
21 }
22 }

```

## Implicit treap

- Split by size

```

1 struct Node {
2     Node *l, *r;
3     int s, sz;
4     // int lazy = 0;
5     ll w;
6
7     Node(int _s) : l(nullptr), r(nullptr), s(_s), sz(1),
8     ↪ w(rnd()) {}
9     void apply() {
10         // for lazy propagation
11         // lazy ^= 1;
12     }
13     void push() {
14         // for lazy propagation
15         // if (lazy) {
16         //     swap(l, r);
17         //     if (l != nullptr) l->apply();
18         //     if (r != nullptr) r->apply();
19         //     lazy = 0;
20         // }
21     }
22     void pull() { sz = 1 + (l ? l->sz : 0) + (r ? r->sz : 0); }
23 };
24
25 std::pair<Node *, Node *> split(Node *t, int v) {
26     // first->sz == v
27     if (t == nullptr) return {nullptr, nullptr};
28     t->push();
29     int left_sz = t->l ? t->l->sz : 0;
30     if (left_sz < v) {
31         auto [x, y] = split(t->r, v - left_sz - 1);
32         t->r = x;
33         t->pull();
34         return {t, y};
35     } else {
36         auto [x, y] = split(t->l, v);
37         t->l = y;
38         t->pull();
39         return {x, t};
40     }
41 }
42
43 Node *merge(Node *p, Node *q) {
44     if (p == nullptr) return q;
45     if (q == nullptr) return p;
46     if (p->w < q->w) {
47         p->push();
48         p->r = merge(p->r, q);
49         p->pull();
50         return p;
51     } else {
52         q->push();
53         q->l = merge(p, q->l);
54         q->pull();
55         return q;
56     }
57 }

```

## Persistent implicit treap

```

1 pair<Node *, Node *> split(Node *t, int v) {
2     // first->sz == v
3     if (t == nullptr) return {nullptr, nullptr};
4     t->push();
5     int left_sz = t->l ? t->l->sz : 0;
6     t = new Node(*t);
7     if (left_sz < v) {
8         auto [x, y] = split(t->r, v - left_sz - 1);
9         t->r = x;
10        t->pull();
11        return {t, y};
12    } else {
13        auto [x, y] = split(t->l, v);
14        t->l = y;
15        t->pull();
16        return {x, t};
17    }
18 }
19
20 Node *merge(Node *p, Node *q) {
21     if (p == nullptr) return new Node(*q);
22     if (q == nullptr) return new Node(*p);
23     if (p->w < q->w) {
24         p = new Node(*p);
25         p->push();
26         p->r = merge(p->r, q);
27         p->pull();
28         return p;
29     } else {
30         q = new Node(*q);
31         q->push();
32         q->l = merge(p, q->l);
33         q->pull();
34         return q;
35     }
36 }

```

## 2D Sparse Table

- Sorry that this sucks - asked

```

1 template <class T, class Compare = less<T>>
2 struct SparseTable2d {
3     int n = 0, m = 0;
4     T*** table;
5     int* log;
6     inline T choose(T x, T y) {
7         return Compare()(x, y) ? x : y;
8     }
9     SparseTable2d(vector<vector<T>>& grid) {
10         if (grid.empty() || grid[0].empty()) return;
11         n = grid.size(); m = grid[0].size();
12         log = new int[max(n, m) + 1];
13         log[1] = 0;
14         for (int i = 2; i <= max(n, m); i++)
15             log[i] = log[i - 1] + ((i & (i - 1)) > i);
16         table = new T***[n];
17         for (int i = n - 1; i >= 0; i--) {
18             table[i] = new T**[m];
19             for (int j = m - 1; j >= 0; j--) {
20                 table[i][j] = new T*[log[n - i] + 1];
21                 for (int k = 0; k <= log[n - i]; k++) {
22                     table[i][j][k] = new T[log[m - j] + 1];
23                     if (!k) table[i][j][k][0] = grid[i][j];
24                     ↪ else table[i][j][k][0] = choose(table[i][j][k-1][0],
25                     ↪ table[i+(1<<(k-1))][j][k-1][0]);
26                     for (int l = 1; l <= log[m - j]; l++)
27                         table[i][j][k][l] = choose(table[i][j][k][l-1],
28                     ↪ table[i+(1<<(l-1))][j][k][l-1]);
29                 }
30             }
31         }
32         T query(int r1, int r2, int c1, int c2) {
33             assert(r1 >= 0 && r2 < n && r1 <= r2);

```

```

33     assert(c1 >= 0 && c2 < m && c1 <= c2);
34     int r1 = log[r2 - r1 + 1], c1 = log[c2 - c1 + 1];
35     T ca1 = choose(table[r1][c1][r1][c1],
36     ↪ table[r2-(1<<r1)+1][c1][r1][c1]);
37     T ca2 = choose(table[r1][c2-(1<<c1)+1][r1][c1],
38     ↪ table[r2-(1<<r1)+1][c2-(1<<c1)+1][r1][c1]);
39     return choose(ca1, ca2);
40 }
41 };

```

## • USAGE

```

1 vector<vector<int>> test = {
2     {1, 2, 3, 4}, {2, 3, 4, 5}, {9, 9, 9, 9}, {-1, -1, -1, -1}
3 };
4
5 SparseTable2d<int> st(test);           // Range min query
6 SparseTable2d<int, greater<int>> st2(test); // Range max query

```

## K-D Tree

```

1 struct Point {
2     int x, y;
3 };
4 struct Rectangle {
5     int lx, rx, ly, ry;
6 };
7
8 bool is_in(const Point &p, const Rectangle &rg) {
9     return (p.x >= rg.lx && (p.x <= rg.rx) && (p.y >= rg.ly) &&
10    ↪ (p.y <= rg.ry));
11 }
12
13 struct KDTree {
14     vector<Point> points;
15     struct Node {
16         int lc, rc;
17         Point point;
18         Rectangle range;
19         int num;
20     };
21     vector<Node> nodes;
22     int root = -1;
23     KDTree(const vector<Point> &points_) {
24         points = points_;
25         Rectangle range = {-1e9, 1e9, -1e9, 1e9};
26         root = tree_construct(0, (int)points.size(), range, 0);
27     }
28     int tree_construct(int l, int r, Rectangle range, int depth)
29     ↪ {
30         if (l == r) return -1;
31         if (l > r) throw;
32         int mid = (l + r) / 2;
33         auto comp = (depth % 2) ? [](Point &a, Point &b) { return
34         ↪ a.x < b.x; }
35         : [](Point &a, Point &b) { return
36         ↪ a.y < b.y; };
37         nth_element(points.begin() + l, points.begin() + mid,
38         ↪ points.begin() + r, comp);
39         Rectangle l_range(range), r_range(range);
40         if (depth % 2) {
41             l_range.rx = points[mid].x;
42             r_range.lx = points[mid].x;
43         } else {
44             l_range.ry = points[mid].y;
45             r_range.ly = points[mid].y;
46         }
47         Node node = {tree_construct(l, mid, l_range, depth + 1),
48         ↪ tree_construct(mid + 1, r, r_range, depth +
49         ↪ 1), points[mid], range, r - l};
50         nodes.push_back(node);
51         return (int)nodes.size() - 1;
52     }
53
54     int inner_query(int id, const Rectangle &rec, int depth) {
55         if (id == -1) return 0;
56         Rectangle rg = nodes[id].range;

```

```

51         if (rg.lx >= rec.lx && rg.rx <= rec.rx && rg.ly >= rec.ly
52         ↪ && rg.ry <= rec.ry) {
53             return nodes[id].num;
54         }
55         int ans = 0;
56         if (depth % 2) { // pruning
57             if (rec.lx <= nodes[id].point.x) ans +=
58             ↪ inner_query(nodes[id].lc, rec, depth + 1);
59             if (rec.rx >= nodes[id].point.x) ans +=
60             ↪ inner_query(nodes[id].rc, rec, depth + 1);
61         } else {
62             if (rec.ly <= nodes[id].point.y) ans +=
63             ↪ inner_query(nodes[id].lc, rec, depth + 1);
64             if (rec.ry >= nodes[id].point.y) ans +=
65             ↪ inner_query(nodes[id].rc, rec, depth + 1);
66         }
67         if (is_in(nodes[id].point, rec)) ans += 1;
68         return ans;
69     }
70
71     int query(const Rectangle &rec) { return inner_query(root,
72     ↪ rec, 0); }
73 };

```

## Link/Cut Tree

```

1 struct Node {
2     Node *ch[2], *p;
3     int id;
4     bool rev;
5     Node(int id) : ch{nullptr, nullptr}, p(nullptr), id(id),
6     ↪ rev(false) {}
7     friend void reverse(Node *p) {
8         if (p != nullptr) {
9             swap(p->ch[0], p->ch[1]);
10            p->rev ^= 1;
11        }
12    }
13    void push() {
14        if (rev) {
15            reverse(ch[0]);
16            reverse(ch[1]);
17            rev = false;
18        }
19    }
20    void pull() {}
21    bool is_root() { return p == nullptr || p->ch[0] != this &&
22    ↪ p->ch[1] != this; }
23    bool pos() { return p->ch[1] == this; }
24    void rotate() {
25        Node *q = p;
26        bool x = !pos();
27        q->ch[!x] = ch[x];
28        if (ch[x] != nullptr) ch[x]->p = q;
29        p = q->p;
30        if (!q->is_root()) q->p->ch[q->pos()] = this;
31        ch[x] = q;
32        q->p = this;
33        pull();
34        q->pull();
35    }
36    void splay() {
37        vector<Node *> s;
38        for (Node *i = this; !i->is_root(); i = i->p)
39        ↪ s.push_back(i->p);
40        while (!s.empty()) s.back()->push(), s.pop_back();
41        push();
42        while (!is_root()) {
43            if (pos() == p->pos()) {
44                p->rotate();
45            } else {
46                rotate();
47            }
48        }
49    }

```



```

49     pull();
50 }
51 void access() {
52     for (Node *i = this, *q = nullptr; i != nullptr; q = i, i
↪ = i->p) {
53         i->splay();
54         i->ch[1] = q;
55         i->pull();
56     }
57     splay();
58 }
59 void makeroot() {
60     access();
61     reverse(this);
62 }
63 };
64 void link(Node *x, Node *y) {
65     x->makeroot();
66     x->p = y;
67 }
68 void split(Node *x, Node *y) {
69     x->makeroot();
70     y->access();
71 }
72 void cut(Node *x, Node *y) {
73     split(x, y);
74     x->p = y->ch[0] = nullptr;
75     y->pull();
76 }
77 bool connected(Node *p, Node *q) {
78     p->access();
79     q->access();
80     return p->p != nullptr;
81 }

```

## Geometry

### Basic stuff

```

1 using ll = long long;
2 using ld = long double;
3
4 constexpr auto eps = 1e-8;
5 const auto PI = acos(-1);
6 int sgn(ld x) { return (abs(x) <= eps) ? 0 : (x < 0 ? -1 : 1);
↪ }
7
8 struct Point {
9     ld x = 0, y = 0;
10     Point() = default;
11     Point(ld _x, ld _y) : x(_x), y(_y) {}
12     bool operator<(const Point &p) const { return !sgn(p.x - x)
↪ ? sgn(y - p.y) < 0 : x < p.x; }
13     bool operator==(const Point &p) const { return !sgn(p.x - x)
↪ && !sgn(p.y - y); }
14     Point operator+(const Point &p) const { return {x + p.x, y +
↪ p.y}; }
15     Point operator-(const Point &p) const { return {x - p.x, y -
↪ p.y}; }
16     Point operator*(ld a) const { return {x * a, y * a}; }
17     Point operator/(ld a) const { return {x / a, y / a}; }
18     auto operator*(const Point &p) const { return x * p.x + y *
↪ p.y; } // dot
19     auto operator^(const Point &p) const { return x * p.y - y *
↪ p.x; } // cross
20     friend auto &operator>>(istream &i, Point &p) { return i >>
↪ p.x >> p.y; }
21     friend auto &operator<<(ostream &o, Point p) { return o <<
↪ p.x << ' ' << p.y; }
22 };
23
24 struct Line {
25     Point s = {0, 0}, e = {0, 0};
26     Line() = default;
27     Line(Point _s, Point _e) : s(_s), e(_e) {}

```

