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A genetic algorithm for the problem of configuring a hub-and-spoke network for a LTL trucking company in Brazil

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Abstract

A heuristic based on genetic algorithms is proposed to the problem of configuring hub-and-spoke networks for trucking companies that operate less-than-truckload (LTL) services in Brazil. The problem consists of determining the number of consolidation terminals (also known as hubs), their locations and the assignment of the spokes to the hubs, aiming to minimize the total cost, which is composed of fixed and variable costs. The proposed formulation differs from similar formulations found in the literature in the sense that it allows variable scale-reduction factors for the transportation costs according to the total amount of freight between hub terminals, as occurs to less-than-truckload (LTL) freight carriers in Brazil. Our genetic algorithm approach incorporates an efficient local improvement procedure that is applied to each generated individual of the population. Computational results for benchmark problems are presented. A practical application to a real world problem involving one of the top-ten trucking companies in Brazil is also described.

Keywords: Location; Genetic algorithms; Heuristics

1. Introduction

Road transportation plays an important role for both long and short haul freight transportation in Brazil's economy. It is the most widely used mode for moving freight over the country, accounting for over 60% of all nation cargo, despite the continental dimensions of the country, the sixth largest in the world. Rail transportation has been growing in the last years, following the privatization of the railroad sector, but it still requires substantial investments in both infrastructure and rolling stock to steadily increase its current 25% of cargo, what is not expected to happen in the near future. In this

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context, less-than-truckload (LTL) services serve those customers whose shipments, from one origin to one destination, would not fill the truck capacity by weight or volume. By consolidating small shipments for a single trailer, LTL trucking companies can move provide fast and reliable services at competitive prices.

In order to achieve high level of efficiency, as well as improved customer service and short transit times that comply with the actual logistics requirements, many LTL trucking companies operate hub-and-spoke networks. The traffic between two nodes is not shipped directly, but has to be routed via a set of nodes designated as hubs. These networks have been widely used in several contexts, including passenger airlines, postal delivery, and computer and telecommunication networks. A major incentive for a hub-and-spoke system is the economy of scale achieved by consolidating traffic through the hub-to-hub links and thus realizing lower unit costs on these links. Hubs serve as consolidation points for traffic originating at different nodes to the same destination or to destinations close to each other. Due to the increased traffic on linkages between hubs, larger vehicles can be used or capacities of existing vehicles can be utilized more efficiently, resulting in smaller per unit transportation costs.

A hub-and-spoke location problem can be generically defined as a location–allocation problem, which consists of determining the number of consolidation terminals (hubs), their locations and the assignment of the spokes (non-hub nodes) to the hubs, aiming to minimize the total cost, which is composed of fixed and variable costs.

In this paper, we describe an efficient hybrid genetic algorithm (GA) approach for the hub-and-spoke location problem for the LTL trucking industry. The proposed formulation differs from other formulations found in the literature in the sense that its non-linear objective function allows economies of scale for the transportation costs that may vary according to the total amount of freight between hub terminals. This problem can be seen as a modified version of the Uncapacitated Hub Location Problem with Single Allocation (UHP-S), in which the discount factor on the hub-to-hub links is not constant but may vary according to the total amount of freight between hub terminals. It aims to reflect level-of-service, time-guaranteed delivery requirements in the LTL trucking industry in Brazil that impose minimal frequencies in the linkages between hubs; in other words, vehicles are dispatched between hubs according to predefined scheduled frequencies and timetables, even if not enough freight has been accumulated to reach their capacities, in which case sometimes a smaller, less efficient vehicle has to be used instead.

Another contribution of this paper is that the proposed GA differs from others in the literature for the UHP-S in the sense that it incorporates a local improvement heuristic for each individual of the generated offspring. It can also be viewed as a memetic algorithm, since it combines crossover operators with local search heuristics (Moscato, 1989). The efficiency of the proposed algorithm has been evidenced by yielded results that match the best solution found in the literature for the UHP-S with fixed discount factor on the hub-to-hub links. The heuristic was also successfully applied to a real-world UHP-S problem with variable discount factor on hub links related to a nationwide LTL trucking carrier in Brazil. This company is seeking to review and improve its service network in terms of defining the terminals that would act as hubs and the spoke facilities assigned to each of the selected hubs.

