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Reliable facility location design under disruptions

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Abstract

Distribution networks have been facing an increased exposure to risk of unpredicted disruptions causing significant economic forfeitures. At the same time, the existing literature features very few studies which examine the impact of facility fortification for improving network reliability. In this paper, we present two related models for design of reliable distribution networks: a reliable P-median problem (RPMP) and a reliable uncapacitated fixed-charge location problem (RUFL). Both models consider heterogenous facility failure probabilities, one layer of supplier backup, and facility fortification within a finite budget. Both RPMP and RUFL are formulated as nonlinear integer programming models and proved to be $\mathcal{NP}$-hard. We develop Lagrangian relaxation-based (LR) solution algorithms and demonstrate their computational efficiency. We compare the effectiveness of the LR-based solutions to that of the solutions obtained by a myopic policy which aims to fortify most reliable facilities regardless of the demand topology. Finally, we discuss an alternative way to assess the effectiveness of the design solutions by using the rate of return on fortification investment.

Keywords: facility location, distribution networks, design, reliable, disruptions, fortification

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1 Motivation

Distribution networks are referred to the entire chain of intermediaries and transportation logistics for distribution of goods and services from the suppliers to the consumers. Modern distribution networks are complex engineered systems due to their size, span, the nature of customer assignment, and the network flow. At the same time, more and more enterprises have been embracing the philosophy of lean manufacturing with an ever increasing reliance on consolidated suppliers, outsourcing, slim inventories, and just-in-time production and delivery. Inasmuch as such reductionism has boosted the operational efficiency of the companies, it has also elevated their risk exposure to unpredicted disruptions. Such disruptions, as triggered by forces of nature, process hazards, and human intervention, can have a potential to entail staggering economic ramifications. This is evidenced by the following sample of recent multi-billion enterprise forfeitures lost to disrupted distribution networks.

In March of 2000, a fire event halted a Philips's semiconductor plant in New Mexico, U.S. for nine months, causing a \$40 million direct sales loss to Philips and an indirect loss of \$2.34 billion to Ericsson's mobile phone division [28]. In March of 2001, the U.S. banned the meat import from the European Union in fear of potential spread of the foot-and-mouth disease originated in the U.K. The ban was applied to 15 countries and affected four percent of the U.S. pork import [19, 23]. In September 11, 2001, following the terrorist attack, all U.S. borders were closed and all flights canceled for several days. This lockdown forced Ford Motors to idle several assembly lines due to the lack of components supplied from overseas [7, 29]. Two years later, a deadly SARS outbreak disrupted among many other industries the furniture manufacturing sector of China, which accounted for about 15 percent of all furniture sold in the U.S. [11, 12]. More recently, in 2005, the aftermath of hurricane Katrina caused a severe disruption to the crude oil production in the Gulf of Mexico amounting nearly 1.4 million barrels a day [5, 13, 20, 32]. The above and some other examples [1, 2, 10, 27, 34] reveal the acute need for distribution networks designed to effectively balance the efficiency and robustness requirements.

Design for reliability of distribution networks can be accomplished by implementation and integration of both proactive and reactive mitigation options, including incorporation of backup and redundancy measures, investment in reliability improvement of existing facilities, and assuring rapid recovery of disrupted suppliers and distributors. Ideally, network design should evolve with disruptive events by updating the risk profile of the network constituents.

2 Status of Current Literature

Most of the existing literature on design of distribution networks takes its roots in the classical P-median [33] and the uncapacitated fixed-charge location problems [21]. Both of these problems seek to choose facility locations and assignments of customers to minimize the total transportation (and construction) cost. In both original models, all facilities are assumed to be perfectly reliable. However, as evidenced from the above examples, facilities can experience disruptions which can cause re-assignment of customers to more distant available suppliers or forfeiture of their demand. This can substantially increase both the transportation costs and customer dissatisfaction. It is therefore important to consider facility failures and measures for reliability improvement in network design.

The recent literature features a number of studies on facility location in the presence of random disruptions. An excellent comprehensive review of these works can be found in [30]. Below we present an up-to-date summary of the most relevant papers in this area.

In [31], Snyder and Daskin presented two reliability models for facility location: a reliable P-median and a reliable uncapacitated fixed-charge location model. In both models, each customer was assigned a primary supplier and a number of backup suppliers, of which at least one was required to be totally reliable. If the current supplier failed, the customer was served by the next available backup supplier. Facility failure probabilities were assumed to be equal and mutually independent. In [6], Cui *et al.* relaxed the assumption of homogeneous failure probabilities in [31] to location specific probabilities. Li and Ouyang expanded this direction to correlated, site specific failure probabilities [17].

[16] looked at an more integrated facility location design problem and considered the case in which both the supplier and retailers are disrupted randomly. The model sought to determine the optimal locations of retailers, customer assignations and inventory policy. [22] introduced the p-robustness criterion so that the designed network performs well in both disrupted and normal conditions. A hybrid metaheuristic algorithm was proposed.

A few recent papers have taken the analysis one step further and examined the impact of facility fortification for reliability improvement of the network. In [4], Church *et al.* examined two related network interdiction problems: the r-interdiction median and the r-interdiction covering problem. Both models are based on the P-median problem. The r-interdiction median problem seeks to find a subset of $r \leq P$ facilities, which if removed from the network, causes the highest loss of the network throughput. Whereas, the r-interdiction covering problem seeks to find such a subset which results in the maximal network coverage loss. In both models, once the critical subset is identified, some of its members can be fortified, as was done in later papers by Church and Scaparra [3, 25, 26].

So far, to the best of our knowledge, the only work on network design with fortification is by Lim *et al.* [18]. The authors analyzed the uncapacitated fixed-charge facility location model with two types of facilities: unreliable and totally reliable or “hardened”. The facility failure probabilities were assumed to be independent and location specific. The model assumed one primary supplier and one *totally reliable* backup supplier for each customer. The objective of the model was to determine the optimal number and location of both types of facilities as well as the customer assignment. The model was formulated as an integer programming model and a Lagrangian relaxation-based solution algorithm was developed. Although the authors incorporated the fixed cost of locating a reliable facility in the objective function, the total available fortification budget was not considered. In other words, the formulation essentially assumed an unlimited fortification budget. Since this assumption does not restrict the number of reliable facilities, the optimal solution may not fit available fortification resources.

