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Global plant capacity and product allocation with pricing decisions

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Abstract

Trade-offs in global manufacturing decisions involve markets, resource costs, trade-barriers, currency exchange rates, joint ventures and investments. We develop a model that optimizes plant investment decisions, while ensuring that the plant investment overhead is optimally absorbed by products produced from that plant. The model also, simultaneously, determines prices by products and countries. The special structure of the model is exploited to construct a fast solution procedure. The model is used to study the implications of labor cost, transportation cost, demand, and import tariff on production quantities, investment, and overhead absorption pattern. Implications of changes in other global parameters such as local-content rule, local taxes, size of the market in a country, and long-term exchange rates are also studied.

Keywords: Investment; Global manufacturing; Overhead absorption; Customer preferences; Demand curve; Market size; Export/import; Pricing; Production quantity; Plant capacity; Resource-cost; Tariff; Exchange rate; Local content; Nonlinear optimization; Shadow price

1. Introduction

The phenomenal growth in foreign direct investments (FDI) in the last three decades (Ferdows, 1993), points to their overwhelming attractiveness. The pattern of investment, however, has changed from being primarily in the developing countries to mainly in the industrialized countries, in the last 10 years (Flaherty, 1996; Chakravarty, 1999). This shift in investment emphasis has been explained by Dunning (1993), as a shift from a resource-seeking to a market-seeking orientation. However, this is not to imply that market size is all that matters in FDI. Importance of factors such as tariffs, taxes, currency exchange rates, shipping, supplies, trade-barriers, local resources, and local demands have been discussed extensively in the literature (Cohen and Lee, 1989; Tombak, 1995; Dasu and Li, 1997; MacCormack et al., 1994; Hadjinicola and

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Kumar, 2002). Kanter (1995), describing Gillette's operations, reports that 70% of company's sales of $6 billion is outside the US, and that it has 58 facilities in 28 countries to serve markets in 200 countries.

The question, therefore, is how should a company configure its plants around the world to supply a global market with variations in customer preferences, resource availability, and cost structures from country to country (Cohen et al., 1989; Naik and Chakravarty, 1994). It is obvious that such a plant configuration must allow for multiple products, country and product specific pricing, and export and import of products between countries (Grunwald and Flamm, 1985). It is also clear that for investment in a manufacturing plant in a country to be viable, it must be completely recoverable from the outputs of that plant during its life cycle. To ensure investment recovery, accountants allocate overheads to products, based on some formula, and then add a mark-up percentage to determine the product's unit price. Pavia (1995) has shown how overhead allocation can be optimized, based on product demands and unit costs, for given mark-up percentages, and shows that some products may not absorb any overhead at all. Bhutta et al. (2003) considers integrated decision making for investment, production, and distribution, assuming fixed prices. Taylor (1997) discusses a model that integrates product choices with global plant capacities, assuming known unit prices and nonexistence of trade barriers. Unit prices are assumed to be known and global trade-barriers are not included in this model, however. Syam (2000), similarly, assumes fixed unit prices and ignores trade-barriers, in his model of regional capacity planning. These three models are similar in that they assume fixed unit prices, enabling formulation of linear models with mixed integer variables.

We argue, in this paper, that while plant investment creates manufacturing capacity, it also creates an overhead burden. Thus, if a country is positioned to supply to a large regional demand, it must also be prepared to absorb a large overhead. Clearly, a high overhead absorption by a product would be detrimental to its export, especially to countries with high import tariffs. It should, therefore, be apparent that adding capacity might reduce the "real" profit margin, which is inclusive of overhead cost. What may not be obvious, however, is the way the overhead allocation decision is intertwined with decisions related to how much and where (country) to invest, how much of each product to produce, how much of it to export to other countries (which countries?), and what prices to charge for products in different countries.

Simple economic logic tells us that the major reasons for investing in a foreign country are weak currency, low labor cost, high import tariff, low local taxes, and high demand for the product (Dunning, 1993). Local content rules, on the other hand, may lead to increase in investment in some countries at the expense of countries that impose too many barriers. Foreign investments in Asia and in EEC countries are examples of companies seeking low labor cost, and low tariffs, respectively. In certain scenarios, special factors such as first-mover advantage, emerging markets, and joint-venture opportunities, may also impact global investment decisions (Bartlett and Ghoshal, 1991). We do not analyze such scenarios in our model.

The decisions usually under a company's control are (a) where and how much to invest, (b) what quantities of which products to produce in a plant, (c) which products to absorb (and how much) the plant investment overhead, in a country, (d) what quantities of which products to export from a country, and (e) how to price the products in each country of operation. Investment decisions are generally made only once during a plant's life cycle, based on average parameter values. Actual values of parameters may, however, vary during the plant's life cycle. Thus the decisions in (b) to (e) will need to be reestablished each period for *given* plant sizes, in different countries.

In this paper, we first develop and solve a quantitative model to determine specific values related to the above decisions. We then use this model to generate managerial insights, such as, (i) how do tariff rates impact overhead allocation decisions, (ii) should products with low variable cost absorb a higher share of overheads, (iii) during a period of trade liberalization (as is now) should a company increase its foreign direct investment, (iv) when does it make sense to invest in improving plant productivity, and (v) how does a weakening of currency in a country impact investment, production, and export/import decisions from that country. The salient issues in the context of investment in plants, and globalization are discussed in Section 2. In Section 3 we outline the mathematical model, and establish its properties. Approaches for solving the model are discussed in Section 4. Results related to the sensitivity of profit and investment with respect to parameter values are studied in Section 5. In Section 6 we discuss the impact of country specific costs such as fixed cost of setting up a plant, and the exchange rates. Issues related to market size and local content rules are discussed in Section 7, and conclusions are outlined in Section 8.

2. Major issues in investment decisions

As mentioned earlier, the country-specific factors such as resources, markets, and trade-barriers would be crucial to global investment in facilities (Vernon, 1966). The attribute of "resource" is very similar to that of Porter's (1990) factor conditions. In our model we interpret this as direct costs, and it is inclusive of labor and material costs. We model the "market" attribute by three sub-attributes: distance of the country from other countries under consideration, customer-preferences that determine demand of a product in a country, and the size of market (size of the population that is likely to buy the product) (Kotler and Armstrong, 1989). We model the relationship between price and demand using traditional economic theories. The trade-barrier attribute consists of several sub-attributes such as import tariff rates, corporate tax on profits, currency exchange rate, financial business incentives, and local content requirements. As we are interested in long term decisions, we do not model within-plant micro issues such as economics of manufacturing, including inventory.

It is therefore obvious that country attributes (Ferdows, 1997) would determine whether a country becomes a manufacturing hub with exports to other countries, a market for imported goods, or both. In what follows we elaborate these further in terms of manufacturing and marketing issues.

2.1. Plant configuration

Clearly, the two extremes of plant configuration are, (a) to have plants in each country, each selling to its domestic market, and (b) to have a single centralized plant and export from that plant to other countries. The optimal configuration may, however, be the one that has plants in a subset of countries with exports to countries without plants, as shown in Fig. 1. Since manufacturing cost and demand of products need not be

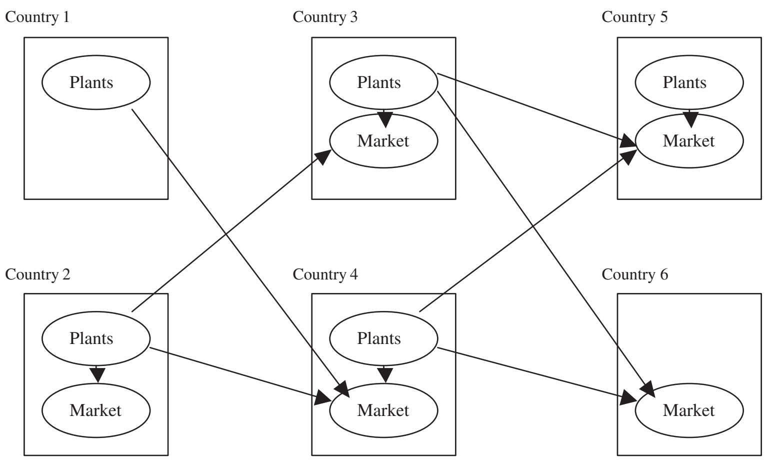

Fig. 1. Sample plant configuration.

the same in all countries, configuration of plant and market for different products need not be identical. Thus the links between plants and markets (shown by arrows in Fig. 1) would be different for different products, depending on whether the demand for a product in a certain country is satisfied with local production or with imported goods.

We denote the set I $(i \in I)$ as the set of products manufactured by the company, in all the countries taken together. Set J $(j \in J))$ includes all countries where the company may have manufacturing operations, and the set K $(k \in K)$ includes all countries where the company may have markets. Thus, we can write, in the context of Fig. 1,

$$I = \{1\}, \quad J = \{1, 2, 3, 4, 5\} \quad \text{and} \quad K = \{2, 3, 4, 5, 6\}.$$

2.2. Manufacturing and distribution costs

2.2.1. Manufacturing

The cost of manufacturing a product in a country will obviously depend on the cost of labor and material, and the quantity of production. To account for the cost of investment (i.e., overhead), as in the accounting practice (Govindrajan and Anthony, 1983), we add a certain amount (to be determined) to the unit variable cost. In some scenarios plants may require additional fixed investments, independent of plant size (e.g. cost of feasibility study, payment for the architect, real estate fees, and licenses and permits related to import and export of goods). As we show in Section 6, a high fixed cost in a country may exclude it from production consideration, due to scale economy.

