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Reliable *p*-Hub Location Problems in Telecommunication Networks

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Hubs are critical elements of telecommunication and transportation networks because they play a vital role in mass traffic movement. The design of more reliable networks in hub-and-spoke systems is a critical issue because current networks, particularly many commercial Internet backbones, are quite vulnerable. In hub-and-spoke-type topologies, any malfunction at a hub may cause degradation of the entire network's ability to transfer flows. This article presents a new hub location problem, termed the reliable p-hub location problem, which focuses on maximizing network performance in terms of reliability by locating hubs for delivering flows among city nodes. Two submodels, the p-hub maximum reliability model and the p-hub mandatory dispersion model, are formulated. Based on hypothetical and empirical analyses using telecommunication networks in the United States, the relationship between network performance and hub facility locations is explored. The results from these models could give useful insights into telecommunication network design.

Introduction

The placement of hubs in telecommunications has been identified as an important operational and defense strategy because the performance of current networks is highly reliant on hub locations (Grubesic, O'Kelly, and Murray 2003; NSTAC 2003). Considering the increasing vulnerability of hub facilities to disruption, designing a reliable hub network is a significant issue in current telecommunication systems (Klincewicz 2006; Skorin-Kapov, Skorin-Kapov, and Boljunčić 2006). Early telecommunication networks were as decentralized as possible to achieve a defensible outcome. For example, a defensive locational philosophy underpinned the earliest designs for ARPANet, where important hubs were strategically dispersed to protect against probable attacks or failures (Boehm and Baran 1964). With

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increasing commercialization, however, networks have evolved into more centralized systems, and there is an intense concentration of key functions at select hubs. Moreover, Internet hubs often are located close to each other for reasons of efficiency (Pastor-Satorras and Vespignani 2004).

As discussed extensively in Murray and Grubesic (2007), this functional and geographical centralization of hubs comes at a time when the need to protect high-speed and broadband technologies and their networks is critical. Even though geographical proximity among hubs reduces the possibility of traffic loss in transmitting aggregated flows, this benefit could in fact be compromised by the increased concentration of hub activities, which could potentially degrade network performance in the face of disruptions (BBC News 2002a, b; Wagner 2002; NCA 2004).

The potential advantage of geographical dispersion of hubs is stressed by O'Kelly and Kim (2007), who analyze the recent cascading failure of Internet services in Korea in terms of network reliability. They demonstrate that the damage to just a single hub can have extensive impacts on an entire network when hub facilities are highly clustered geographically (see also O'Kelly, Kim, and Kim 2006). Another practical example is provided by the recent disruption of Internet service due to the 2006 earthquake in Taiwan, which occurred in an area where undersea cables are highly concentrated, subsequently causing severe telecommunications problems among continents. This latter example highlights the importance of dispersing components to maintain the reliability of telecommunication systems (Heiskanen 2006).

Within this context, we address a variety of new hub location problems, namely the *reliable* p-hub location problem (RPHLP). Specifically, the RPHLP focuses on locating p-hubs on a network to improve network reliability to deliver interacting flows among its set of origin-destination (OD) nodes. The RPHLP consists of two submodels based on consideration of the rationale of dispersed facility location.

The first submodel is the p-hub maximum reliability (PHMR) model. It optimizes a hub network by focusing solely on maximizing the completed flows among the set of nodes via p-hubs based on the level of reliability, which is imposed on such network facilities as hubs and interhub links. The motivation of the PHMR model is to design a reliable hub network by accounting for a network-performance measure in the hub-and-spoke network models. Of particular concern to the PHMR is the exploration of behaviors when reliability-relevant components are embedded in the hub location problem.

The second submodel, based on the first, is the p-hub mandatory dispersion (PHMD) model, which determines the optimal hub locations to improve network reliability while retaining the *mandatory dispersion of hubs* requiring them to be farther apart than a certain minimum separation. Although various types of facility dispersion problems have been addressed in the literature, the rationale for dispersion of *hubs* has rarely been explored (see Curtin and Church 2006 for a recent review of dispersion models). Therefore, the motivation of the PHMD model is to investigate the impacts of hub facility dispersion with respect to network performance, flow patterns, and flow activity levels of hub facilities. It ultimately aims to mitigate potential damage from a possible malfunction on hub facilities with excessive activity levels. As an extension of the PHMD model, we also provide the multiobjective reliable *p*-hub dispersion (MRDI) model, where both objectives—network reliability and level of hub separation—are determined simultaneously.

The term PHMD model used in this article *initially* is different from that of "the *p*-dispersion problem" addressed by Kuby (1987). Conventionally, the *p*-dispersion problem focuses on locating *p*-noxious or undesirable facilities among *n* candidate nodes. This characteristic suggests that the facilities should be located as far away as possible from the nearest one so that an accident at one of the facilities will not impact the others. Moreover, the goal of PHMD models is to avoid concentrating strategic assets in one area to reduce the growing vulnerability that may arise from congestion or disruptions in a network.

Background

The issue of a reliable system has been recognized as a critical factor in the design of network infrastructure (Gavish and Neuman 1992; Herder and Thissen 2003; McGrath 2003; Konak and Smith 2006). In a broad sense, *reliability* refers to the ability of a network to perform a successful operation (Bell and Iida 1997). Because perfect operation of a system free from either random or intended failure is impossible, maintaining *desired* network operations at a high level of reliability is a major challenge (Kansal, Kumar, and Sharma 1995; Lewis 1996; Colbourn 1999).

In general, reliability planners have developed models to maximize system reliability by allocating redundant components to systems with a given set of resources (Mohamed, Leemis, and Ravindran 1992). Many telecommunication network models seek either to maximize network performance for the particular set of origin and destination nodes by increasing the availability of links or to minimize the building costs of a network, subject to reliability-relevant requirements (Gavish et al. 1989; Forsgren and Prytz 2006). Because most reliability optimization problems are difficult to solve due to their computational complexity (see Kuo and Prasad 2000), often practical design approaches are proposed. For example, Shao and Zhao (1997) present a reliable telecommunications network design model that focuses on finding the best linkages to add to the existing backbones so that a network can reach a certain level of reliability while minimizing network costs.

However, many reliable network models in telecommunications are more focused on the topological optimization of networks, and thus complicated network properties often are ignored (Soni, Gupta, and Pirkul 1999; Carello et al. 2004). For example, the main characteristic of a *hub*, which is defined as a set of interacting facilities for the switching and consolidating/concentrating of flows, is often used to be characterized as simply the more important node out of a number of nodes in a system (Campbell 1994a).

In recent research, the idea of a reliable system has been applied in traditional location problems such as the p-median problem (Snyder and Daskin 2007). Even though the need to address a reliable network model in terms of a hub-and-spoke network design has been stressed, few models for this are found in the literature (Klincewicz 1998, 2006). In particular, incorporating performance-related measures (e.g., reliability, traffic loss rate) in a hub network design is a significant issue because ensuring good performance for transferring interacting traffic becomes a critical concern in current telecommunication networks due to its delay-sensitive nature (Klincewicz 2006; Skorin-Kapov, Skorin-Kapov, and Boljunčić 2006).

Model developments

We develop the RPHLP considering the two main variants in hub network design, which are generally referred to as single-assignment (SA) and multiple-assignment (MA) models (Bryan and O'Kelly 1999). The SA model restricts each non-hub node to interact with only one hub so that all flows from an origin must travel to the same hub. In contrast, the MA model is more flexible in routing by allowing each node to interact with more than one hub. Both protocols require all OD flows to be routed through at least one hub, and hubs need to be completely connected with interhub links. To formulate the RPHLP, we first introduce two reliability-relevant components: (1) the routing reliability R_{ijkm} and (2) the reliability factors α and γ, both of which are different compared with conventional hub location models. The data used in our model experiments are described in "Data: reliability matrix and interacting flow matrix."

Routing reliability R_{ijkm}

In telecommunication networks, reliability is defined as the probability of successful communication to deliver traffic without congestion or loss between OD pairs (Bell and Iida 1997). Because here the term "successful communication" is open to interpretation, this article uses the most relevant definition, that is, the probability that a facility, such as a link and hub, performs its operations of transmitting flows for a given period of time without failure (Wakabayashi and Iida 1992). Suppose that the reliability between two nodes is known when they are connected with a linkage; the reliability of the route that consists of n links is calculated as follows (see Colbourn 1987; Shier 1991):

$$R_{\mathrm{OD}} = \prod_{l=1}^{n} \Pr\{r_l\}$$

where R_{OD} is the reliability of a route from origin O to destination D, and r_l is the reliability of link l in route R_{OD} ($l = 1, 2, \ldots, n$).