The UHP-S is related to various hub-and-spoke location problems in the literature. O'Kelly (1987) was the first to formulate the p-Hub Median Problem (p-HMP) as a quadratic integer problem. He showed that this problem, in which the number of hub nodes (p) is given a priori, is NP-hard, and proposed two enumeration-based heuristics. Klincewicz (1991) developed single and double exchange heuristics, as well as a clustering heuristic. In a later work, Klincewicz (1992) proposed two other heuristics based on tabu search (TS) and GRASP (Greedy Randomized Adaptive Search Procedure). Skorin-Kapov and Skorin-Kapov (1994) developed a modified tabu search technique. Skorin-Kapov et al. (1996) obtained exact solutions to p-hub location problem (p-HLP) by using an alternative formulation based on a MIP (mixed integer linear programming) by Campbell (1994).

In the UHP-S, the number of hub nodes is a decision variable in the problem and a fixed cost for establishing a hub is included in the formulation. O'Kelly (1992) proposed a heuristic solution approach to this problem. Abdinnour-Helm and Venkataramanan (1998) developed a branch-and-bound (B&B) and a genetic algorithm (GA) heuristic to solve the UHP-problem. The GA proved to be efficient for large problems, while the exact B&B approach was limited to solve smaller size problems. Klincewicz (1996) proposed a dual algorithm to solve the related Uncapacitated Hub Location problem with Multiple Allocation (UHP-M) in which a spoke can be assigned to more than one hub. Also, Abdinnour-Helm (1998) developed a hybrid GA and TS heuristic for the UHP-S in which GA is used to determine the number and the location of the hubs and TS is used to find the optimal assignment and of spokes to hubs.

More recently, Podnar et al. (2002) and Skorin-Kapov and Skorin-Kapov (2005) addressed a modification of the basic p-HLP, in the sense that there is no special type of nodes (hubs); consequently, flows are not forced to use inter-hub links. Cost of sending flow through each link is discounted by a constant factor α if the corresponding amount of flow exceeds a certain threshold, aiming to construct a network with fewer links than the complete interconnected network. The expected result is that, with large enough economies of scale given by the discounts, a hub-like network will be obtained, once users will be motivated to use a smaller number of discounted links. The objective is to find a feasible flow with minimum cost. According to the authors, applications of this problem arise, for example, in high capacity lines of backbone networks in telecommunications and major airports in air transportation networks.

This paper is organized as follows: in Section 2, we present the mathematical formulation. The proposed GA solution strategy is described in Section 3. In Section 4, we present the computational results used for validating the proposed algorithm using for the CAB data set introduced by O'Kelly (1987) and that has been consistently used in the literature to benchmark heuristics for hub location problems. In Section 5, we describe the application of the proposed approach to a real world problem in a Brazil's LTL trucking company, and in Section 6 we make our final considerations.

2. Mathematical formulation

Let $i = 1, 2, \ldots, N$ be the set of nodes where each node corresponds to origins, destinations and possible hub locations. Let T_{ij} and C_{ij} represent the flow and the cost per unit of flow between nodes i and j, respectively; O_i and D_i represent the total amount of flow originated and destined to node i, respectively; f_j reflects the fixed costs of operating a hub at node j; $\alpha_{km}(T_{km})$ is the discount factor on the linkage between hub nodes k and m. For instance, a discount factor of 0.80 means that transportation cost per unit of flow between these hubs is only 80% of the direct transportation cost between spokes, due to economies-of-scale brought by increased traffic on linkages between hubs. It should be noted that discount factor $\alpha_{km}(T_{km})$ is a function of the total flow T_{km} between hubs k and m, not necessarily restricted to linear or continuous forms; also, no symmetry is assumed, i.e. $\alpha_{km}(T_{km}) \neq \alpha_{mk}(T_{mk})$. Similarly, the cost C_{ij} is not necessarily proportional to the distance between nodes i and j and does not necessarily satisfy the triangle inequality.

As can be seen in the given in Fig. 1, the path from an origin spoke node i to a destination spoke

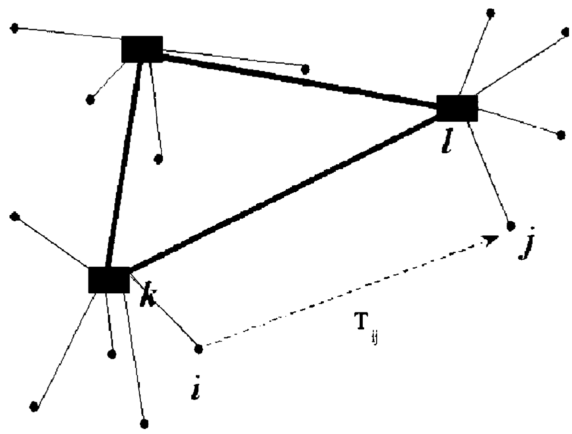

Fig. 1. An example of hub-and-spoke network.

node j includes three components: collection from spoke i to its assigned hub k, transfer between the hub k and hub l, and distribution from hub k to spoke j. Thus, in the UHP-S with variable discount factors on the linkages between hubs three decisions must be made: the number of hubs, the location of the hubs and the assignment of the spokes to the hubs.