We use the following notation to develop cost expressions:

Variable unit cost of product i in country $j = v_{ij}$,
Unit cost with "overhead load" $= x_{ij}$,
Overhead absorbed per unit $= x_{ij} - v_{ij}$,
Quantity of product i produced in country j for country $k = q_{ijk}$.

We can now express cost as

$$\text{Manufacturing cost} = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} q_{ijk} x_{ij}. \tag{1}$$

We assume that the cost (amortization) of investment in a manufacturing plant in country j is F_j. Therefore, to ensure total investment cost recovery, we must have

$$\sum_{i \in I} \sum_{k \in K} (x_{ij} - v_{ij}) q_{ijk} \geqslant F_j. \tag{2}$$

The lower bound on x_{ij}, obviously, is

$$v_{ij} \leqslant x_{ij}. \tag{3}$$

Assuming that each dollar of investment in country j generates w_j units of manufacturing capacity (Bhutta et al., 2003), and each unit of product i requires β_{ij} units of capacity, the capacity constraint can be written as

$$\sum_{i \in I} \sum_{k \in K} \beta_{ij} q_{ijk} \leqslant w_j F_j. \tag{4}$$

Note that β_{ij}, a relative value, can be estimated from the daily production rates of products. Clearly, β_{ij} reflects productivity in country j.

2.2.2. Distribution

Consider product i, manufactured in country j and sold in country k. Observe that the unit cost of the product as it lands in country k is x_{ij}. An import tariff will be levied on this product based on x_{ij}. In addition, the company will incur a shipping cost. We use the following notation:

Tariff in country k expressed as a proportion of unit cost $= t_{ik}$.
Transportation cost per unit $= T_{ijk}$.

It is obvious that $t_{ik} = 0$ if country k does not charge a tariff, or if $k = j$ for all j.

Hence, the cost incurred by the company in paying import tariffs and shipping can be written as

$$\text{Distribution cost} = \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}(t_{ik}x_{ij} + T_{ijk})q_{ijk}. \tag{5}$$

Thus, the total manufacturing and distribution cost will be written as

$$\text{Total cost} = \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\{(1 + t_{ik})x_{ij} + T_{ijk}\}q_{ijk}. \tag{6}$$

2.3. Price and demand

In the economics and marketing literature, the variation of demand with price is described by demand curves. A general form of demand curves used in many applications is

$$\text{Demand} = a(\text{Price})^n,$$

where a and n are given constants and $n < 0$ (Lilien et al., 1992). Simon (1980) discusses many application scenarios where empirical evidence for such demand curves has been found. Henderson and Quandt (1980) discuss the appropriate range of n for different industries. They point out that for most industries $n < -1$. Pavia (1995) has used a similar demand curve for developing an optimization model of overhead allocation to products.

In our model we use a demand curve in which:

D_{ijk} = demand per year in country k for product i, produced in country j,
p_{ijk} = price charged in country k per unit of product i, produced in country j,
a_{ijk} = a nonnegative scaling factor.

Observe that the parameter a_{ijk} models factors such as customer preferences in country k for product i, and the "image" of country j where i is produced. The values of a_{ijk} in a country will be estimated through market research by observing the demand of products imported from other countries, and their corresponding prices. Thus the demand curve in our model is expressed as

$$D_{ijk} = a_{ijk}p_{ijk}^n \quad \text{where } n < 0. \tag{7}$$

The assumption that the exponent n is the same for all products and markets can be easily relaxed by replacing n with n_{ik} throughout. To account for the population size we consider the impact of bounding the total demand of a product in a country by an upper limit, in Section 7.

In our model p_{ijk} is treated (implicitly) as a decision variable. For given values of i, j, and k, the model generates a single value of p_{ijk} for the planning horizon. If prices vary during the planning horizon, we denote the value of p_{ijk} in period m as p_m. Next letting p $(= p_{ijk})$ to denote the price when the m periods are treated as a single period, we can state the equivalence between p and p_m as $p^n = \sum_m p_m^n$; p_m is not an independent decision variable in *strategic* capacity planning. That is, once p is known (from the model), there would be a large number of degrees-of-freedom for determining p_m.

Next, since Revenue = (Price) (Quantity Sold), it is clear that

$$\text{Revenue} = \sum_{j\in J}\sum_{i\in I}\sum_{k\in K} p_{ijk}q_{ijk}. \tag{8}$$

2.4. Profit

We are now in a position to develop an expression for total profit, based on the simple equation

$$\text{Profit} = \text{Revenue} - \text{Cost}.$$

That is,

$$\text{Profit} = \sum_{j\in J}\sum_{i\in I}\sum_{k\in K} p_{ijk}q_{ijk} - \{(1+t_{ik})x_{ij} + T_{ijk}\}q_{ijk}. \tag{9}$$

Note that since plant capacity is being determined in a strategic sense, to satisfy a certain demand, we do not include *inventory-related costs* in this analysis. Without loss of generality, a capacity cushion can be added to balance the cost of inventory with capacity shortage (Hayes and Wheelwright, 1984). Substituting for p_{ijk} from (7) we can rewrite Profit as

$$\text{Profit} = \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\left\{\left(\frac{D_{ijk}}{a_{ijk}}\right)^{1/n} - (1+t_{ik})x_{ij} - T_{ijk}\right\}q_{ijk}.$$

It can be shown, as in Lemma 1 in Appendix A, that $q_{ijk} = D_{ijk}$. Hence,

$$\text{Profit } z = \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\left\{\left(\frac{q_{ijk}}{a_{ijk}}\right)^{1/n} - (1+t_{ik})x_{ij} - T_{ijk}\right\}q_{ijk}. \tag{10}$$

3. Profit maximization

The decision facing a company is how to maximize its profits over a planning horizon, accruing from sales of products manufactured in several countries. We assume that the company has a fixed sum (F) available for investment in plants at the start of the planning horizon.

Based on earlier discussion, the optimization model can now be stated as

Program P_1

$$\text{Maximize} \quad z = \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\left\{\left(\frac{q_{ijk}}{a_{ijk}}\right)^{1/n} - (1+t_{ik})x_{ij} - T_{ijk}\right\}q_{ijk} \tag{11}$$

$$\text{subject to} \quad \sum_{i\in I}\sum_{k\in K}\{x_{ij} - v_{ij}\}q_{ijk} \geqslant F_j, \quad \forall j, \tag{12}$$

$$\sum_{i\in I}\sum_{k\in K}\beta_{ij}q_{ijk} \leqslant w_jF_j, \quad \forall j, \tag{13}$$

$$x_{ij} \geqslant v_{ij}, \quad \forall i \text{ and } j, \tag{14}$$

$$\sum_{j\in J}F_j \leqslant F, \tag{15}$$

where F is the total investment budget.

The decision variables are q_{ijk}, x_{ij}, F_j, and known constants are a_{ijk}, t_{ik}, T_{ijk}, β_{ij}, n, w_j, v_{ij}, and F. We have, at this point, not included the effects of several other variables, such as the size of market in a country, local content rules, local taxes, and currency exchange rates. We shall discuss how the model can be modified to incorporate these variables, in Section 7. Note also that other motivations for setting up plants, such as the first-mover advantage, are not modeled in our model.

Observe that since $x_{ij} - v_{ij}$ is the overhead absorbed per unit of sale, constraint (12) ensures that the cost of investment in a plant in a country (F_j) is *completely* recovered from the sale of products manufactured in *that* plant. This is important as it ensures that there is no avoidance of local taxes and tariffs. Accounting practices that let overheads incurred in one country be charged to plants in high-tax or high-tariff countries, have come under severe criticism lately. We, therefore, do not seek to model such practices. Constraint (13) ensures that the required capacity (for production) in a plant does not exceed the capacity that can be "bought" from the investment amount of F_j. Constraint (14) ensures that the unit cost after overhead allocation stays above its lower limit. Constraint (15) ensures that the total investment budget is not exceeded.

Observe in program P_1 that, the objective function (11) and constraint (12) are nonlinear. It can, however, be verified that P_1 is a convex program so that any solution that satisfies the Kuhn–Tucker conditions will also be the global optimum (Taha, 1982).

The new objective function, inclusive of the Kuhn–Tucker multipliers, is written as

$$L = \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\left\{\left(\frac{q_{ijk}}{a_{ijk}}\right)^{1/n} - (1+t_{ik})x_{ij} - T_{ijk}\right\}q_{ijk} + \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\lambda_j\{(x_{ij}-v_{ij})q_{ijk} - F_j\}$$
$$- \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\mu_j\{\beta_{ij}q_{ijk} - w_jF_j\} + \sum_{i\in I}\sum_{j\in J}\delta_{ij}(x_{ij}-v_{ij}) - \psi\left(\sum_{j\in J}F_j - F\right), \tag{16}$$

where the nonnegative multipliers λ_j, μ_j, δ_{ij}, and ψ correspond to the constraints (12) to (15), respectively.