```

28     friend auto &operator>>(istream &i, Line &l) { return i >>
↪ l.s >> l.e; } // ((x1, y1), (x2, y2))
29 };
30
31 struct Segment : Line {
32     using Line::Line;
33 };
34
35 struct Circle {
36     Point o = {0, 0};
37     ld r = 0;
38     Circle() = default;
39     Circle(Point _o, ld _r) : o(_o), r(_r) {}
40 };
41
42 auto dist2(const Point &a) { return a * a; }
43 auto dist2(const Point &a, const Point &b) { return dist2(a -
↪ b); }
44 auto dist(const Point &a) { return sqrt(dist2(a)); }
45 auto dist(const Point &a, const Point &b) { return
↪ sqrt(dist2(a - b)); }
46 auto dist(const Point &a, const Line &l) { return abs((a -
↪ l.s) ^ (l.e - l.s)) / dist(l.s, l.e); }
47 auto dist(const Point &p, const Segment &l) {
48     if (l.s == l.e) return dist(p, l.s);
49     auto d = dist2(l.s, l.e), t = min(d, max((ld)0, (p - l.s) *
↪ (l.e - l.s)));
50     return dist((p - l.s) * d, (l.e - l.s) * t) / d;
51 }
52 /* Needs is_intersect
53 auto dist(const Segment &l1, const Segment &l2) {
54     if (is_intersect(l1, l2)) return (ld)0;
55     return min({dist(l1.s, l2), dist(l1.e, l2), dist(l2.s, l1),
↪ dist(l2.e, l1)});
56 } */
57
58 Point perp(const Point &p) { return Point(-p.y, p.x); }
59
60 auto rad(const Point &p) { return atan2(p.y, p.x); }

```

### Transformation

```

1 Point project(const Point &p, const Line &l) {
2     return l.s + ((l.e - l.s) * ((l.e - l.s) * (p - l.s))) /
↪ dist2(l.e - l.s);
3 }
4
5 Point reflect(const Point &p, const Line &l) {
6     return project(p, l) * 2 - p;
7 }
8
9 Point dilate(const Point &p, ld scale_x = 1, ld scale_y = 1) {
10     return Point(p.x * scale_x, p.y * scale_y); }
11 Line dilate(const Line &l, ld scale_x = 1, ld scale_y = 1) {
12     return Line(dilate(l.s, scale_x, scale_y), dilate(l.e,
↪ scale_x, scale_y)); }
13 Segment dilate(const Segment &l, ld scale_x = 1, ld scale_y =
↪ 1) { return Segment(dilate(l.s, scale_x, scale_y),
↪ dilate(l.e, scale_x, scale_y)); }
14 vector<Point> dilate(const vector<Point> &p, ld scale_x = 1,
↪ ld scale_y = 1) {
15     int n = p.size();
16     vector<Point> res(n);
17     for (int i = 0; i < n; i++)
18         res[i] = dilate(p[i], scale_x, scale_y);
19     return res;
20 }
21
22 Point rotate(const Point &p, ld a) { return Point(p.x * cos(a)
↪ - p.y * sin(a), p.x * sin(a) + p.y * cos(a)); }
23 Line rotate(const Line &l, ld a) { return Line(rotate(l.s, a),
↪ rotate(l.e, a)); }
24 Segment rotate(const Segment &l, ld a) { return
↪ Segment(rotate(l.s, a), rotate(l.e, a)); }
25 Circle rotate(const Circle &c, ld a) { return
↪ Circle(rotate(c.o, a), c.r); }
26 vector<Point> rotate(const vector<Point> &p, ld a) {

```

```

25     int n = p.size();
26     vector<Point> res(n);
27     for (int i = 0; i < n; i++)
28         res[i] = rotate(p[i], a);
29     return res;
30 }
31
32 Point translate(const Point &p, ld dx = 0, ld dy = 0) { return
    ↪ Point(p.x + dx, p.y + dy); }
33 Line translate(const Line &l, ld dx = 0, ld dy = 0) { return
    ↪ Line(translate(l.s, dx, dy), translate(l.e, dx, dy)); }
34 Segment translate(const Segment &l, ld dx = 0, ld dy = 0) {
    ↪ return Segment(translate(l.s, dx, dy), translate(l.e, dx,
    ↪ dy)); }
35 Circle translate(const Circle &c, ld dx = 0, ld dy = 0) {
    ↪ return Circle(translate(c.o, dx, dy), c.r); }
36 vector<Point> translate(const vector<Point> &p, ld dx = 0, ld
    ↪ dy = 0) {
37     int n = p.size();
38     vector<Point> res(n);
39     for (int i = 0; i < n; i++)
40         res[i] = translate(p[i], dx, dy);
41     return res;
42 }

```

## Relation

```

1  enum class Relation { SEPARATE, EX_TOUCH, OVERLAP, IN_TOUCH,
    ↪ INSIDE };
2  Relation get_relation(const Circle &a, const Circle &b) {
3      auto c1c2 = dist(a.o, b.o);
4      auto r1r2 = a.r + b.r, diff = abs(a.r - b.r);
5      if (sgn(c1c2 - r1r2) > 0) return Relation::SEPARATE;
6      if (sgn(c1c2 - r1r2) == 0) return Relation::EX_TOUCH;
7      if (sgn(c1c2 - diff) > 0) return Relation::OVERLAP;
8      if (sgn(c1c2 - diff) == 0) return Relation::IN_TOUCH;
9      return Relation::INSIDE;
10 }
11
12 auto get_cos_from_triangle(ld a, ld b, ld c) { return (a * a +
    ↪ b * b - c * c) / (2.0 * a * b); }
13
14 bool on_line(const Line &l, const Point &p) { return !sgn((l.s
    ↪ - p) ^ (l.e - p)); }
15
16 bool on_segment(const Segment &l, const Point &p) {
17     return !sgn((l.s - p) ^ (l.e - p)) && sgn((l.s - p) * (l.e -
    ↪ p)) <= 0;
18 }
19
20 bool on_segment2(const Segment &l, const Point &p) { // assume
    ↪ p on Line l
21     if (l.s == p || l.e == p) return true;
22     if (min(l.s, l.e) < p && p < max(l.s, l.e)) return true;
23     return false;
24 }
25
26 bool is_parallel(const Line &a, const Line &b) { return
    ↪ !sgn((a.s - a.e) ^ (b.s - b.e)); }
27 bool is_orthogonal(const Line &a, const Line &b) { return
    ↪ !sgn((a.s - a.e) * (b.s - b.e)); }
28
29 int is_intersect(const Segment &a, const Segment &b) {
30     auto d1 = sgn((a.e - a.s) ^ (b.s - a.s)), d2 = sgn((a.e -
    ↪ a.s) ^ (b.e - a.s));
31     auto d3 = sgn((b.e - b.s) ^ (a.s - b.s)), d4 = sgn((b.e -
    ↪ b.s) ^ (a.e - b.s));
32     if (d1 * d2 < 0 && d3 * d4 < 0) return 2; // intersect at
    ↪ non-end point
33     return (d1 == 0 && sgn((b.s - a.s) * (b.s - a.e)) <= 0) ||
34         (d2 == 0 && sgn((b.e - a.s) * (b.e - a.e)) <= 0) ||
35         (d3 == 0 && sgn((a.s - b.s) * (a.s - b.e)) <= 0) ||
36         (d4 == 0 && sgn((a.e - b.s) * (a.e - b.e)) <= 0);
37 }
38
39 int is_intersect(const Line &a, const Segment &b) {

```

```

40     auto d1 = sgn((a.e - a.s) ^ (b.s - a.s)), d2 = sgn((a.e -
    ↪ a.s) ^ (b.e - a.s));
41     if (d1 * d2 < 0) return 2; // intersect at non-end point
42     return d1 == 0 || d2 == 0;
43 }
44
45 Point intersect(const Line &a, const Line &b) {
46     auto u = a.e - a.s, v = b.e - b.s;
47     auto t = ((b.s - a.s) ^ v) / (u ^ v);
48     return a.s + u * t;
49 }
50
51 int is_intersect(const Circle &c, const Line &l) {
52     auto d = dist(c.o, l);
53     return sgn(d - c.r) < 0 ? 2 : !sgn(d - c.r);
54 }
55
56 vector<Point> intersect(const Circle &a, const Circle &b) {
57     auto relation = get_relation(a, b);
58     if (relation == Relation::INSIDE || relation ==
    ↪ Relation::SEPARATE) return {};
59     auto vec = b.o - a.o;
60     auto d2 = dist2(vec);
61     auto p = (d2 + a.r * a.r - b.r * b.r) / ((long double)2 *
    ↪ d2), h2 = a.r * a.r - p * p * d2;
62     auto mid = a.o + vec * p, per = perp(vec) * sqrt(max((long
    ↪ double)0, h2) / d2);
63     if (relation == Relation::OVERLAP)
64         return {mid + per, mid - per};
65     else
66         return {mid};
67 }
68
69 vector<Point> intersect(const Circle &c, const Line &l) {
70     if (!is_intersect(c, l)) return {};
71     auto v = l.e - l.s, t = v / dist(v);
72     Point a = l.s + t * ((c.o - l.s) * t);
73     auto d = sqrt(max((ld)0, c.r * c.r - dist2(c.o, a)));
74     if (!sgn(d)) return {a};
75     return {a - t * d, a + t * d};
76 }
77
78 int in_poly(const vector<Point> &p, const Point &a) {
79     int cnt = 0, n = (int)p.size();
80     for (int i = 0; i < n; i++) {
81         auto q = p[(i + 1) % n];
82         if (on_segment(Segment(p[i], q), a)) return 1; // on the
    ↪ edge of the polygon
83         cnt ^= ((a.y < p[i].y) - (a.y < q.y)) * ((p[i] - a) ^ (q -
    ↪ a)) > 0;
84     }
85     return cnt ? 2 : 0;
86 }
87
88 int is_intersect(const vector<Point> &p, const Line &a) {
89     // 1: touching, >=2: intersect count
90     int cnt = 0, edge_cnt = 0, n = (int)p.size();
91     for (int i = 0; i < n; i++) {
92         auto q = p[(i + 1) % n];
93         if (on_line(a, p[i]) && on_line(a, q)) return -1; //
    ↪ infinity
94         auto t = is_intersect(a, Segment(p[i], q));
95         (t == 1) && edge_cnt++, (t == 2) && cnt++;
96     }
97     return cnt + edge_cnt / 2;
98 }
99
100 vector<Point> tangent(const Circle &c, const Point &p) {
101     auto d = dist(c.o, p), l = c.r * c.r / d, h = sqrt(c.r * c.r
    ↪ - l * l);
102     auto v = (p - c.o) / d;
103     return {c.o + v * l + perp(v) * h, c.o + v * l - perp(v) *
    ↪ h};
104 }
105
106 Circle get_circumscribed(const Point &a, const Point &b, const
    ↪ Point &c) {

```

```

107 Line u((a + b) / 2, ((a + b) / 2) + perp(b - a));
108 Line v((b + c) / 2, ((b + c) / 2) + perp(c - b));
109 auto o = intersect(u, v);
110 return Circle(o, dist(o, a));
111 }
112
113 Circle get_inscribed(const Point &a, const Point &b, const
    ↪ Point &c) {
114     auto l1 = dist(b - c), l2 = dist(c - a), l3 = dist(a - b);
115     Point o = (a * l1 + b * l2 + c * l3) / (l1 + l2 + l3);
116     return Circle(o, dist(o, Line(a, b)));
117 }
118
119 pair<ld, ld> get_centroid(const vector<Point> &p) {
120     int n = (int)p.size();
121     ld x = 0, y = 0, sum = 0;
122     auto a = p[0], b = p[1];
123     for (int i = 2; i < n; i++) {
124         auto c = p[i];
125         auto s = area({a, b, c});
126         sum += s;
127         x += s * (a.x + b.x + c.x);
128         y += s * (a.y + b.y + c.y);
129         swap(b, c);
130     }
131     return {x / (3 * sum), y / (3 * sum)};
132 }

```

## Area

