One and two sample hotelling T^2-test

Multivariate Statistic

Made by:
Lasse Göransson, Marc Evald, Anne-Charlotte Poulsen & Aske Møller

SDU Robotics
The Maersk Mc-Kinney Moller Institute
University of Southern Denmark

SDU

Agenda

- Motivation
- One-sample
- Two-sample
- Bartlett

Motivation

- Compare μ vector.

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Assumptions

- Obs. IID
- MVN distributed

Hypothesis

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

$$T_0^2 = (\bar{x} - \mu_0)^T \left(\frac{S}{n}\right)^{-1} (\bar{x} - \mu_0) \sim \frac{p(n-1)}{n-p} F_{p,n-p}$$

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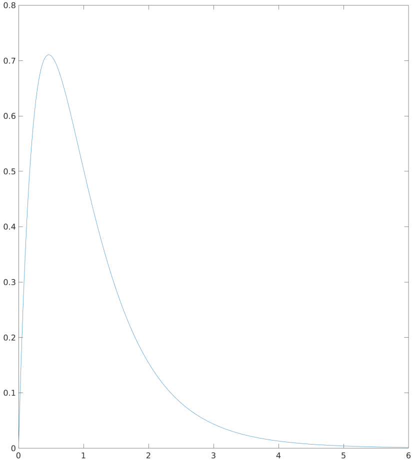
$$t_0^2 > \frac{p(n-1)}{n-p} F_{p,n-p,\alpha}$$

Reject

$$P(F_{p,n-p} > \frac{n-p}{p(n-1)} t_0^2)$$

Hypothesis Testing on μ
One-sample



Assumptions.
Model:

- Pair wise observations. X and Y.

Difference model:

- $D = (X - Y) \sim N(\mu_D, \Sigma_D)$

Hypothesis

$$H_0 : \mu_D = \delta$$
$$H_1 : \mu_D \neq \delta$$

Hypothesis Testing
Paired comparison

Estimates:

- $\bar{D} = \frac{1}{n} \sum_{j=1}^{n} D_j$
- $S_d = \frac{1}{n-1} \sum_{j=1}^{n} \left(D_j - \bar{D}\right)^T \left(D_j - \bar{D}\right)$

Test Statistic:

$$T_0^2 = \left(\bar{D} - \delta_0\right)^T \left(\frac{S_d}{n}\right)^{-1} \left(\bar{D} - \delta_0\right)$$

Reject:

- p-value
- Critical value

Hypothesis Testing on μ

- Equal Σ

$$S_p = \frac{(n_1 - 1)\, S_1 + (n_2 - 1)\, S_2}{(n_1 - 1) + (n_2 - 1)}$$

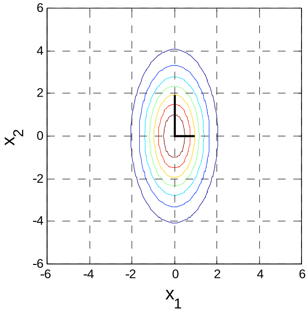
- Unequal Σ

$$T^2 = \left(\bar{D} - \delta\right)^T \left(\frac{S_1}{n_1} + \frac{S_2}{n_2}\right)^{-1} \left(\bar{D} - \delta\right)$$

Confidence regions

Confidence region for δ

$$\forall \delta \in \mathbb{R}^P : \left(\bar{D} - \delta\right)^T \left(\frac{S_d}{n}\right)^{-1} \left(\bar{D} - \delta\right) < \frac{p\,(n-1)}{n-p} F_{p,n-p,\alpha}$$



Bartlett test

Hypothesis:

$$H_0 : \Sigma_X = \Sigma_Y$$
$$H_1 : \Sigma_X \neq \Sigma_Y$$

$$\Lambda = \frac{\underset{\Sigma}{max} L\left(\Sigma\right)}{\underset{\Sigma_X, \Sigma_Y}{max} L\left(\Sigma_X, \Sigma_Y\right)}$$

$$-2 \log \Lambda = (n_X + n_Y - 2) \cdot \log |S_p| - (n_X - 1) \cdot \log |S_X| - (n_Y - 1) \cdot \log |S_Y|$$

$$-2 \log \Lambda \underset{approx}{\sim} \chi^2_{p\frac{p+1}{2}}$$