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A MULTI-COMMODITY, MULTI-PLANT, CAPACITATED FACILITY LOCATION PROBLEM: FORMULATION AND EFFICIENT HEURISTIC SOLUTION

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Scope and Purpose—The PLANWAR model is a new formulation to the multi-commodity, multi-plant, capacitated facility location problem that seeks to locate a number of production plants and distribution centers so that total operating costs for the distribution network are minimized. An important component of a firm's physical distribution strategy is its network strategy, which defines the physical flow of commodities (products) from production plants to warehouses and from warehouses to customer zones. A frequent objective when designing a network strategy is to determine the least total cost system design such that all customer demand is satisfied subject to limitations imposed by production capacities for plants and storage capacities for warehouses. We present a mixed integer programming model, PLANWAR, and provide an efficient heuristic solution procedure for this supply chain management problem.

Abstract—Distribution system design problems commonly occurs in the following form: A number of production plants supply warehouses with multiple products which in turn distribute these products to customer outlets based on their specified demand quantities of the different products. It is required to select the optimum set of plants and warehouses from a potential set and plan production capacities, warehouse capacities and quantities shipped so that the total operating costs of the distribution network are minimized. This paper is a computational study to investigate the value of coordinating production and distribution planning. We present a mixed integer programming formulation for the capacitated plant and warehouse supply chain management problem and propose an efficient heuristic based on Lagrangian relaxation of the problem.

1. INTRODUCTION AND BACKGROUND

The challenge of where to best site warehouses and manufacturing plants has inspired a rich and colorful body of literature and research that spans well over two decades. It is becoming increasingly clear that companies like Procter and Gamble and Wal-Mart have teamed up to efficiently handle the flow of products and are exploring closer coordination along the production and distribution chain. Distribution system design problems commonly occur in the following form: A number of production plants supply the warehouses with multiple products which in turn supply customers (or retail outlets) with specified demand quantities of the different products. Companies now will need to make the required organizational changes that will facilitate coordination of these operational functions and develop an ability to make more complex decisions within this structure. In this paper we will use the terms warehouses and distribution centers interchangeably.

There are several problems in Operations management and Operations research that require decisions to be made in two stages (e.g., Scheduling of machines, Concentrator location problem, Bin packing problem). In the first stage a choice of the subset of concentrators (or bins, or machines) is made. In the second stage the terminals (or items, or parts) are assigned to the chosen subset of concentrators (or bins, or machines). The Capacitated Plant Location Problem (CPLP) also shares the same common features in terms of the decision making in two stages. In the first stage a choice is made of the subset of plants to be opened and in the second stage the assignment of the customers to these plants is made.

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In the last several years, many papers were proposed on solving the CPLP by using both approximate and exact solutions [1–6]. Sridharan [7] provided a review of the various solution methods for the capacitated plant location problem and also covered various heuristics and exact procedures that have appeared in the literature. When we add the restriction that each customer zone is served from a single plant we get the Capacitated Plant Location Problem with Single Source Constraints (CPLPSS). Some papers that address single-source models include [8–10]. A basic model was formulated by Balinski [11] and Manne [12] for the simple plant location problem. This problem consists of locating plants and warehouses among a set of given locations in order to satisfy a given demand at minimum cost. Efroymson and Ray [13] and Spielberg [14] are some examples of work that have provided some heuristics and exact algorithms to the problem. Elson [15] proposed a multicommodity version of the plant-warehouse facility logistics problem, concentrating on a single echelon of transshipment stocking points.

Considerable attention has been devoted to discrete models for the location of plants and warehouses. Researchers have worked on developing both heuristic solution methods and exact algorithms to solve CPLP. While exact algorithms can solve medium sized problems within reasonable computer effort, heuristics are required to solve realistic-sized problems. Geoffrion and Graves [16] solved a multicommodity location problem that optimized flow of the product over the plant-to-customer path via a warehouse. The model dealt with facility location and customer assignment and employed Benders' partitioning procedure to solve the distribution design problem. However, the model does not employ any variable to include selection among alternative plant sites and also does not incorporate any fixed cost for annual possession and operating costs for plants.

