

# Fortcoders Code Library

askd, yangster67, Nea1

April 29th 2022

# Contents

## Intro

Main template	1
Fast IO	1
Pragmas (lol)	1

## Data Structures

Segment Tree	2
Recursive	2
Iterating	2
Union Find	3
Fenwick Tree	4
PBDS	4
Treap	4
Implicit treap	5
Persistent implicit treap	5
2D Sparse Table	6
K-D Tree	6
Link/Cut Tree	7

## Geometry

Basic stuff	7
Transformation	7
Relation	8
Area	9
Convex	10
Basic 3D	11
Miscellaneous	11

## Graph Theory

Max Flow	11
PushRelabel Max-Flow (faster)	12
Min-Cost Max-Flow	12
Heavy-Light Decomposition	13
General Unweight Graph Matching	13
Maximum Bipartite Matching	13
2-SAT and Strongly Connected Components	14
Enumerating Triangles	14
Tarjan	14
Kruskal reconstruct tree	15

## Math

Inverse	15
Mod Class	15
NTT, FFT, FWT	16
Polynomial Class	16
Sieve	18
Gaussian Elimination	18
is_prime	19
Radix Sort	19

## String

AC Automaton	20
KMP	20
Z function	20
General Suffix Automaton	20
Manacher	21
Lyndon	21

# Intro

## Main template

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define FOR(x,n) for(int x=0;x<n;x++)
5 #define forn(i, n) for (int i = 0; i < int(n); i++)
6 #define all(v) v.begin(),v.end()
7 using ll = long long;
8 using ld = long double;
9 using pii = pair<int, int>;
10 const char nl = '\n';
11
12 int main() {
13     cin.tie(nullptr)->sync_with_stdio(false);
14     cout << fixed << setprecision(20);
15     // mt19937
16     ↪ rng(chrono::steady_clock::now().time_since_epoch().count());
17 }
```

## Fast IO

```
1 namespace io {
2     constexpr int SIZE = 1 << 16;
3     char buf[SIZE], *head, *tail;
4     char get_char() {
5         if (head == tail) tail = (head = buf) + fread(buf, 1, SIZE,
6             ↪ stdin);
7         return *head++;
8     }
9     ll read() {
10         ll x = 0, f = 1;
11         char c = get_char();
12         for (; !isdigit(c); c = get_char()) (c == '-') && (f = -1);
13         for (; isdigit(c); c = get_char()) x = x * 10 + c - '0';
14         return x * f;
15     }
16     string read_s() {
17         string str;
18         char c = get_char();
19         while (c == ' ' || c == '\n' || c == '\r') c = get_char();
20         while (c != ' ' && c != '\n' && c != '\r') str += c, c =
21             ↪ get_char();
22         return str;
23     }
24     void print(int x) {
25         if (x > 9) print(x / 10);
26         putchar(x % 10 | '0');
27     }
28     void println(int x) { print(x), putchar('\n'); }
29     struct Read {
30         Read& operator>>(ll& x) { return x = read(), *this; }
31         Read& operator>>(long double& x) { return x =
32             ↪ stold(read_s()), *this; }
33     } in;
34 } // namespace io
```

## Pragmas (lol)

```
1 #pragma GCC optimize(2)
2 #pragma GCC optimize(3)
3 #pragma GCC optimize("Ofast")
4 #pragma GCC optimize("inline")
5 #pragma GCC optimize("-fgcse")
6 #pragma GCC optimize("-fgcse-lm")
7 #pragma GCC optimize("-fipa-sra")
8 #pragma GCC optimize("-ftree-pre")
9 #pragma GCC optimize("-ftree-vrp")
10 #pragma GCC optimize("-fpeephole2")
11 #pragma GCC optimize("-ffast-math")
12 #pragma GCC optimize("-fsched-spec")
13 #pragma GCC optimize("unroll-loops")
14 #pragma GCC optimize("-falign-jumps")
15 #pragma GCC optimize("-falign-loops")
```

```

16 #pragma GCC optimize("-falign-labels")
17 #pragma GCC optimize("-fdevirtualize")
18 #pragma GCC optimize("-fcallee-saves")
19 #pragma GCC optimize("-fcrossjumping")
20 #pragma GCC optimize("-fthread-jumps")
21 #pragma GCC optimize("-funroll-loops")
22 #pragma GCC optimize("-fwhole-program")
23 #pragma GCC optimize("-freorder-blocks")
24 #pragma GCC optimize("-fschedule-insns")
25 #pragma GCC optimize("inline-functions")
26 #pragma GCC optimize("-ftree-tail-merge")
27 #pragma GCC optimize("-fschedule-insns2")
28 #pragma GCC optimize("-fstrict-aliasing")
29 #pragma GCC optimize("-fstrict-overflow")
30 #pragma GCC optimize("-falign-functions")
31 #pragma GCC optimize("-fcse-skip-blocks")
32 #pragma GCC optimize("-fcse-follow-jumps")
33 #pragma GCC optimize("-fsched-interblock")
34 #pragma GCC optimize("-fpartial-inlining")
35 #pragma GCC optimize("no-stack-protector")
36 #pragma GCC optimize("-freorder-functions")
37 #pragma GCC optimize("-findirect-inlining")
38 #pragma GCC optimize("-fhoist-adjacent-loads")
39 #pragma GCC optimize("-frerun-cse-after-loop")
40 #pragma GCC optimize("inline-small-functions")
41 #pragma GCC optimize("-finline-small-functions")
42 #pragma GCC optimize("-ftree-switch-conversion")
43 #pragma GCC optimize("-foptimize-sibling-calls")
44 #pragma GCC optimize("-fexpensive-optimizations")
45 #pragma GCC optimize("-funsafe-loop-optimizations")
46 #pragma GCC optimize("inline-functions-called-once")
47 #pragma GCC optimize("-fdelete-null-pointer-checks")
48 #pragma GCC
    ↪ target("sse,sse2,sse3,ssse3,sse4.1,sse4.2,avx,avx2,popcnt,tun

```

```

34 };

```

- Persistent implicit, range query + point update

```

1 struct Node {
2     int lc = 0, rc = 0, p = 0;
3 };
4
5 struct SegTree {
6     vector<Node> t = {{}}; // init all
7     SegTree() = default;
8     SegTree(int n) { t.reserve(n * 20); }
9     int modify(int p, int l, int r, int x, int v) {
10         // p: original node, update a[x] -> v
11         t.push_back(t[p]);
12         int u = (int)t.size() - 1;
13         if (r - l == 1) {
14             t[u].p = v;
15         } else {
16             int m = (l + r) / 2;
17             if (x < m) {
18                 t[u].lc = modify(t[p].lc, l, m, x, v);
19                 t[u].rc = t[p].rc;
20             } else {
21                 t[u].lc = t[p].lc;
22                 t[u].rc = modify(t[p].rc, m, r, x, v);
23             }
24             t[u].p = t[t[u].lc].p + t[t[u].rc].p;
25         }
26         return u;
27     }
28     int query(int p, int l, int r, int x, int y) {
29         // query sum a[x]...a[y-1] rooted at p
30         // t[p] holds the info of [l, r)
31         if (x <= l && r <= y) return t[p].p;
32         int m = (l + r) / 2, res = 0;
33         if (x < m) res += query(t[p].lc, l, m, x, y);
34         if (y > m) res += query(t[p].rc, m, r, x, y);
35         return res;
36     }
37 };

```

## Data Structures

### Segment Tree

#### Recursive

- Implicit segment tree, range query + point update

```

1 struct Node {
2     int lc, rc, p;
3 };
4
5 struct SegTree {
6     vector<Node> t = {{}};
7     SegTree(int n) { t.reserve(n * 40); }
8     int modify(int p, int l, int r, int x, int v) {
9         int u = p;
10        if (p == 0) {
11            t.push_back(t[p]);
12            u = (int)t.size() - 1;
13        }
14        if (r - l == 1) {
15            t[u].p = t[p].p + v;
16        } else {
17            int m = (l + r) / 2;
18            if (x < m) {
19                t[u].lc = modify(t[p].lc, l, m, x, v);
20            } else {
21                t[u].rc = modify(t[p].rc, m, r, x, v);
22            }
23            t[u].p = t[t[u].lc].p + t[t[u].rc].p;
24        }
25        return u;
26    }
27    int query(int p, int l, int r, int x, int y) {
28        if (x <= l && r <= y) return t[p].p;
29        int m = (l + r) / 2, res = 0;
30        if (x < m) res += query(t[p].lc, l, m, x, y);
31        if (y > m) res += query(t[p].rc, m, r, x, y);
32        return res;
33    }

```

#### Iterating

- Iterating, range query + point update

```

1 struct Node {
2     ll v = 0, init = 0;
3 };
4
5 Node pull(const Node &a, const Node &b) {
6     if (!a.init) return b;
7     if (!b.init) return a;
8     Node c;
9     return c;
10 }
11
12 struct SegTree {
13     ll n;
14     vector<Node> t;
15     SegTree(ll _n) : n(_n), t(2 * n){};
16     void modify(ll p, const Node &v) {
17         t[p += n] = v;
18         for (p /= 2; p; p /= 2) t[p] = pull(t[p * 2], t[p * 2 +
    ↪ 1]);
19     }
20     Node query(ll l, ll r) {
21         Node left, right;
22         for (l += n, r += n; l < r; l /= 2, r /= 2) {
23             if (l & 1) left = pull(left, t[l++]);
24             if (r & 1) right = pull(t[--r], right);
25         }
26         return pull(left, right);
27     }
28 };

```

- Iterating, range query + range update

```

1 struct SegTree {
2     ll n, h = 0;
3     vector<Node> t;
4     SegTree(ll _n) : n(_n), h((ll)log2(n)), t(n * 2) {}
5     void apply(ll x, ll v) {
6         if (v == 0) {
7             t[x].one = 0;
8         } else {
9             t[x].one = t[x].total;
10        }
11        t[x].lazy = v;
12    }
13    void build(ll l) {
14        for (l = (l + n) / 2; l > 0; l /= 2) {
15            if (t[l].lazy == -1) {
16                t[l] = pull(t[l * 2], t[l * 2 + 1]);
17            }
18        }
19    }
20    void push(ll l) {
21        l += n;
22        for (ll s = h; s > 0; s--) {
23            ll i = l >> s;
24            if (t[i].lazy != -1) {
25                apply(2 * i, t[i].lazy);
26                apply(2 * i + 1, t[i].lazy);
27            }
28            t[i].lazy = -1;
29        }
30    }
31    void modify(ll l, ll r, int v) {
32        push(l), push(r - 1);
33        ll l0 = l, r0 = r;
34        for (l += n, r += n; l < r; l /= 2, r /= 2) {
35            if (l & 1) apply(l++, v);
36            if (r & 1) apply(--r, v);
37        }
38        build(l0), build(r0 - 1);
39    }
40    Node query(ll l, ll r) {
41        push(l), push(r - 1);
42        Node left, right;
43        for (l += n, r += n; l < r; l /= 2, r /= 2) {
44            if (l & 1) left = pull(left, t[l++]);
45            if (r & 1) right = pull(t[--r], right);
46        }
47        return pull(left, right);
48    }
49 };

```

- AtCoder Segment Tree (recursive structure but iterative)

```

1 template <class T> struct PointSegmentTree {
2     int size = 1;
3     vector<T> tree;
4     PointSegmentTree(int n) : PointSegmentTree(vector<T>(n)) {}
5     PointSegmentTree(vector<T>& arr) {
6         while(size < (int)arr.size())
7             size <<= 1;
8         tree = vector<T>(size << 1);
9         for(int i = size + arr.size() - 1; i >= 1; i--)
10            if(i >= size) tree[i] = arr[i - size];
11            else consume(i);
12    }
13    void set(int i, T val) {
14        tree[i += size] = val;
15        for(i >= 1; i >= 1; i >= 1)
16            consume(i);
17    }
18    T get(int i) { return tree[i + size]; }
19    T query(int l, int r) {
20        T resl, resr;
21        for(l += size, r += size + 1; l < r; l >= 1, r >= 1) {
22            if(l & 1) resl = resl * tree[l++];
23            if(r & 1) resr = tree[--r] * resr;
24        }
25        return resl * resr;
26    }

```

```

27     T query_all() { return tree[1]; }
28     void consume(int i) { tree[i] = tree[i << 1] * tree[i << 1 + 1]; }
29 };
30
31 struct SegInfo {
32     ll v;
33     SegInfo() : SegInfo(0) {}
34     SegInfo(ll val) : v(val) {}
35     SegInfo operator*(SegInfo b) {
36         return SegInfo(v + b.v);
37     }
38 };
39

```

## Union Find

```

1 vector<int> p(n);
2 iota(p.begin(), p.end(), 0);
3 function<int(int)> find = [&](int x) { return p[x] == x ? x :
4     (p[x] = find(p[x])); };
5 auto merge = [&](int x, int y) { p[find(x)] = find(y); };

```

- Persistent version

```

1 struct Node {
2     int lc, rc, p;
3 };
4
5 struct SegTree {
6     vector<Node> t = {{0, 0, -1}}; // init all
7     SegTree() = default;
8     SegTree(int n) { t.reserve(n * 20); }
9     int modify(int p, int l, int r, int x, int v) {
10        // p: original node, update a[x] -> v
11        t.push_back(t[p]);
12        int u = (int)t.size() - 1;
13        if (r - l == 1) {
14            t[u].p = p;
15        } else {
16            int m = (l + r) / 2;
17            if (x < m) {
18                t[u].lc = modify(t[p].lc, l, m, x, v);
19                t[u].rc = t[p].rc;
20            } else {
21                t[u].lc = t[p].lc;
22                t[u].rc = modify(t[p].rc, m, r, x, v);
23            }
24            t[u].p = t[t[u].lc].p + t[t[u].rc].p;
25        }
26        return u;
27    }
28    int query(int p, int l, int r, int x, int y) {
29        // query sum a[x]...a[y-1] rooted at p
30        // t[p] holds the info of [l, r)
31        if (x <= l && r <= y) return t[p].p;
32        int m = (l + r) / 2, res = 0;
33        if (x < m) res += query(t[p].lc, l, m, x, y);
34        if (y > m) res += query(t[p].rc, m, r, x, y);
35        return res;
36    }
37 };
38
39 struct DSU {
40     int n;
41     SegTree seg;
42     DSU(int _n) : n(_n), seg(n) {}
43     int get(int p, int x) { return seg.query(p, 0, n, x, x + 1); }
44     int set(int p, int x, int v) { return seg.modify(p, 0, n, x, v); }
45     int find(int p, int x) {
46         int parent = get(p, x);
47         if (parent < 0) return x;
48         return find(p, parent);
49     }

```

