Reliable p-median facility location problem: two-stage robust models and algorithms

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December, 2012

Abstract

In this paper, we propose a set of two-stage robust optimization models to design reliable p-median facility location networks subject to disruptions. A customized *column-and-constraint generation* approach is implemented and shown to be more effective than Benders cutting plane method. Numerical experiments are performed on real data and management insights on system design are presented. Our study also demonstrates the strong modeling capability of two-stage robust optimization scheme by including two practical issues, i.e., facility capacities and demand losses due to disruptions, which receive little attention in reliable network design research. Results show the significant influence of the demand loss factor on the network configuration.

keyword: facility location problem, reliable network design, two-stage robust optimization, demand loss

1 Introduction

The selection of facility locations and customer assignments are among the most crucial issues in designing an efficient distribution network. To address these issues, various facility location models are formulated and studied for decades, including those based on p-median and fixed-charge facility location formulations and their extensions (Daskin, 1995; Drezner, 1995; Revelle et al., 2008; Melo et al., 2009). The applications of those facility location models can be found in various industries, including manufacturing, retail and healthcare (Barahona and Jensen, 1998; Teo and Shu, 2004; Jia et al., 2007). Although it is expected by designers that the distribution network works reliably, network components could lose their functions and become unavailable in practice. For example, some facilities may be disrupted by natural disasters, labor strikes or terrorism threats, and become non-operable. Since the material or information flows are generated, processed, and distributed by facilities, facility disruptions could significantly deteriorate the performance of the whole network and result in enormous economic losses, see the descriptions in Snyder et al. (2010) and references therein.

To address this reliability issue, several recent studies, including Snyder and Daskin (2005), Berman et al. (2007), Cui et al. (2010), Li and Ouyang (2010), Lim et al. (2010), Li et al. (2012), and Peng et al. (2011) propose to proactively consider network disruptions and the incurred cost of countermeasures in the system design stage. The countermeasures, i.e., mitigation or recourse operations, are to reassign clients to survived facilities such that they can be served and the impact of disruptions can be minimized. Hence, the objective of system design is to minimize the (weighted) overall cost, including the operation cost in the normal situation when all facilities function properly, and the cost of mitigation in disruptive situations. To analytically represent this new design scheme, based on the explicit probabilistic information, several compact (nonlinear) mixed integer programs or scenario-based two-stage stochastic programming formulations are developed and customized exact or approximation algorithms are designed to solve real instances. (Snyder and Daskin, 2005; Cui et al., 2010; Li and Ouyang, 2010; Lim et al., 2010; Shen et al., 2011; Peng et al., 2011).

Nevertheless, in many situations, either accurate method does not exist or sufficient data are lack to exactly characterize probability distributions, or data are contaminated to provide precise information. Under such situations, probabilistic models, e.g., the aforementioned two types of models, could be inappropriate or lead to infeasible solutions. To address this challenge, robust optimization (RO) method, which simply assumes an uncertainty set to capture random data, is developed to provide solutions that are robust to any perturbations within the uncertainty set. To model the situation where some decisions can be made and implemented after the uncertainty is revealed, robust optimization is extended to include the second stage recourse decisions so that the available information can be fully utilized to produce a less conservative solution. After their introduction, original robust optimization method and its two-stage extension have been applied in many operational and engineering areas (Ben-Tal et al., 2009; Bertsimas et al., 2011), such as facility location problems with random demands (Atamturk and Zhang, 2007; Baron et al., 2011; Gabrel et al., 2011). In fact, comparing with demand uncertainty, facility disruptions are often less likely to be described by accurate probabilistic information. For example, earthquakes in California or hurricanes in Florida could cause facilities in those regions to be disrupted. However, it is very difficult to estimate the number of earthquakes or hurricanes in next 10 years based on historical/statistical data. Hence, in this paper, we apply the concept of uncertainty set to capture the possible site disruptions and employ robust optimization method to study reliable facility location problems.

Specifically, we adopt two-stage robust optimization approach to investigate reliable p-median problem, where location decisions are made before (*here-and-now*) and recourse (mitigation) decisions are made after disruptions are revealed (*wait-and-see*). We mention that such a modeling framework exactly captures the decision making sequence in real operations. In particular, due to its strong modeling capability, we are able to extend our study to consider facility capacities and demand losses due to disruptions. We note that the former situation is very challenging for probabilistic models while the latter has not been investigated in existing literature. Then, classical and customized solution algorithms, i.e., *Benders decomposition* and *column-and-constraint generation* methods, are implemented and a set of numerical experiments are performed to generate insights on the algorithm performance and the network design.

The rest of the paper is organized as follows. In Section 2, we review relevant literature on probabilistic models and two-stage robust optimization models. In Section 3, we introduce our two-stage robust optimization reliable p-median models and analyze their properties. In Section 4 we describe our solution algorithms. In Section 5, we present numerical results and provide insights on system design. In Section 6, we conclude this paper with a discussion on future research directions.

