A New Model for Reliable Facility Location Problem under Random Disruptions

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Abstract – Supply chains are subject to various types of disruptions. In this paper, we analyze the problem of locating facilities in a supply chain under random disruptions on facilities. We present a new reliable location model called 'the α-reliable Minimax regret' which we apply it to the capacitated reliable fixed charge location problem. The model minimizes the worst-case regret with respect to a subset of worst-case scenarios whose collective probability of occurrence is at least α. The model is solved by CPLEX. Computational Results show it is efficient.

Keywords – *supply chain; disruption; reliability; scenario planning.*

I. INTRODUCTION

Facility location problem so far has been studied for various applications. Classical facility location problem implicitly assume that all facilities are quite reliable and obtain the optimal solution under this ideal situation. However, some of the constructed facilities may become unavailable due to disruptions caused by natural disasters, strikes and so on. When a facility failure occurs, customers may have to be reassigned from their original facilities to others that require higher transportation costs. An example of disruptions caused by natural disasters is the recent massive earthquake and tsunami in Japan. This tragedy greatly affected the supply chains of many international companies e.g. General Motors and Toyota. Thus today's designing a reliable supply chain is very important.

The reliable location model was first introduced by Snyder and Daskin [1] to handle facility disruption. They consider a supply network that serves 49 cities, consisting of all state capitals of the continental United States and Washington, DC.

Recently, researchers have dealt with the uncertainties by defining a number of possible future scenarios. In the problems of locating facilities in a supply chain in the presence of disruptions, the availability of facilities is uncertain therefore scenario defining can use in these problems. The available facilities differ between any two scenarios. Facility sites that either optimize the expected performance or optimize the worst-case performance over all the scenarios are recommended. However, such approaches may not be practical since in real life; facilities are not typically designed for the average case or the worst-case scenario. For example airports are never sized for the peak travel day or average volume since doing so would be expensive. Daskin et.al. present a Minimax regret model for problems in which demand and transportation costs are uncertain.

In this paper we present a new reliable location model called the α-reliable Minimax regret which we apply it to the capacitated reliable fixed charge location problem (CRFLP). In this model, the maximum regret is computed over an endogenously selected subset of scenarios, called the reliability set, whose collective probability of occurrence is at least some user-defined value α. By minimizing the α-reliable maximum regret, the planner can be $100\alpha\%$ sure that the regret realized will be no more than that found by the model.

The remainder of this article is organized as follows: in Section 2, we review the relevant literature. In Section 3, we formulate the α–reliable Minimax regret model. We present numerical results and computational experiments in section 4. Finally we summarize discussions and conclusions of the study.

II. LITERATURE REVIEW

Researchers use scenario planning to deal with the uncertainties in strategic facility location. In scenario planning, the decision maker specifies a number of future possible scenarios and estimates the probability of each scenario occurring. The objective is to find solutions which perform well under all scenarios.

Scenario planning is chosen primarily because it is very flexible and can formulate various problems.

Snyder and Daskin present a scenario-based model for reliable uncapacitated fixed charge location problem (RUFLP) [2]. Gade consider the model similar to Snyder and Daskin's but assumes facilities are capacitated [3]. Shen et.al. also use scenario planning to model reliable fixed charge location problem [4]. Peng et.al. extend a scenario-based model for p-robust logistic network design problem [5].

Totally there are at least three approaches to incorporating scenario planning into location modeling [6].

- Optimizing the expected performance over all scenarios
- Optimizing the worst-case performance
- Minimizing the expected or worst-case regret across all scenarios

All the works cited above optimize the expected performance over all scenarios while in this paper, we focus on regret-based approaches. In the context of stochastic facility location, the regret associated with each scenario under a given siting plan is usually defined as the difference between the objective function value when the siting plan is chosen to optimal solution for that scenario and the objective function value when the siting plan is chosen to be the given siting plan [7].

III. THE MINIMAX REGRET MODEL

In this section we present the *α*-reliable Minimax regret model. We define the following notation:

I: set of customers, indexed by i

J: set of potential facilities, indexed by j

S: set of possible scenarios, indexed by s

f_j : fixed cost to construct facility j

k_j : the capacity of facility j

h_i : the demand of customer i

d_{ij} : unit transportation cost from customer i to potential facility j

$$a_{js} = \begin{cases} 1: \text{ if facility j is disrupted in scenario s} \\ 0: \text{ otherwise} \end{cases}$$

c_s^* : optimal objective value (the sum of fixed costs and transportation costs) that can be obtained under scenario s

q_s : the probability that scenario s will occur

M: a large constant

α: desired reliability level

In this model we can plan against an endogenously determined subset of the scenarios, called the reliability set, whose combined probability is at least *α*. Let R_s be the regret associated with scenario s and the current solution (x,y) (may not be optimal for scenario s). It is calculated as follows:

$$R_s = \sum_{j \in J} f_j\, x_j + \sum_{i \in I} \sum_{j \in J} h_i\, d_{ij}\, y_{ijs} - c_s^* \qquad (1)$$

We define the following decision variables:

$$x_j = \begin{cases} 1: \text{ if facility is opend on loaction j} \\ 0: \text{ otherwise} \end{cases}$$

$$y_{ijs} = \begin{cases} 1: \text{ if customer i is asssigned to facility j} \\ \quad \text{under scenario s} \\ 0: \text{ otherwise} \end{cases}$$

$$W_s = \begin{cases} 1: \text{ if scenario s is included in the set over which} \\ \quad \text{the maximum regret is minimized} \\ 0: \text{ otherwise} \end{cases}$$

