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The impact of cost uncertainty on the location of a distribution center

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ABSTRACT

The location of a distribution center (DC) is a key consideration for the design of supply chain networks. When deciding on it, firms usually allow for transportation costs, but not supplier prices. We consider simultaneously the location of a DC and the choice of suppliers offering different, possibly random, prices for a single product. A buying firm attempts to minimize the sum of the price charged by a chosen supplier, and inbound and outbound transportation costs. No costs are incurred for switching suppliers. We first derive a closed-form optimal location for the case of a demand-populated unit line between two suppliers offering deterministic prices. We then let one of the two suppliers offer a random price. If the price follows a symmetric and unimodal distribution, the optimal location is closer to the supplier with a lower mean price. We also show the dominance of high variability: the buyer can decrease the total cost more for higher price variability for any location. The dominance result holds for normal, uniform, and gamma distributions. We propose an extended model with more than two suppliers on a plane and show that the dominance result still holds. From numerical examples for a line and a plane, we observe that an optimal location gets closer to the center of gravity of demands as the variability of any supplier's price increases.
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1. Introduction

As companies continue to pursue worldwide growth, the supply chains that support globalization are covering greater distance and gaining in complexity. Distribution, a key component of these global logistics operations, is also becoming more complex, and increasingly important to the overall success of supply chains. The U.S. Department of Transportation, Bureau of Transportation Statistics has tracked this major trend. For example, inflation-adjusted U.S. commercial transportation costs were 7.2, 8.5, and 10.5 trillion dollars in 1993, 1997, and 2002, respectively (http://www.bts.gov/publications/freight_shipments_in_america/html/figure_01.html, accessed 29.04.11). A significant part of the total cost of a product can be attributed to distribution. Transportation and distribution costs comprise one-fifth or more of sales for a typical U.S. company (Schechter and Sander, 2002). In some companies moving products can account for more than half of total costs. According to information provided recently by Aguas Danone in Argentina, for example, transportation represents 25–60% of total costs for a number of its products.

An important determinant of distribution cost is the location of DCs. The traditional approach to deciding where to establish a DC takes account of the inbound (from supplier to DC) and outbound (from DC to end customer) transportation costs involved. However, the impact of supplier prices is usually not included in the analysis.

This is a significant omission. The location of a DC influences supplier choice, and since the prices charged by vendors can vary greatly, these sourcing decisions have a bearing on total distribution cost. Moreover, supplier-related costs have become more significant over recent years with increasing market volatility. Managing fluctuations in materials prices is a key element of supply chain risk mitigation strategies.

Supplier prices vary in accordance with differences between the cost structures of these enterprises. The variables include raw materials prices, the capital intensiveness of the business, labor relations, and the regulatory regime. In global supply chains, even if a foreign supplier considers its prices to be stable, from the buyer's perspective there might be considerable fluctuation owing to changes in external variables such as exchange rates and tax codes.

Despite these issues and the potential impact on distribution costs, to the best of our knowledge none of the models described in the literature explicitly incorporate the impact of supplier price uncertainty on distribution network design.

Recognizing the increasing importance of distribution within supply chains and the apparent lack of knowledge about the affect of supplier prices on DC location, we are developing an analytical methodology that enables decision makers to consider the choice of suppliers when selecting a site for a DC.

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We aim to determine an optimal DC location in situations where there is uncertainty over the prices set by multiple suppliers. The objective is to minimize the total cost as the sum of the price charged by a supplier, the supplier-to-DC transportation cost, and the DC-to-customer transportation cost.

A number of different sourcing methods are in use, but for the purposes of this analysis, we first consider two generic types, *fixed fraction* and *free switching*.

To a buying firm ordering a fixed fraction of its total purchase from one supplier, low price might not be a major consideration in the process of selecting suppliers. To achieve competitive advantage driven by greater responsiveness to customers, the firm typically works closely with the supplier as part of a long-term commitment to the vendor.

In the main analysis, however, we focus on the free switching. Firms that employ this sourcing method are free to choose among suppliers at any time to reduce the procurement cost which is the sum of the price charged by a chosen supplier and the supplier-to-DC transportation cost. This benefit has driven the current popularity of business-to-business (B2B) e-markets. The use of free switching implies an arm's length relationship between buyer and supplier. Free switching is prevalent in the sourcing of commodity products such as oil and other raw materials. In certain circumstances the method is also used to procure non-commodity products.

We first explore a basic model for locating a single DC on a unit line with the assumptions: (i) demands are populated uniformly on the line; (ii) the prices offered by two pre-located suppliers (suppliers L and R) are deterministic.

