Modification of Hybridized Particle Swarm Optimization Algorithms Applying to Facility Location Problems

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Abstract. *J. Kennedy and R. Eberhart first introduced the concept of so called Particle Swarm Optimization (PSO), applied it to optimization of continuous nonlinear functions and showed the effectiveness of the algorithm. Since then various researchers have been challenging to try to apply the concept to a variety of optimization problems and obtaining reasonable results. In PSO, individuals exchange simple information and communize information each other. Furthermore, the information among individuals is communized in the swarm and the information between individuals and their swarm is also shared. Finally, the swarm approaches to the optimal behavior.*

Facility location problems are to arrange facilities on appropriate locations so that utility of customers becomes the maximum. There exist various kinds of facility location problems, for instance, a single facility location problem, a multiple facility location problem, a newly open facility location problem among group facilities or among rival facilities, and others. We have already proposed simple hybridized algorithms to solve these kinds of problems. The hybridized algorithms timely switch the procedure between the essential Particle Swarm Optimization method and other methods. In the algorithms, our numerical experiments show that convergent time on reasonable results depends on various predetermined factors, such as the number of iterations, changing timing between hybridized algorithms, the number of swarms and their initial locations, etc. In this paper, we clarify properties of these factors and evaluate them through numerical experiments.

Keywords: *Particle Swarm Optimization, Meta-Heuristics, Swarm Intelligence, Facility Location*

1. INTRODUCTION

These years, meta-heuristics, for example, genetic algorithm (GA), evolutionary programming (EP), simulated annealing (SA), etc., are applied to optimization of various complex systems instead of strict optimization methods. Recently, swarm intelligence so called Particle Swarm Optimization (PSO) is tried to apply to various optimization problems. Several years ago, PSO was proposed by J. Kennedy and R. C. Eberhart (Kennedy et al. 1995) as a new meta-heuristic. PSO is a little different from existing meta-heuristic algorithms. It was developed from social behaviors of animals and plants, and physical phenomena, however, it has connection with evolutionary computation and associates with GA and EP.

For a long time, many scientists tried to simulate behaviors of bird flocking and fish schooling on computers. Especially, C. W. Reynolds was enamored with beauty of

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bird flocking (Reynolds 1987). F. Heppner and U. Grenander were interested in finding out the rules of the behavior of bird flocking (Heppner *et al*. 1990). They simulated bird flock behavior on computers by maintaining an optimal distance between their neighbors and obtained very good results. It will be possible that this rule is applied to animal social behavior. E. O. Wilson described in his article as follows, "In theory at least, individual members of the school can profit from the discoveries and previous experience of all other members of the school during the search for food. This advantage can become decisive, outweighing the disadvantage of competition for food items, whenever the resource is unpredictably distributed in patches" (Wilson 1975). That is, social information between members offers an evolutionary advantage. This hypothesis constructs the basis of particle swarm optimization. In 1995, J. Kennedy and R. C. Eberhart proposed PSO using ideas of bird flock behavior described above. After then, some researchers compared particle swarm optimization with genetic algorithm and reported reasonably good results.

Even though each individual in a swarm has low intelligence, its swarm often expresses high performance of ability and shows intensive united behavior. Recently, such swarm intelligence is tried to apply to various optimization problems. PSO has roots in two main component methodologies; artificial life in general, and swarming theory in particular. Individuals exchange simple information and communize information each other. Furthermore, the information among individuals is communized in the swarm and the information between individuals and their swarm is also shared. Finally, the swarm approaches to the optimal behavior. PSO utilizes properties that the individuals act simple behaviors with simple information and the swarm, as an aggregate of individuals, acts the optimal movement with integrated high intelligent information.

J. Kenndy and R. Eberhart first introduced this concept, applied it to optimization of continuous nonlinear functions and showed the effectiveness of the algorithm. Because each individual acts simple movement with easy information, the computation time will be very short when applying it to optimization problems. In addition, because many individuals simultaneously search better solutions, reasonable solutions can be obtained quickly. As PSO is not popular so far, many researchers are trying to search its application areas at present time.

The authors already proposed procedure to apply PSO to facility location problems, and obtained reasonable results (Toyoda *et al*. 2008). Here, we treat the following two kinds of problems; (1) equalization of clients in multi facility location problems and (2) problems of a facility location among rival facilities. To approach to these problems, we hybridize PSO with simple allocation algorithms. In the process to obtain solutions using the hybridized method, several parameters have to determined, and we know the parameter values greatly affect the accuracy of solutions and speed of convergence to the optimal solutions. In this paper, we discuss how these parameters affect solutions and how to determine the parameter values. We show the propriety of our proposal using numerical experiments.

2. EQUALIZATION OF CLIENTS IN MULTI FACILITY LOCATION PROBLEM

2.1 Nature of Problem

Here, the objective function is to minimize the total distance between a facility and clients that the facility takes care of. The Authors already proposed simple procedure how to locate multi facilities to minimize the objective function using PSO.