In general, reliability between two nodes on Internet protocol (IP)–based networks decreases with the length (distance) of a path because the transmission time of packet data or the traffic delay rate is highly correlated with geographical distance and the congestion levels in the route (Agrawal 1997; Choi et al. 2004;

Crovella and Krishnamurthy 2006). Thus, of great concern is determining the routes whose reliabilities maximize interacting flows delivered among a number of sources and destinations on a network (Stavroulakis 2003). In this article, the routing reliability R_{ijkm} is defined as the probability of the successful delivery of flows for the routing variable X_{ijkm}, which is generally used in many hub location models to represent the path along which a flow from origin i to destination j is to be routed via hubs k and m ($i \rightarrow k \rightarrow m \rightarrow j$) (Bryan and O'Kelly 1999).

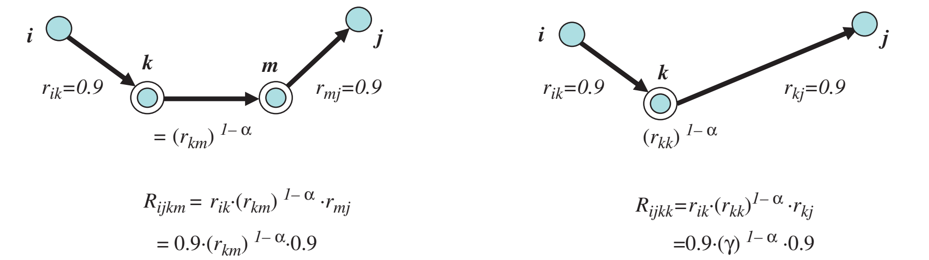
Routing scheme: reliability factors α and γ

As indicated by X_{ijkm}, the routing reliability R_{ijkm} basically is calculated by sequentially multiplying the reliabilities of each link r_{ik}, r_{km}, and r_{mj}. However, performance-related facility conditions need to be reflected in telecommunication networks because the physical aspects of network facilities are related to network performance. For instance, a link with a larger capacity or bandwidth can transmit traffic for a longer distance without the loss of flow, noise, and attenuation of signals, all of which are expected to increase with geographical distance. The capacity and technological level of hub facilities are designed to keep the reliability of traffic at a desired level (Pióro and Medhi 2004; Zheng et al. 2005). To reflect performance-related facility conditions in telecommunication networks, the reliability factors α and γ are introduced into the RPHLP.

The reliability factor α represents the degree of benefit from enhancing the reliability of interhub links when traffic utilizes interhub links. This factor is set as a power parameter of the term $(r_{km})^{1-\alpha}$ ($0 \leq \alpha \leq 1$, $k \neq m$). Thus, the computation of R_{ijkm} is written as $R_{ijkm} = r_{ik} r_{km}^{(1-\alpha)} r_{mj}$. For example, for known link reliabilities $r_{ik} = 0.9$, $r_{km} = 0.8$, and $r_{mj} = 0.9$ with $\alpha = 0.9$, the R_{ijkm} is computed as 0.792 ($= r_{ik} r_{km}{}^{0.10} r_{mj}$), indicating that the route X_{ijkm} can deliver 79.2% of the traffic between i and j via hub k and m without delay or congestion (see Fig. 1a). When the value of α is closer to 1.0, the value of R_{ijkm} improves because the reliability of the interhub link ($r_{km}{}^{1-\alpha}$) is enhanced. In contrast, a smaller α indicates that a smaller benefit obtains when flows utilize the interhub link.

In the case of the one-hub stop route X_{ijkk} ($i \rightarrow k \rightarrow j$), the routing reliability is calculated as $r_{ik}(r_{kk})^{1-\alpha} r_{kj}$ according to the notational order. In this routing, the reliability of a hub (r_{kk}) plays a significant role in transmitting the flows to the destination. To reflect this routing characteristic, we introduce the reliability factor $\gamma(0 \leq \gamma \leq 1)$ for routing computations. Specifically, the reliability factor γ is defined as the ability of a hub to transmit traffic without delay or congestion, which could represent the performance level of intrahub communication. This factor is the value of r_{kk} or r_{mm}, which is the diagonal element in a reliability OD matrix, and the factor is applied when a routing reliability such as R_{ijkk} or R_{ijii} is calculated. A larger γ indicates a better capability for transmission facilities, including the router, repeater, and amplifier in the hub itself.

Note that the route decision for an ij pair is endogenously made depending on R_{ijkm}, whose value is calculated based on the magnitude of reliability factors. As



(a) Interhub route R_{ijkm}

(b) One-hub-stop route R_{ijkk}

Parameters	Reliability of X_{ijkm}
(a) $\alpha = 0.9$, $r_{km} = 0.8$	$R_{ijkm} = 0.792$

Parameters	Reliability of X_{ijkk}
(b1) $\alpha = 0.9$, $\gamma = 0.7$	$R_{ijkk} = 0.782$
(b2) $\alpha = 0.9$, $\gamma = 0.9$	$R_{ijkk} = 0.802$

Figure 1. Route choice based on reliability factor α and reliability factor γ.

demonstrated in Fig. 1, if the same level of α is assumed for both routing schemes (i.e., $\alpha = 0.9$) and γ is <0.8 (i.e., $\gamma = 0.7$ in b1) for a one-hub stop route, then utilizing the interhub (or two-hub stop) route X_{ijkm} (Fig. 1a) is the more reliable route. However, if γ is >0.8 (i.e., $\gamma = 0.9$ in b2), as in Fig. 1b, then the one-hub stop route X_{ijkk} becomes a better routing strategy ($R_{ijkm} < R_{ijkk}$).[1] Because the optimal route is chosen from a number of possible routes, numerous solutions that maximize the sum of traffic flows over OD pairs can be explored for various values of both factors based on the assignment type.

Data: reliability matrix and interacting flow matrix

Two OD matrices, the reliability and the interacting OD matrices (hereafter *R*-matrix and *W*-matrix, respectively) are needed for model experiments here. We construct the *R*-matrix utilizing the empirical *traffic loss rate* among 14 U.S. city nodes monitored by the SAVVIS Network (http://ipsla.savvis.net/) from March to October of 2005. Traffic loss rate is a standard measure to indicate how stably a network can provide quality of service on telecommunication networks, and it can be easily converted into reliability measures (AT&T 2003; Ciabattone et al. 2003). Generally, reliability and traffic latency are related to geographical distance (Murnion and Healey 1998; Pióro and Medhi 2004; Crovella and Krishnamurthy 2006). As illustrated by Fig. 2, the reliabilities for 14 city node pairs (91cases) in the empirical *R*-matrix are not strong but are considerably correlated with their geographical distances.[2] For the *W*-matrix, two data sets were prepared for testing purposes. Set I uses equal flows for all *ij* pairs (i.e., $W_{ij} = 1.0$). The purpose of Set I is to investigate model behaviors by suppressing size effects of interacting flows. In Set II, the potential interacting flows are estimated based on the amount of telecommunications demand and supply between *i* and *j*. The W_{ij} in Set II is calculated as follows, using the data of Zook (2000) and Atkinson and Gottlieb (2001)[3]:

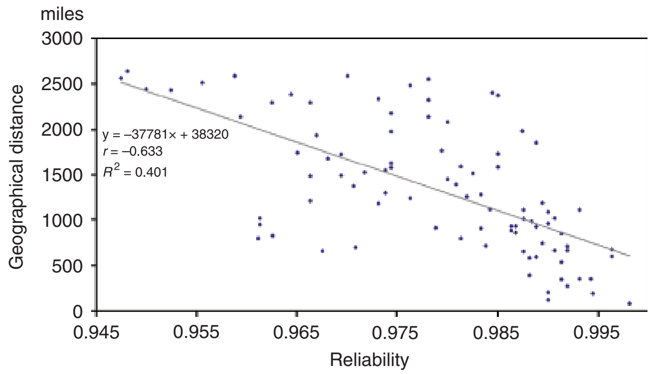


Figure 2. The relationship between city node reliabilities and geographical distance in the *R*-matrix.

$$W_{ij} = k(D_i S_j + D_j S_i)$$

where W_{ij} is the amount of flow traveling between city nodes i and j; D_i is the amount of online population in city node I; S_j is the number of domains in city node j, and k is a scaling factor (10^{-9}).

Model formulations

As mentioned previously, the PHMR has two different variants, MRSA and MRMA, based on what assignment scheme is considered in the model formulation. Both models have the same objective function, but different constraints are used. The following two subsections provide the MRSA and the MRMA model formulations.

