Web-based Supporting Materials for "Bayesian quantile regression joint models: inference and dynamic predictions" by Ming Yang, Sheng Luo and Stacia DeSantis

1 Distribution comparison

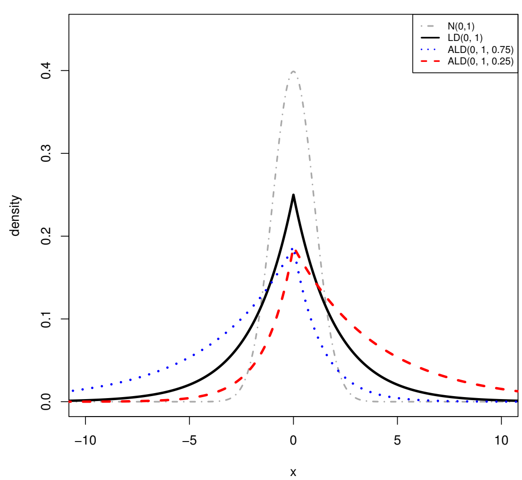


Figure 1: A comparison of normal, Laplace and asymmetric Laplace distributions.

- Laplace distribution (LD) with location at 0 and scale parameter equals 1 is symmetrical about 0. It has heavier tail compared with standard normal distribution.
- Asymmetric Laplace distribution (ALD) is either positively or negatively skewed and the direction and degree of skewness are control by the skewness parameter $\tau \in [0, 1]$.

2 Additional simulation results

Table 1: Simulation result in Simulation I Scenario 2 in which random errors are generated from ALD with $\tau = 0.5$.

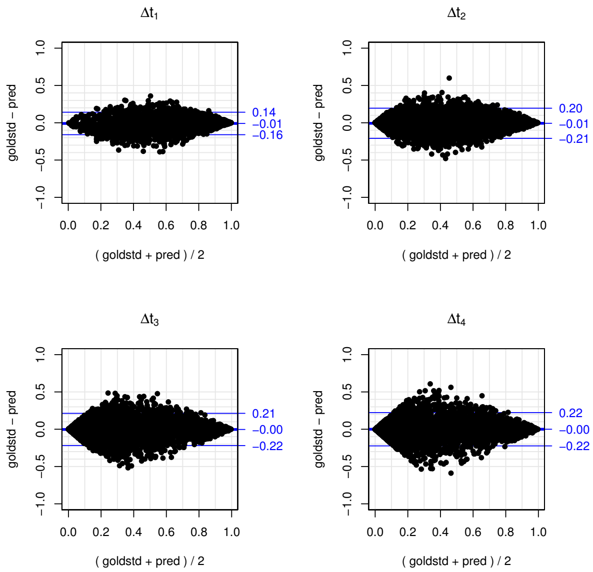
	QRJM ($\tau = 0.5$)				LMJM			
	Bias	SE	MSE	CP	Bias	SE	MSE	CP
Coefficients for longitudinal process								
β_0	-0.006	0.069	0.009	0.960	0.013	0.093	0.017	0.960
β_1	0.008	0.060	0.008	0.900	0.018	0.079	0.018	0.880
β_2	0.014	0.075	0.011	0.950	0.031	0.093	0.021	0.940
Coefficients for survival process								
γ_1	0.008	0.055	0.006	0.940	0.014	0.058	0.007	0.950
γ_2	0.007	0.055	0.007	0.950	0.013	0.057	0.007	0.950
α	-0.001	0.071	0.011	0.930	-0.028	0.101	0.086	0.920

Table 2: Simulation result in Simulation I Scenario 3 in which random errors are generated from $N(0, 1)$.