The second model incorporates the variability of price with the assumptions: (i) demands are uniformly distributed on the line; (ii) supplier L's price is deterministic and yet supplier R's follows a distribution. This setting is still restrictive, but it allows us to obtain an analytical expression and also some structural observations on an optimal location. Assuming that supplier R's price follows a symmetric and unimodal distribution, we show that an optimal location is closer to the supplier with a lower mean price. If supplier R's price follows normal or uniform or gamma distribution, we then show the *dominance of high variability* that, for any location of DC, the total cost can decrease more from higher variability in the price. To help managers understand these implications, we use numerical examples to investigate the behavior of optimal locations under various parameter settings. Numerical results show that, for a larger (smaller) variability of supplier R, an optimal location is closer to (farther away from) the *center of gravity* (CG) of demands, which is the average location of a demand-populated line or plane. This model gives a general idea of the issues involved in similar settings and it is not difficult to see its connection to other practical situations.

We then extend the second model to the third with relaxed assumptions: (i) demands follow a general distribution on a two-dimensional plane; (ii) the price offered by each of n suppliers follows a distribution. We also show that the dominance result still holds in this extended setting. Some numerical examples are explored for more managerial insights.

The remainder of the paper is organized as follows. In Section 2, we review the relevant literature. In Section 3, the basic setting of a location model for a DC with two suppliers is introduced. We develop a special case of the model, in which the two prices of a product are deterministic. In Section 4, the model extends to a probabilistic version allowing variability in supplier R's price while supplier L's still remains fixed. In Section 5, we include parametric analysis and its results. In Section 6, we consider relaxed assumptions to extend the basic model for a two-dimensional plane. Finally, in Section 7, we conclude with our contributions, managerial insights, and suggestions for future research.

2. Literature review

Our proposed models fall into the category of stochastic single-facility location models, which determine the location of the facility to optimize a performance measure which is typically a function of the distance from the facility to demand points. General problems of finding optimal facility location and resource allocation in a stochastic environment belong to the class of problems known as "Location Problems with Stochastic Demands and Congestion (LPSDC)" (Berman and Krass, 2002). Snyder (2006) also provided a survey of location models with uncertainty.

Location problems on a line or a plane are particularly relevant to our models in that they share a similar set of assumptions. Hotelling (1929) introduced a prototype model of spatial competition for the location game between two sellers on a unit line. Alonso (1965) considered the location of an ice-cream seller on a beach, where customers are uniformly dispersed. Geoffrion (1976) considered the problem of determining which warehouses to use for the delivery of a single product, and how to design service areas to minimize the sum of inbound, outbound, and warehouse-related costs. Denardo et al. (1982) considered an n-facility location problem on a line segment to minimize a cost function. De Palma et al. (1994) dealt with an optimal location of a given number of facilities when customer behavior is described by a probabilistic choice model. Drezner and Wesolowsky (1996) developed a location-allocation model by making the cost charged to users by a facility a function of the total number of users patronizing the facility. To select a facility, users consider facility charges and transportation costs. Mirchandani et al. (1996) examined capacitated facility location problems on a straight line. The fixed costs of locating facilities and the unit production costs of serving a customer from a facility can depend upon their locations on the line. Carrizosa et al. (2002) found the optimal location of a mobile server whose position is uniformly distributed over a patrolling area on a line and minimized its average distance to randomly-distributed demand points on the line. Naseraldin and Herer (2008) developed an inventory-location model on a line integrating the number and location of retail outlets with inventory replenishment decisions.

In our location models, we address coordinated planning between procurement and distribution functions in a supply chain. Comprehensive reviews of supply chain coordination mechanisms are available in the literature (Thomas and Griffin, 1996; Li and Wang, 2007). Our model belongs to supply-chain location models with uncertainty. See Melo et al. (2009) for a review of location models in supply chain management. While supply-chain location models inherit the primitives of the classical location models that attempt to optimize certain performance measures in terms of the proximity of facilities to customers or suppliers, they incorporate explicit demand uncertainty or supply uncertainty, or both. Under demand uncertainty, Shen et al. (2003) demonstrated risk-pooling benefits achieved by the economy of scale from consolidation. They showed that the number of DCs decreases as safety stock holding costs increase. On the other hand, when supply uncertainty due to potential disruptions is present, the number of facilities increases through the effect of risk-diversification (Snyder and Daskin, 2005). Shen and Daskin (2005) developed a nonlinear model, which simultaneously determines the location of DCs and the assignment of demand nodes to DCs to optimize cost and service objectives. Shu et al. (2005) studied the stochastic transportation-inventory network design problem involving one supplier and multiple retailers. The problem is to determine which retailers should serve as DCs and how to allocate other retailers to the DCs. Berman et al. (2006) addressed the effects of strategic centralization and co-location of unreliable facilities in p-median problems in the context of supply chain disruptions in health services.

Snyder et al. (2007) presented a stochastic location model with risk pooling to find solutions that minimize the expected total cost (including location, transportation, and inventory costs) of the system across all discrete scenarios. The location model handles the economies of scale and risk-pooling effects that result from consolidating inventory sites.