MRSA (the PHMR model with single assignment)

Maximize

$$\Omega = \sum_i \sum_j \sum_k \sum_m W_{ij} R_{ijkm} X_{ijkm} \tag{1}$$

subject to

$$\sum_k Z_{kk} = p \quad (2 \leq p) \tag{2}$$

$$\sum_k Z_{ik} = 1, \quad \forall i \tag{3}$$

$$Z_{ik} - Z_{kk} \leq 0, \quad \forall i,\ k(i \neq k) \tag{4}$$

$$\sum_m X_{ijkm} - Z_{ik} = 0, \quad \forall j > i;\ k \tag{5}$$

$$\sum_k X_{ijkm} - Z_{jm} = 0, \quad \forall j > i;\ m \tag{6}$$

$$Z_{ik} \in \{0, 1\} \tag{7}$$

$$0 \leq X_{ijkm} \leq 1 \tag{8}$$

where p is the number of hubs to be located; W_{ij} is the amount of flow to travel between i and j; X_{ijkm} is the fraction of flow from origin i to destination j via hub k and m in that order ($i \rightarrow k \rightarrow m \rightarrow j$); R_{ijkm} is the routing reliability for the route X_{ijkm}; Z_{ik} is 1 if node i is allocated to hub k, 0 otherwise; Z_{kk} is 1 if node k is a hub, 0 otherwise; α is interhub reliability factor ($0 \leq \alpha \leq 1$); and γ is intrahub reliability factor ($0 \leq \gamma \leq 1$), $\gamma = r_{kk}$ or r_{mm}.

The objective function (1) maximizes the total network flow that can be transported based on the computed reliability R_{ijkm} for route X_{ijkm} over each OD pair. Constraint (2) requires p-hubs to be open, and constraint (3) forces each node to be assigned to only a single hub. Constraint (4) requires a hub to be opened before a node is allocated to hub k, denoted as Z_{kk}. Taken together, constraints (5) and (6) guarantee that flow i–j should not be routed via hubs k and m unless origin i is linked to hub k and j is linked to hub m. In constraint (7), Z_{ik} is the integer variable that prevents partial facility location.

MRMA (the PHMR model with multiple assignments)
Maximize

$$\Omega = \sum_i \sum_j \sum_k \sum_m W_{ij} R_{ijkm} X_{ijkm} \tag{9}$$

subject to

$$\sum_k Z_k = p \quad (2 \leq p) \tag{10}$$

$$\sum_k \sum_m X_{ijkm} = 1, \quad \forall j > i \tag{11}$$

$$\sum_m X_{ijkm} - Z_k \leq 0, \quad \forall j > i;\ k \tag{12}$$

$$\sum_k X_{ijkm} - Z_m \leq 0, \quad \forall j > i;\ m \tag{13}$$

$$Z_k \in \{0, 1\} \tag{14}$$

$$0 \leq X_{ijkm} \leq 1 \tag{15}$$

In the MRMA, Z_k represents the facility location variables instead of the variables Z_{kk} in the MRSA because the variables Z_{ik} are not needed in the MRMA. The objective function value from the MRMA is always greater or equal to that of the MRSA because the flexibility of assignment to hubs allows more strategic options when determining the optimal route. In other words, each origin *i* can route its interactions with nodes *j* through different hubs to give the most reliable route. Constraint (10) ensures that *p*-hubs are selected. Constraint (11) ensures that all flows from *i* to *j* should travel through hub(s) *k* and *m*, and *k* can be equal to *m*. Together, constraints (12) and (13) prevent the flow between *i* and *j* from being routed through non-hub nodes.

MDSA (the PHMD model with single assignment)
Maximize

$$\Omega = \sum_i \sum_j \sum_k \sum_m W_{ij} R_{ijkm} X_{ijkm} \tag{16}$$

subject to constraints (2)–(8) and

$$D_{\text{man}} \leq d_{km} + M(1 - Z_{kk}) + M(1 - Z_{mm}), \quad \forall m > k \tag{17}$$

where d_{km} is geographical distance (the length of interhub link) between potential hubs *k* and *m*; D_{man} is mandatory separation distance between *p*-hubs ($0 < D_{\text{man}} \leq D_{\text{max}}$); D_{max} the *maximin* distance from any pair of nodes, $D_{\text{max}} = Max_{i,j}[Min\{d_{ij}\}]$; M is a large number greater than $Max_{k,m}\{d_{km}\}$.

The MDSA has the same objective function and employs constraints (2)–(8) used in the MRSA as well as constraint (17), which are the mandatory dispersion constraints (MDC). These last constraints ensure that the two hubs that are closely located within D_{man} cannot both be members of a feasible *p*-hub set. The value D_{man} is exogenously given, but it should be bounded by D_{max} to avoid an infeasible solution. The value of D_{max} can be preobtained by running Kuby's *p*-dispersion model based on the given nodes and the distances among them (Kuby 1987, p. 319).

MDMA (the PHMD model with multiple assignments)
Maximize

$$\Omega = \sum_i \sum_j \sum_k \sum_m W_{ij} R_{ijkm} X_{ijkm} \tag{18}$$

subject to constraints (10)–(15) and

$$D_{\text{man}} \leq d_{km} + M(1 - Z_k) + M(1 - Z_m), \quad \forall m > k \tag{19}$$

The MDMA is a simple extension of the MRMA, in which adding MDC constraint (19) is designed to keep a mandatory separation level among potential hub facilities. Note that the distance constraints in MDC in the PHMD have a similar characteristic with *p*-hub center or covering problems, where distance-based constraints (maximum distance) are used either for an entire route, each link in a route, or only for the spoke links (Campbell 1994b). However, the PHMD applies the minimum distance constraint only to the interhub links for the *mandatory dispersion of hubs*.

MRDI model

A multiobjective type of model provides a tradeoff among conflicting goals with noncommensurate units that cannot be simply incorporated into a single objective model (Cohon 1978). Instead of setting the level of mandatory separation as constraints, the MRDI combines two problems directly to solve both simultaneously: the maximization of network reliability and the maximization of geographical separation among hubs. In this article, the MRDI is formulated based on multiple assignments:
Maximize

$$\Omega_1 = \sum_i \sum_j \sum_k \sum_m W_{ij} R_{ijkm} X_{ijkm} \tag{20}$$

Maximize

$$\Omega_2 = D \tag{21}$$

subject to constraints (10)–(15) and

$$D \le d_{km} + M(1 - Z_k) + M(1 - Z_m), \quad \forall m > k \tag{22}$$

$$0 \le D \le D_{\max} \tag{23}$$

where D is the smallest separation distance among any pair of p hubs; $D_{\max}$ is the *maximin* distance preobtained by the p-dispersion model.

Using the weighting method, the objective of the MRDI is restated as follows: maximize

$$\Omega = w\Omega_1 + (1 - w)\Omega_2 \tag{24}$$

The objective function (21) is essentially a maximin dispersion objective, which is to maximize D, the minimum distance between *open* hub facilities (Kuby 1987). Constraint (22) is written similarly to constraint (19) in the MDMA, but D plays the role of an upper bound, where the distance separating any pair of hubs should be greater than the maximum value of D. Constraint (23) is optional, but it helps to reduce the computational effort because the value $D_{\max}$ plays the role of an upper bound of the solution space D. Note that in objective function (24), given the weight $w = 1$, this model is the same as the MRMA, but when $w = 0$ is assumed, the model is analogous to the classical p-dispersion model, which finds only the smallest separation distance among opened facilities.

Model experiments

Generally, reducing the problem size is an important issue in hub location models. Solving a problem to optimality is of special concern for a clear examination of model results. Thus, we make two assumptions to reduce the problem size. First, the amount of interacting flows and reliabilities between two nodes is assumed to be symmetric, that is, $W_{ij} = W_{ji}$ and $R_{ij} = R_{ji}$. As mentioned above in the ''Data: reliability matrix and interacting flow matrix'' section, the assumption of symmetric flows is based on the way telecommunication potentials are calculated between i and j by considering the amount of demand and supply in both nodes. Second, exclusion of impractical routes from the set of routing variables X_{ijkm} (e.g., $i \to k \to i \to j$ or $i \to j \to m \to j$) results in a practical reduction in the number of variables as well as constraints. As illustrated in Fig. 3, the enumerated routing variables are expressed as a set, $S = \{(i, j, k, m) | (j > i) \cap [(k = i) \cup (k = m = j) \cup (k \neq i \cap k \neq j \cap m \neq j)]\}$. Accordingly, the number of routing variables X_{ijkm} reduces to $n(n-1)(n^2-2n+3)/2$ from the n^4 that are generated in the full-sized model according to the combination of indices (i, j, k, m). The problem sizes for n nodes according to each model formulation are summarized in Table 1. As demonstrated by O'Kelly et al. (1996), these variable reduction techniques are effective when solving the hub location problem efficiently by reducing the computational complexity, while generating the same results as the original problems.