In our paper, we develop two related models for facility location design under the risk of disruptions: a reliable P-median problem (RPMP, Section 3) and a reliable uncapacitated fixed-charge location problem (RUFL, Section 4). Similar to [6, 18], in both our models, we assume that the facility failure probabilities are independent and location specific. As in [18], we also assume one layer of supplier backup. To further enhance the network reliability, we incorporate fortification of selected facilities. As a result of fortification, the facility reliability is improved at some cost. The cost of facility fortification is considered to be location specific and made up of two components: a fixed setup cost and a variable cost for reliability improvement. In both models, we assume that if fortified, the facility becomes totally reliable. Both models incorporate a finite fortification budget constraint. Both models seek to choose the optimal facility location and fortification strategy as well as the assignment of customers.

Both the RPMP and RUFL problems are formulated as nonlinear integer programming models which are shown to be $\mathcal{NP}$-hard. For both models, we develop Lagrangian relaxation-based solution algorithms (Sections 3, 4). We present computational results demonstrating the efficiency of the developed algorithms (Sections 5.2, 5.3). We compare the effectiveness of the LR-based solutions to that of the solutions generated by a myopic policy which aims to fortify most reliable facilities regardless of the demand topology (Section 5.4). The comparison is done at different levels of the fortification budget. Finally, we discuss an alternative way to assess the effectiveness of the design solutions by determining the rate of return on fortification investments (Section 5.5).

Comparing to Lim *et al.* [18] and Cui *et al.* [6], our paper presents the following *main advances*:

(i) Our model incorporates a fortification budget constraint. As a result, it provides a more realistic decision support for network design and *assures that the optimal solution is matched to available reliability improvement resources*, no matter how scarce or abundant these resources are.

(ii) Our formulation enables the strategic decision maker to assess the rate of return on fortification investment and compare it to that of alternative investment opportunities. For example, a company may choose to invest in network fortification only if the rate of return exceeds the minimum acceptable rate of return (MARR [24]).

(iii) Our model allows periodic fortification upgrades whereby reliability of an existing network can be improved as additional fortification budget becomes available. Examples include gradual release of fortification resources or availability of excess cash flow which can be channeled into fortification. To allocate additional fortification budget for an existing network, the model has to be re-solved with fixed facility location decision variables. The ability of our model to support gradual fortification results from incorporation of the budget constraint and separation of the location selection and fortification decision variables, which are combined in Lim *et al.* [18].

3 The Reliable P-Median Problem (RPMP)

The model extends the reliable P-median facility location problem introduced by Snyder [31] by considering heterogeneous facility failure probabilities and facility fortification. The model seeks to minimize the total expected transportation cost by optimally locating P facilities, allocating a finite fortification budget, and assigning the customers. We first formulate this problem as a nonlinear integer programming model and then develop a Lagrangian relaxation-based solution algorithm.

3.1 Problem Formulation

We define I to be the set of customers, J the set of potential facility locations, and P the number of facilities to open. Each customer $i \in I$ has demand h_i. Let $d_{ij} \geq 0$ be the cost of transporting one unit of demand from facility location $j \in J$ to customer i (with the convention that $d_{ii} = 0 \; \forall i$). Associated with each facility j is the failure probability $0 \leq q_j \leq 1$. Each customer is assigned a primary supplier and a different backup supplier (as in [18]). While Lim *et al.* [18] required each backup facility to be "totally reliable" (i.e., available at all times), we stipulate that for any customer, the probability of a simultaneous failure of its primary and backup supplier is negligible.

Our model incorporates facility fortification whereby reliability of facilities can be improved at some cost. The total cost c_j of fortifying facility j includes the setup cost and the variable cost components. The setup cost s_j is a fixed cost required to implement facility fortification (examples include the costs of R&D, contract negotiation, overhead, personnel training, etc.). The variable fortification cost varies with the amount of reliability improvement of the facility. Examples include the cost of acquiring and installing the units of protective measures, the cost of procurement and storage of backup inventory, and the cost of hiring extra workforce. We define r_j as the cost associated with the unit reduction in the failure probability of facility j. Our model incorporates a total available fortification budget B.

Model assumptions. We impose the following set of assumptions: 1) the events of facility failures are independent (as in [6, 18, 31]), 2) for any customer, if the primary supplier fails, the backup supplier is available, 3) if a facility is fortified, it becomes non-failable, 4) if a facility fails, it becomes unavailable, and 5) all suppliers have unlimited capacities (as in [6, 18, 31]).

Our model incorporates the following *decision variables*:

$$X_j = \begin{cases} 1, & \text{if a facility is opened at location } j; \\ 0, & \text{otherwise,} \end{cases}$$

$$Y_{ij0} = \begin{cases} 1, & \text{if customer } i \text{ has facility } j \text{ as its primary supplier;} \\ 0, & \text{otherwise,} \end{cases}$$

$$Y_{ij1} = \begin{cases} 1, & \text{if customer } i \text{ has facility } j \text{ as its backup supplier;} \\ 0, & \text{otherwise,} \end{cases}$$

$$Z_j = \begin{cases} 1, & \text{if facility } j \text{ is fortified;} \\ 0, & \text{otherwise.} \end{cases}$$

$$\textbf{(RPMP)} \text{ minimize } \sum_{i\in I}\sum_{j\in J}\left[h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j)) + h_i d_{ij} Y_{ij1} \sum_{r\in J,\, r\neq j} q_r Y_{ir0}(1-Z_r)\right]$$

subject to

$$\sum_{j\in J} Y_{ij0} = 1, \ \forall i \in I \tag{1a}$$

$$\sum_{j\in J} Y_{ij1} = 1, \ \forall i \in I \tag{1b}$$

$$Y_{ij0} + Y_{ij1} \leq X_j, \ \forall i \in I, \ j \in J \tag{1c}$$

$$\sum_{j\in J} X_j = P \tag{1d}$$

$$\sum_{j\in J} (s_j + r_j q_j)\, Z_j \leq B \tag{1e}$$

$$X_j, Z_j \in \{0, 1\}, \ \forall j \in J \tag{1f}$$

$$Y_{ij0}, Y_{ij1} \in \{0, 1\}, \ \forall i \in I, \ j \in J. \tag{1g}$$

The objective function of the RPMP is the expected total transportation cost associated with satisfying the demands of all customers. The term $\sum_{j\in J} h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j))$ represents the part of the expected transportation cost associated with customer i served by its primary supplier, where $(1-q_j(1-Z_j))$ is the probability that the supplier is available. The term $\sum_{j\in J} h_i d_{ij} Y_{ij1} \sum_{r\neq j} q_r Y_{ir0}(1-Z_r)$ is the cost of customer i served by its backup supplier, where $\sum_{r\neq j, r\in J} q_r Y_{ir0}(1-Z_r)$ is the probability that the primary supplier failed (recall that in this case, the backup facility is assumed to be available).

