
Parameters of the Just-Jahreiß model of the Milky Way disk¹

(python package `jjmodel`²)

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1. General information

There are two parameter files to be prepared for the Just-Jahreiß (JJ) model:

1. **parameters**. This file contains most of the model parameters and must be filled for every run. Parameters are organized in rows.
2. **sfrd_peaks_parameters**. Contains parameters which describe special thin-disk populations (extra peaks on the star formation rate function, SFR, and kinematics of these stars). These parameters need to be specified only when these special model features are activated (with parameter `pkey`, see Sect. 2.5.1). In this file, parameters are organized in columns, and the number of rows corresponds to the number of additional peaks added to the thin-disk SFR.

List of acronyms and paper aliases:

AMR	age-metallicity relation	Paper I	Just and Jahreiß (2010)
AVR	age - W-velocity dispersion relation	Paper II	Just et al. (2011)
DM	dark matter	Paper III	Rybizki and Just (2015)
IMF	initial mass function	Paper IV	Sysoliatina and Just (2021)
MAP	mono-age subpopulation	Paper V	Sysoliatina and Just (2022), in prep.
MD	metallicity distribution		
MW	Milky Way		
SF	star formation		
SFR	star formation rate		
SA	stellar assembly		

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²<https://github.com/askenja/jjmodel>

2. File parameters

2.1. Run settings

run_mode : Running mode (int, Dimensionless[see description])

There are two options how to use the `jjmodel` package: when `run_mode = 0`, only the Solar neighborhood is modeled (then not all of the parameters listed in this manual must be given, see Sect. 4 for details); when `run_mode = 1`, in addition to the Solar radius, the Milky Way (MW) disk is modeled in a range of chosen Galactocentric distances (see `Rmin` and `Rmax`).

nprocess : Number of independent processes (int, Dimensionless[see description])

The `jjmodel` package supports multiprocessing. It is used in two cases: (1) when the Poisson-Boltzmann Eq. is iteratively solved at each Galactocentric distance R (each R -bin of the MW disk is processed independently from other bins), and (2) during the generation of the so-called stellar assemblies (populations of the same metallicities, ages, and masses, SA) at a given radius. In the first case, the maximum reasonable value of `nprocess` is $(R_{\text{max}} - R_{\text{min}})/dR$ (see Sect. 2.3). Though, setting a somewhat higher value is not forbidden as this does not slow down the calculation, as far as we know. In the second case, the larger `nprocess` is, the better, because the number of SA, which can be processed independently, can be very large ($10^3 - 10^5$, depending on the MW component and mass resolution of isochrones). In practice, however, the number of independent processes is limited by the number of available CPU cores (and the code will not allow to set `nprocess` larger than the number of CPUs).

out_dir : Suffix of the output directory name (string)

The default name of the output directory is constructed according to the formula:

`default_name = 'Rsun' + Rsun` if `run_mode = 0`, or `default_name = Rmin + 'R' + Rmax + '_dR' + dR` otherwise.

The full directory name is then constructed as `default_name + '_' + out_dir`. Thus, with the help of this parameter user can customize the output directory name. This helps to distinguish between the different model realizations.

out_mode : Save mode (int, Dimensionless[see description])

Prescribes how the output is saved when the output directory already exists. If `out_mode = 0`, all existing files in the folder will be removed before saving the new ones. If `out_mode = 1`, the files will be overwritten. This second option is useful when a user is sure that the change in parameters will have an impact only on some of the output quantities (e.g. potential, scale heights, density profiles), but cannot affect others (e.g. computationally expensive SA tables, such that it is not necessary to re-calculate them). However, if the parameter impact is understood wrongly, this will result in a mixture of two different model realizations in one folder, so use this option cautiously. If uncertain, set `out_mode = 0` or create a unique folder name for each run with `out_dir` parameter.

2.2. Solar coordinates and velocity

Rsun : Solar Galactocentric distance (float, Distance[kpc])

Galactocentric distance of the Sun. For the most recent and precise value, see e.g. [Gravity Collaboration et al. \(2019\)](#). Does not influence the local potential-density pair. When a range of Galactocentric distances is modeled, `Rsun` plays a role of scaling parameter for the radial density laws of the MW components, and therefore, impacts potential and density at other R . When the JJ model is compared to some data in terms of (Galactocentric) distance-dependent quantities (e.g. stellar positions or velocities in Galactic coordinates), be sure that you use the same `Rsun` both in the model and in the data.

zsun : Solar distance from the Galactic plane (float, Distance[pc])

Distance of the Sun from the Galactic plane (positive, as the Sun is located “above” the plane). In the literature, this value varies in the range of approximately 0 – 20 pc, depending on the method. In the JJ model, we usually assume `zsun` value of ~ 20 pc (e.g. see [Bovy, 2017](#)). Does not influence potential-density pairs at any R , but may become important when the model predictions are compared to the data in the local volume, especially close to the Galactic plane. In this case, be sure that the value of `zsun` is the same when the compared quantities are calculated from the observational data and the model.

Vsun : Solar peculiar velocity component (float, Velocity[km s⁻¹])

The tangential component of the Solar peculiar velocity. The value of 12.24 km s^{-1} from [Schönrich et al. \(2010\)](#) is used most often, but also significantly larger and smaller values are reported in the literature (e.g. [Sysoliatina et al., 2018](#)). Does not influence the local potential-density pair, but implicitly affects the scaling parameter of the radial profile of the dark matter (DM) halo, and thus, impacts the vertical potential and density laws at other R (specifically, it is used to calculate the local circular speed: $v_c(R_\odot) = v_\odot - V_\odot$, where the Solar tangential speed v_\odot is consistent with the assumed Solar Galactocentric distance R_\odot and the measured proper motion of SgrA* from [Reid and Brunthaler, 2005](#); the scaling parameter of the DM cored isothermal sphere, a_h , is then adapted to reproduce $v_c(R_\odot)$).

