1 Instructions

Perform the numerical and analytical exercises below. Write up your results (preferably using LaTeX or a Jupyter notebook) in a document. Include all relevant figures and explanations. Include your codes at the end of the document.

2 Exercises

1. Consider the inviscid Burger Equation

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial \left(u^2\right)}{\partial x}$$

with initial data

$$u(x,0) = \begin{cases} 1, & x < 0 \\ 0, & x > 0 \end{cases}.$$

Show that the following is a weak solution to this problem

$$u(x,t) = \begin{cases} 1 & x < \frac{1}{2}t, \\ 0 & x > \frac{1}{2}t \end{cases}.$$

2. Consider the inviscid Burgers equation with the initial data

$$u(x,0) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases}.$$

Show that the following is a weak solution

$$u(x,t) = \begin{cases} 0 & x < \frac{1}{2}t, \\ 1 & x > \frac{1}{2}t \end{cases}.$$

3. Argue that the solution for problem 1 is the *correct* solution, while the solution in problem 2 is not *correct* by considering a family of initial data with

$$u(x,0) = \begin{cases} U_L, & x < -h, \\ U_R, & x > h, \\ U_L + \frac{x+h}{2h}(U_R - U_L), & -h < x < h \end{cases},$$

where h is taken to be successively smaller. Hint: We really only need to consider the solution well inside $-h < x < h$, so just consider the problem where

$$u(x,0) = ax,$$

where a is a constant.