Constraints (1a) and (1b) respectively assure that each customer is assigned only one primary and one backup supplier. Constraint (1c) serves two purposes. First, it guarantees that only an open facility can serve as a supplier. It also assures that for each customer, its primary and backup suppliers are different facilities. Constraint (1d) demands P facilities to be opened. Constraint (1e) is the total fortification budget constraint. Finally, (1f) and (1g) are the integrality constraints.

The next theorem shows that the model above is $\mathcal{NP}$-hard.

Theorem 1 *The RPMP is $\mathcal{NP}$-hard.*

Proof. We prove this by showing that a special case of the RPMP is $\mathcal{NP}$-hard. Consider $I = J$ and $P = ||I||$. It then follows that there will be a facility open at each customer location. Assume that one of the facilities is totally reliable; assign index s to it (i.e., $q_s = 0$). We also assume that for each customer location $i \neq s$, $d_{is} < \infty$ and $d_{ij} = \infty$ for $j \neq s$ (recall that $d_{ii} = 0\ \forall i$).

Note that since $q_s = 0$ and $d_{ss} = 0$, customer s is assigned facility s as the primary supplier and no backup supplier is needed in this case. Also note that for each customer $i \neq s$, its primary and backup supplier will be assigned as facility i or facility s. In the case when facility i is chosen as the primary supplier and facility s as the backup, the expected transportation cost associated with customer i becomes $h_i d_{is} q_i$. In the other case, the cost is $h_i d_{is}$. Since $h_i d_{is} q_i \leq h_i d_{is}$, supplier i and s will be chosen as the primary and backup supplier, respectively.

At this point, since facility locations and customer assignments are determined, the problem reduces to selecting facilities for fortification. Note that fortification of facility i eliminates the need for its backup supplier, which results in the expected reward $h_i\, d_{is}\, q_i$. The problem then is to maximize the total expected reward gained from fortification subject to the fortification budget availability:

maximize $\big\{ \sum_{i\in I} h_i\, d_{is}\, q_i\, Z_i : \sum_{i\in I} (s_i + r_i\, q_i)\, Z_i \leqslant B, Z_i \in \{0,1\} \big\}$, which is the 0-1 knapsack problem. □

3.2 RPMP: Lagrangian Relaxation

As shown above, the RPMP is $\mathcal{NP}$-hard and has a nonlinear objective function. One possible solution is to linearize the model by introducing new variables, $U_{ij0} = Y_{ij0}\, Z_j$, $V_{ijr} = Y_{ij1}\, Y_{ir0}$, and $W_{ijr} = V_{ijr}\, Z_j = Y_{ij1}\, Y_{ir0}\, Z_j$, with necessary constraints. However, the resultant problem becomes excessively large even for moderately sized networks, which makes solving such cases using commercial solvers challenging (see also § 5). This motivates us to develop a Lagrangian relaxation-based algorithm.

3.2.1 Lower Bound

Relaxing the set of constraints (1c) using Lagrange multipliers u_{ij} yields the following subproblem.

(RPMP-LG)

$$\min \sum_{i\in I}\sum_{j\in J}\big[h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j)) + h_i d_{ij} Y_{ij1}\sum_{r\in J, r\neq j} q_r Y_{ir0}(1-Z_r)\big] + \sum_{i\in I}\sum_{j\in J} u_{ij}(Y_{ij0}+Y_{ij1}-X_j)$$

subject to (1a), (1b), (1d) – (1g), and $Y_{ij0}+Y_{ij1}\le 1$.

The objective function above can be rewritten as follows.

$$\begin{aligned}
&\sum_{i\in I}\sum_{j\in J}\big[h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j)) + h_i d_{ij} Y_{ij1}\sum_{r\neq j, r\in J} q_r Y_{ir0}(1-Z_r)\big] + \sum_{i\in I}\sum_{j\in J} u_{ij}(Y_{ij0}+Y_{ij1}-X_j)\\
=& \sum_{i\in I}\sum_{j\in J}\big[h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j)) + h_i d_{ij} Y_{ij1}\sum_{r\neq j, r\in J} q_r Y_{ir0}(1-Z_r) + u_{ij}(Y_{ij0}+Y_{ij1})\big] - \sum_{j\in J}\sum_{i\in I} u_{ij}X_j\\
=& \sum_{i\in I}\sum_{j\in J}\big\{\big[h_i d_{ij}(1-q_j(1-Z_j)) + u_{ij}\big]Y_{ij0} + \big[h_i d_{ij}\sum_{r\neq j, r\in J} q_r Y_{ir0}(1-Z_r) + u_{ij}\big]Y_{ij1}\big\} - \sum_{j\in J}\sum_{i\in I} u_{ij}X_j.
\end{aligned}$$

For a given $\mathbf{u}$, the optimal value of $\mathbf{X}$ can be found by ranking the values of $(-\sum_{i\in I} u_{ij})$ for all j and setting $X_j = 1$ if $(-\sum_i u_{ij})$ is among the P smallest ranked values, and setting $X_j = 0$ otherwise.

To solve the rest of the problem, we first consider the case when fortification budget is zero ($B = 0$). Then $\mathbf{Z} = \mathbf{0}$ and constraint (1e) can be eliminated. The simplified problem is shown below.

(M1)

$$\min \sum_{i\in I}\sum_{j\in J}\big\{\big[h_i d_{ij}(1-q_j) + u_{ij}\big]Y_{ij0} + \big[h_i d_{ij}\sum_{r\in J, r\neq j} q_r Y_{ir0} + u_{ij}\big]Y_{ij1}\big\}$$

subject to (1a), (1b), (1g).

