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# An integrated approach for reliable facility location/network design problem with link disruption

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## Abstract

Proposing a robust designed facility location is one of the most effective ways to hedge against unexpected disruptions and failures in a transportation network system. This paper considers the combined facility location/network design problem with regard to transportation link disruptions and develops a mixed integer linear programming formulation to model it. With respect to the probability of link disruptions, the objective function of the model minimizes the total costs, including location costs, link construction costs and also the expected transportation costs. An efficient hybrid algorithm based on LP relaxation and variable neighborhood search metaheuristic is developed in order to solve the mathematical model. Numerical results demonstrate that the proposed hybrid algorithm has suitable efficiency in terms of duration of solution time and determining excellent solution quality.

**Keywords**: Facility location; Network design; Reliability; Link disruption; LP relaxation; Variable neighborhood search.

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## 1. Introduction

Nowadays, there is an increasing attention to efficient ways to reduce the total costs. Accordingly, improving the performance is one of the strategic and serious problems for the stakeholders in the supply chain, including manufacturers, distributors and retailers in order to stay in today competitive market world. On the other hand, the worldwide economic downturn in recent years causes that many companies, especially in industrial countries, face numerous undesirable events in their systems and supply chains. Although some of the companies eliminate the effects of risks and disruptions arising at the operational level, many of them cannot manage the unexpected risks and face with significant financial losses. However, improvement and optimization of each part of the supply chain may help the companies manage a number of risks facing in their system. Facility location and allocation topic is one of the fundamental strategic decisions which may affect the cost of an integrated system such as initial investment and transportation costs. As a result, employing an effective and practical model may mitigate the problem. Therefore, a more practical solution is required to reduce cost and increase efficiency.

In this paper, we develop a new integrated approach to facility location problems with respect to the subjects of network design and system reliability as it can be named reliable facility location/network design problem (RFLNDP) in order to improve the efficiency of transportation network systems. The motivation of this research is to consider simultaneously two practical factors (network design and system reliability) to develop the mathematical modeling of facility location problems, which has not been considered until now based on the authors' best knowledge. Moreover, there are numerous practical instances of facility location problems in which simultaneous consideration of network design and system reliability can lead to more practical and realistic mathematical modeling of the problem. Locating gas compressor stations, designing water tubing networks in industries, and regional planning and locating schools, health care service centers and airline networks in service systems are the most obvious and practical paradigms in which simultaneous considering of network design and system reliability in locating facilities and assigning demands plays a critical role in order to improve the efficiency, practicality and also reliability of system. Therefore, study of facility location problems with such constitution can practically improve solving of the mentioned problems in supply chain systems, including industrial factories and service centers, by obtaining more effective and accurate solutions. In addition, with respect to the huge investment for facility location and network design, the attention has been paid to the failure of a system based on several disruptions in facility locating and network design has been increased recently (Qi & Shen, 2007; Qi, Shen, & Snyder, 2010; L.V. Snyder & Daskin, 2007), (Karimi-Nasab et al. 2013).

In the following section, an overview of the previous research will be given. Section 2 reviews the literature and describes the existing research gap. Section 3 describes the problem and also represents the model formulation. The problem-solving methodology is explained in Section 4 and the computational results are discussed in Section 5. Finally, the conclusion is presented in Section 6.

## 2. Review of the Related Literature

An overview of the previous studies will be primarily given in this section. We briefly review two main streams of literature that may be of interest for comparison: the literature on a facility location problem regarding to network design and also on facility location problem with respect to system reliability. The research gaps will be then highlighted.

### 2.1. Facility Location models and network design

As we know, facility location considers the optimization of the predefined objective functions, such as minimizing the operational cost or maximizing the area covering, in locating facilities and allocating customers to them. As a general view, the facility location problems can be classified according to different objective functions such as the *P*-median and *P*-center problems (Hakimi, 1964), the uncapacitated facility location problems (Kuehn & Hamburger, 1963), the maximum covering location problems (Church & ReVelle, 1974) and the set covering location problems (Toregas, Swain, ReVelle, & Bergman, 1971). On the other hand, in the network design, the basic problem is to optimally construct a network that enables some kind of flow, and possibly that satisfies some additional constraints. The nodes usually are given and the network is constructed from a set of potential links.

All of the aforementioned classical models locate facilities on a predetermined network. However, the topology of the underlying network may profoundly influence the optimal facility locations and can have many applications in industries and services. Also, it can affect some studies obviously illustrated its undeniable effect on improvement of the objective function value (Melkote, 1996; Melkote & Daskin, 2001a, 2001b). In other words, simultaneously considering the facility location and network design, the proposed problem can be described in more realistic formulation and modeling.

In the literature review, it is evident that Daskin et al. in 1993 introduced the first initial model of facility location/network design problem (FLNDP) (Daskin, Hurter, & VanBuer, 1993). They presented some preliminary results which showed the effect of network design topic in mathematical modeling of facility location problems and their optimal solution. Later, Melkote (Melkote, 1996) in his doctoral thesis developed three models for the FLNDP including uncapacitated FLNDP (UFLNDP), the capacitated FLNDP (CFLNDP), and the maximum covering location/network design problem (MCLNDP). The results of their thesis were published in (Melkote & Daskin, 2001a, 2001b). Drezner and Wesolowsky (Z. Drezner & Wesolowsky, 2003) proposed a new network design problem with potential links, each of which could be either constructed at a given cost. Moreover, each transportation link could be constructed as either a one-way or two-way link. They developed four basic problems subject to two objective functions: finally, they solved the problems by a descent algorithm, a simulated annealing (SA), a tabu search (TS), and a genetic algorithm (GA) as main solution procedures. In another study, Cocking (Cocking, 2008; Cocking & Reinelt, 2009) expanded some efficient approaches to solve the static budget constrained FLNDP. Some useful algorithms were developed to find good upper bounds and good lower bounds on the optimal solution. Simple greedy heuristics, a local search heuristic, metaheuristics including SA and variable neighborhood search (VNS), as well as a custom heuristic based on the problem-specific structure of FLNDP were the main heuristics and meta-heuristics which were proposed in a study conducted by Cocking's. Furthermore, a branch-and-cut algorithm were developed by applying the heuristic solutions as upper bounds, and cutting planes to improve the lower bound of the problem. The method reduced the number of nodes which were needed to approach optimality. Recently, Bigotte et al. (Bigotte, Krass, Antunes, & Berman, 2010) studied the FLNDP in which, the multiple levels of urban centers and multiple levels of network links were considered simultaneously for developing of a mixed integer mathematical model. In fact, in order to enhance the access of all kinds of facilities, the best transfers of urban centers and network links to a new level of hierarchy are determined. Jabalameli and Mortezaei (JabalAmeli & Mortezaei, 2011) proposed an extension of the CFLNDP in which the maximum the amount of demands can be carried by a link is limited. They presented a bi-objective mixed integer programming formulation of the problem and developed a hybrid algorithm to solve the problem. Contreras and Fernandez (Contreras & Fernández, 2011) reviewed the relevant modeling aspects, alternative formulations and several algorithmic strategies for the FLNDP. In fact, they studied general network design problems in which, design decisions to locate facilities and to select links on an underlying network are integrated with operational allocation and routing decisions to satisfy demands. Contreras et al. (Contreras, Fernández, & Reinelt, 2012) presented a combined FLNDP to minimize the maximum customer-facility travel time. They developed and compered two mixed integer programming formulations by generalizing the model of the classical *P*-center problem so that the models simultaneously consider the location of facilities and the design of its underlying network. Rahmaniani and Ghaderi (Rahmaniani & Ghaderi, 2013) assumed that there are several types of links in which their capacity, transportation and construction costs are different for connecting two nodes. They proposed a mixed-integer model to optimize the location of facilities and the underlying transportation network simultaneously in order to minimize the total transportation and operating costs.