The Kuhn–Tucker conditions can now be written as

$$\frac{\partial L}{\partial x_{ij}} = -\sum_{k\in K}(1+t_{ik})q_{ijk} + \lambda_j\sum_{k\in K}q_{ijk} + \delta_{ij} = 0, \quad \forall i,j, \tag{17}$$

$$\frac{\partial L}{\partial q_{ijk}} = \frac{(n+1)}{n}\left(\frac{q_{ijk}}{a_{ijk}}\right)^{1/n} - (1+t_{ik})x_{ij} - T_{ijk} + \lambda_j(x_{ij}-v_{ij}) - \mu_j\beta_{ij} = 0, \quad \forall i,j,k, \tag{18}$$

$$\frac{\partial L}{\partial F_j} = -\lambda_j + w_j\mu_j - \psi = 0, \quad \forall j, \tag{19}$$

$$q_{ijk} \geqslant 0, \quad \forall i,j,k, \tag{20}$$

$$F_j \geqslant 0, \quad \forall j, \tag{21}$$

$$\lambda_j\left\{\sum_{i\in I}\sum_{k\in K}(x_{ij}-v_{ij})q_{ijk} - F_j\right\} = 0, \quad \forall j, \tag{22}$$

$$\mu_j\left(\sum_{i\in I}\sum_{k\in K}\beta_{ij}q_{ijk} - w_jF_j\right) = 0, \quad \forall j, \tag{23}$$

$$\delta_{ij}(x_{ij}-v_{ij}) = 0, \quad \forall i,j, \tag{24}$$

$$\psi\left(\sum_{j\in J} F_j - F\right) = 0. \tag{25}$$

Observe that nonnegativity of q_{ijk}, F_j, λ_j, μ_j, δ_{ij}, and ψ are incorporated in Eqs. (17) to (19). The condition $x_{ij} > v_{ij}$ is accounted for by δ_{ij} in Eq. (17). Eqs. (22) to (25) are the so-called orthogonality conditions representing the property of complementary slackness of each constraint. Eqs. (17) to (19) can be rewritten, respectively, as

$$\lambda_j = \frac{\sum_{k\in K}(1+t_{ik})q_{ijk} - \delta_{ij}}{\sum_{k\in K} q_{ijk}}, \quad \text{for all } i, \tag{26}$$

$$q_{ijk} = (a_{ijk})\left(\frac{n}{n+1}\right)^n \{(1+t_{ik})x_{ij} + T_{ijk} - \lambda_j(x_{ij} - v_{ij}) + \mu_j\beta_{ij}\}^n, \quad \text{for all } i, j, \text{ and } k, \tag{27}$$

$$\mu_j = \frac{(\psi + \lambda_j)}{w_j}, \quad \text{for all } j. \tag{28}$$

Note that λ_j represents the marginal cost (shadow price) of increasing the overhead burden (F_j), and μ_j the marginal cost (shadow price) of decreasing production capacity. Also note that if the size of the total investment budget (F) is less than its optimal value, $\psi > 0$. Therefore, from (28) $(\mu_j w_j - \lambda_j = \psi)$ it follows that the cost of reducing F_j $(= \mu_j w_j)$ would exceed that of increasing it $(= \lambda_j)$. That is the country will stand to gain by increasing F_j, and hence F.

The optimality conditions (17) to (25) cannot be solved easily as most of them are nonlinear equations. However, these equations possess a special structure, which can be exploited for developing effective solution strategies. We next explore this structure.

Note that from (26) we have

$$\lambda_j = \frac{\sum_{k\in K}(1+t_{ik})q_{ijk}}{\sum_{k\in K} q_{ijk}} - \frac{\delta_{ij}}{\sum_{k\in K} q_{ijk}}. \tag{29}$$

Using (29) we can establish Theorem 1 below.

Theorem 1. *λ_j will be obtained in the range*

$$1 \leqslant \lambda_j \leqslant 2, \tag{30}$$

if t_{ik} is in the range $0 \leqslant t_{ik} \leqslant 1$.

Proof. (See Appendix A)

Note that for t_{ik} to exceed 1.0 the rate of import tariff would have to exceed 100%, a rarity in today's world of declining tariffs. Therefore, we limit our analysis to $0 \leqslant t_{ik} \leqslant 1$. Note, as mentioned earlier, if a country chosen for business levies no tariff, $t_{ik} = 0$. The MFN (most favored nation) designation that the United States has with some countries, and the trading pacts such as NAFTA are attempts at satisfying this condition.

It is clear that using Theorem 1 the search for optimal λ_j can be contained between the values of 1.0 and 2.0, greatly speeding up the solution procedure.

4. Solving the optimization model for tariff and no-tariff scenarios

One of the objectives of this research is to investigate whether or not the global plant location problem requires new modeling and/or solution approaches. Note that by assuming the tariff rate t_{ik} to be zero for all i and k, our model can be applied directly to a domestic (one country with multiple regions) setting. We next investigate the implications of setting $t_{ik} = 0$ to the model, and its solution approach.

4.1. Zero tariff

First, observe in (29) that we would now have

$$\lambda_j = 1 - \frac{\delta_{ij}}{\sum q_{ijk}}, \quad \delta_{ij} \geqslant 0. \tag{31}$$

If $\lambda_j < 1$ it would follow from (31) that $\delta_{ij} > 0$ for all i, which in turn would imply (using (24)) that $x_{ij} = v_{ij}$ for all i. This is not feasible, as (12) will be violated. Hence $\lambda_j = 1$ and $\delta_{ij} = 0$ for all i. Substituting $\lambda_j = 1$, $\delta_{ij} = 0$ and $t_{ik} = 0$ in (16) and simplifying we obtain

$$\begin{aligned} L = {} & \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\left\{\left(\frac{q_{ijk}}{a_{ijk}}\right)^{\frac{1}{n}} - v_{ij} - T_{ijk}\right\}q_{ijk} - \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\mu_j\beta_{ij}q_{ijk} \\ & - \sum_{j\in J}(1-\mu_j w_j)F_j - \psi\left(\sum_{j\in J}F_j - F\right). \end{aligned} \tag{32}$$

Eq. (27) would be modified to

$$q_{ijk} = (a_{ijk})\left(\frac{n}{n+1}\right)^n\{v_{ij} + T_{ijk} + \mu_j\beta_{ij}\}^n. \tag{33}$$

Since $\lambda_j > 0$, we have from (22),

$$\sum_{i\in I}\sum_{k\in K}(x_{ij} - v_{ij})q_{ijk} = F_j. \tag{34}$$

Notice that x_{ij} gets eliminated from the objective function (32), and from the expression for q_{ijk} in (33). Therefore the constraint (34) ceases to influence the optimal solution, and becomes irrelevant to the optimization process. F_j, however, remains relevant, as (13) must still be satisfied. Clearly, we can solve for q_{ijk} in (33) if μ_j, which is a function of ψ as in (28), is known.

Since $\psi \geqslant 0$, we investigate solutions with $\psi = 0$ and $\psi > 0$. If $\psi = 0$, from (28) we would have $\mu_j = \frac{1}{w_j}$. The modified expressions for q_{ijk} would be

$$q_{ijk} = (a_{ijk})\left(\frac{n}{n+1}\right)^n\left\{v_{ij} + T_{ijk} + \frac{\beta_{ij}}{w_j}\right\}^n.$$

Since $\mu_j = \frac{1}{w_j} > 0$, (13) will be satisfied as an equation, so that we can express F_j as

$$F_j = \frac{1}{w_j}\sum_{i\in I}\sum_{k\in K}\beta_{ij}(a_{ijk})\left(\frac{n}{n+1}\right)^n\left\{v_{ij} + T_{ijk} + \frac{\beta_{ij}}{w_j}\right\}^n. \tag{35}$$

Thus, closed-form expressions for q_{ijk} and F_j would exist. Note that $\psi = 0$ also implies that the constraint (15) is irrelevant, which will be the case when the value of F is high.

Next, if $\psi > 0$, we have from (28),

$$\mu_j = \frac{1+\psi}{w_j} > 0. \tag{36}$$

Hence, it would follow from (23) that

$$\sum_{i\in I}\sum_{k\in K}\beta_{ij}q_{ijk} = w_jF_j. \tag{37}$$

Substituting for q_{ijk} from (33) in (37), and using (36) to eliminate μ_j, we have

$$\sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\frac{\beta_{ij}}{w_j}(a_{ijk})\left(\frac{n}{n+1}\right)^n\left\{v_{ij}+T_{ijk}+\frac{\beta_{ij}}{w_j}(1+\psi)\right\}^n = F,$$

which can be solved for ψ.

Once ψ is known, we determine μ_j from (28), q_{ijk} from (33), and F_j from (37). Again, we see that optimal solutions can be determined easily.

Observe that $\delta_{ij}=0$ implies that the optimal solution is not sensitive to x_{ij} even if does not exceed v_{ij}. That is, allocation of overhead to products will be exogenous to the optimization problem, and accepted accounting practices (Govindrajan and Anthony, 1983) can be used to satisfy (34). Clearly, x_{ij} (for given i and j) can have a large number of possible solutions, including the one where $x_{ij} > v_{ij}$ for all i and j. That is, all products from all plants may absorb overhead.

4.2. Nonzero tariff

For the case, $t_{ik} \geqslant 0$, if there is at least a single $t_{ik} \neq 0$, we would have from (26),

$$\lambda_j = 1+\vartheta-\frac{\delta_{ij}}{\sum_{k\in K}q_{ijk}},$$

where $0<\vartheta<1$.

Clearly, now δ_{ij} can exceed zero for one or more values of i. Hence the expression for L, unlike (32), cannot be simplified.