The model experiments are organized into the next two subsections according to testing purposes. The first section is designed to examine the behaviors of the MRSA and MRMA models. While various parametric combinations of reliability factors α and γ are possible, larger and smaller values of α and γ are reported that allow the model behaviors to be examined (Table 2). Case I has very small reliability factors ($\alpha = 0.001$ and $\gamma = 0.10$). In contrast, Case IV assumes highly reliable conditions on both facilities ($\alpha = 0.99$ and $\gamma = 0.99$). Cases II and III represent the conditions of extreme differences between the two factors. The combination of reliability factors reflects a level of technology in telecommunication systems. For ex-

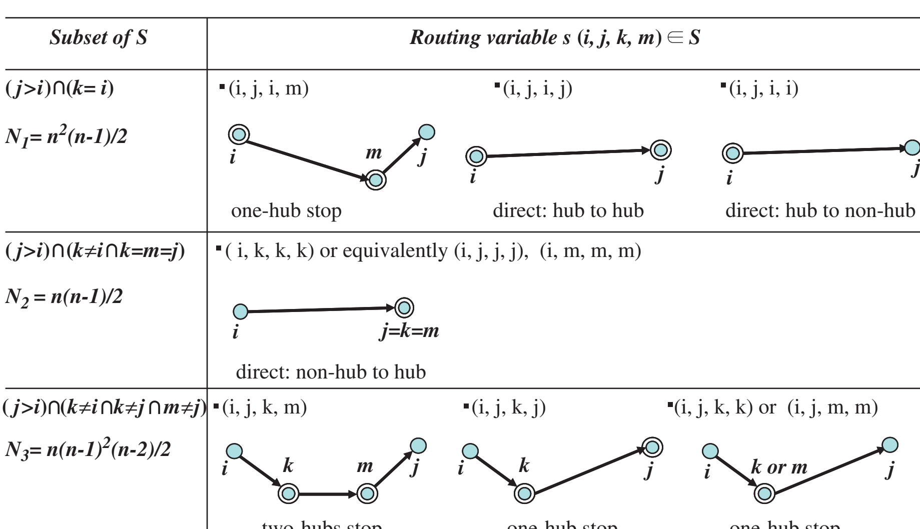


Figure 3. The enumerated routing variable set S. *Note:* Total number of variables X_{ijkm} is $n(n-1)(n^2-2n+3)/2$, which is the sum of each subset N_1, N_2, and N_3.

Table 1 Problem Sizes for Model Formulations

Models	Number of variables	Number of constraints
MRSA	$(n^4-3n^3+7n^2-3n)/2$	n^3+1
MRMA	$(n^4-3n^3+5n^2-n)/2$	$(2n^3-n^2-n+2)/2$
MDSA	$(n^4-3n^3+7n^2-3n)/2$	$(2n^3+n^2-n+2)/2$
MDMA	$(n^4-3n^3+5n^2-n)/2$	n^3-n+1
MRDI	$(n^4-3n^3+5n^2-n+2)/2$	n^3-n+2

MRSA, PHMR model with single assignment; MRMA, PHMR model with multiple assignments; MDSA, PHMD model with single assignment; MDMA, PHMD model with multiple assignments; MRDI, multiobjective reliable *p*-hub dispersion model.

ample, Case I represents traditional telecommunication systems with "copper-based technologies," which resulted in frequent disruptions of service due to less-reliable facilities and limited performance. In contrast, Case IV is associated with "fiber-optic technologies" in current IP-based telecommunication systems, where highly reliable network facilities are established (Grötschel, Monma, and Stoer 1995; Pastor-Satorras and Vespignani 2004). All models were solved using CPLEX 8.1 on a Dual Intel Pentium III Xeon 733 MHz with 1 GB RAM with the Windows NT operating system. The models were solved to optimality, and the solution times ranged from several seconds for most problems to a few minutes for some MRDI problems.

Model behaviors of the MRSA and MRMA

Fig. 4 presents the results of optimal configurations of hub networks for $p=4$ using W-matrix Set I. As expected, an increase in the objective function values is found for both models as one or both reliability factors increase. The objective function value for the MRMA is always greater than that for the MRSA, although the difference in network reliability (i.e., the objective function value) between the two models decreases as both reliability factors become larger. Notice that the MRMA has more linkages than the MRSA due to its routing flexibility. With respect to network cost, this property indicates that the MRMA would be a *less* cost-effective design approach than the MRSA if budgetary resources to construct the linkages are considered in the model.[4]

Table 2 Testing Scheme

Reliability factor γ	Reliability factor α	
	Small α ($\alpha=0.001$) (low transmission ability of interhub links)	Large α ($\alpha=0.99$) (high transmission ability of interhub links)
Small γ ($\gamma=0.10$) Low intrahub reliability	Case I	Case III
Large γ ($\gamma=0.99$) High intrahub reliability	Case II	Case IV

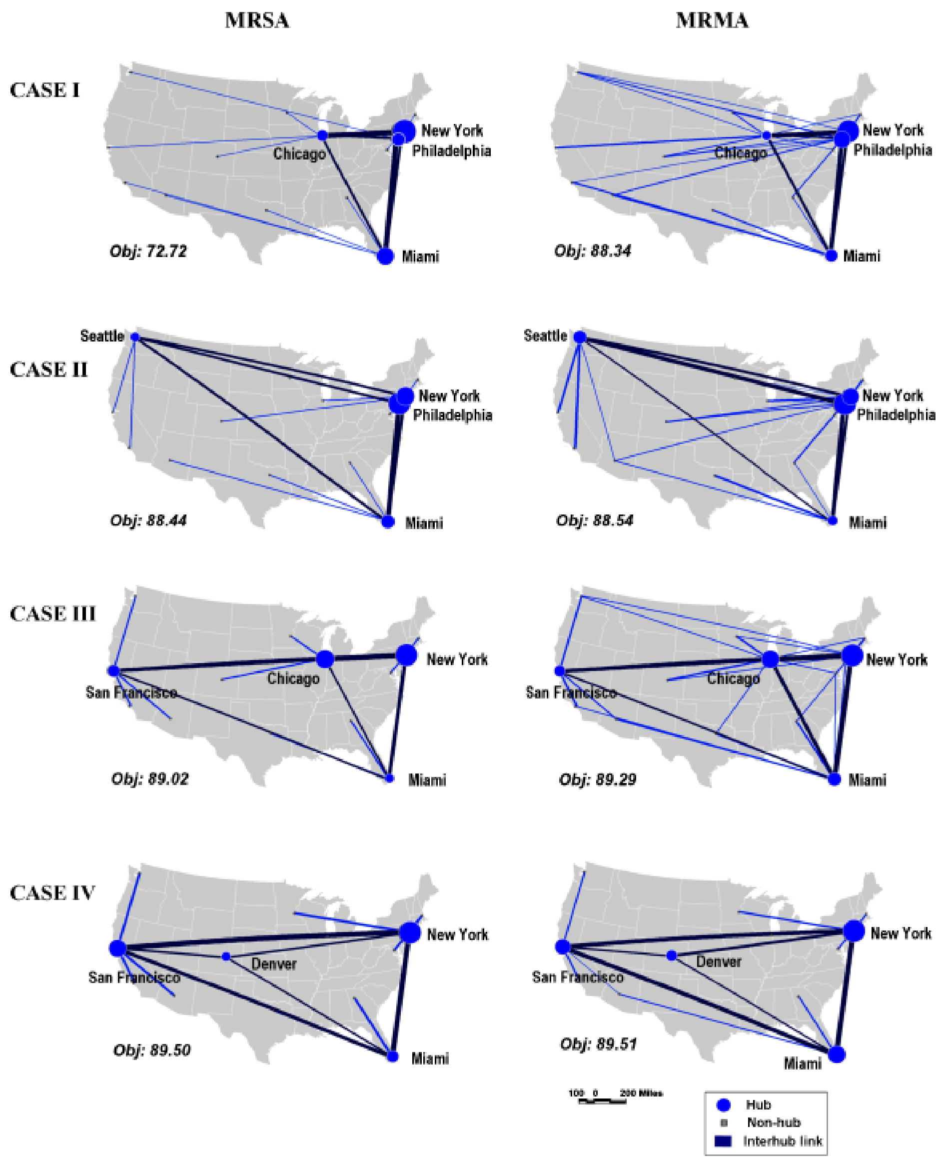


Figure 4. Model behaviors: MRSA (left) and MRMA (right) for $p = 4$ using Set I. *Note:* The size of a hub is proportional to the amount of the flows.

As implicitly discussed by O'Kelly (1986) and O'Kelly et al. (1996), the spatial pattern of hubs is related to the distance of routes and the level of benefit imposed on a facility (e.g., interhub link discounts in air transportation). In this context, Fig. 4 demonstrates that the distinct change of hub locations and allocations relies on the different levels of the reliability factors. In Case I, hubs are located in close proximity but become dispersed as one of the reliability factors improves (Case II or III).