```

1 auto area(const vector<Point> &p) {
2     int n = (int)p.size();
3     long double area = 0;
4     for (int i = 0; i < n; i++) area += p[i] ^ p[(i + 1) % n];
5     return area / 2.0;
6 }
7
8 auto area(const Point &a, const Point &b, const Point &c) {
9     return ((long double)((b - a) ^ (c - a))) / 2.0;
10 }
11
12 auto area2(const Point &a, const Point &b, const Point &c) {
13     ↪ return (b - a) ^ (c - a); }
14
15 auto area_intersect(const Circle &c, const vector<Point> &ps)
16 ↪ {
17     int n = (int)ps.size();
18     auto arg = [&](const Point &p, const Point &q) { return
19 ↪ atan2(p ^ q, p * q); };
20     auto tri = [&](const Point &p, const Point &q) {
21         auto r2 = c.r * c.r / (long double)2;
22         auto d = q - p;
23         auto a = d * p / dist2(d), b = (dist2(p) - c.r * c.r) /
24 ↪ dist2(d);
25         long double det = a * a - b;
26         if (sgn(det) <= 0) return arg(p, q) * r2;
27         auto s = max((long double)0, -a + sqrt(det)); t =
28 ↪ min((long double)1, -a + sqrt(det));
29         if (sgn(t) < 0 || sgn(1 - s) <= 0) return arg(p, q) * r2;
30         auto u = p + d * s, v = p + d * t;
31         return arg(p, u) * r2 + (u ^ v) / 2 + arg(v, q) * r2;
32     };
33     long double sum = 0;
34     for (int i = 0; i < n; i++) sum += tri(ps[i] - c.o, ps[(i +
35 ↪ 1) % n] - c.o);
36     return sum;
37 }
38
39 auto adaptive_simpson(ld _l, ld _r, function<ld(ld)> f) {
40     auto simpson = [&](ld l, ld r) { return (r - l) * (f(l) + 4
    ↪ * f((l + r) / 2) + f(r)) / 6; };
41     function<ld(ld, ld, ld)> asr = [&](ld l, ld r, ld s) {
42         auto mid = (l + r) / 2;
43         auto left = simpson(l, mid), right = simpson(mid, r);
44         if (!sgn(left + right - s)) return left + right;
45         return asr(l, mid, left) + asr(mid, r, right);
46     };

```

```

41     return asr(_l, _r, simpson(_l, _r));
42 }
43
44 vector<Point> half_plane_intersect(vector<Line> &L) {
45     int n = (int)L.size(), l = 0, r = 0; // [left, right]
46     sort(L.begin(), L.end(),
47         [](const Line &a, const Line &b) { return rad(a.s -
    ↪ a.e) < rad(b.s - b.e); });
48     vector<Point> p(n), res;
49     vector<Line> q(n);
50     q[0] = L[0];
51     for (int i = 1; i < n; i++) {
52         while (1 < r && sgn((L[i].e - L[i].s) ^ (p[r - 1] -
    ↪ L[i].s)) <= 0) r--;
53         while (1 < r && sgn((L[i].e - L[i].s) ^ (p[l] - L[i].s))
    ↪ <= 0) l++;
54         q[++r] = L[i];
55         if (sgn((q[r].e - q[r].s) ^ (q[r - 1].e - q[r - 1].s)) ==
    ↪ 0) {
56             r--;
57             if (sgn((q[r].e - q[r].s) ^ (L[i].s - q[r].s)) > 0) q[r]
    ↪ = L[i];
58         }
59         if (1 < r) p[r - 1] = intersect(q[r - 1], q[r]);
60     }
61     while (1 < r && sgn((q[l].e - q[l].s) ^ (p[r - 1] - q[l].s))
    ↪ <= 0) r--;
62     if (r - l <= 1) return {};
63     p[r] = intersect(q[r], q[l]);
64     return vector<Point>(p.begin() + l, p.begin() + r + 1);
65 }

```

## Convex

```

1 vector<Point> get_convex(vector<Point> &points, bool
    ↪ allow_collinear = false) {
2     // strict, no repeat, two pass
3     sort(points.begin(), points.end());
4     points.erase(unique(points.begin(), points.end()),
    ↪ points.end());
5     vector<Point> L, U;
6     for (auto &t : points) {
7         for (ll sz = L.size(); sz > 1 && (sgn((t - L[sz - 2]) ^
    ↪ (L[sz - 1] - L[sz - 2])) >= 0);
8             L.pop_back(), sz = L.size()) {
9         }
10        L.push_back(t);
11    }
12    for (auto &t : points) {
13        for (ll sz = U.size(); sz > 1 && (sgn((t - U[sz - 2]) ^
    ↪ (U[sz - 1] - U[sz - 2])) <= 0);
14            U.pop_back(), sz = U.size()) {
15        }
16        U.push_back(t);
17    }
18    // contain repeats if all collinear, use a set to remove
19    ↪ repeats
20    if (allow_collinear) {
21        for (int i = (int)U.size() - 2; i >= 1; i--)
22            ↪ L.push_back(U[i]);
23    } else {
24        set<Point> st(L.begin(), L.end());
25        for (int i = (int)U.size() - 2; i >= 1; i--) {
26            if (st.count(U[i]) == 0) L.push_back(U[i]),
27            ↪ st.insert(U[i]);
28        }
29    }
30    return L;
31 }
32
33 vector<Point> get_convex2(vector<Point> &points, bool
    ↪ allow_collinear = false) { // strict, no repeat, one pass
34     nth_element(points.begin(), points.begin(), points.end());
35     sort(points.begin() + 1, points.end(), [&](const Point &a,
    ↪ const Point &b) {
36         int rad_diff = sgn((a - points[0]) ^ (b - points[0]));

```

```

34     return !rad_diff ? (dist2(a - points[0]) < dist2(b -
↪ points[0])) : (rad_diff > 0);
35 });
36 if (allow_collinear) {
37     int i = (int)points.size() - 1;
38     while (i >= 0 && !sgn((points[i] - points[0]) ^ (points[i]
↪ - points.back())) i--;
39     reverse(points.begin() + i + 1, points.end());
40 }
41 vector<Point> hull;
42 for (auto &t : points) {
43     for (ll sz = hull.size();
44         sz > 1 && (sgn((t - hull[sz - 2]) ^ (hull[sz - 1] -
↪ hull[sz - 2])) >= allow_collinear);
45         hull.pop_back(), sz = hull.size()) {
46     }
47     hull.push_back(t);
48 }
49 return hull;
50 }
51
52 vector<Point> get_convex_safe(vector<Point> points, bool
↪ allow_collinear = false) {
53     return get_convex(points, allow_collinear);
54 }
55
56 vector<Point> get_convex2_safe(vector<Point> points, bool
↪ allow_collinear = false) {
57     return get_convex2(points, allow_collinear);
58 }
59
60 bool is_convex(const vector<Point> &p, bool allow_collinear =
↪ false) {
61     int n = p.size();
62     int lo = 1, hi = -1;
63     for (int i = 0; i < n; i++) {
64         int cur = sgn((p[(i + 2) % n] - p[(i + 1) % n]) ^ (p[(i +
↪ 1) % n] - p[i]));
65         lo = min(lo, cur); hi = max(hi, cur);
66     }
67     return allow_collinear ? (hi - lo) < 2 : (lo == hi && lo);
68 }
69
70 auto rotating_calipers(const vector<Point> &hull) {
71     // use get_convex2
72     int n = (int)hull.size(); // return the square of longest
↪ dist
73     assert(n > 1);
74     if (n <= 2) return dist2(hull[0], hull[1]);
75     ld res = 0;
76     for (int i = 0, j = 2; i < n; i++) {
77         auto d = hull[i], e = hull[(i + 1) % n];
78         while (area2(d, e, hull[j]) < area2(d, e, hull[(j + 1) %
↪ n])) j = (j + 1) % n;
79         res = max(res, max(dist2(d, hull[j]), dist2(e, hull[j])));
80     }
81     return res;
82 }
83
84 // Find polygon cut to the left of l
85 vector<Point> convex_cut(const vector<Point> &p, const Line
↪ &l) {
86     int n = p.size();
87     vector<Point> cut;
88     for (int i = 0; i < n; i++) {
89         auto a = p[i], b = p[(i + 1) % n];
90         if (sgn((l.e - l.s) ^ (a - l.s)) >= 0)
91             cut.push_back(a);
92         if (sgn((l.e - l.s) ^ (a - l.s)) * sgn((l.e - l.s) ^ (b -
↪ l.s)) == -1)
93             cut.push_back(intersect(Line(a, b), l));
94     }
95     return cut;
96 }
97
98 // Sort by angle in range [0, 2pi)
99 template <class RandomIt>

```

```

100 void polar_sort(RandomIt first, RandomIt last, Point origin =
↪ Point(0, 0)) {
101     auto get_quad = [&](const Point& p) {
102         Point diff = p - origin;
103         if (diff.x > 0 && diff.y >= 0) return 1;
104         if (diff.x <= 0 && diff.y > 0) return 2;
105         if (diff.x < 0 && diff.y <= 0) return 3;
106         return 4;
107     };
108     auto polar_cmp = [&](const Point& p1, const Point& p2) {
109         int q1 = get_quad(p1), q2 = get_quad(p2);
110         if (q1 != q2) return q1 < q2;
111         return ((p1 - origin) ^ (p2 - origin)) > 0;
112     };
113     sort(first, last, polar_cmp);
114 }

```

## Basic 3D

```

1 using ll = long long;
2 using ld = long double;
3
4 constexpr auto eps = 1e-8;
5 const auto PI = acos(-1);
6 int sgn(ld x) { return (abs(x) <= eps) ? 0 : (x < 0 ? -1 : 1);
↪ }
7
8 struct Point3D {
9     ld x = 0, y = 0, z = 0;
10     Point3D() = default;
11     Point3D(ld _x, ld _y, ld _z) : x(_x), y(_y), z(_z) {}
12     bool operator<(const Point3D &p) const { return !sgn(p.x -
↪ x) ? (!sgn(p.y - y) ? sgn(p.z - z) < 0 : y < p.y) : x <
↪ p.x; }
13     bool operator==(const Point3D &p) const { return !sgn(p.x -
↪ x) && !sgn(p.y - y) && !sgn(p.z - z); }
14     Point3D operator+(const Point3D &p) const { return {x + p.x,
↪ y + p.y, z + p.z}; }
15     Point3D operator-(const Point3D &p) const { return {x - p.x,
↪ y - p.y, z - p.z}; }
16     Point3D operator*(ld a) const { return {x * a, y * a, z *
↪ a}; }
17     Point3D operator/(ld a) const { return {x / a, y / a, z /
↪ a}; }
18     auto operator*(const Point3D &p) const { return x * p.x + y
↪ * p.y + z * p.z; } // dot
19     Point3D operator^(const Point3D &p) const { return {y * p.z
↪ - z * p.y, z * p.x - x * p.z, x * p.y - y * p.x}; } //
↪ cross
20     friend auto &operator>>(istream &i, Point3D &p) { return i
↪ >> p.x >> p.y >> p.z; }
21 };
22
23 struct Line3D {
24     Point3D s = {0, 0, 0}, e = {0, 0, 0};
25     Line3D() = default;
26     Line3D(Point3D _s, Point3D _e) : s(_s), e(_e) {}
27 };
28
29 struct Segment3D : Line3D {
30     using Line3D::Line3D;
31 };
32
33 auto dist2(const Point3D &a) { return a * a; }
34 auto dist2(const Point3D &a, const Point3D &b) { return
↪ dist2(a - b); }
35 auto dist(const Point3D &a) { return sqrt(dist2(a)); }
36 auto dist(const Point3D &a, const Point3D &b) { return
↪ sqrt(dist2(a - b)); }
37 auto dist(const Point3D &a, const Line3D &l) { return dist((a
↪ - l.s) ^ (l.e - l.s)) / dist(l.s, l.e); }
38 auto dist(const Point3D &p, const Segment3D &l) {
39     if (l.s == l.e) return dist(p, l.s);
40     auto d = dist2(l.s, l.e), t = min(d, max((ld)0, (p - l.s) *
↪ (l.e - l.s)));
41     return dist((p - l.s) * d, (l.e - l.s) * t) / d;
42 }

```

## Miscellaneous