```

50     int is_same(int p, int x, int y) { return find(p, x) ==
    ↪ find(p, y); }
51     int merge(int p, int x, int y) {
52         int rx = find(p, x), ry = find(p, y);
53         if (rx == ry) return -1;
54         int rank_x = -get(p, rx), rank_y = -get(p, ry);
55         if (rank_x < rank_y) {
56             p = set(p, rx, ry);
57         } else if (rank_x > rank_y) {
58             p = set(p, ry, rx);
59         } else {
60             p = set(p, ry, rx);
61             p = set(p, rx, -rx - 1);
62         }
63         return p;
64     }
65 };

```

## Fenwick Tree

- askd version

```

1  template <typename T> struct FenwickTree {
2      int size = 1, high_bit = 1;
3      vector<T> tree;
4      FenwickTree(int _size) : size(_size) {
5          tree.resize(size + 1);
6          while((high_bit << 1) <= size) high_bit <<= 1;
7      }
8      FenwickTree(vector<T>& arr) : FenwickTree(arr.size()) {
9          for(int i = 0; i < size; i++) update(i, arr[i]);
10     }
11     int lower_bound(T x) {
12         int res = 0; T cur = 0;
13         for(int bit = high_bit; bit > 0; bit >>= 1) {
14             if((res|bit) <= size && cur + tree[res|bit] < x) {
15                 res |= bit; cur += tree[res];
16             }
17         }
18         return res;
19     }
20     T prefix_sum(int i) {
21         T ret = 0;
22         for(i++; i > 0; i -= (i & -i)) ret += tree[i];
23         return ret;
24     }
25     T range_sum(int l, int r) { return (l > r) ? 0 :
    ↪ prefix_sum(r) - prefix_sum(l - 1); }
26     void update(int i, T delta) { for(i++; i <= size; i += (i &
    ↪ -i)) tree[i] += delta; }
27 };

```

- Neal version

```

1  template <typename T>
2  struct Fenwick {
3      const int n;
4      vector<T> a;
5      Fenwick(int n) : n(n), a(n) {}
6      void add(int x, T v) {
7          for (int i = x + 1; i <= n; i += i & -i) {
8              a[i - 1] += v;
9          }
10     }
11     T sum(int x) {
12         T ans = 0;
13         for (int i = x; i > 0; i -= i & -i) {
14             ans += a[i - 1];
15         }
16         return ans;
17     }
18     T rangeSum(int l, int r) { return sum(r) - sum(l); }
19 };

```

## PBDS

```

1  #include <bits/stdc++.h>
2  #include <ext/pb_ds/assoc_container.hpp>
3  using namespace std;
4  using namespace __gnu_pbds;
5  template<typename T>
6  using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    ↪ tree_order_statistics_node_update>;
7  template<typename T, typename X>
8  using ordered_map = tree<T, X, less<T>, rb_tree_tag,
    ↪ tree_order_statistics_node_update>;
9  template<typename T, typename X>
10 using fast_map = cc_hash_table<T, X>;
11 template<typename T, typename X>
12 using ht = gp_hash_table<T, X>;
13 mt19937_64
    ↪ rng(chrono::steady_clock::now().time_since_epoch().count());
14
15 struct splitmix64 {
16     size_t operator()(size_t x) const {
17         static const size_t fixed =
    ↪ chrono::steady_clock::now().time_since_epoch().count();
18         x += 0x9e3779b97f4a7c15 + fixed;
19         x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
20         x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
21         return x ^ (x >> 31);
22     }
23 };

```

## Treap

- (No rotation version)

```

1  struct Node {
2      Node *l, *r;
3      int s, sz;
4      // int t = 0, a = 0, g = 0; // for lazy propagation
5      ll w;
6
7      Node(int _s) : l(nullptr), r(nullptr), s(_s), sz(1),
    ↪ w(rng()) {}
8      void apply(int vt, int vg) {
9          // for lazy propagation
10         // s -= vt;
11         // t += vt, a += vg, g += vg;
12     }
13     void push() {
14         // for lazy propagation
15         // if (l != nullptr) l->apply(t, g);
16         // if (r != nullptr) r->apply(t, g);
17         // t = g = 0;
18     }
19     void pull() { sz = 1 + (l ? l->sz : 0) + (r ? r->sz : 0); }
20 };
21
22 std::pair<Node *, Node *> split(Node *t, int v) {
23     if (t == nullptr) return {nullptr, nullptr};
24     t->push();
25     if (t->s < v) {
26         auto [x, y] = split(t->r, v);
27         t->r = x;
28         t->pull();
29         return {t, y};
30     } else {
31         auto [x, y] = split(t->l, v);
32         t->l = y;
33         t->pull();
34         return {x, t};
35     }
36 }
37
38 Node *merge(Node *p, Node *q) {
39     if (p == nullptr) return q;
40     if (q == nullptr) return p;
41     if (p->w < q->w) swap(p, q);
42     auto [x, y] = split(q, p->s + rng() % 2);

```

```

43     p->push();
44     p->l = merge(p->l, x);
45     p->r = merge(p->r, y);
46     p->pull();
47     return p;
48 }
49
50 Node *insert(Node *t, int v) {
51     auto [x, y] = split(t, v);
52     return merge(merge(x, new Node(v)), y);
53 }
54
55 Node *erase(Node *t, int v) {
56     auto [x, y] = split(t, v);
57     auto [p, q] = split(y, v + 1);
58     return merge(merge(x, merge(p->l, p->r)), q);
59 }
60
61 int get_rank(Node *t, int v) {
62     auto [x, y] = split(t, v);
63     int res = (x ? x->sz : 0) + 1;
64     t = merge(x, y);
65     return res;
66 }
67
68 Node *kth(Node *t, int k) {
69     k--;
70     while (true) {
71         int left_sz = t->l ? t->l->sz : 0;
72         if (k < left_sz) {
73             t = t->l;
74         } else if (k == left_sz) {
75             return t;
76         } else {
77             k -= left_sz + 1, t = t->r;
78         }
79     }
80 }
81
82 Node *get_prev(Node *t, int v) {
83     auto [x, y] = split(t, v);
84     Node *res = kth(x, x->sz);
85     t = merge(x, y);
86     return res;
87 }
88
89 Node *get_next(Node *t, int v) {
90     auto [x, y] = split(t, v + 1);
91     Node *res = kth(y, 1);
92     t = merge(x, y);
93     return res;
94 }

```

## • USAGE

```

1 int main() {
2     cin.tie(nullptr)->sync_with_stdio(false);
3     int n;
4     cin >> n;
5     Node *t = nullptr;
6     for (int op, x; n--;) {
7         cin >> op >> x;
8         if (op == 1) {
9             t = insert(t, x);
10        } else if (op == 2) {
11            t = erase(t, x);
12        } else if (op == 3) {
13            cout << get_rank(t, x) << "\n";
14        } else if (op == 4) {
15            cout << kth(t, x)->s << "\n";
16        } else if (op == 5) {
17            cout << get_prev(t, x)->s << "\n";
18        } else {
19            cout << get_next(t, x)->s << "\n";
20        }
21    }
22 }

```

## Implicit treap

### • Split by size

```

1 struct Node {
2     Node *l, *r;
3     int s, sz;
4     // int lazy = 0;
5     ll w;
6
7     Node(int _s) : l(nullptr), r(nullptr), s(_s), sz(1),
8     ↪ w(rnd()) {}
9     void apply() {
10         // for lazy propagation
11         // lazy ^= 1;
12     }
13     void push() {
14         // for lazy propagation
15         // if (lazy) {
16         //     swap(l, r);
17         //     if (l != nullptr) l->apply();
18         //     if (r != nullptr) r->apply();
19         //     lazy = 0;
20         // }
21     void pull() { sz = 1 + (l ? l->sz : 0) + (r ? r->sz : 0); }
22 };
23
24 std::pair<Node *, Node *> split(Node *t, int v) {
25     // first->sz == v
26     if (t == nullptr) return {nullptr, nullptr};
27     t->push();
28     int left_sz = t->l ? t->l->sz : 0;
29     if (left_sz < v) {
30         auto [x, y] = split(t->r, v - left_sz - 1);
31         t->r = x;
32         t->pull();
33         return {t, y};
34     } else {
35         auto [x, y] = split(t->l, v);
36         t->l = y;
37         t->pull();
38         return {x, t};
39     }
40 }
41
42 Node *merge(Node *p, Node *q) {
43     if (p == nullptr) return q;
44     if (q == nullptr) return p;
45     if (p->w < q->w) {
46         p->push();
47         p->r = merge(p->r, q);
48         p->pull();
49         return p;
50     } else {
51         q->push();
52         q->l = merge(p, q->l);
53         q->pull();
54         return q;
55     }
56 }

```

## Persistent implicit treap

```

1 pair<Node *, Node *> split(Node *t, int v) {
2     // first->sz == v
3     if (t == nullptr) return {nullptr, nullptr};
4     t->push();
5     int left_sz = t->l ? t->l->sz : 0;
6     t = new Node(*t);
7     if (left_sz < v) {
8         auto [x, y] = split(t->r, v - left_sz - 1);
9         t->r = x;
10        t->pull();
11        return {t, y};
12    } else {
13        auto [x, y] = split(t->l, v);

```

```

14     t->l = y;
15     t->pull();
16     return {x, t};
17 }
18 }
19
20 Node *merge(Node *p, Node *q) {
21     if (p == nullptr) return new Node(*q);
22     if (q == nullptr) return new Node(*p);
23     if (p->w < q->w) {
24         p = new Node(*p);
25         p->push();
26         p->r = merge(p->r, q);
27         p->pull();
28         return p;
29     } else {
30         q = new Node(*q);
31         q->push();
32         q->l = merge(p, q->l);
33         q->pull();
34         return q;
35     }
36 }

```

## 2D Sparse Table

- Sorry that this sucks - askd

```

1 template <class T, class Compare = less<T>>
2 struct SparseTable2d {
3     int n = 0, m = 0;
4     T**** table;
5     int* log;
6     inline T choose(T x, T y) {
7         return Compare()(x, y) ? x : y;
8     }
9     SparseTable2d(vector<vector<T>>& grid) {
10         if(grid.empty() || grid[0].empty()) return;
11         n = grid.size(); m = grid[0].size();
12         log = new int[max(n, m) + 1];
13         log[1] = 0;
14         for(int i = 2; i <= max(n, m); i++)
15             log[i] = log[i - 1] + ((i ^ (i - 1)) > i);
16         table = new T***[n];
17         for(int i = n - 1; i >= 0; i--) {
18             table[i] = new T**[m];
19             for(int j = m - 1; j >= 0; j--) {
20                 table[i][j] = new T*[log[n - i] + 1];
21                 for(int k = 0; k <= log[n - i]; k++) {
22                     table[i][j][k] = new T[log[m - j] + 1];
23                     if(!k) table[i][j][k][0] = grid[i][j];
24                     else table[i][j][k][0] = choose(table[i][j][k-1][0],
25     table[i][j+(1<<(k-1))][j][k-1][0]);
26                     for(int l = 1; l <= log[m - j]; l++)
27                         table[i][j][k][l] = choose(table[i][j][k][l-1],
28     table[i][j+(1<<(l-1))][k][l-1]);
29                 }
30             }
31         }
32         T query(int r1, int r2, int c1, int c2) {
33             assert(r1 >= 0 && r2 < n && r1 <= r2);
34             assert(c1 >= 0 && c2 < m && c1 <= c2);
35             int r1 = log[r2 - r1 + 1], c1 = log[c2 - c1 + 1];
36             T ca1 = choose(table[r1][c1][r1][c1],
37     table[r2-(1<<r1)+1][c1][r1][c1]);
38             T ca2 = choose(table[r1][c2-(1<<c1)+1][r1][c1],
39     table[r2-(1<<r1)+1][c2-(1<<c1)+1][r1][c1]);
40             return choose(ca1, ca2);
41         }
42     };
43 };

```

- USAGE

```

1 vector<vector<int>> test = {
2     {1, 2, 3, 4}, {2, 3, 4, 5}, {9, 9, 9, 9}, {-1, -1, -1, -1}
3 };

```

```

4
5 SparseTable2d<int> st(test); // Range min query
6 SparseTable2d<int,greater<int>> st2(test); // Range max query

```

## K-D Tree

```

1 struct Point {
2     int x, y;
3 };
4 struct Rectangle {
5     int lx, rx, ly, ry;
6 };
7
8 bool is_in(const Point &p, const Rectangle &rg) {
9     return (p.x >= rg.lx) && (p.x <= rg.rx) && (p.y >= rg.ly) &&
10     (p.y <= rg.ry);
11 }
12
13 struct KDTree {
14     vector<Point> points;
15     struct Node {
16         Point point;
17         Rectangle range;
18         int num;
19     };
20     vector<Node> nodes;
21     int root = -1;
22     KDTree(const vector<Point> &points_) {
23         points = points_;
24         Rectangle range = {-1e9, 1e9, -1e9, 1e9};
25         root = tree_construct(0, (int)points.size(), range, 0);
26     }
27     int tree_construct(int l, int r, Rectangle range, int depth)
28     {
29         if (l == r) return -1;
30         if (l > r) throw;
31         int mid = (l + r) / 2;
32         auto comp = (depth % 2) ? [](Point &a, Point &b) { return
33     a.x < b.x; }
34     : [](Point &a, Point &b) { return
35     a.y < b.y; };
36         nth_element(points.begin() + l, points.begin() + mid,
37     points.begin() + r, comp);
38         Rectangle l_range(range), r_range(range);
39         if (depth % 2) {
40             l_range.rx = points[mid].x;
41             r_range.lx = points[mid].x;
42         } else {
43             l_range.ry = points[mid].y;
44             r_range.ly = points[mid].y;
45         }
46         Node node = {tree_construct(l, mid, l_range, depth + 1),
47     tree_construct(mid + 1, r, r_range, depth +
48     1), points[mid], range, r - l};
49         nodes.push_back(node);
50         return (int)nodes.size() - 1;
51     }
52
53     int inner_query(int id, const Rectangle &rec, int depth) {
54         if (id == -1) return 0;
55         Rectangle rg = nodes[id].range;
56         if (rg.lx >= rec.lx && rg.rx <= rec.rx && rg.ly >= rec.ly
57     && rg.ry <= rec.ry) {
58             return nodes[id].num;
59         }
60         int ans = 0;
61         if (depth % 2) { // pruning
62             if (rec.lx <= nodes[id].point.x) ans +=
63     inner_query(nodes[id].lc, rec, depth + 1);
64             if (rec.rx >= nodes[id].point.x) ans +=
65     inner_query(nodes[id].rc, rec, depth + 1);
66         } else {
67             if (rec.ly <= nodes[id].point.y) ans +=
68     inner_query(nodes[id].lc, rec, depth + 1);
69             if (rec.ry >= nodes[id].point.y) ans +=
70     inner_query(nodes[id].rc, rec, depth + 1);
71         }
72     }
73 };

```