Our models are relevant to supply (or upstream) uncertainty. Qi and Shen (2007) proposed an integrated inventory-location model for a three-tiered supply chain network with one supplier, one or more facilities and retailers to analyze the impacts of supply uncertainty on supply chain design decisions. However, our models focus on the uncertainty of prices set by suppliers, not on the quantity of supply. Supply uncertainty has been mostly discussed in inventory models (Ramasesh et al., 1991; Anupindi and Akella, 1993; Lau and Zhao, 1993, 1994; Sedarage et al., 1999; Chen et al., 2001). See Tajbakhsh et al. (2007) for a structured review of different studies on combined multiple sourcing and upstream uncertainty. They pointed out that multiple sourcing in the presence of random prices set by suppliers, one of the main issues in this paper, had received very little attention in the literature on inventory.

3. Model structure

We introduce an analytical framework for designing a supply chain network for a single product defined on a unit line between two suppliers, as depicted in Fig. 1. The unit line is considered as a market, and two end points represent two distant suppliers.

3.1. Overview and primitives of the model

We assume that demands follow a distribution on the line. Two suppliers, L and R, are pre-located at the two end points of the line with L at the leftmost point and R at the rightmost. These two suppliers are able to produce an identical product, yet they set different, and possibly random, unit prices for the product. A *firm* as a buyer of the product buys from either supplier L or supplier R. Due to free switching, the firm has greater flexibility in sourcing.

The firm locates a DC at distance x away from supplier L in Fig. 1 and meets all demands generated from any points on the line. For each unit of product, the firm pays the price (C_L or C_R) charged by a chosen supplier, the unit supplier-to-DC transportation cost s, and the unit DC-to-customer transportation cost t for one unit of distance. We assume that the price charged by a supplier is adjusted by exchange rate and tax. The charged price is translated as a cost to the buying firm. Thus, by "cost uncertainty", we mean the variability of the price offered by a supplier. Supplier L offers a fixed price, C_L, for the product, which may be guaranteed by a long-term contract, as described in Yi and Scheller-Wolf (2003). Supplier R offers a random price, C_R, drawn from distribution function F with mean $\overline{C}_R$ and standard deviation σ. Our analysis highlighting the impact of σ on a DC location, along with these simplified assumptions, yields insightful results which can be extended to a more general setting.

From the viewpoint of the buying firm, we refer to the expected unit DC-to-customer transportation cost as the *outbound transportation cost* ($H(x)$), the expected unit supplier-to-DC transportation cost as the *inbound transportation cost* ($I(x)$), and the expected unit price set by a supplier as the *expected price* ($C(x)$). Note that all the three costs are functions of location x. The objective is to minimize the sum of the three costs which is referred to as the *total cost* ($TC(x) = H(x) + I(x) + C(x)$). In particular, we refer to the sum of the inbound transportation cost and the expected price as the *procurement cost* ($G(x) = I(x) + C(x)$), based on which the firm chooses among suppliers. Our analysis proceeds with the derivation of $H(x)$, $I(x)$, and $C(x)$ for deterministic and probabilistic versions of our models.

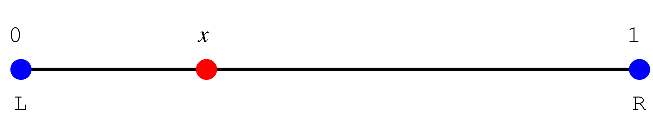

Fig. 1. An example of a unit line with two suppliers at the end points.

3.2. The deterministic model: a special case

We first investigate a special case with the assumptions: (i) demands are populated uniformly on the line; (ii) the prices offered by two pre-located suppliers (suppliers L and R) are deterministic, i.e., C_L and C_R are constants. Later, we extend this deterministic model to a probabilistic version where C_R follows a distribution. Given that the DC is located at distance x from supplier L, the firm orders from supplier L if the procurement cost incurred by supplier L is not higher than that incurred by supplier R, i.e.,

$$C_L + xs \leqslant C_R + (1 - x)s, \tag{1}$$

or

$$0 \leqslant x \leqslant \frac{C_R - C_L + s}{2s}. \tag{2}$$

The outbound transportation cost is given as

$$H(x) = \int_0^x (x - y)t\,dy + \int_x^1 (y - x)t\,dy = \left(x^2 - x + \frac{1}{2}\right)t. \tag{3}$$

Given the condition of (2), the total cost is

$$TC(x) = xs + C_L + \left(x^2 - x + \frac{1}{2}\right)t. \tag{4}$$

An optimal location can be found by the theorem below.

Theorem 1. *Assume that both C_L and C_R are fixed. If $C_L \leqslant C_R$, an optimal DC location is $x^* = \max(0, t - s)/(2t)$.*

If $C_L \leqslant C_R$, $x^* = \max(0, t - s)/(2t) \leqslant 0.5$ lies on the left half of the unit line and is obviously closer to C_L. Note that the specific location of x^* is determined solely by t and s, regardless of C_L and C_R. However, its objective function value still depends on them.

