Facility Location and Covering Problems

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ABSTRACT
We discuss the connection between facility location problems and covering problems, and present approaches to solving facility location problems by reducing them to covering problems. In particular, we discuss the following NP-hard optimization problems: the set cover and the related set cover facility location problem, the k-center and k-suppliers problem, the dominating set problem, the maximum independent set problem, and the maximum covering problem.

Keywords
combinatorial optimization, approximation algorithms, facility location, covering problems

1. INTRODUCTION
Facility location is an important research area of environment engineering, regional and city planning, management and transportation science and other fields with a lot of applications [2, 4, 11]. Often facility location and covering problems are NP-hard. Thus, exact solutions are not expected to be found easily. Thus, finding several different ways of solving these problems is important in view of the efficiency of the solutions found. In this article we present several facility location problems and a way of solving them by using (i.e. transforming to) the covering problems.

Our view of the facility location problems is graph-theoretical, although the problems can also be defined in continuous or discrete domain [2]. Usually, the environment of the facility location problem is defined as a simple clique $G = (V, E)$, i.e. a complete graph with no loops. Edges $e \in E$ of G have weights $w(e)$, which may represent travel distances or service times from one vertex to another, i.e. from a facility to a client. Such a weighted graph is called network. Often the *triangle inequality* is required, i.e. $w(e_{uv}) \leq w(e_{uw}) + w(e_{wv})$, which is usually a realistic assumption.

In this article we show the connection between facility location problems and the covering problems, i.e. the way the covering problems and the results about them can be used for solving the facility location problems. Sometimes these reductions are very straightforward. In the next section we describe several facility location problems and their relations to some covering problems. In particular, we discuss the set cover and the related facility location problem, the k-center and k-suppliers problem, the dominating set problem, the independent set problem, and the maximum covering problem.

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2. FACILITY LOCATION AND COVERING
In this section we present several ways of solving facility location problems using methods for solving covering problems.

2.1 Facility Location and Set Cover Problem
Consider the following facility location problem [2, 4]. Given a complete network $G = (V, E)$ and a coverage distance D, find the minimum cardinality set $S \subseteq V$, such that for every $v \in V - S$ there is some $u \in S$ so that $d(u, v) \leq D$, i.e. find the minimum cardinality subset S of vertices V such that the distance from every vertex in V to some vertex in S is at most D. We call this problem the *set cover facility location problem.* In the following we show that it is actually the set cover problem defined in the graph-theoretical domain.

To obtain the *set cover* problem from the set cover facility location problem, we construct the family of sets $\{S_1, S_2, \ldots, S_n\}$ where $S_u = \{v \in V | d(u, v) \leq D\}$, i.e. S_u contains all vertices which are covered by the vertex u. Notice that $V = \cup_{u \in V} S_u$. The corresponding set cover problem is to find the minimum cardinality set $\mathcal{S} \subseteq \{S_1, S_2, \ldots, S_n\}$ such that all vertices $v \in V$ are covered, i.e. $V = \cup_{S_u \in \mathcal{S}} S_u$.

Using this reduction one can also easily solve the set cover facility location problem with additional constraint where vertices of possible locations of facilities are specified. To do this, we construct the family of sets $\{S_1, S_2, \ldots, S_{|F|}\}$, where $F \subseteq V$ is the set of vertices representing possible facility locations and $S_u = \{v \in F | d(u, v) \leq D\}$, and use it with the corresponding set cover problem.

Additionally, one can also specify the price w_v, $v \in V$ of opening a facility at the vertex v. Then by using the weighted set cover problem one can minimize the total price of opened facilities in order to cover all clients.

2.2 k-Center and Set Cover Problem
In this subsection we describe the reduction from the *k-center facility location* problem to the set cover facility location problem [2, 4, 10].

Given a complete network $G = (V, E)$, the k-center problem is to find $S \subseteq V$, where $|S| \leq k$, which minimizes $\max_{v \in V} \min_{u \in S} d(u, v)$.

In the following we describe the so-called *bottleneck technique* [6], which is often used to solve the k-center problem. We assume w.l.g. that edges $E = \{e_1, e_2, \ldots, e_m\}$ are sorted in nondecreasing order by their weights, i.e. $w(e_i) \leq w(e_{i+1})$. Let the *bottleneck graph* be the graph $G_i = (V, E_i)$, where $E_i = \{e_1, e_2, \ldots, e_i\}$, i.e. G_i is a subgraph of G containing only edges whose weights are at most $w(e_i)$. The bottleneck technique is to iteratively consider bottleneck graphs $G_1, G_2, \ldots, G_m$. Instead of iterative search often binary search is used to speed up the search process through the bottleneck graphs. At each step of the search (i.e. for each bottleneck graph G_i), we examine G_i for the feasible solution(s).