2.3. Rz-grid and radial profiles of the Milky Way components

Rmin : Minimal Galactocentric distance (float, Distance[kpc])

Center of the innermost radial bin. The JJ model is not applicable for the central MW region with the bulge and bar, so the model must not be applied to distances smaller than 4 kpc, i.e., $R_{\min} \geq 4 \text{ kpc}$.

Rmax : Maximal Galactocentric distance (float, Distance[kpc])

Center of the outermost radial bin. As the disk warp is not included in the JJ model, we recommend using $R_{\max} \lesssim 14\text{--}15 \text{ kpc}$.

dR : Resolution in Galactocentric distance (float, Distance[kpc])

Width of the radial bins. Make sure that $(R_{\max} - R_{\min})/dR$ is an integer.

zmax : Maximum height (float, Distance[pc])

Maximum distance from the Galactic plane up to which the vertical gravitational potential and density profiles are reconstructed at each radius. The standard (and maximal allowed) value is 2000 pc.

dz : Resolution in height (float, Distance[pc])

Step for the vertical z-grid. Should not be too large ($\lesssim 10 \text{ pc}$), especially when near-plane regions have to be investigated, as it defines how accurately the shape of the vertical gravitational potential is reconstructed. On the other side, setting very small values can significantly slow down the calculation. Optimal value lays in the range of 1 – 5 pc.

Rd : Thin-disk radial scale length (float, Distance[kpc])

Exponential scale length of the overall thin-disk radial density profile. See [Bland-Hawthorn and Gerhard \(2016\)](#) for an overview of R_d determination methods and its best value.

Rt : Thick-disk radial scale length (float, Distance[kpc])

Exponential scale length of the overall thick-disk radial density profile (see [Bland-Hawthorn and Gerhard, 2016](#)).

Rg1 : Radial scale length of molecular gas (float, Distance[kpc])

Exponential scale length of the radial profile of molecular gas. Surface density profile of molecular gas peaks at $\sim 4 \text{ kpc}$ and declines at larger radii with a short scale length of $\sim 2 - 3 \text{ kpc}$ ([Kramer and Randall, 2016](#)) which we adopt for our modeling.

Rg2 : Radial scale length of atomic gas (float, Distance[kpc])

Exponential scale length of the radial profile of atomic gas. According to [Kramer and Randall \(2016\)](#), the HI surface density remains roughly constant in the wide distance range of $R \approx 4 - 14 \text{ kpc}$ and then declines with a scale length of $\sim 4.5 \text{ kpc}$. Similarly, [Kalberla and Dedes \(2008\)](#) describe HI radial distribution as approximately constant in the inner disk and declining exponentially in the outer disk (with a scale length of $\sim 3.75 \text{ kpc}$ for $12.5 \text{ kpc} < R < 30 \text{ kpc}$). When the global JJ model is built for $4 \text{ kpc} < R \lesssim 14 \text{ kpc}$, we often put $R_{g2} = 10 \text{ kpc}$ to describe the atomic gas as a component with a slow decline of surface density with radius which is broadly consistent with the mentioned observational results.

Rg10 : Radius of the H₂ inner hole (float, Distance[kpc])

Radius of an inner hole in the molecular gas disk. According to observations (see [Kramer and Randall, 2016](#) for an overview), the H₂ gas density peaks at ~ 4 kpc and quickly decreases at smaller R, therefore, this parameter may be important for modeling the innermost radial bins. Also, it is necessary to account for the inner hole in gas distribution to get a correct enclosed gas mass for the prediction of the MW rotation curve. Currently, Rg10 can be ≤ 4 kpc.

Rg20 : Radius of the HI inner hole (float, Distance[kpc])

Radius of an inner hole in the atomic gas disk. See [Kramer and Randall \(2016\)](#) for an overview of measured HI gas distribution across the disk. In comparison to molecular gas, atomic hydrogen density peaks at somewhat larger radius, ~ 6 kpc instead of ~ 4 kpc, and the density decrease at smaller R is not so quick as for H₂. With the current model parameters, Rg2 and Rg20, this gradual change in the gas density in the inner disk cannot be reproduced, so it is possible that at $R \approx 5 - 6$ kpc the assumed HI surface density will be under- or overestimated, depending on whether the inner hole is added or not. Parameter Rg20 is mainly needed for predicting a correct enclosed mass of the atomic gas needed for the calculation of the MW rotation curve. For now, mainly for the technical reasons, Rg20 must be ≤ 4 kpc.

Rf : Radius where the thin-disk flaring begins (float, Distance[kpc])

Galactocentric distance where the thin disk starts to flare. As it is common to assume a constant disk thickness, disk flaring can be switched off by simply setting this value larger than the outermost radial bin, $Rf > R_{\max}$.

Rdf : Thin-disk flaring scale length (float, Distance[kpc])

Exponential scale length of the thin-disk thickness radial profile. Motivated by [Kalberla et al. \(2014\)](#) where authors report a common flaring of stars and HI gas across the MW disk and propose a simple exponential law to describe the change of the gaseous and stellar disk's thickness. Note that [Kalberla et al. \(2014\)](#) assume a common flaring law for the atomic gas and stars. The radial profile of the HI gas scale height is the JJ model input (data from [Nakanishi and Sofue, 2016](#)). If the flaring is allowed in the model, a preliminary check for consistency between the assumed Rdf and the input HI scale height profile may be needed. If flaring is switched off by parameter Rf, Rdf will not impact the model predictions.

a_in : Power index of the inner halo profile (float, Dimensionless[see description])

Power index of the radial density profile of the inner halo. Does not influence the model predictions for the Solar neighborhood, but impacts the potential-density pair at other radii (though very weakly as the halo contribution is very small). Also, enters the expression for the MW rotation curve. Note that if the model predictions for halo need to be extrapolated to $z > z_{\max}$, (e.g. to get star counts integrated over the line of sight perpendicular to the Galactic plane). Then also power index of the outer halo **a_out** and break radius **r_br** should be assumed, but this we leave to a user.