### 2.2. Facility location models and system reliability

Another significant subject that can affect facility location and allocation is reliability. Most of the earlier studies assume that all parts of the considered system are always available and unfailable. However, these assumptions could affect the flexibility of their designs and significantly reduce their efficiency especially when some disruptions occurred. The terrorist attacks of 9/11, the catastrophic devastation caused by Hurricane Katrina (Barrionuevo, September 1 (2005); Latour, January 29 (2001); Mouawad, September 4 (2005)), and the Japan's tragic earthquake and the following tsunami in 2011 (Clark, March 12 (2011)) are the most obvious examples. These recent events show higher risks occurred because of disruptions which may cause change in the factors contributing to the modern business world. Although most of the designers and experts believe that the existing international supply chains are strong and reliable, in reality, many are fragile and easily disrupted when the unexpected events happen. Paying huge fines by Boeing Company in compensation for postponing the delivery of the Dreamliner 787 is an evident example (Bathgate, April 10 (2008)). Several potential threats can lead to disruptions in supply chain systems, e.g., natural disasters such as avalanche, hurricane, volcano, heavy rain or snow, industrial accidents; operational eventualities such as equipment failures or supplier discontinuities; power outages; labor strikes; and terrorism. Although these disruption events may only lead to short-term facility contingencies, they can also cause not only serious operational consequences, such as higher transportation costs, order delays, inventory shortages, loss of market shares, and so on, but also extended negative financial impacts. An empirical study by (Hendricks, 2005) has illustrated that over the time period of 1989–2000, the abnormal stock returns of firms that have been influenced by disruptions were nearly 40%. Evidence has also showed that these firms had a hard time recovering from the negative effects of disruptions and that their equity risk significantly increased after that. Similar findings are found in the study conducted by (Hicks, 2002).

It is mentioned that in a supply chain system, when a facility failure occurs, as an efficient solution, customers might be reassigned from their primary facilities to the other available facilities; likewise, when a link disrupts, extra transportation costs may be paid for transferring by the disrupted link because of several difficulties. As another solution, demands may be reassigned to the alternative links. In the above mentioned conditions, the transportation costs certainly increased.

In the traditional locational analysis literature, Drezner (Zvi Drezner, 1987) was one of the first researchers who proposed the mathematical models for facility location with unreliable suppliers. He studied the unreliable *P*-median and (*P*,*q*) center location problems, in which a facility has a predetermined probability of becoming inactive. In the following research way, Snyder and Daskin (L. Snyder & Ülker, 2005; L.V. Snyder, 2003; L. V. Snyder & Daskin, 2005) proposed an implicit formulation of the stochastic *P*-median and fixed charge problems based on level assignments, in which the candidate sites are subject to random disruptions with equal probability. Shen et al. (Shen, 2009) and Berman et al. (Berman, Krass, & Menezes, 2007) relaxed the assumption of uniform failure probabilities, modeled the stochastic fixed-charged facility location problem as a nonlinear mixed integer program, and proposed several heuristic solution algorithms. Berman et al. (Berman, et al., 2007) supposed on an asymptotic property of the problem and verified that the solution to the stochastic *P*-median problem coincides with the deterministic problem as the failure probabilities approach zero. They also proposed some efficient heuristics with bounds on the worst-case performance. Lim et al. (Lim, Bassamboo, & Chopra, 2009) presented a reliability continuum approximation (CA) approach for facility location problems with uniform customer density. For simplification, a specific form of failure-proof facility was proposed; a customer is always reassigned to a failure-proof facility after its nearest regular facility failed, regardless of other regular facilities. Cui et al. (2010) (Cui, Ouyang, & Shen, 2010) proposed a mixed integer program (MIP) formulation and a continuum approximation (CA) model to study the reliable uncapacitated fixed charge location problem (RUFL) which seeks to optimize the initial set up costs and expected transportation costs in normal and failure scenarios. The MIP determines the optimal facility locations as well as the optimal customer assignments, and the MIP is solved using a custom-designed Lagrangian Relaxation (LR) algorithm.

Hanley and Church (O'Hanley & Church, 2011) developed a new facility location–interdiction covering model for finding a robust alignment of the facilities that has a suitable efficiency in the worst situations of facility loss. They formulated the problem as two mathematical models. In the first model, all possible interdiction patterns are considered and a standard MIP formulation is proposed. In the second model, the optimal interdiction pattern is implicitly defined in terms of the chosen facility location, layout and more compact bi-level programming formulation was developed. Peng et al. (Peng, Snyder, Lim, & Liu, 2011) considered the effect of considering the reliability topic on logistic networks design with facility disruptions and presented that applying a reliable network design is often possible with negligible increases in total location and allocation costs which depends on decision makers opinion. They studied the commodity production/delivery system regardless of the open/close decisions on the arcs of supply chain system and with the application of the *p*-robustness criterion (which bounds the cost in disruption scenarios), the nominal cost (the cost when no disruptions occur), and reduce the disruption risk are minimized simultaneously. Recently, Liberatore et al. (Liberatore, Scaparra, & Daskin, 2012) introduced the problem of optimizing fortification plans in median distribution systems in the face of disruptions that involve large areas. They developed an effective exact solution algorithm to solve it optimally. Also, they showed empirically that ignoring correlation effects in a system can lead to suboptimal protection plans that result in an unnecessary increase in the system cost when disruptions take place. Moreover, Jabbarzadeh et al. (Jabbarzadeh, Jalali Naini, Davoudpour, & Azad, 2012) studied a supply chain design problem with the risk of disruptions at facilities and formulated the problem as a mixed-integer nonlinear program which maximizes the total profit for the whole system. The proposed model simultaneously determines the number and locations of facilities, the subset of customers to serve, the assignment of customers to facilities, and the cycle-order quantities at facilities. Shishebori et al. (Shishebori, Jabalameli, & Jabbarzadeh, 2013) considered facility disruptions as a constraint for the maximum allowable disruption cost of the system. They proposed a MINLP model for the problem and studied it as a case study. Li et al.

(2013) proposed two mathematical models, as a MINLP, for design of reliable distribution networks: a reliable *P*-median problem (RPMP) and a reliable uncapacitated fixed-charge location problem (RUFL). Also, in another study, Shishebori et al. (2014) formulated an efficient mathematical model which not only takes into account facility location costs, link construction costs, and transportation costs, but also constrains the maximum allowable disruption cost of the system, as well as the investment in facility location and transportation link construction. Shishebori and Yousefi Babadi (2015) proposed an efficient MIP model for a robust and reliable medical service center location network design problem, which simultaneously takes uncertain parameters, system disruptions, and investment budget constraint into account. They formulated the proposed model based on an efficient robust optimization approach to protect the network against uncertainty. Also, a MIP model with augmented $P_R$-robust constraints is proposed to control the system reliability under unpredictable situations.