As shown in Case IV, if both larger reliability factors are given, the model output locates hubs such that they are widely spaced with a regular separation (i.e., uniform spacing).

The model behaviors in Fig. 4 can be explained by the types of routings used in each phase. Table 3 shows how routing choices vary as the reliability factors change in accordance with the results in Fig. 4. As mentioned previously, if no reliability factors are considered in a model, each interacting flow between i and j tends to utilize the shortest route via hub(s) because the coefficient R_{ijkm} declines as a function of the length (distance) of a route. However, more weight on a reliability factor produces a tendency to use the route whose R_{ijkm} is enhanced by that weight. A large α on interhub links increases the chance to utilize two-hub stop routes, and the larger number of one-hub stop (R_{ijkk}) or direct (R_{ikkk}) routes are produced as the weight γ increases. As shown in the routing choices of Cases I and III, the use of two-hub stop routes is encouraged more if an extremely small γ value is applied in a routing scheme because the route choice simply tends to avoid the routings in which the small γ is involved. However, using one-hub stop or direct routes becomes a better routing choice than a two-hub stop route when incentive γ is larger. This tendency of model behaviors is more clearly observed in the MRMA because of more flexible routing choices (see the result of Cases II and IV).

The behavior of hub selection and facility activities is also influenced by the different size of interacting flows for OD pairs. Table 4 summarizes the optimal hub locations as well as facility activity levels of the MRSA and the MRMA for $p = 4$ and 5 using Set II for the testing scheme and three additional phases, which apply moderate levels of reliability factors (i.e., Mod I, II, and III with α and $\gamma = 0.5$, 0.7, and 0.9, respectively). The hub locations with Set II are less responsive to the change of reliability factors compared with the response of hub locations with

Table 3 Routing Choices for $p = 4$ Using Set I

Model	Scheme	Two-hub stop	One-hub stop	Direct route*
MRSA	Case I	60	25	6
	Case II	55	30	6
	Case III	62	23	6
	Case IV	31	44	16
MRMA	Case I	91	0	0
	Case II	66	20	5
	Case III	91	0	0
	Case IV	31	45	15

*"Direct route" include the "nonhub to hub" and "hub-to-hub" routes, which are expressed as X_{ijjj}, X_{ijii}, and X_{ijij} in Fig. 3. Notice that the flow in the category of direct routes looks similar in that the route delivers traffic directly without any intermediate hub, but the origin and destination are different in notation. For instance, the destination of the route X_{ijjj} is a hub while the origin is a hub in the route X_{ijii}.
MRSA, PHMR model with single assignment; MRMA, PHMR model with multiple assignments.

Table 4 Hub Locations and Facility Activity Levels for $p = 4$ and 5 Using Set II

	Objective value	Hubs	The largest interhub flow	The largest intrahub flow
MRSA				
$p = 4$				
Case I	18,912.15	PHX MIA MIN SEA	PHX–MIA	MIA
Case II	28,481.02	CHI PHL MIA SEA	PHL–SEA	PHL
Case III	28,730.20	NY PHL MIA SEA	NY–PHL	NY
Case IV	28,871.42	PHX DEN PHL SEA	PHL–PHX	PHL
Mod I*	23,608.05	PHX MIA MIN SEA	MIA–SEA	MIA
Mod II	25,894.19	PHX MIA MIN SEA	MIA–MIN	MIN
Mod III	27,962.03	PHX PHL MIN SEA	PHL–PHX	PHL
$p = 5$				
Case I	18,484.27	PHX DEN MIA MIN SEA	MIN–SEA	MIN
Case II	28,489.98	PHX CHI PHL MIA SEA	PHX–PHL	PHL
Case III	28,782.25	PHX MIA NY PHL SEA	NY–PHL	NY
Case IV	28,901.97	PHX DEN PHL MIA SEA	PHL–PHX	PHL
Mod I	23,001.83	PHX DEN MIA MIN SEA	MIN–SEA	SEA
Mod II	25,425.06	PHX DEN MIA MIN SEA	PHX–MIA	MIA
Mod III	27,815.28	PHX DEN MIA PHL SEA	PHX–PHL	PHL
MRMA				
$p = 4$				
Case I	28,453.43	NY CHI PHL SEA	NY–PHL	PHL
Case II	28,537.44	NY PHL MIA SEA	NY–PHL	PHL
Case III	28,815.71	PHX PHL MIA SEA	PHL–MIA	PHL
Case IV	28,873.56	NY PHL MIA SEA	NY–PHL	SEA
Mod I	28,628.48	CHI MIA PHL SEA	SEA–PHL	PHL
Mod II	28,703.25	CHI MIA PHL SEA	SEA–PHL	PHL
Mod III	28,778.32	CHI MIA PHL SEA	SEA–PHL	PHL
$p = 5$				
Case I	28,510.98	CHI MIA NY PHL SEA	NY–PHL	PHL
Case II	28,560.92	PHX MIA NY PHL SEA	NY–PHL	PHL
Case III	28,863.55	PHX CHI MIA PHL SEA	PHL–MIA	PHL
Case IV	28,905.54	DEN MIA NY PHL SEA	NY–PHL	SEA
Mod I	28,675.81	CHI MIA NY PHL SEA	NY–PHL	PHL
Mod II	28,745.85	PHX CHI MIA PHL SEA	PHL–MIA	PHL
Mod III	28,826.04	PHX CHI MIA PHL SEA	PHL–MIA	PHL

*Mods I, II, and III apply the moderate level of reliability factors in the model (both α and $\gamma = 0.5$, 0.7, and 0.9, respectively).

CHI, Chicago; DEN, Denver; MIA, Miami; MIN, Minneapolis; NY, New York; PHL, Philadelphia; PHX, Phoenix; SEA, Seattle; MRSA, PHMR model with single assignment; MRMA, PHMR model with multiple assignments.

identical flows (Set I). The impact of different sizes of flows is more clearly observed in the MRMA. As shown in the testing scheme results in Table 4, the MRMA open hubs at city nodes such as New York and Philadelphia frequently have larger interactions with other nodes, whereas a different set of hub locations is found in the MRSA models with distinct changes of reliability factors. As indicated in columns 4 and 5, if those hubs are placed close to each other, then the largest concentration of interactions takes place at their interhub links and hubs, producing large variances in flow activity levels to other facilities. The peripheral hub (e.g., Seattle) can be selected as a hub with a high concentration of flows if the number of non-hub nodes to be served increases. The different behavior between the MRSA and the MRMA is due to the allocation scheme. Like other multiple-assignment hub location models, the MRMA tends to change allocations of non-hub nodes to hubs to find optimal routes between *i* and *j*, rather than to move the location of hubs (O'Kelly et al. 1996).

Impact of hub dispersion

The benefits from operating hub networks could be compromised by increased congestion levels for particular hub and interhub facilities (O'Kelly 1986). The basic rationale of hub dispersion is to redistribute flows by separating facilities appropriately *if* an excessively amassed flow traveling on particular interhub link(s) or hub(s) is identified in the PHMR models.

Fig. 5 demonstrates how inter- and intrahub activity levels are changed when clustered hubs are dispersed geographically, based on the results of a MRMA model. These results are obtained using Set II for $p = 3$, and a moderate level of reliability factors (both α and $\gamma = 0.7$) is applied. The clustered hub network (Fig. 5a) is the optimal solution of the MRMA model, and the dispersed network (Fig. 5b) is made by relocating one of the hubs (Philadelphia) to Dallas (Fig. 5a). To compare the flow activity levels, we devised two indicators, the *interhub* and the *intrahub dependency rates* (hereafter INTERD and INTRAD, respectively), which are defined as the ratio of the largest interhub flow or the largest intrahub flow to the total sum of inter- or intrahub flows, respectively. A larger value indicates that a higher concentration of flow exists in the particular inter- or intrahub. Even though the gap of objective function values between the two networks is small, the differing flow activity levels on the facilities are quite evident (Fig. 5). In terms of both indicators, a large concentration of flows is observed in particular hubs and their interhub links in a clustered hub network configuration (Fig. 5a). However, dispersing hubs from the clustered configuration (Fig. 5b) results in a decrease in both dependency rates, from 0.50 to 0.41 (INTERD) and from 0.47 to 0.38 (INTRAD), implying that an appropriate separation of hubs could help mitigate any excess concentration of flows occurring in particular hub facilities. This result is due to the non-hub node reallocations in response to the dispersed hub(s), resulting in a more equitable flow distribution on the network, although it might not be the optimal hub location in terms of network reliability.[5]

If excessive flow dependencies are identified in a network due to clustered hubs, the benefit from a *mandatory* dispersion of hubs is clearly observed. Fig. 6 shows the impact of hub dispersion based on the results of the MDSA and the