Mb : Mass of the bulge (float, Mass[M_⊙])

Mass of the MW bulge. This parameter is used only in the calculation of the MW rotation curve, where the bulge is included simply as a point mass. Strictly speaking, Mb is the *difference* between the total mass of the bulge and the enclosed disk mass within $\lesssim 4$ kpc, as we do not cut off the disk profile in the inner Galaxy.

2.4. Local surface densities

sigmad : Thin-disk surface density (float, Surface density[M_⊙ pc⁻²])

Total surface density of the thin disk in the Solar neighborhood (if there are additional peaks at the thin-disk SFR, sigmad is still the total surface density – the surface densities of these peaks' populations are included into it, see parameter sigmap in Sect. 3).

sigmat : Thick-disk surface density (float, Surface density[M_⊙ pc⁻²])

Local surface density of the thick disk.

sigmag1 : Molecular gas surface density (float, Surface density[M_⊙ pc⁻²])

Local surface density of the molecular gas.

sigmag2 : Atomic gas surface density (float, Surface density[$M_{\odot} \text{ pc}^{-2}$])

Local surface density of the atomic gas.

sigmadh : DM halo surface density (float, Surface density[$M_{\odot} \text{ pc}^{-2}$])

Local surface density of the DM halo (only up to z_{max}).

sigmash : Stellar halo surface density (float, Surface density[$M_{\odot} \text{ pc}^{-2}$])

Local surface density of the halo stars.

2.5. Star formation rate

2.5.1. Thin disk

td1 : Galactic time of the beginning of the thin-disk formation (float, Time[Gyr])

The standard thin-disk SFR of the JJ model (smooth function with a single peak ~ 10 Gyr ago) is given by Eq.(10) in [Paper IV](#). By default, $\text{td1} = 0$ Gyr, i.e., there is no time delay between the beginning of the formation of the thick and thin disk (see [Beraldo e Silva et al., 2021](#) for the discussion of the co-evolution of both disks). A sequential formation of the two disks is possible within the JJ model, for this use $\text{td1} > 0$ Gyr.

td2 : SFR parameter (float, Time[Gyr])

Thin-disk SFR parameter at R_{sun} . An increase of td2 results in a slower increase of SFR at old ages, a shift of the peak to younger ages, and a slower SFR decline after the peak (Eq.(10) in [Paper IV](#)). The default value is $\text{td2} = 7.8$ Gyr.

dzeta : SFR power index (float, Dimensionless[see description])

Thin-disk SFR parameter at R_{sun} . An increase of parameter dzeta has the same effect as an increase of td2 : SFR grows slower at old ages, peaks later (peak shifts to younger ages), and declines slower after the peak (Eq.(10) in [Paper IV](#)).

eta : SFR power index (float, Dimensionless[see description])

Thin-disk SFR parameter at R_{sun} . An increase of parameter eta has the opposite effect with respect to an increase of td2 or dzeta : SFR grows faster at old ages, peaks earlier (peaks shifts to older ages), and declines faster after the peak (Eq.(10) in [Paper IV](#)).

pkey : Mode of thin-disk SFR and kinematics (int, Dimensionless[see description])

This parameter can take three values: 0, 1, or 2. Option $\text{pkey} = 0$ corresponds to the thin-disk SFR given by Eq.(10) in [Paper IV](#). If $\text{pkey} = 1$, the thin-disk SFR is allowed to have an *arbitrary* number of Gaussian peaks on top of the underlying continuum, which is described by a generalized Eq. (12) from [Paper IV](#), where two additional peaks were introduced. Also, in this case the stellar populations associated with the star formation (SF) excess in the peaks have special vertical kinematics. Namely, their W -velocity dispersion σ_W is different from σ_W prescribed for the thin-disk populations of the same age by the age-velocity dispersion relation (AVR). When $\text{pkey} = 2$, extra peaks can be added to the SFR, but all thin-disk stars follow AVR. Parameters describing the peak shapes ($\text{pkey} = 1, 2$) and kinematics ($\text{pkey} = 1$) must be specified in the second parameter file `sfrd_peaks_parameters` (Sect. 3).

k_td2 : Power index of the radial law of the thin-disk SFR parameter td2 (float, Dimensionless[see description])

To introduce a variation of the thin-disk SFR with Galactocentric distance R , parameters td2 , dzeta , and eta are assumed to have power-law radial profiles, i.e., $\sim \text{td2} \cdot (R/R_{\text{sun}})^{k_{\text{td2}}}$. An example of a combination of these SFR parameters' radial slopes which allows to mimic the inside-out disk growth is $(k_{\text{td2}}, k_{\text{dzeta}}, k_{\text{eta}}) = (1.5 \text{ Gyr}, 0.18, -0.1)$.

k_dzeta : Power index of the radial law of the thin-disk SFR parameter dzeta (float, Dimensionless[see description])

Same as parameter `k_td2`, but for `dzeta`.

k_eta : Power index of the radial law of the thin-disk SFR parameter `eta` (float, Dimensionless[see description])

Same as parameter `k_td2`, but for `eta`.

2.5.2. Thick disk

tt1 : Thick-disk SFR parameter (float, Time[Gyr])

Thick-disk SFR is given by Eqs. (7) and (8) in [Paper IV](#), and is a combined power-exponential function. Parameter `tt1` is related to a starting value of SF at $t = 0$ Gyr. The default value is `tt1 = 0.1` Gyr. The thick-disk SFR shape is assumed to be the same at all Galactocentric distances, so parameters `tt1`, `tt2`, `gamma`, `beta` are the same at all radii.

tt2 : Galactic time when the thick-disk formation stops (float, Time[Gyr])

Galactic time, when the formation of the thick disk stops (Eqs. (7) and (8) in [Paper IV](#)). The MW thick disk has formed quickly, so `tt2` cannot be very large, our default value is 4 Gyr (but in fact, the SF drops to almost zero even earlier due to the exponential decline).

gamma : Thick-disk SFR exponential index (float, Dimensionless[see description])

Controls steepness of the thick-disk SFR increase at the oldest ages (Eqs. (7) and (8) in [Paper IV](#)). The default value is `gamma = 2`.

beta : Thick-disk SFR exponential index (float, Inverse time[Gyr⁻¹])

Controls the declining part of the thick-disk SFR (Eqs. (7) and (8) in [Paper IV](#)). The default value is `beta = 3.5` Gyr⁻¹.