```
1 tuple<int,int,ld> closest_pair(vector<Point> &p) {
2     using Pt = pair<Point,int>;
3     int n = p.size();
4     assert(n > 1);
5     vector<Pt> pts(n), buf;
6     for (int i = 0; i < n; i++) pts[i] = {p[i], i};
7     sort(pts.begin(), pts.end());
8     buf.reserve(n);
9     auto cmp_y = [](const Pt& p1, const Pt& p2) { return
10     ↪ p1.first.y < p2.first.y; };
11     function<tuple<int,int,ld>(int, int)> recurse = [&](int l,
12     ↪ int r) -> tuple<int,int,ld> {
13         int i = pts[l].second, j = pts[l + 1].second;
14         ld d = dist(pts[l].first, pts[l + 1].first);
15         if (r - l < 5) {
16             for (int a = l; a < r; a++) for (int b = a + 1; b < r;
17             ↪ b++) {
18                 ld cur = dist(pts[a].first, pts[b].first);
19                 if (cur < d) { i = pts[a].second; j = pts[b].second; d
20                 ↪ = cur; }
21             }
22             sort(pts.begin() + l, pts.begin() + r, cmp_y);
23         }
24         else {
25             int mid = (l + r)/2;
26             ld x = pts[mid].first.x;
27             auto [li, lj, ldist] = recurse(l, mid);
28             auto [ri, rj, rdist] = recurse(mid, r);
29             if (ldist < rdist) { i = li; j = lj; d = ldist; }
30             else { i = ri; j = rj; d = rdist; }
31             inplace_merge(pts.begin() + l, pts.begin() + mid,
32             ↪ pts.begin() + r, cmp_y);
33             buf.clear();
34             for (int a = l; a < r; a++) {
35                 if (abs(x - pts[a].first.x) >= d) continue;
36                 for (int b = buf.size() - 1; b >= 0; b--) {
37                     if (pts[a].first.y - buf[b].first.y >= d) break;
38                     ld cur = dist(pts[a].first, buf[b].first);
39                     if (cur < d) { i = pts[a].second; j = buf[b].second;
40                     ↪ d = cur; }
41                 }
42                 buf.push_back(pts[a]);
43             }
44             return {i, j, d};
45         }
46     };
47     return recurse(0, n);
48 }
49
50 Line abc_to_line(ld a, ld b, ld c) {
51     assert(!sgn(a) || !sgn(b));
52     if (a == 0) return Line(Point(0, -c/b), Point(1, -c/b));
53     if (b == 0) return Line(Point(-c/a, 0), Point(-c/a, 1));
54     Point s(0, -c/b), e(1, (-c - a)/b), diff = e - s;
55     return Line(s, s + diff/dist(diff));
56 }
```

```
57 tuple<ld,ld,ld> line_to_abc(const Line& l) {
58     Point diff = l.e - l.s;
59     return {-diff.y, diff.x, -(diff ^ l.s)};
60 }
```

## Graph Theory

### Max Flow

```
1 struct Edge {
2     int from, to, cap, remain;
3 };
4
5 struct Dinic {
6     int n;
7     vector<Edge> e;
8     vector<vector<int>> g;
```

```
9     vector<int> d, cur;
10     Dinic(int _n) : n(_n), g(n), d(n), cur(n) {}
11     void add_edge(int u, int v, int c) {
12         g[u].push_back((int)e.size());
13         e.push_back({u, v, c, c});
14         g[v].push_back((int)e.size());
15         e.push_back({v, u, 0, 0});
16     }
17     ll max_flow(int s, int t) {
18         int inf = 1e9;
19         auto bfs = [&]() {
20             fill(d.begin(), d.end(), inf), fill(cur.begin(),
21             ↪ cur.end(), 0);
22             d[s] = 0;
23             vector<int> q{s}, nq;
24             for (int step = 1; q.size(); swap(q, nq), nq.clear(),
25             ↪ step++) {
26                 for (auto& node : q) {
27                     for (auto& edge : g[node]) {
28                         int ne = e[edge].to;
29                         if (!e[edge].remain || d[ne] <= step) continue;
30                         d[ne] = step, nq.push_back(ne);
31                         if (ne == t) return true;
32                     }
33                 }
34             }
35             return false;
36         };
37         function<int(int, int)> find = [&](int node, int limit) {
38             if (node == t || !limit) return limit;
39             int flow = 0;
40             for (int i = cur[node]; i < g[node].size(); i++) {
41                 cur[node] = i;
42                 int edge = g[node][i], oe = edge ^ 1, ne = e[edge].to;
43                 if (!e[edge].remain || d[ne] != d[node] + 1) continue;
44                 if (int temp = find(ne, min(limit - flow,
45                 ↪ e[edge].remain))) {
46                     e[edge].remain -= temp, e[oe].remain += temp, flow
47                     ↪ += temp;
48                 } else {
49                     d[ne] = -1;
50                 }
51                 if (flow == limit) break;
52             }
53             return flow;
54         };
55         ll res = 0;
56         while (bfs())
57             while (int flow = find(s, inf)) res += flow;
58         return res;
59     }
60 }
```

### • USAGE

```
1 int main() {
2     int n, m, s, t;
3     cin >> n >> m >> s >> t;
4     Dinic dinic(n);
5     for (int i = 0; u, v, c; i < m; i++) {
6         cin >> u >> v >> c;
7         dinic.add_edge(u - 1, v - 1, c);
8     }
9     cout << dinic.max_flow(s - 1, t - 1) << '\n';
10 }
```

### PushRelabel Max-Flow (faster)

```
1 //
2     ↪ https://github.com/kth-competitive-programming/kactl/blob/main/con
3     #define rep(i, a, b) for (int i = a; i < (b); ++i)
4     #define all(x) begin(x), end(x)
5     #define sz(x) (int)(x).size()
6     typedef long long ll;
7     typedef pair<int, int> pii;
8     typedef vector<int> vi;
```



```

9 struct PushRelabel {
10     struct Edge {
11         int dest, back;
12         ll f, c;
13     };
14     vector<vector<Edge>> g;
15     vector<ll> ec;
16     vector<Edge*> cur;
17     vector<vi> hs;
18     vi H;
19     PushRelabel(int n) : g(n), ec(n), cur(n), hs(2 * n), H(n) {}
20
21     void addEdge(int s, int t, ll cap, ll rcap = 0) {
22         if (s == t) return;
23         g[s].push_back({t, sz(g[t]), 0, cap});
24         g[t].push_back({s, sz(g[s]) - 1, 0, rcap});
25     }
26
27     void addFlow(Edge& e, ll f) {
28         Edge& back = g[e.dest][e.back];
29         if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
30         e.f += f;
31         e.c -= f;
32         ec[e.dest] += f;
33         back.f -= f;
34         back.c += f;
35         ec[back.dest] -= f;
36     }
37     ll calc(int s, int t) {
38         int v = sz(g);
39         H[s] = v;
40         ec[t] = 1;
41         vi co(2 * v);
42         co[0] = v - 1;
43         rep(i, 0, v) cur[i] = g[i].data();
44         for (Edge& e : g[s]) addFlow(e, e.c);
45
46         for (int hi = 0;;) {
47             while (hs[hi].empty())
48                 if (!hi--) return -ec[s];
49             int u = hs[hi].back();
50             hs[hi].pop_back();
51             while (ec[u] > 0) // discharge u
52                 if (cur[u] == g[u].data() + sz(g[u])) {
53                     H[u] = 1e9;
54                     for (Edge& e : g[u])
55                         if (e.c && H[u] > H[e.dest] + 1) H[u] = H[e.dest]
56 + 1, cur[u] = &e;
57                     if (++co[H[u]], !--co[hi] && hi < v)
58                         rep(i, 0, v) if (hi < H[i] && H[i] < v)--
59 + co[H[i]], H[i] = v + 1;
60                     hi = H[u];
61                 } else if (cur[u]->c && H[u] == H[cur[u]->dest] + 1)
62                     addFlow(*cur[u], min(ec[u], cur[u]->c));
63                 else
64                     ++cur[u];
65             }
66         }
67     }
68     bool leftOfMinCut(int a) { return H[a] >= sz(g); }
69 };

```

## Min-Cost Max-Flow

```

1 class MCMF {
2 public:
3     static constexpr int INF = 1e9;
4     const int n;
5     vector<tuple<int, int, int>> e;
6     vector<vector<int>> g;
7     vector<int> h, dis, pre;
8     bool dijkstra(int s, int t) {
9         dis.assign(n, INF);
10        pre.assign(n, -1);
11        priority_queue<pair<int, int>, vector<pair<int, int>>,
12 + greater<>> que;
13        dis[s] = 0;
14        que.emplace(0, s);

```

```

14        while (!que.empty()) {
15            auto [d, u] = que.top();
16            que.pop();
17            if (dis[u] != d) continue;
18            for (int i : g[u]) {
19                auto [v, f, c] = e[i];
20                if (c > 0 && dis[v] > d + h[u] - h[v] + f) {
21                    dis[v] = d + h[u] - h[v] + f;
22                    pre[v] = i;
23                    que.emplace(dis[v], v);
24                }
25            }
26        }
27        return dis[t] != INF;
28    }
29    MCMF(int n) : n(n), g(n) {}
30    void add_edge(int u, int v, int fee, int c) {
31        g[u].push_back(e.size());
32        e.emplace_back(v, fee, c);
33        g[v].push_back(e.size());
34        e.emplace_back(u, -fee, 0);
35    }
36    pair<ll, ll> max_flow(const int s, const int t) {
37        int flow = 0, cost = 0;
38        h.assign(n, 0);
39        while (dijkstra(s, t)) {
40            for (int i = 0; i < n; ++i) h[i] += dis[i];
41            for (int i = t; i != s; i = get<0>(e[pre[i] ^ 1])) {
42                --get<2>(e[pre[i]]);
43                ++get<2>(e[pre[i] ^ 1]);
44            }
45            ++flow;
46            cost += h[t];
47        }
48        return {flow, cost};
49    }
50 };

```

## Max Cost Feasible Flow

```

1 struct Edge {
2     int from, to, cap, remain, cost;
3 };
4
5 struct MCMF {
6     int n;
7     vector<Edge> e;
8     vector<vector<int>> g;
9     vector<ll> d, pre;
10    MCMF(int _n) : n(_n), g(n), d(n), pre(n) {}
11    void add_edge(int u, int v, int c, int w) {
12        g[u].push_back((int)e.size());
13        e.push_back({u, v, c, c, w});
14        g[v].push_back((int)e.size());
15        e.push_back({v, u, 0, 0, -w});
16    }
17    pair<ll, ll> max_flow(int s, int t) {
18        ll inf = 1e18;
19        auto spfa = [&]() {
20            fill(d.begin(), d.end(), -inf); // important!
21            vector<int> f(n), seen(n);
22            d[s] = 0, f[s] = 1e9;
23            vector<int> q{s}, nq;
24            for (; q.size(); swap(q, nq), nq.clear()) {
25                for (auto& node : q) {
26                    seen[node] = false;
27                    for (auto& edge : g[node]) {
28                        int ne = e[edge].to, cost = e[edge].cost;
29                        if (!e[edge].remain || d[ne] >= d[node] + cost)
30 + continue;
31                        d[ne] = d[node] + cost, pre[ne] = edge;
32                        f[ne] = min(e[edge].remain, f[node]);
33                        if (!seen[ne]) seen[ne] = true, nq.push_back(ne);
34                    }
35                }
36            }
37            return f[t];
38        };

```

```

37     };
38     ll flow = 0, cost = 0;
39     while (int temp = spfa()) {
40         if (d[t] < 0) break; // important!
41         flow += temp, cost += temp * d[t];
42         for (ll i = t; i != s; i = e[pre[i]].from) {
43             e[pre[i]].remain -= temp, e[pre[i] ^ 1].remain +=
↪ temp;
44         }
45     }
46     return {flow, cost};
47 }
48 };

```

## Heavy-Light Decomposition

```

1 int root = 0, cur = 0;
2 vector<int> parent(n), deep(n), hson(n, -1), top(n), sz(n),
↪ dfn(n, -1);
3 function<int(int, int, int)> dfs = [&](int node, int fa, int
↪ dep) {
4     deep[node] = dep, sz[node] = 1, parent[node] = fa;
5     for (auto &ne : g[node]) {
6         if (ne == fa) continue;
7         sz[node] += dfs(ne, node, dep + 1);
8         if (hson[node] == -1 || sz[ne] > sz[hson[node]]) hson[node]
↪ = ne;
9     }
10    return sz[node];
11 };
12 function<void(int, int)> dfs2 = [&](int node, int t) {
13     top[node] = t, dfn[node] = cur++;
14     if (hson[node] == -1) return;
15     dfs2(hson[node], t);
16     for (auto &ne : g[node]) {
17         if (ne == parent[node] || ne == hson[node]) continue;
18         dfs2(ne, ne);
19     }
20 };
21 // read in graph as vector<vector<int>> g(n)
22 dfs(root, -1, 0), dfs2(root, root);

```

### • USAGE: get LCA

