

New Evolutionary Algorithm Applying to a Type of Facility Location Problem

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Abstract: Mathematical model is built for solving a type of Facility Location Problem (FLP) in this study first. Then, genetic algorithm using symbolic coding is proposed. Based on this GA, a new evolutionary algorithm is proposed using of the basic idea of Particle Swarm Optimization (PSO). Symbolic coding method is still used in the new algorithms, which makes the model scale decrescent and reflects its characteristics. But the selection operator and mutation operator are all abandoned here. Furthermore, a type of total probability crossover is performed and the evolutionary policy of particle swarm optimization is absorbed into the new algorithm, which reduces the complexity and enhance the efficiency greatly. The model and the algorithm have been applied to a government-funded traffic project. The process of constructing the evolutionary algorithm based on total probability crossover dispensed with any especial condition, so our algorithm is universal to all facility location problem.

Key words: Genetic algorithms, particle swarm optimization, total probability crossover, elitist model

INTRODUCTION

Nowadays, road transport has increased dramatically throughout the world and transport station becomes more important for it is the key point of the transport net. The stations, including passenger transport station and highway freight station, are important place where the passengers or goods are collected, dispatched and transshipped. Station programming is special programming, which guarantee unobstructed and convenient to transportation. Furthermore, Station selection is the key issues in the progress of station programming. In the theory filed, station selection is essentially leads to Facility Location Problem (FLP), or Location Allocation Problem (LAP). This problem is obviously a type of optimization problem and it can be defined as follows:

Subjecting to the supply and demand relation and considering to the distribution of all demand points, how to determine the number and the situation of all facilities in order to make the total index optimal.

There are many different types of the branch problems of FLP that they can be applied to many fields, such as transportation, logistics and so on (Klose and

Drexl, 2004). A concrete FLP would be studied in this research and then the mathematics model is presented. For the NP-hard nature, exact algorithms for this problem may be used, but the heuristic method would be the advisable choice for larger instances. Indeed, much progress has been made in terms of approximation algorithms for the uncapacitated or capacitated version of this problem (Ghosh, 2003; Mauricio and Renato, 2006). Many heuristic algorithms for this problem have been applied, such as simulated annealing (Alves and Almeida, 1992), genetic algorithms (Kratka *et al.*, 2001), tabu search (Al-Sultan and Al-Fawzan, 1999), Particle Swarm Optimization (Guner and Sevcli, 2008) and so on. These algorithms have shown good performance.

In this study, a Genetic Algorithm (GA) using symbolic coding is also proposed that can be even better to our problem. Furthermore, based on this GA, a new evolutionary algorithm is proposed using of the basic idea of Particle Swarm Optimization (PSO), which reduce the complexity and enhance the efficiency greatly. Last, the model and the algorithm are applied to a government-funded traffic project; the outcome indicates that the new algorithm is efficient and effective.

MODEL BUILD

Mathematics model: First, we name the problem in this paper Facility Location Problem with Volume Constraints (FLPVC) and define it as follows:

For given set of potential facility points and demand points in one field, the passenger send volume in any station is constrained and all demand volume in the demand point keeps invariable respectively. In order to minimize the cost including transportation and construction and all demand in the demand point must be satisfied; the problem is how to determine the number of the stations which can be constructed in the fields of the potential points. Here, the demand point means the center of the region of population and potential facility point indicates the concrete place where the passenger transport station could be set up.

Note the minimum total costs as objective function and then build the mathematics model of FLPVC as follows:

$$\text{Min } \sum_{i \in I} \sum_{j \in J} c_{ij} d_j x_{ij} + \sum_{i \in I} f_i y_i \quad (1)$$

$$\text{S.t.: } \sum_{i \in I} x_{ij} = 1 \quad \forall j \in J \quad (2)$$

$$x_{ij} \leq y_i \quad \forall i \in I; \forall j \in J \quad (3)$$

$$N_l \leq \sum_{i \in I} y_i \leq N_u \quad (4)$$

$$\sum_{j \in J} d_j x_{ij} \leq V_i y_i \quad \forall i \in I \quad (5)$$

$$V_i^l \leq V_i y_i \leq V_i^u \quad \forall i \in I \quad (6)$$

$$x_{ij}, y_i \in \{0, 1\} \quad \forall i \in I; \forall j \in J \quad (7)$$

where, x_{ij} notes two-value decision variable, which indicates demand point j would be distributed to station i if the value of x_{ij} is 1, wouldn't be if the value of x_{ij} is 0. y_i is also two-value decision variable, which indicates one station would be set up at potential point i if the value of y_i is 1, wouldn't be if the value of y_i is 0.