### 2.3. Research Gap

Although the literature on facility location problems is abundant, it seems that few studies have addressed facility location problems in which facility location problems were considered regarding to network design and system reliability topics simultaneously. However, numerous examples of practical problems can be referred in which simultaneously considering facility location, network design, and also system reliability is very important in improving the efficiency and security of the system. These examples include pipelines for gas and water, infrastructure for airline and railroad networks, and systems for delivering services such as health care and education. It is mentioned that the link construction may represent route establishment for medical transport vehicles or school buses, or may illustrate the construction of new roads to access the facilities, e.g., in underdeveloped regions.

As a result, it appears that this area of investigation has received scant attention to manage the practical facility location problems. Accordingly, proposing a new mathematical model formulation, which can obtain optimal facility location and link construction under some special conditions such as system reliability, can lead decision makers to more accurate solutions for the considered problem. In other words, an integrated comprehensive model provides an enough effective and a confidant approach to be applied by different decision makers, especially facility location planners for locating several facilities and constructing potential links in order to improve the efficiency and responsibility of supply chain.

## 3. Problem definition and model formulation

### 3.1. Definition

Suppose that in a geographical region, a set of demand nodes exists and a set of roads as transportation links that contains existing and new candidate links is defined to construct a transportation network in the mentioned region. Likewise, a set of facilities exists in the region and a number of new facilities are determined to locate in the region. The location cost of existing facilities and construction cost of existing links are zero. Because of the geographical situation of the region and its mountainous paths, all of the transportation links (containing existing and new links) are not reliable. In fact, due to some unexpected events such as heavy rain or snow, avalanche, hurricane and volcano, they occasionally disrupt and become very difficult to use. Accordingly, the assigned demands of disrupted links may be transferred by having some delays or installing some spare equipment, such as car tire chain, and slow transferring, or even finding some further alternative links. However, in the link disruption conditions, all of the possible solutions lead to increase in the delivery time or the traveled distances by demand nodes and raise the transportation costs as an extra mobilizing cost, delay cost or shortage cost. If the increase in transportation costs is considered as failure cost, then a part of objective function can be defined as “failure cost” and the total nominal cost (including the investment cost and the transportation cost of nominal (normal) conditions) together with the failure cost (transportation cost of disruption conditions) can be simultaneously considered and optimized. In the other words, it is clearly desired to locate a set of new facilities and construct new candidate links so that the total investment costs (including locating facilities, constructing links) and total expected operational costs (including transportation costs in nominal and disruption conditions) are minimized.

The problem is to determine: (1) the optimum locations of new facilities with regard to network design and disruption probability of transportation links, (2) the new transportation links that should be constructed in the proposed network, (3) the amount of demands of nodes that should be transported by the transportation links, and, (4) the fraction of every demand that should be supplied by new and exciting facilities.

Assumptions:

The assumptions for RFLNDP can be described as follows:

1. Each node of network illustrates a demand point.
2. The facilities and network links (transportation roads) are uncapacitated.
3. New facilities can only be located on the nodes of the network and may not be located on the links of network.
4. At most only one new facility can be located on each node.
5. The general structure of the network is planned based on a *customer-to-server* system, which means that the demands themselves travel to the relevant facilities in order to be served.
6. All travel costs are symmetric.
7. All network links are directed.
8. All of the facilities (including existing and new facilities) are reliable.
9. Based on the geographical situation, the transportation links sometimes will disrupt with a specified probability.
10. The disruption of each link leads to increase its transportation cost as a constant coefficient of nominal transportation cost.
11. It may happen that several links simultaneously have disruptions and not be available at a time.
12. Locating new facilities and allocating demand nodes are considered so that the facilities location costs and link construction costs as well as the expected transportation costs (in the nominal and disruption conditions of transportation links) are simultaneously optimized, subject to if any link disrupts, then, by paying extra costs for transferring related demands, the resulting cost are known as the link disruption costs and additional objective function.

Notifications:

Parameters:

$P$ number of new facility to open, $(P \geq 2)$
$N$ set of demand nodes in the network
$M$ set of transportation links in the network (including existing and new candidate links)

$d_i$ demand at node $i \in N$

$f_i$ fixed cost of locating a facility at node $i \in N$
$q_{ij}$ probability that the transportation link $(i , j)$ will disrupt ($0 \le q_{ij} \le 1$)
$u_{ij}$ increasing coefficient of transportation cost of link $(i , j)$ in disruption conditions
$c_{ij}$ cost of constructing link $(i , j)$
$t_{ij}^0$ transportation cost of a unit flow on link $(i , j)$
$t_{ij}^l$ transportation cost of a unit flow of demand node $l$ on link $(i , j) = t_{ij}^0\, d_l$

We assume all parameters are integer-valued except all kinds of costs. It is mentioned that $t_{ij}^0(t_{ij}^l)$ presents a link-specific transportation cost, not an origin-destination transportation cost and we have to utilize link-specific transportation cost as an initial parameter of RFLNDP model because in RFLNDP, unlike RFLP, the network is not known in advance. Hence, we cannot calculate origin-destination transportation costs.

Variables:

$Z_i = 1$ if a facility is located at node $i$, 0 otherwise
$X_{ij} = 1$ if link $(i , j)$ is constructed, 0 otherwise
$Y_{ij}^l$ = fraction of demand of node $l$ that flows on link $(i , j) \in M$
$Y_{ij}^i = X_{ij} \quad (i , j) \in M$
$W_i^l$ = fraction of demand of node $l$ that is served by a facility at node $i \in N$
$W_i^i = Z_i \quad i \in N$

### 3.2. Model formulation

Using these notations and assumptions, the mathematical formulation of the RFLNDP is shown as follows:
(RFLNDP)

$$\begin{aligned} \textit{Minimize} \quad & \sum_{i \in N} f_i Z_i + \sum_{(i,j) \in M} c_{ij} X_{ij} + \sum_{(i,j) \in M} (1-q_{ij}) t_{ij}^i X_{ij} + \sum_{(i,j) \in M} q_{ij} u_{ij} t_{ij}^i X_{ij} \\ & + \sum_{(i,j) \in M} (1-q_{ij}) t_{ij}^l Y_{ij}^l + \sum_{(i,j) \in M} q_{ij} u_{ij} t_{ij}^l Y_{ij}^l \end{aligned} \tag{1}$$