```

1 function<int(int, int)> lca = [&](int x, int y) {
2     while (top[x] != top[y]) {
3         if (deep[top[x]] < deep[top[y]]) swap(x, y);
4         x = parent[top[x]];
5     }
6     return deep[x] < deep[y] ? x : y;
7 };
8
9 vector<ll> light(n);
10 SegTree heavy(n), form_parent(n);
11 // cin >> x >> y, x--, y--;
12 int z = lca(x, y);
13 while (x != z) {
14     if (dfn[top[x]] <= dfn[top[z]]) {
15         // [dfn[z], dfn[x]], from heavy
16         heavy.modify(dfn[z], dfn[x], 1);
17         break;
18     }
19     // x -> top[x];
20     heavy.modify(dfn[top[x]], dfn[x], 1);
21     light[parent[top[x]]] += a[top[x]];
22     x = parent[top[x]];
23 }
24 while (y != z) {
25     if (dfn[top[y]] <= dfn[top[z]]) {
26         // [dfn[z], dfn[y]], from heavy
27         form_parent.modify(dfn[z] + 1, dfn[y] + 1, 1);
28         break;
29     }
30     // y -> top[y];
31     form_parent.modify(dfn[top[y]], dfn[y] + 1, 1);
32     y = parent[top[y]];
33 }

```

## General Unweight Graph Matching

### • Complexity: $O(n^3)$ (?)

```

1 struct BlossomMatch {
2     int n;
3     vector<vector<int>> e;
4     BlossomMatch(int _n) : n(_n), e(_n) {}
5     void add_edge(int u, int v) { e[u].push_back(v),
↪ e[v].push_back(u); }
6     vector<int> find_matching() {
7         vector<int> match(n, -1), vis(n), link(n), f(n), dep(n);
8         function<int(int)> find = [&](int x) { return f[x] == x ?
↪ x : (f[x] = find(f[x])); };
9         auto lca = [&](int u, int v) {
10             u = find(u), v = find(v);
11             while (u != v) {
12                 if (dep[u] < dep[v]) swap(u, v);
13                 u = find(link[match[u]]);
14             }
15             return u;
16 };
17 queue<int> que;
18 auto blossom = [&](int u, int v, int p) {
19     while (find(u) != p) {
20         link[u] = v, v = match[u];
21         if (vis[v] == 0) vis[v] = 1, que.push(v);
22         f[u] = f[v] = p, u = link[v];
23     }
24 };
25 // find an augmenting path starting from u and augment (if
↪ exist)
26 auto augment = [&](int node) {
27     while (!que.empty()) que.pop();
28     iota(f.begin(), f.end(), 0);
29     // vis = 0 corresponds to inner vertices, vis = 1
↪ corresponds to outer vertices
30     fill(vis.begin(), vis.end(), -1);
31     que.push(node);
32     vis[node] = 1, dep[node] = 0;
33     while (!que.empty()) {
34         int u = que.front();
35         que.pop();
36         for (auto v : e[u]) {
37             if (vis[v] == -1) {
38                 vis[v] = 0, link[v] = u, dep[v] = dep[u] + 1;
39                 // found an augmenting path
40                 if (match[v] == -1) {
41                     for (int x = v, y = u, temp; y != -1; x = temp,
↪ y = x == -1 ? -1 : link[x]) {
42                         temp = match[y], match[x] = y, match[y] = x;
43                     }
44                     return;
45                 }
46                 vis[match[v]] = 1, dep[match[v]] = dep[u] + 2;
47                 que.push(match[v]);
48             } else if (vis[v] == 1 && find(v) != find(u)) {
49                 // found a blossom
50                 int p = lca(u, v);
51                 blossom(u, v, p), blossom(v, u, p);
52             }
53         }
54     }
55 };
56 // find a maximal matching greedily (decrease constant)
57 auto greedy = [&]() {
58     for (int u = 0; u < n; ++u) {
59         if (match[u] != -1) continue;
60         for (auto v : e[u]) {
61             if (match[v] == -1) {
62                 match[u] = v, match[v] = u;
63                 break;
64             }
65         }
66     }
67 };
68 greedy();
69 for (int u = 0; u < n; ++u)

```

```

70     if (match[u] == -1) augment(u);
71     return match;
72 }
73 };

```

## Maximum Bipartite Matching

- Needs dinic, complexity  $\approx O(n + m\sqrt{n})$

```

1 struct BipartiteMatch {
2     int l, r;
3     Dinic dinic = Dinic(0);
4     BipartiteMatch(int _l, int _r) : l(_l), r(_r) {
5         dinic = Dinic(l + r + 2);
6         for (int i = 1; i <= l; i++) dinic.add_edge(0, i, 1);
7         for (int i = 1; i <= r; i++) dinic.add_edge(l + i, l + r +
↵ 1, 1);
8     }
9     void add_edge(int u, int v) { dinic.add_edge(u + 1, l + v +
↵ 1, 1); }
10    ll max_matching() { return dinic.max_flow(0, l + r + 1); }
11 };

```

## 2-SAT and Strongly Connected Components

```

1 void scc(vector<vector<int>>& g, int* idx) {
2     int n = g.size(), ct = 0;
3     int out[n];
4     vector<int> ginv[n];
5     memset(out, -1, sizeof out);
6     memset(idx, -1, n * sizeof(int));
7     function<void(int)> dfs = [&](int cur) {
8         out[cur] = INT_MAX;
9         for (int v : g[cur]) {
10             ginv[v].push_back(cur);
11             if (out[v] == -1) dfs(v);
12         }
13         ct++; out[cur] = ct;
14     };
15     vector<int> order;
16     for (int i = 0; i < n; i++) {
17         order.push_back(i);
18         if (out[i] == -1) dfs(i);
19     }
20     sort(order.begin(), order.end(), [&](int& u, int& v) {
21         return out[u] > out[v];
22     });
23     ct = 0;
24     stack<int> s;
25     auto dfs2 = [&](int start) {
26         s.push(start);
27         while (!s.empty()) {
28             int cur = s.top();
29             s.pop();
30             idx[cur] = ct;
31             for (int v : ginv[cur])
32                 if (idx[v] == -1) s.push(v);
33         }
34     };
35     for (int v : order) {
36         if (idx[v] == -1) {
37             dfs2(v);
38             ct++;
39         }
40     }
41 }
42
43 // 0 => impossible, 1 => possible
44 pair<int, vector<int>> sat2(int n, vector<pair<int, int>>&
↵ clauses) {
45     vector<int> ans(n);
46     vector<vector<int>> g(2*n + 1);
47     for (auto [x, y] : clauses) {
48         x = x < 0 ? -x + n : x;
49         y = y < 0 ? -y + n : y;
50         int nx = x <= n ? x + n : x - n;

```

```

51         int ny = y <= n ? y + n : y - n;
52         g[nx].push_back(y);
53         g[ny].push_back(x);
54     }
55     int idx[2*n + 1];
56     scc(g, idx);
57     for (int i = 1; i <= n; i++) {
58         if (idx[i] == idx[i + n]) return {0, {}};
59         ans[i - 1] = idx[i + n] < idx[i];
60     }
61     return {1, ans};
62 }

```

## Enumerating Triangles

- Complexity:  $O(n + m\sqrt{m})$

```

1 void enumerate_triangles(vector<pair<int, int>>& edges,
↵ function<void(int, int, int)> f) {
2     int n = 0;
3     for (auto [u, v] : edges) n = max({n, u + 1, v + 1});
4     vector<int> deg(n);
5     vector<int> g[n];
6     for (auto [u, v] : edges) {
7         deg[u]++;
8         deg[v]++;
9     }
10    for (auto [u, v] : edges) {
11        if (u == v) continue;
12        if (deg[u] > deg[v] || (deg[u] == deg[v] && u > v))
13            swap(u, v);
14        g[u].push_back(v);
15    }
16    vector<int> flag(n);
17    for (int i = 0; i < n; i++) {
18        for (int v : g[i]) flag[v] = 1;
19        for (int v : g[i]) for (int u : g[v]) {
20            if (flag[u]) f(i, v, u);
21        }
22        for (int v : g[i]) flag[v] = 0;
23    }
24 }

```

## Tarjan

- shrink all circles into points (2-edge-connected-component)

```

1 int cnt = 0, now = 0;
2 vector<ll> dfn(n, -1), low(n), belong(n, -1), stk;
3 function<void(ll, ll)> tarjan = [&](ll node, ll fa) {
4     dfn[node] = low[node] = now++; stk.push_back(node);
5     for (auto& ne : g[node]) {
6         if (ne == fa) continue;
7         if (dfn[ne] == -1) {
8             tarjan(ne, node);
9             low[node] = min(low[node], low[ne]);
10        } else if (belong[ne] == -1) {
11            low[node] = min(low[node], dfn[ne]);
12        }
13    }
14    if (dfn[node] == low[node]) {
15        while (true) {
16            auto v = stk.back();
17            belong[v] = cnt;
18            stk.pop_back();
19            if (v == node) break;
20        }
21        ++cnt;
22    }
23 };

```

- 2-vertex-connected-component / Block forest

```

1 int cnt = 0, now = 0;
2 vector<vector<ll>> e1(n);

```



```

3 vector<ll> dfn(n, -1), low(n), stk;
4 function<void(ll)> tarjan = [&](ll node) {
5     dfn[node] = low[node] = now++, stk.push_back(node);
6     for (auto& ne : g[node]) {
7         if (dfn[ne] == -1) {
8             tarjan(ne);
9             low[node] = min(low[node], low[ne]);
10            if (low[ne] == dfn[node]) {
11                e1.push_back({});
12                while (true) {
13                    auto x = stk.back();
14                    stk.pop_back();
15                    e1[n + cnt].push_back(x);
16                    // e1[x].push_back(n + cnt); // undirected
17                    if (x == ne) break;
18                }
19                e1[node].push_back(n + cnt);
20                // e1[n + cnt].push_back(node); // undirected
21                cnt++;
22            }
23        } else {
24            low[node] = min(low[node], dfn[ne]);
25        }
26    }
27 };

```

## Kruskal reconstruct tree

```

1 int _n, m;
2 cin >> _n >> m; // _n: # of node, m: # of edge
3 int n = 2 * _n - 1; // root: n-1
4 vector<array<int, 3>> edges(m);
5 for (auto& [w, u, v] : edges) {
6     cin >> u >> v >> w, u--, v--;
7 }
8 sort(edges.begin(), edges.end());
9 vector<int> p(n);
10 iota(p.begin(), p.end(), 0);
11 function<int(int)> find = [&](int x) { return p[x] == x ? x :
12     ↪ (p[x] = find(p[x])); };
13 auto merge = [&](int x, int y) { p[find(x)] = find(y); };
14 vector<vector<int>> g(n);
15 vector<int> val(m);
16 val.reserve(n);
17 for (auto [w, u, v] : edges) {
18     u = find(u), v = find(v);
19     if (u == v) continue;
20     val.push_back(w);
21     int node = (int)val.size() - 1;
22     g[node].push_back(u), g[node].push_back(v);
23     merge(u, node), merge(v, node);
24 }

```

## Math

### Inverse

```

1 ll inv(ll a, ll m) { return a == 1 ? 1 : ((m - m / a) * inv(m
2     ↪ % a, m) % m); }
3 // or
4 power(a, MOD - 2)

```

- USAGE: get factorial

```

1 vector<Z> f(MAX_N, 1), rf(MAX_N, 1);
2 for (int i = 2; i < MAX_N; i++) f[i] = f[i - 1] * i % MOD;
3 rf[MAX_N - 1] = power(f[MAX_N - 1], MOD - 2);
4 for (int i = MAX_N - 2; i > 1; i--) rf[i] = rf[i + 1] * (i +
5     ↪ 1) % MOD;
6 auto binom = [&](ll n, ll r) -> Z {
7     if (n < 0 || r < 0 || n < r) return 0;
8     return f[n] * rf[n - r] * rf[r];
9 };

```

## Mod Class