Consider the scenario where the worldwide tariffs increase. From (26) it is clear that either λ_j increases, or more of the δ_{ij} values become positive (to hold λ_j constant). First consider the case of increased λ_j. It can be seen from (28) that ψ will decrease, or μ_j will increase, either or both indicate that F_j will be reduced. Finally if we let δ_{ij} increase, holding λ_j constant, corresponding x_{ij} would be reduced to v_{ij}, which again will lead to a reduction in F_j.

Clearly, one of the ways of coping with a reduced F_j is to decrease β_{ij} (that is, increase productivity in country j). Obviously the production system must become more efficient, indicating that *efficiency improvement* measures may become critical in a rising tariff scenario.

With the presence of x in the objective function, it is not possible to establish simple expressions for optimal q, x, and F, as in the zero-tariff case. However, using Theorem 2 below we can develop an efficient search procedure to solve for the optimal values.

Theorem 2. *The search for the optimal solution will entail a search in only λ_j, ψ, and x_{ij} dimensions, with a computational complexity of $O(\eta)$, where $\eta = |J|$.*

Proof. (See Appendix A)

In general, to solve the Kuhn–Tucker equations (which are nonlinear), a search in all x_{ij}, q_{ijk}, λ_j, μ_j, δ_{ij}, and ψ may be required. Theorem 2 simplifies the search greatly by limiting it to only three independent variables, λ_j, x_{ij}, and ψ. Also note that while κ (number of products) can be large, the value of η (the number of countries of interest) will be relatively small, keeping the computational complexity manageable.

The solution procedure is based upon the properties outlined in Theorem 2. The grid search starts with starting values of ψ and λ_j set equal to 0 and 1.5 (since $1 \leqslant \lambda_j \leqslant 2$). The values of λ_j and ψ help in (a) determining q_{ijk} and δ_{ij} (in sequence) using appropriate equations, and (b) identifying products that require search in x_{ij} (i.e., $\delta_{ij} \leqslant 0$). The investment cost F_j that would provide sufficient overhead recovery, and adequate manufacturing capacity can then be determined separately. Note that for the appropriate value of λ_j these two values of F_j will be equal. Similarly, for the appropriate value of ψ the sum of all F_j will equal F. This procedure is outlined in detail in Appendix B.

It is thus clear that with nonzero tariffs, both the model and its solution procedure change significantly.

5. Profit and investment sensitivity

To study the variations in profit and investment with parameter values, a small experiment with ten products and five countries was conducted. It is assumed that a prescreen of potential sites disqualified two of the five possible sites from plant location. The product could be sold in all five countries, however. The parameter values were generated randomly within specified ranges. The chosen ranges were: v_{ij} (5–15); T_{ijk} (2–8); a_{ijk} (500,000–1,000,000); and β_{ij} (4–10).

Variations in profit and investment in the first three countries were observed. For products sold in countries 4 and 5, all profits were credited to the countries from where the products were imported. We are also interested in the pattern of overhead allocation to products in different countries.

In Table 1, the solution $x_{ij} = v_{ij}$ is shown as "$x = v$", and if x_{ij} exceeds v_{ij}, it is indicated as "$x > v$" in appropriate cells. Observe in Table 1 that in country 1, products 3, 5, and 9 absorb overhead, whereas in country 3, the corresponding products are 2 and 6. In country 2, four products (1, 6, 8, and 9) absorb overhead. Also observe that products 4, 7, and 10 are special in that they absorb no overhead in any country.

To understand how the values of x_{ij} are influenced by the country and/or product characteristics we revisit expressions (11) and (12) (we ignore constraints (13) to (15) to keep our discussion simple). It is clear from Appendix C that z_j (profit accruing in country j) decreases, and L_j (overhead absorbed in country j) increases, with x_{ij}. Since L_j must exceed or be equal to F_j, we would be interested in maximizing L_j for any value of z_j, by appropriately choosing x_{ij}. That is, the ratio, $R_j = L_j/z_j$ will be maximized. As shown in Appendix C, assuming $\lambda_j = 0$, we can express the differential coefficient, when $x_{ij} = v_{ij}$, as

Table 1
Unit cost with overhead load, x_{ij}

Product	Country		
	1	2	3
1	$x = v$	$x > v$	$x = v$
2	$x = v$	$x = v$	$x > v$
3	$x > v$	$x = v$	$x = v$
4	$x = v$	$x = v$	$x = v$
5	$x > v$	$x = v$	$x = v$
6	$x = v$	$x > v$	$x > v$
7	$x = v$	$x = v$	$x = v$
8	$x = v$	$x > v$	$x = v$
9	$x > v$	$x > v$	$x = v$
10	$x = v$	$x = v$	$x = v$

$$\frac{\partial R_j}{\partial x_{ij}} = \frac{-(n+1)\sum_k a_{ijk}\{(1+t_{ik})v_{ij}+T_{ijk}\}^n}{\sum_i\sum_k a_{ijk}\{(1+t_{ik})v_{ij}+T_{ijk}\}^{n+1}}.$$

Clearly if $\lambda_j > 0$, $L_j = F_j$, and $\frac{\partial R_j}{\partial x_{ij}} = 0$.

Thus, for $n \leqslant -1$,

$$\frac{\partial R_j}{\partial x_{ij}} \geqslant 0.$$

Next, note that as v_{ij} is increased the numerator of $\frac{\partial R_j}{\partial x_{ij}}$ will decrease faster than the denominator, and so $\frac{\partial R_j}{\partial x_{ij}}$ will decrease. With a similar reasoning we may also conclude that $\frac{\partial R_j}{\partial x_{ij}}$ will decrease with T_{ijk} and t_{ik}, and increase with a_{ijk}. Therefore, a product in country j will be expected to absorb more of the overhead cost (i.e., $x_{ij} > v_{ij}$) if its unit labor cost content (v_{ij}) or its unit transportation cost (T_{ijk}) are low, or if the tariff levied on this product in other countries is low. While this is intuitive, the expression for $\frac{\partial R_j}{\partial x_{ij}}$ tells us how these variables may interact. Thus, a unit increase in v_{ij} would have the same impact on $\frac{\partial R_j}{\partial x_{ij}}$, as $1 + t_{ik}$ units increase in T_{ijk}.

In the context of different countries, assume that v_{i2} is higher than v_{i1}. Assuming all other parameter values to be identical, we would have $\frac{\partial R_2}{\partial x_{i2}} \leqslant \frac{\partial R_1}{\partial x_{i1}}$. That is product i, while absorbing overhead in country 1, may not do so in country 2.

Thus we conclude that (a) the pattern of optimal overhead allocation to products is influenced by country characteristics, and (b) some products may never absorb any overhead in any country. This may also suggest that v_{4j}, v_{7j} and v_{10j} are relatively high in all three countries.

Next, the model was run by varying the parameters of tariff, demand scaling factor, production efficiency and unit transportation cost, in country 2, by up to ±75% from their respective values in the base case. For the experiment with demand scaling, scenarios beyond +25% variation became infeasible. The investment in plants and the profits are shown, by country, in Figs. 2 to 9.

Observe in Fig. 2 that as the import tariff in country 2 decreases investment in that country decreases and investments in other countries increase. That is, the company would manufacture in a high tariff country (other things being equal) to reduce its cost of imports to that country. Observe in Fig. 3, that the higher investments are just enough to hold the profits constant in countries 1 and 3. The higher sales in countries 1 and 3 are generated by cutting unit price, which reduces profit margins. The profit-drop in country 2 is brought about mainly by reduced revenue from the sale of locally produced goods, as investment in plants drops. Clearly, for the parameter values in our experiment, this reduction more than offsets the increased profits from the sale of imported goods to country 2.

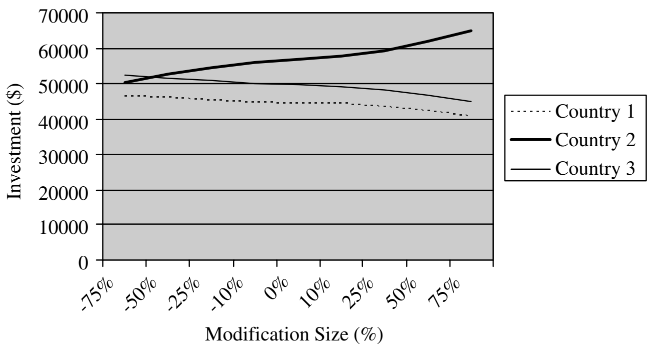

Fig. 2. Variation in plant investment with tariff.

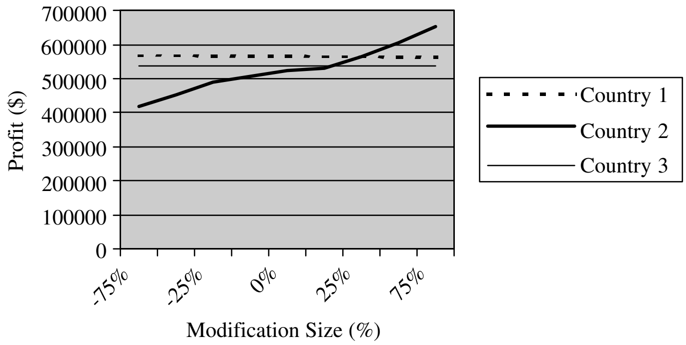

Fig. 3. Variation in profit with tariff.