Subject to:

$$Z_i + \sum_{j \in N} X_{ij} = 1 \qquad \forall i \in N \tag{2}$$

$$X_{li} + \sum_{j \in N: j \ne l} Y_{ji}^l = \sum_{j \in N} Y_{ij}^l + W_i^l \qquad \forall i, l \in N: i \ne l, \forall (l, i) \in M \tag{3}$$

$$\sum_{j \in N: j \ne l} Y_{ji}^l = \sum_{j \in N} Y_{ij}^l + W_i^l \qquad \forall i, l \in N: i \ne l, \forall (l, i) \notin M \tag{4}$$

$$Z_l + \sum_{i \in N: i \ne l} W_i^l = 1 \qquad \forall l \in N \tag{5}$$

$$Y_{ij}^l \le X_{ij} \qquad \forall (i, j) \in M, \quad \forall l \in N: i \ne l \tag{6}$$

$$W_i^l \le Z_i \qquad \forall i, l \in N: i \ne l \tag{7}$$

$$X_{ij} + X_{ji} \le 1 \qquad \forall (i, j) \in M \tag{8}$$

$$\sum_{i \in N} Z_i = P \qquad (9)$$

$$Y_{ij}^{l} \geq 0 \qquad \forall (i, j) \in M \quad , \quad l \in N \; : \; l \neq i \qquad (10)$$

$$W_i^{l} \geq 0 \qquad \forall i, l \in N \qquad (11)$$

$$Z_i \in \{0,1\} \qquad \forall i \in N \qquad (12)$$

$$X_{ij} \in \{0,1\} \qquad \forall (i, j) \in M \qquad (13)$$

In this formulation, the objective function (1) minimizes the total investment cost (including facility location cost and link construction cost) as well as the expected value of transportation cost (in nominal and disruption conditions). In general, constraints (2-5) consider the rational conditions of the transportation flow between demand nodes and facilities. Specifically, Constraints (2) ensure that demand at $i$ is either served by a facility at $i$ or by transporting on some links out of $i$. Constraints (3) and (4) state conservation of flow for transshipped demand. Constraints (5) impose that the demand of node $l$ must find a destination, whether it is estimated by node $l$ itself ($z_{lk}$) or by the other nodes $i$ ($W_i^l$). Constraints (6) and (7) guarantee that potential links and facilities are not used if they are not constructed. Constraints (8) emphasize that on any given link, an optimal solution flow will be in only one direction; Therefore, both links ($i$ , $j$) and ($j$ , $i$) cannot be constructed. Constraint (9) restricts the total number of newly located facilities to the predetermined facilities of $P$. Constraints (10) and (11) force the flow variables to be non-negative; while, Constraints (12) and (13) enforce the binary restriction on the facility location and link construction decision variables.

As it was mentioned, according to the single assignment property, every demand of node is completely assigned to the closest single facility. That is, nothing is gained by “splitting up” a demand and sending parts of it to different facilities. Therefore, the fractions of demands, which were served by a single facility, are integer-valued, while $W_i^l$ and $Y_{ij}^l$ are integral (Melkote, 1996).

### 3.3. Complexity

Property 1 emphasizes that the RFLNDP is NP-hard, since it has the UFLP, which is itself NP-hard, as a special case.

**Property 1**. *The RFLNDP is NP-hard.*
**Proof:** If the values of $q_{ij}$s are set to zero and also all of the $u_{ij}$s are equal to 1, then the RFLNDP will be reduced to UFLNDP. Also UFLNDP is known as a generalization of the UFLP in which link additions are allowed. Since The UFLP is NP-hard (Cornuejols, 1990), so the RFLNDP is a more general of UFLP and certainly it is NP-hard.

## 4. Problem-solving approach

The proposed model (RFLNDP) was coded in GAMS 23.3 and regarding its efficiency in solving of different mathematical models, the CPLEX solver was used and it can find a suitable solution with a very good efficiency for small-scale instances. However, with respect to the property 1, the performance of CPLEX is not desirable to solve the medium and large scale instances. As a considerable point, even finding a feasible solution for large scale instances is challenging and difficult. Thus, it seems that an efficient solution algorithm is necessary to solve the model (RFLNDP) in different scales. Fig. 1 and Fig. 2 illustrate the percentage of applying several solution algorithms for FLNDP and reliable facility location problems respectively.

As displayed in Figure1 and Figure2, it can be concluded that the custom heuristic algorithms based on the problem-specific structure form a remarkable part of solution algorithms for both FLNDP and reliable facility location problems. In addition, previous studies showed that an efficient customized heuristic algorithm can give effective solutions in reasonable solving time. On the other hand, the meta-heuristic algorithms form a significant section of solution methods. Therefore, in this paper, an integrated hybrid algorithm is proposed in which a LP relaxation heuristic and variable neighborhood search (VNS), as an efficient robust innovative meta-heuristic, are combined together and applied to solve the model (RFLNDP).