```

1 constexpr ll norm(ll x) { return (x % MOD + MOD) % MOD; }
2 template <typename T>
3 constexpr T power(T a, ll b, T res = 1) {
4     for (; b; b /= 2, (a *= a) %= MOD)
5         if (b & 1) (res *= a) %= MOD;
6     return res;
7 }
8 struct Z {
9     ll x;
10    constexpr Z(ll _x = 0) : x(norm(_x)) {}
11    // auto operator<=>(const Z &); const = default; // cpp20
12    ↪ only
13    Z operator-() const { return Z(norm(MOD - x)); }
14    Z inv() const { return power(*this, MOD - 2); }
15    Z &operator*=(const Z &rhs) { return x = x * rhs.x % MOD,
16    ↪ *this; }
17    Z &operator+=(const Z &rhs) { return x = norm(x + rhs.x),
18    ↪ *this; }
19    Z &operator-=(const Z &rhs) { return x = norm(x - rhs.x),
20    ↪ *this; }
21    Z &operator/=(const Z &rhs) { return *this *= rhs.inv(); }
22    Z &operator%=(const ll &rhs) { return x %= rhs, *this; }
23    friend Z operator*(Z lhs, const Z &rhs) { return lhs * rhs;
24    ↪ }
25    friend Z operator+(Z lhs, const Z &rhs) { return lhs += rhs;
26    ↪ }
27    friend Z operator-(Z lhs, const Z &rhs) { return lhs -= rhs;
28    ↪ }
29    friend Z operator/(Z lhs, const Z &rhs) { return lhs /= rhs;
30    ↪ }
31    friend Z operator%(Z lhs, const ll &rhs) { return lhs %=
32    ↪ rhs; }
33    friend auto &operator>>(istream &i, Z &z) { return i >> z.x;
34    ↪ }
35    friend auto &operator<<(ostream &o, const Z &z) { return o
36    ↪ << z.x; }
37 };

```

- large mod (for NTT to do FFT in ll range without modulo)

```

1 using ll = long long;
2 using i128 = __int128;
3 constexpr i128 MOD = 9223372036737335297;
4
5 constexpr i128 norm(i128 x) { return x < 0 ? (x + MOD) % MOD :
6     ↪ x % MOD; }
7 template <typename T>
8 constexpr T power(T a, i128 b, T res = 1) {
9     for (; b; b /= 2, (a *= a) %= MOD)
10         if (b & 1) (res *= a) %= MOD;
11     return res;
12 }
13 struct Z {
14     i128 x;
15     constexpr Z(i128 _x = 0) : x(norm(_x)) {}
16     Z operator-() const { return Z(norm(MOD - x)); }
17     Z inv() const { return power(*this, MOD - 2); }
18     // auto operator<=>(const Z&) const = default;
19     Z &operator*=(const Z &rhs) { return x = x * rhs.x % MOD,
20     ↪ *this; }
21     Z &operator+=(const Z &rhs) { return x = norm(x + rhs.x),
22     ↪ *this; }
23     Z &operator-=(const Z &rhs) { return x = norm(x - rhs.x),
24     ↪ *this; }
25     Z &operator/=(const Z &rhs) { return *this *= rhs.inv(); }
26     Z &operator%=(const i128 &rhs) { return x %= rhs, *this; }
27     friend Z operator*(Z lhs, const Z &rhs) { return lhs * rhs;
28     ↪ }
29     friend Z operator+(Z lhs, const Z &rhs) { return lhs += rhs;
30     ↪ }
31     friend Z operator-(Z lhs, const Z &rhs) { return lhs -= rhs;
32     ↪ }
33     friend Z operator/(Z lhs, const Z &rhs) { return lhs /= rhs;
34     ↪ }
35 };

```

```

27     friend Z operator%(Z lhs, const i128 &rhs) { return lhs %=
    ↪ rhs; }
28 };

    • fastest mod class! be careful with overflow, only use
      when the time limit is tight

1  constexpr int MOD = 998244353;
2
3  constexpr int norm(int x) {
4      if (x < 0) x += MOD;
5      if (x >= MOD) x -= MOD;
6      return x;
7  }
8  template <typename T>
9  constexpr T power(T a, int b, T res = 1) {
10     for (; b; b /= 2, (a *= a) %= MOD)
11         if (b & 1) (res *= a) %= MOD;
12     return res;
13 }
14 struct Z {
15     int x;
16     constexpr Z(int _x = 0) : x(norm(_x)) {}
17     // constexpr auto operator<=>(const Z &) const = default; //
    ↪ cpp20 only
18     constexpr Z operator-(const) { return Z(norm(MOD - x)); }
19     constexpr Z inv() const { return power(*this, MOD - 2); }
20     constexpr Z &operator*=(const Z &rhs) { return x = ll(x) *
    ↪ rhs.x % MOD, *this; }
21     constexpr Z &operator+=(const Z &rhs) { return x = norm(x +
    ↪ rhs.x), *this; }
22     constexpr Z &operator-=(const Z &rhs) { return x = norm(x -
    ↪ rhs.x), *this; }
23     constexpr Z &operator/=(const Z &rhs) { return *this *=
    ↪ rhs.inv(); }
24     constexpr Z &operator%=(const ll &rhs) { return x %= rhs,
    ↪ *this; }
25     constexpr friend Z operator*(Z lhs, const Z &rhs) { return
    ↪ lhs * rhs; }
26     constexpr friend Z operator+(Z lhs, const Z &rhs) { return
    ↪ lhs + rhs; }
27     constexpr friend Z operator-(Z lhs, const Z &rhs) { return
    ↪ lhs - rhs; }
28     constexpr friend Z operator/(Z lhs, const Z &rhs) { return
    ↪ lhs / rhs; }
29     constexpr friend Z operator%(Z lhs, const ll &rhs) { return
    ↪ lhs %= rhs; }
30     friend auto &operator>>(istream &i, Z &z) { return i >> z.x;
    ↪ }
31     friend auto &operator<<(ostream &o, const Z &z) { return o
    ↪ << z.x; }
32 };

```

## Cancer mod class

- Explanation: for some prime modulo  $p$ , maintains numbers of form  $p\hat{x} * y$ , where  $y$  is a nonzero remainder mod  $p$
- Be careful with calling `Cancer(x, y)`, it doesn't fix the input if  $y > p$

```

1  struct Cancer {
2      ll x; ll y;
3      Cancer() : Cancer(0, 1) {}
4      Cancer(ll _y) {
5          x = 0, y = _y;
6          while(y % MOD == 0) {
7              y /= MOD;
8              x++;
9          }
10     }
11     Cancer(ll _x, ll _y) : x(_x), y(_y) {}
12     Cancer inv() { return Cancer(-x, power(y, MOD - 2)); }
13     Cancer operator*(const Cancer &c) { return Cancer(x + c.x,
    ↪ (y * c.y) % MOD); }

```

```

14     Cancer operator*(ll m) {
15         ll p = 0;
16         while(m % MOD == 0) {
17             m /= MOD;
18             p++;
19         }
20         return Cancer(x + p, (m * y) % MOD);
21     }
22     friend auto &operator<<(ostream &o, Cancer c) { return o <<
    ↪ c.x << ' ' << c.y; }
23 };

```

## NTT, FFT, FWT

- ntt

```

1  void ntt(vector<Z> &a, int f) {
2      int n = int(a.size());
3      vector<Z> w(n);
4      vector<int> rev(n);
5      for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] / 2) | ((i
    ↪ & 1) * (n / 2));
6      for (int i = 0; i < n; i++) {
7          if (i < rev[i]) swap(a[i], a[rev[i]]);
8      }
9      Z wn = power(f ? (MOD + 1) / 3 : 3, (MOD - 1) / n);
10     w[0] = 1;
11     for (int i = 1; i < n; i++) w[i] = w[i - 1] * wn;
12     for (int mid = 1; mid < n; mid *= 2) {
13         for (int i = 0; i < n; i += 2 * mid) {
14             for (int j = 0; j < mid; j++) {
15                 Z x = a[i + j], y = a[i + j + mid] * w[n / (2 * mid) *
    ↪ j];
16                 a[i + j] = x + y, a[i + j + mid] = x - y;
17             }
18         }
19     }
20     if (f) {
21         Z iv = power(Z(n), MOD - 2);
22         for (auto& x : a) x *= iv;
23     }
24 }

```

- USAGE: Polynomial multiplication

```

1  vector<Z> mul(vector<Z> a, vector<Z> b) {
2      int n = 1, m = (int)a.size() + (int)b.size() - 1;
3      while (n < m) n *= 2;
4      a.resize(n), b.resize(n);
5      ntt(a, 0), ntt(b, 0);
6      for (int i = 0; i < n; i++) a[i] *= b[i];
7      ntt(a, 1);
8      a.resize(m);
9      return a;
10 }

```

- FFT (should prefer NTT, only use this when input is not integer)

```

1  const double PI = acos(-1);
2  auto mul = [&](const vector<double> &aa, const vector<double> &
    ↪ bb) {
3      int n = (int)aa.size(), m = (int)bb.size(), bit = 1;
4      while ((1 << bit) < n + m - 1) bit++;
5      int len = 1 << bit;
6      vector<complex<double>> a(len), b(len);
7      vector<int> rev(len);
8      for (int i = 0; i < n; i++) a[i].real(aa[i]);
9      for (int i = 0; i < m; i++) b[i].real(bb[i]);
10     for (int i = 0; i < len; i++) rev[i] = (rev[i >> 1] >> 1) |
    ↪ ((i & 1) << (bit - 1));
11     auto fft = [&](vector<complex<double>> &p, int inv) {
12         for (int i = 0; i < len; i++)
13             if (i < rev[i]) swap(p[i], p[rev[i]]);
14         for (int mid = 1; mid < len; mid *= 2) {
15             auto w1 = complex<double>(cos(PI / mid), (inv ? -1 : 1)
    ↪ * sin(PI / mid));

```

```

16     for (int i = 0; i < len; i += mid * 2) {
17         auto wk = complex<double>(1, 0);
18         for (int j = 0; j < mid; j++, wk = wk * w1) {
19             auto x = p[i + j], y = wk * p[i + j + mid];
20             p[i + j] = x + y, p[i + j + mid] = x - y;
21         }
22     }
23 }
24 if (inv == 1) {
25     for (int i = 0; i < len; i++) p[i].real(p[i].real() /
↪ len);
26 }
27 };
28 fft(a, 0), fft(b, 0);
29 for (int i = 0; i < len; i++) a[i] = a[i] * b[i];
30 fft(a, 1);
31 a.resize(n + m - 1);
32 vector<double> res(n + m - 1);
33 for (int i = 0; i < n + m - 1; i++) res[i] = a[i].real();
34 return res;
35 };

```

## Polynomial Class

```

1 using ll = long long;
2 constexpr ll MOD = 998244353;
3
4 ll norm(ll x) { return (x % MOD + MOD) % MOD; }
5 template <class T>
6 T power(T a, ll b, T res = 1) {
7     for (; b; b /= 2, (a *= a) %= MOD)
8         if (b & 1) (res *= a) %= MOD;
9     return res;
10 }
11
12 struct Z {
13     ll x;
14     Z(ll _x = 0) : x(norm(_x)) {}
15     // auto operator<=>(const Z &) const = default;
16     Z operator-() const { return Z(norm(MOD - x)); }
17     Z inv() const { return power(*this, MOD - 2); }
18     Z &operator*=(const Z &rhs) { return x = x * rhs.x % MOD,
↪ *this; }
19     Z &operator+=(const Z &rhs) { return x = norm(x + rhs.x),
↪ *this; }
20     Z &operator-=(const Z &rhs) { return x = norm(x - rhs.x),
↪ *this; }
21     Z &operator/=(const Z &rhs) { return *this *= rhs.inv(); }
22     Z &operator%=(const ll &rhs) { return x %= rhs, *this; }
23     friend Z operator*(Z lhs, const Z &rhs) { return lhs * rhs;
↪ }
24     friend Z operator+(Z lhs, const Z &rhs) { return lhs + rhs;
↪ }
25     friend Z operator-(Z lhs, const Z &rhs) { return lhs - rhs;
↪ }
26     friend Z operator/(Z lhs, const Z &rhs) { return lhs / rhs;
↪ }
27     friend Z operator%(Z lhs, const ll &rhs) { return lhs %=
↪ rhs; }
28     friend auto &operator>>(istream &i, Z &z) { return i >> z.x;
↪ }
29     friend auto &operator<<(ostream &o, const Z &z) { return o
↪ << z.x; }
30 };
31
32 void ntt(vector<Z> &a, int f) {
33     int n = (int)a.size();
34     vector<Z> w(n);
35     vector<int> rev(n);
36     for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] / 2) | ((i
↪ & 1) * (n / 2));
37     for (int i = 0; i < n; i++)
38         if (i < rev[i]) swap(a[i], a[rev[i]]);
39     Z wn = power(ll(f ? (MOD + 1) / 3 : 3), (MOD - 1) / n);
40     w[0] = 1;
41     for (int i = 1; i < n; i++) w[i] = w[i - 1] * wn;
42     for (int mid = 1; mid < n; mid *= 2) {

```