Most of the solution algorithms used in the literature use branch and bound techniques and Benders decomposition-based algorithms. Although these techniques are well known, they do have some inherent disadvantages [17]. Benders decomposition methods exploit only the primal structure of the problem. However, it has been observed that many mixed integer programming problems have both easy-to-solve primal and dual subproblems. Techniques like branch-and-bound expend considerable amount of time and effort to find an optimal solution to realistic size problems.

In this paper, we consider the multi commodity version of the plant and warehouse logistics problem. This particular model has not been solved before. We are not aware of any existing solution procedure that could be directly adapted to the formulation of this model and produce a feasible solution in a realistic amount of computational time. In this capacitated plant and warehouse location model, customers typically demand multiple units of different products that are distributed to customer outlets from open warehouses while warehouses receive these products from several manufacturing plants. The objective function in our model minimizes the sum of the fixed cost of establishing and operating the plants and the warehouses plus the variable cost of transporting units of products from the plants to the warehouses and distributing the products from the warehouses to the customer in order to satisfy the multiple demands of the customers. Both plants as well as warehouses are capacitated.

In the next section, a formulation for the PLANWAR (PLANt and WARehouse) location problem is presented. We then provide a Lagrangian relaxation of the model and also the necessary steps required to find a lower bound at every iteration of the problem. A heuristic solution procedure for solving the problem is provided. The gap between the heuristic solution procedure and the Lagrangian based lower bound provides an estimate of the solution quality. Computational results are reported and discussed. In the last section, conclusions and summary are provided.

2. MODEL FORMULATION

PLANWAR aims to locate a given number of plants and warehouses subject to warehouse and plant capacity restrictions. We assume that the customer zone locations and their demand for multiple products are known in advance. The potential warehouse and plant locations as well as their maximum capacities are also known.

Consider a set of customer zones, $i \varepsilon I$, and a set of potential warehouse sites, $j \varepsilon J$, where the throughput limit of each warehouse site is W_j. Let the set of potential plant locations, $k \varepsilon K$, have a capacity of D_k. Each product $l \varepsilon L$ is related to the capacity limit of the plants by the quantity q_l. Each warehouse site incurs a fixed cost g_j to open and operate while each plant site also incurs a fixed cost f_k to open and operate. There is a demand for each product that is placed by the set of customer zones (a_{il}) and is satisfied by open warehouse sites. There is a variable cost, C_{ijl}, to distribute a unit of product l from an open warehouse j to a customer zone i, while there is also a variable cost, T_{jkl}, to transport a unit of product l to an open warehouse j from an open plant k. Each product is related to the throughput limit of the warehouse by the quantity s_l, while there is an upper limit on the number of warehouses that can be opened, W, and an upper limit on the number of plants that can be opened, P.

Let X_{ijl} be a decision variable that indicates the total number of units of product l distributed to customer zone i from open warehouse j and Y_{jkl} be the total number of units of product l that is shipped to open warehouse j from open plant k. Let Z_j indicate whether a warehouse is open ($Z_j = 1$) or not ($Z_j = 0$) and P_k indicate whether a plant is open ($P_k = 1$) or not ($P_k = 0$). The problem is, then, to choose some subset of plants and warehouses to open; for each open plant and warehouse, a decision must be made on the total number of units of products that need to be transported from open plants to warehouses and the total number of units of products that need to be distributed from open warehouses based on demand requirements imposed by customer zones.

3. MODEL PLANWAR

Problem LR

$$\text{Min } Z = \sum_i \sum_j \sum_l C_{ijl} X_{ijl} + \sum_j \sum_k \sum_l T_{jkl} Y_{jkl} + \sum_k f_k P_k + \sum_j g_j Z_j$$

subject to

$$\sum_j X_{ijl} = a_{il} \quad \text{for all } i \text{ and } l \tag{1}$$

$$\sum_i \sum_l s_l X_{ijl} \leq Z_j W_j \quad \text{for all } j \tag{2}$$

$$\sum_j Z_j \leq W \tag{3}$$

$$\sum_i X_{ijl} \leq \sum_k Y_{jkl} \quad \text{for all } j \text{ and } l \tag{4}$$

$$\sum_j \sum_l q_l Y_{jkl} \leq D_k P_k \quad \text{for all } k \tag{5}$$

$$\sum_k P_k \leq P \tag{6}$$

$$P_k,\ Z_j = \{0,\ 1\} \quad \text{for all } j \text{ and } k. \tag{7}$$

$$X_{ijl},\ Y_{jkl} \geq 0 \quad \text{for all } i, j, k \text{ and } l \tag{8}$$