I and J notes the set of potential points and demand points respectively, c_{ij} notes the unit transport costs from point i to point j , f_i notes the construction costs if one station is set up at potential point i . V_i notes the passenger send volume of station i . In the objective function, the first part notes the total transport costs and the second notes the total construction costs.

In those constraints, Eq. 2 guarantee that one demand point would be only distributed to one station,

Eq. 3 indicates that only when one station was set up would demand point be distributed to this station. Equation 4 presents a limit to the number of stations, where N_l notes the minimum number in the interval and N_u notes the maximum one. Equation 5 indicates that the volume of one station must larger than or equate the total demand this station should to deal with. Equation 6 presents a limit to the send volume of stations, where V_i^u and V_i^l note respectively the minimum and the maximum number in the interval.

GA design: Because FLPVC is NP-hard (Shmoys *et al.*, 1997), a Genetic Algorithm (GA) based on symbol operator is established first in this study (Aytug and Saydam, 2002), which can guarantee the global optimality of the solution. Some key designs in GA are presented as follows:

Symbolic coding: Symbolic coding method is used in this part. Based on the characteristic of the problem, if passage station would be set up at one potential point, we sign it as 1, otherwise sign it as 0. In this way, the search space would be compacted to minimum and the algorithm complexity also would be reduced (Zhou and Sun, 2002). That is to say, one individual or chromosome should be represented by the symbolic codes 1 and 0. Then two samples of symbolic coding can be displayed as follows:

Improved crossover: Unlike the similar problem without volume constrains, in order to keep the chromosome valid, the number of the code 1 or 0 in one chromosome must be restricted on a range. Otherwise, the related solution may be unfeasible.

For two parent chromosomes, such as those in Fig. 1, choose a point fixedly and set it as the crosspoint, then operate as usual one-point crossover. The cross result can be displayed in Fig. 2.

Here, in the both sides of the crosspoint, the number of the code 1 or 0 in one initial chromosome must be fixed, so the code 1 or 0 would be unchanged after crossover operating.

1	0	1	1	0	1	0	0	1	0
0	1	1	0	0	1	0	1	1	0

Fig. 1: Samples of symbolic coding

1	0	1	0	0	1	0	1	1	0
0	1	1	1	0	1	0	0	1	0

Fig. 2: Two children chromosomes after crossover

Mutation: In order to keep the number of 1 and 0 unchanged, mutation should happen to two random genes at same time, for example, one change from 1 to 0, then the other change from 0 to 1.

For the volume constrains, some invalid chromosomes may be generated, such as the total send volume is less than the total demand. In this case, mutation would be performed compulsorily to increase the number of code 1. In a word, all genetic operators should guarantee the feasibility of the related solution.

Selection method: Take the usual proportional selection model.

Fitness function: The target of FLPVC is to minimum the total costs, so the fitness function can be got by some reversible operation.

$$F(Z) = M - f(Z) \quad (8)$$

The character M in the right part of above equation means to a number large enough.

NEW EVOLUTIONARY ALGORITHM BASED ON THE TOTAL PROBABILITY CROSSOVER

GA designed above could be used to solve FLPVC, but it involves many operators and isn't efficient when the problem scale is larger. So, a new evolutionary algorithm is designed here.

Particle Swarm Optimization (PSO) is a population based stochastic optimization technique (Kennedy and Eberhart, 1995), inspired by social behavior of bird flocking or fish schooling. PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation, so there are few parameters to adjust. On one hand, PSO gets better results in a faster, cheaper way compared with GA. On the other hand, PSO leads to local optimal solution in some cases (Yang and Li, 2004).