Note that relaxing constraints (1c) allows a customer to be assigned to a facility which is not open. Constraints (1a) and (1b) still assure that each customer is assigned only one primary and one backup supplier. Note that (M1) is separable in i, so that in order to solve the problem, it suffices to optimally assign a primary and a backup supplier to each customer. For a given customer i, if facility v and w are selected as the primary and backup supplier, respectively, the objective function of (M1) associated with customer i becomes $\Phi_i(v,w) = h_i d_{iv}(1-q_v) + u_{iv} + h_i d_{iw} q_v + u_{iw}$. To find the optimal assignment, we enumerate the values of $\Phi_i(v,w)$ for all $v, w \in J$ to find $\Phi_i^* = \min_{v,w}\{\Phi_i(v,w)\}$.

Now we consider the case $B > 0$. Suppose again that customer i is assigned facility v and w as the primary and backup supplier, respectively. Suppose now that facility v is fortified (i.e., $Z_v = 1$). The objective function of the simplified problem (for customer i) then becomes $\Psi_i(v,w) = h_i d_{iv} + u_{iv} + u_{iw}$. Let $\Psi_i^*(v) = \min_w\{\Psi_i(v,w)\}$.

We now let $E_i(v) = \max\{\Phi_i^* - \Psi_i^*(v), 0\}$. In other words, $E_i(v)$ is an improvement, if any, gained from fortifying facility v, for customer i. Then the objective is to maximize the utilization of the fortification budget over all $v \in J$, for all customers $i \in I$. For this purpose, we first introduce a new variable K_{ij} as follows.

$$K_{ij} = \begin{cases} 1, & \text{if customer } i \text{ is assigned a fortified primary supplier } j;\\ 0, & \text{otherwise.}\end{cases}$$

Then the problem becomes as following.

(M2)

$$\max \sum_{i \in I} \sum_{j \in J} E_i(j) K_{ij}$$

subject to

$$K_{ij} \leq Z_j, \;\; \forall i \in I, \; \forall j \in J \tag{2a}$$

$$\sum_{j \in J} K_{ij} \leq 1, \;\; \forall i \in I \tag{2b}$$

$$\sum_{j \in J} (s_j + r_j q_j)\, Z_j \leq B, \tag{2c}$$

$$K_{ij} \in \{0, 1\}, \;\; \forall i \in I, \;\; \forall j \in J \tag{2d}$$

$$Z_j \in \{0, 1\}, \;\; \forall j \in J. \tag{2e}$$

The objective function of (M2) is to maximize the total improvement from fortification when compared to the optimal objective function value of (M1). Subtracting the optimal objective function value of (M2) from the optimal objective function value of (M1) gives the optimal objective function value of the (RPMP-LG). The set of constraints (2a) assures that facility j, as the primary supplier of customer i, must be fortified in order to realize improvement $E_i(j)$. The set of constraints (2b) guarantees that customer i gets assigned to no more than one fortified supplier. Constraint (2c) is the fortification budget constraint. Constraints (2d) and (2e) are standard integrality constraints. (M2) is solved by using CPLEX in our algorithm.

3.2.2 Upper Bound

At each iteration of the Lagrangian procedure, a lower bound and an upper bound for (RPMP) are obtained. The solution to (RPMP-LG) provides a lower bound. If the solution is feasible, it also provides an upper bound, which is then optimal for (RPMP). Otherwise, if the solution is infeasible, we construct a feasible solution which becomes an upper bound. We apply the following *heuristic*.

In the solution of (RPMP-LG), exactly P facilities are open. For each customer, we select the closest and second closest open facility as the primary and backup supplier, respectively. To decide which facilities to fortify, we let $G(j)$ be the set of customers who have facility j as the primary supplier.

For each customer $i \in G(j)$, if facility j is not fortified, the corresponding expected transportation cost is $h_i d_{ij}(1 - q_j) + h_i d_{ir} q_j$, where r represents its backup supplier. If facility j is fortified, the expected cost becomes $h_i d_{ij}$. The total expected cost reduction from fortifying facility j is then $\varphi_j = \sum_{i \in G(j)} h_i (d_{ir} - d_{ij}) q_j$. Thus, the objective is to maximize the utilization of the fortification budget over P open facilities. The problem becomes as follows.

$$\max\Big\{ \sum_{j \in J} \varphi_j Z_j : \sum_{j \in J} c_j\, Z_j \leqslant B, Z_j \in \{0, 1\}\Big\}.$$

This is a knapsack problem which is solved by CPLEX in our algorithm rather easily. The described heuristic performed well in the computational tests (see § 5).

3.2.3 Multiplier Initiation and Updating

As discussed in [31], the performance of Lagrangian relaxation algorithms can be sensitive to the choice of initial multipliers. In order to obtain a good initial multiplier, we examined the final multipliers of the cases where (RPMP) was solved to optimality. We found that the formula $u_{ij} = h_i/\|I\|$ generated efficient initial multipliers for our problem.

Once the algorithm starts running, at each iteration k, we use subgradient optimization [9] to update $\mathbf{u}$ by setting

$$u_{ij}^{k+1} = u_{ij}^{k} + t_k\left(Y_{ij0} + Y_{ij1} - X_j\right),$$

where t_k is a step size, $t_k = \dfrac{\lambda_k(z^* - z(\mathbf{u}^k))}{\| Y_{ij0} + Y_{ij1} - X_j \|^2}$.

In the formula above, λ_k is a constant at iteration k, initially set to $\lambda_0 = 2$, as in [9]. We divide the values of λ_k by 2 when every 60 consecutive iterations fail to improve the lower bound. Also, z^* is the best known upper bound, and $z(\mathbf{u}^k)$ is the lower bound when the multipliers are equal to $\mathbf{u}^k$.

The algorithm terminates when any of the following criteria are met:

- $(z^* - z(\mathbf{u}^k))/z^* \le \epsilon$, for some optimality tolerance ϵ, specified by the user,
- $k > k_{max}$, for some iteration limit k_{max}.

4 The Reliable Uncapacitated Fixed-Charge Facility Location Model

The RUFL model can increase the network reliability of RPMP by relaxing the restriction on the number of open facilities. Our RUFL model extends the reliable fixed-charge facility location problem introduced by Snyder [31] by considering heterogeneous facility failure probabilities and facility fortification. The model seeks to minimize the sum of the total facility construction cost and the expected transportation cost by optimally selecting facility locations, allocating a finite fortification budget, and assigning the customers. RUFL is formulated as a nonlinear integer programming model.