Even if the total distance between the facilities and clients is the minimum, utility of the facilities and clients is not always the maximum. If a facility has to take care of too many clients, service of the facility to the clients the facility takes care of may be insufficient. Therefore, equalization of the number of clients each facility takes care of is sometimes necessary.

Once the positions of facilities are determined after the predetermined number of iteration in PSO algorithm, the number of clients each facility takes care of may be biased. To equalize this number, we also proposed the following procedure.

First, we define symbols:
A_a: the position of the client, a,
R_r: the position of the facility, r,
M: the standard number of clients a facility can take care of,
N_r: the number of clients the facility, r, takes care of,
C_r: $\{a\}$ where a facility, r, takes care of a client, a,
D_α: $\{r\}$ where $N_r>M$, and
D_β: $\{r\}$ where $N_r<M$.
Then,

$$C_p \leftarrow C_p - \{a\},\ C_q \leftarrow C_p + \{a\},\ N_p \leftarrow N_p - 1,\ N_q \leftarrow N_q + 1,$$

where

$$\min_{p,q,a}\left\{\left\|A_a - R_q\right\| - \left\|A_a - R_p\right\|\right\},$$

$$p \in D_\alpha,\ q \in D_\beta,\ a \in C_p. \qquad (1)$$

Repeat this procedure until the number of clients at each facility is equalized. We call this method the Differential Method.

Here, we proposed another more simplified algorithm to find a client to move from a crowded facility to a less crowded facility.

$$C_p \leftarrow C_p - \{a\},\ C_q \leftarrow C_q + \{a\},\ N_p \leftarrow N_p - 1,\ N_q \leftarrow N_q + 1,$$

where

$$\min_{q,a} \left\| A_a - R_q \right\|, \; q \in D_\beta, \; a \in C_u \qquad (2)$$

Repeat this procedure until the number of clients at each facility is equalized. We call this method the Simplified Method.

We hybridized this equalizing procedure with PSO and repeat these procedures alternately until the predetermined stopping rule is satisfied.

To perform these procedures, we have to determine several parameters. We already know that those parameter values have important role to obtain good solutions in short calculation time. The following numerical experiments show the effectiveness of the parameters.

2.2 Numerical Experiments

Our method using PSO was programmed in Digital Visual Fortran, version 6.0 and ran on the computer, Dell Optiplex 755, Intel® Core™2 DUO E6850 processor, 2GB memory with Microsoft Windows XP Professional SP2.

Our procedure involves several parameters that affect its ability to reach better solutions in a shorter computation time. Among those parameters, 2 parameters regarding PSO iterations are important in particular. Those are as follows.

V: the number of repeated iterations of the whole procedure
S: the number of PSO iterations for every facility

To evaluate the affection of the combination of the values V and S to the obtained solutions by our procedure mentioned above, we examined 6 different combinations of (V, S) such as (2 50), (5, 20), (10, 10), (20, 5), (50, 2), and (100, 1). Therefore, every combination totally has 100 iterations.

50 different sets of initial particles are generated by random numbers and we solve 50 times using these initial data. The numbers of clients are 150, 450 and 750 with 5 facilities in the numbers of particles, 10, 30 and 50.

Figure 1 shows one of 50 trial solutions to the problem of 150 clients with 5 facilities before equalization of the numbers of the clients when the combination (V, S) is (10, 10) and the particle member size is 50. Figure 2 and 3 show the equalizing results for Figure 1 by Differential Method and Simplified Method. Figure 4 shows one of 50 trials to the problem of 750 clients with 5 facilities before equalization of the numbers of the clients under using the same parameter values of Figure 1. Figure 5 and 6 show the equalizing results for Figure 4 by both the methods.

Table 1 shows each of the best solutions in 50 trials by Differential Method on each combination when the number of clients is 150 with the numbers of particles, 10, 30 and 50. We could obtain almost the same results when our procedure is finished. Table 2 shows each average solution in 50 trials by Simplified Method when the combinations of (V, S) are (1,100), (2, 50), (50, 2) and (100, 1), the average solutions are not good.

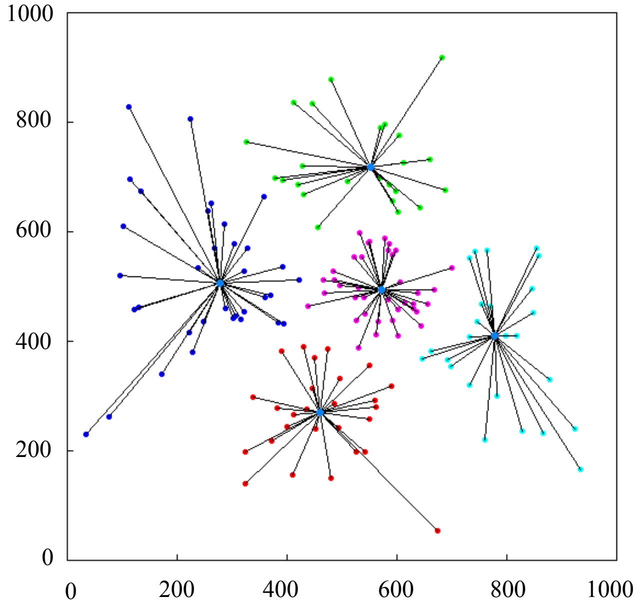

Figure 1: A result of a 5 facility location problem with 150 clients before equalization