2 Literature review

In this section, we briefly review two types of relevant studies on the facility location problem: probability based reliable facility location models and (two-stage) robust optimization ones. Results on classical and deterministic facility location problems can be found in Daskin (1995) and Drezner (1995). For problems with uncertain demands and cost, readers are referred to a comprehensive review in Snyder (2006).

The research by Drezner (1987) is probably the first one studying facility location problem with unreliable facilities while Snyder and Daskin (2005) present the first reliable facility location models with inclusion of mitigation/recourse operations and costs. Lagrangian relaxation algorithms are implemented within a branch-and-bound scheme to solve the resulting linear mixed integer programs for real instances. By relaxing the assumption that all sites share the same failure rate, Cui et al. (2010) build a nonlinear mixed integer program and develop both Lagrangian relaxation and continuum approximation algorithms for this challenging problem. To reduce the complexity of the nonlinear form, Lim et al. (2010) study a simplified model where clients will be assigned to (unreliable) facilities and reliable backup facilities if needed. Shen et al. (2011) present both scenario-based stochastic programming and a nonlinear mixed integer programming model and show that they are generally equivalent. Also, a constant-ratio approximation algorithm for the case where all failure rates are identical is proposed. Li and Ouyang (2010) study a problem with correlated probabilistic disruptions and solve their model by a continuum approximation algorithm. Li et al. (2012) consider problems with a fortification budget in the first stage so that unreliable facilities can be fortified by hardening operations. Recently, this line of research is extended to investigate more general reliable network design problems. Peng et al. (2011) consider a reliable multiple-echelon logistics network design problem where disruptions can happen in multiple echelons. An et al. (2011) study reliable hub-and-spoke network design problems in which hubs could be disrupted and affected flows will be rerouted through survived operational hubs. From those aforementioned studies, we observe that (i) either complicated nonlinear mixed integer programs or large-scale scenario-based stochastic programs are necessary to build the model. When professional solvers are not efficient to deal with those models, customerized algorithms, either analytical or heuristic ones, will be developed; (ii) some practical situations are not sufficiently investigated. For example, very limited research is done for capacitated models except Peng et al. (2011) and no exact algorithm has been developed. Also, demand losses due to disrupted clients have not been modeled, which could affect the location of facilities.

Different from nonlinear mixed integer programs or scenario-based stochastic programs that are developed based on precise probabilistic information, robust optimization based location models, including those developed with two-stage robust optimization method, assume a probability-free uncertainty set and seek to determine locations that are robust to any perturbations in that uncertainty set. Baron et al. (2011) build a multi-period capacitated fixed charge (single-stage) robust location model and investigate the impact of different uncertainty sets on facility locations. Gülpınar et al. (2012) propose to use tractable (single-stage) robust optimization method to approximately solve stochastic facility location problem with a chance constraint. Atamturk and Zhang (2007), Gabrel et al. (2011), and Zeng and Zhao (2011) develop two-stage robust optimization formulations for location-transportation problems where locations and capacities must be determined in the first stage and transportation decisions can be adjusted after demand is realized. Different solution algorithms are proposed by them respectively, including an approximation algorithm (Atamturk and Zhang, 2007), Benders cutting plane algorithm (Gabrel et al., 2011), and the *column-and-constraint* *generation* (C&CG) algorithm (Zeng and Zhao, 2011). We mention that the *column-and-constraint generation* algorithm demonstrates a superior computational performance over Benders cutting plane method in the two-stage facility location problem and power system scheduling problems (Zhao and Zeng, 2010).

In order to make this paper focused, we restrict this study to p-median problem and leave the study of two-stage RO formulations for another classical model, i.e., the fixed-charge facility location problem, as a future research direction. Research presented in this paper makes the following contributions to the literature.

(i) To the best of our knowledge, no research has been done to apply two-stage RO to formulate reliable facility location design problems with consideration of disruptions. Hence, this paper presents the first set of reliable facility location formulations using two-stage robust optimization tools.

(ii) Because of the modeling advantages of two-stage RO, we consider real features that have received very limited or no attention. They are finite capacities of facilities and demand losses due to disruptions.

(iii) In addition to some analytical study on those models, we customize and implement solution algorithms to perform numerical experiments. We also present management insights based on the numerical results from instances with real data.

3 Two-stage robust p-median reliable models

In this section, we present our formulations on two-stage RO reliable p-median facility location problem. We first consider uncapacitated robust models and then extend our work to consider capacitated cases. Existing research generally ignores the demand losses due to disruptions. We show that, by using the two-stage robust optimization framework, demand losses can be easily incorporated. We also derive structural properties of those models.