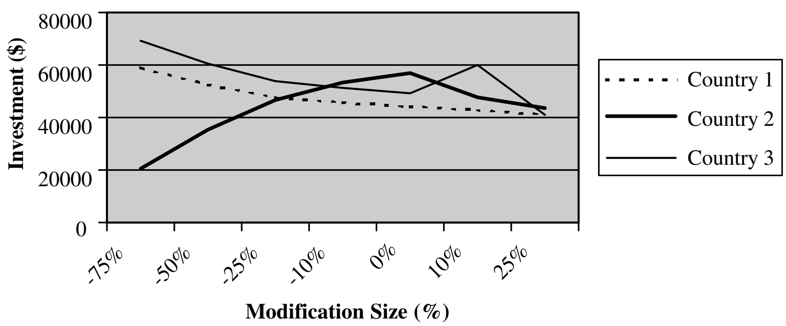

Fig. 4. Variation of investment with demand elasticity.

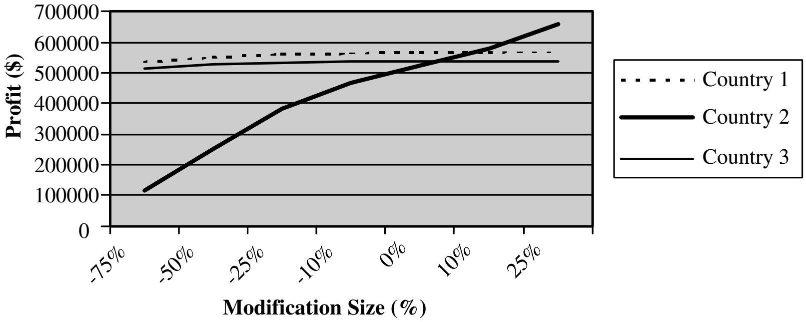

Fig. 5. Variation of profit with demand elasticity.

Next, the results of increasing the scale factor (a_{ijk}) in country 2 are shown in Figs. 4 and 5. Since increasing scale factor is equivalent to increasing demand in that country, investment in country 2 increases. The investment in country 2 reaches a peak and then drops slightly; the drop is offset by increase in investment in country 3. The manufacturing efficiency $(1/\beta)$ in country 3 (not shown) is higher than its value in country 2, and it appears to become a significant factor beyond a certain level of investment. Fig. 5 shows that the profit keeps increasing even after the investment in country 2 has peaked. This is explained

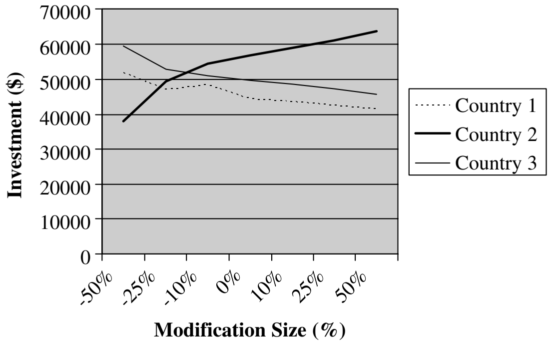

Fig. 6. Variation of investment with productivity.

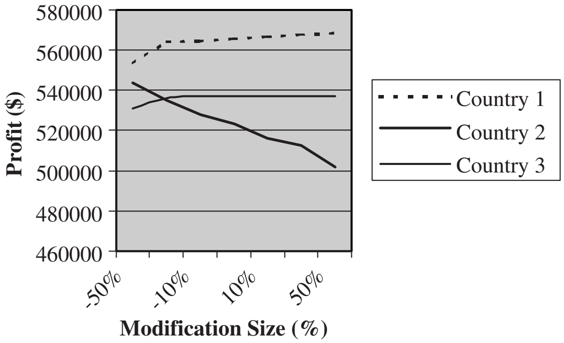

Fig. 7. Variation of profit with productivity.

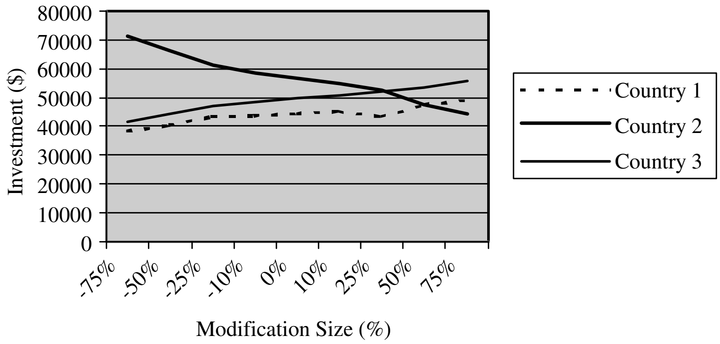

Fig. 8. Variation of investment with unit transportation cost.

by the fact that any increase in demand (higher a_{ijk}) in country 2 (after investment in plants has peaked) increases unit prices, and so the profit increases. Observe also that the variation in sales in country 3 is being offset by an almost synchronous variation in profit margins (since profit in country 3 is constant).

The variations in investment and profit with β_{ij} in country 2 are shown in Figs. 6 and 7. Since β_{ij} is the required production capacity per unit of production, needed investment in production capacity increases

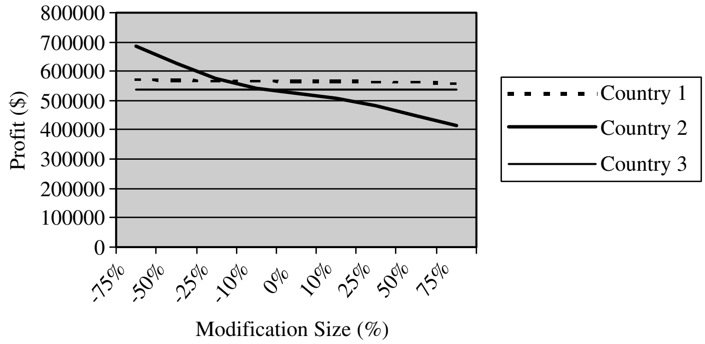

Fig. 9. Variation of profit with unit transportation cost.

with increasing β_{ij} as in Fig. 6. The production quantities in country 2 decrease, but not sufficiently to offset the increase in β_{ij}. There is an offsetting drop in investment in countries 1 and 3. The profit in country 2 drops, as the products absorb a higher overhead cost of investment. Observe that profits in countries 1 and 3 increase slightly although investments in those two countries decrease. The counterintuitive observation can be explained by observing that if β_{ij}/w_j is small compared to $x_{ij} - v_{ij}$, a reduction in F_j will also result in a reduction in x_{ij} (follows from Eqs. (2) and (4)). Therefore, for a sufficiently low value of β_{ij}/w_j, the reduction in x_{ij} may more than offset the reduction in q_{ijk}, increasing profit, z_j.

Finally, as shown in Fig. 8, investment in country 2 drops sharply with increase in transportation cost from that country, as would be expected. The profit from country 2 also drops, in pace with the drop in investment, as shown in Fig. 9. Although there is an increase in investment in countries 1 and 3, profits from these two countries are almost unchanged, due to the reduced profit margins on products imported from country 2. Thus, all variations shown in Figs. 2 to 9 can be explained.

6. Country specific costs

6.1. Fixed cost of setting up plants

Assume that there is a fixed cost of A_j for setting up a plant in country j (Hodder, 1984). It is clear that the objective function (11) will be augmented as

$$\text{Maximize } y = z - \sum_{j\in J} A_j\theta_j,$$

where θ_j is a 0,1 binary variable. $\theta_j = 1$ if a plant is constructed in country j; 0 otherwise. In addition to constraints (12) to (15) we would have

$$F_j \leqslant M\theta_j,$$

where M is a large number.

Unlike program P_1, where it was optimal to produce in all three countries considered, the impact of including the fixed cost A_j, would be to not produce in country j, if A_j is high. Hence with N possible countries, it may be optimal to produce in only r countries where $r = 1, 2, 3, \ldots, N$. Since r countries can be selected from N countries in several different ways, we use the tuple r, m to denote the mth selection of r countries from N.

The objective function z in (11) is modified to

$$z(J_{rm}) = \sum_{i\in I}\sum_{j\in J_{rm}}\sum_{k\in K}\left\{\left(\frac{q_{ijk}}{a_{ijk}}\right)^{\frac{1}{n}} - (1+t_{ik})x_{ij} - T_{ijk}\right\}q_{ijk},$$

where J_{rm} is the mth set of r countries chosen for manufacturing. With N possible country locations, m will assume values $1, 2, \ldots, {}^{N}C_r\left(=\frac{N!}{r!(N-r)!}\right)$.

Inclusive of fixed cost, the objective function (to be maximized) would be

$$y(J_{rm}) = z(J_{rm}) - S(J_{rm}), \quad \text{for all } r \text{ and } m,$$

where

$$S(J_{rm}) = \sum_{j\in J_{rm}} A_j.$$

Note that for a given J_{rm}, $S(J_{rm})$ is fixed (a constant). Hence, for a known J_{rm}, $\operatorname{Max} y(J_{rm})$ is equivalent to

$$\underset{\mathrm{q_{ijk}, x_{ij}, F_j}}{\operatorname{Max}}\{z(J_{rm})\}.$$

Therefore, denoting optimal values by asterisks,

$$y^*(J_{rm}) = z^*(J_{rm}) - S(J_{rm}).$$

Since $z^*(J_{rm})$ can be determined by solving program P_1, independently of $S(J_{rm})$, the optimal values of q_{ijk}, x_{ij}, and F_j *are not impacted* by values of A_j, for any choice of r and m. The value of z and the optimal choice, however, would be.