The binary decision variable X_{ik} is equal to one if node i is assigned to the hub located at node k, and it is equal to zero otherwise. Each hub is assigned to itself; that is, if node j is a hub, $X_{jj} = 1$.

The mathematical formulation of the UHP-S with variable discount factors on the linkages between hubs can be written as

$$\text{minimize} \quad \sum_i \sum_k X_{ik} d_{ik}(O_i + D_i) + \sum_j \sum_m X_{jm} \times \sum_i \sum_k X_{ik}(T_{ij} d_{km} \alpha_{km}(T_{km})) + \sum_j X_{jj} f_j \tag{1}$$

$$\text{subject to} \quad X_{ij} \leqslant X_{jj} \quad \text{for all } i, j \in N \tag{2}$$

$$\sum_j X_{ij} = 1 \quad \text{for all } i \in N \tag{3}$$

$$X_{ij} \in \{0, 1\} \quad \text{for all } i, j \in N \tag{4}$$

The objective function (1) seeks to minimize the sum of the linear transportation costs between hubs and spokes, the non-linear inter-hub costs and the fixed costs of the selected hubs. Discount factor $\alpha_{km}(T_{km})$ may assume any function of the total flow T_{km} between hubs k and m; if, otherwise, all discount factors are all equal and constant, i.e. $\alpha_{km}(T_{km}) = \alpha$, as originally proposed by O'Kelly (1992), the objective function reduces to a quadratic form. Constraints (2) ensure that no spoke node is assigned to a location unless a hub is opened at that site. Constraints (3) ensure that each node is assigned to exactly one hub. Constraints (4) enforce all decision variables to be binary.

3. The genetic algorithm heuristic

Genetic algorithms (GAs) are one of the most popular heuristic algorithms that represent a powerful and robust approach for developing heuristic for complex and large-scale combinatorial optimization problems. A GA can be described as a probabilistic search, which imitates the process of natural selection and evolution to evolve a population of initial solutions. Each solution of a problem is treated as an individual, whose fitness is governed by the corresponding objective function value and some penalization to infeasibility. Pairs of individuals of a given population are selected to act as parents and reproduce to generate the next population of better individuals through a structured yet randomized information exchange known as crossover operator. Diversity is added to the population by randomly changing some genes (mutation operator). As new "offspring" are generated, unfit individuals in the population are replaced using the concept of survival of the fittest. This evaluation–selection–reproduction cycle is repeated until a satisfactory solution is found or other stopping criteria are met.

GAs can be implemented in a variety of ways. The excellent books by Goldberg (1989), Davis (1991) and Holland (1975) describe many possible variants of GAs. We also refer to these books for various GA definitions and notations as chromosomes, alleles, genes, reproduction, etc., as well as for other problem specific operators.

Our GA heuristic is based on a chromosome representation in which the solution structure is a string of zeros (0) and ones (1) with length equal to the number of nodes in the hub-and-spoke network under consideration. The value 1 indicates that the corresponding node is a hub and the value 0 indicates otherwise.

The GA approach seems to be appealing for the UHP-S since it allows high degree of flexibility in the definition of the function that relates transportation cost to the amount of traffic between each pair of hubs, including non-continuous and non-linear functions. In addition to the standard GA procedures, and inspired on the work of Beasley (1999), our GA heuristic for the UHP-S with variable discount factors on the linkages between hubs also incorporates an efficient local improvement procedure that is applied to each new generated individual. This improvement procedure aims to overcome the deficiency of the GA heuristic proposed by Abdinnour-Helm and Venkataramanan (1998) for the UHP-S, which proved to be successful to select the number and the locations of the hubs, but failed to effectively assign the spokes to the hubs based on "distance based" rule (assigning a spoke to the nearest hub). This rule had also been applied by O'Kelly (1987) and Klincewicz (1992) and failed to yield to the optimal assignment of spokes to hubs, since it does not take into consideration the economies of scale in the traffic between hubs.