3.1 Robust p-median models without demand losses

Different from stochastic programming models that explicitly consider all possible uncertain scenarios, (two-stage) robust optimization models use an uncertainty set to describe the concerned possible scenarios without depending on probability information. Specifically, assume that all sites in set J are homogeneous and consider all possible scenarios with up to k simultaneous disruptions. Then, the uncertainty set, i.e., the disruption set in this paper, can be represented as

$$A = \{\mathbf{z} \in \{0,1\}^{|J|} : \sum_{j \in J} z_j \leq k\}. \tag{1}$$

where z_j is the indicator variable for site j, i.e., $z_j = 1$ if site j is disrupted and $z_j = 0$ otherwise. Note that, although there may exist an exponential number of disruptive scenarios, this formulation provides an implicit but compact algebraic format to capture all of them. In the remainder of this paper, unless explicitly mentioned, we employ this disruption set to perform our study.

Next, we develop our two-stage RO reliable p-median facility location models. Let I be the set of clients (customer nodes) and $J \subseteq I$ be the set of potential facility nodes. Without loss of generality, we assume that $I = J$. Each client $i \in I$ has a demand d_i and the unit cost of serving i by the facility at $j \in J$ is $c_{ij} \geq 0$ with $c_{ii} = 0$. We use $\mathbf{y}$ and $\mathbf{x}$ to denote the first stage (the normal situation without disruptions) decision variables: $y_j = 1$ means that a facility is located at j, $y_j = 0$ otherwise; $x_{ij} \in [0, 1]$ represents the portion of i's demand served by j in the normal situation. Note that the first stage decision variables are to be fixed before any disruptive scenario $\mathbf{z}$ in set A is realized. In a disruptive scenario, as in Snyder and Daskin (2005) and Cui et al. (2010), a disrupted facility at j can not serve any client. However, system reliability can be achieved by implementing recourse or mitigation operations such as re-assigning customers to survived facilities. So, we introduce $\mathbf{w}$ and $\mathbf{q}$ to represent the second stage recourse operation decisions in a disruptive scenario, where $w_{ij} \in [0, 1]$ represents the portion of demand d_i served by the survived facility at j and $q_i \in [0, 1]$ represents the unsatisfied portion. Each unit of unsatisfied demand of d_i will incur a penalty M.

Similar to all existing studies on reliable facility location models, we first assume that there is no demand loss in any disruptive scenario, i.e., disruptions only affect the function of facilities and all clients keep generating demands as usual. In the following, we present the two-stage RO reliable p-median facility location formulation with up to k simultaneous disruptions and no demand loss.
RO-PMP$_0$

$$V_0(p, k, \rho) = \min_{\mathbf{x},\mathbf{y}} \ (1-\rho)\sum_i \sum_j c_{ij} d_i x_{ij} + \rho \max_{\mathbf{z}\in A} \min_{(\mathbf{w},\mathbf{q})\in S_0(\mathbf{y},\mathbf{z})} (\sum_i \sum_j c_{ij} d_i w_{ij} + \sum_i M d_i q_i) \tag{2}$$

$$\text{s.t.} \quad x_{ij} \le y_j, \qquad \forall i, j \tag{3}$$

$$\sum_j x_{ij} \ge 1, \qquad \forall i \tag{4}$$

$$\sum_j y_j = p, \tag{5}$$

$$x_{ij} \ge 0, \ \forall i, j; \ y_j \in \{0, 1\}, \ \forall j \tag{6}$$

where

$$S_0(\mathbf{y}, \mathbf{z}) = \{w_{ij} \le 1 - z_j, \qquad \forall i, j \tag{7}$$

$$w_{ij} \le y_j, \qquad \forall i, j \tag{8}$$

$$\sum_j w_{ij} + q_i \ge 1, \qquad \forall i \tag{9}$$

$$w_{ij} \ge 0, \ \forall i, j; q_i \ge 0, \ \forall i \ \} \tag{10}$$

In this formulation, the objective function in (2) seeks to minimize the weighted sum of the operation costs in the normal disruption-free situation and in the worst disruptive scenarios in A. The weight $\rho \in [0, 1]$ is a parameter that reflects the system designer's attitude towards the disruption cost. Clearly, a larger ρ indicates that the designer is more conservative and willing to configure the system in a way such that less recourse/mitigation operation costs will incur in disruptive situations. Constraints in (3)-(5) are from the classical p-median model and simply mean that a customer site can be assigned to a facility only if the facility is built, the entire demand of a customer site has to be served, and the total number of facilities is p, respectively.