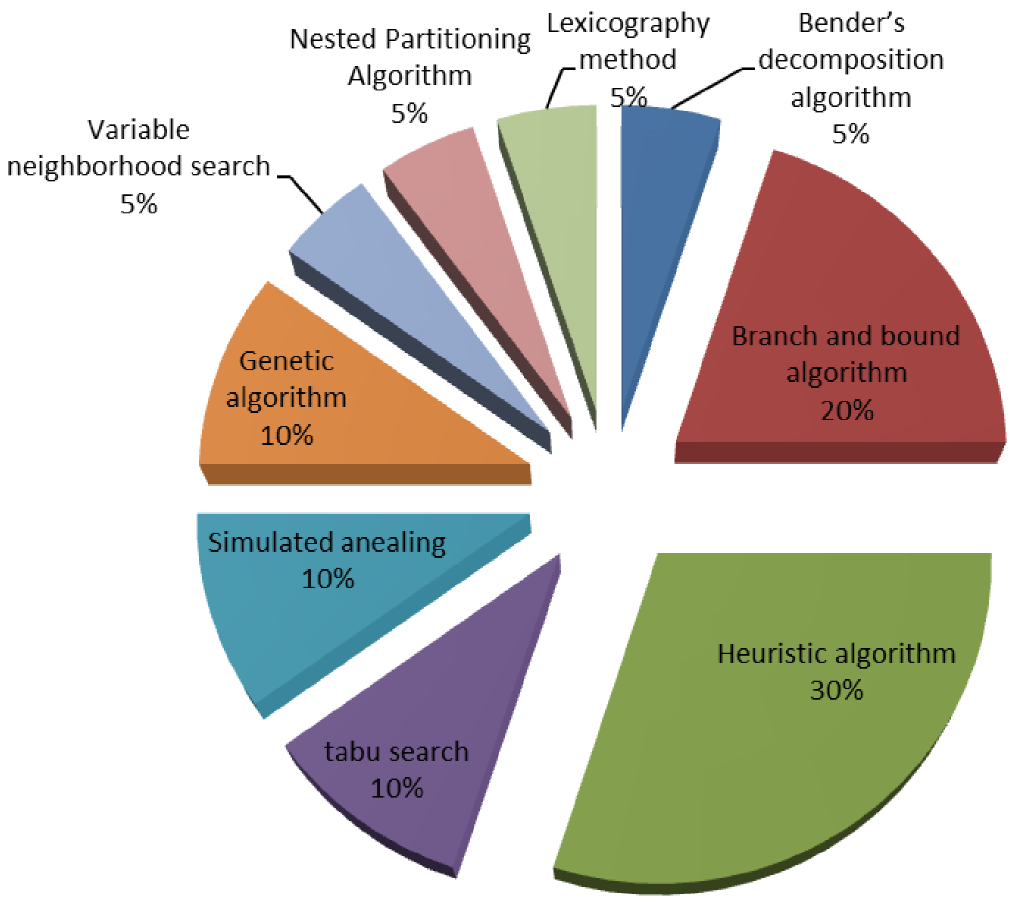


**Figure 1.** Percentage of applying several solution methods for FLNDP

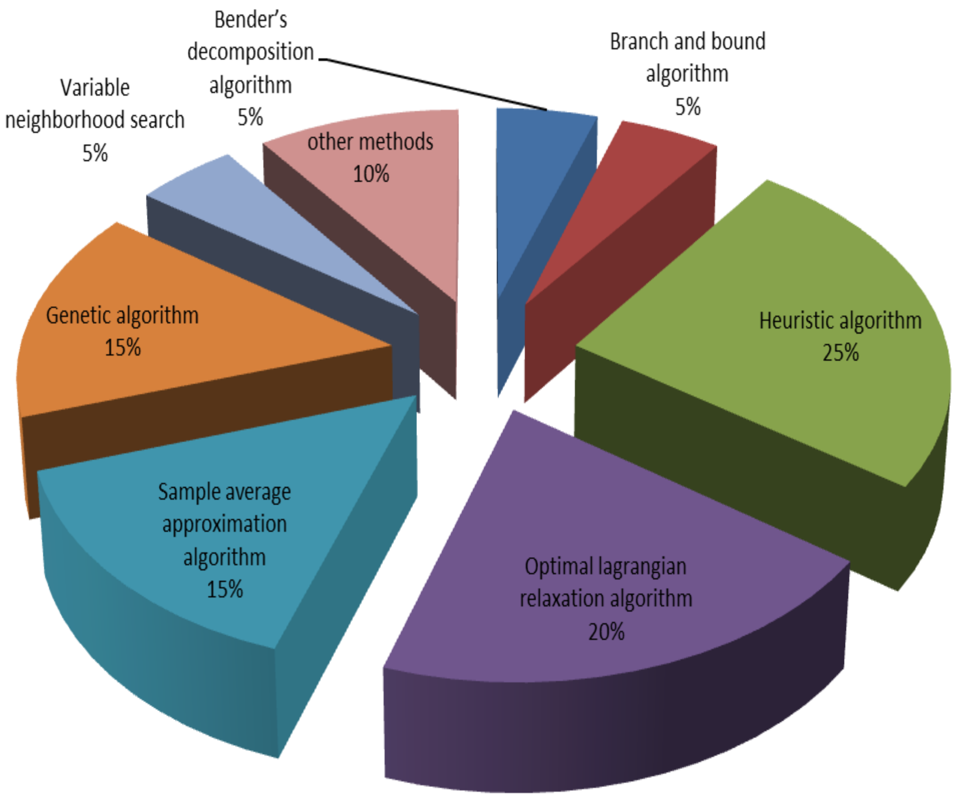


**Figure 2.** Percentage of applying several solution methods for reliable facility location problem

As a brief review, variable neighborhood search (VNS) is a meta-heuristic for creating heuristics whose basic idea is a systematic variation of neighborhood structures within the local search algorithm without guaranteeing the solution's optimality (MladenoviÄ‡, UroÅ¡eviÄ‡, Hanafi, & IliÄ‡, 2012; Mladenović & Hansen, 1997). In recent years, a large variety of VNS strategies have been proposed. The Variable Neighborhood Descent (VND), Reduced VNS (RVNS), Basic VNS (BVNS), Skewed VNS (SVNS), General VNS (GVNS), Variable Neighborhood Decomposition Search (VNDS) and Reactive VNS, are some of the most important VNS strategies (Duarte, et al.). We refer the reader to (Hansen, Mladenović, & Moreno Pérez, 2010) for a complete review of this methodology and its different variants. In this paper, we focus on the Reduced VNS variant, see (Mladenović & Hansen, 1997) for the details, which uses only shaking phase while exploring the solutions.

### 4.1. Motivation

In the proposed model, when the location of facilities is known and predetermined, it is easier to find an optimal solution for the obtained sub-problem. As an innovative idea, the hybrid algorithm in this paper applies a LP relaxation heuristic as the initial solution of RVNS. In other words, as an initial step, by solving the LP relaxation of model (FLNDP), the proposed hybrid algorithm fixes $P$ locations of $P$ new facilities and then the optimal underlying network is determined to construct new links and allocate demands to facilities. In this study, the software that makes an interface between MATLAB and GAMS is applied to obtain the local optimum from CPLEX. In other words, the algorithm is coded in the MATLAB software which generates the moves according to the RVNS scheme and brings them into the GAMS software and then the CPLEX solves the problem. Accordingly, the location variables (i.e. $Z_i$ ) are fixed. This integrated approach is so simple yet efficient.

In what follows, the main materials of the hybrid RVNS algorithm, applied for RFLNDP, are explained in detail. Then, the main procedure of the algorithm is presented.

### 4.2. Initial solution

Using the best recorded solution in generating next neighborhoods is one of most efficient advantages of using the VNS algorithm. As a result, initial solution has a great impact on a successful VNS implementation. As an initial step, by solving the LP relaxation of model (RFLNDP), the proposed hybrid algorithm generates an initial vector for location of facilities ($Z$) and due to the $P$s largest elements of initial vector; it is converted to a binary vector and applied as an initial solution of RVNS algorithm. The general structure of the heuristic method is described in subsection 5-5 as Algorithm 1.

### 4.3. Shaking

Suppose that $N_k$, for $k = 1, \ldots, k_{max}$ illustrates a finite set of pre-selected neighborhood structures, and the $N_k(Z)$ shows the set of solutions in the $k^{th}$ neighborhood of $Z$. These neighborhoods are used within the hybrid RVNS algorithm for diversification purposes (i.e., in the Shaking step). The Shaking step consists of generating random point from $N_k(Z)$ ($k = 1; 2; \ldots ; k_{max}$) used as a stimulant solution for RVND.

### 4.4. Stopping condition

As a stopping condition for “while” loop, depending on the size of the problem, the hybrid RVNS algorithm stops after meeting one of the following conditions: (1) obtaining to the optimal solution (2) elapsing considered CPU time (3) after 5*|$N$| iteration without improvement.

### 4.5. Main procedure

The main procedure of the algorithm is illustrated in Algorithm 1. At the initial step, the algorithm generates an initial binary vector, named $Z$, by solving the LP relaxation of the model (RFLNDP). Afterwards, in the improvement step, by Shaking operation, the binary vector $Z$ is changed and set as an input data of the model (RFLNDP). In the follows, the obtained model is solved by the CPLEX solver in order to find the optimal solution of the sub-problem, this procedure in improvement step will be continued until the stopping condition is met.

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Algorithm 1: Main procedure of hybrid RVNS algorithm

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*Data Step*:

Read the input data:

- ✓ parameters –$P$ , $M$ , $N$–
- ✓ vectors –$d$ , $f$ –
- ✓ matrics – $q$ , $u$ , $c$ , $t$ –

Define neighborhood structures $N_k$ ($k$ = 1,...,$k_{max}$)

*Initial Step*:

---

Generate initial solution $Z$ as follows:

- ✓ Solve the LP relaxation of the mathematical model (RFLNDP) to obtain the facility location vector ($Z^{LP}$)
- ✓ Select the $P$s largest elements of vector $Z^{LP}$ and set 1 for values of them
- ✓ Set 0 for other elements of vector $Z^{LP}$
- ✓ Set the new binary vector $Z^{LP}$ in model (RFLNDP) as an input parameter, and solve it.
- ✓ Put the new vector $Z^{LP}$ as the initial solution $Z$ and set $obj(Z)$ as the best objective function value of model (RFLNDP) named $obj(Z_{best})$

*Improvement Step*:
While stopping condition is not met to do
$k \leftarrow 1$
while $k \leq k_{max}$ do
$Z' \leftarrow$ Shake($Z$), $Z' \in N_k(Z)$
Set $s$' as the input data of model (RFLNDP) and solve it
if ($obj(Z') < obj(Z_{best})$)
$Z \leftarrow Z'$
$obj(Z_{best}) \leftarrow obj(Z')$
$k \leftarrow 1$
else
$k \leftarrow k+1$
end-while
end-while
End-Procedure

## 5. Computational results

A series of numerical experiments was performed to evaluate the performance of the proposed hybrid RVNS algorithm. The algorithm was coded in MATLAB R2011b and GAMS 23.3 and executed on a server with an AMD Opteron 2.0 GHz (×16) and 32GB RAM, operating under Linux.