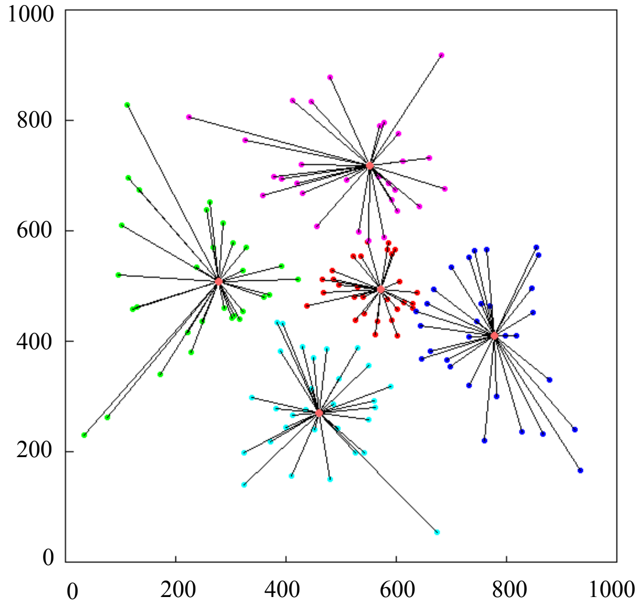

Figure 2: The equalization result of a 5 facility location problem with 150 clients by Differential Method

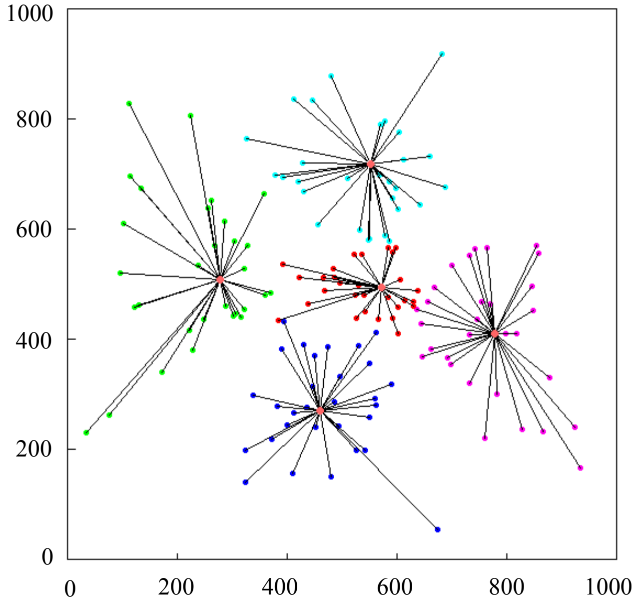

Figure 3: The equalization result of a 5 facility location problem with 150 clients by Simplified Method

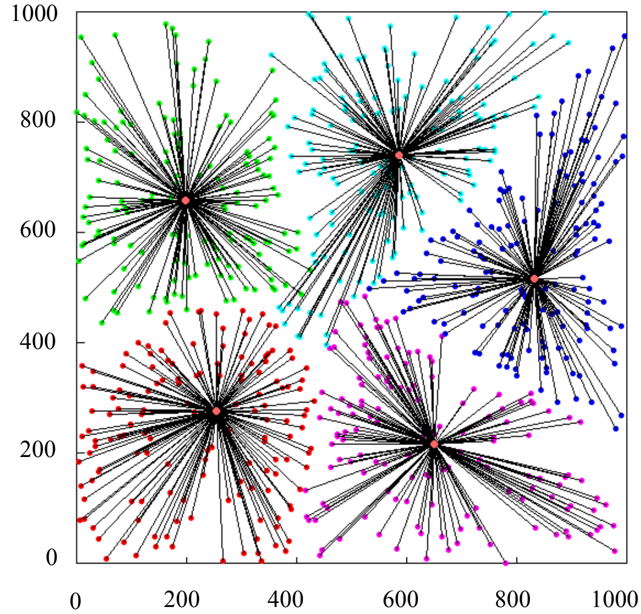

Figure 5: The equalization result of a 5 facility location problem with 150 clients by Differential Method