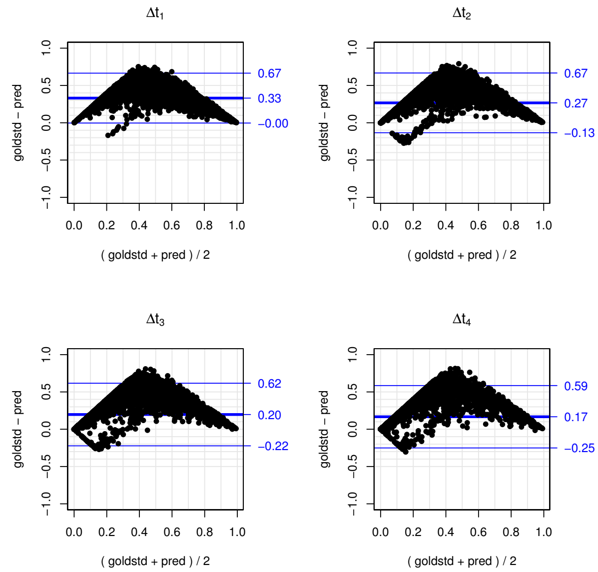
	QRJM ($\tau = 0.5$)				LMJM			
	Bias	SE	MSE	CP	Bias	SE	MSE	CP
Coefficients for longitudinal process								
β_0	0.015	0.037	0.003	0.950	0.000	0.035	0.002	0.980
β_1	0.004	0.034	0.002	0.960	-0.003	0.033	0.002	0.950
β_2	0.013	0.050	0.005	0.950	0.006	0.049	0.005	0.950
Coefficients for survival process								
γ_1	0.008	0.055	0.006	0.920	0.003	0.054	0.006	0.900
γ_2	0.015	0.055	0.007	0.920	0.010	0.054	0.006	0.920
α	-0.013	0.055	0.006	0.950	0.007	0.055	0.006	0.950

Table 3: Prediction results in Simulation II Scenario 3 in which random errors are generated from $N(0,1)$.

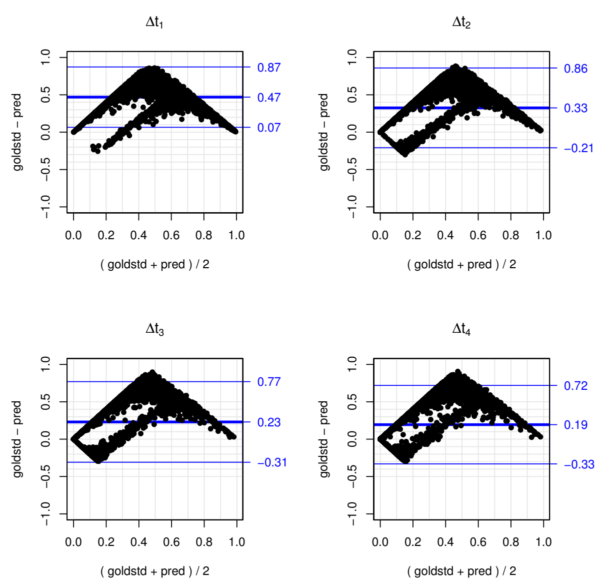
t	Δt	true $\text{AUC}_t^{\Delta t}$	QRJM ($\tau = 0.5$) $S(t+\Delta t\|t)_{MSE}$	QRJM ($\tau = 0.5$) predicted $\text{AUC}_t^{\Delta t}$	LMJM $S(t+\Delta t\|t)_{MSE}$	LMJM predicted $\text{AUC}_t^{\Delta t}$
0.25	0.25	0.812	0.005	0.800	0.005	0.799
	1	0.867	0.009	0.868	0.009	0.869
(subjects left: 47.87%)	2	0.918	0.011	0.898	0.010	0.900
	3	0.936	0.011	0.916	0.011	0.917
0.5	0.25	0.817	0.005	0.807	0.003	0.804
	1	0.868	0.012	0.851	0.008	0.849
(subjects left: 34.78%)	2	0.907	0.014	0.893	0.011	0.890
	3	0.929	0.015	0.908	0.012	0.907
0.75	0.25	0.807	0.008	0.801	0.002	0.787
	1	0.849	0.019	0.847	0.007	0.827
(subjects left: 27.71%)	2	0.892	0.021	0.890	0.011	0.876
	3	0.921	0.022	0.906	0.012	0.898



(2.1) QRJM with $\tau = 0.25$ (True model)