2.6. Initial mass function

imfkey : IMF input mode (int, Dimensionless[see description])

When `imfkey = 0`, the initial mass function (IMF) is a four-slope broken power law, and its shape is described by parameters (`a0`, `a1`, `a2`, `a3`, `m0`, `m1`, `m2`). Another option is `imfkey = 1`, which corresponds to a custom IMF shape, which has to be specified by introducing custom IMF parameters (in this parameter file, or directly in a script). Class `jjmodel.funcs.IMF` has pre-defined Kroupa93 and Chabrier03 IMF.

a0 : IMF slope (float, Dimensionless[see description])

IMF slope for masses $m < m0$.

a1 : IMF slope (float, Dimensionless[see description])

IMF slope for masses $m0 \leq m < m1$.

a2 : IMF slope (float, Dimensionless[see description])

IMF slope for masses $m1 \leq m < m2$.

a3 : IMF slope (float, Dimensionless[see description])

IMF slope for masses $m \geq m2$.

m0 : IMF break point mass (float, Mass[M_⊙])

Break point between the IMF slopes `a0` and `a1`.

m1 : IMF break point mass (float, Mass[M_{\odot}])

Break point between the IMF slopes a1 and a2.

m2 : IMF break point mass (float, Mass[M_{\odot}])

Break point between the IMF slopes a2 and a3.

2.7. Age-metallicity relation and metallicity distributions

dFeHdt : Dispersion of the disk MAP's MDs (float, Metallicity[dex])

If dFeHdt = 0, there is a unique correspondence between age and metallicity in the model, which is prescribed by the thin-disk and thick-disk AMR functions. However, we know from observations that the AMR has a significant scatter in metallicity, i.e., a range of different metallicities can correspond to the same age (radial migration effect). If dFeHdt > 0, this scatter is modeled by assuming for each mono-age subpopulation (MAP) a Gaussian metallicity distribution (MD). Mean value for such MD at a fixed age is given by metallicity from the AMR, and dFeHdt is MD dispersion. This metallicity spread is applied to both the thin and thick disk.

n_FeHdt : Number of subpopulations for sampling the disk MAP's MDs (int, Dimensionless[see description])

The number of subpopulations used to sample disk MAP MDs (see parameter dFeHdt). n_FeHdt = 1 corresponds to no scatter in AMR. Obviously, if the scatter is added (dFeHdt > 0), then n_FeHdt must be > 1 to sample the assumed MAP MDs. n_FeHdt has to be an uneven number (to sample the MD mean value). Do not use too large values ($\gtrsim 9$ or so), as the overall SA number increases approximately proportionally to n_FeHdt, i.e., the calculation of different quantities will take $\sim n_FeHdt$ times longer. E.g., use n_FeHdt = 7 to sample the mean MD value $\langle [Fe/H] \rangle = AMR(t)$ and values $\pm 1\sigma$, $\pm 2\sigma$, $\pm 3\sigma$ from it, where $\sigma = dFeHdt$.

2.7.1. Thin disk

fehkey : Thin-disk AMR input mode (integer, Dimensionless[see description])

When fehkey = 0, the thin-disk AMR is described by Eq. (21)-(22) in [Paper IV](#) and parameters FeHd0, FeHdp, q, and rd must be given. If run_mode = 1, the local AMR is extended to other radii with the help of parameters k_FeHd0, k_FeHdp, k_q, and k_rd. When fehkey = 1, the AMR equation is Eq. (22)-(23) from Paper V. In this case, extension to outer radii is done by extending AMR parameters with linear or broken linear laws (Table 2 in Paper V). In overall, the following thin-disk AMR parameters must be specified then: FeHd0, rd1, rd2, Rbr1, Rbr2, Rbr3, k1_FeHdp, b1_FeHdp, k_alpha, b_alpha, Rbr2, Rbr3, k1_FeHdp, b1_FeHdp, k_alpha, b_alpha, k1_t01, b1_t01, k2_t01, b2_t01, k3_t01, b3_t01, k1_t02, b1_t02, k2_t02, b2_t02, k3_t02, b3_t02. Alternatively, user can also set fehkey = 2 to use the radial extension of AMR from Paper V with the best parameters. Then only FeHd0 has to be given in parameter file.

FeHd0 : Initial thin-disk metallicity (float, Metallicity[dex])

FeHd0 is metallicity of the oldest thin-disk subpopulation at the Solar radius R_{sun} . In the AMR, metallicity is assumed to increase monotonously with Galactic time at all radii. Many observations imply that the local value FeHd0 belongs to the range of $(-0.8, -0.6)$ dex.

FeHdp : Present-day thin-disk metallicity (float, Metallicity[dex])

Metallicity of the youngest thin-disk subpopulation at the Solar radius R_{sun} . The value of FeHdp belongs to the range of $(0.1, 0.3)$ dex. Must be given only if fehkey = 0.

rd : Thin-disk AMR power index (float, Dimensionless[see description])

Parameter controlling the thin-disk AMR shape. Increase of rd results in a slower increase of metallicity with Galactic time. Must be given only if fehkey = 0.

q : Thin-disk AMR parameter (float, Dimensionless[see description])

Parameter controlling the thin-disk AMR shape. Increase of this parameter has an opposite effect with respect to an increase of `rd`: AMR then has a faster chemical enrichment at old ages. Must be given only if `fehkey = 0`.

k_FeHd0 : Power index of the radial law of FeHd0 (float, Dimensionless[see description])

