1 Plan for simulation study

The simulation study is designed to check the validity of our propsed joint model in modeling the longitudinal outcome and the informative drop-out event time. Our focus of the simulation results lies on the accuracy of our estimation, i.e. bias, and the precision, i.e. standard deviation, of the samples from posterior distribution. Comparision will be made between our posprosed model against the model that simply ignores the underlying informative drop-out mechanism.

$$\begin{cases} y_{it} = \boldsymbol{\beta}^\top \boldsymbol{X}_{it} + \boldsymbol{\delta}^\top \boldsymbol{H}_{it} + \boldsymbol{u}_i^\top \boldsymbol{Z}_{it} + \varepsilon_{it} = \widetilde{\tau}_{it} + \varepsilon_{it} \\ h(T_i|\mathcal{T}_{iT_i}, \boldsymbol{W}_i; \boldsymbol{\gamma}, \alpha_1, \alpha_2) = h_0(T_i)\exp(\boldsymbol{\gamma}^\top \boldsymbol{W}_i + \alpha_1 \boldsymbol{\delta}^\top \boldsymbol{H}_{iT_i} + \alpha_2 \boldsymbol{u}_i^\top \boldsymbol{Z}_{iT_i}) \end{cases} \tag{1}$$

2 Simulation settings

Following (Farcomeni and Viviani, 2014), by varying the values of the association parameters α_1 and α_2 in our model (1), we will have four different settings of simulation study, which are:

1. $(\alpha_1, \alpha_2) = (0, 0)$, the two models are indepdent with each other
2. $(\alpha_1, \alpha_2) = (1, 0)$, the two models are related only through the observed heterogeneity in some covarites, $\boldsymbol{H}_{it}$ in our model
3. $(\alpha_1, \alpha_2) = (0, 1)$, the two models are related only through the unobserved heterogeneity, i.e. the random effects
4. $(\alpha_1, \alpha_2) = (1, 1)$, the depdence of the two models is explained by both observed and unobserved heterogeneity

In model (1) note that we have specific covariates $\boldsymbol{X}_{it}$ only for the longitudinal model and covariates $\boldsymbol{W}_{it}$ only for the survival model.

Under different combinations of $(\alpha_1,\ \alpha_2)$, for the regression coefficients we choose $\boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{\gamma} = (1,1)^\top$, the covriates $\boldsymbol{Z}_{it} = (1,t)^\top$, $\boldsymbol{H}_{it} = (h_{i1}, h_{i2} * t)^\top$, $\boldsymbol{X}_{it} = (1, x_i)^\top$, and $\boldsymbol{W}_i = (w_{i1}, w_{i2})^\top$ with $h_{i1}, h_{i2}, x_i, w_{i1}$ and w_{i2} generated from independent standard normal distributions, and the random effects $\boldsymbol{u}_i$ from bivariate normal with mean 0, standard deviations equal 0.3 and correlation 0.16. We also fix $\sigma = 1$ and vary the quantile τ among $\{0.25, 0.5, 0.75\}$ for the ALD specification when simulating longitudinal data.

To simulate the survival time data, for simplicity, we fix $h_0(s) = 1$ and obtain the survival distribution as

$$S(t|\boldsymbol{u}_i, \boldsymbol{H}_{it}, \boldsymbol{W}_i) = \exp\left\{ -\frac{e^{\alpha_1(\delta_1 H_{i1} + \delta_2 H_{i2t}) + \alpha_2(u_{i1} + u_{i2}t) + \boldsymbol{\gamma}^\top \boldsymbol{W}_i} - e^{\alpha_1 \delta_1 H_{i1} + \alpha_2 u_{i1} + \boldsymbol{\gamma}^\top \boldsymbol{W}_i}}{\alpha_2 u_{i2} + \alpha_1 \delta_2 h_{i2}} \right\}$$

when $\alpha_1 \neq 0$ or $\alpha_2 \neq 0$ and

$$S(t|\boldsymbol{u}_i, \boldsymbol{H}_{it}, \boldsymbol{W}_i) = \exp\{-te^{\boldsymbol{\gamma}^\top \boldsymbol{W}_i}\}$$

when $\alpha_1 = \alpha_2 = 0$. We then can obtain event time T_i by inverting above survival function after generating n random variates from standard uniform distribution. To obtain a censoring proportion around 25%, we choose the censoring time $C_i/5$ be distributed according to $beta(4, 1)$.

To simulate the longitudinal data, we draw them independently from the ALD for the $\tau-$th quantile, centered on

$$\boldsymbol{\beta}^\top \boldsymbol{X}_{it} + \boldsymbol{\delta}^\top \boldsymbol{H}_{it} + \boldsymbol{u}_i^\top \boldsymbol{Z}_{it},$$

and with dispersion parameter σ. We keep maximum six observations for each subject at follow-up time $t = (0, 0.25, 0.5, 0.75, 1, 3)$ respectively, after incorporating the drop-out information.

3 Result

			τ=0.25															
			β_1		β_2		δ_1		δ_2		γ_1		γ_2		α_1		α_2	
n	α_1	α_2	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.
250	0	0	0.05	0.108	0.00	0.106	0.02	0.108	0.00	0.131								
250	1	0	0.077	0.119	0.011	0.111	0.001	0.094	0.037	0.096	0.033	0.078	0.003	0.077	0.014	0.109	0.024	0.190
250	0	1																
250	1	1																
			τ=0.5															
			β_1		β_2		δ_1		δ_2		γ_1		γ_2		α_1		α_2	
n	α_1	α_2	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.
250	0	0	0.05	0.108	0.00	0.106	0.02	0.108	0.00	0.131								
250	1	0	0.077	0.119	0.011	0.111	0.001	0.094	0.037	0.096	0.033	0.078	0.003	0.077	0.014	0.109	0.024	0.190
250	0	1																
250	1	1																
			τ=0.75															
			β_1		β_2		δ_1		δ_2		γ_1		γ_2		α_1		α_2	
n	α_1	α_2	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.	bias	s.d.
250	0	0	0.05	0.108	0.00	0.106	0.02	0.108	0.00	0.131								
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References

Alessio Farcomeni and Sara Viviani. Longitudinal quantile regression in presence of informative drop-out through longitudinal-survival joint modeling. *arXiv preprint arXiv:1404.1175*, 2014.