4. The model and analysis

We now consider a probabilistic version of our model with the assumptions: (i) demands are uniformly distributed on a unit line; (ii) supplier L offers a deterministic price C_L and supplier R a random price C_R with mean $\overline{C}_R$, standard deviation σ, and the coefficient of variation $cv = \sigma/\overline{C}_R$. Since demands follow a uniform distribution in this section, the center of gravity (CG) of demands is $x = 0.5$. We also let C_R be drawn from a differentiable distribution function F on support $[C_L - a, C_L + b]$. We assume that $a, b > 0$ since all other cases are trivial. Note that a and b can be ∞.

As in (1), the firm orders from supplier L if $C_L + xs \leqslant C_R + (1 - x)s$ or

$$C_R \geqslant C_L - s + 2sx.$$

The inbound transportation cost is given as

$$\begin{aligned} I(x) &= (1 - x)sF(C_L - s + 2sx) + xs[1 - F(C_L - s + 2sx)] \\ &= s[x + (1 - 2x)F(C_L - s + 2sx)], \end{aligned} \tag{5}$$

where $F(C_L - s + 2sx)$ is the probability that the procurement cost incurred by supplier R is lower than that incurred by supplier L. The expected price is then

$$C(x) = \int_{C_L - a}^{C_L - s + 2sx} y f(y) dy + C_L[1 - F(C_L - s + 2sx)], \quad (6)$$

where $y f(y)$ is a supplier R's price multiplied by the probability density of that particular price being offered. However, with probability $1 - F(C_L - s + 2sx)$, the procurement cost incurred by supplier L is lower and thus, the firm pays C_L for one unit of the product.

As in (3), the outbound transportation cost $H(x) = (x^2 - x + 0.5)t$ is convex in x and is minimized at $x = 0.5$ (the CG of demands), regardless of t. However, the procurement cost is concave in x by the lemma below.

Lemma 1. *The procurement cost $G(x) = I(x) + C(x)$ is concave in x.*

$G(x)$ is concave in x according to Lemma 1 and thus is minimized at an end point. However, $H(x)$ is convex in x and is minimized at an end point or a stationary point. Although a closed-form optimal location can be obtained in some special cases, for example, when C_R is uniformly distributed, this is not the case in general. We thus characterize an optimal location for a certain class of distributions of C_R. In particular, we refer to symmetric and unimodal distributions as type-$\mathcal{P}$ distributions. Type-$\mathcal{P}$ distributions include, for example, uniform and normal distributions which are widely used. We first need an intermediate result below.

Lemma 2. *If a probability density (or mass) function of type-$\mathcal{P}$ has a mean, μ, smaller than a constant k, for any δ, $F(k - \delta) + F(k + \delta) > 1$.*

We use the result to find an intuitive observation below.

Theorem 2. *Assume that C_R is drawn from a type-$\mathcal{P}$ distribution. If $C_L \geqslant \overline{C}_R$, an optimal location x^* is on the right half of the line; otherwise, it is on the left half of the line.*

Shown in Fig. 2, the objective function $TC(x)$ is neither convex nor concave in general.

However, it is not hard to see that $TC(x)$ is still convex for $0 \leqslant x \leqslant \max(0, 1/2 - a/(2s))$ or $\min(1/2 + b/(2s), 1) \leqslant x \leqslant 1$. We can find local optimal solutions easily by solving two convex programming problems on the two intervals. For $\max(0, 1/2 - a/(2s)) \leqslant x \leqslant \min(1/2 + b/(2s), 1)$, we need to minimize $TC(x) = H(x) + G(x)$ as a sum of convex and concave functions, which can be treated as the difference of convex (d.-c.) functions. D.-c. programming is a global optimization technique useful for an objective function as a difference of convex functions. See Tuy (1987), Horst and Tuy (1993), and Chen et al. (1998) for more details. For an approximate solution, we can simply enumerate $x \in [\max(0, 1/2 - a/(2s)), \min(1/2 + b/(2s), 1)]$ with a very small increment to compare $TC(x)$.

In Fig. 2, we see that a higher variability in price yields a lower total cost. Specifically, with all the other parameters unchanged, a higher cv yields a lower $TC(x^*)$. Intuitively, a higher cv of C_R would lead to a thicker left-tail of its distribution. The firm is then more likely to be offered a lower price, for example, one below 150. However, a distribution of C_R with a smaller cv has a smaller probability of a price offered below 150. The obvious downside of a larger cv is the possibility of higher prices since the right-tail of the distribution is also thicker. Suppose now $C_L = 220$. The *option* to buy at the price of $C_L = 220$ mitigates the impact of the right-tail. That is, whenever $C_R > 220$, the firm freely switches to supplier L, as it may exercise this option at any time.