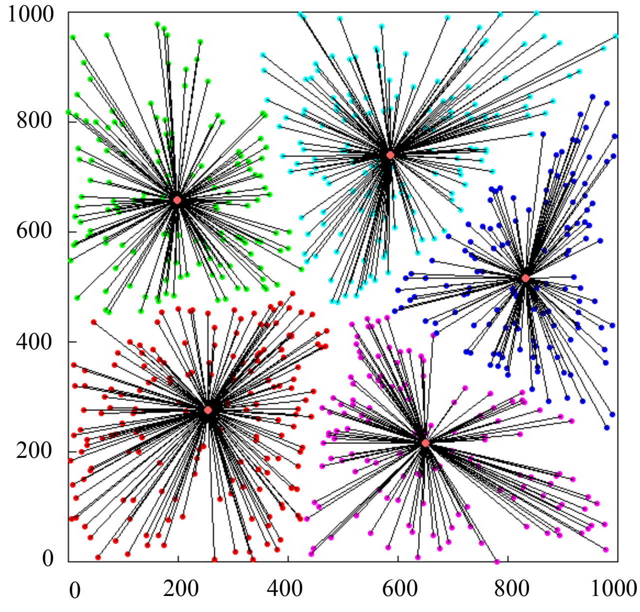

Figure 4: A result of a 5 facility location problem with 750 clients before equalization

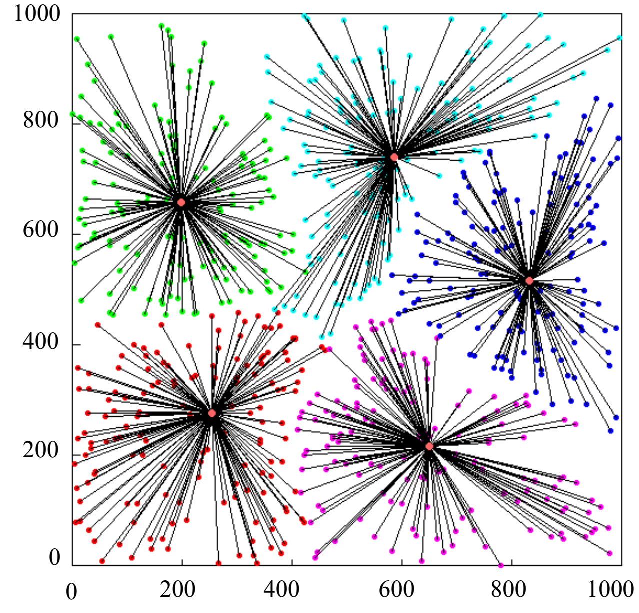

Figure 6: The equalization result of a 5 facility location problem with 150 clients by Simplified Method

Table 3 shows each of the best solutions in 50 trials by Differential Method on each combination when the number of clients is 150 with the number of particles, 10, 30 and 50. When the number of particle is lager such as 30 or 50, we could obtain almost the same solution at the combination of (10, 10) and (5, 20). The smaller particle sizes are not good for this method. Table 4 shows each average solution of 50 trials by Simplified Method. The average

Table 1: The best solutions in 50 trials by Differential Method on various combinations of (V, S) when the number of clients is 150 with the number of particles, 10, 30 and 50

Particle	(1,100)	(2,50)	(5,20)	(10,10)	(20,5)	(50, 2)	(100,1)
10	18678.44	17223.22	17093.01	17185.65	17186.12	17199.85	17199.71
30	18433.28	17216.70	17129.45	17200.14	17200.24	17200.45	17200.24
50	18432.73	17217.83	17133.41	17200.51	17200.46	17200.53	17200.58

Table 2: The best solutions of 50 trials by Simplified Method on various combinations of (V, S) when the number of clients is 150 with the number of particles, 10, 30, and 50

Particle	(1,100)	(2,50)	(5,20)	(10,10)	(20,5)	(50, 2)	(100,1)
10	19220.43	17267.34	17275.66	17431.63	17431.55	17431.64	17431.86
30	19220.12	17270.96	17243.99	17431.20	17431.55	17431.61	17431.22
50	18785.60	17314.17	17262.83	17399.66	17400.49	17431.21	17430.88

Table 3: The average solutions of 50 trials by Differential Method on various combinations of (V, S) when the number of clients is 150 with the number of particles, 10, 30, and 50

Particle	(1,100)	(2,50)	(5,20)	(10,10)	(20,5)	(50, 2)	(100,1)
10	32510.84	18494.12	17616.55	17289.30	17318.87	17882.42	18057.87
30	32447.47	18174.47	17227.33	17201.10	17201.22	17358.92	17709.12
50	32411.53	18234.84	17272.01	17201.28	17201.14	17262.70	17902.75

Table 4: The average solutions of 50 trials by Simplified Method on various combinations of (V, S) when the number of clients is 150 with the numbers of particles, 10, 30 and 50

Particle	(1,100)	(2,50)	(5,20)	(10,10)	(20,5)	(50, 2)	(100,1)
10	33646.58	18787.62	17913.11	17541.85	17587.29	18187.19	18325.31
30	33939.41	18477.90	17431.63	17432.41	17432.49	17623.16	17954.69
50	33813.96	18513.65	17483.19	17432.64	17432.51	17512.39	18136.59