The PLANWAR model minimizes the sum of: the costs to distribute products from open warehouses to customers; the costs for transporting units of different commodities from plants to warehouses; the fixed cost associated with locating and operating manufacturing plants and warehouses. Constraint set equation (1) ensures that all the demand of customers are satisfied by open warehouses. Constraint set equation (2) ensures that the customer demands that are distributed from open warehouses do not exceed warehouse throughput limit. Constraint equation (3) ensures that we locate at most W warehouses. Constraint set equation (4) ensures that all the demand of customer i for product l is balanced by the total units of product l available at warehouse j that has been transported from open plants. Constraint equation (5) represents the capacity restriction of plant k in terms of the amount of demand it can handle. Constraint set equation (6) locates at most P plants. Constraint set equation (7) enforces the binary nature of the decision variables and constraint equation (8) enforces the non-negativity restriction on the decision variables used in the PLANWAR model.

We have provided a new formulation to solve the capacitated version of the plant and warehouse location problem. The capacitated p-median problem is a special case of the problem considered in this paper and it is known that the capacitated p-median problem is NP-Complete [18]. Hence the decision analog of PLANWAR model is NP-Complete. Commercial general purpose mathematical programming codes can solve small instances of this problem; however, computational times with such codes become prohibitive for reasonably sized problems. Hence, in the next section we will present a Lagrangian relaxation of the model and develop a heuristic solution procedure which uses the information provided by this relaxation to generate good feasible solutions.

4. SOLUTION PROCEDURE

The Lagrangian relaxation scheme has been used successfully in various operations management problems [19, 20]. Many of these problems can be viewed as easy problems complicated by a set of side constraints that makes the problem difficult. The Lagrangian relaxation schema is based on just this observation. The Lagrangian relaxation method tries to simplify this problem by dualizing the complicated constraints and introducing them into the objective function with a penalty function. In the PLANWAR model, constraints Equations (1) and (4) in the model make the problem difficult to solve. If these constraints were absent, then the problem breaks down into subproblems that are relatively simple to solve. For this reason, we use the Lagrangian method to relax these constraints. The reader is referred to a book by Reeves [21] for detailed discussion on the Lagrangian relaxation methodology.

4.1. A lagrangian relaxation of the model

The following Lagrangian relaxation of the problem is obtained by relaxing constraint sets Equations (1) and (4):

$$\text{Min } Z_{\text{LR}} = \sum_i \sum_j \sum_l C_{ijl} X_{ijl} + \sum_j \sum_k \sum_l T_{jkl} Y_{jkl} + \sum_k f_k P_k + \sum_j g_j Z_j + \sum_i \sum_l \gamma_{il} \left(\sum_j X_{ijl} - a_{il} \right) + \sum_j \sum_l \beta_{jl} \left(\sum_i X_{ijl} - \sum_k Y_{jkl} \right) \tag{9}$$

subject to

Constraints Equations (2), (3) and (5)–(8).

Problem LR can be further decomposed into two subproblems LR^1 and LR^2:

LR^1

$$\text{Min } Z_{\text{LR}^1} = \sum_i \sum_j \sum_l (C_{ijl} + \beta_{jl} + \gamma_{il}) X_{ijl} + \sum_j g_j Z_j - \sum_i \sum_l a_{il} \gamma_{il} \tag{10}$$

subject to

$$\sum_i \sum_l s_l X_{ijl} \le Z_j W_j \quad \text{for all } j \tag{11}$$

$$\sum_j Z_j \le W \tag{12}$$

$$Z_j = \{0,\ 1\} \quad \text{for all } j; \quad X_{ijl} \geq 0 \quad \text{for all } i, j \text{ and } l \tag{13}$$