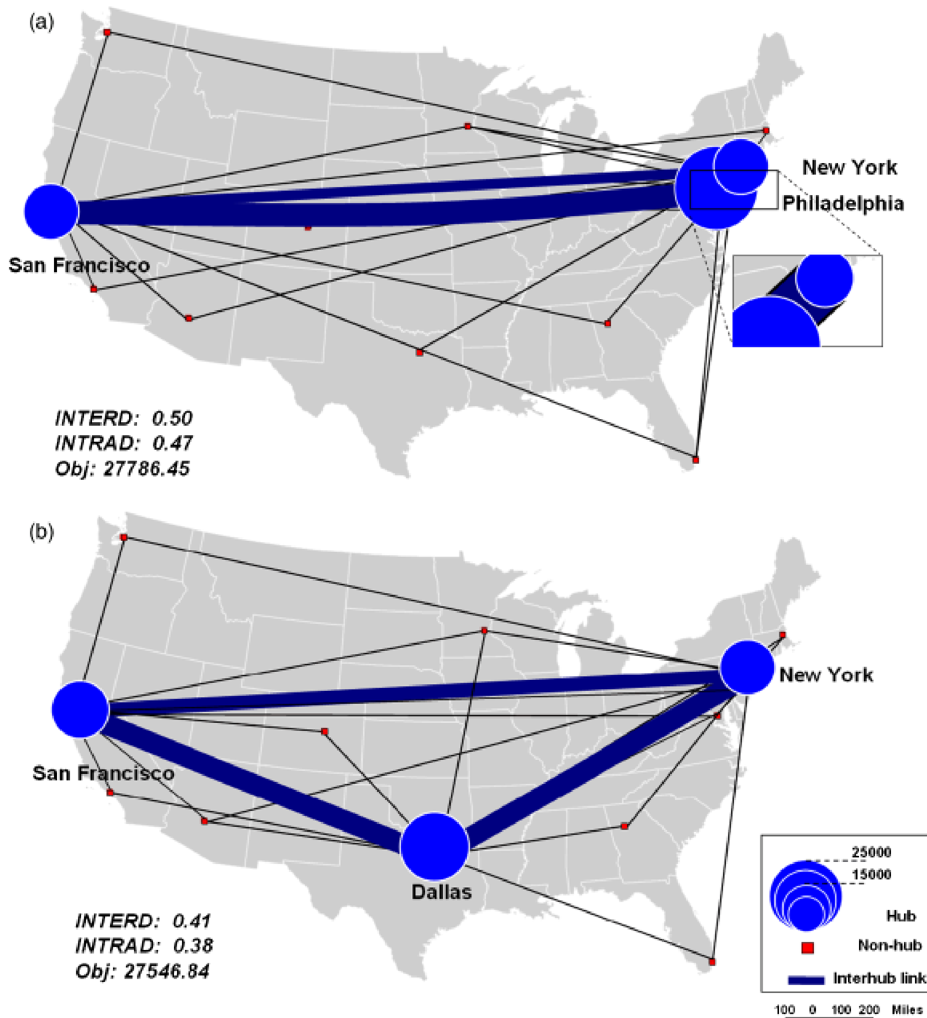


Figure 5. Comparison of facility activity level for a clustered (a) and dispersed (b) hub network at a moderate level of reliability factors ($p = 3$, both α and $\gamma = 0.7$).

MDMA ($p = 5$) for different mandatory dispersion levels with $D_{\text{man}} = 0$ and 1100 miles. The experiments are made for Cases I and IV, where *both* dependency rates INTERD and INTRAD are higher than other model results reported in Table 4. Our interest lies in the variation of the objective function values and dependency rates as the mandatory dispersion level increases. First, as the mandatory separation D_{man} increases, the objective function value of the model diminishes because the number of feasible hub sets is limited for a given D_{man}. Second, mitigation of the high-flow dependencies on particular inter- and intrahubs is observed with an increase in hub dispersion, resulting in a smaller variance among facilities. In

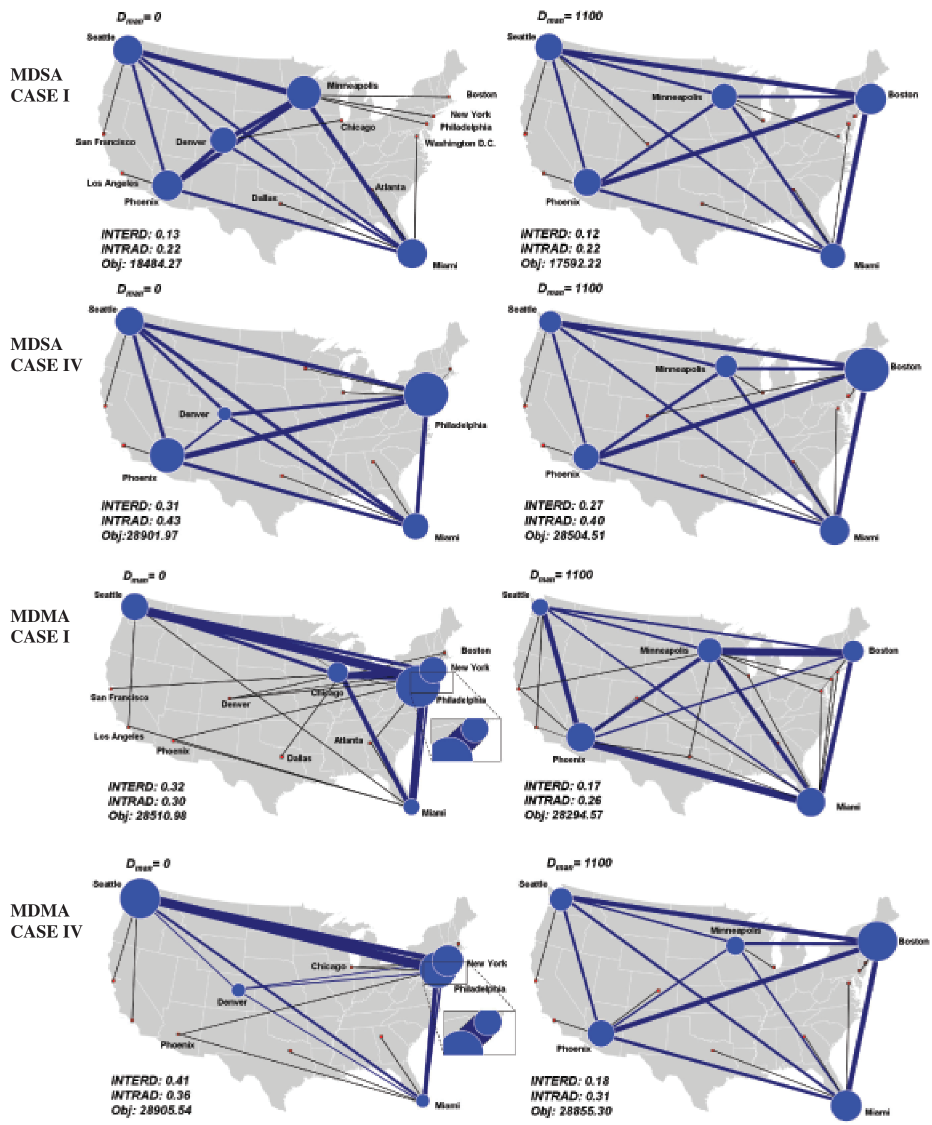


Figure 6. The solutions to the MDSA and MDMA for Cases I and IV ($p = 5$, $D_{man} = 0$, and 1100 miles). *Note:* For visual comparison, the size of hub and interhub links is proportional to the amount of flows on the facilities.

particular, the impact of hub dispersion is more clearly identified in the MDMA, where excessive flow dependencies are formed between the hubs (i.e., New York–Philadelphia) due to their close geographical proximity (Fig. 6).[6]

Table 5 presents the solution characteristics of the MRDI for the results of Cases I and IV ($p = 4$). As expected, network reliability declines as the level of optimal facility separation D increases, signifying that there is an intrinsic trade-off between

Table 5 Noninferior Solutions for the MRDI for Cases I and IV ($p = 4$, Set II)

Weight w	Objective value (Ω_1)	Separation (D) (miles)	Hubs	Iterations	Branches	Solution times (s)
Case I						
1.00	28,453.43	80	CHI NY PHL SEA	1122	8	16.80
0.95	28,447.88	664	PHL CHI MIA SEA	2481	19	28.17
0.90	28,409.67	1021	PHL PHX MIA SEA	5535	233	47.94
0.60	28,358.87	1114	PHX CHI MIA SEA	9614	327	83.17
0.01	28,246.98	1120	MIN BOS MIA PHX	10,339	544	90.42
0.00	28,145.88	1120	MIN BOS MIA SF	1074	35	16.73
Case IV						
1.00	28,873.56	80	NY PHL MIA SEA	687	0	1.88
0.95	28,873.21	1019	DEN PHL MIA SEA	702	1	2.26
0.60	28,830.68	1114	PHX BOS MIA SEA	2795	132	41.28
0.01	28,824.29	1120	MIN BOS MIA SEA	2379	95	31.22
0.00	28,803.31	1120	MIN BOS MIA SF	1074	35	17.05