### 5.1. Experimental design

In order to verify the performance of the proposed hybrid RVNS algorithm, we solved 30 test problems included different size problems. These problems were generated randomly according to similar way in literature (Melkote, 1996; Melkote & Daskin, 2001a, 2001b). For a test problem generation, the following assumptions were considered: the transportation cost for each client in kilometer is randomly generated subject to a uniform distribution in [30, 100]. The construction costs of new links are calculated as a coefficient of transportation cost like $u$ for per kilometer (the $u$ can be considered as a discrete uniform distribution in [15, 30]). The demand at each node is drawn uniformly from [10, 150] and is also rounded to the nearest integer. Also, the fixed cost of opening facility is drawn uniformly from [1200, 3000].

### 5.2. Algorithm performance

Table 1 indicates the characteristics of test problems and summarizes the results of the efficiency of the proposed hybrid RVNS algorithm with that of the CPLEX. For each algorithm, three columns report the run time (‘‘Time’’), objective value (‘‘Cost’’) and optimality gap vs. the CPLEX lower bound (‘‘Gap’’) that are calculated as follows:

$$Gap_{CPLEX}\,(\%) = \frac{Cost_{CPLEX} - LB_{CPLEX}}{Cost_{CPLEX}} \times 100 \quad ; \quad Gap_{HRVNS}\,(\%) = \frac{Cost_{HRVNS} - LB_{HRVNS}}{Cost_{HRVNS}} \times 100$$

The CPU times for the hybrid RVNS algorithm include the time required to calculate the LP relaxation bound and execute RVNS. The two algorithms are compared in the (‘‘% DIFF’’) column, where the (‘‘Time’’) column gives the percentages of CPLEX’s CPU time required by the hybrid RVNS algorithm and the (‘‘Cost’’) gives the percent difference between the objective function values. A value less than 100% in the (‘‘Cost’’) column indicates that the proposed algorithm found a better solution, while a value less than 100% in the (‘‘Time’’) column illustrates that the algorithm was faster, which occurs in all instances.

As shown in Table 1, comparing the CPLEX, the hybrid RVNS algorithm was able to find the same or better solutions for 16 (or 53.333%) of the 30 test problems, while taking only approximately a 2/3 fraction of CPLEX’s time (61.459% on average).

Moreover, Fig. 3 and Fig. 4 illustrate the performance of the hybrid RVNS algorithm vs. the CPLEX graphically. As shown in Fig. 3, it can be concluded that the proposed hybrid algorithm can obtain the same or a little worse solution in comparison with CPLEX. Also Fig. 4 demonstrates that the CPU time of CPLEX will remarkably increase, especially as the number of demand nodes increases.

**Table1.** Comparison of hybrid RVNS algorithm performance vs. CPLEX

| **Sample number** | ***N*** | ***P*** | | **CPLEX** | | | |
|---|---|---|---|---|---|---|---|
| | | | | **Cost** | **LB** | **Gap (%)** | **Time** |
| **TP1** | 10 | 2 | | 36,452.800 | 36,452.800 | 0.000 | 0.261 |
| **TP2** | | 2 | | 92,800.200 | 92,800.200 | 0.000 | 0.268 |
| **TP3** | | 3 | | 42,009.400 | 42,009.400 | 0.000 | 0.266 |
| **AVG** | | | | | | | 0.265 |
| **TP4** | 20 | 2 | | 305,604.400 | 305,604.400 | 0.000 | 0.826 |
| **TP5** | | 3 | | 330,705.000 | 330,705.850 | 0.000 | 0.799 |
| **TP6** | | 3 | | 378,857.400 | 378,857.400 | 0.000 | 0.808 |
| **AVG** | | | | | | | 0.811 |
| **TP7** | 30 | 2 | | 566,826.360 | 538,831.750 | 4.939 | 6.636 |
| **TP8** | | 3 | | 817,214.240 | 782,740.902 | 4.218 | 4.002 |
| **TP9** | | 4 | | 398,019.800 | 383,254.485 | 3.710 | 2.402 |
| **AVG** | | | | | | | 4.347 |
| **TP10** | 40 | 3 | | 560,630.760 | 549,885.435 | 1.917 | 6.008 |
| **TP11** | | 4 | | 1,079,885.160 | 1,043,812.910 | 3.340 | 5.685 |
| **TP12** | | 5 | | 361,267.940 | 354,308.400 | 1.926 | 5.760 |

**Table1.** Continued

| Sample number | *N* | *P* | | CPLEX | | | |
|---|---|---|---|---|---|---|---|
| | | | | **Cost** | **LB** | **Gap (%)** | **Time** |
| **AVG** | | | | | | | 5.818 |
| **TP13** | 50 | 3 | | 1,320,188.500 | 1,280,782.810 | 2.985 | 11.271 |
| **TP14** | | 4 | | 2,833,688.040 | 2,833,688.040 | 0.000 | 11.117 |
| **TP15** | | 5 | | 4,466,527.800 | 4,466,527.800 | 0.000 | 10.704 |
| **AVG** | | | | | | | 11.031 |
| **TP16** | 60 | 3 | | 2,730,993.360 | 2,730,993.360 | 0.000 | 18.455 |
| **TP17** | | 4 | | 8,610,322.300 | 8,610,322.300 | 0.000 | 18.649 |
| **TP18** | | 5 | | 4,201,946.200 | 4,201,946.200 | 0.000 | 18.485 |
| **AVG** | | | | | | | 18.530 |
| **TP19** | 70 | 3 | | 4,404,315.450 | 4,404,315.450 | 0.000 | 30.483 |
| **TP20** | | 4 | | 5,551,433.900 | 5,551,433.900 | 0.000 | 30.122 |
| **TP21** | | 5 | | 6,832,597.600 | 6,832,597.600 | 0.000 | 29.936 |
| **AVG** | | | | | | | 30.180 |
| **TP22** | 80 | 3 | | 5,856,458.700 | 5,856,458.700 | 0.000 | 45.607 |
| **TP23** | | 4 | | 5,266,439.700 | 5,266,439.700 | 0.000 | 45.761 |
| **TP24** | | 5 | | 1,396,959.100 | 1,396,959.100 | 0.000 | 45.165 |
| **AVG** | | | | | | | 45.511 |
| **TP25** | 90 | 4 | | 3,198,584.600 | 3,198,584.600 | 0.000 | 66.257 |
| **TP26** | | 5 | | 2,814,861.600 | 2,814,861.600 | 0.000 | 70.068 |
| **TP27** | | 6 | | 2,503,643.600 | 2,503,643.600 | 0.000 | 67.526 |
| **AVG** | | | | | | | 67.950 |
| **TP28** | 100 | 5 | | 333,439.660 | 333,439.660 | 0.000 | 93.645 |
| **TP29** | | 6 | | 1,233,162.630 | 1,233,162.630 | 0.000 | 95.272 |
| **TP30** | | 7 | | 1,336,804.030 | 1,336,804.030 | 0.000 | 96.957 |
| **AVG** | | | | | | | 95.291 |
| **Total AVG** | | | | | | | **27.973** |