This combination of genetic algorithms with local search heuristics is sometimes referred to as *hybrid genetic algorithm* or *memetic algorithm* (Moscato, 1989) in the literature, though not all authors make this distinction (for instance, Beasley and Chu, 1996; Chu and Beasley, 1997).

The steps involved in our GA heuristic for the UHP-S with variable discount factors on the linkages between hubs are as follows:

1. Generate an initial population of randomly constructed solutions. Each of the solutions is generated so that a bit position in a given string can become a hub, i.e. take the value 1, with a given probability p greater than zero; we adopted $p = 0.15$ based on preliminary experiments undertaken.
2. Select two parent solutions for reproduction. Some different methods were tried. We chose to use the binary tournament selection method, since it provided the best results in terms of speed, requiring less number of iterations to converge to the best solutions, according to some preliminary experiments undertaken. In this method, two individuals are chosen randomly from the population. The fitter (smaller fitness value) individual is then allocated a reproductive trial. In order to produce a child, two binary tournaments are held, each one producing one parent.
3. Generate a child solution by first applying a crossover operator to the selected parents. We use a single two-point crossover operator, in which two crossover points $p \in \{1, \ldots, N-1\}$ and $q \in \{p+1, \ldots, N\}$ are selected, and the child solution will consist of the first p genes taken from the first parent, the next $(q-p)$ genes taken from the second parent, and the remaining $(|N|-q)$ genes taken from the first parent, or vice-versa with equal probabilities. The crossover procedure is followed by a mutation procedure. Each bit in the offspring can be mutated (changed from 0 to 1 or vice versa) with some low probability.
4. For each generated offspring solution, determine the initial assignment of the spokes to the corresponding nearest hubs and then calculate the fitness, taking into consideration the flows between hubs to determine the proper discount factor.
5. Apply a local search heuristic to each generated individual in order to improve the assignment of the spokes to the hubs.
6. Evaluate the new population and replace an individual in the solution by the child solution. We use an *elitist generation replacement*, in which only the n best individuals (n is the population size) are taken into the next generation.
7. Steps 2–6 are repeated until a pre-defined number of iterations is reached.

With respect to Step 5, initially a simple and straightforward local search heuristic was developed, which consisted of simply reassigning spokes to hubs through shift and swap movements until no improvement movement could be found. Though being sufficiently fast to be applied to a large number of individuals generated at each iteration, it was noted that this heuristic frequently got trapped into local optima and was not very effective into leading to better solutions. To overcome this deficiency, a fast and simplified mechanism was incorporated to allow non-improving solutions to be accepted with a given probability. This mechanism is inspired by the work of Metropolis et al. (1953) which led to the development of simulated annealing (Kirkpatrick et al., 1983; Reeves, 1993). Our mechanism, which is applied to each generated individual, has the following high-level description:

1. Make all shift and swap movements that improve the solution. Let the final cost be C_{old}. Make this solution the current one.
2. For each spoke node, calculate the cost C_{new} of shifting it from the current assigned hub to each of the selected hubs in the solution.

3. Determine the difference $d = C_{\text{new}} - C_{\text{old}}$.
4. If $d \leqslant 0$, the new assignment is accepted. Update the current solution and C_{old}. Go to step 6.
5. If $d > 0$, determine the probability of the new assignment being accepted: $p = \exp(-d/t)$, where t is the temperature control parameter. To accomplish this, generate a random number r between 0 and 1; if $r \leqslant p$, the new assignment is accepted and made current, i.e., C_{new} and the current solution are both updated accordingly; otherwise (if $r > p$) keep the current assignment.
6. Repeat steps 2–5 until all spoke nodes have been evaluated.

In the simulated annealing (SA), the temperature t suffers a slow cooling, which is motivated by the equivalence of the annealing of solids. Our experiments showed the search with variable temperature parameter t resulted in undesirable high CPU times, since it is applied to each individual of the offspring. Our experiments also indicated that keeping this temperature fixed to a properly defined value resulted in very good solutions, comparable to those obtained using the cooling scheme, but in shorter CPU times.

4. Computational experiments

The algorithm presented was coded in C++ and run on Pentium IV 1.7 GHz with 256 MB RAM. A data set introduced by O'Kelly (1987) was used to validate the proposed GA heuristic for the UHP-S with variable discount factors on the linkages between hubs. This data has been used to benchmark heuristics for various hub location problems. It is based on airline passenger flow between 25 US cities in 1970 as evaluated by the Civil Aeronautics Board (the CAB data set). The 25 cities are part of a larger data set consisting of the flows between 100 US cities. The subset of 25 cities is chosen to account for 51% of the flows observed.

