GLMM ①

$$\begin{cases} \text{logit}[p(y_{ij}=1)] = \beta_0 + \beta_1 x_i + u_i & i=1,\cdots,I \\ u_i \sim N(0, 1/\tau_u) & j=1,\cdots,J \end{cases}$$

let $\underset{\sim}{\theta} = (\beta_0, \beta_1, \tau_u)$

$$P(\underline{\theta} \mid \underset{\sim}{u}, \text{data}) = \prod_{i=1}^{I} \prod_{j=1}^{J} \frac{[\exp(\beta_0 + \beta_1 x_i + u_i)]^{I(y_{ij}=1)}}{1 + \exp(\beta_0 + \beta_1 x_i + u_i)}$$

$$P(\underset{\sim}{\theta}, \underset{\sim}{u} \mid \text{data}) \propto \prod_{i=1}^{I} \left\{ \prod_{j=1}^{J} \frac{[\exp(\beta_0 + \beta_1 x_i + u_i)]^{I(y_{ij}=1)}}{1 + \exp(\beta_0 + \beta_1 x_i + u_i)} \right.$$

$$\left. \cdot \ \tau_u^{\frac{1}{2}} \exp\left\{ - \frac{u_i^2 \cdot \tau_u}{2} \right\} \right\}.$$

$$\log P(\underset{\sim}{\theta}, \underset{\sim}{u} \mid \text{data}) \propto \sum_{i=1}^{I} \Big(\sum_{j=1}^{J} \Big\{ I(y_{ij}=1)(\beta_0 + \beta_1 x_i + u_i)$$

$$- \log[1 + \exp(\beta_0 + \beta_1 x_i + u_i)] \Big\}$$

$$+ \frac{1}{2} \log \tau_u - \frac{u_i^2 \tau_u}{2} \Big)$$

$$\propto \sum_{i=1}^{I} \Big(\sum_{j=1}^{J} \Big\{ I(y_{ij}=1)(\beta_0 + \beta_1 x_i + u_i) - \log[1 + \exp(\beta_0 + \beta_1 x_i + u_i)] \Big\} \Big)$$

$$+ \frac{I}{2} \log \tau_u - \frac{\sum_{i=1}^{I} u_i^2}{2} \tau_u.$$

Full conditionals

$$\beta_0, \beta_1 \mid \text{others} \propto \sum_{i=1}^{I}\left(\sum_{j=1}^{J}\left\{I(y_{ij}=1)(\beta_0+\beta_1 x_i+u_i) - \log[1+\exp(\beta_0+\beta_1 x_i+u_i)]\right\}\right)$$

$$\tau_u \mid \text{others} \propto \frac{I}{2}\log\tau_u - \frac{\sum_{i=1}^{I} u_i^2}{2}\tau_u$$

$$\propto \left(\frac{I}{2}+1-1\right)\log\tau_u - \frac{\sum_{i=1}^{I} u_i^2}{2}\tau_u$$

$$\therefore \tau_u \mid \text{others} \sim Gam\left(\frac{I}{2}+1,\ 0.5\sum_{i=1}^{I} u_i^2\right)$$

$$u_i \mid \text{others} \propto \sum_{j=1}^{J}\left\{I(y_{ij}=1)(\beta_0+\beta_1 x_i+u_i) - \log[1+\exp(\beta_0+\beta_1 x_i+u_i)]\right\} - \frac{u_i^2}{2}\tau_u \quad \text{for } i=1,\dots,I.$$