The *max* operator identifies the disruptive scenario(s) in A yielding the largest operation cost, given the location $\mathbf{y}$. The second *min* seeks for the least costly mitigation solution while the set $S_0(\mathbf{y}, \mathbf{z})$ defines the possible recourse operations. That is, given the definition of y_j and z_j, constraints (7) and (8) ensure that in any disruptive scenario, a client i's demand can only be assigned to established *and* survived facilities. Then, constraints in (9) represents that except the lost portion q_i, the rest of i's demand, $1 - q_i$, has to be served. In this paper, our research focuses on the nontrivial cases where $k \le p - 1$. Otherwise, there will be no mitigation operations in any worst disruption scenario and the problem reduces to the p-median formulation.

Although this robust formulation is a complicated tri-level optimization problem, we can derive some structural properties by analyzing the underlying location problem. Specifically, we consider the case where the penalty coefficient M is sufficiently large, e.g., $M \geq \max_{i,j}\{c_{ij}\}$. It implies that all customer demands will be served in any disruptive situation because the penalty cost is higher than the largest serving cost. Actually, every individual customer's demand will be served by an available facility that is closest to him, in both normal and disruptive situations. Clearly, given that some customer's initially assigned facility becomes unavailable in a disruptive situation, his demand will be served by a survived facility that is definitely further. Hence, we have the following result.

Lemma 1. *For given a facility location $\mathbf{y}_0$, let $C_r(\mathbf{y}_0)$ and $C_{\mathbf{z}}(\mathbf{y}_0)$ be the operating costs in the normal situation and a disruptive situation $\mathbf{z}$, respectively. When M is sufficiently large, we have $C_{\mathbf{z}}(\mathbf{y}_0) \geq C_r(\mathbf{y}_0)$.*

Consequently, the following results can be proven easily.

Proposition 1. *When M is sufficiently large, the function $V_0(p,k,\rho)$ is (i) non-increasing with respect to p; (ii)non-decreasing with respect to k; and (iii) non-decreasing with respect to ρ;*

Proof. Statements in (i) and (ii) are very easy to prove. We give the proof for the statement in (iii). Consider $\rho_1 \leq \rho_2$ and their corresponding optimal facility locations $\mathbf{y}_1$ and $\mathbf{y}_2$.

Clearly, as $\mathbf{y}_2$ may not be optimal when $\rho = \rho_1$, we have

$$V_0(p,k,\rho_1) = V_0(p,k,\rho_1|\mathbf{y}=\mathbf{y}_1) \leq V_0(p,k,\rho_1|\mathbf{y}=\mathbf{y}_2).$$

Given that $\rho_1 \leq \rho_2$, it follows from Lemma 1 that

$$V_0(p,k,\rho_1|\mathbf{y}=\mathbf{y}_2) \leq V_0(p,k,\rho_2|\mathbf{y}=\mathbf{y}_2).$$

Therefore, we have

$$V_0(p,k,\rho_1) = V_0(p,k,\rho_1|\mathbf{y}=\mathbf{y}_1) \leq V_0(p,k,\rho_2|\mathbf{y}=\mathbf{y}_2) = V_0(p,k,\rho_2)$$

□

Note from constraints (3)-(10) that the two-stage RO is a very adaptive modeling framework. By using $\mathbf{y}$ and $\mathbf{z}$, we can impose logic or physical restrictions on recourse decisions without enumerating all their possible values. In the following, we show that a more involved situation can be formulated compactly.

3.2 Robust p-median problem with demand losses

Note that the aforementioned formulation assumes that a disrupted site keeps generating demands as usual while a facility on it, if exists, loses the function. However, as an example, demands of non-essential or luxury products often vanish in disruptions. To the best of our knowledge, no existing work on the reliable facility location problem studies the impact of demand losses due to site disruptions on the system design. One possible reason is that, if the demand loss factor is considered, classical probabilistic models have to evaluate all possible scenarios while different scenarios will have different coefficients in their objective functions, which makes it very challenging to have a compact and tractable formulation. Nevertheless, the two-stage robust optimization scheme provides us a convenient modeling framework to address this issue. Specifically, given the interpretation of z_i in (1), we can simply add "$-z_i$" on the right-hand-side of (9) to model the demand loss at a disrupted site.

<u>RO-PMP$_l$</u>

$$V_l(p,k,\rho) = \min_{\mathbf{x},\mathbf{y}} (1-\rho)\sum_i \sum_j c_{ij} d_i x_{ij} + \rho \max_{\mathbf{z}\in A} \min_{(\mathbf{w},\mathbf{q})\in S_l(\mathbf{y},\mathbf{z})} \Big(\sum_i \sum_j c_{ij} d_i w_{ij} + \sum_i M d_i q_i\Big) \tag{11}$$

s.t. (3) – (6),

with

$$S_l(\mathbf{y},\mathbf{z}) = \{(7) - (8),\ (10)$$

$$\sum_j w_{ij} + q_i \geq 1 - z_i, \quad \forall i\} \tag{12}$$

Note from (12) that when $z_i = 1$, the minimization recourse problem will drive $w_{ij} = 0$ for all j and $q_i = 0$, i.e., there is no demand at i to serve.