The optimization problem can now be stated as

$$\underset{r,m}{\operatorname{Max}}\{y^*(J_{rm})\} = \underset{r,m}{\operatorname{Max}}\{z^*(J_{rm}) - S(J_{rm})\}.$$

Note that if $N = 3$, m values can be from 1 to 7. The seven country combinations would be (1, 2, 3); (1, 2); (2, 3); (1, 3); (1); (2); (3), and program P_1 will be solved for each of these seven cases individually.

Example

We solve a system with $N = 3$ for the following scenarios: use a plant in any one country ($r = 1$), use plants in any two countries ($r = 2$), and use plants in all three countries ($r = 3$). Total investment $\sum F_j$ equals \$150,000. It is clear from Table 2 that the optimal solutions would be

Number of plants (r)	Countries (J_{rm})	Profit
1	2	187,391
2	1,2	327,326
3	1,2,3	442,909

Table 2
Solution of $z(J_{rm})$

Number of plants	$r = 1$			$r = 2$			$r = 3$
Countries	1	2	3	1,2	2,3	1,3	1,2,3
Profit	156,107	187,391	131,744	327,326	252,140	292,711	442,909

Including fixed costs the net profit for the three solutions would be $187{,}391 - A_2, 327{,}326 - (A_1 + A_2)$, and $442{,}909 - (A_1 + A_2 + A_3)$. For the special case when $A_i = A$ for all i, the decision maker can determine the threshold values of A where each of the above three solutions dominate. This is shown graphically in Fig. 10, where for the 3-plant, 2-plant and 1-plant solutions, the profit lines corresponds to $442{,}909 - 3A$, $327{,}326 - 2A$, and $187{,}391 - A$, respectively. From this graph we can establish the ranges of dominance of the solutions, as shown in Table 3.

Note that the number of possible solutions will increase rapidly with N. Therefore if N is large we first group the countries into a smaller number of clusters (called regions), and solve the investment problem ($N =$ number of regions) to select a subset of regions for investment. For each of the regions selected we then identify the countries qualifying for investment solving the investment problem again ($N_m =$ number of countries in region m). Clearly, optimality of the final solution will be a function of the procedure for forming regional clusters.

6.2. Exchange rate and local taxes

In this section we explore the impact of issues such as currency exchange rate, and local taxes (Kogut and Kulatilaka, 1994). We first study the implications of currency exchange rates. Assume that e_j is the number of units of local currency in country j equivalent to one unit of a standard currency (say, US\$). We are only interested in the long term variations in exchange rates, as the short term fluctuations should not impact long term facility investment decisions.

Currency exchange rates impact the revenues in country k, and the manufacturing and investment costs in country j. Thus, the revised value of L, corresponding to (16) is written as

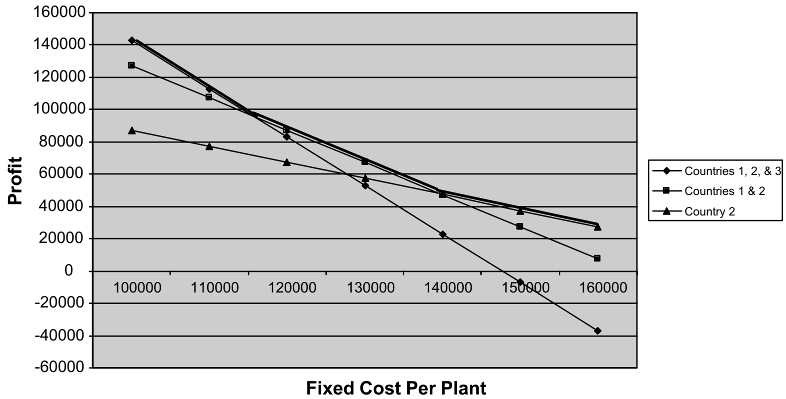

Fig. 10. Profit and fixed cost.

Table 3
Optimal plants with fixed cost

Number of plants	Optimality condition
3	$A \leqslant 115{,}583$
2	$115{,}583 \leqslant A \leqslant 139{,}935$
1	$139{,}935 \leqslant A \leqslant 187{,}391$
0	$A \geqslant 187{,}391$

$$L = \sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\left[\frac{1}{e_k}\left(\frac{q_{ijk}}{a_{ijk}}\right)^{1/n} - \frac{1}{e_j}\{(1+t_{ik})x_{ij}+T_{ijk}\}\right]q_{ijk} + \sum_{j\in J}\sum_{i\in I}\sum_{k\notin K}\lambda_j\{(x_{ij}-v_{ij})q_{ijk}-F_j\}$$
$$-\sum_{j\in J}\sum_{i\in I}\sum_{k\in K}\mu_j\{\beta_{ij}q_{ijk}-w_jF_j\} + \sum_{i\in I}\sum_{j\in J}\delta_{ij}(x_{ij}-v_{ij}) - \psi\left(\sum_{j\in J}F_j - F\right).$$

Note that both F and F_j are expressed in standard currency units (US\$). We transform λ_j and μ_j as $\lambda'_j = \lambda_j e_j$ and $\mu'_j = \mu_j e_j$, so that the solution procedure established in program P_1 can be used. Setting up Kuhn–Tucker equations it can be verified that:

$$\lambda'_j = \frac{\sum_{k\in K}(1+t_{ik})q_{ijk}}{\sum_{k\in K}q_{ijk}} - \frac{\delta_{ij}e_j}{\sum_{k\in K}q_{ijk}}, \tag{38}$$

$$q_{ijk} = a_{ijk}\left(\frac{n}{n+1}\frac{e_k}{e_j}\right)^n\{(1+t_{ik})x_{ij}+T_{ijk}-\lambda'_j(x_{ij}-v_{ij})+\mu'_j\beta_{ij}\}^n \tag{39}$$

and

$$e_j\psi = w_j\mu'_j - \lambda'_j. \tag{40}$$

Eqs. (38) to (40), with equations corresponding to (26) to (28) can be solved as program P_1 using the algorithm developed earlier. The value of e_j is assumed known.

Next observe from (39) that if e_k decreases (with e_j held constant), q_{ijk} would decrease. That is, production in country k decreases if its currency value strengthens. This is intuitive, as products produced in a weak currency country can be sold at relatively low prices in a strong currency country, and so they become very competitive.

Including tax, on profits earned in a country, the first term on the RHS of the objective function will be multiplied by ϵ_k (Hodder, 1984), where

$$\epsilon_k = 1 - \frac{\text{local tax rate (in percent)in country } k}{100}.$$

Expressions (38) and (39) would be modified to

$$\lambda'_j = \frac{\sum_{k\in K}(1+t_{ik})\epsilon_k q_{ijk}}{\sum_{k\in K}q_{ijk}} - \frac{\delta_{ij}e_j}{\sum_{k\in K}q_{ijk}}$$

and

$$q_{ijk} = (a_{ijk})^{1+n}\left(\frac{n}{n+1}\frac{e_k}{e_j}\right)^n\left\{(1+t_{ik})x_i + T_{ijk} - \frac{\lambda'_j}{\epsilon_k}(x_{ij}-v_{ij}) + \frac{\mu'_j}{\epsilon_k}\beta_{ij}\right\}^n.$$

Note that the quantity q_{ijk} sold in country k (produced in country j) will decrease if the tax rate in country k increases, provided $\mu'_j\beta_{ij} \geqslant \lambda'_j(x_{ij}-v_{ij})$.

7. Market size and local content

Observe that the demand curve used in the model, $D_{ijk} = a_{ijk}p^n_{ijk}$, is asymptotic in both demand and price axes. That is, demand shoots up rapidly with decreasing price. Although, a_{ijk} can be estimated for the demand curve to hold over a wide range of price, the predicted demand at very low prices may be absurd, as the size of the buying population (market size) may vary tremendously from country to country. In any case, the total demand may not exceed the country's population. For example, the maximum demand in China today would be much larger than, say, Malaysia.

To overcome this problem we introduce a new constraint limiting the total quantity of products that can be sold in a country. Thus, for a market size M_k in country k, we express it as

$$\sum_{i\in I}\sum_{j\in J} q_{ijk} \leqslant M_k. \tag{41}$$

The other restriction on sales in a country arises from the so-called local-content requirement. To ensure local economic development, most countries require that a company's total production in a country exceed a certain proportion of its sales in that country. Letting this proportion to be γ_r for country r, we express this local-content requirement as

$$\gamma_r \sum_{i\in I}\sum_{j\in J} q_{ijr} \leqslant \sum_{i\in I}\sum_{k\in K} q_{irk}, \quad \text{for all } r \in J. \tag{42}$$

The LHS of (42) is the total quantity sold in country r, irrespective of where they are produced. The term on the RHS is the total quantity produced in country r, irrespective of where they are sold.