LR^2

$$\text{Min } Z_{\mathrm{LR}^2} = \sum_j \sum_k \sum_l (T_{jkl} - \beta_{jl}) Y_{jkl} + \sum_k f_k P_k \tag{14}$$

subject to

$$\sum_j \sum_l s_l Y_{jkl} \leq D_k P_k \quad \text{for all } k \tag{15}$$

$$\sum_k P_k \leq P \tag{16}$$

$$P_k = \{0,\ 1\} \quad \text{for all } k; \quad Y_{jkl} \geq 0 \quad \text{for all } j,\ k \text{ and } l \tag{17}$$

Leaving constraint equation (12) aside, we can decompose problem LR^1 into J subproblems, one for each warehouse j. For each of the J subproblems, Z_j is either equal to 0 or 1. If Z_j is equal to 0, then X_{ijl} is equal to 0 for all i, j and l because of constraint equation (11). If Z_j is equal to 1, then the subproblem becomes a continuous knapsack problem. There exist efficient solution procedures to solve continuous knapsack problems. Constraint equation (12) can then be enforced by choosing the W "best" solutions obtained after solving the J subproblems.

Leaving constraint equation (16) aside, subproblem LR^2 can be further decomposed into k subproblems, one for each plant site k. For each of the K subproblems, P_k is either equal to 0 or 1. If P_k is equal to 0, then Y_{jkl} is equal to 0 for all j, k and l because of constraint equation (15). However, if P_k is equal to 1, then subproblem LR^2 becomes a continuous knapsack problem. Constraint equation (16) can then be enforced by choosing the P "best" (minimum cost) solutions obtained after solving the K subproblems.

The solution to the Lagrangian subproblems, LR^1 and LR^2, provides a lower bound to the optimal solution for model PLANWAR Fig. 1.

In this paper, we use the subgradient method to derive the bounds for Lagrangian problem. In the next section, we describe a heuristic solution procedure that is used in conjunction with the Lagrangian subproblem to generate a feasible solution to the model.

5. A HEURISTIC SOLUTION PROCEDURE

We describe below a heuristic procedure to solve model PLANWAR that utilizes the solution to the Lagrangian relaxation discussed in the previous section. This heuristic solution procedure attempts to generate a feasible solution to model PLANWAR at every iteration of the subgradient optimization algorithm. The best feasible solution is retained at the termination of the algorithm. In our heuristic solution procedure, we consider the set of open warehouses that are obtained by solving subproblem LR^1 and the set of open plants that are obtained by solving subproblem LR^2. To obtain a feasible solution to model PLANWAR, we need to complete the assignment of customers to open warehouses based on their demand for products and assign these open warehouses to open plants.

Heuristic procedure:

(1) *Non-assignment penalty calculations*: We consider the set of open warehouses and the set of customers who have not yet been assigned to these warehouses. We first calculate the penalty of not assigning customer i to the cheapest warehouse for product l.

(2) *Ratio calculations*: Take the ratio of penalty to the demand requirement of customer i for product l. This ratio attempts to weigh the penalty of assigning customer i to the second cheapest available warehouse by capacity usage of the warehouse.

(3) *Customer assignment*: The customer with the highest ratio is assigned to the cheapest warehouse to satisfy his demand for product l. If the customer's demand for product l is not fully satisfied from the cheapest warehouse, we repeat the steps and supply that customer the remain-