By analogy to the thin-disk SFR, to introduce a variation of the thin-disk AMR with Galactocentric distance R , parameters FeHd0, FeHdp, `rd`, and `q` are assumed to vary with R according to power laws, i.e., $\sim \text{FeHd0} \cdot (R/R_{\text{sun}})^{k_{\text{FeHd0}}}$. An example of such combination of the AMR parameters' radial slopes which corresponds to a faster enrichment in the inner disk is $(\text{FeHd0}, \text{FeHdp}, \text{rd}, q) = (-0.025, -0.0875, 0.005, -0.03)$. Must be given only if `fehkey = 0` and `run_mode = 1`.

k_FeHdp : Power index of the radial law of FeHdp (float, Dimensionless[see description])

Same as parameter `k_FeHd0`, but for FeHdp. Must be given only if `fehkey = 0` and `run_mode = 1`.

k_rd : Power index of the radial law of `rd` (float, Dimensionless[see description])

Same as parameter `k_FeHd0`, but for `rd`. Must be given only if `fehkey = 0` and `run_mode = 1`.

k_q : Power index of the radial law of `q` (float, Dimensionless[see description])

Same as parameter `k_FeHd0`, but for `q`. Must be given only if `fehkey = 0` and `run_mode = 1`.

rd1 : First thin-disk AMR power index (float, Dimensionless[see description])

Power index of the first tanh-term of the thin-disk AMR in Eq. (22)-(23) from Paper V. Must be given only if `fehkey = 1`.

rd2 : Second thin-disk AMR power index (float, Dimensionless[see description])

Power index of the second tanh-term of the thin-disk AMR in Eq. (22)-(23) from Paper V. Must be given only if `fehkey = 1`.

Rbr1 : First break point for AMR parameters (float, Distance[kpc])

Position of the first break point for the AMR parameters' radial profiles. Must be given only if `fehkey = 1`.

Rbr2 : Second break point for AMR parameters (float, Distance[kpc])

Position of the second break point for the AMR parameters' radial profiles. Must be given only if `fehkey = 1`.

Rbr3 : Third break point for AMR parameters (float, Distance[kpc])

Position of the third break point for the AMR parameters' radial profiles. Must be given only if `fehkey = 1`.

k1_FeHdp : Power index of the radial law of FeHdp (float, Metallicity gradient[dex kpc⁻¹])

Power index which together with `b1_FeHdp` defines radial profile of FeHdp for $R > R_{\text{br1}}$ (at smaller R , FeHdp is constant). Must be given only if `fehkey = 1`.

b1_FeHdp : Intercept of the radial law of FeHdp (float, Metallicity[dex])

Intercept which together with `k1_FeHdp` defines radial profile of FeHdp for $R > R_{\text{br1}}$ (at smaller R , FeHdp is constant). Must be given only if `fehkey = 1`.

k_alphaw : Power index of the radial law of `alphaw` (float, Gradient of dimensionless quantity[kpc⁻¹])

Power index which together with `b_alphaw` defines radial profile of `alphaw`. Must be given only if `fehkey = 1`.

b_alphaw : Intercept of the radial law of `alphaw` (float, Dimensionless[see description])

Intercept which together with k_{α} defines radial profile of α . Must be given only if $fehkey = 1$.

k1_t01 : Power index of the radial law of t_{01} (float, Age gradient[Gyr kpc⁻¹])

Power index which together with $b1_t01$ defines radial profile of t_{01} for $R \leq Rbr2$. Must be given only if $fehkey = 1$.

b1_t01 : Intercept of the radial law of t_{01} (float, Age[Gyr])

Intercept which together with $k1_t01$ defines radial profile of t_{01} for $R \leq Rbr2$. Must be given only if $fehkey = 1$.

k2_t01 : Power index of the radial law of t_{01} (float, Age gradient[Gyr kpc⁻¹])

Same as $texttt{k1_t01}$, but for $Rbr2 < R \leq Rbr3$.

b2_t01 : Intercept of the radial law of t_{01} (float, Age[Gyr])

Same as $texttt{b1_t01}$, but for $Rbr2 < R \leq Rbr3$.

k3_t01 : Power index of the radial law of t_{01} (float, Age gradient[Gyr kpc⁻¹])

Same as $texttt{k1_t01}$, but for $R > Rbr3$.

b3_t01 : Intercept of the radial law of t_{01} (float, Age[Gyr])

Same as $texttt{b1_t01}$, but for $R > Rbr3$.

k1_t02 : Power index of the radial law of t_{02} (float, Age gradient[Gyr kpc⁻¹])

Power index which together with $b1_t02$ defines radial profile of t_{02} for $R \leq Rbr1$. Must be given only if $fehkey = 1$.

b1_t02 : Intercept of the radial law of t_{02} (float, Age[Gyr])

Intercept which together with $k1_t02$ defines radial profile of t_{02} for $R \leq Rbr1$. Must be given only if $fehkey = 1$.

k2_t02 : Power index of the radial law of t_{02} (float, Age gradient[Gyr kpc⁻¹])

Same as $texttt{k1_t02}$, but for $Rbr1 < R \leq Rbr3$.

b2_t02 : Intercept of the radial law of t_{02} (float, Age[Gyr])

Same as $texttt{b1_t02}$, but for $Rbr1 < R \leq Rbr3$.

k3_t02 : Power index of the radial law of t_{02} (float, Age gradient[Gyr kpc⁻¹])

Same as $texttt{k1_t02}$, but for $R > Rbr3$.

b3_t02 : Intercept of the radial law of t_{02} (float, Age[Gyr])

Same as $texttt{b1_t02}$, but for $R > Rbr3$.

2.7.2. Thick disk

FeHt0 : Initial thick-disk metallicity (float, Metallicity[dex])

$FeHt0$ is metallicity of the oldest thick-disk subpopulation, this is one of four parameters of the thick-disk AMR which is given by Eq. (21)-(22) in [Paper IV](#). Belongs to the range of $(-1, -0.8)$ (same at all R , no variation of the thick-disk AMR with radius is assumed).

