Homework 1:

1 Simpson's rule

Simpson's rule is a method for approximating the integral

$$\int_a^b f(x) = dx.$$

It takes the form

$$S_n = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_4) + \cdots + 4f(x_{n-1}) + f(x_n)\right),$$

where $\Delta x = \frac{b-a}{n}$, $x_j = a + j\Delta x$, and n is a positive even integer.
Consider the integral

$$\int_0^{1/\sqrt{2}} \sqrt{1-x^2}dx.$$

Write a program in C that performs the following tasks:

1. Reads an integer n from the command line and checks that n is positive and even (you should use the function ATOI).
2. Allocates space for an array of doubles of length $n+1$ (you should use the function MALLOC).
3. Calls another function to fill in the array with $f(x_0)$, $f(x+1)$ etc.
4. Passes the filled in array to another function that implements Simpson's rule. This function should return S_n.
5. Finally, print the numbers Δx, S_n, and the error in S_n to stdout

2 Plotting the result

Suppose you named your program SIMPSON. You can generate a table of grid spacing and errors with the following shell commands

```
for n in 100 200 400 800 1600
do
  ./simpson $n >> output
done
```

This will append (or create) a file called output which will have three columns. Column 1 will be Δx, column 2 will be the error, and column 3 will be the approximate value of the integral.
Use GNUPLOT or MATPLOTLIB to make a log-log plot of the absolute value of the error versus Δx. Save this plot as a "png" or "pdf" file.
Upload your source code as a single "C" file and the pdf file of the plot.