We let dual variables ρ_k and ϕ_r be associated with constraints (41) and (42), respectively. Note that q_{ijk} is the only decision variable involved in (41) and (42). Hence the impact of the market-size and local-content rules will only be felt in the Kuhn–Tucker condition, specified in (18) as $\frac{\partial L}{\partial q_{ijk}} = 0$. Note that the local content rule will be relevant only in those countries where both production and consumption (of the product) take place. These countries can clearly be identified as k (or j) $\in K \cap J$ if country k (or j) is both a producer and a consumer, and k (or j) $\notin K \cap J$ otherwise. As shown in Appendix D, the RHS of (18) will now be augmented by the term τ_{ijk}, where

$$\tau_{ijk} = \rho_k - \gamma_k\phi_k + \phi_j, \quad \text{for all } i, \tag{43}$$

where $\phi_k = 0$ if $k \notin K \cap J$.

The expression for q_{ijk}, corresponding to (27) will now be written as

$$q_{ijk} = (a_{ijk})\left(\frac{n}{n+1}\right)^n \{(1+t_{ik})x_{ij} + T_{ijk} - \lambda_k(x_{ij} - v_{ij}) + \mu_j\beta_{ij} - \rho_k + \gamma_k\phi_k - \phi_j\}^n, \tag{44}$$

where $\phi_k = 0$ if $k \notin K \cap J$.

To solve the above augmented model, the algorithm as in Appendix B is modified to insert step 2a, following step 2, as outlined below:

Step 2: Compute q_{ijk} using Eq. (44).

Step 2a: Check constraints (41) and (42). If (41) is violated increase ρ_k, and if (42) is violated increase ϕ_r, and go to step 2. If both constraints are satisfied, go to step 3. Observe from (41) and (42) that if

$$M_r \leqslant \frac{1}{\gamma_r}\sum_{i\in I}\sum_{k\in K} q_{irk},$$

Eq. (41) will dominate (42). In that case if $\rho_k > 0$ $(k \in K \cap J)$ then $\phi_k = 0$. This would vastly reduce the number of variables to work with, in the algorithm.

Example

Consider now the example in Appendix D with $J = \{1,2\}$, and $K = \{1,2,3\}$. Assume that the local content rule is active in country 1, and inactive in country 2. Obviously, $\phi_1 > 0$ and $\phi_2 = 0$. It can be seen from the expressions for $\frac{\partial L_A}{\partial q_{i11}}$, $\frac{\partial L_A}{\partial q_{i12}}$, $\frac{\partial L_A}{\partial q_{i13}}$, and $\frac{\partial L_A}{\partial q_{i21}}$ (in Appendix D) that q_{i11}, q_{i12}, and q_{i13} will increase, whereas q_{i21} will decrease (there will be no change in q_{i22} and q_{i23}). That is, production in country 1 $(q_{i11} + q_{i12} + q_{i13})$ increases. Correspondingly, the sale of country 1's products in all three countries would increase. However, since $\gamma_j < 1$, q_{i12} and q_{i13} increase more than q_{i11}, which facilitates compliance with the local-content rule in country 1. The sale of products from country 2, in country 1, q_{i21} decreases, to make it easier to satisfy the local-content rule.

If country 2 now activates its local-content rule, ϕ_2 becomes positive and, as is clear from the expression for $\frac{\partial L_A}{\partial q_{ijk}}$, import from country 1 to country 2 will decrease and the export to country 1 (from country 2) will increase, depending on the relative values of ϕ_1, and ϕ_2. Observe that ϕ_2 does not impact sales of country 1's product in country 1, and vastly increases sale of country 2's product in country 2. Also observe that neither ϕ_1 nor ϕ_2 affects the sales of goods, from the other country, in country 3 where no production takes place.

Expressions such as $\frac{\partial L_A}{\partial q_{ijk}}$ are means of assessing the trade off of the local-content rules in two countries. From the example scenario it is clear that the impact of local-content rules is to (a) increase sale of homemade products (observe that $\phi_j(1-\gamma_j)$ term in $\frac{\partial L_A}{\partial q_{ijk}}$ is nonnegative), (b) increase trade between countries (observe that the terms $\phi_1 - \gamma_2\phi_2$ and $\phi_2 - \gamma_1\phi_1$, appearing in $\frac{\partial L}{\partial q_{i12}}$ and $\frac{\partial L}{\partial q_{i21}}$, respectively, cannot become zero simultaneously), and (c) sales in third countries where local-content rules are not restrictive always increase. Note that sales cannot increase indefinitely, as the value of ρ_k in that country eventually applies the brakes, by becoming positive (when the country demand is saturated).

It should be clear that to increase sales in a country, as above, companies increase demand of their products by cutting price. Since such a price reduction is involuntary (forced by the local-content rules) the company's overall profit would decrease. Our model helps in minimizing this drop in profit by adjusting the production, sales, export, and import quantities.

8. Conclusions

We have clearly demonstrated how plant location, production quantities and export/import quantities can be determined simultaneously with pricing (inclusive of overhead allocation). We have also established why plant related fixed cost do not directly influence the above relationship. The clear distinction between domestic and international plant location scenarios and their solution approaches is striking.

In a domestic setting (Section 4) where tariffs are zero, we show that the optimal production quantities are independent of overhead allocation. The accepted accounting practice, which allocates overhead based on known quantities, will now be consistent with the profit maximization objective. In a global setting, on the other hand, where the shadow price of overhead burden exceeds 1, delinking overhead allocation from profit maximization will clearly obtain sub-optimal profits.

For the nonzero tariff scenario, all shadow prices and optimal quantities are implicit functions of one another. Hence the pattern of variations in optimal quantities with respect to parameter values is not as well established, as in a domestic setting. For example, optimal production quantities and investments decrease with unit production cost in a domestic setting, but it need not be so in a global setting.

The dominance properties in Theorem 2 establish that (a) it is sufficient to have only three independent decision variables, and hence a grid search in three variables is sufficient, and (b) the marginal cost (shadow price) of the overhead burden is bounded within a narrow range, thus expediting the search for optimality.

The role of the import tariffs, as revealed in our analysis, is indeed interesting. In general, tariffs behave like market imperfections, setting up arbitrage-like opportunities, with overhead allocation as the instrument of arbitrage. That is companies can compete effectively by judiciously choosing overhead allocation to products based on where they are produced and where they are sold. Specifically, we show in Section 4.1 that if tariff barriers in the world were eliminated, overhead allocation to products would be arbitrary. But if the tariff rate varies with product type (a more likely scenario), overhead allocation to products must be selective. Note, as we have established in Section 5, products with low values of unit variable cost, unit transportation cost, and tariff rates, absorb most of the overhead.

Three categories of products emerge in terms of overhead absorption: category 1 products with no overhead absorption in any country, category 2 products with overhead absorption in a subset of countries, and category 3 products with overhead absorption in all countries. It is clear that the subset of products absorbing overheads may be different from one country to another. Hence country managers must be made aware of operating policies that would be *specific* to these *product subsets*. They must also monitor the situation closely, as the overhead-absorbing product-set may change in time, with changes in demand, cost, and product preference.

In the context of local content rules, note that the shadow prices (ϕ_r) become instrumental in assessing adjustments in production and trade between countries, when local-content rules change. For a 3-country example, we show that implementing the local content rule in a country would increase total production in that country, and increase the total trade between any two countries with such rules.

The values of the parameter related to tariff, productivity, transportation cost, and customer preferences can change with time, during the life cycle of a plant. Therefore, our model will first be run using average values (estimated for the life cycle) of the above four parameters, and the value of optimal investments (by country) will be determined. Next, in each time period, program P_1 will be run with the above values of investments and the current values of the parameter values. Thus the optimal values of production quantities and prices will be made to respond to the changing parameter values.

Further research is required to adapt our model to the scenario where semi-finished products can be transported between plants. A similar adaptation can be made to incorporate a supply chain for raw materials. Modeling of scenarios with competition and price-inflation would also be of interest in this context.

Appendix A. Proofs of theorems and lemma

Lemma 1. *Production quantity q_{ijk} cannot be less than the demand.*

Proof. Let $q_{ijk} < D_{ijk}$.

It follows from (7) that D_{ijk} can be reduced by increasing p_{ijk}. Since for given values of q_{ijk}, x_{ijk}, t_{ik}, and T_{jk} profit as in (9) will increase monotonically with p_{ijk}, it is clear that p_{ijk} will be increased until $D_{ijk} = q_{ijk}$. □

Theorem 1. $1 \leqslant \lambda_j \leqslant 2$, *if* $0 \leqslant t_{ik} \leqslant 1$.

Proof. It follows from Eq. (17) that

$$\lambda_j = \frac{\sum_{k\in K}(1+t_{ik})q_{ijk}}{\sum\limits_{k\in K} q_{ijk}} - \frac{\delta_{ij}}{\sum\limits_{k\in K} q_{ijk}}, \tag{A.1}$$

which can be written as

$$\lambda_j = \lambda_{ij} - \frac{\delta_{ij}}{\sum\limits_{k\in K} q_{ijk}}, \tag{A.2}$$

where

$$\lambda_{ij} = 1 + \sum_{k\in K} r_{ijk}t_{ik}, \quad \text{and} \quad r_{ijk} = \frac{q_{ijk}}{\sum_{k\in K} q_{ijk}}, \quad \text{so that}$$

$$\sum_{k\in K} r_{ijk} = 1, \quad \text{and} \quad 0 \leqslant \sum_{k\in K} r_{ijk}t_{ik} \leqslant 1 \quad (\text{since } 0 \leqslant t_{ik} \leqslant 1).$$

Hence,

$$\lambda_{ij} \leqslant 2 \quad \text{and} \quad \lambda_{ij} \geqslant 1. \tag{A.3}$$

Assume $\lambda_j > 2$. It follows from (A.2) and (A.3), since $\lambda_{ij} \leqslant 2$, that $\delta_{ij} \geqslant 0$ for all i, and that $\lambda_j \leqslant 2$. Therefore, $\lambda_j \leqslant 2$.