FeHtp : Present-day thick-disk metallicity (float, Metallicity[dex])

Metallicity of the youngest thick-disk subpopulation, same at all radii.

rt : Thick-disk AMR power index (float, Dimensionless[see description])

Thick-disk AMR parameter controlling the AMR shape, same at all radii. An increase of **rt** results in a slower increase of metallicity with Galactic time.

t0 : Thick-disk AMR scaling parameter (float, Time[Gyr])

Thick-disk AMR parameter controlling the AMR shape, same at all radii. An increase of this parameter has an opposite effect with respect to an increase of **rt**: AMR then prescribes a faster chemical enrichment at old ages.

2.7.3. Halo

FeHsh : Mean metallicity of the stellar halo MD (float, Metallicity[dex])

Stellar halo is represented by a single-age subpopulation with a Gaussian MD. **FeHsh** is its mean value.

dFeHsh : Metallicity dispersion of the stellar halo MD (float, Metallicity[dex])

Metallicity dispersion of the stellar halo MD.

n_FeHsh : Number of the stellar halo MAPs (int, Dimensionless[see description])

The number of subpopulations used to sample the halo MD (analog of **n_FeHdt** parameter for the disk). An uneven number (to sample the MD mean value). E.g., use **n_FeHsh** = 7 to sample $\langle [\text{Fe}/\text{H}]_{\text{sh}} \rangle = \text{FeHsh}$ and $\pm 1\sigma$, $\pm 2\sigma$, $\pm 3\sigma$ values from it, where $\sigma = \text{dFeHsh}$.

2.8. W-velocity dispersions

alpha : AVR power index (float, Dimensionless[see description])

The thin-disk AVR power index (AVR analytic form is given by Eq.(13) in [Paper IV](#)). Assumed to be constant at all radii.

sig : AVR scaling parameter (float, Velocity[km s⁻¹])

The thin-disk AVR scaling parameter at R_{sun} , corresponds to the W-velocity dispersion of the oldest thin-disk subpopulation. A variation of **sig** with radius (and thus, a variation of the AVR shape) is determined within the code using the prescribed disk thickness (see parameters **Rf** and **Rdf**) as a “boundary condition”.

sigt : Thick-disk W-velocity dispersion (float, Velocity[km s⁻¹])

The W-velocity dispersion of the thick disk. No variation with age or radius is assumed.

sigdh : DM halo W-velocity dispersion (float, Velocity[km s⁻¹])

W-velocity dispersion of the DM halo (same at all radii).

sigsh : Stellar halo W-velocity dispersion (float, Velocity[km s⁻¹])

W-velocity dispersion of the stellar halo (same at all radii).

3. File `sfrd_peaks_parameters`

Each additional thin-disk SFR peak is described by the following five (`pkey = 2`) or six (`pkey = 1`) parameters.

sigmap : Amplitude-related parameter of the *i*-th peak (float, Surface density [$M_{\odot} \text{ pc}^{-2}$])

Parameter related to the amplitude of the thin-disk SFR peak at R_{sun} . For the exact expression linking this parameter to the real amplitude of the peak see Sect. 3.1 in [Paper IV](#). Note that the surface density of the peak does not add to the thin-disk surface density `sigmad`, but is included into it.

taup : Mean age of the *i*-th peak (float, Age[Gyr])

The mean age of the thin-disk SFR peak, same at all radii. Formally, can be also negative which corresponds to the future.

dtau : Age dispersion of the *i*-th peak (float, Age[Gyr])

The age dispersion of the thin-disk SFR peak, same at all radii.

Rp : Mean radius of the *i*-th peak (float, Distance[kpc])

The mean Galactocentric distance of the thin-disk SFR peak (Gaussian distribution across *R* is assumed).

dRp : Radial dispersion of the *i*-th peak (float, Distance[kpc])

The radial dispersion of the thin-disk SFR peak, defines which interval of Galactocentric distances is affected by the presence of the special populations associated with this extra SF event.

sigp : W-velocity dispersion of the *i*-th peak's populations (float, Velocity[km s^{-1}])

The W-velocity dispersion of the thin-disk stellar populations associated with the SFR peak (same for all ages of these special populations). Must be specified only when `pkey = 1`.

4. Summary table of the JJ model parameters

Table 1: Summary of the JJ model parameters. Colors indicate whether a parameter is related to the Solar neighborhood (red) or to the extension of the local model to other radii (green). Parameters that must be specified regardless of `run_mode` (local or global model) are also highlighted with red. Parameters are classified as follows: (a) constrained independently from the JJ model, (b) uncalibrated, (c) calibrated against Gaia DR2 in Paper IV, (d) calibrated against APOGEE RC in Paper IV, (e) calibrated against APOGEE RC in Paper V, (f) boolean (switch on and off different JJ model features), and (g) technical (do not influence the JJ model predictions or influence their quality). In case of the technical and boolean parameters, as well as parameters which values are not yet constrained, an example of possible values is given in the column “Value”. For the rest of parameters the best values are given (determined in the framework of the JJ model or independently from it).

References: [1] Gravity Collaboration et al. (2019), [2] Bennett and Bovy (2019), [3] Schönrich et al. (2010), [4] Bland-Hawthorn and Gerhard (2016), [5] Kramer and Randall (2016), [6] Kalberla and Dedes (2008), [7] Kalberla et al. (2014), [8] Nakanishi and Sofue (2016), [9] Paper IV, [10] Paper I, [11] McKee et al. (2015) [12] Paper II, [13] Paper III, [14] Rybizki (2018), [15] Conroy et al. (2019), [16] Paper V.