```

61     }
62     if (is_in(nodes[id].point, rec)) ans += 1;
63     return ans;
64 }
65 int query(const Rectangle &rec) { return inner_query(root,
↪ rec, 0); }
66 };

```

## Link/Cut Tree

```

1 struct Node {
2     Node *ch[2], *p;
3     int id;
4     bool rev;
5     Node(int id) : ch{nullptr, nullptr}, p(nullptr), id(id),
↪ rev(false) {}
6     friend void reverse(Node *p) {
7         if (p != nullptr) {
8             swap(p->ch[0], p->ch[1]);
9             p->rev ^= 1;
10        }
11    }
12    void push() {
13        if (rev) {
14            reverse(ch[0]);
15            reverse(ch[1]);
16            rev = false;
17        }
18    }
19    void pull() {}
20    bool is_root() { return p == nullptr || p->ch[0] != this &&
↪ p->ch[1] != this; }
21    bool pos() { return p->ch[1] == this; }
22    void rotate() {
23        Node *q = p;
24        bool x = !pos();
25        q->ch[!x] = ch[x];
26        if (ch[x] != nullptr) ch[x]->p = q;
27        p = q->p;
28        if (!q->is_root()) q->p->ch[q->pos()] = this;
29        ch[x] = q;
30        q->p = this;
31        pull();
32        q->pull();
33    }
34    void splay() {
35        vector<Node*> s;
36        for (Node *i = this; !i->is_root(); i = i->p)
↪ s.push_back(i->p);
37        while (!s.empty()) s.back()->push(), s.pop_back();
38        push();
39        while (!is_root()) {
40            if (!p->is_root()) {
41                if (pos() == p->pos()) {
42                    p->rotate();
43                } else {
44                    rotate();
45                }
46            }
47            rotate();
48        }
49        pull();
50    }
51    void access() {
52        for (Node *i = this, *q = nullptr; i != nullptr; q = i, i
↪ = i->p) {
53            i->splay();
54            i->ch[1] = q;
55            i->pull();
56        }
57        splay();
58    }
59    void makeroot() {
60        access();
61        reverse(this);
62    }

```

```

63 };
64 void link(Node *x, Node *y) {
65     x->makeroot();
66     x->p = y;
67 }
68 void split(Node *x, Node *y) {
69     x->makeroot();
70     y->access();
71 }
72 void cut(Node *x, Node *y) {
73     split(x, y);
74     x->p = y->ch[0] = nullptr;
75     y->pull();
76 }
77 bool connected(Node *p, Node *q) {
78     p->access();
79     q->access();
80     return p->p != nullptr;
81 }

```

## Geometry

### Basic stuff

```

1 using ll = long long;
2 using ld = long double;
3
4 constexpr auto eps = 1e-8;
5 const auto PI = acos(-1);
6 int sgn(ld x) { return (abs(x) <= eps) ? 0 : (x < 0 ? -1 : 1);
↪ }
7
8 struct Point {
9     ld x = 0, y = 0;
10    Point() = default;
11    Point(ld _x, ld _y) : x(_x), y(_y) {}
12    bool operator<(const Point &p) const { return !sgn(p.x - x)
↪ ? sgn(y - p.y) < 0 : x < p.x; }
13    bool operator==(const Point &p) const { return !sgn(p.x - x)
↪ && !sgn(p.y - y); }
14    Point operator+(const Point &p) const { return {x + p.x, y +
↪ p.y}; }
15    Point operator-(const Point &p) const { return {x - p.x, y -
↪ p.y}; }
16    Point operator*(ld a) const { return {x * a, y * a}; }
17    Point operator/(ld a) const { return {x / a, y / a}; }
18    auto operator*(const Point &p) const { return x * p.x + y *
↪ p.y; } // dot
19    auto operator^(const Point &p) const { return x * p.y - y *
↪ p.x; } // cross
20    friend auto &operator>>(istream &i, Point &p) { return i >>
↪ p.x >> p.y; }
21    friend auto &operator<<(ostream &o, Point p) { return o <<
↪ p.x << ' ' << p.y; }
22 };
23
24 struct Line {
25     Point s = {0, 0}, e = {0, 0};
26     Line() = default;
27     Line(Point _s, Point _e) : s(_s), e(_e) {}
28     friend auto &operator>>(istream &i, Line &l) { return i >>
↪ l.s >> l.e; } // ((x1, y1), (x2, y2))
29 };
30
31 struct Segment : Line {
32     using Line::Line;
33 };
34
35 struct Circle {
36     Point o = {0, 0};
37     ld r = 0;
38     Circle() = default;
39     Circle(Point _o, ld _r) : o(_o), r(_r) {}
40 };
41
42 auto dist2(const Point &a) { return a * a; }

```



```

2  auto dist2(const Point &a, const Point &b) { return dist2(a -
    ↪ b); }
3  auto dist(const Point &a) { return sqrt(dist2(a)); }
4  auto dist(const Point &a, const Point &b) { return
    ↪ sqrt(dist2(a - b)); }
5  auto dist(const Point &a, const Line &l) { return abs((a -
    ↪ l.s) ^ (l.e - l.s)) / dist(l.s, l.e); }
6  auto dist(const Point &p, const Segment &l) {
7      if (l.s == l.e) return dist(p, l.s);
8      auto d = dist2(l.s, l.e), t = min(d, max((ld)0, (p - l.s) *
    ↪ (l.e - l.s)));
9      return dist((p - l.s) * d, (l.e - l.s) * t) / d;
10 }
11 /* Needs is_intersect
12 auto dist(const Segment &l1, const Segment &l2) {
13     if (is_intersect(l1, l2)) return (ld)0;
14     return min({dist(l1.s, l2), dist(l1.e, l2), dist(l2.s, l1),
    ↪ dist(l2.e, l1)});
15 } */
16
17 Point perp(const Point &p) { return Point(-p.y, p.x); }
18
19 auto rad(const Point &p) { return atan2(p.y, p.x); }

```

## Transformation

```

1  Point project(const Point &p, const Line &l) {
2      return l.s + ((l.e - l.s) * ((l.e - l.s) * (p - l.s))) /
    ↪ dist2(l.e - l.s);
3  }
4
5  Point reflect(const Point &p, const Line &l) {
6      return project(p, l) * 2 - p;
7  }
8
9  Point dilate(const Point &p, ld scale_x = 1, ld scale_y = 1) {
    ↪ return Point(p.x * scale_x, p.y * scale_y); }
10 Line dilate(const Line &l, ld scale_x = 1, ld scale_y = 1) {
    ↪ return Line(dilate(l.s, scale_x, scale_y), dilate(l.e,
    ↪ scale_x, scale_y)); }
11 Segment dilate(const Segment &l, ld scale_x = 1, ld scale_y =
    ↪ 1) { return Segment(dilate(l.s, scale_x, scale_y),
    ↪ dilate(l.e, scale_x, scale_y)); }
12 vector<Point> dilate(const vector<Point> &p, ld scale_x = 1,
    ↪ ld scale_y = 1) {
13     int n = p.size();
14     vector<Point> res(n);
15     for (int i = 0; i < n; i++)
16         res[i] = dilate(p[i], scale_x, scale_y);
17     return res;
18 }
19
20 Point rotate(const Point &p, ld a) { return Point(p.x * cos(a)
    ↪ - p.y * sin(a), p.x * sin(a) + p.y * cos(a)); }
21 Line rotate(const Line &l, ld a) { return Line(rotate(l.s, a),
    ↪ rotate(l.e, a)); }
22 Segment rotate(const Segment &l, ld a) { return
    ↪ Segment(rotate(l.s, a), rotate(l.e, a)); }
23 Circle rotate(const Circle &c, ld a) { return
    ↪ Circle(rotate(c.o, a), c.r); }
24 vector<Point> rotate(const vector<Point> &p, ld a) {
25     int n = p.size();
26     vector<Point> res(n);
27     for (int i = 0; i < n; i++)
28         res[i] = rotate(p[i], a);
29     return res;
30 }
31
32 Point translate(const Point &p, ld dx = 0, ld dy = 0) { return
    ↪ Point(p.x + dx, p.y + dy); }
33 Line translate(const Line &l, ld dx = 0, ld dy = 0) { return
    ↪ Line(translate(l.s, dx, dy), translate(l.e, dx, dy)); }
34 Segment translate(const Segment &l, ld dx = 0, ld dy = 0) {
    ↪ return Segment(translate(l.s, dx, dy), translate(l.e, dx,
    ↪ dy)); }
35 Circle translate(const Circle &c, ld dx = 0, ld dy = 0) {
    ↪ return Circle(translate(c.o, dx, dy), c.r); }

```

```

36 vector<Point> translate(const vector<Point> &p, ld dx = 0, ld
    ↪ dy = 0) {
37     int n = p.size();
38     vector<Point> res(n);
39     for (int i = 0; i < n; i++)
40         res[i] = translate(p[i], dx, dy);
41     return res;
42 }

```

## Relation

```

1  enum class Relation { SEPARATE, EX_TOUCH, OVERLAP, IN_TOUCH,
    ↪ INSIDE };
2  Relation get_relation(const Circle &a, const Circle &b) {
3      auto c1c2 = dist(a.o, b.o);
4      auto r1r2 = a.r + b.r, diff = abs(a.r - b.r);
5      if (sgn(c1c2 - r1r2) > 0) return Relation::SEPARATE;
6      if (sgn(c1c2 - r1r2) == 0) return Relation::EX_TOUCH;
7      if (sgn(c1c2 - diff) > 0) return Relation::OVERLAP;
8      if (sgn(c1c2 - diff) == 0) return Relation::IN_TOUCH;
9      return Relation::INSIDE;
10 }
11
12 auto get_cos_from_triangle(ld a, ld b, ld c) { return (a * a +
    ↪ b * b - c * c) / (2.0 * a * b); }
13
14 bool on_line(const Line &l, const Point &p) { return !sgn((l.s
    ↪ - p) ^ (l.e - p)); }
15
16 bool on_segment(const Segment &l, const Point &p) {
17     return !sgn((l.s - p) ^ (l.e - p)) && sgn((l.s - p) * (l.e -
    ↪ p)) <= 0;
18 }
19
20 bool on_segment2(const Segment &l, const Point &p) { // assume
    ↪ p on Line l
21     if (l.s == p || l.e == p) return true;
22     if (min(l.s, l.e) < p && p < max(l.s, l.e)) return true;
23     return false;
24 }
25
26 bool is_parallel(const Line &a, const Line &b) { return
    ↪ !sgn((a.s - a.e) ^ (b.s - b.e)); }
27 bool is_orthogonal(const Line &a, const Line &b) { return
    ↪ !sgn((a.s - a.e) * (b.s - b.e)); }
28
29 int is_intersect(const Segment &a, const Segment &b) {
30     auto d1 = sgn((a.e - a.s) ^ (b.s - a.s)), d2 = sgn((a.e -
    ↪ a.s) ^ (b.e - a.s));
31     auto d3 = sgn((b.e - b.s) ^ (a.s - b.s)), d4 = sgn((b.e -
    ↪ b.s) ^ (a.e - b.s));
32     if (d1 * d2 < 0 && d3 * d4 < 0) return 2; // intersect at
    ↪ non-end point
33     return (d1 == 0 && sgn((b.s - a.s) * (b.s - a.e)) <= 0) ||
    ↪ (d2 == 0 && sgn((b.e - a.s) * (b.e - a.e)) <= 0) ||
    ↪ (d3 == 0 && sgn((a.s - b.s) * (a.s - b.e)) <= 0) ||
    ↪ (d4 == 0 && sgn((a.e - b.s) * (a.e - b.e)) <= 0);
34 }
35
36 int is_intersect(const Line &a, const Segment &b) {
37     auto d1 = sgn((a.e - a.s) ^ (b.s - a.s)), d2 = sgn((a.e -
    ↪ a.s) ^ (b.e - a.s));
38     if (d1 * d2 < 0) return 2; // intersect at non-end point
39     return d1 == 0 || d2 == 0;
40 }
41
42 Point intersect(const Line &a, const Line &b) {
43     auto u = a.e - a.s, v = b.e - b.s;
44     auto t = ((b.s - a.s) ^ v) / (u ^ v);
45     return a.s + u * t;
46 }
47
48 int is_intersect(const Circle &c, const Line &l) {
49     auto d = dist(c.o, l);
50     return sgn(d - c.r) < 0 ? 2 : !sgn(d - c.r);
51 }
52
53 }
54
55

```