We mention that considering demand losses complicates the problem structure as the customer demand in a disruptive situation is depending on $\mathbf{z}$ now. Nevertheless, we note that in RO-PMP$_l$, if M is sufficiently large, the operation cost of the *worst* disruptive situation is still larger than that of the normal situation.

Lemma 2. *When M is sufficiently large, consider a given facility location $\mathbf{y}^*$ and the disruption set A. We have (i) the worst case disruptions and therefore demand losses happen only at facility sites, i.e., those with $y^*_j = 1$; (ii) $\hat{C}_A(\mathbf{y}^*) \geq C_r(\mathbf{y}^*)$ where $\hat{C}_A(\mathbf{y}^*)$ is the operating cost of the worst case scenario in A.*

Proof. For (i), we prove it by contradiction. Consider the worst case disruptive situation $\mathbf{z}^1$ where a disruption happens at site j_0, on which there is no facility, i.e., $z^1_{j_0} = 1$ and $y^*_{j_0} = 0$. Let C^1 be the operation cost under this disruptive situation.

As $p - k \geq 1$, there exists a facility, say j_1 with $y^*_{j_1} = 1$, survived in the disruptive situation $\mathbf{z}^1$. Consider two disruptive situations: $\mathbf{z}'$ where $z'_{j_0} = 0$, and $z'_j = z^1_j$ for $j \neq j_0$, and $\mathbf{z}^2$ where $z^2_{j_0} = 0$, $z^2_{j_1} = 1$ and $z^2_j = z^1_j$ for $j \neq j_0$ and $j \neq j_1$. Denote the operation cost under $\mathbf{z}'$ by C', and that under $\mathbf{z}^2$ by C^2.

First, it is clear that $C' \geq C^1$, because demand from customer site j_0 must be served by some facility in $\mathbf{z}'$ while this demand is lost in $\mathbf{z}^1$ and does not incur any cost.

Second, under the disruptive situation $\mathbf{z}^2$, because the facility at j_1 is not available, all its served customer demands, except the demand from j_1, will be served by other survived facilities, which will be further and more costly. Because $c_{j_1 j_1} = 0$, the demand from site j_1 will not incur any service cost in $\mathbf{z}'$ and $\mathbf{z}^2$. So, we have $C^2 \geq C'$.

Because $C^2 \geq C' \geq C^1$, we have the desired contradiction. Also, based on the argument of $C' \leq C^2$, it is easy to see that $\hat{C}_A(\mathbf{y}^*)$ is non-decreasing with respect to k and therefore (ii) follows. □

As a result, similar to those presented in Proposition 1, the following properties for RO-PMP$_l$ can be obtained based on Lemma 2.

Proposition 2. *When M is sufficiently large, the function $V_l(p,k,\rho)$ is (i) non-increasing with respect to p; (ii) non-decreasing with respect to k; and (iii) non-decreasing with respect to ρ.*

Next, we extend our study to capacitated facility location problem, whose reliable models have received little research attention.

3.3 The robust capacitated facility location model

The capacitated p-median facility location (CPMP) problem is an extension of the classical facility location model. Besides the same objective function and decision variables as in the classical uncapacitated facility location problem, it assumes that each potential facility has a capacity, i.e., an upper bound on the amount of demand that it can serve (Sridharan, 1995). Let C_j denote the capacity of site j. The two-stage robust capacitated p-median facility location problem without demand loss is shown as follows:
RO-CPMP$_0$

$$VC_0(p,k,\rho) = \min_{\mathbf{x},\mathbf{y}} (1-\rho)\sum_i\sum_j c_{ij}d_ix_{ij} + \rho \max_{\mathbf{z}\in A} \min_{(\mathbf{w},\mathbf{q})\in S_0(\mathbf{y},\mathbf{z})} (\sum_i\sum_j c_{ij}d_iw_{ij} + \sum_i Md_iq_i) \quad (13)$$

$$\text{s.t.} \quad (3)-(6)$$

$$\sum_i d_ix_{ij} \le C_jy_j, \qquad \forall j \quad (14)$$

with

$$S_0(\mathbf{y},\mathbf{z}) = \{(7)-(9),(10)$$

$$\sum_i d_iw_{ij} \le C_jy_j, \qquad \forall j\} \quad (15)$$

Constraints (14) ensure that the total demand served by facility j does not exceed its capacity C_j. Constraints (15) impose the similar requirement on the survived facility j.