4.1 Problem Formulation

The formulation is similar to RPMP with an addition of the cost f_j of constructing facility j.

$$\textbf{(RUFL)}\ \text{minimize} \sum_{j\in J} f_j X_j + \sum_{i\in I}\sum_{j\in J}\left[h_i d_{ij} Y_{ij0}(1 - q_j(1 - Z_j)) + h_i d_{ij} Y_{ij1} \sum_{r\in J,\, r\neq j} q_r Y_{ir0}(1 - Z_r)\right]$$

subject to

$$\sum_{j\in J} Y_{ij0} = 1,\ \forall i \in I \tag{3a}$$

$$\sum_{j\in J} Y_{ij1} = 1,\ \forall i \in I \tag{3b}$$

$$Y_{ij0} + Y_{ij1} \le X_j,\ \forall i \in I, j \in J \tag{3c}$$

$$\sum_{j\in J} c_j Z_j \leqslant B, \tag{3d}$$

$$X_j, Z_j \in \{0, 1\},\ \forall j \in J \tag{3e}$$

$$Y_{ij0}, Y_{ij1} \in \{0, 1\},\ \forall i \in I,\ j \in J. \tag{3f}$$

The formulation is similar to (RPMP) except that in (RUFL), the total construction cost is included in the objective function and the number of facilities to be opened is not restricted to P. Similar to [6, 18, 31], our formulation does not consider a construction budget.

Theorem 2 *The RUFL is $\mathcal{NP}$-hard.*

Proof: Note that when $B = 0$ and $q_j = 0$ for all $j \in J$, the RUFL becomes the classical uncapacitated fixed-charge location problem. □

4.2 Solution Method

Lower bound. We relax constraints (3c) to obtain the following Lagrangian subproblem.

(RUFL-LG)

$$\min \sum_{j\in J} f_j X_j + \sum_{i\in I}\sum_{j\in J}\big[h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j)) + h_i d_{ij} Y_{ij1} \sum_{r\neq j, r\in J} q_r Y_{ir0}(1-Z_r)\big]$$
$$+ \sum_{i\in I}\sum_{j\in J} u_{ij}(Y_{ij0} + Y_{ij1} - X_j)$$

subject to (3a), (3b), (3d) – (3f), and $Y_{ij0} + Y_{ij1} \leq 1$.

The objective function can be rewritten as follows.

$$\min \sum_{j\in J}(f_j - \sum_{i\in I} u_{ij})X_j + \sum_{i\in I}\sum_{j\in J}\big(\big[h_i d_{ij}(1-q_j(1-Z_j)) + u_{ij}\big]Y_{ij0} + \big[h_i d_{ij} \sum_{r\neq j, r\in J} q_r Y_{ir0}(1-Z_r) + u_{ij}\big]Y_{ij1}\big)$$

For a given $\mathbf{u}$, the optimal value of $\mathbf{X}$ is found by setting $X_j = 1$, if $(f_j - \sum_{i\in I} u_{ij}) < 0$ and $X_j = 0$ otherwise. Customer assignment and facility fortification are solved as described in § 3.2.1.

A simple heuristic is used to obtain an initial upper bound. Starting with facilities opened at all locations, an iterative procedure is used to drop one facility at a time in a greedy manner. At each dropping iteration, customer assignment and facility fortification are done as in § 3.2.2. The solution with the minimum objective function value is then used as the initial upper bound. Once the Lagrangian procedure starts, at each iteration, an upper bound is obtained using the same heuristic as in § 3.2.2. The multiplier updating is conducted in a manner described in § 3.2.3.

5 Computational Results

5.1 Experimental Design

We tested the performance of both the RPMP and RUFL solution algorithms on four datasets [14, 15] containing 30, 49, 100, and 150 nodes, respectively. The last three datasets were adapted from [31] whereas the 30-node dataset was generated by arbitrarily selecting 30 nodes from the 49-node dataset. Demands h_i were taken from [31]. The Euclidean distance between nodes i and j was used as the transportation cost d_{ij}. We let the sets I and J be equal. The failure probabilities q_j were randomly generated from $U \sim [0, 0.05]$. The fortification setup cost s_j was set to 30. The variable fortification cost r_j (associated with the unit reduction in the failure probability) was randomly generated from $U \sim [0, 3000]$. We tested the RPMP algorithm for $P = 5$ and $P = 8$. The values of P were matched to those in [31] in an attempt to benchmark the results to assess algorithmic effectiveness. Since we consider designing distribution networks for a single product, having eight distributors (and up to 150 centers of customers) of the same product may represent a reasonably large regional network (depending on the industry and the type of economy (centralized vs. decentralized)). Modeling distribution networks with a large number of suppliers is an interesting problem from the computational perspective but probably less so from the point of network robustness to disruptions.

To test the RUFL algorithm, the facility construction cost f_j was randomly drawn from $U \sim [500, 1500]$. Both algorithms were tested for the values of fortification budget B ranging between 0 and 360. The algorithms were coded in C++ and were run on a Windows XP SP3 PC with a 2.2 GHz Duo core CPU and 2.0 GB of physical RAM. **(M2)** and the knapsack problem in the heuristic was solved using CPLEX10.1 within the algorithms. The gap tolerance and the maximum number of iterations were set to 0.5% and 3000, respectively.

5.2 Performance of the RPMP and RUFL Algorithms

Results for the RPMP and RUFL algorithms using four datasets are listed in Table 1 and Table 2, respectively. The abbreviations LB and UB stand for the lower bound and the upper bound, respectively. The gap is the difference (in %) between the upper and lower bounds.

The algorithms solved 87 out of a total of 96 cases to 0.5% optimality (not optimal). For both algorithms, the computing time increases substantially with the size of the problem. This can be partially explained by noting that at each iteration, the number of enumerations required for solving **M1** increases quadratically with the problem size and so does the size of **M2**. In general, solving the RUFL model is more time demanding which can be attributed to the fact that its underlying fixed-charge facility location problem is harder to solve. This can also be the reason that the gaps for the 150-node cases did not improve after the algorithm exhausted 3,000 iterations.