**Table1.** Continued

| Sample number | *N* | *P* | | HRVNS | | | |
|---|---|---|---|---|---|---|---|
| | | | | Cost | LB | Gap (%) | Time |
| **TP1** | 10 | 2 | | 36,452.800 | 36,452.800 | 0.000 | 0.215 |
| **TP2** | | 2 | | 92,800.200 | 92,800.200 | 0.000 | 0.224 |
| **TP3** | | 3 | | 42,009.400 | 42,009.400 | 0.000 | 0.198 |
| **AVG** | | | | | | | 0.212 |
| **TP4** | 20 | 2 | | 305,604.400 | 305,604.400 | 0.000 | 0.512 |
| **TP5** | | 3 | | 330,705.850 | 330,705.850 | 0.000 | 0.524 |
| **TP6** | | 3 | | 378,857.400 | 378,857.400 | 0.000 | 0.751 |
| **AVG** | | | | | | | 0.596 |
| **TP7** | 30 | 2 | | 557,638.900 | 557,638.900 | 0.000 | 4.265 |
| **TP8** | | 3 | | 817,214.240 | 817,214.240 | 0.000 | 2.742 |
| **TP9** | | 4 | | 396,850.450 | 396,850.450 | 0.000 | 1.306 |
| **AVG** | | | | | | | 2.771 |
| **TP10** | 40 | 3 | | 582,898.140 | 572,720.760 | 0.017 | 3.982 |
| **TP11** | | 4 | | 1,062,869.320 | 1,062,869.320 | 0.000 | 3.125 |
| **TP12** | | 5 | | 361,267.940 | 361,267.940 | 0.000 | 3.621 |
| **AVG** | | | | | | | 3.576 |
| **TP13** | 50 | 3 | | 1,316,583.700 | 1,316,583.700 | 0.000 | 7.698 |
| **TP14** | | 4 | | 2,883,688.040 | 2,833,688.040 | 0.017 | 6.134 |
| **TP15** | | 5 | | 4,496,527.800 | 4,466,527.800 | 0.007 | 5.730 |
| **AVG** | | | | | | | 6.521 |
| **TP16** | 60 | 3 | | 2,930,993.360 | 2,730,993.360 | 0.068 | 11.567 |
| **TP17** | | 4 | | 9,819,003.150 | 9,819,003.150 | 0.000 | 10.103 |
| **TP18** | | 5 | | 4,201,946.200 | 4,201,946.200 | 0.000 | 10.021 |
| **AVG** | | | | | | | 10.564 |
| **TP19** | 70 | 3 | | 4,424,315.450 | 4,404,315.450 | 0.005 | 16.957 |
| **TP20** | | 4 | | 5,561,433.900 | 5,551,433.900 | 0.002 | 17.564 |
| **TP21** | | 5 | | 7,232,597.600 | 6,832,597.600 | 0.055 | 16.212 |
| **AVG** | | | | | | | 16.911 |
| **TP22** | 80 | 3 | | 6,156,458.700 | 5,856,458.700 | 0.049 | 26.548 |
| **TP23** | | 4 | | 5,766,439.700 | 5,266,439.700 | 0.087 | 24.669 |
| **TP24** | | 5 | | 1,406,959.100 | 1,396,959.100 | 0.007 | 24.226 |
| **AVG** | | | | | | | 25.148 |
| **TP25** | 90 | 4 | | 4,043,349.100 | 4,043,349.100 | 0.000 | 36.133 |
| **TP26** | | 5 | | 3,656,356.100 | 3,656,356.100 | 0.000 | 37.534 |
| **TP27** | | 6 | | 2,503,643.600 | 2,503,643.600 | 0.000 | 36.744 |
| **AVG** | | | | | | | 36.804 |
| **TP28** | 100 | 5 | | 333,439.660 | 333,439.660 | 0.000 | 51.138 |
| **TP29** | | 6 | | 1,457,431.130 | 1,457,431.130 | 0.000 | 51.835 |
| **TP30** | | 7 | | 1,336,804.300 | 1,336,804.030 | 0.000 | 53.669 |
| **AVG** | | | | | | | 52.214 |
| **Total AVG** | | | | | | | **15.532** |

**Table1.** Continued

| Sample number | *N* | *P* | | DIFF (%) | |
|---|---|---|---|---|---|
| | | | | Time (%) | Cost (%) |
| **TP1** | 10 | 2 | | 82.375 | 100.000 |
| **TP2** | | 2 | | 83.582 | 100.000 |
| **TP3** | | 3 | | 74.436 | 100.000 |
| **AVG** | | | | 80.131 | 100.000 |
| **TP4** | 20 | 2 | | 61.985 | 100.000 |
| **TP5** | | 3 | | 65.582 | 100.000 |
| **TP6** | | 3 | | 92.946 | 100.000 |
| **AVG** | | | | 73.504 | 100.000 |
| **TP7** | 30 | 2 | | 64.271 | 98.379 |
| **TP8** | | 3 | | 68.516 | 100.000 |
| **TP9** | | 4 | | 54.371 | 99.706 |
| **AVG** | | | | 62.386 | 99.362 |
| **TP10** | 40 | 3 | | 66.278 | 103.972 |
| **TP11** | | 4 | | 54.969 | 98.424 |
| **TP12** | | 5 | | 62.865 | 100.000 |
| **AVG** | | | | 61.371 | 100.799 |
| **TP13** | 50 | 3 | | 68.299 | 99.727 |
| **TP14** | | 4 | | 55.177 | 101.764 |
| **TP15** | | 5 | | 53.531 | 100.672 |
| **AVG** | | | | 59.002 | 100.721 |
| **TP16** | 60 | 3 | | 62.677 | 107.323 |
| **TP17** | | 4 | | 54.174 | 114.038 |
| **TP18** | | 5 | | 54.212 | 100.000 |
| **AVG** | | | | 57.021 | 107.120 |
| **TP19** | 70 | 3 | | 55.628 | 100.454 |
| **TP20** | | 4 | | 58.310 | 100.180 |
| **TP21** | | 5 | | 54.156 | 105.854 |
| **AVG** | | | | 56.031 | 102.163 |
| **TP22** | 80 | 3 | | 58.210 | 105.123 |
| **TP23** | | 4 | | 53.908 | 109.494 |
| **TP24** | | 5 | | 53.639 | 100.716 |
| **AVG** | | | | 55.253 | 105.111 |
| **TP25** | 90 | 4 | | 54.535 | 126.411 |
| **TP26** | | 5 | | 53.568 | 129.895 |
| **TP27** | | 6 | | 54.415 | 100.000 |
| **AVG** | | | | 54.172 | 118.768 |
| **TP28** | 100 | 5 | | 54.608 | 100.000 |
| **TP29** | | 6 | | 54.407 | 118.186 |
| **TP30** | | 7 | | 55.353 | 100.000 |
| **AVG** | | | | 54.790 | 106.062 |
| **Total AVG** | | | | **61.459** | **61.459** |