For G_i we consider the solution S of the set cover facility location problem. If $|S| \leq k$, we accept S as the feasible solution of the k-center problem; otherwise, we continue and consider the next bottleneck graph.

2.3 k-Suppliers and Set Cover Problem

The *k-suppliers facility location* problem is similar to the k-center problem [5] and is defined as follows. Given a complete bipartite network $G = (U, V, E)$, where U is the set of possible center locations, and V is the set of clients, find the set $S \subseteq U$ which minimizes $\max_{v \in V} \min_{u \in S} d(u, v)$.

By using the bottleneck technique we can also solve the k-suppliers problem. The only difference is to consider only vertices in U, i.e. for the set cover problem construct only sets S_u where $u \in U$.

There are several ways to solve the set cover problem. One may use greedy algorithm [1], elimination heuristic [7] or zero-one integer linear programming [2, 4].

2.4 k-Center and Dominating Set Problem

In this subsection we describe how to solve the k-center problem using the *dominating set* problem. Let us first define the dominating set problem. Given a graph $G = (V, E)$ the dominating set problem is to find minimum cardinality set $S \subseteq V$, such that every vertex in $V - S$ is adjacent to some vertex in S.

Again, we apply bottleneck technique for solving the k-center problem. The solution of the k-center problem is the dominating set S of the bottleneck graph G_j, where $|S| \leq k$ and j is the smallest index such that G_j contains a dominating set with at most k vertices. Notice that there are well-know algorithms for computing small dominating sets [1, 8, 9].

If the triangle inequality is assumed, one can construct a 2-approximation algorithm for the k-center problem in the following way. Find the *maximal independent set* I in the square G_i^2 of the bottleneck graph G_i. The set I is the approximate solution of the k-center problem. For I and the minimum dominating set D in G_i one can prove [5], that $|I| \leq |D|$. Notice also that the maximum edge weight of G_i^2 is at most twice the maximum edge weight of G_i.

Thus, we can solve the k-center problem by solving series of dominating set problems or, alternatively, maximal independent set problems. The later approach results in a polynomial 2-approximation algorithm since maximal independent set is one of suboptimal solutions of the maximum independent set problem.

2.5 k-Center and Maximum Covering Problem

Let use first define the *maximum covering facility location* problem. Given a complete network $G = (V, E)$ and positive parameters k and D, the task is to find $S \subseteq V$, where $|S| \leq k$, such that S maximizes the number of covered clients, i.e. the number of vertices whose distance from S is at most D.

Again, we use bottleneck technique to solve the k-center problem [3]. For each G_i we find the solution S of the maximum covering facility location problem. If $|S| = n$, i.e. if all clients are covered then S is also a feasible solution of the k-center problem.

3. CONCLUSIONS

In this article we discussed several ways of solving facility location problems by using covering problems. Because all the presented optimization problems are NP-hard, different algorithms for the given problem may considerably differ in the quality of their (suboptimal) solutions.

4. REFERENCES

[1] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein. *Introduction to Algorithms*. The MIT Press, 2nd edition, 2001.

[2] M. S. Daskin. *Network and Discrete Location: Models Algorithms and Applications*. Wiley, New York, 1995.

[3] M. S. Daskin. A new approach to solving the vertex p-center problem to optimality: Algorithm and computational results. *Communications of the Operations Research Society of Japan*, 45:9:428–436, 2000.

[4] Z. Drezner and H. W. Hamacher, editors. *Facility Location: applications and theory*. Springer-Verlah, Berlin, 2002.

[5] D. S. Hochbaum, editor. *Approximation Algorithms for NP-hard Problems*. PWS publishing company, Boston, 1995.

[6] D. S. Hochbaum and D. B. Shmoys. A unified approach to approximation algorithms for bottlneck problems. *J. of ACM*, 33:533–550, 1986. PR.

[7] J. Mihelič. *Hevristično reševanje problemov pokrivanja in razmeščanja*. Fakulteta za računalništvo in informatiko, Univerza v Ljubljani, 2004. (Magistrsko delo).

[8] J. Mihelič and B. Robič. Approximation algorithms for k-center problem: an experimental evaluation. *Proc. OR 2002, Klagenfurt, Austria*, 2002.

[9] J. Mihelič and B. Robič. Solving the k-center problem efficiently with a dominating set algorithm. 2004. (poslano v recenzijo).

[10] E. Minieka. The m-center problem. *SIAM Rev.*, 12:138–139, 1970.

[11] P. B. Mirchandani and R. L. Francis, editors. *Discrete Location Theory*. John Wiley & Sons, New York, 1998.