

# One and two sample hotelling $T^2$ -test

## Multivariate Statistic

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# Agenda



- ▶ Motivation
- ▶ One-sample
- ▶ Two-sample
- ▶ Bartlett

# Motivation



# Motivation



## Assumptions

- ▶ Obs. IID
- ▶ MVN distributed

# Hypothesis Testing on $\mu$

One-sample



## Hypothesis

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu \neq \mu_0$$

$$T_0^2 = (\bar{x} - \mu_0)^T \left( \frac{S}{n} \right)^{-1} (\bar{x} - \mu_0) \sim \frac{p(n-1)}{n-p} F_{p, n-p}$$

# Hypothesis Testing on $\mu$

One-sample



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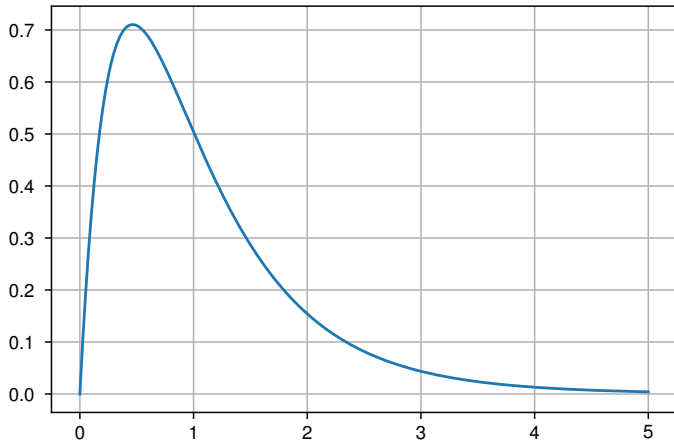
$$t_0^2 > \frac{p(n-1)}{n-p} F_{p, n-p, \alpha}$$

Reject

$$P(F_{p, n-p} > \frac{n-p}{p(n-1)} t_0^2)$$

# Hypothesis Testing on $\mu$

One-sample



# Motivation





# Hypothesis Testing on $\mu$

Two-sample



Assumptions.

Model:

- Pair wise observations.  $X$  and  $Y$ .

Difference model:

- $D = (X - Y) \sim N(\mu_D, \Sigma_D)$

Hypothesis

$$H_0 : \mu_D = \delta$$

$$H_1 : \mu_D \neq \delta$$

# Hypothesis Testing

Paired comparison



Estimates:

$$\blacktriangleright \bar{D} = \frac{1}{n} \sum_{j=1}^n D_j$$

$$\blacktriangleright S_d = \frac{1}{n-1} \sum_{j=1}^n (D_j - \bar{D})^T (D_j - \bar{D})$$

Test Statistic:

$$T_0^2 = (\bar{D} - \delta_0)^T \left( \frac{S_d}{n} \right)^{-1} (\bar{D} - \delta_0)$$

Reject:

- ▶ p-value
- ▶ Critical value

# Hypothesis Testing on $\mu$



## ► Equal $\Sigma$

$$S_p = \frac{(n_1 - 1) S_1 + (n_2 - 1) S_2}{(n_1 - 1) + (n_2 - 1)}$$

## ► Unequal $\Sigma$

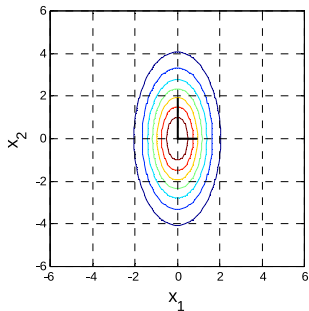
$$T^2 = (\bar{D} - \delta)^T \left( \frac{S_1}{n_1} + \frac{S_2}{n_2} \right)^{-1} (\bar{D} - \delta)$$

# Confidence regions



Confidence region for  $\delta$

$$\forall \delta \in \mathbb{R}^P : (\bar{D} - \delta)^T \left( \frac{S_d}{n} \right)^{-1} (\bar{D} - \delta) < \frac{p(n-1)}{n-p} F_{p, n-p, \alpha}$$



# Bartlett test



Hypothesis:

$$H_0 : \Sigma_X = \Sigma_Y$$

$$H_1 : \Sigma_X \neq \Sigma_Y$$

$$\Lambda = \frac{\max_{\Sigma} L(\Sigma)}{\max_{\Sigma_X, \Sigma_Y} L(\Sigma_X, \Sigma_Y)}$$

$$-2 \log \Lambda = (n_X + n_Y - 2) \cdot \log |S_p| - (n_X - 1) \cdot \log |S_X| - (n_Y - 1) \cdot \log |S_Y|$$

$$-2 \log \Lambda \underset{approx}{\sim} \chi^2_{p \frac{p+1}{2}}$$