1 Instructions

Perform the numerical and analytical exercises below. Write up your results (preferably using LaTeX or a Jupyter notebook) in a document. Include all relevant figures and explanations. Include your codes at the end of the document.

2 Exercises

1. Brent's method is a robust root finding algorithm based on the methods that were discussed (or will be discussing) in class (Bisection, Secant Method, Inverse quadratic interpolation). Develop a similar robust method based on bisection and Newton's method. Your root finding routine should take as input a function, its derivative, and a valid bracket of a root of the function and return the root to maximum accuracy achievable by your algorithm. To test the robustness of the algorithm, use if to find the root of

$$\frac{(x+5)^3+x+5}{(x+5)^4+1}+\left(x+\frac{11}{2}\right)e^{\cos(x+5)}$$

   Note that while you can choose any large a for a starting value, $(-a, a)$ doesn't actually bracket the root if $a \lesssim 5$. Plot the number of iterations required as a function of a for $6 < a < 20$.

2. Using the same function as in problem 1, compare the performance of Newton's method and the Secant method. Rather than bracketing the root, for the Secant method initialize with the points x_0 and $x_0+.0001$. Compare the number of iterations required to solve the problem for both methods. Choose values of x_0 both close to the true root, and relatively far away. Watch out, Newton's method in particular will fail for many different choices of x_0.