R: the maximum regret over all scenarios in the reliability set

The *α*-reliable Minimax regret model is formulated as follows:

$$\min \quad R \qquad (2)$$

Subject to:

$$\sum_{s \in S} q_s W_s \ge \alpha \qquad (3)$$

$$\sum_{j \in J} f_j\, x_j + \sum_{i \in I} \sum_{j \in J} h_i\, d_{ij}\, y_{ijs} - c_s^* - M(1 - W_s) \le R \quad \forall s \in S \qquad (4)$$

$$\sum_{j \in J} y_{ijs} = 1 \qquad \forall i \in I, s \in S \qquad (5)$$

$$\sum_{i\in I} h_i\, y_{ijs} \le (1-a_{js})\, k_j\, x_j \qquad \forall j \in J, s \in S \tag{6}$$

$$y_{ijs} \le x_j \qquad \forall i \in I, j \in J, s \in S \tag{7}$$

$$x_j \in \{0,1\} \tag{8}$$

$$y_{ijs} \in \{0,1\} \qquad \forall i \in I, j \in J, s \in S \tag{9}$$

$$W_s \in \{0,1\} \qquad \forall s \in S \tag{10}$$

$$R \ge 0 \tag{11}$$

The objective function (2) minimizes the maximum regret over all scenarios in the reliability set. Constraint (3) declares the probability of reliability set must be at least α. (4) defines the maximum regret associated with reliability set. Constraints (5) state that each customer must be assigned to exactly one facility in each scenario. Constraints (6) declare the assignments to facility j in scenario s must be less than its capacity in that scenario. Constraints (7) prohibit an assignment to a facility that has not been opened. Constraints (8), (9), (10), (11) define decision variables.

IV. COMPUTATIONAL RESULTS

To test the performance of the model, we used 6 datasets. All of our datasets involve five facilities and 10 customers. We generate different scenarios and test our model. The scenario probability generated with a uniform distribution Uniform (0, 1] and then normalized such that the total probability of all the scenarios is equal to 1.

The models were coded in C++ and tested on a laptop computer with 2.4 GHz Intel Core i5 and 4GB RAM memory. The problems are solved exactly by CPLEX version 10.1 and the operating system is Microsoft Windows 7. All computational times presented are in seconds.

Table 1 shows the results for optimality solving Minimax regret model, with the reliability level α fixed at 95%. CPU time column gives the total number of CPU second required.

It is evident from Table 1; when the number of scenarios is large, the running time increases. This is not surprising, because the problems with more scenarios have more decision making variables and more constraints so finding optimal solution can be difficult.

Figure 1 shows CPU time increased with increasing number of scenarios.

You can find from results that in most cases the CPU time is suitable and α-reliable Minimax regret model is efficient.

TABLE 1 : RESULTS FOR MINIMAX REGRET MODEL

No. of scenarios	CPU time (second)
10	1.00
25	3.35
50	18.14
70	56.21
85	99.40
100	146.57

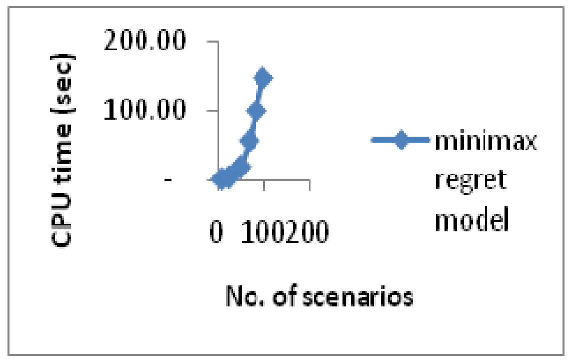

Fig. 1 : solution times for Minimax regret model with α=0.95

V. DISCUSSIONS AND CONCLUSION

In this paper we present the α-reliable Mean-excess model for facility location planning in the supply chain. In this framework, decision makers identify future scenarios and estimate the probability of each scenario occurring. The model then finds a solution that minimizes the worst-case regret with respect to an endogenously selected subset of worst-case scenarios whose combined probability of occurrence is at least 1 – α. We also show the computational efficiency of the model.

We believe that several promising avenues exist for future research in this field. First, it is difficult to consider a crisp disruption probability for each scenario, Therefore the use of fuzzy failure probabilities seems reasonable. It is also valuable to extend a solution algorithm for Minimax regret models.

REFERENCES

[1] L.V. Snyder and M.S. Daskin, (2005) "Reliability models for facility location: the expected failure cost case," Transportation Science, vol. 39, pp. 400-416, 2005.

[2] L.V. Snyder and M.S. Daskin, "Models for Reliable Supply Chain Network Design," 2006.

[3] B.E. Dinakar Gade, "Capacitated Facilities Location Problems with Unreliable Faclilities," MS thesis, Birla Institute of Technology and Science, University of Arkansas, 2007.

[4] Z.J.M. Shen, "The Reliable Facility Location Problem: Formulations, Heuristics, and Approximation Algorithms," Working paper, Department of Industrial Engineering and Operations Research, University of California at Berkeley, 2007.

[5] P. Peng, L.V. Snyder, A. Lim and Z. Liu "Reliable logistics networks design with facility disruptions," Transportation Research, Part B, 2011.

[6] S.H. Owen and M.S. Daskin, "Strategic facility location: A review," European Journal of Operational Research, vol 111, pp. 423447, 1998.

[7] G. chen, M.S. Daskin, Z.M. shen and S. Uryasev, "The α-Reliable Mean-Excess Regret Model for Stochastic Facility Location Modeling," Naval research Logistic, vol. 53, 2006.

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