```

56 vector<Point> intersect(const Circle &a, const Circle &b) {
57     auto relation = get_relation(a, b);
58     if (relation == Relation::INSIDE || relation ==
    ↪ Relation::SEPARATE) return {};
59     auto vec = b.o - a.o;
60     auto d2 = dist2(vec);
61     auto p = (d2 + a.r * a.r - b.r * b.r) / ((long double)2 *
    ↪ d2), h2 = a.r * a.r - p * p * d2;
62     auto mid = a.o + vec * p, per = perp(vec) * sqrt(max((long
    ↪ double)0, h2) / d2);
63     if (relation == Relation::OVERLAP)
64         return {mid + per, mid - per};
65     else
66         return {mid};
67 }
68
69 vector<Point> intersect(const Circle &c, const Line &l) {
70     if (!is_intersect(c, l)) return {};
71     auto v = l.e - l.s, t = v / dist(v);
72     Point a = l.s + t * ((c.o - l.s) * t);
73     auto d = sqrt(max((ld)0, c.r * c.r - dist2(c.o, a)));
74     if (!sgn(d)) return {a};
75     return {a - t * d, a + t * d};
76 }
77
78 int in_poly(const vector<Point> &p, const Point &a) {
79     int cnt = 0, n = (int)p.size();
80     for (int i = 0; i < n; i++) {
81         auto q = p[(i + 1) % n];
82         if (on_segment(Segment(p[i], q), a)) return 1; // on the
    ↪ edge of the polygon
83         cnt ^= ((a.y < p[i].y) - (a.y < q.y)) * ((p[i] - a) ^ (q -
    ↪ a)) > 0;
84     }
85     return cnt ? 2 : 0;
86 }
87
88 int is_intersect(const vector<Point> &p, const Line &a) {
89     // 1: touching, >=2: intersect count
90     int cnt = 0, edge_cnt = 0, n = (int)p.size();
91     for (int i = 0; i < n; i++) {
92         auto q = p[(i + 1) % n];
93         if (on_line(a, p[i]) && on_line(a, q)) return -1; //
    ↪ infinity
94         auto t = is_intersect(a, Segment(p[i], q));
95         (t == 1) && edge_cnt++, (t == 2) && cnt++;
96     }
97     return cnt + edge_cnt / 2;
98 }
99
100 vector<Point> tangent(const Circle &c, const Point &p) {
101     auto d = dist(c.o, p), l = c.r * c.r / d, h = sqrt(c.r * c.r
    ↪ - l * l);
102     auto v = (p - c.o) / d;
103     return {c.o + v * l + perp(v) * h, c.o + v * l - perp(v) *
    ↪ h};
104 }
105
106 Circle get_circumscribed(const Point &a, const Point &b, const
    ↪ Point &c) {
107     Line u((a + b) / 2, ((a + b) / 2) + perp(b - a));
108     Line v((b + c) / 2, ((b + c) / 2) + perp(c - b));
109     auto o = intersect(u, v);
110     return Circle(o, dist(o, a));
111 }
112
113 Circle get_inscribed(const Point &a, const Point &b, const
    ↪ Point &c) {
114     auto l1 = dist(b - c), l2 = dist(c - a), l3 = dist(a - b);
115     Point o = (a * l1 + b * l2 + c * l3) / (l1 + l2 + l3);
116     return Circle(o, dist(o, Line(a, b)));
117 }
118
119 pair<ld, ld> get_centroid(const vector<Point> &p) {
120     int n = (int)p.size();
121     ld x = 0, y = 0, sum = 0;
122     auto a = p[0], b = p[1];

```

```

123     for (int i = 2; i < n; i++) {
124         auto c = p[i];
125         auto s = area({a, b, c});
126         sum += s;
127         x += s * (a.x + b.x + c.x);
128         y += s * (a.y + b.y + c.y);
129         swap(b, c);
130     }
131     return {x / (3 * sum), y / (3 * sum)};
132 }

```

## Area

```

1 auto area(const vector<Point> &p) {
2     int n = (int)p.size();
3     long double area = 0;
4     for (int i = 0; i < n; i++) area += p[i] ^ p[(i + 1) % n];
5     return area / 2.0;
6 }
7
8 auto area(const Point &a, const Point &b, const Point &c) {
9     return ((long double)((b - a) ^ (c - a))) / 2.0;
10 }
11
12 auto area2(const Point &a, const Point &b, const Point &c) {
13     ↪ return (b - a) ^ (c - a); }
14
15 auto area_intersect(const Circle &c, const vector<Point> &ps)
    ↪ {
16     int n = (int)ps.size();
17     auto arg = [&](const Point &p, const Point &q) { return
    ↪ atan2(p ^ q, p * q); };
18     auto tri = [&](const Point &p, const Point &q) {
19         auto r2 = c.r * c.r / (long double)2;
20         auto d = q - p;
21         auto a = d * p / dist2(d), b = (dist2(p) - c.r * c.r) /
    ↪ dist2(d);
22         long double det = a * a - b;
23         if (sgn(det) <= 0) return arg(p, q) * r2;
24         auto s = max((long double)0, -a - sqrt(det)), t =
    ↪ min((long double)1, -a + sqrt(det));
25         if (sgn(t) < 0 || sgn(1 - s) <= 0) return arg(p, q) * r2;
26         auto u = p + d * s, v = p + d * t;
27         return arg(p, u) * r2 + (u ^ v) / 2 + arg(v, q) * r2;
28     };
29     long double sum = 0;
30     for (int i = 0; i < n; i++) sum += tri(ps[i] - c.o, ps[(i +
    ↪ 1) % n] - c.o);
31     return sum;
32 }
33
34 auto adaptive_simpson(ld _l, ld _r, function<ld(ld)> f) {
35     auto simpson = [&](ld l, ld r) { return (r - l) * (f(l) + 4
    ↪ * f((l + r) / 2) + f(r)) / 6; };
36     function<ld(ld, ld, ld)> asr = [&](ld l, ld r, ld s) {
37         auto mid = (l + r) / 2;
38         auto left = simpson(l, mid), right = simpson(mid, r);
39         if (!sgn(left + right - s)) return left + right;
40         return asr(l, mid, left) + asr(mid, r, right);
41     };
42     return asr(_l, _r, simpson(_l, _r));
43 }
44
45 vector<Point> half_plane_intersect(vector<Line> &L) {
46     int n = (int)L.size(), l = 0, r = 0; // [left, right]
47     sort(L.begin(), L.end(),
    ↪ [&](const Line &a, const Line &b) { return rad(a.s -
    ↪ a.e) < rad(b.s - b.e); });
48     vector<Point> p(n), res;
49     vector<Line> q(n);
50     q[0] = L[0];
51     for (int i = 1; i < n; i++) {
52         while (l < r && sgn((L[i].e - L[i].s) ^ (p[r - 1] -
    ↪ L[i].s)) <= 0) r--;
53         while (l < r && sgn((L[i].e - L[i].s) ^ (p[l] - L[i].s))
    ↪ <= 0) l++;
54         q[++r] = L[i];

```

```

55     if (sgn((q[r].e - q[r].s) ^ (q[r - 1].e - q[r - 1].s)) ==
↪ 0) {
56         r--;
57         if (sgn((q[r].e - q[r].s) ^ (L[i].s - q[r].s)) > 0) q[r]
↪ = L[i];
58     }
59     if (1 < r) p[r - 1] = intersect(q[r - 1], q[r]);
60 }
61 while (1 < r && sgn((q[l].e - q[l].s) ^ (p[r - 1] - q[l].s))
↪ <= 0) r--;
62 if (r - 1 <= 1) return {};
63 p[r] = intersect(q[r], q[l]);
64 return vector<Point>(p.begin() + 1, p.begin() + r + 1);
65 }

```

## Convex

```

1 vector<Point> get_convex(vector<Point> &points, bool
↪ allow_collinear = false) {
2     // strict, no repeat, two pass
3     sort(points.begin(), points.end());
4     points.erase(unique(points.begin(), points.end()),
↪ points.end());
5     vector<Point> L, U;
6     for (auto &t : points) {
7         for (ll sz = L.size(); sz > 1 && (sgn((t - L[sz - 2]) ^
↪ (L[sz - 1] - L[sz - 2])) >= 0);
8             L.pop_back(), sz = L.size()) {
9         }
10        L.push_back(t);
11    }
12    for (auto &t : points) {
13        for (ll sz = U.size(); sz > 1 && (sgn((t - U[sz - 2]) ^
↪ (U[sz - 1] - U[sz - 2])) <= 0);
14            U.pop_back(), sz = U.size()) {
15        }
16        U.push_back(t);
17    }
18    // contain repeats if all collinear, use a set to remove
↪ repeats
19    if (allow_collinear) {
20        for (int i = (int)U.size() - 2; i >= 1; i--)
↪ L.push_back(U[i]);
21    } else {
22        set<Point> st(L.begin(), L.end());
23        for (int i = (int)U.size() - 2; i >= 1; i--) {
24            if (st.count(U[i]) == 0) L.push_back(U[i]),
↪ st.insert(U[i]);
25        }
26    }
27    return L;
28 }
29
30 vector<Point> get_convex2(vector<Point> &points, bool
↪ allow_collinear = false) { // strict, no repeat, one pass
31     nth_element(points.begin(), points.begin(), points.end());
32     sort(points.begin() + 1, points.end(), [&](const Point &a,
↪ const Point &b) {
33         int rad_diff = sgn((a - points[0]) ^ (b - points[0]));
34         return !rad_diff ? (dist2(a - points[0]) < dist2(b -
↪ points[0])) : (rad_diff > 0);
35     });
36     if (allow_collinear) {
37         int i = (int)points.size() - 1;
38         while (i >= 0 && !sgn((points[i] - points[0]) ^ (points[i]
↪ - points.back())) i--);
39         reverse(points.begin() + i + 1, points.end());
40     }
41     vector<Point> hull;
42     for (auto &t : points) {
43         for (ll sz = hull.size();
44             sz > 1 && (sgn((t - hull[sz - 2]) ^ (hull[sz - 1] -
↪ hull[sz - 2])) >= allow_collinear);
45             hull.pop_back(), sz = hull.size()) {
46         }
47         hull.push_back(t);
48     }

```

```

49     return hull;
50 }
51
52 vector<Point> get_convex_safe(vector<Point> points, bool
↪ allow_collinear = false) {
53     return get_convex(points, allow_collinear);
54 }
55
56 vector<Point> get_convex2_safe(vector<Point> points, bool
↪ allow_collinear = false) {
57     return get_convex2(points, allow_collinear);
58 }
59
60 bool is_convex(const vector<Point> &p, bool allow_collinear =
↪ false) {
61     int n = p.size();
62     int lo = 1, hi = -1;
63     for (int i = 0; i < n; i++) {
64         int cur = sgn((p[(i + 2) % n] - p[(i + 1) % n]) ^ (p[(i +
↪ 1) % n] - p[i]));
65         lo = min(lo, cur); hi = max(hi, cur);
66     }
67     return allow_collinear ? (hi - lo) < 2 : (lo == hi && lo);
68 }
69
70 auto rotating_calipers(const vector<Point> &hull) {
71     // use get_convex2
72     int n = (int)hull.size(); // return the square of longest
↪ dist
73     assert(n > 1);
74     if (n <= 2) return dist2(hull[0], hull[1]);
75     ld res = 0;
76     for (int i = 0, j = 2; i < n; i++) {
77         auto d = hull[i], e = hull[(i + 1) % n];
78         while (area2(d, e, hull[j]) < area2(d, e, hull[(j + 1) %
↪ n])) j = (j + 1) % n;
79         res = max(res, max(dist2(d, hull[j]), dist2(e, hull[j])));
80     }
81     return res;
82 }
83
84 // Find polygon cut to the left of l
85 vector<Point> convex_cut(const vector<Point> &p, const Line
↪ &l) {
86     int n = p.size();
87     vector<Point> cut;
88     for (int i = 0; i < n; i++) {
89         auto a = p[i], b = p[(i + 1) % n];
90         if (sgn((l.e - l.s) ^ (a - l.s)) >= 0)
91             cut.push_back(a);
92         if (sgn((l.e - l.s) ^ (a - l.s)) * sgn((l.e - l.s) ^ (b -
↪ l.s)) == -1)
93             cut.push_back(intersect(Line(a, b), l));
94     }
95     return cut;
96 }
97
98 // Sort by angle in range [0, 2pi)
99 template <class RandomIt>
100 void polar_sort(RandomIt first, RandomIt last, Point origin =
↪ Point(0, 0)) {
101     auto get_quad = [&](const Point& p) {
102         Point diff = p - origin;
103         if (diff.x > 0 && diff.y >= 0) return 1;
104         if (diff.x <= 0 && diff.y > 0) return 2;
105         if (diff.x < 0 && diff.y <= 0) return 3;
106         return 4;
107     };
108     auto polar_cmp = [&](const Point& p1, const Point& p2) {
109         int q1 = get_quad(p1), q2 = get_quad(p2);
110         if (q1 != q2) return q1 < q2;
111         return ((p1 - origin) ^ (p2 - origin)) > 0;
112     };
113     sort(first, last, polar_cmp);
114 }

```

## Basic 3D

```
1 using ll = long long;
2 using ld = long double;
3
4 constexpr auto eps = 1e-8;
5 const auto PI = acos(-1);
6 int sgn(ld x) { return (abs(x) <= eps) ? 0 : (x < 0 ? -1 : 1);
  ↪ }
7
8 struct Point3D {
9     ld x = 0, y = 0, z = 0;
10     Point3D() = default;
11     Point3D(ld _x, ld _y, ld _z) : x(_x), y(_y), z(_z) {}
12     bool operator<(const Point3D &p) const { return !sgn(p.x -
  ↪ x) ? (!sgn(p.y - y) ? sgn(p.z - z) < 0 : y < p.y) : x <
  ↪ p.x; }
13     bool operator==(const Point3D &p) const { return !sgn(p.x -
  ↪ x) && !sgn(p.y - y) && !sgn(p.z - z); }
14     Point3D operator+(const Point3D &p) const { return {x + p.x,
  ↪ y + p.y, z + p.z}; }
15     Point3D operator-(const Point3D &p) const { return {x - p.x,
  ↪ y - p.y, z - p.z}; }
16     Point3D operator*(ld a) const { return {x * a, y * a, z *
  ↪ a}; }
17     Point3D operator/(ld a) const { return {x / a, y / a, z /
  ↪ a}; }
18     auto operator*(const Point3D &p) const { return x * p.x + y
  ↪ * p.y + z * p.z; } // dot
19     Point3D operator^(const Point3D &p) const { return {y * p.z
  ↪ - z * p.y, z * p.x - x * p.z, x * p.y - y * p.x}; } //
  ↪ cross
20     friend auto &operator>>(istream &i, Point3D &p) { return i
  ↪ >> p.x >> p.y >> p.z; }
21 };
22
23 struct Line3D {
24     Point3D s = {0, 0, 0}, e = {0, 0, 0};
25     Line3D() = default;
26     Line3D(Point3D _s, Point3D _e) : s(_s), e(_e) {}
27 };
28
29 struct Segment3D : Line3D {
30     using Line3D::Line3D;
31 };
32
33 auto dist2(const Point3D &a) { return a * a; }
34 auto dist2(const Point3D &a, const Point3D &b) { return
  ↪ dist2(a - b); }
35 auto dist(const Point3D &a) { return sqrt(dist2(a)); }
36 auto dist(const Point3D &a, const Point3D &b) { return
  ↪ sqrt(dist2(a - b)); }
37 auto dist(const Point3D &a, const Line3D &l) { return dist((a
  ↪ - l.s) ^ (l.e - l.s)) / dist(l.s, l.e); }
38 auto dist(const Point3D &p, const Segment3D &l) {
39     if (l.s == l.e) return dist(p, l.s);
40     auto d = dist2(l.s, l.e), t = min(d, max((ld)0, (p - l.s) *
  ↪ (l.e - l.s)));
41     return dist((p - l.s) * d, (l.e - l.s) * t) / d;
42 }
```

## Miscellaneous