5.3 Comparison with CPLEX Solver

To compare the performance of our algorithms to that of the CPLEX solver, the RPMP and RUFL models were linearized using the method described in § 3.2. The CPLEX code was written in C++ using the CPLEX Concert Technology. For the purpose of comparison, only the CPLEX solver CPU time was measured. The comparison of the performance of both algorithms and the CPLEX solver was done on the same computer with version 10.1 of CPLEX. We used a total of fourteen 30-node cases solvable by CPLEX (CPLEX failed to solve larger size cases due to insufficient memory). The comparison of the computation times between CPLEX and both algorithms is shown in Table 3. It can be observed that both our algorithms are significantly faster than CPLEX where the total computation time for all fourteen cases is only 20 seconds versus 4344 seconds. Compared to the optimal solutions obtained by CPLEX, the final feasible solutions by using our algorithms are also optimal.

5.4 Comparison with a Myopic Policy

To illustrate the effectiveness of the LR-based design solutions, we compared them to the solutions obtained by using a myopic policy which allocates fortification budget to the most reliable facilities first. By doing so, the policy does not account for the demand topology, hence is the name myopic.

		P=5				P=8			
Nodes	B	LB	UB	Gap, %	Time, s	LB	UB	Gap, %	Time, s
30	0	3694.2	3694.2	-	0.7	2192.5	2201.0	0.33	0.5
30	30	3694.2	3694.2	-	0.7	2192.5	2201.0	0.33	0.5
30	60	3573.8	3573.8	-	0.8	2144.1	2150.1	0.28	0.9
30	120	3502.5	3502.5	-	0.6	2096.8	2102.7	0.28	1.6
30	180	3366.1	3382.1	0.47	0.7	2044.4	2053.7	0.45	0.6
30	240	3327.9	3344.4	0.50	0.9	2014.8	2024.7	0.49	3.0
30	300	3309.7	3309.7	-	0.9	1980.5	1990.5	0.50	1.3
30	360	3283.5	3299.2	0.48	0.4	1981.4	1991.3	0.49	1.2
49	0	8826.3	8870.4	0.50	3.4	5874.6	5903.2	0.48	2.1
49	30	8826.3	8870.4	0.50	3.3	5874.6	5903.2	0.48	2.0
49	60	8704.7	8736.3	0.36	8.8	5772.2	5801.1	0.50	3.1
49	120	8625.0	8653.5	0.33	4.9	5724.6	5752.8	0.49	2.3
49	180	8538.4	8538.5	-	29.3	5678.3	5705.6	0.48	3.5
49	240	8417.0	8459.0	0.50	45.1	5638.9	5647.3	0.14	5.2
49	300	8360.0	8401.9	0.50	24.3	5625.1	5627.0	0.03	8.0
49	360	8325.4	8366.9	0.50	10.2	5580.9	5608.1	0.48	27.0
100	0	17594.4	17682.8	0.49	56.3	12711.3	12775.1	0.49	27.3
100	30	17594.4	17682.8	0.49	59.7	12711.3	12775.1	0.49	30.3
100	60	17328.8	17380.3	0.30	27.1	12571.7	12593.1	0.16	32.3
100	120	17086.4	17172.2	0.50	146.4	12431.5	12491.3	0.47	96.4
100	180	16977.0	17061.6	0.50	64.1	12311.3	12372.8	0.49	27.6
100	240	16897.5	16927.0	0.17	66.7	12283.6	12339.3	0.45	63.9
100	300	16847.2	16847.2	-	48.7	12230.6	12252.4	0.17	44.9
100	360	16810.0	16886.6	0.45	31.1	12198.9	12198.9	-	46.8
150	0	20136.7	20237.0	0.49	98.3	14682.7	14755.9	0.49	143.2
150	30	20136.7	20237.0	0.49	104.4	14682.7	14755.9	0.49	151.7
150	60	19881.7	19968.5	0.43	120.8	14659.3	14685.7	0.17	190.4
150	120	19760.8	19829.1	0.34	208.4	14494.1	14566.7	0.49	263.9
150	180	19653.5	19747.2	0.47	221.3	14436.3	14508.8	0.49	406.9
150	240	19494.4	19592.3	0.50	188.9	14389.3	14462.3	0.50	783.3
150	300	19451.0	19547.3	0.49	189.2	14385.3	14440.8	0.38	159.0
150	360	19512.0	19589.9	0.40	171.7	14389.7	14401.5	0.66	872.2

Table 1: Testbed performance results for the RPMP algorithm

Two myopic policies were implemented for a separate comparison with the RPMP and RUFL optimal (within 0.5% gap) strategies. For a RPMP-type myopic policy, P most reliable facilities were open. For each customer, the closest and the second closest open facility were assigned as the primary and the backup supplier, respectively. Facilities were fortified starting from the most reliable until either the fortification budget was used up or all open facilities were fortified. For a RUFL-type myopic policy, for each level of B, the number of facilities to open N was not fixed but varied from one to $|J|$. For each value of N, facility fortification and customer assignment were done in the same way as for the RPMP-type myopic policy. For a fixed B, the value of the RUFL-type myopic policy was chosen to be the minimum of the total expected cost over the range of values of N.

Figures 1(a) and 1(b) show the results of the performance comparison of the optimal RPMP and