We now characterize a class of probability distributions for which the dominance of high variability holds, provided that all the other parameters remain unchanged. Let C_R follow a type-$\mathcal{F}$ distribution with mean $\overline{C}_R$ and standard deviation σ. We define a distribution to be type-$\mathcal{F}$ if its loss function (Cachon and Terwiesch, 2006) is increasing in the standard deviation while the mean remains fixed.

Theorem 3. *If C_R is drawn from a type-$\mathcal{F}$ distribution, TC(x) is nonincreasing in σ.*

In the three properties below, we prove that type-$\mathcal{F}$ distributions include some widely used distributions such as normal, uniform, and gamma distributions.

Property 1. *The normal distribution is a type-$\mathcal{F}$ distribution.*

Property 2. *The uniform distribution is a type-$\mathcal{F}$ distribution.*

Property 3. *The gamma distribution is a type-$\mathcal{F}$ distribution.*

5. Parametric analysis

We start by considering the impact of $\overline{C}_R$ and σ on an optimal location x^*. Fig. 3 shows the curves of x^* (shown as Opt. Location) for different values of σ over a range of $\overline{C}_R$. The curve for $\sigma = 0.3\mu$ represents the case where σ increases with $\overline{C}_R$ (shown also as μ) and thus cv remains constant. The two other lines represent cases where, as $\overline{C}_R$ increases for a fixed σ, cv decreases.

From Fig. 3, we find three observations. First, x^* is highly sensitive to changes in $\overline{C}_R$, particularly for its values around $C_L = 1$. Second, x^* approaches to the optimal location for $\overline{C}_R = 4$ even more rapidly for a smaller σ. Third, for a fixed value of $\overline{C}_R, x^*$ is closer to (farther away from) the CG of demands ($x = 0.5$) for a larger (smaller) σ. The intuition behind the third observation is that a

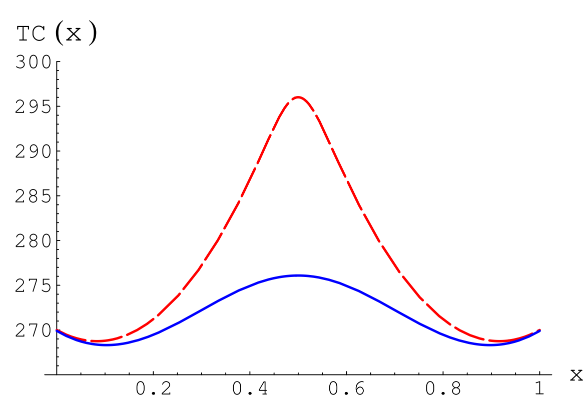

Fig. 2. Total cost ($TC(x)$) ($C_L = 180$, $s = 150$, $t = 180$, C_R normally distributed with $\overline{C}_R = 180, cv = 1/18$ for dashed line, and $cv = 1/3$ for solid line).

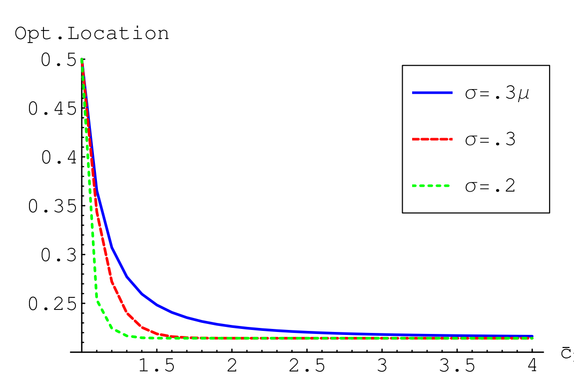

Fig. 3. Optimal location (x^*) as a function of $\overline{C}_R$ ($C_L = 1, s = 0.4, t = 0.7, C_R$ normally distributed).

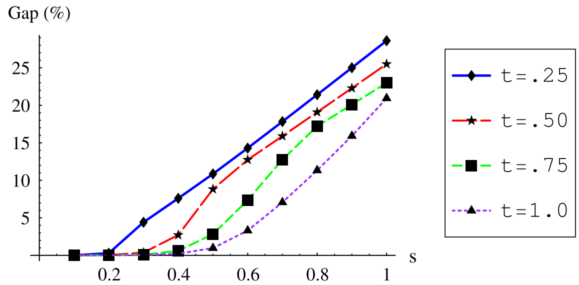

Fig. 4. Gap $((TC^u - TC^*)/TC^*)$ due to the lack of coordination ($C_L = 1$, C_R normally distributed with $\overline{C}_R = 0.9$ and $cv = 1/3$).

larger σ decreases $C(x)$ (and also $G(x)$) under free switching and thus it would be beneficial to get closer to the CG of demands, to further reduce $H(x)$.