```
1 tuple<int,int,ld> closest_pair(vector<Point> &p) {
2     using Pt = pair<Point,int>;
3     int n = p.size();
4     assert(n > 1);
5     vector<Pt> pts(n), buf;
6     for (int i = 0; i < n; i++) pts[i] = {p[i], i};
7     sort(pts.begin(), pts.end());
8     buf.reserve(n);
9     auto cmp_y = [&](const Pt& p1, const Pt& p2) { return
  ↪ p1.first.y < p2.first.y; };
10     function<tuple<int,int,ld>(int, int)> recurse = [&](int l,
  ↪ int r) -> tuple<int,int,ld> {
11         int i = pts[l].second, j = pts[l + 1].second;
12         ld d = dist(pts[l].first, pts[l + 1].first);
```

```
13         if (r - l < 5) {
14             for (int a = l; a < r; a++) for (int b = a + 1; b < r;
  ↪ b++) {
15                 ld cur = dist(pts[a].first, pts[b].first);
16                 if (cur < d) { i = pts[a].second; j = pts[b].second; d
  ↪ = cur; }
17             }
18             sort(pts.begin() + l, pts.begin() + r, cmp_y);
19         }
20         else {
21             int mid = (l + r) / 2;
22             ld x = pts[mid].first.x;
23             auto [li, lj, ldist] = recurse(l, mid);
24             auto [ri, rj, rdist] = recurse(mid, r);
25             if (ldist < rdist) { i = li; j = lj; d = ldist; }
26             else { i = ri; j = rj; d = rdist; }
27             inplace_merge(pts.begin() + l, pts.begin() + mid,
  ↪ pts.begin() + r, cmp_y);
28             buf.clear();
29             for (int a = l; a < r; a++) {
30                 if (abs(x - pts[a].first.x) >= d) continue;
31                 for (int b = buf.size() - 1; b >= 0; b--) {
32                     if (pts[a].first.y - buf[b].first.y >= d) break;
33                     ld cur = dist(pts[a].first, buf[b].first);
34                     if (cur < d) { i = pts[a].second; j = buf[b].second;
  ↪ d = cur; }
35                 }
36                 buf.push_back(pts[a]);
37             }
38             return {i, j, d};
39         }
40     };
41     return recurse(0, n);
42 }
43
44 Line abc_to_line(ld a, ld b, ld c) {
45     assert(!sgn(a) || !sgn(b));
46     if (a == 0) return Line(Point(0, -c/b), Point(1, -c/b));
47     if (b == 0) return Line(Point(-c/a, 0), Point(-c/a, 1));
48     Point s(0, -c/b), e(1, (-c - a)/b), diff = e - s;
49     return Line(s, s + diff/dist(diff));
50 }
51
52 tuple<ld,ld,ld> line_to_abc(const Line& l) {
53     Point diff = l.e - l.s;
54     return {-diff.y, diff.x, -(diff ^ l.s)};
55 }
```

## Graph Theory

### Max Flow

```
1 struct Edge {
2     int from, to, cap, remain;
3 };
4
5 struct Dinic {
6     int n;
7     vector<Edge> e;
8     vector<vector<int>> g;
9     vector<int> d, cur;
10     Dinic(int _n) : n(_n), g(n), d(n), cur(n) {}
11     void add_edge(int u, int v, int c) {
12         g[u].push_back((int)e.size());
13         e.push_back({u, v, c, c});
14         g[v].push_back((int)e.size());
15         e.push_back({v, u, 0, 0});
16     }
17     ll max_flow(int s, int t) {
18         int inf = 1e9;
19         auto bfs = [&]() {
20             fill(d.begin(), d.end(), inf), fill(cur.begin(),
  ↪ cur.end(), 0);
21             d[s] = 0;
22             vector<int> q{s}, nq;
```

```

23     for (int step = 1; q.size(); swap(q, nq), nq.clear(),
↳ step++) {
24         for (auto& node : q) {
25             for (auto& edge : g[node]) {
26                 int ne = e[edge].to;
27                 if (!e[edge].remain || d[ne] <= step) continue;
28                 d[ne] = step, nq.push_back(ne);
29                 if (ne == t) return true;
30             }
31         }
32     }
33     return false;
34 };
35 function<int(int, int)> find = [&](int node, int limit) {
36     if (node == t || !limit) return limit;
37     int flow = 0;
38     for (int i = cur[node]; i < g[node].size(); i++) {
39         cur[node] = i;
40         int edge = g[node][i], oe = edge ^ 1, ne = e[edge].to;
41         if (!e[edge].remain || d[ne] != d[node] + 1) continue;
42         if (int temp = find(ne, min(limit - flow,
↳ e[edge].remain))) {
43             e[edge].remain -= temp, e[oe].remain += temp, flow
↳ += temp;
44         } else {
45             d[ne] = -1;
46         }
47         if (flow == limit) break;
48     }
49     return flow;
50 };
51 ll res = 0;
52 while (bfs())
53     while (int flow = find(s, inf)) res += flow;
54 return res;
55 }
56 };

```

## • USAGE

```

1 int main() {
2     int n, m, s, t;
3     cin >> n >> m >> s >> t;
4     Dinic dinic(n);
5     for (int i = 0, u, v, c; i < m; i++) {
6         cin >> u >> v >> c;
7         dinic.add_edge(u - 1, v - 1, c);
8     }
9     cout << dinic.max_flow(s - 1, t - 1) << '\n';
10 }

```

## PushRelabel Max-Flow (faster)

```

1 //
↳ https://github.com/kth-competitive-programming/kactl/blob/main/content/flow/pushrelabel.cpp
2 #define rep(i, a, b) for (int i = a; i < (b); ++i)
3 #define all(x) begin(x), end(x)
4 #define sz(x) (int)(x).size()
5 typedef long long ll;
6 typedef pair<int, int> pii;
7 typedef vector<int> vi;
8
9 struct PushRelabel {
10     struct Edge {
11         int dest, back;
12         ll f, c;
13     };
14     vector<vector<Edge>> g;
15     vector<ll> ec;
16     vector<Edge*> cur;
17     vector<vi> hs;
18     vi H;
19     PushRelabel(int n) : g(n), ec(n), cur(n), hs(2 * n), H(n) {}
20
21     void addEdge(int s, int t, ll cap, ll rcap = 0) {
22         if (s == t) return;
23         g[s].push_back({t, sz(g[t]), 0, cap});

```

```

24         g[t].push_back({s, sz(g[s]) - 1, 0, rcap});
25     }
26
27     void addFlow(Edge& e, ll f) {
28         Edge& back = g[e.dest][e.back];
29         if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
30         e.f += f;
31         e.c -= f;
32         ec[e.dest] += f;
33         back.f -= f;
34         back.c += f;
35         ec[back.dest] -= f;
36     }
37
38     ll calc(int s, int t) {
39         int v = sz(g);
40         H[s] = v;
41         ec[t] = 1;
42         vi co(2 * v);
43         co[0] = v - 1;
44         rep(i, 0, v) cur[i] = g[i].data();
45         for (Edge& e : g[s]) addFlow(e, e.c);
46
47         for (int hi = 0;;) {
48             while (hs[hi].empty())
49                 if (!hi--) return -ec[s];
50             int u = hs[hi].back();
51             hs[hi].pop_back();
52             while (ec[u] > 0) // discharge u
53                 if (cur[u] == g[u].data() + sz(g[u])) {
54                     H[u] = 1e9;
55                     for (Edge& e : g[u])
56                         if (e.c && H[u] > H[e.dest] + 1) H[u] = H[e.dest]
↳ + 1, cur[u] = &e;
57                     if (++co[H[u]], !--co[hi] && hi < v)
58                         rep(i, 0, v) if (hi < H[i] && H[i] < v)--
↳ co[H[i]], H[i] = v + 1;
59                     hi = H[u];
60                 } else if (cur[u]->c && H[u] == H[cur[u]->dest] + 1)
61                     addFlow(*cur[u], min(ec[u], cur[u]->c));
62                 else
63                     ++cur[u];
64             }
65         }
66         bool leftOfMinCut(int a) { return H[a] >= sz(g); }

```

## Min-Cost Max-Flow

```

1 struct MinCostFlow {
2     static constexpr int INF = 1e9;
3     const int n;
4     vector<tuple<int, int, int>> e;
5     vector<vector<int>> g;
6     vector<int> h, dis, pre;
7     MinCostFlow(int n, int s, int t) {
8         dis.assign(n, INF);
9         pre.assign(n, -1);
10        priority_queue<pair<int, int>, vector<pair<int, int>>,
↳ greater<>> que;
11        dis[s] = 0;
12        que.emplace(0, s);
13        while (!que.empty()) {
14            auto [d, u] = que.top();
15            que.pop();
16            if (dis[u] != d) continue;
17            for (int i : g[u]) {
18                auto [v, f, c] = e[i];
19                if (c > 0 && dis[v] > d + h[u] - h[v] + f) {
20                    dis[v] = d + h[u] - h[v] + f;
21                    pre[v] = i;
22                    que.emplace(dis[v], v);
23                }
24            }
25        }
26        return dis[t] != INF;
27    }
28    MinCostFlow(int _n) : n(_n), g(n) {}

```



```

29 void addEdge(int u, int v, int f, int c) {
30     g[u].push_back((int)e.size());
31     e.emplace_back(v, f, c);
32     g[v].push_back((int)e.size());
33     e.emplace_back(u, -f, 0);
34 }
35 pair<int, int> minCostMaxFlow(const int s, const int t) {
36     int flow = 0, cost = 0;
37     h.assign(n, 0);
38     while (dijkstra(s, t)) {
39         for (int i = 0; i < n; ++i) h[i] += dis[i];
40         for (int i = t; i != s; i = get<0>(e[pre[i] ^ 1])) {
41             --get<2>(e[pre[i]]);
42             ++get<2>(e[pre[i] ^ 1]);
43         }
44         ++flow;
45         cost += h[t];
46     }
47     return {flow, cost};
48 }
49 };

```

## Heavy-Light Decomposition

```

1 int root = 0, cur = 0;
2 vector<int> parent(n), deep(n), hson(n, -1), top(n), sz(n),
   ↳ dfn(n, -1);
3 function<int(int, int, int)> dfs = [&](int node, int fa, int
   ↳ dep) {
4     deep[node] = dep, sz[node] = 1, parent[node] = fa;
5     for (auto &ne : g[node]) {
6         if (ne == fa) continue;
7         sz[node] += dfs(ne, node, dep + 1);
8         if (hson[node] == -1 || sz[ne] > sz[hson[node]]) hson[node]
   ↳ = ne;
9     }
10    return sz[node];
11 };
12 function<void(int, int)> dfs2 = [&](int node, int t) {
13     top[node] = t, dfn[node] = cur++;
14     if (hson[node] == -1) return;
15     dfs2(hson[node], t);
16     for (auto &ne : g[node]) {
17         if (ne == parent[node] || ne == hson[node]) continue;
18         dfs2(ne, ne);
19     }
20 };
21 // read in graph as vector<vector<int>> g(n)
22 dfs(root, -1, 0), dfs2(root, root);

```

- USAGE: get LCA

```

1 function<int(int, int)> lca = [&](int x, int y) {
2     while (top[x] != top[y]) {
3         if (deep[top[x]] < deep[top[y]]) swap(x, y);
4         x = parent[top[x]];
5     }
6     return deep[x] < deep[y] ? x : y;
7 };

```

## General Unweight Graph Matching

- Complexity:  $O(n^3)$  (?)

```

1 struct BlossomMatch {
2     int n;
3     vector<vector<int>> e;
4     BlossomMatch(int _n) : n(_n), e(_n) {}
5     void add_edge(int u, int v) { e[u].push_back(v),
   ↳ e[v].push_back(u); }
6     vector<int> find_matching() {
7         vector<int> match(n, -1), vis(n), link(n), f(n), dep(n);
8         function<int(int)> find = [&](int x) { return f[x] == x ?
   ↳ x : (f[x] = find(f[x])); };
9         auto lca = [&](int u, int v) {
10            u = find(u), v = find(v);
11            while (u != v) {

```

```

12            if (dep[u] < dep[v]) swap(u, v);
13            u = find(link[match[u]]);
14        }
15        return u;
16    };
17    queue<int> que;
18    auto blossom = [&](int u, int v, int p) {
19        while (find(u) != p) {
20            link[u] = v, v = match[u];
21            if (vis[v] == 0) vis[v] = 1, que.push(v);
22            f[u] = f[v] = p, u = link[v];
23        }
24    };
25    // find an augmenting path starting from u and augment (if
   ↳ exist)
26    auto augment = [&](int node) {
27        while (!que.empty()) que.pop();
28        iota(f.begin(), f.end(), 0);
29        // vis = 0 corresponds to inner vertices, vis = 1
   ↳ corresponds to outer vertices
30        fill(vis.begin(), vis.end(), -1);
31        que.push(node);
32        vis[node] = 1, dep[node] = 0;
33        while (!que.empty()) {
34            int u = que.front();
35            que.pop();
36            for (auto v : e[u]) {
37                if (vis[v] == -1) {
38                    vis[v] = 0, link[v] = u, dep[v] = dep[u] + 1;
39                    // found an augmenting path
40                    if (match[v] == -1) {
41                        for (int x = v, y = u, temp; y != -1; x = temp,
   ↳ y = x == -1 ? -1 : link[x]) {
42                            temp = match[y], match[x] = y, match[y] = x;
43                        }
44                        return;
45                    }
46                    vis[match[v]] = 1, dep[match[v]] = dep[u] + 2;
47                    que.push(match[v]);
48                } else if (vis[v] == 1 && find(v) != find(u)) {
49                    // found a blossom
50                    int p = lca(u, v);
51                    blossom(u, v, p), blossom(v, u, p);
52                }
53            }
54        }
55    };
56    // find a maximal matching greedily (decrease constant)
57    auto greedy = [&]() {
58        for (int u = 0; u < n; ++u) {
59            if (match[u] != -1) continue;
60            for (auto v : e[u]) {
61                if (match[v] == -1) {
62                    match[u] = v, match[v] = u;
63                    break;
64                }
65            }
66        }
67    };
68    greedy();
69    for (int u = 0; u < n; ++u)
70        if (match[u] == -1) augment(u);
71    return match;
72 }
73 };

```

## Maximum Bipartite Matching

- Needs dinic, complexity  $\approx O(n + m\sqrt{n})$