Considering on the character of FLPVC, a new evolutionary algorithm based on PSO and GA is designed. This algorithm performs total probability crossover, which means all individuals must be crossed with the best one in any generation. So, we call this algorithm Evolutionary Algorithm based on total probability crossover (EATPC). Few parameters need to be determined in EATPC. Furthermore, when performing EATPC, the code method and fitness function keep unchanged, but following operators should be emphasized.

Table 1: Total probability crossover and random crosspoint

```

For i = 1 To PopSize - 1
    Randomize
    p1 = Rnd
    CrossPoint = Int(CHROMLENGTH * Rnd)
    point = CrossPoint
    If p1 < 0.5 Then
        For j = 1 To point
            population(i).chrom(j) = currentbest.chrom(j)
        Next j
    Else
        For j = point To CHROMLENGTH
            population(i).chrom(j) = currentbest.chrom(j)
        Next j
    End If
Next i

```

Table 2: The gene adjustment procedure

```

If FacilityNUM < LowNUM Then
    For i = 1 To CrossPoint
        If Fac(i).CodeName = "0" And FacilityNUM < LowNUM Then
            Fac(i).CodeName = "1"
            FacilityNUM = FacilityNUM + 1
        ElseIf FacilityNUM >= LowNUM Then Exit For
        End If
    Next I
End If
If FacilityNUM > UpNUM Then
    For i = CrossPoint To CHROMLENGTH
        If Fac(i).CodeName = "1" And FacilityNUM > UpNUM Then
            Fac(i).CodeName = "0"
            FacilityNUM = FacilityNUM - 1
        ElseIf FacilityNUM <= UpNUM Then Exit For
        End If
    Next i
End If

```

Total probability crossover: This operator is similar to particle flying in PSO, but belongs to GA operator, which may transform the good character from best individual to all new individual in next generation.

Random crosspoint: Based on one-point crossover, the crosspoint is set at random, which guarantees the variety of new individual. The procedure may be given as Table 1.

Gene adjustment: For the volume constrains to station, the number of code 1 must be restricted on a range. So crossover at random may lead to invalid individual. Furthermore, the number of code 1 should be monitored. When this number jumps out of the range, some adjustment must be performed. This procedure may be stated as Table 2.

Elitist model: To perform evolution operation based on elitist model, which means that the worst individual of this generation would be replaced by the current best one. Furthermore, the best individual and the best feasible individual of this generation are kept simultaneously and they would be compared with the best one and the best

Table 3: The elitist keep procedure

```

bestindividual = population(1)
worstindividual = population(1)
For i = 2 To PopSize
    If population(i).fitness > bestindividual.fitness Then
        bestindividual = population(i)
        best_index = i
    ElseIf population(i).fitness < worstindividual.fitness Then
        worstindividual = population(i)
        worst_index = i
    End If
Next i
End If
If generation = 0 Then
    currentbest = bestindividual
ElseIf bestindividual.fitness >= currentbest.fitness Then
    currentbest = bestindividual
If bestindividual.fitness > currentbest.fitness Then
    currentbest = population(best_index)
Else population(worst_index) = currentbest
End If

```

feasible one of next generation. The nicer one will be kept as the current best one. This procedure may be written as shown in Table 3. Evolution operation using elitist model is performed, which guarantee the global optimality of the solution.

PRACTICAL APPLICATION AND SIMULATION RESULTS

Input data: In this study, the GA and EATPC would be applied respectively to the practical station programming.

For one western city in China, a group data is presented in Table 4 after analysis with population, transportation and economy. Some parameters could be gotten from this table. In actual calculation, the coordinates of demand points and potential facility points are zoomed by the ratio 1:1000. The demand is given according to the local situation actually. If the station is new one, the investment costs would be always in direct proportion to its scale. Otherwise, like reconstruction or extension, the investment costs would be determined specifically.

Algorithm comparison: According to the characters of the problem and using Design of Experimental (DOE) method (Coy, 2000), determine the basis parameters presented in Table 5.

In Table 5, the other parameters, like Number of potential facility points, are determined in one experimental relatively.

Distributions: How to distribute the demand points to stations? The principle of the nearest and demand satisfaction is accepted in this study. The procedure may be stated as Table 6.