Thus from (A.2) and (A.3) we have $\lambda_j \leqslant \lambda_{ij} \leqslant 2$.

Assume next that $\lambda_j < 1$. Since $\lambda_{ij} \geqslant 1$, we have, from (A.2), $\delta_{ij} > 0$. Therefore from Eq. (24), $x_{ij} = v_{ij}$ for all i, which clearly is not possible as constraint (12) will be violated. Hence $\lambda_j \geqslant 1$. Thus $1 \leqslant \lambda_j \leqslant 2$. □

Theorem 2. *The search for the optimal solution will entail a search in only λ_j, ψ, and x_{ij} dimensions, with computational complexity would be $O(\eta)$, where $\eta = |J|$.*

Proof. Note that in the three equations (26) to (28) we have six unknowns (q_{ijk}, λ_j, ψ, μ_j, x_{ij}, and δ_{ij}). However, the only independent variable is λ_j, since the variations in q_{ijk}, ψ, μ_j, x_{ij}, and δ_{ij} are all driven by the variations in λ_j. Hence the computational complexity would be $O(\eta)$, where $\eta = |J|$.

Appendix B

Algorithm

Step 0: Set initial values of $\lambda_j = 1.5$ for all j, and $\psi = 0$. Set $x_{ij} = v_{ij}$ for all i and j. Set ϖ, the step size adjustment in λ_j, to 0.1.

Step 1: Compute μ_j, using Eq. (28).

Step 2: Compute q_{ijk}, using Eq. (27).

Step 3: Compute δ_{ij} from Eq. (27). If δ_{ij} is negative, set it to zero.

Step 4: Reset the value of x_{ij} as
- (a) If $\delta_{ij} > 0$, $x_{ij} = v_{ij}$,
- (b) If $\delta_{ij} = 0$, increase x_{ij} by 0.1.

Step 5: Compute the investment in capacity, FC_j, and the amount of absorbed overhead, FO_j as

$$FC_j = \frac{1}{w_j}\sum_{i\in I}\sum_{k\in K}\beta_{ij}q_{ijk},$$

$$FO_j = \sum_{i\in I}\sum_{k\in K}(x_{ij} - v_{ij})q_{ijk}.$$

If $|FC_j - FO_j| \leqslant$ threshold-value, go to step 6.
If $FC_j - FO_j$, changes sign from negative to positive (or vice versa) reduce ϖ by a factor 2.
If $FC_j < FO_j$, reduce λ_j by ϖ and go to step 1.
If $FC_j > FO_j$, increase λ_j by ϖ and go to step 1.

Step 6: (a) If $|\sum_j FC_j - F| <$ threshold-value, EXIT.
(b) If $\sum_j FC_j > F$, increase ψ and go to step 1.
(c) If $\sum_j FC_j < F$, decrease ψ and go to step 1.

In all computation runs the above algorithm converged quickly since λ's are bounded. However, in certain scenarios that the number of grid points to be evaluated can be large, slowing the rate of convergence.

Appendix C

From (11) and (12), we write

$$z_j = \sum_{i\in I}\sum_{k\in K}\left\{\left(\frac{q_{ijk}}{a_{ijk}}\right)^{1/n} - (1+t_{ik})x_{ij} - T_{ijk}\right\}q_{ijk}$$

and

$$L_j = \sum_{i\in I}\sum_{k\in K}(x_{ij} - v_{ij})q_{ijk}.$$

From (27) ignoring λ_j and μ_j we have

$$q_{ijk} = a_{ijk}\left(\frac{n}{n+1}\right)^n\{(1+t_{ik})x_{ij} + T_{ijk}\}^n.$$

Substituting for q_{ijk} in z_j and L_j we can express the ratio, R_j as

$$R_j = \frac{L_j}{z_j} = \frac{\sum_i\sum_k(x_{ij} - v_{ij})a_{ijk}\{(1+t_{ik})x_{ij} + T_{ijk}\}^n}{\left(-\frac{1}{n+1}\right)\sum_i\sum_k a_{ijk}\{(1+t_{ik})x_{ij} + T_{ijk}\}^{n+1}}.$$

Letting $u_{ijk} = (1+t_{ik})x_{ij} + T_{ijk}$, we can write

$$\frac{\partial R_j}{\partial x_{ij}} = -(n+1)(B_1 - B_2),$$

where

$$B_1 = \frac{\sum_k a_{ijk}\{u_{ijk} + n(1+t_{ik})(x_{ij} - v_{ij})\}u_{ijk}^{n-1}}{\sum_i\sum_k a_{ijk}u_{ijk}^{n+1}},$$

$$B_2 = \frac{\left[\sum_i\sum_k a_{ijk}(x_{ij} - v_{ij})u_{ijk}^n\right]\left[\sum_k a_{ijk}(n+1)(1+t_{ik})u_{ijk}^n\right]}{\left\{\sum_i\sum_k a_{ijk}u_{ijk}^{n+1}\right\}^2}.$$

If $x_{ij} = v_{ij}$, the above simplifies to

$$\frac{\partial R_j}{\partial x_{ij}} = -(n+1)\frac{\sum_k a_{ijk}\{(1+t_{ik})v_{ij} + T_{ijk}\}^n}{\sum_i\sum_k a_{ijk}\{(1+t_{ik})v_{ij} + T_{ijk}\}^{n+1}}.$$

Appendix D

To incorporate the constraints (41) and (42) in (16), we need to augment L by L_A, where

$$L_A = -\sum_{k\in K}\rho_k \sum_{i\in I}\sum_{j\in J}(q_{ijk} - M_k) - \sum_{r\in J}\phi_r\left\{\gamma_r \sum_{i\in I}\sum_{j\in J} q_{ijr} - \sum_{i\in I}\sum_{k\in K} q_{irk}\right\}.$$

Hence

$$\frac{\partial L_A}{\partial q_{ijk}} = -\rho_k - \frac{\partial}{\partial q_{ijk}}\left\{\sum_{r\in J}\phi_r\gamma_r \sum_{i\in I}\sum_{j\in J} q_{ijr}\right\} + \frac{\partial}{\partial q_{ijk}}\sum_{r\in J}\sum_{i\in I}\sum_{k\in K}\phi_r q_{irk}.$$

We next use a simple example to motivate further simplification of the above expression.

Example

Consider $J = \{1,2\}$, and $K = \{1,2,3\}$, so that $K \cap J = \{1.2\}$. Constraint (41) would be written for a specific i, as

$$q_{i11} + q_{i21} \leqslant M_1,$$
$$q_{i12} + q_{i22} \leqslant M_2,$$
$$q_{i13} + q_{i23} \leqslant M_3.$$

The associated dual values would be ρ_1, ρ_2, and ρ_3 respectively. Constraint (42) would be written, for a specific i, as

$$\gamma_1(q_{i11} + q_{i21}) \leqslant q_{i11} + q_{i12} + q_{i13},$$
$$\gamma_2(q_{i12} + q_{i22}) \leqslant q_{i21} + q_{i22} + q_{i23}.$$

The associated dual values would be ϕ_1 and ϕ_2, respectively.

$$\begin{aligned} L_A = {} & -\rho_1(q_{i11} + q_{i21} - M_1) - \rho_2(q_{i12} + q_{i22} - M_2) - \rho_3(q_{i13} + q_{i23} - M_3) \\ & - \phi_1\{\gamma_1(q_{i11} + q_{i21}) - (q_{i11} + q_{i12} + q_{i13})\} - \phi_2\{\gamma_2(q_{i12} + q_{i22}) - (q_{i21} + q_{i22} + q_{i23})\}. \end{aligned}$$

Hence

$$\frac{\partial L_A}{\partial q_{i11}} = -\rho_1 - \gamma_1\phi_1 + \phi_1, \quad \frac{\partial L_A}{\partial q_{i21}} = -\rho_1 - \gamma_1\phi_1 + \phi_2,$$

$$\frac{\partial L_A}{\partial q_{i12}} = -\rho_2 - \gamma_2\phi_2 + \phi_1, \quad \frac{\partial L_A}{\partial q_{i22}} = -\rho_2 - \gamma_2\phi_2 + \phi_2,$$

$$\frac{\partial L_A}{\partial q_{i13}} = -\rho_3 + \phi_1, \quad \frac{\partial L_A}{\partial q_{i23}} = -\rho_3 + \phi_2.$$

Clearly, if $k \notin K \cap J$, $\frac{\partial q_{ijr}}{\partial q_{ijk}} = 0$ since $r \in J$. Also $\frac{\partial q_{irk}}{\partial q_{ijk}} = 1$ if $r = j$, and $\frac{\partial q_{irk}}{\partial q_{ijk}} = 0$ if $r \neq j$. Hence we express $\frac{\partial L_A}{\partial q_{ijk}}$ as

$$\frac{\partial L_A}{\partial q_{ijk}} = -\rho_k - \phi_k\gamma_k + \phi_j$$

where $k \in K$, $j \in J$, and $\phi_k = 0$ if $k \notin K \cap J$.

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