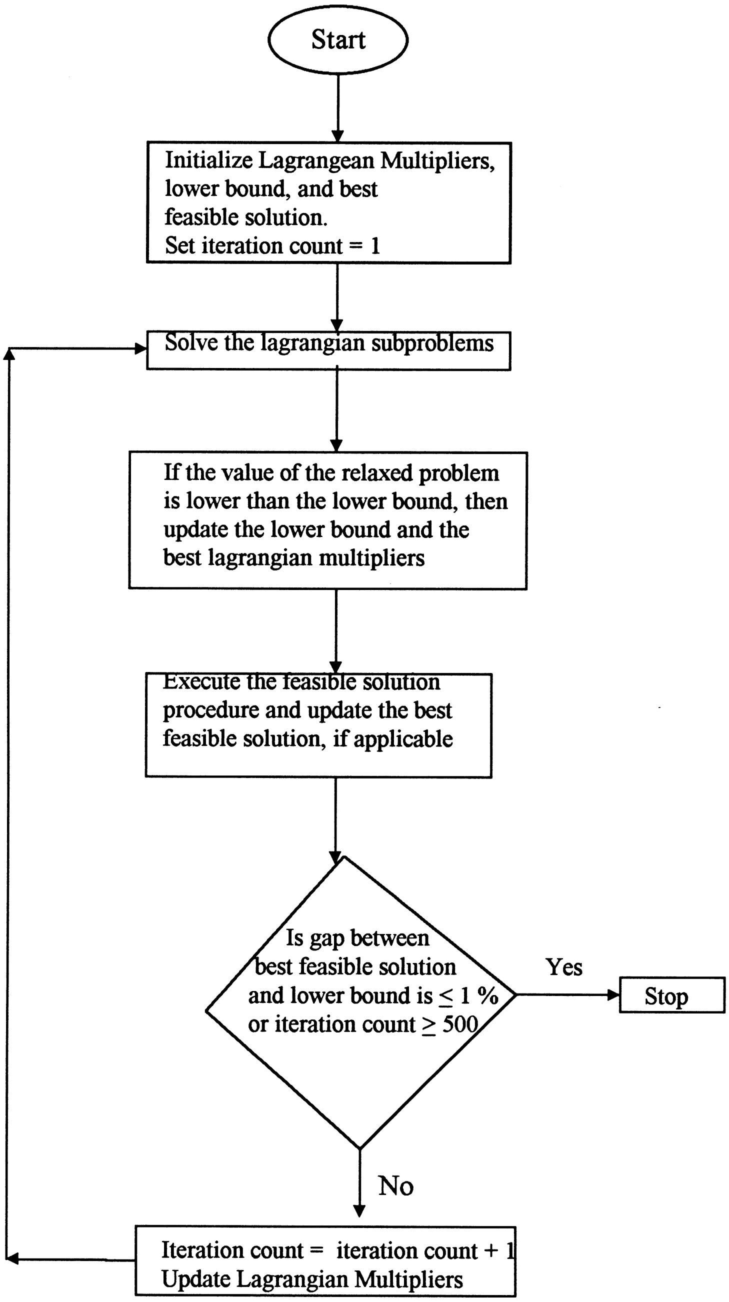

Fig. 1. Overall solution procedure.

ing demand from the second cheapest warehouse. We then reset unassigned customers and adjust available warehouse capacity.

(4) Repeat steps 1 through 3 until all customers have satisfied their demand for all products.

At this stage, we know the assignment of customers to open warehouses.

(5) *Customer list calculations*: We consider the set of open plants and open warehouses and form a list of customers who have been assigned to these open warehouses. Calculate the demand of product l for each open warehouse j by considering Σa_{il} over all customer i for every product l.

(6) *Cost ratio calculations*: Calculate the cheapest ratio of cost over demand requirement for each open plant-warehouse link. Denote the first member in the list as j_{open1}–k_{open1} to denote open warehouse j_{open1} and open plant k_{open1}.

(7) *Demand allocation*: Supply as much as possible of the demand of warehouse j_{open1} for product l from the cheapest plant k_{open1} as long as there is enough capacity at the plant k_{open1}.

(8) If we are still not satisfying the demand requirement of warehouse j_{open1} for product l, move down the list to the second cheapest (j_{open1}–k_{open2}) combination and try to supply the demand requirements of warehouse j_{open1} for product l subject to available capacity at the open plant k_{open2}.

(9) We keep repeating this procedure until we are done with supplying the demand requirements of all open warehouses for the multiple products.

We employ this heuristic procedure to provide a feasible solution to model PLANWAR.