```

1 struct BipartiteMatch {
2     int l, r;
3     Dinic dinic = Dinic(0);
4     BipartiteMatch(int _l, int _r) : l(_l), r(_r) {
5         dinic = Dinic(l + r + 2);
6         for (int i = 1; i <= l; i++) dinic.add_edge(0, i, 1);

```

```

7     for (int i = 1; i <= r; i++) dinic.add_edge(l + i, l + r +
↪ 1, 1);
8 }
9 void add_edge(int u, int v) { dinic.add_edge(u + 1, l + v +
↪ 1, 1); }
10 ll max_matching() { return dinic.max_flow(0, l + r + 1); }
11 };

```

## 2-SAT and Strongly Connected Components

```

1 void scc(vector<vector<int>>& g, int* idx) {
2     int n = g.size(), ct = 0;
3     int out[n];
4     vector<int> ginv[n];
5     memset(out, -1, sizeof out);
6     memset(idx, -1, n * sizeof(int));
7     function<void(int)> dfs = [&](int cur) {
8         out[cur] = INT_MAX;
9         for(int v : g[cur]) {
10             ginv[v].push_back(cur);
11             if(out[v] == -1) dfs(v);
12         }
13         ct++; out[cur] = ct;
14     };
15     vector<int> order;
16     for(int i = 0; i < n; i++) {
17         order.push_back(i);
18         if(out[i] == -1) dfs(i);
19     }
20     sort(order.begin(), order.end(), [&](int& u, int& v) {
21         return out[u] > out[v];
22     });
23     ct = 0;
24     stack<int> s;
25     auto dfs2 = [&](int start) {
26         s.push(start);
27         while(!s.empty()) {
28             int cur = s.top();
29             s.pop();
30             idx[cur] = ct;
31             for(int v : ginv[cur])
32                 if(idx[v] == -1) s.push(v);
33         }
34     };
35     for(int v : order) {
36         if(idx[v] == -1) {
37             dfs2(v);
38             ct++;
39         }
40     }
41 }
42
43 // 0 => impossible, 1 => possible
44 pair<int, vector<int>> sat2(int n, vector<pair<int, int>>&
↪ clauses) {
45     vector<int> ans(n);
46     vector<vector<int>> g(2*n + 1);
47     for(auto [x, y] : clauses) {
48         x = x < 0 ? -x + n : x;
49         y = y < 0 ? -y + n : y;
50         int nx = x <= n ? x + n : x - n;
51         int ny = y <= n ? y + n : y - n;
52         g[nx].push_back(y);
53         g[ny].push_back(x);
54     }
55     int idx[2*n + 1];
56     scc(g, idx);
57     for(int i = 1; i <= n; i++) {
58         if(idx[i] == idx[i + n]) return {0, {}};
59         ans[i - 1] = idx[i + n] < idx[i];
60     }
61     return {1, ans};
62 }

```

## Enumerating Triangles

- Complexity:  $O(n + m\sqrt{m})$

```

1 void enumerate_triangles(vector<pair<int, int>>& edges,
↪ function<void(int, int, int)> f) {
2     int n = 0;
3     for(auto [u, v] : edges) n = max({n, u + 1, v + 1});
4     vector<int> deg(n);
5     vector<int> g[n];
6     for(auto [u, v] : edges) {
7         deg[u]++;
8         deg[v]++;
9     }
10    for(auto [u, v] : edges) {
11        if(u == v) continue;
12        if(deg[u] > deg[v] || (deg[u] == deg[v] && u > v))
13            swap(u, v);
14        g[u].push_back(v);
15    }
16    vector<int> flag(n);
17    for(int i = 0; i < n; i++) {
18        for(int v : g[i]) flag[v] = 1;
19        for(int v : g[i]) for(int u : g[v]) {
20            if(flag[u]) f(i, v, u);
21        }
22        for(int v : g[i]) flag[v] = 0;
23    }
24 }

```

## Tarjan

- shrink all circles into points (2-edge-connected-component)

```

1 int cnt = 0, now = 0;
2 vector<ll> dfn(n, -1), low(n), belong(n, -1), stk;
3 function<void(ll, ll)> tarjan = [&](ll node, ll fa) {
4     dfn[node] = low[node] = now++; stk.push_back(node);
5     for (auto& ne : g[node]) {
6         if (ne == fa) continue;
7         if (dfn[ne] == -1) {
8             tarjan(ne, node);
9             low[node] = min(low[node], low[ne]);
10        } else if (belong[ne] == -1) {
11            low[node] = min(low[node], dfn[ne]);
12        }
13    }
14    if (dfn[node] == low[node]) {
15        while (true) {
16            auto v = stk.back();
17            belong[v] = cnt;
18            stk.pop_back();
19            if (v == node) break;
20        }
21        ++cnt;
22    }
23 };

```

- 2-vertex-connected-component / Block forest

```

1 int cnt = 0, now = 0;
2 vector<vector<ll>> e1(n);
3 vector<ll> dfn(n, -1), low(n), stk;
4 function<void(ll)> tarjan = [&](ll node) {
5     dfn[node] = low[node] = now++; stk.push_back(node);
6     for (auto& ne : g[node]) {
7         if (dfn[ne] == -1) {
8             tarjan(ne);
9             low[node] = min(low[node], low[ne]);
10            if (low[ne] == dfn[node]) {
11                e1.push_back({});
12                while (true) {
13                    auto x = stk.back();
14                    stk.pop_back();
15                    e1[n + cnt].push_back(x);
16                    // e1[x].push_back(n + cnt); // undirected

```



```

17         if (x == ne) break;
18     }
19     e1[node].push_back(n + cnt);
20     // e1[n + cnt].push_back(node); // undirected
21     cnt++;
22 }
23 } else {
24     low[node] = min(low[node], dfn[ne]);
25 }
26 }
27 };

```

## Kruskal reconstruct tree

```

1  int _n, m;
2  cin >> _n >> m; // _n: # of node, m: # of edge
3  int n = 2 * _n - 1; // root: n-1
4  vector<array<int, 3>> edges(m);
5  for (auto& [w, u, v] : edges) {
6      cin >> u >> v >> w, u--, v--;
7  }
8  sort(edges.begin(), edges.end());
9  vector<int> p(n);
10 iota(p.begin(), p.end(), 0);
11 function<int(int)> find = [&](int x) { return p[x] == x ? x :
    ↪ (p[x] = find(p[x])); };
12 auto merge = [&](int x, int y) { p[find(x)] = find(y); };
13 vector<vector<int>> g(n);
14 vector<int> val(m);
15 val.reserve(n);
16 for (auto [w, u, v] : edges) {
17     u = find(u), v = find(v);
18     if (u == v) continue;
19     val.push_back(w);
20     int node = (int)val.size() - 1;
21     g[node].push_back(u), g[node].push_back(v);
22     merge(u, node), merge(v, node);
23 }

```

## Math

### Inverse

```

1  ll inv(ll a, ll m) { return a == 1 ? 1 : ((m - m / a) * inv(m
    ↪ % a, m) % m); }
2  // or
3  power(a, MOD - 2)

```

- USAGE: get factorial

```

1  vector<ll> f(MAX_N, 1), rf(MAX_N, 1);
2  for (int i = 1; i < MAX_N; i++) f[i] = (f[i - 1] * i) % MOD;
3  for (int i = 1; i < MAX_N; i++) rf[i] = (rf[i - 1] * inv(i,
    ↪ MOD)) % MOD;
4  // or (the later one should be preferred)
5  vector<ll> f(MAX_N, 1), rf(MAX_N, 1);
6  for (int i = 2; i < MAX_N; i++) f[i] = f[i - 1] * i % MOD;
7  rf[MAX_N - 1] = power(f[MAX_N - 1], MOD - 2);
8  for (int i = MAX_N - 2; i > 1; i--) rf[i] = rf[i + 1] * (i +
    ↪ 1) % MOD;

```

### Mod Class

```

1  constexpr ll norm(ll x) { return (x % MOD + MOD) % MOD; }
2  template <typename T>
3  constexpr T power(T a, ll b, T res = 1) {
4      for (; b; b /= 2, (a *= a) %= MOD)
5          if (b & 1) (res *= a) %= MOD;
6      return res;
7  }
8  struct Z {
9      ll x;
10     constexpr Z(ll _x = 0) : x(norm(_x)) {}
11     // auto operator<=>(const Z &) const = default; // cpp20
    ↪ only

```

```

12     Z operator-() const { return Z(norm(MOD - x)); }
13     Z inv() const { return power(*this, MOD - 2); }
14     Z &operator+=(const Z &rhs) { return x = x * rhs.x % MOD,
    ↪ *this; }
15     Z &operator+=(const Z &rhs) { return x = norm(x + rhs.x),
    ↪ *this; }
16     Z &operator-=(const Z &rhs) { return x = norm(x - rhs.x),
    ↪ *this; }
17     Z &operator/=(const Z &rhs) { return *this *= rhs.inv(); }
18     Z &operator%=(const ll &rhs) { return x %= rhs, *this; }
19     friend Z operator*(Z lhs, const Z &rhs) { return lhs * rhs;
    ↪ }
20     friend Z operator+(Z lhs, const Z &rhs) { return lhs += rhs;
    ↪ }
21     friend Z operator-(Z lhs, const Z &rhs) { return lhs -= rhs;
    ↪ }
22     friend Z operator/(Z lhs, const Z &rhs) { return lhs /= rhs;
    ↪ }
23     friend Z operator%(Z lhs, const ll &rhs) { return lhs %=
    ↪ rhs; }
24     friend auto &operator>>(istream &i, Z &z) { return i >> z.x;
    ↪ }
25     friend auto &operator<<(ostream &o, const Z &z) { return o
    ↪ << z.x; }
26 };

```

- large mod (for NTT to do FFT in ll range without modulo)

```

1  using ll = long long;
2  using i128 = __int128;
3  constexpr i128 MOD = 9223372036737335297;
4
5  constexpr i128 norm(i128 x) { return x < 0 ? (x + MOD) % MOD :
    ↪ x % MOD; }
6  template <typename T>
7  constexpr T power(T a, i128 b, T res = 1) {
8      for (; b; b /= 2, (a *= a) %= MOD)
9          if (b & 1) (res *= a) %= MOD;
10     return res;
11 }
12 struct Z {
13     i128 x;
14     constexpr Z(i128 _x = 0) : x(norm(_x)) {}
15     Z operator-() const { return Z(norm(MOD - x)); }
16     Z inv() const { return power(*this, MOD - 2); }
17     // auto operator<=>(const Z &) const = default;
18     Z &operator+=(const Z &rhs) { return x = x * rhs.x % MOD,
    ↪ *this; }
19     Z &operator+=(const Z &rhs) { return x = norm(x + rhs.x),
    ↪ *this; }
20     Z &operator-=(const Z &rhs) { return x = norm(x - rhs.x),
    ↪ *this; }
21     Z &operator/=(const Z &rhs) { return *this *= rhs.inv(); }
22     Z &operator%=(const i128 &rhs) { return x %= rhs, *this; }
23     friend Z operator*(Z lhs, const Z &rhs) { return lhs * rhs;
    ↪ }
24     friend Z operator+(Z lhs, const Z &rhs) { return lhs += rhs;
    ↪ }
25     friend Z operator-(Z lhs, const Z &rhs) { return lhs -= rhs;
    ↪ }
26     friend Z operator/(Z lhs, const Z &rhs) { return lhs /= rhs;
    ↪ }
27     friend Z operator%(Z lhs, const i128 &rhs) { return lhs %=
    ↪ rhs; }
28 };

```

- fastest mod class! be careful with overflow, only use when the time limit is tight

```

1  constexpr int MOD = 998244353;
2
3  constexpr int norm(int x) {
4      if (x < 0) x += MOD;
5      if (x >= MOD) x -= MOD;
6      return x;
7  }
8  template <typename T>

```

```

9  constexpr T power(T a, int b, T res = 1) {
10     for (; b; b /= 2, (a *= a) %= MOD)
11         if (b & 1) (res *= a) %= MOD;
12     return res;
13 }
14 struct Z {
15     int x;
16     constexpr Z(int _x = 0) : x(norm(_x)) {}
17     // constexpr auto operator<=>(const Z & const = default; //
18     ↪ cpp20 only
19     constexpr Z operator-() const { return Z(norm(MOD - x)); }
20     constexpr Z inv() const { return power(*this, MOD - 2); }
21     constexpr Z &operator*=(const Z &rhs) { return x = ll(x) *
22     ↪ rhs.x % MOD, *this; }
23     constexpr Z &operator+=(const Z &rhs) { return x = norm(x +
24     ↪ rhs.x), *this; }
25     constexpr Z &operator-=(const Z &rhs) { return x = norm(x -
26     ↪ rhs.x), *this; }
27     constexpr Z &operator/=(const Z &rhs) { return *this *=
28     ↪ rhs.inv(); }
29     constexpr Z &operator%=(const ll &rhs) { return x %= rhs,
30     ↪ *this; }
31     constexpr friend Z operator*(Z lhs, const Z &rhs) { return
32     ↪ lhs * rhs; }
33     constexpr friend Z operator+(Z lhs, const Z &rhs) { return
34     ↪ lhs + rhs; }
35     constexpr friend Z operator-(Z lhs, const Z &rhs) { return
36     ↪ lhs - rhs; }
37     constexpr friend Z operator/(Z lhs, const Z &rhs) { return
38     ↪ lhs / rhs; }
39     constexpr friend Z operator%(Z lhs, const ll &rhs) { return
40     ↪ lhs % rhs; }
41     friend auto &operator>>(istream &i, Z &z) { return i >> z.x;
42     ↪ }
43     friend auto &operator<<(ostream &o, const Z &z) { return o
44     ↪ << z.x; }
45 };

```

## NTT, FFT, FWT

### • ntt