Table 5: The best solutions of 50 trials by both methods on various combinations of (V, S) when the number of clients is 450 with the number of particles, 30 and 50

Method	Particle	(1,100)	(2,50)	(5,20)	(10,10)	(20,5)	(50, 2)	(100,1)
Differential	30	79364.85	77402.56	76581.54	76873.09	76927.80	76928.17	76926.71
	50	79363.01	77401.12	76673.35	76926.04	76928.13	76925.98	76927.85
Simplified	30	79498.45	78006.40	76743.95	77203.67	77279.05	77279.61	77277.46
	50	79493.80	78004.89	77092.24	77276.87	77278.90	77276.76	77279.28

Table 6: The average solutions of 50 trials by both methods on various combinations of (V, S) when the number of clients is 450 with the numbers of particles 30 and 50

Method	Particle	(1,100)	(2,50)	(5,20)	(10,10)	(20,5)	(50, 2)	(100,1)
Differential	30	114343.51	82409.04	78728.07	78705.41	78587.62	78634.22	78432.27
	50	115890.30	82576.03	78704.76	78349.81	78504.98	78488.20	78283.42
Simplified	30	117074.73	83264.39	79196.64	79197.38	78988.22	79084.71	78948.29
	50	118376.08	83388.75	79237.48	78792.67	79033.86	79062.43	78790.16

Table 7: The best and average solutions of 50 trials by both methods on various combinations of (V, S) when the number of clients is 750 with the number of particles, 50

Method	(V, S)	(1,100)	(2,50)	(5,20)	(10,10)	(20,5)	(50, 2)	(100,1)
Differential	Best	131315.61	128916.33	128113.26	128342.84	128448.95	129265.30	129273.55
	Average	196046.70	136507.64	130309.82	130437.70	132905.79	130706.48	130862.77
Simplified	Best	132124.02	130096.67	128967.72	129150.11	129596.46	129823.63	129831.95
	Average	199991.27	138015.68	131951.26	132763.79	135225.62	133073.05	133480.14

solutions by Simplified Method are not better than those of Differential Method.

From all of these results, when the combination of (V, S) is (1,100), (2, 50), (50, 2), (5, 20) or (100, 1) and smaller particle sizes are not preferable for both the methods.

Table 5 and 6 show the best solution and average solutions of 50 trials by both the methods on each combination when the number of clients is 450 with the number of particles, 30 and 50. Table 7 shows the best and average solutions of 50 trials by both the methods on each combination when the number of clients is 750 with the number of particles, 50.

From these results, when the combinations (V, S) are (5, 20) and (10, 10), we could obtain desirable solutions by Differential and Simplified Methods. The smaller numbers of particles are not good to solve these facility location problems by using our proposed methods.

3. FACILITY LOCATION AMONG RIVAL FACILITIES

3.1 Nature of Problem

This problem is to snatch clients from other rival facilities as many as possible. As a client belongs to the closest facility in this paper, locate the new facility, so that the distances between clients and the facility are closer than that between the clients and the facility they presently belong to. Therefore, maximize N, where $N \leftarrow N+1$, $C \leftarrow C+\{a\}$, $N_p \leftarrow N_p-1$, $C_p \leftarrow C_p-\{a\}$, if $\|R - A_a\| < \|R_p - A_a\|$ for every $p \in C_p$. Here, N and C describe the number of the clients and a set of clients the new facility takes care of respectively. We try to put the new facility on an appropriate location using PSO, so that N becomes the maximum. The procedure is repeated until the predetermined stopping rule is satisfied. In this method, there exist several parameters that should be predetermined and the values effect the precision and computation time of solutions. In the following section, we make the role of those parameters clear.

3.2 Numerical Experiments

In our numerical experiments, the numbers of clients are 150, 450 and 750 with 5 rival facilities while the numbers of particles are 30 and 50. We generated 50 different sets of initial particles by random numbers and we solved 50 times using these initial data.

Table 7 shows the number of solutions that reach the maximum number of snatched clients when the 5 rival facility locations are allocated by random numbers and V=10, 20, 40, 60, 80 and 100 (in this problem, S=1). 66 out of 150 clients are snatched to the new facility on both the number of particles as the maximum solution. In the case of 450 clients, 91 is the maximum number as the number of the snatched clients. In the case of 750 clients, 151 is the maximum number. In every case, as increasing the number of the iterations, the number of solutions that reach the maximum number of the snatched clients is getting larger. More than 96% of 50 solutions reach the maximum number at the number of the iteration of 100.

