1 Instructions

Perform the numerical and analytical exercises below. Write up your results (preferably using LaTeX or a Jupyter notebook) in a document. Include all relevant figures and explanations. Include your codes at the end of the document.

2 Exercises

1. Develop an adaptive Runge-Kutta algorithm based on the standard fourth-order accurate RK4 algorithm. To estimate the local error, calculate $f(x_0 + h)$ using RK4 with stepsize h and with two RK4 steps using stepsize $h/2$. The difference between these two will be your estimate of the local error. Test your code by solving $y''(x) = -y$, $y(0) = 10$, $y'(0) = 1$, for $0 \leq x \leq 10$. Measure the L^∞ norm of the error. Note that your code should automatically choose the stepsize h based on the user-specified tolerance. Try to set the tolerance to just above the roundoff limit.

2. Express the equations of motion for three gravitating point masses in Newtonian gravity in first order form. Implement your system of equations within the adaptive RK4 integrator you developed in problem 1. Finally, using your code, solve the problem of three masses initially at rest and arranged on the vertices of a right triangle whose sides have length 3ℓ, 4ℓ, and ℓ. Choose the masses to be proportional to the side opposite the given vertex. Confirm that you can rescale the space and time coordinates so that the masses are 3, 4, and 5, the lengths to be 3, 4, and 5, and the gravitational constant to be $G = 1$ and solve this problem numerically. Plot the trajectories of the three masses on a single plot. In addition, measure the total energy, linear, and angular momentum and determine how well these are conserved as a function of the tolerance.