As in Abdinnour-Helm (1998), the experimental design consists of running the GA heuristic on subsets of 10, 15, and 20, and also on the full CAB data set of 25 nodes. The constant discount factor on the hub-to-hub links is considered at four levels: 1, 0.8, 0.6 and 0.4. The fixed cost of establishing a hub is taken at four levels: 250, 200, 150 and 100.

The parameters for the GA are: population size of 100; crossover probability of 0.99; mutation probability of 0.01; and 300 generations produced. Based on preliminary experiments undertaken, we found that the best value for the temperature factor t used in the local search improvement heuristic should be set equal to the product of the fixed cost of establishing a hub and an acceptance rate of 0.40.

The results presented in Tables 1–4 are summarized according to the number of cities (nodes) in the CAB data set, corresponding to 10, 15, 20 and 25 cities, respectively. The best solutions listed were obtained from Abdinnour-Helm (1998) and used to determine the corresponding relative deviations. The results are also compared to the GA approach proposed by Abdinnour-Helm and Venkataramanan (1998), denoted by GA-AH&V.

It should be noted that the best solutions for comparison were derived from a study of Skorin-Kapov et al. (1996). The study found the optimal solution to the p-Hub Median Problem for values of p equal to 2, 3 and 4. For a given problem, an optimal solution to the UHP-S can be derived by simply comparing the p-HMP solutions after adding the corresponding amount (p^*fixed_cost) to each, and then selecting the minimum.

The results indicate the effectiveness of the proposed GA Heuristic. The best solutions have been reached in all 57 of 64 problem instances. It clearly outperforms the GA-AH&V, which found the best solution for only 48 problem instances. For those instances in which the best solutions have not been found, the average deviation of the proposed GA heuristic was only 0.13% and the maximum deviation was only 0.35%, while for GA-AH&V the corresponding deviations were 0.85% and 2.05%, respectively. These good results were obtained in reasonably short CPU times, even considering that the local improvement heuristic is applied to each generated offspring. Thus, our GA overcomes the major weakness of the GA-AH&V, as pointed out by Abdinnour-Helm (1998): the assignment of the spokes to the selected hubs.

Table 1
Computational results for 10 cities

α	Fixed cost	Best solution	GA-AH&V		Proposed GA heuristic		
			Best value	% dev	Best value	% dev	CPU time (seconds)
1	250	1181.05	1181.05	0.00	1181.05	0.00	3.24
	200	1131.05	1131.05	0.00	1131.05	0.00	3.27
	150	1081.05	1081.05	0.00	1081.05	0.00	3.30
	100	1031.05	1031.05	0.00	1031.05	0.00	3.29
0.8	250	1181.05	1181.05	0.00	1181.05	0.00	3.34
	200	1131.05	1131.05	0.00	1131.05	0.00	3.36
	150	1081.05	1081.05	0.00	1081.05	0.00	3.35
	100	990.94	990.94	0.00	990.94	0.00	3.84
0.6	250	1181.05	1181.05	0.00	1181.05	0.00	3.31
	200	1131.05	1131.05	0.00	1131.05	0.00	3.34
	150	1032.62	1032.62	0.00	1032.62	0.00	3.68
	100	932.62	932.62	0.00	932.62	0.00	4.12
0.4	250	1174.30	1174.30	0.00	1174.30	0.00	3.32
	200	1074.30	1074.30	0.00	1074.30	0.00	3.76
	150	974.30	974.30	0.00	974.30	0.00	3.83
	100	867.91	867.91	0.00	867.91	0.00	4.18

Table 2
Computational results for 15 cities

α	Fixed cost	Best solution	GA-AH&V		Proposed GA heuristic		
			Best value	% dev	Best value	% dev	CPU time (seconds)
1	250	1556.66	1556.66	0.00	1556.66	0.00	5.89
	200	1506.66	1506.66	0.00	1506.66	0.00	5.99
	150	1456.66	1456.66	0.00	1456.66	0.00	6.05
	100	1406.66	1406.66	0.00	1406.66	0.00	6.20
0.8	250	1556.66	1556.66	0.00	1556.66	0.00	6.15
	200	1506.66	1506.66	0.00	1506.66	0.00	6.25
	150	1456.66	1456.66	0.00	1456.66	0.00	6.27
	100	1390.76	1390.76	0.00	1390.76	0.00	6.44
0.6	250	1556.66	1556.66	0.00	1556.66	0.00	6.19
	200	1506.66	1506.66	0.00	1506.66	0.00	6.33
	150	1456.66	1456.66	0.00	1456.66	0.00	6.35
	100	1309.92	1317.42	0.57	1310.21	0.02	8.79
0.4	250	1556.66	1556.66	0.00	1556.66	0.00	6.28
	200	1462.62	1492.54	2.05	1462.62	0.00	7.31
	150	1355.09	1376.44	1.58	1358.31	0.24	8.17
	100	1179.71	1186.69	0.59	1181.96	0.19	10.98