Consider a disruptive situation, when M is sufficiently large, demands will either be completely served; or if the remaining capacity is not sufficient, some of them will be lost and penalized with M per unit. So, we can derive some insights on RO-CPMP$_0$, although it has a more complicated structure. Specifically, for a given facility location $\mathbf{y}_0$ and a disruptive situation $\mathbf{z}$, since $\mathbf{x}$, $\mathbf{w}$, and $\mathbf{q}$ are continuous, we need to solve two linear programs to determine the operation costs $C_r(\mathbf{y}_0)$ and $C_{\mathbf{z}}(\mathbf{y}_0)$. It is easy to see that an optimal solution of the recourse problem of the disruptive situation is either a feasible solution to the first-stage problem or can be converted into a feasible solution by assigning the lost demand to facilities.

Lemma 3. *When M is sufficiently large, for a given facility solution $\mathbf{y}_0$ and a disruptive situation $\mathbf{z}$, if there is no demand loss due to disruptions, we have $C_{\mathbf{z}}(\mathbf{y}_0) \ge C_r(\mathbf{y}_0)$.*

Similarly, we have the following properties on $VC_0(p,k,\rho)$.

Proposition 3. *When M is sufficiently large, the function $VC_0(p,k,\rho)$ is (i) non-increasing with respect to p; (ii)non-decreasing with respect to k; and (iii) non-decreasing with respect to ρ;*

We can also extend our study with a little modification to model demand losses if a disrupted site does not generate demand. The formulation is

RO-CPMP$_l$

$$VC_l(p,k,\rho) = \min_{\mathbf{x},\mathbf{y}} (1-\rho)\sum_i\sum_j c_{ij}d_ix_{ij} + \rho\max_{\mathbf{z}\in A}\min_{(\mathbf{w},\mathbf{q})\in S_l(\mathbf{y},\mathbf{z})}(\sum_i\sum_j c_{ij}d_iw_{ij} + \sum_i Md_iq_i) \quad (16)$$

$$\text{s.t.} \quad (3)-(6),(14)$$

with

$$S_l(\mathbf{y},\mathbf{z}) = \{(7),(8),(10),(12),(15)\}$$

Because of the demand losses and facility capacity constraints, RO-CPMP$_l$ is less trackable than previously studied models. Nevertheless, under some mild assumptions, properties similar to those models can be derived.

Specifically, we assume that (i) $C_j \geq d_j$ for all j, and (ii) the service costs satisfy the triangular inequality. The first assumption indicates that in both normal and disruptive situations, it is feasible to serve the whole demand of a (survived) facility site by the facility itself. The second assumption shows that it always leads to less cost to serve the demand of a (survived) facility site by the facility on it. Finally, given that $c_{jj} = 0$, it can be proven, by the same line of Lemma 2, that a customer site disruption actually decreases the operation cost while the disruption of one facility site will lead to more operation cost. Hence, we have the following property similar to Lemma 2.

Lemma 4. *Under assumptions (i) and (ii), when M is sufficiently large, consider a given facility solution $\mathbf{y}^*$ and a disruption set A. We have that (i) the worst case disruptions and therefore demand losses happen only at facility sites, i.e., those with $y_j^* = 1$; (ii) $\hat{C}_A(\mathbf{y}^*) \geq C_r(\mathbf{y}^*)$ where $\hat{C}_A(\mathbf{y}^*)$ is the operating cost in the worst disruptive situation in A.*

Finally, we have

Proposition 4. *Under assumptions (i) and (ii), when M is sufficiently large, the function $VC_l(p,k,\rho)$ is (i) non-increasing with respect to p; (ii) non-decreasing with respect to k; and (iii) non-decreasing with respect to ρ.*

4 Solution algorithms

Two-stage RO models in general are very difficult to solve (Ben-Tal et al., 2004). When the second stage mitigation problem is a linear program (LP), as in each of the models we introduced so far, Benders decomposition method can be employed to derive optimal solutions (Bertsimas et al., 2012; Jiang et al., 2011). However, Benders method is not efficient in dealing with real size instances. A different solution method, the *column-and-constraint generation* algorithm, denoted by C&CG, was developed in Zeng and Zhao (2011) recently, which shows a superior performance over Benders method in solving practical problems. In this paper, we adopt C&CG method as the primary solution method to solve two-stage RO reliable p-median facility location models. We first provide details of a customized C&CG method for our robust models and then present a set of improvement strategies. We also briefly discuss the implementation of Benders decomposition method. Our computational study also confirm the efficiency of C&CG algorithm over Benders decomposition method.

4.1 Implementation of $C\&CG$ algorithm

We select RO-PMP$_l$ to describe the development of the customized C&CG algorithm. Because other three robust models are of similar structures, C&CG can be implemented with minor modifications to solve them.