Table 8 shows the number of solutions that reach the maximum number of snatched clients when the 5 rival facility locations are allocated by our PSO procedure and V=10, 20, 40, 60, 80 and 100. 23 out of 150 clients are assigned to the new facility as the maximum solution when the numbers of particles are 30 and 50. In the case of 450 clients, the best solution is that 99 clients are reassigned to the new facility. That 160 clients out of 750 are reassigned to the new facility is the best solution in the 750 client problem. In the case of 750 client problem, less than 35% of the solutions reach the maximum solution on both 30 and 50 particles. However, more than 60% of the solutions reach the solution that 159 clients are snatched to the new facility on both 30 and 50 particles.

Figure 7 shows one of the assignment results for 150 clients before snatching. The 5 rival facility locations are generated by random numbers. Figure 8 shows the result of new assignment of 150 clients to the new facility and the existent rival facilities. 66 clients are reassigned to the new facility.

Table 7: The number of solutions that reach the maximum number of the snatched clients at each iteration when the 5 rival facility locations are allocated by random numbers

The number of clients	The number of particles	The maximum number of snatched clients	The number of solutions at each iteration					
			S=10	S=20	S=40	S=60	S=80	S=100
150	30	66	6	36	43	43	48	48
	50	66	9	38	47	48	48	49
450	30	91	10	33	38	43	46	48
	50	91	23	43	47	48	49	49
750	30	151	4	24	38	40	46	46
	50	151	5	30	47	48	46	50

Table 8: The number of solutions that reach the maximum number of the snatched clients at each iteration when the 5 rival facility locations are allocated by our PSO procedure

The number of clients	The number of particles	The maximum number of snatched clients	The number of solutions at each iteration					
			S=10	S=20	S=40	S=60	S=80	S=100
150	30	23	1	12	24	23	24	29
	50	23	5	16	24	31	32	33
450	30	99	3	19	34	32	40	43
	50	99	6	28	41	42	44	45
750	30	160	0	0	6	6	8	12
	50	160	0	1	9	9	15	15

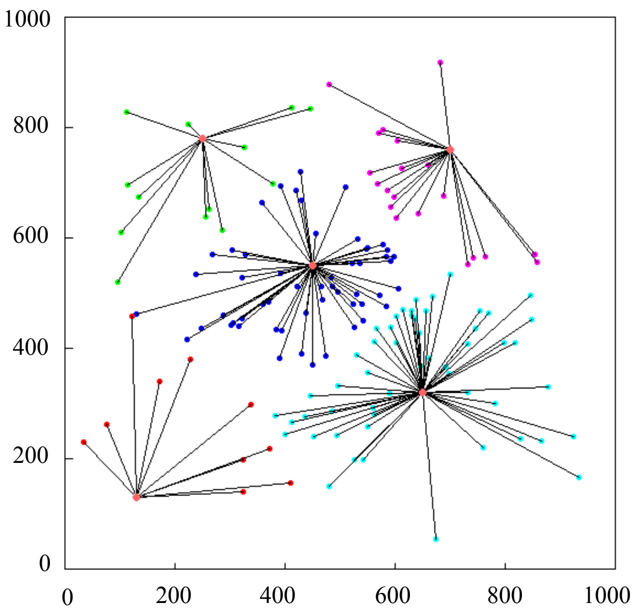

Figure 7: The assignment of 150 clients to the nearest facilities whose locations are generated by random numbers

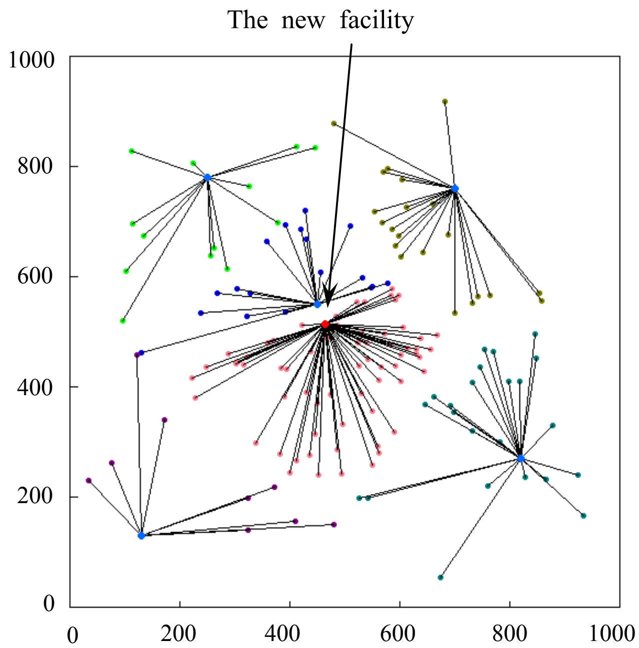

Figure 8: The reassignment of Figure 7 to the new facility and the 5 rival facilities