Inefficiencies in a supply chain can originate from a lack of coordination between the decisions taken in different functions. Specifically, we consider a case where the firm considers only C_L or C_R set by a supplier, but not the inbound transportation cost $I(x)$, given that the DC is optimally located. For example, supplier R is chosen whenever $C_R \leqslant C_L$. While this sourcing decision is obviously non-optimal, it is not as rare as one might think, and we have seen examples of this practice. Fig. 4 shows the *gap* $(TC^u - TC^*)/TC^*$ between the optimal objective function value (TC^*) and the objective function value TC^u of this *uncoordinated* practice. We omit the detailed derivation of TC^u. Note that a larger s (t) makes $I(x)$ ($H(x)$) larger. In Fig. 4, the gap is larger for a larger s ($I(x)$) and for a smaller t ($H(x)$). Thus, the firm should pay a special attention to sourcing when $I(x)$ is a relatively large component of $TC(x)$ compared to $H(x)$.

Finally, we consider the *responsiveness* related to an optimal location. We refer to the average distance travelled from the DC to customers as the average outbound distance. Let d_P be the average outbound distance when the DC is located at x^*, and d_M be the corresponding distance when it is located at $x = 0.5$ (the median). Since the median location minimizes the average outbound distance, the lower bound for d_P is *0.25*. The upper bound of 0.5 can be obtained when the DC is located at one of the end points of the unit line. We refer to the gap of the two average outbound distances as the response gap, which is defined as $(d_P - d_M)/d_M$. Fig. 5 shows the response gap for different ratios of s/t.

In Fig. 5, C_R is drawn from a gamma distribution with parameters $(0.9cv^2, cv^{-2})$. Keeping $\overline{C}_R = 0.9$ fixed, we change only σ over different values of cv. Hence, the response gap is a proxy for the loss of responsiveness due to an optimal location. Fig. 5 shows that the larger the ratio s/t is, the larger the loss of responsiveness is. Special attention should be paid to the cases where $I(x)$ is relatively larger than $H(x)$. When σ is not clearly known, which is often the case, one could estimate a reasonable range of σ values for which the firm needs to investigate what happens to responsiveness. For example, in Fig. 5, response gaps get much larger for a relative short range of cv values around 0.4. For such cv values, the DC location can be non-optimal and also the response time to customers may be significantly long.

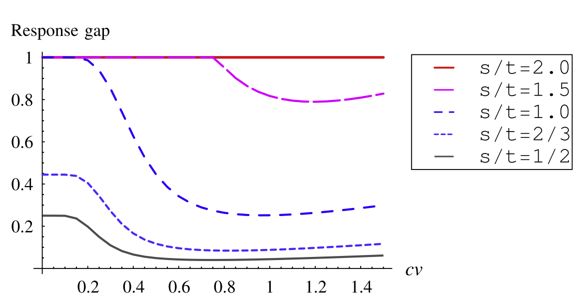

Fig. 5. Response gap $((d_P - d_M)/d_M)$ between x^* and the median location of $x = 0.5$ ($C_L = 1$, $s = 0.5$, C_R gamma-distributed with $(0.9cv^2, cv^{-2})$).

6. Beyond the basic model

In this section, we generalize some results found in previous sections with the following relaxed assumptions: (i) demands follow a general distribution on a unit line; (ii) both of the prices offered by two suppliers (C_L and C_R) follow distributions. Thus, the results in this section hold also for non-uniform demand distributions. Recall that the buying firm orders from supplier L if $C_L + xs \leqslant C_R + (1 - x)s$. Let $g(x, C_L) = C_L + x\ s$ and $g(x, C_R) = C_R + (1 - x)s$, where $g(x, C_i)$ is the procurement cost given that supplier i is chosen. The total cost is then

$$TC(x) = H(x) + G(x)$$
$$= H(x) + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\min_{i\in\{L,R\}} g(x, C_i)\right] f(C_L, C_R)dC_L dC_R, \tag{7}$$

where f is the joint density function of suppliers' prices. Since $g(x, C_L)$ and $g(x, C_R)$ are linear in x, $\min_{i\in\{L,R\}} g(x, C_i)$ is concave in x and thus $G(x)$ in (7) is also concave. Obviously, $H(x)$ is convex. Hence, the total cost $TC(x) = H(x) + G(x)$ is the sum of a convex function and a concave function. The d.-c. programming is still an option as a solution approach. Again, we can simply enumerate x with a very small increment for an approximate solution.

In addition to the two assumptions above, we also relax the assumption that two suppliers are pre-located at both ends of the unit line $[0, 1]$ by letting suppliers L and R be located at S_L and S_R, where $0 \leqslant S_L < S_R \leqslant 1$. Define $E_L = \min(S_L, CG)$ and $E_R = \max(CG, S_R)$. It is not hard to see that $x' \in [0, E_L) \cup (E_R, 1]$ is not optimal since $H(x') \geqslant H(x)$ and $G(x') \geqslant G(x)$ for some $x \in [E_L, E_R]$. Note that this holds for any demand distributions as long as their CGs of demands are available. Hence, we can limit the search for an optimal location within the interval, $[E_L, E_R]$.