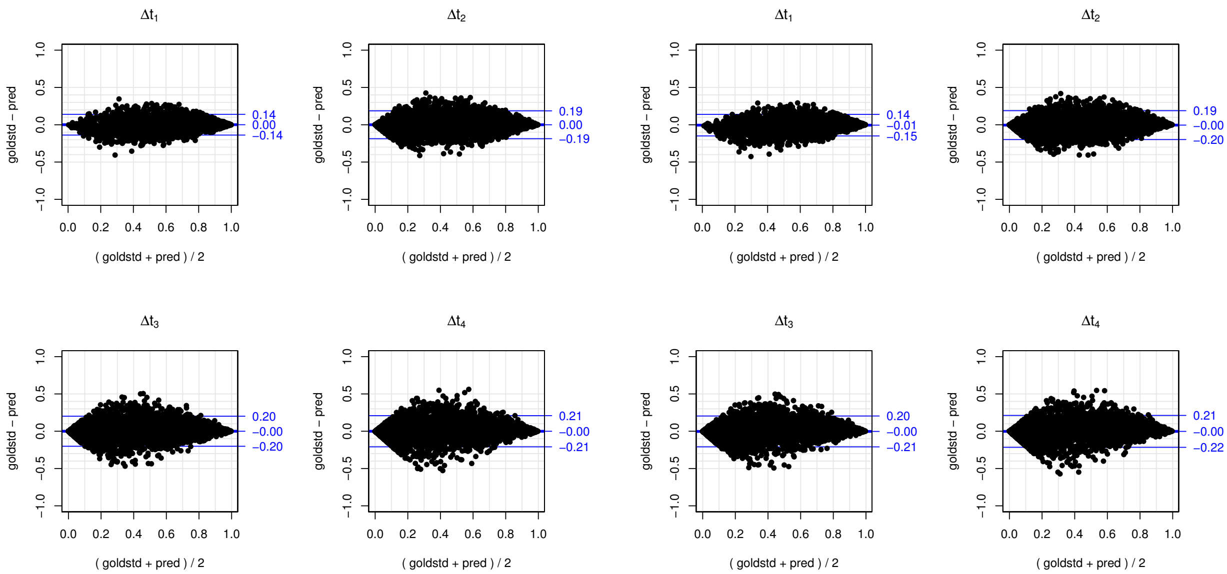


(2.2) QRJM with $\tau = 0.5$



(2.3) LMJM

Figure 2: Prediction results in Simulation II Scenario 1: Bland-Altman plot (bias and 95% limits of agreement) of gold standard versus model predictions at $t = 0.25$ for four prediction time intervals $(\Delta t_1 < \Delta t_2 < \Delta t_3 < \Delta t_4)$.



(3.1) LMJM (True model)

(3.2) QRJM with $\tau = 0.5$

Figure 3: Prediction results in Simulation II Scenario 3: Bland-Altman plot (bias and 95% limits of agreement) of gold standard versus model predictions at $t = 0.25$ for four different prediction time intervals ($\Delta t_1 < \Delta t_2 < \Delta t_3 < \Delta t_4$).

3 JAGS code to fit the QRJM in simulation study

```
model{
  k1 <- (1-2*qt)/(qt*(1-qt))
  k2 <- 2/(qt*(1-qt))
  for (i in 1:I){
      # prior for random effects
      u[i, 1:2] ~ dmnorm(zero[], precision[,])
      # longitudinal process
      for (j in 1:J[i]){
          er[i,j] ~ dexp(sigma)
          mu[i,j] <- beta + u[i,1] + u[i,2]*t[j] + time[1]*X[i,1]
                    + time[2]*X[i,2]*t[j] + k1*er[i,j]
          prec[i,j] <- sigma/(k2*er[i,j])
          y[i,j] ~ dnorm(mu[i,j], prec[i,j])
      } #end of j loop
      # time-to-event process
      A[i] <- assoct.*u[i,2] + assoct.*time[2]*X[i,2]
      B[i] <- assoct.*(time[1]*X[i,1] + u[i,1] + beta) + inprod(gamma, W[i,])
      S[i] <- exp(-c*exp(B[i])*(exp(A[i]*Ti[i])-1)/A[i])
      h[i] <- c*exp(inprod(gamma, W[i,]) + assoct.*(beta +
            time[1]*X[i,1] + time[2]*X[i,2]*Ti[i] + u[i,1] + u[i,2]*Ti[i]))
      L[i] <- pow(h[i], event[i])*S[i]/1E+08
      phi[i] <- -log(L[i])
      zeros[i] ~ dpois(phi[i])
  }#end of i loop
  precision[1:2, 1:2] ~ dwish(Omega[,], 3)
  Sigma[1:2,1:2] <- inverse(precision[,])
  Omega[1,1] <- 1
  Omega[2,2] <- 1
  Omega[1,2] <- 0
  Omega[2,1] <- 0
  # priors for other parameters
  assoct. ~ dnorm(0, 0.001)
  int. ~ dnorm(0, 0.001)
  time[1] ~ dnorm(0, 0.001)
  time[2] ~ dnorm(0, 0.001)
  gamma[1] ~ dnorm(0, 0.001)
  gamma[2] ~ dnorm(0, 0.001)
  sigma ~ dgamma(0.001, 0.001)
  c ~ dunif(0.01, 10)
}
```