BOS, Boston; CHI, Chicago; DEN, Denver; MIA, Miami; MIN, Minneapolis; NY, New York; PHL, Philadelphia; PHX, Phoenix; SEA, Seattle; SF, San Francisco.

the same directional problems. By incrementing weights, this multiobjective model identifies a couple of noninferior solutions for the particular weights. For example, given the weight $w = 1$, where only the importance of the reliability of a network is considered, the results are the same as those found for a very small separation level ($D = 80$ miles, New York–Philadelphia) in both Cases I and IV. As more weight is imposed on the geographical separation, the hubs are dispersed whenever the model reaches a certain threshold of a given weight. Interestingly, the same separation level ($D = 1120$ miles) but different optimal hub locations are identified as noninferior solutions at $w = 0.0$ and 0.01 (Table 5) because the MRDI is forced to identify the best hubs that can maximize network reliability among probable alternative optima once a very small weight is imposed on the objective function Ω_1. The MRDI would be helpful for decision makers to compromise between desired levels of network reliability and an appropriate separation of hubs.

Conclusions

The results from these models could give useful insights into telecommunication network design. First, the PHMR demonstrates how optimal hub locations can be determined under various reliability conditions on hubs and interhub links. Second, like the well-recognized important design aspect in telecommunication networks (Medhi 1999; Ball and Vakhutinsky 2001), the dispersion of hub facilities can be considered in hub network design that can protect the excessive concentration of interacting flows from particular hub facilities. This prescription could be helpful by achieving a more equitable flow distribution on network facilities not only to prevent networks from potential congestion and degradation that can arise from clustered hubs, but also, ultimately, to reduce damage when any malfunctions or attacks occur in highly active hubs or interhub links.

Given the focus here on models that provide accurate behavior using a small data set, further research needs to expand upon the general problem addressed. First, future models could take into account other constraints or objectives that are commonly addressed in hub network design, such as costs, the capacity of facilities, and the minimum threshold of flows on links. For example, considering the importance of network costs for constructing hubs and links, the RPHLPs can be extended to fixed cost hub location problems or to uncapacitated hub location problems where the number of hubs is determined implicitly as a part of optimization in a model. Second, here the RPHLPs are presented based on simple assumptions used in standard hub location models. However, the RPHLPs can be developed considering other characteristics of telecommunication networks where interhub links may not be established to connect all hubs or where different assignment schemes (i.e., direct routes between non-hub nodes) are required. In terms of reliability factors, the RPHLP can employ *flow-dependent* reliability factors whose reliability levels are determined based on the activity level at hub facilities (Campbell, Ernst, and Krishinamoorthy 2002). Another avenue for expanding the RPHLP entails exploring model behaviors by applying other network performance-related measures, such as *traffic latency rate*, in network design (Crovella and Krishnamurthy 2006; Forsgren and Prytz 2006). Model behavior with different reliability measures could provide valuable insight into developing other network design models. Finally, the development of efficient algorithms or heuristics that can be applied to real networks is essential, because hub location problems are known to be difficult to solve for substantially larger numbers of nodes and links.

Notes

1 It might be possible to apply the intrahub factor γ in computing the reliability of two-hub stop routes. However, in this article, γ is considered only if R_{ijkm} involves r_{kk} or r_{mm} according to notational order, because considering γ in computing two-hub routes might mitigate the effect of α. Technically, the reliability of two-hub stop routes in telecommunication networks is more influenced by the transferability of the interhub links where the transferability of inbound flows via hubs can be enhanced, whereas a one-hub stop only depends on the reliability of a hub (Pióro and Medhi 2004).

2 The 14 city nodes whose network performances are monitored by the SAVVIS network are Phoenix, Atlanta, Boston, Chicago, Dallas, Washington DC, Denver, Los Angeles, Miami, Minneapolis, New York, Philadelphia, Seattle, and San Francisco.

3 This article assumes that the empirical data rate and potential interactions represent theoretical reliabilities and the actual interacting flows. For readers interested in the theoretical quantification of reliability and estimation of OD flow in telecommunication networks, see Feldmann et al. (2001) and Zhang et al. (2005). The size of interactions between city nodes (W_{ij}) can be estimated differently, depending on which data resource is used to calculate the telecommunication potentials.

4 In theory, the MRSA always constructs a hub network with the fixed number of linkages ($n-p+p(p-1)/2$) for n nodes and p hubs, but the MRMA has at least the same number of linkages as the MRSA, and up to ($p(n-p)+p(p-1)/2$) linkages, which is the case when every non-hub node is allocated to all p hubs. Thus, the assumption of multiple assignments might not be appropriate for telecommunication network designs, depending on construction costs.

5 Although beyond the scope of this article, either imposing capacities on links or upgrading hubs with better transferability would be an effective prescription to respond to possible congestion in classical hub location models. However, to make these capacity constraints effective, the constraint fixing of the exact number of p hubs is generally dropped (Bryan 1998; Campbell, Ernst, and Krishinamoorthy 2002).

6 The dispersion of hubs does not necessarily indicate the dispersion of the arcs. For example, New York is frequently assigned to Boston if Boston is selected as a hub because of its close proximity on the network, and the separated hubs might use the same interhub or spoke links even when the hubs are dispersed. However, as shown in Fig. 6, the activity level of inter- and intrahub flows is highly influenced by the dispersion of hubs because each OD would change its optimal route in response to a change in hub locations.

References

Agrawal, G. P. (1997). *Fiber-Optic Communication Systems*. New York: Wiley.

AT&T. (2003). ''The Quality of Internet Service: AT&T's Global IP Network Performance Measurements.'' AT&T report. Available at http://ipnetwork.bgtmo.ip.att.net/pws/paper.pdf (accessed Oct. 5, 2006).

Atkinson, R. D., and P. D. Gottlieb. (2001). ''The Metropolitan New Economy Index: Benchmarking Economic Transformation in the Nation's Metropolitan Area.'' The Progressive Policy Institute (PPI) report. Available at http://www.neweconomyindex.org/metro/metro_3mb.pdf (accessed Aug. 5, 2006).

Ball, M. O., and A. Vakhutinsky. (2001). ''Fault-Tolerant Virtual Path Layout in ATM Networks.'' *INFORMS Journal on Computing* 13, 76–94.

BBC News. (2002a). ''Net Weathers WorldCom Fallout.'' *BBC News*, July 22, 2002.

BBC News. (2002b). ''Attack on the Wires.'' *BBC News*, August 5, 2002.

Bell, M. G. H., and Y. Iida. (1997). *Transportation Network Analysis*. New York: Wiley.

Boehm, S., and P. Baran. (1964). *On Distributed Communications II: Digital Simulation of Hot-Potato Routing in a Broadband Distributed Communications Network, Memorandum, RM-3103-PR*. Santa Monica, CA: The RAND Corporation.

Bryan, D. (1998). ''Extensions to the Hub Location Problem: Formulations and Numerical Examples.'' *Geographical Analysis* 30, 315–30.

Bryan, D., and M. E. O'Kelly. (1999). ''Hub and Spoke Networks in Air Transportation: An Analytical Review.'' *Journal of Regional Science* 39, 275–95.

Campbell, J. F. (1994a). ''A Survey of Network Hub Location.'' *Studies in Locational Analysis* 6, 31–49.

Campbell, J. F. (1994b). ''Integer Programming Formulations of Discrete Hub Location Problems.'' *European Journal of Operational Research* 72, 387–405.

Campbell, J. F., A. T. Ernst, and M. Krishinamoorthy. (2002). ''Hub Location Problems.'' In *Facility Location*, 373–407, edited by Z. Drezner and H. W. Hamacher. Berlin, Germany: Springer-Verlag.

Carello, G., F. Della Croce, M. Ghirardi, and R. Tadei. (2004). ''Solving the Hub Location Problem in Telecommunication Network Design: A Local Search Approach.'' *Networks* 44, 94–105.

Choi, B.-Y., S. Moon, Z. Zhang, K. Papagiannaki, and C. Diot. (2004). ''Analysis of Point-to-Point Packet Delay in an Operational Network.'' *Proceeding of IEEE INFOCOM*, 1797–807.

Ciabattone, L., A. Morton, G. Ramachandran, and A. T. Laboratories. (2003). ''Standardized Active Measurements on a Tier 1 IP Backbone.'' *IEEE Communications Magazine* 41(6), 90–97.