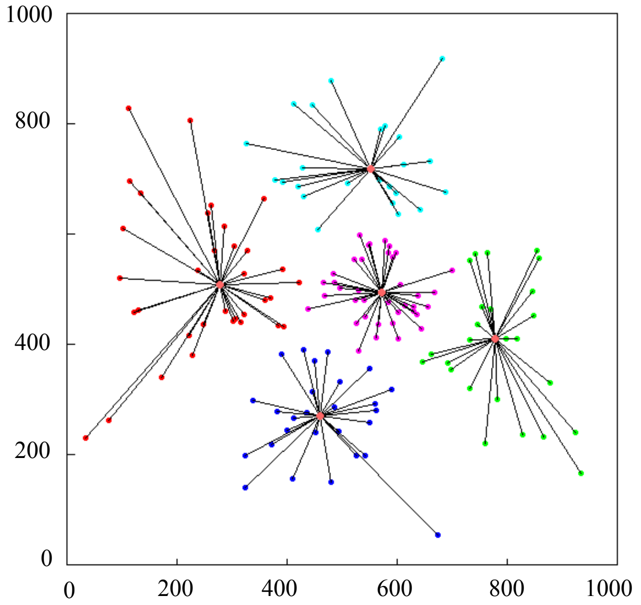

Figure 9: The assignment of 150 clients to the nearest facilities using our PSO procedure

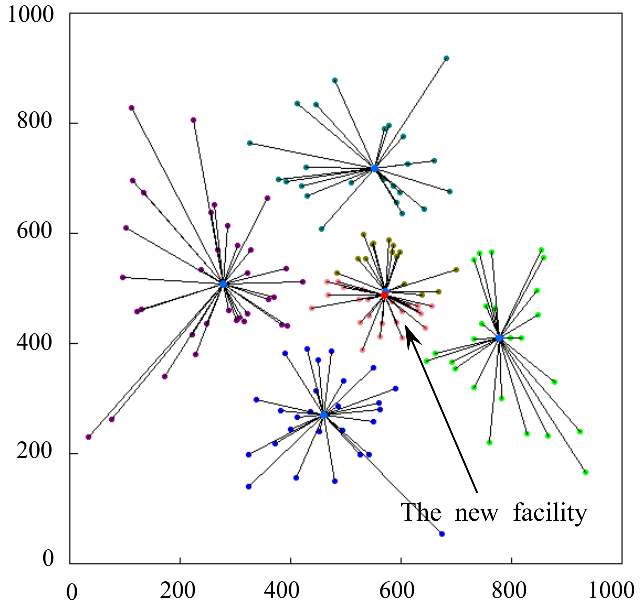

Figure 10: The reassignment of Figure 9 to the new facility and the 5 rival facilities

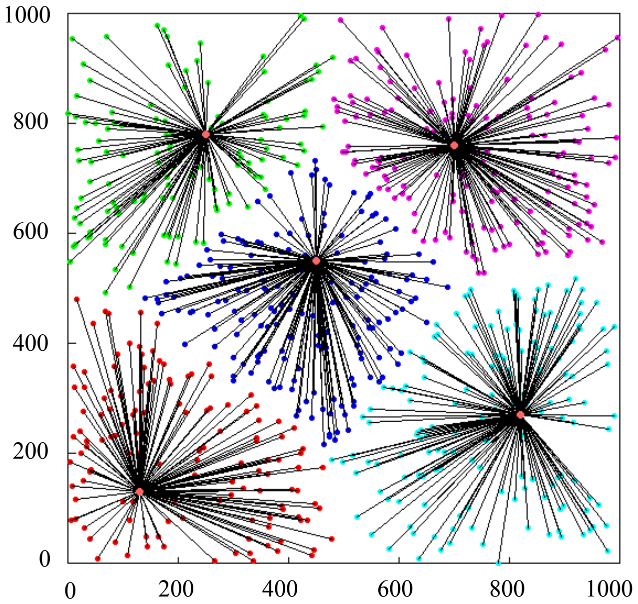

Figure 11: The assignment of 750 clients to the nearest facilities whose locations are generated by random numbers

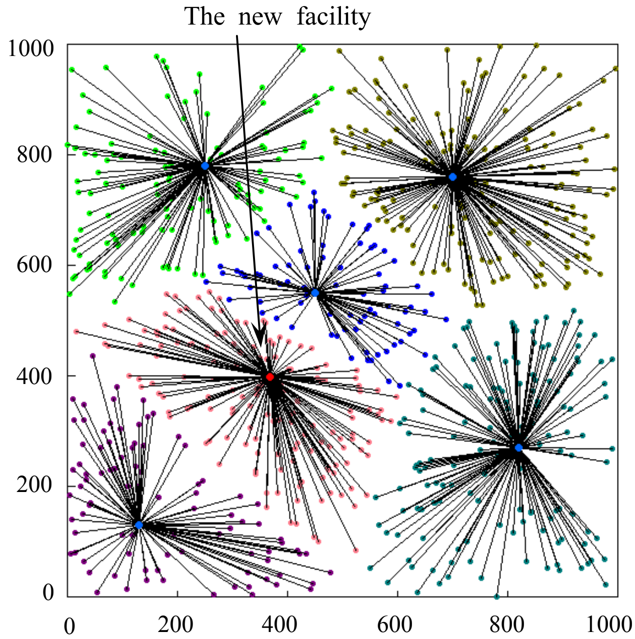

Figure 12: The reassignment of Figure 11 to the new facility and the 5 rival facilities