We are now interested in a DC location on a two-dimensional plane with the following more generalized assumptions: (i) demands follow a general distribution on the plane; (ii) the prices offered by n suppliers follow distributions. In order to formalize the problem, we need modified definitions using two-dimensional coordinates. We assume that $S = \{1, \ldots, n\}$ is a set of n suppliers offering random prices $C_1, \ldots, C_n$ with means $\overline{C}_1, \ldots, \overline{C}_n$ and standard deviations $\sigma_1, \ldots, \sigma_n$. The problem is to find an optimal location (x^*, y^*) to minimize the total cost $TC(x, y)$ detailed as follows:

$$TC(x, y) = H(x, y) + G(x, y)$$
$$= H(x, y)$$
$$+ \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left[\min_{i\in S} g(x, y, C_i)\right] f(C_1, \ldots, C_n)dC_1\cdots dC_n, \tag{8}$$

where f is the joint density function of suppliers' prices. Note that $g(x, y, C_i)$ is convex because the Euclidean distance between (x, y) and supplier i's location is convex. Since $H(x, y)$ and $g(x, y, C_i)$ are all convex, the d.-c. approach cannot be used because the minimum of convex functions is neither convex nor concave. Still, we are able to show the dominance of high variability in a supplier's price for any number of suppliers, any demand distributions, and any price distributions, as long as their first and second moments are well defined.

Theorem 4. *If supplier i's price is independent of the other suppliers' and follows a type-$\mathcal{F}$ distribution, $TC(x, y)$ is decreasing in σ_i.*

Theorem 4 is the planar version of Theorem 3. Note that Theorem 4 holds under the independence of suppliers' prices. For example, in a global supply chain, international suppliers facing different business environments are more likely to offer independent prices. However, we acknowledge that the prices of a product can be dependent and leave the proof in this case for future research.

It is challenging to find an optimal location on a plane. Instead of developing solution methodologies, we focus on gaining some insights from the behavior of optimal locations as a function of the variability of a supplier's price. Thus, our observations below are based on some numerical examples and we have not been able to prove them. We first consider examples with two suppliers on a plane and then build insights by relating them to cases with more than two suppliers. In Fig. 6, suppliers i and j are represented by twin rectangles. Optimal locations are tracked against a fixed single point d, the CG of demands. As σ_i increases, an optimal location (x^*,y^*) shown as the center of an oval also approaches d. Note that a larger oval corresponds to a higher σ_i. This observation is supported also in Fig. 3. A larger σ_i makes $G(x,y)$ smaller and then $H(x,y)$ gets relatively larger. Thus, (x^*,y^*) moves towards d to further reduce $H(x,y)$. Fig. 6 confirms again the implication in Theorem 2. Since $\overline{C}_i \leqslant \overline{C}_j$, (x^*,y^*) is still closer to i than to j for any σ_i.

To understand better the behavior of (x^*,y^*) on a plane with more than two suppliers, we consider again the proof of Theorem 4 by creating *dummy* supplier j as a proxy for all suppliers but i. Suppose that supplier j is located at the point where she can offer the minimum of procurement costs incurred by all suppliers but i. Let random variables V and W be the procurement costs incurred by supplier i and supplier j, respectively. Let also W have a mean of $\overline{W}$. For any fixed d and (x,y), the total cost can be written as

$$TC(x,y) = H(x,y) + \overline{W} + \int_{-\infty}^{\overline{W}} (v - \overline{W}) p(v)\, dv. \tag{9}$$

For more details on (9), see equation (11) in the proof of Theorem 4 in Appendix. An optimal location (x^*,y^*) is determined by a balance between the three costs: the outbound transportation cost $H(x,y)$ minimized at d, the procurement costs V and W incurred by supplier i and supplier j. The relationship between the three *forces* can be demonstrated again in Fig. 6, where supplier $i(j)$ is located at $i(j)$ and m is the geometric median of angle $\angle idj$. If σ_i increases, V is more often smaller than $\overline{W}$ which is fixed. While $H(x,y)$ remains the same, the integral in (9) decreases. Note that this last term is negative. To further decrease $H(x,y)$, (x^*,y^*) moves closer to d. On the other hand, if σ_i decreases, (x^*,y^*) moves away from d. If $\sigma_i = 0$, (x^*,y^*) lies at i. Since $\overline{C}_i \leqslant \overline{C}_j$, as σ_i approaches 0, (x^*,y^*) moves closer to i than to j. In other words, the force driving (x^*,y^*) toward supplier i is greater than that towards supplier j. Hence, (x^*,y^*) lies somewhere in the triangle $\triangle dim$. Similarly, if $\overline{C}_i > \overline{C}_j$, (x^*,y^*) lies in