It should be emphasized that our ultimate aim is not to improve the previous results but to find a good solution algorithm for the general version of the UHP-S with variable discount factors on the hub-to-hub links, which depends on the total amount of freight between hub terminals. No

Table 3
Computational results for 20 cities

α	Fixed cost	Best solution	GA-AH&V		Proposed GA heuristic		
			Best value	% dev	Best value	% dev	CPU time (seconds)
1	250	1570.91	1570.91	0.00	1570.91	0.00	11.05
	200	1520.91	1520.91	0.00	1520.91	0.00	11.06
	150	1470.91	1470.91	0.00	1470.91	0.00	11.55
	100	1410.07	1414.38	0.31	1410.07	0.00	13.18
0.8	250	1570.91	1570.91	0.00	1570.91	0.00	11.37
	200	1520.91	1520.91	0.00	1520.91	0.00	11.64
	150	1469.52	1469.52	0.00	1469.52	0.00	11.96
	100	1369.52	1369.52	0.00	1369.52	0.00	14.86
0.6	250	1570.91	1570.91	0.00	1570.91	0.00	11.60
	200	1506.04	1506.04	0.00	1506.04	0.00	11.75
	150	1406.04	1406.04	0.00	1406.04	0.00	14.03
	100	1269.15	1286.14	1.34	1269.15	0.00	19.22
0.4	250	1542.56	1542.56	0.00	1542.56	0.00	11.49
	200	1442.56	1442.56	0.00	1442.56	0.00	14.40
	150	1297.76	1315.04	1.33	1297.76	0.00	16.92
	100	1127.09	1142.07	1.33	1127.09	0.00	21.35

Table 4
Computational results for 25 cities

α	Fixed cost	Best solution	GA-AH&V		Proposed GA heuristic		
			Best value	% dev	Best value	% dev	CPU time (seconds)
1	250	1740.57	1740.57	0.00	1740.57	0.00	19.49
	200	1690.57	1690.57	0.00	1690.57	0.00	19.68
	150	1640.57	1640.57	0.00	1640.57	0.00	20.20
	100	1556.63	1565.22	0.55	1562.15	0.35	25.52
0.8	250	1740.57	1740.57	0.00	1740.57	0.00	19.80
	200	1690.57	1690.57	0.00	1690.57	0.00	20.39
	150	1594.08	1603.07	0.56	1594.08	0.00	24.90
	100	1458.83	1470.46	0.80	1459.74	0.06	30.36
0.6	250	1701.2	1702.35	0.07	1701.2	0.00	22.97
	200	1601.2	1602.35	0.07	1601.2	0.00	24.21
	150	1483.56	1491.87	0.56	1483.99	0.03	28.70
	100	1333.56	1341.87	0.62	1333.99	0.03	34.02
0.4	250	1601.62	1601.62	0.00	1601.62	0.00	26.68
	200	1501.62	1501.62	0.00	1501.62	0.00	28.02
	150	1351.69	1361.24	0.71	1351.69	0.00	30.81
	100	1187.51	1195.99	0.71	1187.51	0.00	35.70

specific rule or function is imposed for the variation of the discount factor. Thus, the proposed GA approach is particularly suitable for this problem, since it can easily handle more general, nonlinear and even non-continuous objective functions that may arise.

5. Real world application

The main motivation of the proposed GA heuristic is its application to a real world problem involving a LTL trucking company that has been among the top ten best companies in the sector in Brazil for the last three years.

Currently this company operates a hub-and-spoke network whose configuration has been defined and updated over time based on empirical decisions and motivated by expanding its geographic coverage across the nation's immense continental territory (Brazil's area is about 8.6 million km^2, the sixth world's largest country in area).