```

43         for (int i = 0; i < n; i += 2 * mid) {
44             for (int j = 0; j < mid; j++) {
45                 Z x = a[i + j], y = a[i + j + mid] * w[n / (2 * mid) *
↪ j];
46                 a[i + j] = x + y, a[i + j + mid] = x - y;
47             }
48         }
49     }
50     if (f) {
51         Z iv = power(Z(n), MOD - 2);
52         for (int i = 0; i < n; i++) a[i] *= iv;
53     }
54 }
55
56 struct Poly {
57     vector<Z> a;
58     Poly() {}
59     Poly(const vector<Z> &a) : a(a) {}
60     int size() const { return (int)a.size(); }
61     void resize(int n) { a.resize(n); }
62     Z operator[](int idx) const {
63         if (idx < 0 || idx >= size()) return 0;
64         return a[idx];
65     }
66     Z &operator[](int idx) { return a[idx]; }
67     Poly mulxk(int k) const {
68         auto b = a;
69         b.insert(b.begin(), k, 0);
70         return Poly(b);
71     }
72     Poly modxk(int k) const { return Poly(vector<Z>(a.begin(),
↪ a.begin() + min(k, size()))); }
73     Poly divxk(int k) const {
74         if (size() <= k) return Poly();
75         return Poly(vector<Z>(a.begin() + k, a.end()));
76     }
77     friend Poly operator+(const Poly &a, const Poly &b) {
78         vector<Z> res(max(a.size(), b.size()));
79         for (int i = 0; i < (int)res.size(); i++) res[i] = a[i] +
↪ b[i];
80         return Poly(res);
81     }
82     friend Poly operator-(const Poly &a, const Poly &b) {
83         vector<Z> res(max(a.size(), b.size()));
84         for (int i = 0; i < (int)res.size(); i++) res[i] = a[i] -
↪ b[i];
85         return Poly(res);
86     }
87     friend Poly operator*(Poly a, Poly b) {
88         if (a.size() == 0 || b.size() == 0) return Poly();
89         int n = 1, m = (int)a.size() + (int)b.size() - 1;
90         while (n < m) n *= 2;
91         a.resize(n), b.resize(n);
92         ntt(a, 0), ntt(b, 0);
93         for (int i = 0; i < n; i++) a[i] *= b[i];
94         ntt(a, 1);
95         a.resize(m);
96         return a;
97     }
98     friend Poly operator*(Z a, Poly b) {
99         for (int i = 0; i < (int)b.size(); i++) b[i] *= a;
100         return b;
101     }
102     friend Poly operator*(Poly a, Z b) {
103         for (int i = 0; i < (int)a.size(); i++) a[i] *= b;
104         return a;
105     }
106     Poly &operator+=(Poly b) { return (*this) = (*this) + b; }
107     Poly &operator-=(Poly b) { return (*this) = (*this) - b; }
108     Poly &operator*=(Poly b) { return (*this) = (*this) * b; }
109     Poly deriv() const {
110         if (a.empty()) return Poly();
111         vector<Z> res(size() - 1);
112         for (int i = 0; i < size() - 1; ++i) res[i] = (i + 1) *
↪ a[i + 1];
113         return Poly(res);
114     }

```

```

115 Poly integr() const {
116     vector<Z> res(size() + 1);
117     for (int i = 0; i < size(); ++i) res[i + 1] = a[i] / (i +
↪ 1);
118     return Poly(res);
119 }
120 Poly inv(int m) const {
121     Poly x({a[0].inv()});
122     int k = 1;
123     while (k < m) {
124         k *= 2;
125         x = (x * (Poly({2}) - modxx(k) * x)).modxx(k);
126     }
127     return x.modxx(m);
128 }
129 Poly log(int m) const { return (deriv() *
↪ inv(m)).integr().modxx(m); }
130 Poly exp(int m) const {
131     Poly x({1});
132     int k = 1;
133     while (k < m) {
134         k *= 2;
135         x = (x * (Poly({1}) - x.log(k) + modxx(k))).modxx(k);
136     }
137     return x.modxx(m);
138 }
139 Poly pow(int k, int m) const {
140     int i = 0;
141     while (i < size() && a[i].x == 0) i++;
142     if (i == size() || 1LL * i * k >= m) {
143         return Poly(vector<Z>(m));
144     }
145     Z v = a[i];
146     auto f = divxx(i) * v.inv();
147     return (f.log(m - i * k) * k).exp(m - i * k).mulxx(i * k)
↪ * power(v, k);
148 }
149 Poly sqrt(int m) const {
150     Poly x({1});
151     int k = 1;
152     while (k < m) {
153         k *= 2;
154         x = (x + (modxx(k) * x.inv(k)).modxx(k)) * ((MOD + 1) /
↪ 2);
155     }
156     return x.modxx(m);
157 }
158 Poly mult(Poly b) const {
159     if (b.size() == 0) return Poly();
160     int n = b.size();
161     reverse(b.a.begin(), b.a.end());
162     return ((*this) * b).divxx(n - 1);
163 }
164 Poly divmod(Poly b) const {
165     auto n = size(), m = b.size();
166     auto t = *this;
167     reverse(t.a.begin(), t.a.end());
168     reverse(b.a.begin(), b.a.end());
169     Poly res = (t * b.inv(n)).modxx(n - m + 1);
170     reverse(res.a.begin(), res.a.end());
171     return res;
172 }
173 vector<Z> eval(vector<Z> x) const {
174     if (size() == 0) return vector<Z>(x.size(), 0);
175     const int n = max(int(x.size()), size());
176     vector<Poly> q(4 * n);
177     vector<Z> ans(x.size());
178     x.resize(n);
179     function<void(int, int, int)> build = [&](int p, int l,
↪ int r) {
180         if (r - l == 1) {
181             q[p] = Poly({1, -x[l]});
182         } else {
183             int m = (l + r) / 2;
184             build(2 * p, l, m), build(2 * p + 1, m, r);
185             q[p] = q[2 * p] * q[2 * p + 1];
186         }

```

```

187     };
188     build(1, 0, n);
189     auto work = [&](auto self, int p, int l, int r, const Poly
↪ &num) -> void {
190         if (r - l == 1) {
191             if (l < int(ans.size())) ans[l] = num[0];
192         } else {
193             int m = (l + r) / 2;
194             self(self, 2 * p, l, m, num.mult(q[2 * p + 1]).modxx(m
↪ - l));
195             self(self, 2 * p + 1, m, r, num.mult(q[2 * p]).modxx(r
↪ - m));
196         }
197     };
198     work(work, 1, 0, n, mult(q[1].inv(n)));
199     return ans;
200 }
201 };

```

## Sieve

### • linear sieve

```

1 vector<int> min_primes(MAX_N), primes;
2 primes.reserve(1e5);
3 for (int i = 2; i < MAX_N; i++) {
4     if (!min_primes[i]) min_primes[i] = i, primes.push_back(i);
5     for (auto& p : primes) {
6         if (p * i >= MAX_N) break;
7         min_primes[p * i] = p;
8         if (i % p == 0) break;
9     }
10 }

```

### • mobius function

```

1 vector<int> min_p(MAX_N), mu(MAX_N), primes;
2 mu[1] = 1, primes.reserve(1e5);
3 for (int i = 2; i < MAX_N; i++) {
4     if (!min_p[i]) {
5         min_p[i] = i;
6         primes.push_back(i);
7         mu[i] = -1;
8     }
9     for (auto p : primes) {
10         if (i * p >= MAX_N) break;
11         min_p[i * p] = p;
12         if (i % p == 0) {
13             mu[i * p] = 0;
14             break;
15         }
16         mu[i * p] = -mu[i];
17     }
18 }

```

### • Euler's totient function

```

1 vector<int> min_p(MAX_N), phi(MAX_N), primes;
2 phi[1] = 1, primes.reserve(1e5);
3 for (int i = 2; i < MAX_N; i++) {
4     if (!min_p[i]) {
5         min_p[i] = i;
6         primes.push_back(i);
7         phi[i] = i - 1;
8     }
9     for (auto p : primes) {
10         if (i * p >= MAX_N) break;
11         min_p[i * p] = p;
12         if (i % p == 0) {
13             phi[i * p] = phi[i] * p;
14             break;
15         }
16         phi[i * p] = phi[i] * phi[p];
17     }
18 }

```

## Gaussian Elimination

```
1 bool is_0(Z v) { return v.x == 0; }
2 Z abs(Z v) { return v; }
3 bool is_0(double v) { return abs(v) < 1e-9; }
4
5 // 1 => unique solution, 0 => no solution, -1 => multiple
   ↪ solutions
6 template <typename T>
7 int gaussian_elimination(vector<vector<T>> &a, int limit) {
8     if (a.empty() || a[0].empty()) return -1;
9     int h = (int)a.size(), w = (int)a[0].size(), r = 0;
10    for (int c = 0; c < limit; c++) {
11        int id = -1;
12        for (int i = r; i < h; i++) {
13            if (!is_0(a[i][c]) && (id == -1 || abs(a[id][c]) <
   ↪ abs(a[i][c]))) {
14                id = i;
15            }
16        }
17        if (id == -1) continue;
18        if (id > r) {
19            swap(a[r], a[id]);
20            for (int j = c; j < w; j++) a[id][j] = -a[id][j];
21        }
22        vector<int> nonzero;
23        for (int j = c; j < w; j++) {
24            if (!is_0(a[r][j])) nonzero.push_back(j);
25        }
26        T inv_a = 1 / a[r][c];
27        for (int i = r + 1; i < h; i++) {
28            if (is_0(a[i][c])) continue;
29            T coeff = -a[i][c] * inv_a;
30            for (int j : nonzero) a[i][j] += coeff * a[r][j];
31        }
32        ++r;
33    }
34    for (int row = h - 1; row >= 0; row--) {
35        for (int c = 0; c < limit; c++) {
36            if (!is_0(a[row][c])) {
37                T inv_a = 1 / a[row][c];
38                for (int i = row - 1; i >= 0; i--) {
39                    if (is_0(a[i][c])) continue;
40                    T coeff = -a[i][c] * inv_a;
41                    for (int j = c; j < w; j++) a[i][j] += coeff *
   ↪ a[row][j];
42                }
43                break;
44            }
45        }
46    } // not-free variables: only it on its line
47    for (int i = r; i < h; i++) if (!is_0(a[i][limit])) return 0;
48    return (r == limit) ? 1 : -1;
49 }
50
51 template <typename T>
52 pair<int, vector<T>> solve_linear(vector<vector<T>> a, const
   ↪ vector<T> &b, int w) {
53    int h = (int)a.size();
54    for (int i = 0; i < h; i++) a[i].push_back(b[i]);
55    int sol = gaussian_elimination(a, w);
56    if (!sol) return {0, vector<T>()};
57    vector<T> x(w, 0);
58    for (int i = 0; i < h; i++) {
59        for (int j = 0; j < w; j++) {
60            if (!is_0(a[i][j])) {
61                x[j] = a[i][w] / a[i][j];
62                break;
63            }
64        }
65    }
66    return {sol, x};
67 }
```

## is\_prime

- (Miller–Rabin primality test)

