1 Instructions

Perform the numerical and analytical exercises below. Write up your results (preferably using LaTeX or a Jupyter notebook) in a document. Include all relevant figures and explanations. Include your codes at the end of the document.

2 Exercises

In class, we demonstrated a code that can be used to solve the Poisson problem in spherical symmetry using Chebyshev polynomials. Modify that code, or create your own to solve the following problem

$$\frac{d^2\psi}{dr^2} + \frac{2}{r}\frac{d\psi}{dr} + 4\pi\rho(r) = 0, \quad 0 < r < r_{\max} \tag{1}$$

$$\frac{d\psi}{dr}|_{r=0} = 0, \tag{2}$$

$$r_{\max}\frac{d\psi}{dr}|_{r_{\max}} + \psi(r_{\max}) = 0, \tag{3}$$

where

$$\rho(r) = \begin{cases} 1 & 0 \le r \le 1 \\ e^{-r+1} & r \ge 1 \end{cases}. \tag{4}$$

Here, you should consider two different domain decompositions. The first consists of a single subdomain $[0, r_{\max}]$, and the second two subdomains $[0, 1]$ and $[1, r_{\max}]$. Choose $r_{\max}$ such that $\rho(r_{\max}$ is small. This problem has an analytic solution. Measure the inifnity norm of the error in both cases versus the number of Chebyshev coefficients. For latter, choose an equal number of coefficients for the two subdomains.