This trucking company, founded more than 60 years ago, operates facilities in 46 different locations, and serves more than 6000 municipalities in different 20 states using more than 1000 vehicles for both long distance transfers between hubs and short distance hauls between hubs and spokes. About 55% of the fleet is tracked using GPS based systems and satellites for on-line real-time location and communication.

The challenges brought about by increasing level-of-service requirements from customers, especially shorter transit-time and delivery deadlines that do not allow freight to wait long for full-load consolidation, as well as rising competition between LTL operators that lowered prices and, consequently margins and profits, required this company to review its network in order to improve its operations and to survive the rising competition. Thus, the company wanted to determine the optimal network configuration in terms of total cost minimization, comprising the determination of the number of consolidation terminals (hubs), their locations and the assignment of the spokes to the hubs.

The application of the proposed algorithm required a comprehensive data collection, treatment and analysis regarding historic flows that were consolidated into an 8-month period, considered relevant to adequately represent the year's seasonality. This data was treated and aggregated and then used for projecting the future scenarios. Transportation costs were also estimated for both long distance hauls involving transfer between hubs and short distance hauls between hubs and spokes. It should be noted that transportation costs had to be estimated for each origin–destination pair, since they cannot be directly approximated to distances due to several economic and regional aspects that interfere into the final freight rates and costs. Costs also suffer variations along the year. Contrarily to what occurs in the US and in other developed countries, a substantial part of the road freight transportation is done by third-party drivers that own their vehicles and are hired on a short time period basis. This short-term commitment model frequently leads to higher level of fluctuation in transportation costs, influenced by momentarily relative equilibrium between supply and demand.

The fixed costs of establishing a hub in a given facility were estimated based on the cost of additional labor, vehicles, equipments and eventually area required to operate each facility as a consolidation center, and also taking into consideration historic data costs. A comprehensive sensitivity analysis was also carried out in order to measure the influence of changes in input data in the final solution, as well as its robustness. We considered variations of ±10% and +20% in the unity transportation cost between nodes i and j (C_{ij}); similarly, we also evaluated the influence of the fixed costs in the final solution, given by a change of ±20% and ±50% in the base monthly costs. Variations on flows T_{ij} between nodes i and j (±10% and +20%) were also evaluated.

As mentioned before, due to delivery time requirements, the company operates daily services linking all hubs. The values considered for the discount factors on the linkage between each pair of hub nodes are:

Daily quantity (kg/day)	$\alpha(T)$
$\leqslant$1000	1.00
1001–3000	0.95
3001–6000	0.90
6001–12,000	0.80
>12,000	0.70

These discount factors reflect a general current market practice in Brazil, given the context mentioned above, in which a substantial part of the road freight transportation is done by third-party drivers that own their vehicles and are hired on a trip-to-trip basis. The above values resulted from a negotiation agreement between the LTL company and vehicle owners/drivers that are hired on a regular basis, in order to take into consideration the use of larger, more efficient and economic (in terms of unit costs) vehicles. A more complex and detailed procedure or function was contrary to the current practice in this market and thus discarded by both parts, given the undesired complexity added, making it more difficult to the drivers to understand it, and for both sides to verify, keep track and process data and calculate the due payments on a daily basis.

Tables 5 and 6 present the results from the application of the proposed GA heuristic to the LTL trucking company for different scenarios with respect to flows on links and cost per unit of flow, respectively. In both tables, different fixed costs are considered. For each entry, the first row indicates the selected hubs in the best solution found and the second the total savings with respect to the current network operated by the company. The results evidence that the minimum cost configuration corresponds to two hubs (1 and 45), instead of the eight hubs currently operated by the company, and yields to annual savings of 28.4%. The configuration is unchanged, even when demands, fixed costs and variable costs are altered, allowing savings above 20% in all cases. The only exception in terms of selected hubs occurs when fixed costs are reduced by 50%, a quite unlikely situation; in this case, more hubs may be selected (due to the much lower fixed costs of opening and operating a hub), though hubs 1 and 45 are always included in the corresponding solutions.