6. COMPUTATIONAL RESULTS

The heuristic procedure was coded in Pascal as an integral part of the subgradient procedure algorithm. A series of computational experiments was carried out on an IBM RISC 6000 machine.

The problem sets were generated randomly but systematically to capture a wide range of problem structures. The performance of the solution procedure is reported in Tables 1–3. Many problems with the same input structure were solved in order to achieve a reasonable level of confidence about the performance and validation of the solution procedure. The cost coefficients C_{ijl} and C_{jkl} were generated from a uniform distribution between 0 and 200. The demand requirements of the multiple products for the customer zones were drawn from a uniform distribution between 10 and 99 and the space occupied by the products were drawn from a uniform distribution between 5 and 20 units. The fixed costs for opening plants and warehouses and the capacities of the warehouses and plants were varied over a range of values to provide a realistic scenario for the plant and warehouse logistics problem.

A comparison of the bounds obtained by the Lagrangian relaxation and the linear programming relaxation is reported in Table 1.

The Lagrangian bound produces much tighter bounds on the optimal solution values. It should be noted that since the LP bound is far worse than the Lagrangian bound from Table 1, it is apparent that any attempt to obtain an optimal solution for every instance of the problem will require unacceptable levels of computing time. The standard mathematical programming packages are not expected to perform well as the linear programming bounds for this problem are very bad. We solved a number of problems by creating an MPS input of the linear programming relaxation and used a relatively efficient integer programming code Sciconic [22]. Even for small sized problems consisting of 15 customers, 5 warehouses, 3 plants and 2 products combination, the time taken for the LP bound was 100 to 200 times slower than the Lagrangian bound. Hence the necessity to develop an efficient heuristic to solve the plant–warehouse distribution logistics problem.

The results of the performance of the solution procedure are reported in Table 2. The capacities of the warehouses and the plants as well as the fixed costs of the warehouses and plants were also changed to capture more realism with respect to the logistics model. The gap between the best feasible solution and the lower bound is used to judge the quality of the solution procedure. The gaps were expressed as a percentage of the lower bound. The warehouse load ratio (W.L.R.) which is defined to be the ratio of the total demand of customers to the total capacity of the W open warehouses was also varied between 0.80 to 0.85 and the plant load ratio

Table 1. Comparison of Lagrangian bound with LP bound for plant and warehouse location problem

CUST (I)	WARE (J)	PLANTS (K)	PROD, (L)	W.L.R.	P.L.R.	GAP (improvement over LP bound) (%)	CPU time (s)	
							LP	LAG
15	5	3	2	0.82	0.76	19.53	372	3.22
15	5	3	2	0.85	0.71	13.79	134	2.08
15	5	3	2	0.87	0.70	10.86	357	3.62
15	5	3	2	0.88	0.77	7.44	989	6.47
15	5	3	2	0.93	0.82	6.97	1246	9.21
15	5	3	2	0.95	0.89	7.54	2073	10.09

Warehouse load ratio (W.L.R.) = total demand of customers/total capacity of W warehouses.
Plant load ratio (P.L.R.) = total capacity of W warehouses/total capacity of P plants.
GAP = (Lagrangian bound − LP bound)/LP bound*100.

Table 2. Performance of solution procedure for plant and warehouse location problem

