Regression and the General Linear Model

Multivariate Statistic

Made by:

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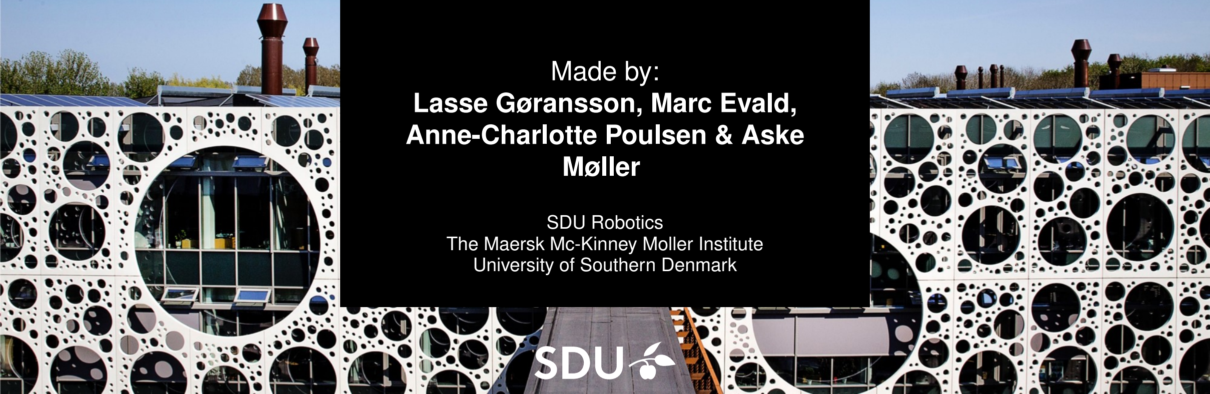
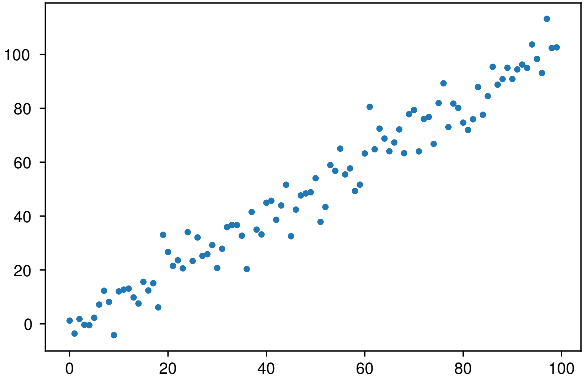


Agenda

- Motivation
- UMLR
  - Model
  - Estimation
  - Model Check
  - Inference on β
  - Model reduction
- MMLR

Motivation



Assumptions

- $$Y = z\beta + \epsilon$$
- $$E\left[\epsilon\right] = 0$$
- $$V\left[\epsilon\right] = \sigma^2$$

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & z_{11} & z_{12} & \cdots & z_{1r} \\ 1 & z_{21} & z_{22} & & \vdots \\ \vdots & & & \ddots & \\ 1 & z_{n1} & \cdots & & z_{nr} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_r \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}$$

Estimation of Coefficients

$$\left(z^T z\right)^{-1} \quad z^T Y = \hat{\beta}_{LS}$$

$$\left(z^T z\right)^{+} \quad z^T Y = \hat{\beta}_{LS}$$

Model check

$$\sum_{j=1}^{n}\left(Y_j-\bar{Y}\right)\left(Y_j-\bar{Y}\right)^T=\sum_{j=1}^{n}\left(Y_j-\hat{Y}_j\right)\left(Y_j-\hat{Y}_j\right)^T+\sum_{j=1}^{n}\left(\hat{Y}_j-\bar{Y}\right)\left(\hat{Y}_j-\bar{Y}\right)^T$$

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$$SST=SSE+SSR$$

$$R^2=\frac{SSR}{SST}$$

Model check

$$\sum_{j=1}^{n} \left(Y_j - \bar{Y}\right)\left(Y_j - \bar{Y}\right)^T = \sum_{j=1}^{n} \left(Y_j - \hat{Y}_j\right)\left(Y_j - \hat{Y}_j\right)^T \quad + \sum_{j=1}^{n} \quad \left(\hat{Y}_j - \bar{Y}\right)\left(\hat{Y}_j - \bar{Y}\right)^T$$

$$SST = SSE \qquad\qquad\qquad\qquad + SSR$$

$$R^2 = \frac{SSR}{SST}$$

$$R^2_{adj} = 1 - \frac{(n-1)\,SSE}{(n-r-1)\,SST}$$

Inference on β

New assumption.

$$\left(\hat{\beta} - \beta^*\right)^T \left(\frac{Z^T Z}{(r+1)\,\sigma^2}\right) \left(\hat{\beta} - \beta^*\right) \sim F_{r+1,n-(r+1)}$$

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- Hypothesis Test
- Confidence Region

Model reduction

$$\beta = \begin{bmatrix} \beta_{(1)} \\ \beta_{(2)} \end{bmatrix} \begin{array}{l} (q+1) \\ (r-q) \end{array}$$

Model reduction

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$$ESS = \Delta SSR = SSR_{\beta} - SSR_{\beta_{(1)}}$$

$$\frac{1}{\sigma^2} \cdot \frac{ESS}{r-q} \sim F_{r-q,n-(r+1)}$$

$$\begin{bmatrix} y_{11} & \cdots & y_{1p} \\ \vdots & & \vdots \\ y_{n1} & \cdots & y_{np} \end{bmatrix} = \begin{bmatrix} 1 & z_{11} & z_{12} & \cdots & z_{1r} \\ 1 & z_{21} & z_{22} & & \vdots \\ \vdots & & & \ddots & \\ 1 & z_{n1} & \cdots & & z_{nr} \end{bmatrix} \begin{bmatrix} \beta_{01} & \cdots & \beta_{0p} \\ \vdots & & \vdots \\ \beta_{0r} & \cdots & \beta_{rp} \end{bmatrix} + \begin{bmatrix} \epsilon_{11} & \cdots & \epsilon_{1p} \\ \vdots & & \vdots \\ \epsilon_{n1} & \cdots & \epsilon_{np} \end{bmatrix}$$

$$\Lambda = \frac{\max L\left(\beta_{(1)}, \Sigma\right)}{\max L\left(\beta, \Sigma\right)} = \left(\frac{|\hat{\Sigma}|}{|\hat{\Sigma}_{(1)}|}\right)^{\frac{n}{2}}$$

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$$-2\log\left(\Lambda\right) = -n\log\left(\frac{|\hat{\Sigma}|}{|\hat{\Sigma}_{(1)}|}\right) \underset{approx}{\sim} \chi^2_{(r-q)p}$$

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$$-2\log\left(\Lambda\right) = -\left(n-(r+1)-\frac{p-r+q+1}{2}\right)\log\left(\frac{|\hat{\Sigma}|}{|\hat{\Sigma}_{(1)}|}\right) \underset{approx}{\sim} \chi^2_{(r-q)p}$$