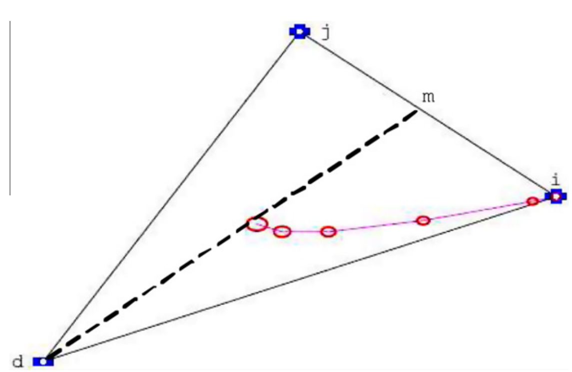

Fig. 6. Optimal location (x^*,y^*) as a function of σ_i for $\overline{C}_i \leqslant \overline{C}_j$.

$\triangle djm$. We leave the following conjecture as a summary of our observations:

Conjecture 1. *Assume that the distribution of demands is symmetric around its CG and the price of each supplier follows a type-$\mathcal{P}$ distribution. If $\overline{C}_i \leqslant \overline{C}_j$, (x^*,y^*) lies in $\triangle dim$. Otherwise, (x^*,y^*) lies in $\triangle djm$.*

Conjecture 1 is in accordance with Theorem 2 in the case of $d = m$. If proven, the conjecture would be very helpful in developing an efficient solution procedure for the case of a plane. We feel that this conjecture can be proved when the demand distribution is symmetric around the CG of demands. However, for asymmetric cases, we are not able to provide any conjecture or guiding discussions, and thus recommend problem-specific analysis.

We consider another generalization of our basic models by relaxing the assumption on switching costs. Under free switching, no transaction costs are incurred whenever the firm changes suppliers. Because this might not be the case for some firms or industries, it is worth considering the effect of switching cost ($K > 0$) on both x^* and $TC(x^*)$. Our previous results must be valid for a smaller K. For a larger K, if supplier i offers a price with a smaller $\overline{C}_i$ and a larger σ_i than the other suppliers, a firm would purchase only from supplier i and the other suppliers' locations have no impact on x^*. For the other cases, the model explicitly incorporating K is beyond the scope of our analysis. The main challenge would come from the dependence of consecutive purchases over time. The analysis of this problem would need to track the history of purchases and use a dynamic model employing Brownian motions, for instance.

7. Conclusion

Our generic models, although stylized, represent an evolution of location models for the analysis of the distribution-network design decisions that take both location and sourcing factors into account. Recognizing the growing importance of a firm's distribution functions, our methodology highlights the variability of a supplier's price and its impact on the location of a DC in a supply chain. While more complex settings are possible under more general assumptions, we developed our models in simpler settings to provide some analytical results and insightful observations from numerical examples. We further extended some results from the basic model to more general settings.

The results provide some useful insights for supply chain managers when configuring distribution networks.

1. *Optimal locations tend to gravitate towards suppliers offering lower mean prices.* The analysis provides a mathematical basis for this intuitive observation. We introduced a deterministic model in which both suppliers offer fixed prices and derived a closed-form optimal location. The deterministic model extends to the probabilistic version where one supplier's price is fixed and the other supplier's is random. We showed that an optimal location is closer to the supplier with a lower mean price. This implication holds for a wide range of distributions, specifically, for symmetric and unimodal distributions.
2. *Higher price variability promotes lower costs.* Suppliers with higher price variability are likely to offer lower prices under free switching. High price variability lowers the total cost and can provide some advantage to the buying firm. This dominance result holds for normal, uniform, and gamma distributions. We also extended the result analytically for the case of a two-dimensional plane.
3. *Higher price variability generally moves optimal locations closer to the center of gravity (CG) of demands.* Through the numerical examples, we observed that, as the variability increases,

optimal locations get closer to the CG of demands. High variability decreases the procurement cost because there are more opportunities for buyers to capture lower prices. It is beneficial, therefore, for a firm to locate a DC closer to the CG of demands to further reduce the outbound transportation cost.

We feel that there are rich opportunities to explore variations and extensions of our generic models. One obvious direction for future research is the need to expand the class of probability distributions for which our results still hold. It would be interesting and yet challenging to find an optimal location under another contract type, for example, placing a fixed fraction (α_i) of orders to supplier *i*. Allowing multiple DCs on a plane or a general network offers a more realistic scenario for supply chain managers, but the analysis would be even more complex.

One limitation of our analysis is the independence of suppliers' prices. The analysis of correlations among suppliers' prices is another topic worthy of future study. If supplier prices are negatively correlated, the benefit of free switching would be larger. Two positively correlated prices are not likely to allow opportunities for hedging against a higher price from a supplier and thus the total cost would increase.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2011.11.016.

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