CUST (I)	WARE (J)	PLANTS (K)	PROD (L)	W.L.R.	P.L.R.	FCRW	FCRP	GAP (%)	CPU (s)
20	10	5	2	0.80	0.70	0.30	0.32	0.01	9.76
20	10	5	2	0.80	0.75	0.33	0.32	0.16	21.52
20	10	5	2	0.80	0.80	0.33	0.29	0.22	17.31
20	10	5	3	0.85	0.70	0.33	0.30	0.11	10.23
20	10	5	3	0.85	0.75	0.30	0.31	0.26	9.89
20	10	5	3	0.85	0.80	0.29	0.32	0.78	11.33
30	10	5	2	0.80	0.70	0.32	0.34	0.01	10.04
30	10	5	2	0.80	0.75	0.35	0.32	0.15	10.22
30	10	5	2	0.80	0.80	0.33	0.31	0.09	9.76
30	10	5	3	0.85	0.70	0.32	0.32	0.13	15.98
30	10	5	3	0.85	0.75	0.33	0.31	0.19	21.65
30	10	5	3	0.85	0.80	0.31	0.32	0.28	20.57
40	10	5	2	0.80	0.70	0.32	0.34	0.23	26.29
40	10	5	2	0.80	0.75	0.30	0.31	0.65	21.42
40	10	5	2	0.80	0.80	0.36	0.33	0.99	25.61
40	10	5	3	0.85	0.70	0.31	0.32	0.11	25.37
40	10	5	3	0.85	0.75	0.36	0.33	0.36	27.29
40	10	5	3	0.85	0.80	0.32	0.31	0.57	19.73
40	15	5	3	0.80	0.70	0.28	0.29	0.13	25.59
40	15	5	3	0.80	0.75	0.32	0.31	0.27	27.41
40	15	5	3	0.80	0.80	0.31	0.34	0.67	27.77
40	15	5	3	0.85	0.70	0.30	0.32	0.74	23.82
40	15	5	3	0.85	0.75	0.33	0.31	0.43	25.68
40	15	5	3	0.85	0.80	0.32	0.29	0.55	19.96
50	10	5	3	0.85	0.70	0.31	0.31	0.95	31.11
50	10	5	3	0.85	0.75	0.34	0.31	0.91	30.62
50	10	5	3	0.85	0.80	0.32	0.33	1.01	37.36
50	15	8	3	0.85	0.70	0.33	0.31	0.85	29.82
50	15	8	3	0.85	0.75	0.32	0.32	1.02	31.39
50	15	8	3	0.85	0.80	0.31	0.32	1.09	35.65
75	20	10	3	0.85	0.70	0.32	0.35	1.64	55.03
75	20	10	3	0.85	0.75	0.31	0.33	2.04	52.76
75	20	10	3	0.85	0.80	0.36	0.32	1.33	55.35
100	20	10	3	0.85	0.70	0.33	0.30	1.42	63.09
100	20	10	3	0.85	0.75	0.35	0.31	2.02	65.71
100	20	10	3	0.85	0.80	0.32	0.32	1.41	66.89

Fixed cost ratio for warehouses (FCRW) = total fixed cost of W warehouses/total feas cost.
Fixed cost ratio for plants (FCRP) = total fixed cost of P plants/total feas cost.
Warehouse load ratio (W.L.R.) = total demand of customers/total capacity of W warehouses.
Plant load ratio (P.L.R.) = total capacity of W warehouses/total capacity of P plants.
GAP = (feasible solution value – lower bound)/lower bound*100.

(P.L.R.) which is defined to be the ratio between the total capacity of the W open warehouses and the total capacity of the P open plants were also varied between 0.70 to 0.80 to present a different flavor to the problem under different operating considerations. The solution procedure in Table 2 indicates that the problem behaves very well.

Table 3. Performance of solution procedure with changes in warehouse load ratio and plant load ratio

CUST (I)	WARE (J)	PLANTS (K)	PROD (L)	W.L.R.	P.L.R.	FCRW	FCRP	GAP (%)	CPU (s)
50	10	3	2	0.90	0.80	0.32	0.53	1.03	62.45
50	10	3	2	0.85	0.80	0.33	0.47	0.87	39.78
50	10	3	2	0.80	0.80	0.31	0.49	0.88	53.93
50	10	3	2	0.75	0.80	0.36	0.50	0.79	39.82
50	10	3	2	0.70	0.80	0.33	0.54	0.82	42.75
50	10	3	2	0.65	0.80	0.28	0.51	0.36	41.63
50	10	3	3	0.70	0.80	0.32	0.32	1.01	41.59
50	10	3	3	0.65	0.80	0.30	0.30	0.53	27.01
50	10	3	3	0.60	0.80	0.34	0.32	0.29	29.70
50	10	3	3	0.80	1.00	0.31	0.32	1.01	58.72
50	10	3	3	0.80	0.91	0.32	0.33	1.00	50.18
50	10	3	3	0.80	0.83	0.30	0.31	0.89	47.43
50	10	3	3	0.80	0.70	0.31	0.32	0.94	49.72
50	10	3	3	0.80	0.64	0.33	0.35	0.58	45.14
50	10	5	2	0.85	1.00	0.33	0.33	0.97	51.81
50	10	5	2	0.85	0.91	0.34	0.36	0.91	55.79
50	10	5	2	0.85	0.83	0.33	0.35	0.69	53.25
50	10	5	2	0.85	0.70	0.30	0.32	0.72	48.64
50	10	5	2	0.85	0.64	0.31	0.33	0.55	47.93