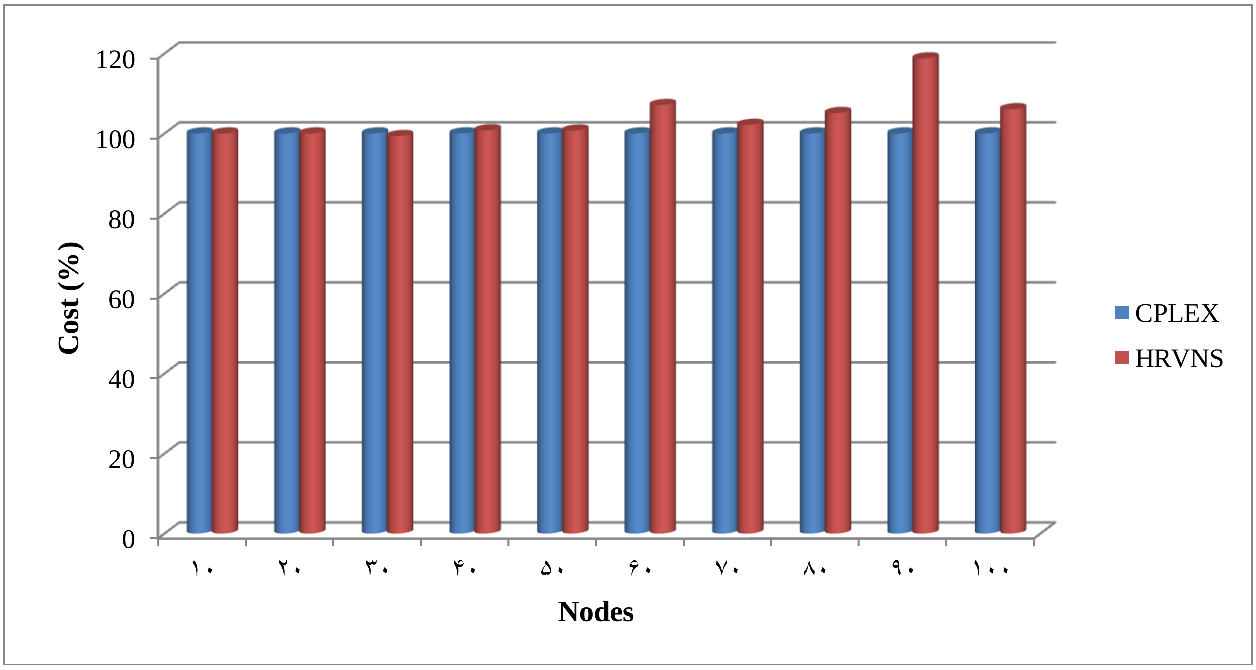


**Figure 3.** The performance of the hybrid RVNS algorithm vs. the CPLEX subject to the value of objective function (cost (%))

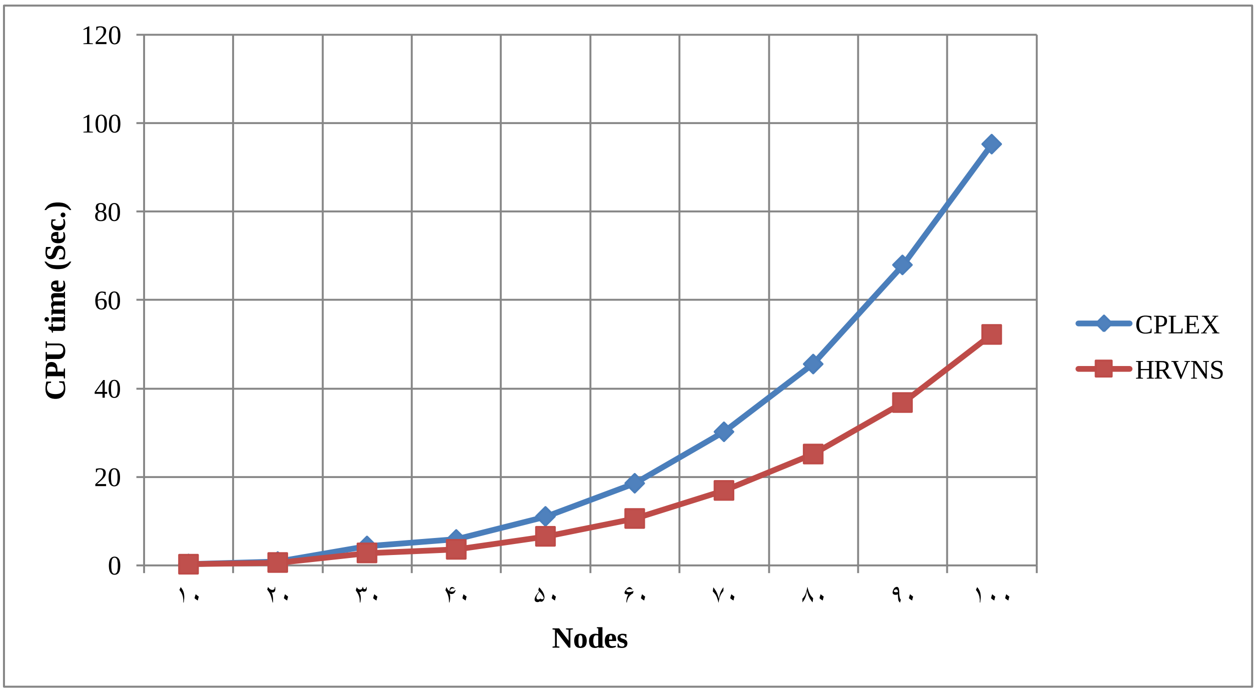


**Figure 4.** The performance of the proposed hybrid RVNS algorithm vs. the CPLEX subject to solution time

## 6. Conclusions and suggestions for future research

Regarding the crucial role of considering several practical factors in efficient designing of facility location and allocating demands to them, in this paper, a mixed integer-linear programming (MILP) model was proposed in order to both minimize the total costs, including location costs and link construction costs, as well as the expected transportation costs in a kind of facility location problem with respect to network design and link disruption topics. The proposed MILP model determines the optimal locating of new facility locations, optimal constructing of transportation links and also optimal allocating demand nodes to facilities regarding to constructed links.

According to the suitable efficiency and successful execution of VNS metaheuristic in solving several NP-hard problems, a hybrid RVNS algorithm was proposed for solving the RFLNDP. The algorithm applies a LP relaxation heuristic as the initial solution of RVNS by solving the LP relaxation of model (RFLNDP). Therefore, the proposed hybrid algorithm fixes $P$ locations for $P$ new facilities and then the optimal underlying network is determined to construct new links and allocate demands to the facilities via them. The location variables (i.e. $Z_i$ ) as a vector are used as initial solution and with each iteration, the RVNS changes the location of facilities and solves the obtained sub problem. This simple integrated approach is efficient. The experimental results illustrated that the proposed hybrid RVNS algorithm has suitable efficiency in solving the model not only finding the high quality solutions but also in the reasonable execution time.

For future studies, in this paper, only the RFLNDP with uncapacitated facilities and links was studied; however, considering the RFLNDP with capacitated system can have more practical application in real industrial and service environments. Moreover, one can seek and test other efficient heuristics and metaheuristics such as tabu search (TS) and particle swarm optimization (PSO) for improving the efficiency of the proposed solution method.

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