1 Instructions

Perform the numerical and analytical exercises below. Write up your results (preferably using LaTeX or a Jupyter notebook) in a document. Include all relevant figures and explanations. Include your codes at the end of the document.

2 Exercises

1. Implement a 4-step Adams Bashforth integrator and use it to solve the 2-body Newtonian problem for a mildly eccentric orbit. Compare the accuracy and speed of this method compared to RK4.

2. In spherical symmetry, the Laplacian takes on the form

$$\nabla^2\psi = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = \frac{d^2\psi}{dr^2} + \frac{2}{r}\frac{d\psi}{dr}.$$

   Using Gauss-Seidel iterations and second-order-accurate finite differencing, solve the following Poisson problem

$$\begin{aligned}
\nabla^2\psi &= -4\pi\rho, \quad 1 < x < 10 \\
\frac{d\psi}{dr}|_{r=1} &= 0, \\
\psi(10) &= 1, \\
\rho(r) &= 1/r^4,
\end{aligned}$$

   Develop your own code to do this. Do not start from the generic code we discussed in class (and located in mycourses).

   (a) Starting with $n = 1024$ cells, plot the residual versus r after 100, 200, and 1000 iterations of Gauss-Seidel (with no over relaxation)

   (b) Repeat the above with $\omega = 1.5$

   (c) This problem can be solved exactly. Using the exact solution, determine the L^∞ norm of the error in the approximate value of ψ over the grid (once the GS algorithm has converged) as a function of the number of cells.