Cohon, J. L. (1978). *Multiobjective Programming and Planning*. New York: Academic Press.

Colbourn, C. J. (1987). *The Combinatorics of Network Reliability*. New York: Oxford University Press.

Colbourn, C. J. (1999). ''Reliability Issues in Telecommunications Network Planning.'' In *Telecommunications Network Planning*, 1–16, edited by B. Sanso and P. Soriano. Boston: Kluwer Academic Press.

Crovella, M., and B. Krishnamurthy. (2006). *Internet Measurement: Infrastructure, Traffic, and Applications*. Oxford, UK: Wiley.

Curtin, K. M., and R. L. Church. (2006). ''A Family of Location Models for Multiple-Type Discrete Dispersion.'' *Geographical Analysis* 38, 248–70.

Feldmann, A. A., C. Greenberg, N. Lund, J. Reingold, J. Rexford, and F. True. (2001). ''Deriving Traffic Demands for Operational IP Networks: Methodology and Experience.'' *IEEE/ACM Transactions on Networking* 9, 265–80.

Forsgren, A., and M. Prytz. (2006). ''Telecommunications Network Design.'' In *Handbook of Optimization in Telecommunications*, 269–90, edited by M. G. C. Resende and P. M. Pardalos. New York: Springer.

Gavish, B., and I. Neuman. (1992). ''Routing in a Network with Unreliable Components.'' *IEEE Transactions on Communications* 40, 1248–58.

Gavish, B., P. Trudeau, M. Dror, M. Gendreau, and L. Mason. (1989). ''Fiberoptic Network Design under Reliability Constraints.'' *IEEE Journal on Selected Areas in Communications* 7, 1181–87.

Grötschel, M., C. L. Monma, and M. Stoer. (1995). ''Design of Survivable Network.'' In *Handbooks in Operations Research and Management Science*, Vol. 7: 617–72, edited by M. O. Ball. Amsterdam: Elsevier.

Grubesic, T. H., M. E. O'Kelly, and A. T. Murray. (2003). ''A Geographic Perspective on Commercial Internet Survivability.'' *Telematics and Informatics* 20, 51–69.

Heiskanen, V. (2006). ''Internet's Vulnerability Underscored by Earthquakes in Taiwan.'' Available at http://www.bloomberg.com/apps/news?pid=20601087&sid=amgFDQ.Pcho4&refer=home (accessed Feb. 11, 2006).

Herder, P. M., and W. A. H. Thissen. (2003). ''Critical Infrastructures: A New and Challenging Research Field.'' In *Critical Infrastructures: State of the Art in Research and Application*, 1–8, edited by W. A. T. Thissen and P. M. Herder. Boston: Kluwer Academic Publishers.

Kansal, M. L., A. Kumar, and P. B. Sharma. (1995). ''Reliability Analysis of Water Distribution Systems Under Uncertainty.'' *Reliability Engineering and System Safety* 50, 51–59.

Klincewicz, J. G. (1998). ''Hub Location in Backbone/Tributary Network Design: A Review.'' *Location Science* 6, 307–35.

Klincewicz, J. G. (2006). ''Optimization Issues in Quality of Service.'' In *Handbook of Optimization in Telecommunications*, 435–58, edited by M. G. C. Resende and P. M. Pardalos. New York: Springer.

Konak, A., and A. E. Smith. (2006). ''Network Reliability Optimization.'' In *Handbook of Optimization in Telecommunications*, 735–60, edited by M. G. C. Resende and P. M. Pardalos. New York: Springer.

Kuby, M. (1987). ''Programming Models for Facility Dispersion: The p-Dispersion and Maxisum Dispersion Problems.'' *Geographical Analysis* 19, 315–29.

Kuo, W., and R. Prasad. (2000). ''An Annotated Overview of System-Reliability Optimization.'' *IEEE Transactions on Reliability* 49, 176–87.

Lewis, E. E. (1996). *Introduction to Reliability Engineering*, 2nd ed. New York: Wiley.

McGrath, D. (2003). ''Measuring the 4:11 Effect: The Power Failure and the Internet.'' *IEEE Security and Privacy Magazine* 1(5), 16–18.

Medhi, D. (1999). ''Network Reliability and Fault Tolerance.'' In *Wiley Encyclopedia of Electrical and Electronics Engineering*, 213–18, edited by J. Webster. New York: Wiley.

Mohamed, A. A., L. M. Leemis, and A. Ravindran. (1992). ''Optimization Techniques for System Reliability: A Review.'' *Reliability Engineering and System Safety* 35, 137–46.

Murnion, S., and R. G. Healey. (1998). ''Modeling Distance Decay Effects in Web Server Information Flows.'' *Geographical Analysis* 30, 285–303.

Murray, A. T., and T. H. Grubesic (eds.). (2007). *Critical Infrastructure: Reliability and Vulnerability*. Berlin, Germany: Springer-Verlag.

National Computerization Agency of Korea. (2004). *Korea Internet White Paper*. Seoul, Korea: Ministry of Information and Communication.

National Security Telecommunications Advisory Committee. (2003). *Internet Peering Security/Vulnerabilities Task Force Report*. Washington, DC: The President's National Security Telecommunications Advisory Committee.

O'Kelly, M. E. (1986). ''Activity Levels at Hub Facilities in Interacting Networks.'' *Geographical Analysis* 18, 343–56.

O'Kelly, M. E., and H. Kim. (2007). ''Survivability of Commercial Backbones with Peering: A Case Study of Korean Networks.'' In *Critical Infrastructure: Reliability and Vulnerability*, 107–28, edited by A. T. Murray and T. H. , Grubesic. Berlin, Germany: Springer-Verlag.

O'Kelly, M. E., D. Bryan, D. Skorin-Kapov, and J. Skorin-Kapov. (1996). ''Hub Network Design with Single and Multiple Allocation: A Computational Study.'' *Location Science* 4(3), 125–38.

O'Kelly, M. E., H. Kim, and C. J. Kim. (2006). ''Internet Reliability with Realistic Peering.'' *Environment and Planning B: Planning and Design* 33, 325–43.

Pastor-Satorras, R., and A. Vespignani. (2004). *Evolution and Structure of the Internet: A Statistical Physics Approach*. New York: Cambridge University Press.

Pióro, M., and D. Medhi. (2004). *Routing, Flow, and Capacity Design in Communication and Computer Networks*. San Francisco: Morgan Kaufmann Publishers.

SAVVIS Network Performance Statistics. Available at http://ipsla.savvis.net/ (accessed Dec. 1, 2005).

Shao, F., and L. Zhao. (1997). ''Optimal Design Improving Communication Network Reliability.'' *Microelectronics and Reliability* 37, 591–95.

Shier, D. R. (1991). *Network Reliability and Algebraic Structures*. New York: Oxford University Press.

Skorin-Kapov, D., J. Skorin-Kapov, and V. Boljunčić. (2006). ''Location Problems in Telecommunications.'' In *Handbook of Optimization in Telecommunications*, 517–44, edited by M. G. C. Resende and P. M. Pardalos. New York: Springer.

Snyder, L. V., and M. S. Daskin. (2007). ''Models for Reliable Supply Chain Network Design.'' In *Critical Infrastructure: Reliability and Vulnerability*, 257–90, edited by A. T. Murray and T. H. Grubesic. Berlin, Germany: Springer-Verlag.

Soni, S., R. Gupta, and H. Pirkul. (1999). ''Survivable Network Design: The State of the Art.'' *Information Systems Frontiers* 1, 303–15.

Stavroulakis, P. (2003). *Reliability, Survivability and Quality of Large Scale Telecommunication Systems*. New York: Wiley.

Wagner, M. (2002). ''Increased Internet Centralization Threatens Reliability.'' Available at http://www.InformationWeek.com (accessed Aug. 25, 2006).

Wakabayashi, H., and Y. Iida. (1992). ''Upper and Lower Bounds of Terminal Reliability of Road Networks: An Efficient Method with Boolean Algebra.'' *Journal of Natural Disaster Science* 14, 29–44.

Zhang, Y., M. Roughan, C. Lund, and D. L. Donoho. (2005). ''Estimating Point-to-Point and Point-to-Multipoint Traffic Matrices: An Information-Theoretic Approach.'' *IEEE-ACM Transactions on Networking* 13, 947–60.

Zheng, H., E. K. Lua, M. Pias, and T. G. Griffin. (2005). ''Internet Routing Policies and Round Trip Times.'' Paper presented at PAM 2005, March 31–April 1, Boston.

Zook, M. A. (2000). ''Internet Metrics: Using Hosts and Domain Counts to Map the Internet Globally.'' *Telecommunications Policy* 24, 613–20.