Table 4: Input data in the station programming

Demand points	
Paris of coordinates	33.2, 5.8, 8.2, 2.7, 15.1, 7.8, 12.8, 7.0, 11.9, 6.3, 10.7, 6.4, 19.4, 6.8, 37, 0, 7.4, 6.5, 9.4, 7.4, 10.5, 7.5, 11.3, 7.5, 13.9, 8.3, 13.9, 7.7, 15.0, 8.0, 15.1, 8.3, 16, 7.4, 17, 7.5, 14.7, 6.7, 12.6, 6.5, 18.5, 7, 29.3, 3.6, 35.1, 1.5, 37.9, 1.8, 34.1, 0, 33.8, 6.5, 33.4, 6.2, 33.0, 5.6, 43.4, 3.9
Demand (Million)	0.66, 0.79, 0.066, 0.052, 0.13, 0.13, 0.066, 0.66, 0.039, 0.026, 0.039, 0.66, 0.092, 0.118, 0.105, 0.066, 0.079, 0.052, 0.197, 0.066, 0.105, 0.144, 0.223, 0.105, 0.066, 0.079, 0.039, 0.052, 0.039, 0.197
Potential facility points	
Paris of coordinates	14.1, 8.4, 12.9, 7.4, 12.0, 8.0, 34.2, 5.1, 13.8, 6.4, 8.9, 2.8, 19.7, 7.0, 44.5, 3.8, 32.0, 3.4, 34.8, 0
Station-level	2, 1, 1, 2, 1, 2, 1, 1, 1, 1

Table 5: Basis parameters used in the programming

Parameter name	In GA	In EATPC	Other parameters	
Scale of population	50	30	No. of potential facility points	10
Selection probability	Random	/	No. of demand points	30
Crossover probability	0.7	1	Lower limit of station number	4
Maximum generation	300	80	Upper limit of station number	6
Mutation probability	0.1	/		

Table 6: The procedure of distribution method

```

For Passger = 1 To N
    For j = 1 To FNumUsed
        If Pas(Passger).DemNum <= Fac(IdxNEW(Passger, j)).BuildSize - Fac(IdxNEW(Passger, j)).FCused
            Then
                Fac(IdxNEW(Passger, j)).FCused =
                    Fac(IdxNEW(Passger, j)).FCused +
                    Pas(Passger).DemNum
                Cost = Cost + DisLast(Passger, j) *
                    Pas(Passger).DemNum *
                    Fac(IdxNEW(Passger, j)).InvestMoney
            Exit For
        End If
    Next j
Next Passger

```

Table 7: Outcomes of the simulations

Station number	1	4	6	10
Station-level	Grade 1	Grade 1	Grade 1	Grade 2
Distribution programs	3-7,	1, 22, 23	2, 9, 10	8, 24-26
Total costs	21-Nov	27-30		

First, distribute the demand points to the station between which the generalized distance is smallest. Here, the distance considering not only space distance, but also transport costs, transport time and so on. Then, if the nearest station volume is not satisfied, the second-nearest station would be considered, the rest may be deduced by analogy until distributing the demand over. All demand would be distributed over reasonably in this way, which satisfies the people's travel habits.

Outcomes: Simulations were programmed with GA and EATPC and the outcomes are listed in Table 7. The outcome is consistent with each other, but the computing time is different extremely. The average computing time performing GA ten times is 12658 m sec; the corresponding time performing EATPC is 6936 m sec.

According to the outcomes listed in Table 7, there are four stations should be set up including three first-grade stations and one second-grade station. The outcomes are received approval from the experts and accepted by the related department.

CONCLUSION

As a part of station programming, the study of FLP is pivotal in road traffic. In order to solve a type of FLP problem, a mathematical model is built in this paper and some measures are taken under this model. The genetic algorithm and a new evolutionary algorithm with symbolic coding are applied in solving this problem and optimization simulation is performed. Simulation result shows that the new algorithm is more effective than GA. The process of constructing the new algorithm doesn't include any especial condition, so EATPC is universal to all FLP and it is useful in the field of station programming and road transportation. Nevertheless, it is not proved whether this algorithm can be used to solve other optimization problem.

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