```

1  void ntt(vector<Z>& a, int f) {
2      int n = int(a.size());
3      vector<Z> w(n);
4      vector<int> rev(n);
5      for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] / 2) | ((i
6      ↪ & 1) * (n / 2));
7      for (int i = 0; i < n; i++) {
8          if (i < rev[i]) swap(a[i], a[rev[i]]);
9      }
10     Z wn = power(f ? (MOD + 1) / 3 : 3, (MOD - 1) / n);
11     w[0] = 1;
12     for (int i = 1; i < n; i++) w[i] = w[i - 1] * wn;
13     for (int mid = 1; mid < n; mid *= 2) {
14         for (int i = 0; i < n; i += 2 * mid) {
15             for (int j = 0; j < mid; j++) {
16                 Z x = a[i + j], y = a[i + j + mid] * w[n / (2 * mid) *
17             ↪ j];
18                 a[i + j] = x + y, a[i + j + mid] = x - y;
19             }
20         }
21     }
22     if (f) {
23         Z iv = power(Z(n), MOD - 2);
24         for (auto& x : a) x *= iv;
25     }
26 }

```

### • USAGE: Polynomial multiplication

```

1  vector<Z> mul(vector<Z> a, vector<Z> b) {
2      int n = 1, m = (int)a.size() + (int)b.size() - 1;
3      while (n < m) n *= 2;
4      a.resize(n), b.resize(n);
5      ntt(a, 0), ntt(b, 0);
6      for (int i = 0; i < n; i++) a[i] *= b[i];

```

```

7      ntt(a, 1);
8      a.resize(m);
9      return a;
10 }

```

- FFT (should prefer NTT, only use this when input is not integer)

```

1  const double PI = acos(-1);
2  auto mul = [&](const vector<double>& aa, const vector<double>&
3  ↪ bb) {
4      int n = (int)aa.size(), m = (int)bb.size(), bit = 1;
5      while ((1 << bit) < n + m - 1) bit++;
6      int len = 1 << bit;
7      vector<complex<double>> a(len), b(len);
8      vector<int> rev(len);
9      for (int i = 0; i < n; i++) a[i].real(aa[i]);
10     for (int i = 0; i < m; i++) b[i].real(bb[i]);
11     for (int i = 0; i < len; i++) rev[i] = (rev[i >> 1] >> 1) |
12     ↪ ((i & 1) << (bit - 1));
13     auto fft = [&](vector<complex<double>>& p, int inv) {
14         for (int i = 0; i < len; i++)
15             if (i < rev[i]) swap(p[i], p[rev[i]]);
16         for (int mid = 1; mid < len; mid *= 2) {
17             auto w1 = complex<double>(cos(PI / mid), (inv ? -1 : 1)
18             ↪ * sin(PI / mid));
19             for (int i = 0; i < len; i += mid * 2) {
20                 auto wk = complex<double>(1, 0);
21                 for (int j = 0; j < mid; j++) wk = wk * w1;
22                 for (int x = p[i + j], y = wk * p[i + j + mid];
23                 ↪ p[i + j] = x + y, p[i + j + mid] = x - y;
24             }
25             if (inv == 1) {
26                 for (int i = 0; i < len; i++) p[i].real(p[i].real() /
27             ↪ len);
28             }
29         }
30     };
31     fft(a, 0), fft(b, 0);
32     for (int i = 0; i < len; i++) a[i] = a[i] * b[i];
33     fft(a, 1);
34     a.resize(n + m - 1);
35     vector<double> res(n + m - 1);
36     for (int i = 0; i < n + m - 1; i++) res[i] = a[i].real();
37     return res;
38 };

```

## Polynomial Class

```

1  using ll = long long;
2  constexpr ll MOD = 998244353;
3
4  ll norm(ll x) { return (x % MOD + MOD) % MOD; }
5  template <class T>
6  T power(T a, ll b, T res = 1) {
7      for (; b; b /= 2, (a *= a) %= MOD)
8          if (b & 1) (res *= a) %= MOD;
9      return res;
10 }
11
12 struct Z {
13     ll x;
14     Z(ll _x = 0) : x(norm(_x)) {}
15     // auto operator<=>(const Z & const = default;
16     Z operator-() const { return Z(norm(MOD - x)); }
17     Z inv() const { return power(*this, MOD - 2); }
18     Z &operator*=(const Z &rhs) { return x = x * rhs.x % MOD,
19     ↪ *this; }
20     Z &operator+=(const Z &rhs) { return x = norm(x + rhs.x),
21     ↪ *this; }
22     Z &operator-=(const Z &rhs) { return x = norm(x - rhs.x),
23     ↪ *this; }
24     Z &operator/=(const Z &rhs) { return *this *= rhs.inv(); }
25     Z &operator%=(const ll &rhs) { return x %= rhs, *this; }
26     friend Z operator*(Z lhs, const Z &rhs) { return lhs * rhs;
27     ↪ }

```

```

24     friend Z operator+(Z lhs, const Z &rhs) { return lhs += rhs; }
25     friend Z operator-(Z lhs, const Z &rhs) { return lhs -= rhs; }
26     friend Z operator/(Z lhs, const Z &rhs) { return lhs /= rhs; }
27     friend Z operator%(Z lhs, const ll &rhs) { return lhs %=
28     rhs; }
29     friend auto &operator>>(istream &i, Z &z) { return i >> z.x; }
30     friend auto &operator<<(ostream &o, const Z &z) { return o
31     << z.x; }
32     void ntt(vector<Z> &a, int f) {
33         int n = (int)a.size();
34         vector<Z> w(n);
35         vector<int> rev(n);
36         for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] / 2) | ((i
37         & 1) * (n / 2));
38         for (int i = 0; i < n; i++)
39             if (i < rev[i]) swap(a[i], a[rev[i]]);
40         Z wn = power(f ? (MOD + 1) / 3 : 3, (MOD - 1) / n);
41         w[0] = 1;
42         for (int i = 1; i < n; i++) w[i] = w[i - 1] * wn;
43         for (int mid = 1; mid < n; mid *= 2) {
44             for (int i = 0; i < n; i += 2 * mid) {
45                 for (int j = 0; j < mid; j++) {
46                     Z x = a[i + j], y = a[i + j + mid] * w[n / (2 * mid) *
47                     j];
48                     a[i + j] = x + y, a[i + j + mid] = x - y;
49                 }
50             }
51             if (f) {
52                 Z iv = power(Z(n), MOD - 2);
53                 for (int i = 0; i < n; i++) a[i] *= iv;
54             }
55         }
56         struct Poly {
57             vector<Z> a;
58             Poly() {}
59             Poly(const vector<Z> &a) : a(a) {}
60             int size() const { return (int)a.size(); }
61             void resize(int n) { a.resize(n); }
62             Z operator[](int idx) const {
63                 if (idx < 0 || idx >= size()) return 0;
64                 return a[idx];
65             }
66             Z &operator[](int idx) { return a[idx]; }
67             Poly mulxk(int k) const {
68                 auto b = a;
69                 b.insert(b.begin(), k, 0);
70                 return Poly(b);
71             }
72             Poly modxk(int k) const { return Poly(vector<Z>(a.begin(),
73             a.begin() + min(k, size()))); }
74             Poly divxk(int k) const {
75                 if (size() <= k) return Poly();
76                 return Poly(vector<Z>(a.begin() + k, a.end()));
77             }
78             friend Poly operator+(const Poly &a, const Poly &b) {
79                 vector<Z> res(max(a.size(), b.size()));
80                 for (int i = 0; i < (int)res.size(); i++) res[i] = a[i] +
81                 b[i];
82                 return Poly(res);
83             }
84             friend Poly operator-(const Poly &a, const Poly &b) {
85                 vector<Z> res(max(a.size(), b.size()));
86                 for (int i = 0; i < (int)res.size(); i++) res[i] = a[i] -
87                 b[i];
88                 return Poly(res);
89             }
90             friend Poly operator*(Poly a, Poly b) {
91                 if (a.size() == 0 || b.size() == 0) return Poly();
92                 int n = 1, m = (int)a.size() + (int)b.size() - 1;
93                 while (n < m) n *= 2;
94                 a.resize(n), b.resize(n);
95                 ntt(a.a, 0), ntt(b.a, 0);
96                 for (int i = 0; i < n; i++) a[i] *= b[i];
97                 ntt(a.a, 1);
98                 a.resize(m);
99                 return a;
100             }
101             friend Poly operator*(Z a, Poly b) {
102                 for (int i = 0; i < (int)b.size(); i++) b[i] *= a;
103                 return b;
104             }
105             friend Poly operator*(Poly a, Z b) {
106                 for (int i = 0; i < (int)a.size(); i++) a[i] *= b;
107                 return a;
108             }
109             Poly &operator+=(Poly b) { return (*this) = (*this) + b; }
110             Poly &operator-=(Poly b) { return (*this) = (*this) - b; }
111             Poly &operator*=(Poly b) { return (*this) = (*this) * b; }
112             Poly deriv() const {
113                 if (a.empty()) return Poly();
114                 vector<Z> res(size() - 1);
115                 for (int i = 0; i < size() - 1; ++i) res[i] = (i + 1) *
116                 a[i + 1];
117                 return Poly(res);
118             }
119             Poly integr() const {
120                 vector<Z> res(size() + 1);
121                 for (int i = 0; i < size(); ++i) res[i + 1] = a[i] / (i +
122                 1);
123                 return Poly(res);
124             }
125             Poly inv(int m) const {
126                 Poly x({a[0].inv()});
127                 int k = 1;
128                 while (k < m) {
129                     k *= 2;
130                     x = (x * (Poly({2}) - modxk(k) * x)).modxk(k);
131                 }
132                 return x.modxk(m);
133             }
134             Poly log(int m) const { return (deriv() *
135             inv(m)).integr().modxk(m); }
136             Poly exp(int m) const {
137                 Poly x({1});
138                 int k = 1;
139                 while (k < m) {
140                     k *= 2;
141                     x = (x * (Poly({1}) - x.log(k) + modxk(k))).modxk(k);
142                 }
143                 return x.modxk(m);
144             }
145             Poly pow(int k, int m) const {
146                 int i = 0;
147                 while (i < size() && a[i].x == 0) i++;
148                 if (i == size() || 1LL * i * k >= m) {
149                     return Poly(vector<Z>(m));
150                 }
151                 Z v = a[i];
152                 auto f = divxk(i) * v.inv();
153                 return (f.log(m - i * k) * k).exp(m - i * k).mulxk(i * k)
154                 * power(v, k);
155             }
156             Poly sqrt(int m) const {
157                 Poly x({1});
158                 int k = 1;
159                 while (k < m) {
160                     k *= 2;
161                     x = (x + (modxk(k) * x.inv(k)).modxk(k)) * ((MOD + 1) /
162                     2);
163                 }
164                 return x.modxk(m);
165             }
166             Poly mulT(Poly b) const {
167                 if (b.size() == 0) return Poly();
168                 int n = b.size();
169                 reverse(b.a.begin(), b.a.end());

```

```

162     return ((*this) * b).divvxk(n - 1);
163 }
164 Poly divmod(Poly b) const {
165     auto n = size(), m = b.size();
166     auto t = *this;
167     reverse(t.a.begin(), t.a.end());
168     reverse(b.a.begin(), b.a.end());
169     Poly res = (t * b.inv(n)).modxxk(n - m + 1);
170     reverse(res.a.begin(), res.a.end());
171     return res;
172 }
173 vector<Z> eval(vector<Z> x) const {
174     if (size() == 0) return vector<Z>(x.size(), 0);
175     const int n = max(int(x.size()), size());
176     vector<Poly> q(4 * n);
177     vector<Z> ans(x.size());
178     x.resize(n);
179     function<void(int, int, int)> build = [&](int p, int l,
180 ↪ int r) {
181         if (r - l == 1) {
182             q[p] = Poly({1, -x[l]});
183         } else {
184             int m = (l + r) / 2;
185             build(2 * p, l, m), build(2 * p + 1, m, r);
186             q[p] = q[2 * p] * q[2 * p + 1];
187         }
188     };
189     build(1, 0, n);
190     auto work = [&](auto self, int p, int l, int r, const Poly
191 ↪ &num) -> void {
192         if (r - l == 1) {
193             if (l < int(ans.size())) ans[l] = num[0];
194         } else {
195             int m = (l + r) / 2;
196             self(self, 2 * p, l, m, num.mulT(q[2 * p + 1]).modxxk(m
197 ↪ - l));
198             self(self, 2 * p + 1, m, r, num.mulT(q[2 * p]).modxxk(r
199 ↪ - m));
200         }
201     };
202     work(work, 1, 0, n, mulT(q[1].inv(n)));
203     return ans;
204 }
205 };

```

## Sieve

- linear sieve

```

1 vector<int> min_primes(MAX_N), primes;
2 primes.reserve(1e5);
3 for (int i = 2; i < MAX_N; i++) {
4     if (!min_primes[i]) min_primes[i] = i, primes.push_back(i);
5     for (auto& p : primes) {
6         if (p * i >= MAX_N) break;
7         min_primes[p * i] = p;
8         if (i % p == 0) break;
9     }
10 }

```

- mobius function

```

1 vector<int> min_p(MAX_N), mu(MAX_N), primes;
2 mu[1] = 1, primes.reserve(1e5);
3 for (int i = 2; i < MAX_N; i++) {
4     if (min_p[i] == 0) {
5         min_p[i] = i;
6         primes.push_back(i);
7         mu[i] = -1;
8     }
9     for (auto p : primes) {
10         if (i * p >= MAX_N) break;
11         min_p[i * p] = p;
12         if (i % p == 0) {
13             mu[i * p] = 0;
14             break;
15         }

```

```

16         mu[i * p] = -mu[i];
17     }
18 }

```

- Euler's totient function

```

1 vector<int> min_p(MAX_N), phi(MAX_N), primes;
2 phi[1] = 1, primes.reserve(1e5);
3 for (int i = 2; i < MAX_N; i++) {
4     if (min_p[i] == 0) {
5         min_p[i] = i;
6         primes.push_back(i);
7         phi[i] = i - 1;
8     }
9     for (auto p : primes) {
10         if (i * p >= MAX_N) break;
11         min_p[i * p] = p;
12         if (i % p == 0) {
13             phi[i * p] = phi[i] * p;
14             break;
15         }
16         phi[i * p] = phi[i] * phi[p];
17     }
18 }

```

## Gaussian Elimination