```
1 i128 power(i128 a, i128 b, i128 MOD = 1, i128 res = 1) {
2     for (; b; b /= 2, (a *= a) %= MOD)
3         if (b & 1) (res *= a) %= MOD;
4     return res;
5 }
6
7 bool is_prime(ll n) {
8     if (n < 2) return false;
9     static constexpr int A[] = {2, 3, 5, 7, 11, 13, 17, 19, 23};
10    int s = __builtin_ctzll(n - 1);
11    ll d = (n - 1) >> s;
12    for (auto a : A) {
13        if (a == n) return true;
14        ll x = (ll)power(a, d, n);
15        if (x == 1 || x == n - 1) continue;
16        bool ok = false;
17        for (int i = 0; i < s - 1; ++i) {
18            x = ll((i128)x * x % n); // potential overflow!
19            if (x == n - 1) {
20                ok = true;
21                break;
22            }
23        }
24        if (!ok) return false;
25    }
26    return true;
27 }
28
29 ll pollard_rho(ll x) {
30     ll s = 0, t = 0, c = rng() % (x - 1) + 1;
31     ll stp = 0, goal = 1, val = 1;
32     for (goal = 1;; goal *= 2, s = t, val = 1) {
33         for (stp = 1; stp <= goal; ++stp) {
34             t = ll(((i128)t * t + c) % x);
35             val = ll(((i128)val * abs(t - s) % x);
36             if ((stp % 127) == 0) {
37                 ll d = gcd(val, x);
38                 if (d > 1) return d;
39             }
40         }
41         ll d = gcd(val, x);
42         if (d > 1) return d;
43     }
44 }
45
46 ll get_max_factor(ll _x) {
47     ll max_factor = 0;
48     function<void(ll)> fac = [&](ll x) {
49         if (x <= max_factor || x < 2) return;
50         if (is_prime(x)) {
51             max_factor = max_factor > x ? max_factor : x;
52             return;
53         }
54         ll p = x;
55         while (p >= x) p = pollard_rho(x);
56         while ((x % p) == 0) x /= p;
57         fac(x), fac(p);
58     };
59     fac(_x);
60     return max_factor;
61 }
```

## Radix Sort

```
1 struct identity {
2     template<typename T>
3     T operator()(const T &x) const {
4         return x;
5     }
6 };
7
8 // A stable sort that sorts in passes of `bits_per_pass` bits
   ↪ at a time.
9 template<typename T, typename T_extract_key = identity>
10 void radix_sort(vector<T> &data, int bits_per_pass = 10, const
   ↪ T_extract_key &extract_key = identity()) {
```

```

11     if (int64_t(data.size()) * (64 -
↳ __builtin_clzll(data.size())) < 2 * (1 << bits_per_pass))
12     {
13         stable_sort(data.begin(), data.end(), [&](const T &a,
↳ const T &b) {
14             return extract_key(a) < extract_key(b);
15         });
16         return;
17     }
18     using T_key = decltype(extract_key(data.front()));
19     T_key minimum = numeric_limits<T_key>::max();
20
21     for (T &x : data)
22         minimum = min(minimum, extract_key(x));
23
24     int max_bits = 0;
25
26     for (T &x : data) {
27         T_key key = extract_key(x);
28         max_bits = max(max_bits, key == minimum ? 0 : 64 -
↳ __builtin_clzll(key - minimum));
29     }
30
31     int passes = max((max_bits + bits_per_pass / 2) /
↳ bits_per_pass, 1);
32
33     if (64 - __builtin_clzll(data.size()) <= 1.5 * passes) {
34         stable_sort(data.begin(), data.end(), [&](const T &a,
↳ const T &b) {
35             return extract_key(a) < extract_key(b);
36         });
37         return;
38     }
39
40     vector<T> buffer(data.size());
41     vector<int> counts;
42     int bits_so_far = 0;
43
44     for (int p = 0; p < passes; p++) {
45         int bits = (max_bits + p) / passes;
46         counts.assign(1 << bits, 0);
47
48         for (T &x : data) {
49             T_key key = T_key(extract_key(x) - minimum);
50             counts[(key >> bits_so_far) & ((1 << bits) -
↳ 1)]++;
51         }
52
53         int count_sum = 0;
54
55         for (int &count : counts) {
56             int current = count;
57             count = count_sum;
58             count_sum += current;
59         }
60
61         for (T &x : data) {
62             T_key key = T_key(extract_key(x) - minimum);
63             int key_section = int((key >> bits_so_far) & ((1
↳ << bits) - 1));
64             buffer[counts[key_section]++] = x;
65         }
66
67         swap(data, buffer);
68         bits_so_far += bits;
69     }
70 }

```

## • USAGE

```

1 radix_sort(edges, 10, [&](const edge &e) -> int { return
↳ abs(e.weight - x); });

```

## lucas

```

1 ll lucas(ll n, ll m, ll p) {
2     if (m == 0) return 1;
3     return (binom(n % p, m % p, p) * lucas(n / p, m / p, p)) %
↳ p;
4 }

```

## parity of n choose m

```

1 (n & m) == m <=> odd

```

## sosdp

### subset sum

```

1 auto f = a;
2 for (int i = 0; i < SZ; i++) {
3     for (int mask = 0; mask < (1 << SZ); mask++) {
4         if (mask & (1 << i)) f[mask] += f[mask ^ (1 << i)];
5     }
6 }

```

## String

## AC Automaton

```

1 struct AC_automaton {
2     int sz = 26;
3     vector<vector<int>> e = {vector<int>(sz)}; // vector is
↳ faster than unordered_map
4     vector<int> fail = {0}, end = {0};
5     vector<int> fast = {0}; // closest end
6
7     int insert(string& s) {
8         int p = 0;
9         for (auto c : s) {
10             c -= 'a';
11             if (!e[p][c]) {
12                 e.emplace_back(sz);
13                 fail.emplace_back();
14                 end.emplace_back();
15                 fast.emplace_back();
16                 e[p][c] = (int)e.size() - 1;
17             }
18             p = e[p][c];
19         }
20         end[p] += 1;
21         return p;
22     }
23
24     void build() {
25         queue<int> q;
26         for (int i = 0; i < sz; i++)
27             if (e[0][i]) q.push(e[0][i]);
28         while (!q.empty()) {
29             int p = q.front();
30             q.pop();
31             fast[p] = end[p] ? p : fast[fail[p]];
32             for (int i = 0; i < sz; i++) {
33                 if (e[p][i]) {
34                     fail[e[p][i]] = e[fail[p]][i];
35                     q.push(e[p][i]);
36                 } else {
37                     e[p][i] = e[fail[p]][i];
38                 }
39             }
40         }
41     }
42 };

```



## KMP

- `nex[i]`: length of longest common prefix & suffix for `pat[0..i]`

```
1 vector<int> get_next(vector<int> &pat) {
2     int m = (int)pat.size();
3     vector<int> nex(m);
4     for (int i = 1, j = 0; i < m; i++) {
5         while (j && pat[j] != pat[i]) j = nex[j - 1];
6         if (pat[j] == pat[i]) j++;
7         nex[i] = j;
8     }
9     return nex;
10 }
```

- kmp match for txt and pat

```
1 auto nex = get_next(pat);
2 for (int i = 0, j = 0; i < n; i++) {
3     while (j && pat[j] != txt[i]) j = nex[j - 1];
4     if (pat[j] == txt[i]) j++;
5     if (j == m) {
6         // do what you want with the match
7         // start index is `i - m + 1`
8         j = nex[j - 1];
9     }
10 }
```

## Z function

- `z[i]`: length of longest common prefix of `s` and `s[i:]`

```
1 vector<int> z_function(string s) {
2     int n = (int)s.size();
3     vector<int> z(n);
4     for (int i = 1, l = 0, r = 0; i < n; ++i) {
5         if (i <= r) z[i] = min(r - i + 1, z[i - l]);
6         while (i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z[i];
7         if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
8     }
9     return z;
10 }
```

## General Suffix Automaton

```
1 constexpr int SZ = 26;
2
3 struct GSAM {
4     vector<vector<int>> e = {vector<int>(SZ)}; // the labeled
5     // edges from node i
6     vector<int> parent = {-1}; // the parent of
7     // i
8     vector<int> length = {0}; // the length of
9     // the longest string
10
11     GSAM(int n) { e.reserve(2 * n), parent.reserve(2 * n),
12     // length.reserve(2 * n); };
13     int extend(int c, int p) { // character, last
14         bool f = true; // if already exist
15         int r = 0; // potential new node
16         if (!e[p][c]) { // only extend when not exist
17             f = false;
18             e.push_back(vector<int>(SZ));
19             parent.push_back(0);
20             length.push_back(length[p] + 1);
21             r = (int)e.size() - 1;
22             for (; ~p && !e[p][c]; p = parent[p]) e[p][c] = r; //
23             // update parents
24         }
25         if (f || ~p) {
26             int q = e[p][c];
27             if (length[q] == length[p] + 1) {
28                 if (f) return q;
29                 parent[r] = q;
30             } else {
31                 e.push_back(e[q]);
32             }
33         }
34     }
35 }
```

```
27     parent.push_back(parent[q]);
28     length.push_back(length[p] + 1);
29     int qq = parent[q] = (int)e.size() - 1;
30     for (; ~p && e[p][c] == q; p = parent[p]) e[p][c] =
31     // qq;
32     if (f) return qq;
33     parent[r] = qq;
34 }
35 return r;
36 }
37 };
```

- Topo sort on GSAM

```
1 ll sz = gsam.e.size();
2 vector<int> c(sz + 1);
3 vector<int> order(sz);
4 for (int i = 1; i < sz; i++) c[gsam.length[i]]++;
5 for (int i = 1; i < sz; i++) c[i] += c[i - 1];
6 for (int i = 1; i < sz; i++) order[c[gsam.length[i]]--] = i;
7 reverse(order.begin(), order.end()); // reverse so that large
8 // len to small
```

- can be used as an ordinary SAM
- USAGE (the number of distinct substring)

```
1 int main() {
2     int n, last = 0;
3     string s;
4     cin >> n;
5     auto a = GSAM();
6     for (int i = 0; i < n; i++) {
7         cin >> s;
8         last = 0; // reset last
9         for (auto&& c : s) last = a.extend(c, last);
10    }
11    ll ans = 0;
12    for (int i = 1; i < a.e.size(); i++) {
13        ans += a.length[i] - a.length[a.parent[i]];
14    }
15    cout << ans << endl;
16    return 0;
17 }
```

## Manacher

```
1 string longest_palindrome(string& s) {
2     // init "abc" -> "~$a#b#c$"
3     vector<char> t{'^', '#'};
4     for (char c : s) t.push_back(c), t.push_back('#');
5     t.push_back('$');
6     // manacher
7     int n = t.size(), r = 0, c = 0;
8     vector<int> p(n, 0);
9     for (int i = 1; i < n - 1; i++) {
10        if (i < r + c) p[i] = min(p[2 * c - i], r + c - i);
11        while (t[i + p[i] + 1] == t[i - p[i] - 1]) p[i]++;
12        if (i + p[i] > r + c) r = p[i], c = i;
13    }
14    // s[i] -> p[2 * i + 2] (even), p[2 * i + 2] (odd)
15    // output answer
16    int index = 0;
17    for (int i = 0; i < n; i++)
18        if (p[index] < p[i]) index = i;
19    return s.substr((index - p[index]) / 2, p[index]);
20 }
```

## Lyndon

- def:  $\text{suf}(s) > s$

```
1 void duval(const string &s) {
2     int n = (int)s.size();
3     for (int i = 0; i < n; i++) {
4         int j = i, k = i + 1;
5         for (; j < n && s[j] <= s[k]; j++, k++)
```

```

6         if (s[j] < s[k]) j = i - 1;
7
8     while (i <= j) {
9         // cout << s.substr(i, k - j) << '\n';
10        i += k - j;
11    }
12 }
13 }

```

## minimal representation

```

1  int k = 0, i = 0, j = 1;
2  while (k < n && i < n && j < n) {
3      if (s[(i + k) % n] == s[(j + k) % n]) {
4          k++;
5      } else {
6          s[(i + k) % n] > s[(j + k) % n] ? i = i + k + 1 : j = j +
↵      k + 1;
7          if (i == j) i++;
8          k = 0;
9      }
10 }
11 i = min(i, j); // from 0

```