Parameter	Units	Value	Name in ref.	Comment
Run settings				
run_mode	-	1 ^(g)	-	Can be 0 or 1
nprocess	-	4 ^(g)	-	≥ 1 , up to the number of available CPUs
out_dir	-	‘model1’ ^(g)	-	Example of a name
out_mode	-	1 ^(g)	-	Can be 0 or 1
Solar coordinates and velocity				
Rsun	kpc	$8.178 \pm 0.013_{\text{stat}} \pm 0.022_{\text{sys}}$ ^(a)	R_{\odot}	Value from [1], we often use 8.2 kpc
zsun	pc	20.8 ± 0.3 ^(a)	z_{\odot}	Value from [2], we often use 20 pc
Vsun	km s ⁻¹	12.24 ± 0.47 ^(a)	V_{\odot}	Value from [3]
Rz-grid and radial profiles of the MW components				
Rmin	kpc	4.0 ^(g)	R_{min}	Must be ≥ 4 kpc
Rmax	kpc	14.0 ^(g)	R_{max}	Should be $\lesssim 14\text{--}15$ kpc
dR	kpc	0.5 ^(g)	dR	Recommended ≤ 1 kpc
zmax	pc	2000 ^(g)	z_{max}	Must be ≤ 2000 pc
dz	pc	2 ^(g)	dz	Recommended range is (0.5,10) pc
Rd	kpc	2.5 ^(a)	R_{d}	See literature overview in [4]
Rt	kpc	2.0 ^(a)	R_{t}	See literature overview in [4]
Rg1	kpc	3.0 ^(a)	R_{H_2}	See literature overview in [5]
Rg2	kpc	10.0 ^(a)	R_{HI}	See [6] and literature overview in [5]
Rg10	kpc	4.0 ^(a)	$R_{\text{in,H}_2}$	See literature overview in [4]
Rg20	kpc	4.0 ^(a)	$R_{\text{in,HI}}$	See literature overview in [4]
Rf	kpc	7 ^(a)	-	Set $< R_{\text{max}}$ to include disk flaring
Rdf	kpc	9.22 ^(a)	-	See [7], but compare to the model input from [8]
a_in	dim	-2.5 ^(a)	α_{in}	Taken from [4]
Mb	M_{\odot}	$0.8 \cdot 10^{10}$	M_{b}	
Local surface densities				
sigmad	$M_{\odot} \text{ pc}^{-2}$	$29.4^{+2.7}_{-2.7}$ ^(c)	Σ_{d} or $\Sigma_{\text{d}\odot}$	Updated in [9], consistent with [10]
sigmat	$M_{\odot} \text{ pc}^{-2}$	$4.9^{+1.4}_{-1.3}$ ^(c)	Σ_{t} or $\Sigma_{\text{t}\odot}$	Updated in [9], consistent with [10]
sigmag1	$M_{\odot} \text{ pc}^{-2}$	1.7 ^(a)	Σ_{H_2} or $\Sigma_{\text{H}_2\odot}$	Adopted from [11]
sigmag2	$M_{\odot} \text{ pc}^{-2}$	10.86 ^(a)	Σ_{HI} or $\Sigma_{\text{HI}\odot}$	Adopted from [11]
sigmadh	$M_{\odot} \text{ pc}^{-2}$	$51.6^{+9.3}_{-9.3}$ ^(c)	Σ_{dh} or $\Sigma_{\text{dh}\odot}$	Updated in [9], broadly consistent with [10]
sigmash	$M_{\odot} \text{ pc}^{-2}$	$0.49^{+0.14}_{-1.14}$ ^(c)	Σ_{sh} or $\Sigma_{\text{sh}\odot}$	Updated in [9]

Star formation rate				
Thin disk				
td1	Gyr	0.0 ^(b)	t_{d1} or $t_{d1\odot}$	Eq. (10) in [9], Eq. (9) in [16]
td2	Gyr	7.8 ^(b)	t_{d2} or $t_{d2\odot}$	Eq. (10) in [9], Eq. (9) in [16], consistent with [10,12]
dzeta	dim	$0.83^{+0.09}_{-0.09}$ ^(c)	ζ	Eq. (10) in [9], Eq. (9) in [16]
eta	dim	$5.6^{+0.1}_{-0.1}$ ^(c)	η	Eq. (10) in [9], Eq. (9) in [16]
k_td2	dim	1.5 ^(b)	$k_{t_{d2}}$	Table 1 in [16]
k_dzeta	dim	0.18 ^(b)	k_{ζ}	Table 1 in [16]
k_eta	dim	-0.1 ^(b)	k_{η}	Table 1 in [16]
pkey	-	1 ^(f)	-	Can be 0, 1, or 2; for R_{\odot} 1 is preferred [9]
sigmap	$M_{\odot} \text{ pc}^{-2}$	$[3.5^{+2.4}_{-1.8}, 1.3^{+1.3}_{-0.8}]$ ^(c)	$\Sigma_{p,k}$ or $\Sigma_{p\odot,k}$	Eq. (12) in [9], Eqs. (10) and (13) in [16]
taup	Gyr	$[3.0^{+0.8}_{-0.9}, 0.5^{+0.5}_{-0.5}]$ ^(c)	$\tau_{p,k}$	Eq.(12) in [9], Eq. (10) in [16]
dtaup	Gyr	$[0.7^{+0.15}_{-0.14}, 0.25^{+0.05}_{-0.05}]$ ^(c)	$d\tau_{p,k}$	Eq.(12) in [9], Eq. (10) in [16]
Rp	kpc	$[7.5, 10.3]$ ^(b)	$R_{p,k}$	Eq. (13) in [16]
dRp	kpc	$[1.0, 0.5]$ ^(b)	$dR_{p,k}$	Eq. (13) in [16]
Thick disk				
tt1	Gyr	0.1 ^(b)	t_{t1}	Eq.(8) in [9]
tt2	Gyr	4.0 ^(b)	t_{t2}	Eq.(8) in [9]
gamma	dim	3.5 ^(b)	γ	Eq.(8) in [9]
beta	Gyr^{-1}	2.0 ^(b)	β	Eq.(8) in [9]
Initial mass function				
imfkey	-	0 ^(g)	-	Can be 0 or 1
a0	dim	$1.31^{+0.28}_{-0.28}$ ^(c)	α_0	Updated in [9], consistent with [13]
a1	dim	$1.5^{+0.23}_{-0.24}$ ^(c)	α_1	Updated in [9], consistent with [13]
a2	dim	$2.88^{+0.26}_{-0.23}$ ^(c)	α_2	Updated in [9], broadly consistent with [13]
a3	dim	2.28 ^(b)	α_3	Adopted from [14]
m0	M_{\odot}	$0.49^{+0.15}_{-0.15}$ ^(c)	m_0	Poorly constrained, see [9]
m1	M_{\odot}	$1.43^{+0.14}_{-0.14}$ ^(c)	m_1	Updated in [9], consistent with [13]
m2	M_{\odot}	6 ^(b)	m_2	Adopted from [14]
Age-metallicity relation and metallicity distributions				
dFeHdt	dex	0.1 ^(a)	$d[\text{Fe}/\text{H}]_{dt}$	Value from 0 to $\sim 0.2 - 0.25$ dex
n_FeHdt	-	7 ^(g)	-	Uneven value > 1 , values $\gtrsim 11$ not recommended
Thin disk				
fehkey	-	1 ^(g)	-	Can be 0, 1, or 2; preferred mode is 1 [16]
FeHd0	dex	-0.81 ^(e)	$[\text{Fe}/\text{H}]_0$	Eq. (21)-(22) in [9], Eq. (22)-(23) in [16]
FeHdp	dex	0.29 ^(d)	$[\text{Fe}/\text{H}]_p$	Eq. (21)-(22) in [9]
rd	dim	0.34 ^(d)	r_d	Eq. (21)-(22) in [9]
q	dim	-0.72 ^(d)	q	Eq. (21)-(22) in [9]
k_FeHd0	dim	-0.025 ^(b)	$k_{[\text{Fe}/\text{H}]_0}$	Sect. 2.10.1 in [16]
k_FeHdp	dim	-0.0875 ^(b)	$k_{[\text{Fe}/\text{H}]_p}$	Sect. 2.10.1 in [16]
k_rd	dim	0.005 ^(b)	k_{r_d}	Sect. 2.10.1 in [16]
k_q	dim	-0.03 ^(b)	k_q	Sect. 2.10.1 in [16]
rd1	dim	0.5 ^(e)	r_{d1}	Eq. (22)-(23) in [16]
rd2	dim	1.5 ^(e)	r_{d2}	Eq. (22)-(23) in [16]
Rbr1	kpc	6 ^(e)	R_{br1}	Sect. 4.2.4 and Table 2 in [16]
Rbr2	kpc	7.5 ^(e)	R_{br2}	Sect. 4.2.4 and Table 2 in [16]
Rbr3	kpc	9.75 ^(e)	R_{br3}	Sect. 4.2.4 and Table 2 in [16]
k1_FeHdp	dex kpc^{-1}	-0.59 ^(e)	-	Table 2 in [16]