C&CG algorithm is implemented within a two level master-sub problem framework. In the subproblem, for a given solution $(\mathbf{x}^*, \mathbf{y}^*)$ to the first stage decision problem, we solve the remaining *max-min* problem to identify the worst scenario. As the unsatisfied demand will be penalized in any disruptive situation, the second stage mitigation problem is always feasible. Hence, we can take the dual and obtain a *max-max* problem, which is actually a maximization problem. Specifically, let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{s}$ be the dual variables of the constraints (7), (8) and (12) respectively. The resulting nonlinear maximization formula of subproblem is as follows:

<u>NL-SP</u>

$$\max_{\mathbf{z},\mathbf{u},\mathbf{v},\mathbf{s}} \sum_i \sum_j (1 - z_j) u_{ij} + \sum_i \sum_j y_j^* v_{ij} + \sum_i (1 - z_i) s_i \tag{17}$$

$$\text{s.t. } u_{ij} + v_{ij} + s_i \le c_{ij} d_i, \qquad \forall i, j \tag{18}$$

$$s_i \le M d_i, \qquad \forall i \tag{19}$$

$$\sum_{j \in J} z_j \le k, \tag{20}$$

$$u_{ij} \le 0,\ \forall i, j;\ v_{ij} \le 0,\ \forall i, j; s_i \ge 0,\ \forall i; z_j \in \{0, 1\},\ \forall j \tag{21}$$

As the nonlinear terms are the products of a continuous variable and a binary variable, we can linearize this formulation by replacing them with two new variables, i.e., $U_{ij} = u_{ij} z_j$ and $S_i = s_i z_i$, and using *big-M* method. Given the penalty coefficient M in (2), the *big M* for U_{ij} and S_i can be set to Md_i. As a result, the linearized subproblem is:

<u>SP</u>

$$\max_{\mathbf{z},\mathbf{u},\mathbf{v},\mathbf{s},\mathbf{U},\mathbf{S}} \sum_i \sum_j (u_{ij} - U_{ij} + y_j^* v_{ij}) + \sum_i (s_i - S_i) \tag{22}$$

$$\text{s.t. } u_{ij} + v_{ij} + s_i \le c_{ij} d_i, \qquad \forall i, j \tag{23}$$

$$s_i \le M d_i, \qquad \forall i \tag{24}$$

$$U_{ij} \ge u_{ij}, \qquad \forall i, j \tag{25}$$

$$U_{ij} \ge -M d_i z_j, \qquad \forall i, j \tag{26}$$

$$U_{ij} \le u_{ij} + M d_i (1 - z_j), \qquad \forall i, j \tag{27}$$

$$S_i \le s_i, \qquad \forall i \tag{28}$$

$$S_i \le M d_i z_i, \qquad \forall i \tag{29}$$

$$S_i \ge s_i + M d_i (z_i - 1), \qquad \forall i \tag{30}$$

$$\sum_{j \in J} z_j \le k, \tag{31}$$

$$\begin{aligned} &u_{ij} \le 0,\ \forall i, j; U_{ij} \le 0,\ \forall i, j; v_{ij} \le 0,\ \forall i, j; \\ &s_i \ge 0,\ \forall i; S_i \ge 0,\ \forall i; z_j \in \{0, 1\},\ \forall j \end{aligned} \tag{32}$$

Note that the linearized subproblem (SP), which is a MIP problem, can be solved by a professional MIP solver. Next, we describe the details of the *column-and-constraint generation algorithm* along with the formulation of the *master problem*, which will be solved iteratively. In each iteration n, a significant scenario $\mathbf{z}^n$ will be identified through solving the SP. Then, a set of recourse variables $(\mathbf{w}^n, \mathbf{q}^n)$ and corresponding constraints in the forms of (34) and (35)-(37) associated with this particular scenario will be created and added to the *master problem*. Let UB and LB be the upper and lower bounds respectively, *Gap* be the relative gap between UB and LB, n be the iteration index and ϵ be the optimality tolerance.

Column-and-constraint generation algorithm

1. Set $LB = -\infty$, $UB = \infty$, and $n = 0$.

2. Solve the following *master problem* (MP) and obtain an optimal solution $(\mathbf{x}^n, \mathbf{y}^n, \eta^n)$ (a feasible solution if unbounded) and set LB to the optimal value of the MP.
   MP

$$\min\ (1-\rho)\sum_i\sum_j c_{ij}d_i x_{ij} + \rho\eta \tag{33}$$

$$\text{s.t.}\quad (3)-(6)$$

$$\eta \geq \sum_i\sum_j c_{ij}d_i w_{ij}^l + \sum_i M d_i q_i^l, \qquad \forall l = 1,2,...,n \tag{34}$$

$$\sum_j w_{ij}^l \geq 1 - q_i^l - z_i^l, \qquad \forall i, l = 1,2,...,n \tag{35}$$

$$w_{ij}^l \leq 1 - z_j^l, \qquad \forall i,j,l = 1,2,...,n \tag{36}$$

$$w_{ij}^l \leq y_j, \qquad \forall i,j,l = 1,2,...,n \tag{37}$$

$$q_i^l \geq 0,\ \forall i, l = 1,2,...,n;\ w_{ij}^l \geq 0,\ \forall i,j,l = 1,2,...,n. \tag{38}$$