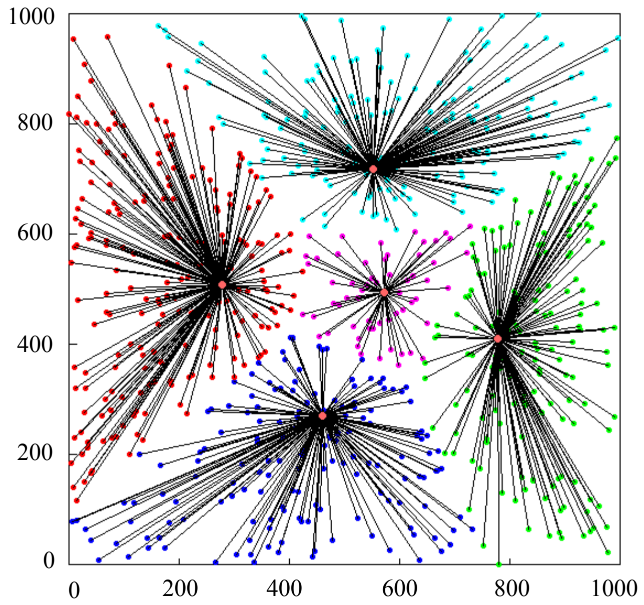

Figure 13: The assignment of 750 clients to the nearest facilities using our PSO procedure

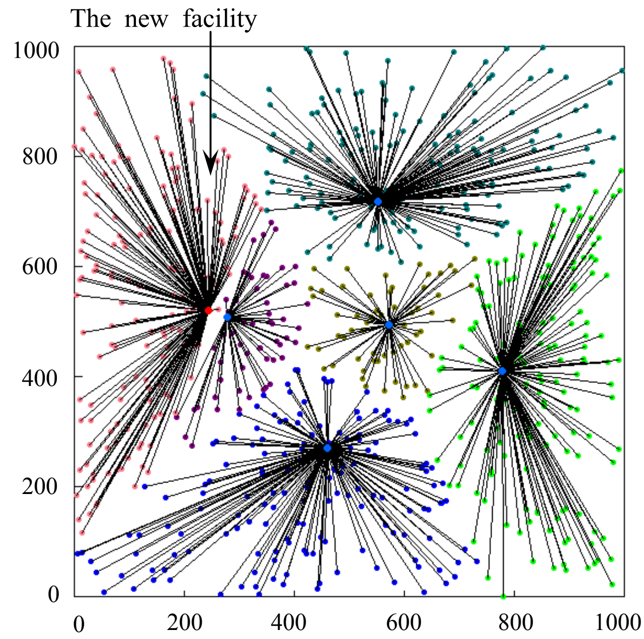

Figure 14: The reassignment of Figure 13 to the new facility and the 5 rival facilities

Figure 9 shows one of assignment results with 5 rival facilities and 150 clients before snatching. The locations of the 5 rival facilities are assigned by our PSO method. Figure 10 shows snatching result of Figure 9. 23 clients are assigned to the new facility.

Figure 11 shows the assignment of 750 clients to the nearest rival facilities. 5 rival facility locations are generated by random numbers. Figure 12 shows one of snatching results in 50 trials. 151 clients are assigned to the new facility.

Figure 13 shows one of assignment results before snatching. 5 rival facility locations are determined by our PSO method. Figure 14 shows the one of snatching results in 50 trials. 160 clients are assigned to the new facility.

After performing the procedure mentioned above, we could almost reach the maximum solutions using any initial particles at 100^{th} iteration. The smaller particle sizes such as 10 or 30 are not preferable to get good solutions.

4. CONCLUSIONS

In this paper, we examined various combinations of parameter values in our procedure using PSO to solve the facility location problems. Through many numerical experiments, to get better solutions in a shorter computation time depends upon appropriate conbinations of (V,S) in equalization problems of the number of clients.

From our numerical experiments, it is preferable that S is up to five times as many iterations as V so that desirable solutions in a acceptable computation time are obtained by Differential and Simplified Methods.

In the case of the rival facility problem,the larger number of partilces shows a trend to converge to a better solution in a shorter computation time.

In conclusion, setting the adequate values to the parameters in our procedure, our PSO method can reach the good soluion in a shorter computation time. At the same time, our results show the effectiveness of our approach to solve these types of facility location problems.

Our procedure can reach the reasonable results in less than 10 seconds even when we solve the client equalization problem with 750 clients and 5 faclitiles using Diffrential method. We are sure this computation time is acceptable to sovle this kind of location problems.

Our further research is to develop the method when two or more facilities should be allocated in the rival facilities.

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