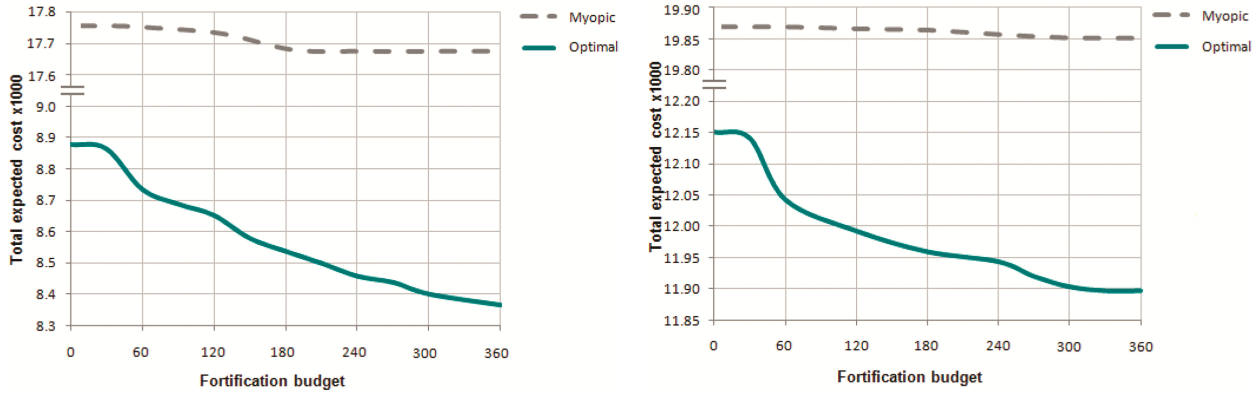
Nodes	B	LB	UB	Gap, %	Time, s	Nodes	B	LB	UB	Gap, %	Time, s
30	0	7963.9	8003.9	0.50	0.9	100	0	17295.3	17380.3	0.49	62.8
30	30	7963.9	8003.9	0.50	1.4	100	30	17295.3	17380.3	0.49	66.3
30	60	7854.1	7886.3	0.41	1.0	100	60	17185.6	17271.4	0.50	63.6
30	120	7751.5	7789.8	0.49	2.4	100	120	17084.9	17178.5	0.54	98.9
30	180	7713.3	7751.1	0.49	2.4	100	180	17056.0	17140.5	0.49	187.8
30	240	7701.0	7734.3	0.43	2.9	100	240	17031.0	17116.8	0.50	198.6
30	300	7697.7	7734.3	0.47	2.2	100	300	16999.8	17082.6	0.48	246.7
30	360	7696.0	7734.3	0.50	1.7	100	360	16971.0	17056.0	0.50	281.3
49	0	12090.9	12151.1	0.50	4.6	150	0	18953.7	19117.4	0.85	810.3
49	30	12090.9	12151.1	0.50	4.9	150	30	18953.7	19117.4	0.85	819.9
49	60	11983.0	12042.5	0.49	5.9	150	60	18840.4	19021.6	0.95	1199.6
49	120	11933.2	11992.4	0.49	11.6	150	120	18838.9	19000.3	0.84	906.4
49	180	11899.6	11959.3	0.50	12.1	150	180	18763.9	18916.9	0.80	1732.6
49	240	11884.4	11943.3	0.49	18.1	150	240	18586.8	18907.5	1.70	587.7
49	300	11846.5	11903.0	0.47	13.3	150	300	18600.2	18841.3	1.28	747.8
49	360	11837.2	11896.6	0.50	19.8	150	360	18545.7	18853.0	1.63	885.9

Table 2: Testbed performance results for the RUFL algorithm

Algorithm	P	B	CPLEX time, s	Alg. time, s
RPMP	5	20	243.1	0.8
	5	120	623.1	0.6
	5	180	282.8	0.7
	8	60	687.7	0.9
	8	120	948.1	1.1
	8	180	807.2	0.5
RUFL		30	93.5	1.4
		60	126.1	1.0
		90	151.6	2.5
		120	148.3	2.4
		180	97.1	2.4
		240	70.4	2.9
		300	50.2	2.2
		360	108.9	1.7

Table 3: Computation time for 30-node cases

RUFL strategies to that of their respective myopic policies. In both cases, the comparison was done for the values of B ranging between 0 and 360. The total expected cost was used as the measure of policy performance. As expected, in both cases, the curves for both optimal and myopic policies generally exhibit a downward trend as the fortification budget increases. For all curves, the presence of “flat” regions for the values of B between 0 and 30 is due to the fact that no facility can be fortified as the budget is consumed by the fixed fortification cost $s_j = 30$. For RPMP- and RUFL-type myopic policy curves, the presence of “flat” regions for the values of B exceeding 180 and 240, respectively is attributed to the fact that all facilities have been fortified and no further improvement can be made.



(a) RPMP case ($P = 5$, 49 nodes)

(b) RUFL case (49 nodes)

Figure 1: Comparison of the optimal RPMP and RUFL and their respective myopic policies

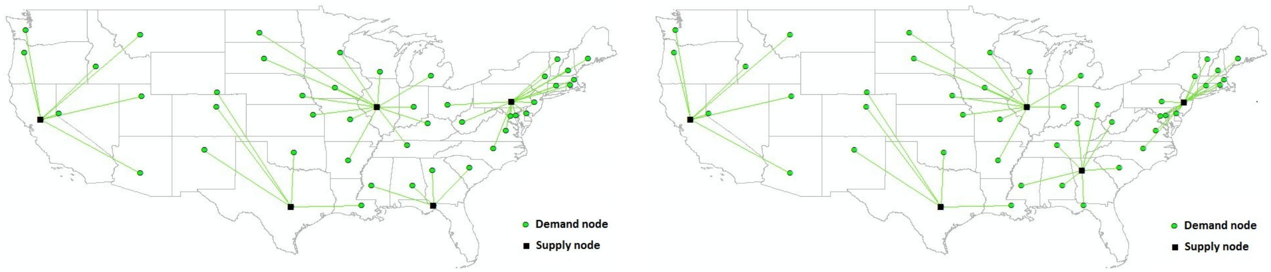
It can be observed that for both RPMP and RUFL optimal policies, the marginal reduction in the policy values diminishes with B. This effect can be explained by noting that in order to attain optimality, the model allocates the fortification budget to the facilities in the decreasing order of reduction in the total expected cost realized from fortification. For the RUFL case, the ROR becomes negative as the reduction in objective function value diminishes so fast and becomes less than the fortification investment after B=180.

It can be observed that in both cases, the optimal policies outperform the respective myopic policies over the entire range of B. Moreover, in both cases, the difference in policy performance widens as B increases. This can be explained as follows. As mentioned earlier, for both myopic policies, the curves flatten starting at certain levels of B as all facilities get fortified. This "rapid" total fortification is possible because myopic policies select most reliable facilities to open and fortify. In the case of the optimal policies, selection of facilities for fortification is governed not only by facility reliability but also by topology of demand. As a result, total fortification generally requires larger levels of B.

Analyzing optimal solutions for both RPMP and RUFL policies we can notice that in general, facility locations are chosen from among demand-heavy nodes. Table 4 shows an example of optimal facility locations for the case of RPMP (numbering of facilities was done in the decreasing order of demand). Figures 2(a) and 2(b) show the topology of the optimal solution for $B = 30$ and $B = 360$, respectively (the figures are used merely to illustrate the effect of fortification at different budget levels; similar figures can be found in [6, 18, 31], the details of the datasets used can be found in [31] and [8]). We observe that optimal locations were chosen from among the top ten demand nodes. This can be explained by noting that in our testbed, customer sites also serve as candidates for facility locations. Since the values of facility reliabilities are chosen from a uniform distribution, the selection of facilities is primarily driven by demand topology.