```

1 bool is_0(Z v) { return v.x == 0; }
2 Z abs(Z v) { return v; }
3 bool is_0(double v) { return abs(v) < 1e-9; }
4
5 // 1 => unique solution, 0 => no solution, -1 => multiple
6 ↪ solutions
7 template <typename T>
8 int gaussian_elimination(vector<vector<T>> &a, int limit) {
9     if (a.empty() || a[0].empty()) return -1;
10     int h = (int)a.size(), w = (int)a[0].size(), r = 0;
11     for (int c = 0; c < limit; c++) {
12         int id = -1;
13         for (int i = r; i < h; i++) {
14             if (!is_0(a[i][c]) && (id == -1 || abs(a[id][c]) <
15 ↪ abs(a[i][c]))) {
16                 id = i;
17             }
18         }
19         if (id == -1) continue;
20         if (id > r) {
21             swap(a[r], a[id]);
22             for (int j = c; j < w; j++) a[id][j] = -a[id][j];
23         }
24         vector<int> nonzero;
25         for (int j = c; j < w; j++) {
26             if (!is_0(a[r][j])) nonzero.push_back(j);
27         }
28         T inv_a = 1 / a[r][c];
29         for (int i = r + 1; i < h; i++) {
30             if (is_0(a[i][c])) continue;
31             T coeff = -a[i][c] * inv_a;
32             for (int j : nonzero) a[i][j] += coeff * a[r][j];
33         }
34         ++r;
35     }
36     for (int row = h - 1; row >= 0; row--) {
37         for (int c = 0; c < limit; c++) {
38             if (!is_0(a[row][c])) {
39                 T inv_a = 1 / a[row][c];
40                 for (int i = row - 1; i >= 0; i--) {
41                     if (is_0(a[i][c])) continue;
42                     T coeff = -a[i][c] * inv_a;
43                     for (int j : nonzero) a[i][j] += coeff *
44 ↪ a[row][j];
45                 }
46                 break;
47             }
48         }
49     }
50     // not-free variables: only it on its line
51     for (int i = r; i < h; i++) if (!is_0(a[i][limit])) return 0;

```

```

48     return (r == limit) ? 1 : -1;
49 }
50
51 template <typename T>
52 pair<int, vector<T>> solve_linear(vector<vector<T>> a, const
    ↪ vector<T> &b, int w) {
53     int h = (int)a.size();
54     for (int i = 0; i < h; i++) a[i].push_back(b[i]);
55     int sol = gaussian_elimination(a, w);
56     if (!sol) return {0, vector<T>()};
57     vector<T> x(w, 0);
58     for (int i = 0; i < h; i++) {
59         for (int j = 0; j < w; j++) {
60             if (!is_0(a[i][j])) {
61                 x[j] = a[i][w] / a[i][j];
62                 break;
63             }
64         }
65     }
66     return {sol, x};
67 }

```

## is\_prime

- (Miller–Rabin primality test)

```

1  i128 power(i128 a, i128 b, i128 MOD = 1, i128 res = 1) {
2      for (; b; b /= 2, (a *= a) %= MOD)
3          if (b & 1) (res *= a) %= MOD;
4      return res;
5  }
6
7  bool is_prime(ll n) {
8      if (n < 2) return false;
9      static constexpr int A[] = {2, 3, 5, 7, 11, 13, 17, 19, 23};
10     int s = __builtin_ctzll(n - 1);
11     ll d = (n - 1) >> s;
12     for (auto a : A) {
13         if (a == n) return true;
14         ll x = (ll)power(a, d, n);
15         if (x == 1 || x == n - 1) continue;
16         bool ok = false;
17         for (int i = 0; i < s - 1; ++i) {
18             x = ll((i128)x * x % n); // potential overflow!
19             if (x == n - 1) {
20                 ok = true;
21                 break;
22             }
23         }
24         if (!ok) return false;
25     }
26     return true;
27 }
28
29 ll pollard_rho(ll x) {
30     ll s = 0, t = 0, c = rng() % (x - 1) + 1;
31     ll stp = 0, goal = 1, val = 1;
32     for (goal = 1;; goal *= 2, s = t, val = 1) {
33         for (stp = 1; stp <= goal; ++stp) {
34             t = ll(((i128)t * t + c) % x);
35             val = ll(((i128)val * abs(t - s) % x);
36             if ((stp % 127) == 0) {
37                 ll d = gcd(val, x);
38                 if (d > 1) return d;
39             }
40         }
41         ll d = gcd(val, x);
42         if (d > 1) return d;
43     }
44 }
45
46 ll get_max_factor(ll _x) {
47     ll max_factor = 0;
48     function<void(ll)> fac = [&](ll x) {
49         if (x <= max_factor || x < 2) return;
50         if (is_prime(x)) {
51             max_factor = max_factor > x ? max_factor : x;

```

```

24     return;
25 }
26 ll p = x;
27 while (p >= x) p = pollard_rho(x);
28 while ((x % p) == 0) x /= p;
29 fac(x), fac(p);
30 };
31 fac(_x);
32 return max_factor;
33 }

```

## Radix Sort

```

1  struct identity {
2      template<typename T>
3      T operator()(const T &x) const {
4          return x;
5      }
6  };
7
8  // A stable sort that sorts in passes of `bits_per_pass` bits
    ↪ at a time.
9  template<typename T, typename T_extract_key = identity>
10 void radix_sort(vector<T> &data, int bits_per_pass = 10, const
    ↪ T_extract_key &extract_key = identity()) {
11     if ((int64_t)(data.size()) * (64 -
    ↪ __builtin_clzll(data.size())) < 2 * (1 << bits_per_pass))
12     {
13         stable_sort(data.begin(), data.end(), [&](const T &a,
    ↪ const T &b) {
14             return extract_key(a) < extract_key(b);
15         });
16         return;
17     }
18     using T_key = decltype(extract_key(data.front()));
19     T_key minimum = numeric_limits<T_key>::max();
20
21     for (T &x : data)
22         minimum = min(minimum, extract_key(x));
23
24     int max_bits = 0;
25
26     for (T &x : data) {
27         T_key key = extract_key(x);
28         max_bits = max(max_bits, key == minimum ? 0 : 64 -
    ↪ __builtin_clzll(key - minimum));
29     }
30
31     int passes = max((max_bits + bits_per_pass / 2) /
    ↪ bits_per_pass, 1);
32
33     if (64 - __builtin_clzll(data.size()) <= 1.5 * passes) {
34         stable_sort(data.begin(), data.end(), [&](const T &a,
    ↪ const T &b) {
35             return extract_key(a) < extract_key(b);
36         });
37         return;
38     }
39
40     vector<T> buffer(data.size());
41     vector<int> counts;
42     int bits_so_far = 0;
43
44     for (int p = 0; p < passes; p++) {
45         int bits = (max_bits + p) / passes;
46         counts.assign(1 << bits, 0);
47
48         for (T &x : data) {
49             T_key key = T_key(extract_key(x) - minimum);
50             counts[(key >> bits_so_far) & ((1 << bits) -
    ↪ 1)]++;
51         }
52
53         int count_sum = 0;
54
55         for (int &count : counts) {

```

```

56         int current = count;
57         count = count_sum;
58         count_sum += current;
59     }
60
61     for (T &x : data) {
62         T_key key = T_key(extract_key(x) - minimum);
63         int key_section = int((key >> bits_so_far) & ((1
↳ << bits) - 1));
64         buffer[counts[key_section]++] = x;
65     }
66
67     swap(data, buffer);
68     bits_so_far += bits;
69 }
70 }

```

## • USAGE

```

1 radix_sort(edges, 10, [&](const edge &e) -> int { return
↳ abs(e.weight - x); });

```

## String

### AC Automaton

```

1 struct AC_automaton {
2     int sz = 26;
3     vector<vector<int>> e = {vector<int>(sz)}; // vector is
↳ faster than unordered_map
4     vector<int> fail = {0};
5     vector<int> end = {0};
6
7     void insert(string& s) {
8         int p = 0;
9         for (auto c : s) {
10             c -= 'a';
11             if (!e[p][c]) {
12                 e.emplace_back(sz);
13                 fail.emplace_back();
14                 end.emplace_back();
15                 e[p][c] = e.size() - 1;
16             }
17             p = e[p][c];
18         }
19         end[p] += 1;
20     }
21
22     void build() {
23         queue<int> q;
24         for (int i = 0; i < sz; i++)
25             if (e[0][i]) q.push(e[0][i]);
26         while (!q.empty()) {
27             int p = q.front();
28             q.pop();
29             for (int i = 0; i < sz; i++) {
30                 if (e[p][i]) {
31                     fail[e[p][i]] = e[fail[p]][i];
32                     q.push(e[p][i]);
33                 } else {
34                     e[p][i] = e[fail[p]][i];
35                 }
36             }
37         }
38     }
39 };

```

### KMP

- nex[i]: length of longest common prefix & suffix for pat[0..i]

```

1 vector<int> get_next(vector<int> &pat) {
2     int m = (int)pat.size();
3     vector<int> nex(m);
4     for (int i = 1, j = 0; i < m; i++) {

```

```

5         while (j && pat[j] != pat[i]) j = nex[j - 1];
6         if (pat[j] == pat[i]) j++;
7         nex[i] = j;
8     }
9     return nex;
10 }

```

- kmp match for txt and pat

```

1 auto nex = get_next(pat);
2 for (int i = 0, j = 0; i < n; i++) {
3     while (j && pat[j] != txt[i]) j = nex[j - 1];
4     if (pat[j] == txt[i]) j++;
5     if (j == m) {
6         // do what you want with the match
7         // start index is `i - m + 1`
8         j = nex[j - 1];
9     }
10 }

```

### Z function

- z[i]: length of longest common prefix of s and s[i:]

```

1 vector<int> z_function(string s) {
2     int n = (int)s.size();
3     vector<int> z(n);
4     for (int i = 1, l = 0, r = 0; i < n; ++i) {
5         if (i <= r) z[i] = min(r - i + 1, z[i - l]);
6         while (i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z[i];
7         if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
8     }
9     return z;
10 }

```

### General Suffix Automaton

```

1 constexpr int SZ = 26;
2
3 struct GSAM {
4     vector<vector<int>> e = {vector<int>(SZ)}; // the labeled
↳ edges from node i
5     vector<int> parent = {-1}; // the parent of
↳ i
6     vector<int> length = {0}; // the length of
↳ the longest string
7
8     GSAM(int n) { e.reserve(2 * n), parent.reserve(2 * n),
↳ length.reserve(2 * n); };
9     int extend(int c, int p) { // character, last
10         bool f = true; // if already exist
11         int r = 0; // potential new node
12         if (!e[p][c]) { // only extend when not exist
13             f = false;
14             e.push_back(vector<int>(SZ));
15             parent.push_back(0);
16             length.push_back(length[p] + 1);
17             r = (int)e.size() - 1;
18             for (; ~p && !e[p][c]; p = parent[p]) e[p][c] = r; //
↳ update parents
19         }
20         if (f || ~p) {
21             int q = e[p][c];
22             if (length[q] == length[p] + 1) {
23                 if (f) return q;
24                 parent[r] = q;
25             } else {
26                 e.push_back(e[q]);
27                 parent.push_back(parent[q]);
28                 length.push_back(length[p] + 1);
29                 int qq = parent[q] = (int)e.size() - 1;
30                 for (; ~p && e[p][c] == q; p = parent[p]) e[p][c] =
↳ qq;
31             if (f) return qq;
32             parent[r] = qq;
33         }
34     }

```



```

35     return r;
36 }
37 };

    • Topo sort on GSAM

```

```

15 int main() {
16     string s;
17     cin >> s;
18     duval(s);
19 }

```

```

1 ll sz = gsam.e.size();
2 vector<int> c(sz + 1);
3 vector<int> order(sz);
4 for (int i = 1; i < sz; i++) c[gsam.length[i]]++;
5 for (int i = 1; i < sz; i++) c[i] += c[i - 1];
6 for (int i = 1; i < sz; i++) order[c[gsam.length[i]]--] = i;
7 reverse(order.begin(), order.end()); // reverse so that large
    ↳ len to small

```

- can be used as an ordinary SAM
- USAGE (the number of distinct substring)

```

1 int main() {
2     int n, last = 0;
3     string s;
4     cin >> n;
5     auto a = GSAM();
6     for (int i = 0; i < n; i++) {
7         cin >> s;
8         last = 0; // reset last
9         for (auto&& c : s) last = a.extend(c, last);
10    }
11    ll ans = 0;
12    for (int i = 1; i < a.e.size(); i++) {
13        ans += a.length[i] - a.length[a.parent[i]];
14    }
15    cout << ans << endl;
16    return 0;
17 }

```

## Manacher

```

1 string longest_palindrome(string& s) {
2     // init "abc" -> "~$a#b#c$"
3     vector<char> t{'^', '#'};
4     for (char c : s) t.push_back(c), t.push_back('#');
5     t.push_back('$');
6     // manacher
7     int n = t.size(), r = 0, c = 0;
8     vector<int> p(n, 0);
9     for (int i = 1; i < n - 1; i++) {
10        if (i < r + c) p[i] = min(p[2 * c - i], r + c - i);
11        while (t[i + p[i] + 1] == t[i - p[i] - 1]) p[i]++;
12        if (i + p[i] > r + c) r = p[i], c = i;
13    }
14    // s[i] -> p[2 * i + 2] (even), p[2 * i + 2] (odd)
15    // output answer
16    int index = 0;
17    for (int i = 0; i < n; i++)
18        if (p[index] < p[i]) index = i;
19    return s.substr((index - p[index]) / 2, p[index]);
20 }

```

## Lyndon

- def: suf(s) > s

```

1 void duval(const string &s) {
2     int n = (int)s.size();
3     for (int i = 0; i < n; i++) {
4         int j = i, k = i + 1;
5         for (; j < n && s[j] <= s[k]; j++, k++)
6             if (s[j] < s[k]) j = i - 1;
7
8         while (i <= j) {
9             // cout << s.substr(i, k - j) << '\n';
10            i += k - j;
11        }
12    }
13 }
14

```