Fixed cost ratio for warehouses (FCRW) = total fixed cost of W warehouses/total feas cost.
Fixed cost ratio for plants (FCRP) = total fixed cost of P plants/total feas cost.
Warehouse load ratio (W.L.R.) = total demand of customers/total capacity of W warehouses.
Plant load ratio (P.L.R.) = total capacity of W warehouses/total capacity of P plants.
GAP = (feasible solution value – lower bound)/lower bound*100.

We report extremely low gaps ranging from 0.01% to 2.04%. The computing times were also found to be quite stable and ranged from 9 s to less than 67 s. The fixed cost ratio for warehouses and the plants are also provided in Table 2 to help the decision maker to better analyze the various input data with respect to cost structures that are part of the problem. These results indicate that the heuristic solution procedure accomplished what was intended; to distribute commodities to customers while minimizing total costs incurred to achieve the goal of designing an efficient distribution design system.

A third computational experiment was designed to further test the distribution model for changes in the plant and warehouse load ratio. We initially varied the warehouse load ratio between 0.65 to 0.90 while keeping the plant load ratio at a fixed ratio of 0.80 in Table 3. Computational times as well as the quality of solutions reported in this table are similar to those reported in Table 2. As the warehouse load ratio was increased, the gaps increased slightly, but tend to remain at a very low value and varied between 0.29% to 1.03%. The solution times also remained quite stable and varied between 27 to 62 s.

Table 3 also provides the performance of the solution procedure with changes in the plant load ratio. The plant load ratio was varied between 0.64 to 1.00 while the warehouse load ratio was fixed at 0.80 and 0.85 for two sets of problems. Again, we report low gaps and found that as the plant load ratio was increased, the gaps tended to increase, but never increased beyond 1.01%. The solution times also remained quite stable and varied between 45 and 59 s.

7. MANAGERIAL IMPLICATIONS

The PLANWAR location model with the effective heuristic solution procedure can provide the decision maker with a good logistics tool in the design of an efficient distribution strategy and planning model that best sites plants and warehouses and the type of products that need to be manufactured at the plants and the type of products that need to be placed in the warehouses in order to effectively satisfy the demand of customers. The design of such distribution systems is one of the complex tasks facing those charged with planning for an organization's needs and the services to meet those demands. The development of a distribution system is a long-term endeavor so that the design of these systems actually falls within the realms of strategic decision making. The information needs for such distribution-planning decisions require a strategic perspective. This perspective is characterized by a high degree of uncertainty that is not addressed by traditional, short-term, predictive approaches.

The PLANWAR model obviously represents an extremely large mixed integer program with well over 20,000 variables and over 21,000 functional constraints. Yet the heuristic was able to find solutions that would have been practically unobtainable with commercial integer programming codes. Solution times were low and varied between 46 to 76 s while the gaps ranged between 0.8% to 2.7% and were well within the range of tractability.

8. CONCLUDING REMARKS

In this paper we present the distribution network strategic design problem and discuss the transportation and distribution issues that exist for the multi-commodity, multi-level logistics problem. We develop a mixed integer programming model for the plant and warehouse location problem to minimize the total distribution and transportation costs and the fixed costs of opening and operating plants and warehouses. We employ Lagrangian relaxation to the model and also present a heuristic to provide an effective feasible solution for the problem. Computational results on a wide variety of the problems are reported and these results indicate that the feasible solution procedure consistently provides stable solutions to the problem. Moreover, the heuristic performs well in terms of both approximation to optimality and solution times regardless of problem size and structure.

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