It should be noted that, in the modeling process, all 46 locations were considered as candidates for new hubs in the new optimized configuration

Table 5
Selected hubs and respective savings for sensitivity analysis considering different fixed costs and demands

Fixed cost	Demand (T_{ij})			
	Base	+10%	+20%	−10%
Base	1, 45 +28.4%	1, 45 +27.3%	1, 45 +26.3%	1, 45 +29.6%
+20%	1, 45 +30.8%	1, 45 +29.6%	1, 45 +28.5%	1, 45 +32.2%
−20%	1, 45 +25.7%	1, 45 +24.8%	1, 45 +23.8%	1, 45 +26.8%
+50%	1, 45 +33.9%	1, 45 +32.7%	1, 45 +31.4%	1, 45 +35.5%
−50%	1, 35, 45 +21.1%	1, 45 +20.6%	1, 19, 42, 45 +20.3%	1, 45 +21.8%

Table 6
Selected hubs and savings for sensitivity analysis considering different fixed and variable costs

Fixed cost	Cost per unit of flow (C_{ij})			
	Base	+10%	+20%	−10%
Base	1, 45 +28.4%	1, 45 +27.2%	1, 45 +26.1%	1, 45 +29.7%
+20%	1, 45 +30.8%	1, 45 +29.5%	1, 45 +28.4%	1, 45 +32.2%
−20%	1, 45 +25.7%	1, 45 +24.6%	1, 45 +23.7%	1, 45 +26.9%

that is sought; similarly, the number of selected hubs was not preset. Results were obtained in less than 300 CPU seconds, which is considered quite satisfactory, given the size of the network (46 nodes), thus allowing the validation procedures, as well as the analysis of different scenarios to be carried out without any difficulty or delays. We allowed the proposed heuristic to run for longer periods of time (up to 3600 seconds), without any improvement in the best solutions found.

When the results were presented to the top managers of the company in charge of the decision on how to reformulate the network configuration, they unveiled their concern with respect to level-of-service issues in terms of average and maximum distances between hubs and spokes. We then could supply them with some other top solutions to be examined, reflecting different levels-of-service. This was possible due to an interesting feature of the implementation of the proposed GA heuristic, since it provides not only the best near-optimal solution, but some good solutions as well. They initially showed some propensity and interest in another configuration, comprising the two hubs found in the best solution and another two additional hubs. This new configuration represented an increase of about 3% in the total cost but a slightly better level-of-service coverage. Due to the relevance of this decision, we decided to formulate new scenarios and undertake further sensitivity analysis regarding the main input parameters, and that are reflected in the results presented. Costs were also calculated for this new configuration for the different scenarios investigated. The results indicated that the configuration with four hubs was not robust with respect to variations in demand, fixed and variable costs; in other words, costs were higher for several alternative scenarios considered. After an extensive and lengthy analysis, the managers finally decided for the original best near-optimal configuration comprising two hubs. This decision was based on the must to minimize costs, given the fierce competition and lowering margins in this segment. It also ensured time-guaranteed delivery requirements in terms of minimal frequencies in the linkages between hubs and also accumulating enough freight to allow the use of more efficient vehicles, though not providing the best coverage.

6. Conclusions

Motivated by a real world application in the LTL trucking industry, in this paper we introduced a genetic algorithm heuristic that solves the UHP-S with variable discount factors on the linkages between hubs by finding the number of hubs, the location of the hubs and the assignment of the spokes to the hubs.

The computational results with the CAB data set clearly demonstrated the efficiency and the effectiveness of the proposed strategy, which relies on an efficient local improvement procedure that is used to improve each new generated individual of the population. This local heuristic is based on a simplified simulated annealing mechanism that allows non-improved solutions to be accepted with a certain probability.

The proposed heuristic was also applied to a real world problem related to major LTL trucking company in Brazil, with very good results and relevant cost reduction. The GA heuristic also provided some elite solutions that helped the company consider and analyze some level-of-service issues, improving its overall decision-making process. The best near-optimal configuration that was selected was essential for the company to reduce costs and survive the competition in this low-margin LTL market in Brazil.

There may be some possibilities for further investigation considering more general expressions for the discount factor on the hub-to-hub links, as well as some other related problems whose mathematical models could be derived from this more general formulation, like, for instance, problems similar to the one addressed by Podnar et al. (2002), where there is no special type of nodes (hubs) and, consequently, flows are not forced to use inter-hub links. Instead of discounting cost through each link by a single constant factor if the corresponding amount of flow exceeds a certain threshold, more general and realistic functions might be considered.

Another area of interest would be to investigate the incorporation of level-of-service constraints into the model formulation, particularly those related to coverage in terms of average and maximum distances between hubs and spokes. Also, heuristics based on GA might be explored for solving real world problems that can be considered as multi-objective in nature, like some location problems. The potential of GA approach is that it provides not only the best near-optimal solution but several good solutions as well, not to mention that it may consider other attributes and constraints that cannot always be incorporated into the mathematical models. For now, we leave all of them as topics of future investigation.

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