B	Locations
30	1, 3, 4, 5, 6
60	1, 3, 5, 6, 11
120	1, 3, 5, 6, 11
180	1, 3, 5, 6, 11
240	1, 2, 3, 6, 11
300	1, 3, 6, 9, 11
360	1, 3, 6, 9, 11

Table 4: Optimal locations (RPMP, 49 nodes)

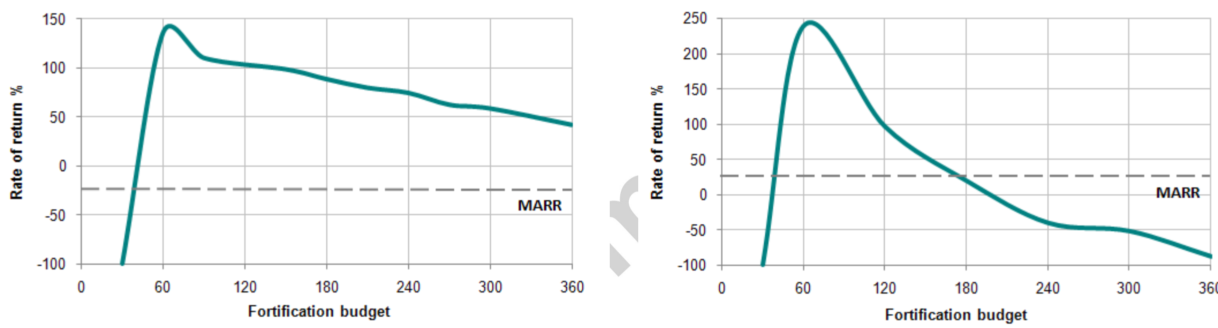


(a) $B = 30$ (b) $B = 360$

Figure 2: The optimal solution to RPMP ($P = 5$, 49 nodes)

5.5 Rate of Return on Fortification Investment

Calculation of the rate of return (ROR) on fortification investment allows a decision maker to assess the effectiveness of such investment when compared to alternative investment opportunities. For instance, a decision maker may choose to invest in network fortification only if the ROR exceeds the minimum acceptable rate of return (MARR) [24]. We illustrate such analysis for both RPMP and RUFL by considering the 49-node case with the values of fortification budget ranging between 0 and 360.



(a) RPMP optimal policy ($P = 5$, 49 nodes) (b) RUFL optimal policy (49 nodes)

Figure 3: Rate of return (ROR) on fortification investment

The optimal objective function value for $B = 0$ was used as the baseline. For each $B > 0$, the value of ROR (in %) was calculated as the overall reduction in the optimal objective function value (compared to the baseline) minus B, all divided by B.

Figures 3(a) and 3(b) depict the values of the ROR for RPMP and RUFL optimal policies, respectively. We can observe that for both policies, the ROR is negative for small values of B. This is due to the fact that no facility can be fortified as the budget is consumed by the fixed fortification cost. For both policies, the curves exhibit a sharp rise until reaching the maximum at $B = 60$, followed by a gradual decrease for higher values of B. The downward trend can be attributed to the fact that both models allocate the fortification budget to the facilities in the decreasing order of reduction in the optimal objective function value. For the RUFL policy, ROR becomes negative starting with $B = 180$ implying that the gain associated with reduction in the objective function does not offset the amount of fortification investment. Note that in the case of our study, for an arbitrarily chosen level of MARR, the RPMP optimal policy has a broader range of attractive fortification investment opportunities.

6 Conclusions

In this paper, we presented two related models for design of distribution networks exposed to risk of disruptions: a reliable P-median problem and a reliable uncapacitated fixed-charge location problem. Both models considered heterogenous facility failure probabilities and one layer of supplier backup. The facility failure probabilities were assumed to be independent and location specific. We considered facility fortification whereby the cost of fortification was assumed to be location specific. We assumed that if fortified, the facility became totally reliable. Both models considered a finite fortification budget.

Summary of the main results. Both RPMP and RUFL were formulated as nonlinear integer programming models which were proven to be $\mathcal{NP}$-hard. Our results showed that the developed Lagrangian relaxation-based algorithms were computationally efficient, demonstrating a distinctively better performance than CPLEX, particularly for solving large scale problems. Our comparison study revealed that both RPMP and RUFL optimal policies outperformed their respective myopic counterparts over the examined range of fortification budget. Both optimal policies exhibited a diminishing marginal reduction in the total expected cost as fortification budget increased. It was shown that facility locations were generally selected from among demand-heavy nodes. In the testbed case, when compared to RUFL, the RPMP optimal policy had a broader range of attractive fortification investment opportunities.

Contributions of the paper. Our RPMP model extends the reliable P-median facility location problem of Snyder *et al.* [31] by considering heterogeneous facility failure probabilities and facility fortification. Comparing to Lim *et al.* [18], our RUFL model incorporates the fortification budget constraint. As such, our model enables a decision maker to match the optimal solution to the available reliability improvement resources. Moreover, our formulation makes it possible to assess the effectiveness of fortification investments (by using ROR) and compare it to that of alternative investment opportunities. Finally, our formulation allows periodic fortification upgrades whereby the reliability of the existing network can be improved as additional fortification resources become available. This ability to support gradual fortification results from incorporation of the budget constraint and separation of the location selection and fortification decisions, which are combined in [18].

Limitations of the model. The principal assumption in our models was that all suppliers had unlimited capacities. Although this assumption is commonly found in reliable facility location models, it may be unrealistic. The practical importance of the capacitated case is also associated with its significant modeling complexity. This makes the capacitated case worthy of future study. We also assumed that facility failures were independent and once the primary supplier failed, the backup supplier was available. Relaxing these assumptions will be the focus of our future efforts. We also did not consider partial fortification, which may be more realistic in scenarios with limited fortification budget. To implement partial fortification, the models have to be redefined (to include new decision variables) and new algorithms have to be developed. The authors are currently working in this direction.

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- We propose a reliable P-median problem (RPMP) model and a reliable uncapacitated fixed-charge location (RUFL) model
- Models consider heterogeneous facility failure probabilities and one layer of supplier backup
- We incorporate facility fortification constrained by fortification budget
- An efficient Lagrangian relaxation solution algorithm is developed
- We include analysis of effectiveness of fortification investment