b1_FeHdp	dex	$0.85^{(e)}$	-	Table 2 in [16]
k_alpha	kpc^{-1}	$-0.43^{(e)}$	-	Table 2 in [16]
b_alpha	dim	$0.99^{(e)}$	-	Table 2 in [16]
k1_t01	Gyr kpc^{-1}	$2.04^{(e)}$	-	Table 2 in [16]
b1_t01	Gyr	$-0.32^{(e)}$	-	Table 2 in [16]
k2_t01	Gyr kpc^{-1}	$-2.71^{(e)}$	-	Table 2 in [16]
b2_t01	Gyr	$4.99^{(e)}$	-	Table 2 in [16]
k3_t01	Gyr kpc^{-1}	$-3.91^{(e)}$	-	Table 2 in [16]
b3_t01	Gyr	$7.14^{(e)}$	-	Table 2 in [16]
k1_t02	Gyr kpc^{-1}	$15.89^{(e)}$	-	Table 2 in [16]
b1_t02	Gyr	$-5.29^{(e)}$	-	Table 2 in [16]
k2_t02	Gyr kpc^{-1}	$7.77^{(e)}$	-	Table 2 in [16]
b2_t02	Gyr	$-3.04^{(e)}$	-	Table 2 in [16]
k3_t02	Gyr kpc^{-1}	$-2.97^{(e)}$	-	Table 2 in [16]
b3_t02	Gyr	$9.96^{(e)}$	-	Table 2 in [16]

Thick disk				
FeHt0	dex	$-0.91^{(e)}$	$[\text{Fe}/\text{H}]_0$	Eq. (21)-(22) in [9], Eq. (22) and (24) in [16]
FeHtp	dex	$0.04^{(e)}$	$[\text{Fe}/\text{H}]_p$	Eq. (21)-(22) in [9], Eq. (22) and (24) in [16]
rt	dim	$1.28^{(e)}$	r_t	Eq. (21)-(22) in [9], Eq. (22) and (24) in [16]
t0	Gyr	$0.9^{(e)}$	t_0	Eq. (21)-(22) in [9], Eq. (22) and (24) in [16]

Halo				
FeHsh	dex	$-1.5^{(a)}$	$[\text{Fe}/\text{H}]_{\text{sh}}$	[4], (-1.2 in [15])
dFeHsh	dex	$0.4^{(a)}$	$\sigma_{[\text{Fe}/\text{H}]_{\text{sh}}}$	See [15]
n_FeHsh	-	$7^{(g)}$	-	Uneven value > 1 , no need for $\gtrsim 15$

W-velocity dispersions				
alpha	dim	$0.409^{+0.046}_{-0.045}{}^{(c)}$	α	Eq. (13) in [9]
sige	km s^{-1}	$25.1^{+1.8}_{-1.7}{}^{(c)}$	σ_e	Eq. (13) in [9]
sigp	km s^{-1}	$[26.3^{+4.4}_{-4.0}, 12.6^{+3.0}_{-2.9}{}^{(c)}]$	$\sigma_{p,k}$ or $\sigma_{p\odot,k}$	Eq. (14) in [16]
sigt	km s^{-1}	$43.3^{+3.7}_{-3.8}{}^{(c)}$	σ_t	Determined in [9], consistent with [12]
sigdh	km s^{-1}	$140^{(b)}$	σ_{dh}	Assumed in [10]
sigsh	km s^{-1}	$100^{(b)}$	σ_{sh}	Assumed in [10]

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