3. Solve SP with respect to $(\mathbf{x}^n, \mathbf{y}^n)$ and derive an optimal solution $(\mathbf{z}^n, \mathbf{u}^n, \mathbf{v}^n, \mathbf{s}^n)$ and its optimal value $\mathcal{R}^n$. Update

$$UB = min\{UB, (1-\rho)\sum_i\sum_j c_{ij}d_i x_{ij}^n + \rho\mathcal{R}^n\}.$$

4. If $Gap = \frac{UB-LB}{LB} \leq \epsilon$, an ϵ-optimal solution is found, and terminate. Otherwise, create recourse variables $(\mathbf{w}^n, \mathbf{q}^n)$ and corresponding constraints associated with $\mathbf{z}^n$ and add them to *MP*. Update $n = n + 1$. Go to Step 2. □

It has been proven in Zeng and Zhao (2011) that C&CG algorithm converges to an optimal solution in finite iterations. Different from C&CG method, after solving SP, Benders decomposition method will iteratively supply a single cutting plane in the following form to its master problem that only carries the first stage decision variables $(\mathbf{x}, \mathbf{y})$

$$\eta \geq \sum_i\sum_j (1-z_j^n)u_{ij}^n + \sum_i\sum_j v_{ij}^n y_j + \sum_i (1-z_i^n)s_i^n$$

Comparing these two types of algorithms, Zeng and Zhao (2011) theoretically show that C&CG method is of a much less computational complexity and its generated constraints are stronger.

4.2 Algorithm improvement

In this section, we study how to improve the computational performance of C&CG method on solving reliable p-median facility location problems. In particular, note that the numbers of variables and constraints in MP will quickly increase over iterations. So, solving MP to optimality may take excessive amount of time for large instances. To address this challenge, a few strategies could be applied to reduce the computational expenses on solving MP.

Adding valid inequalities to MP. Two types of valid inequalities are generated. The first type is generally applicable for any implementation of C&CG. That is, let t denote the objective value of the current MP. Then, the following constraint

$$t \geq LB \tag{39}$$

is valid and can be added to MP to speed up the branch-and-bound procedure of the solver. The second type is specific to our reliable facility location problems. Based on Proposition 1-4, it can be seen that, when M is sufficiently large, t can be bounded from below by the optimal values of instances with smaller k and ρ. Moreover, instances with smaller ρ and k are easier to compute. Therefore, constraint similar to (39) can be added to MP once the optimal value of an instance with smaller ρ or k is available.

Passing significant disruptive scenarios. Similarly, as instances with smaller ρ are easier to compute, we can first solve such an instance (with a small ρ) to obtained a set of disruptive scenarios. Those scenarios, i.e., the corresponding recourse variables and constraints, can be supplied to the MP of an instance with a larger ρ. Intuitively speaking, those scenarios, which yield the most disruptive cost, should also be significant even when ρ is larger. Hence, C&CG algorithm can avoid starting an MP from scratch, less iterations in C&CG can be expected. Nevertheless, we note that it also has a negative effect as MP will be larger and its computational time could be longer.

Obtaining good solutions before convergence. Note that MP gradually evolves into a large and computationally intensive mixed integer program while any of its feasible solutions can be used to generate valid disruptive scenarios. So, when *Gap* is large, it is not necessary to derive an optimal solution of MP and a good feasible solution could be sufficient to generate significant disruptive scenarios. When we employ a solver to compute, we can seek a balance between the solution quality and the computational time by dynamically changing its optimality tolerance. When *Gap* is large at the beginning, we can set a relatively larger optimality tolerance for a good solution to MP. As *Gap* becomes smaller, a smaller optimality tolerance will be adopted for a better solution and a more precise lower bound.

5 Numerical study and analysis

In this section, we first describe data and experimental setup. Then, we provide results of a set of numerical experiments and present our insights on various reliable p-median models.

All of our experiments are performed on the 49-node data set described in Snyder and Daskin (2005), which includes information of demands and site coordinates. We also consider a data set of 25 nodes that are randomly selected from the 49-node data set as shown in Table 1.

In the study, c_{ij} is the Euclidean distance between node i and j obtained from site coordinates. For capacitated models, the capacity of each site is randomly generated between $[D/10, 3D/10]$ where D is the total demand of all nodes. Whenever the random site capacity is smaller than its demand, we set the value of capacity equal to the demand. For all problems with 25 nodes and 49 nodes, we test them with different parameter values, i.e., $\rho = 0.2, 0.4$, $P = 8, 10, 12$, and $k = 1, 2, 3$, totally 36 instances for each model. We consider the penalty coefficient M equal to 15 and $\max_{i,j}\{c_{ij}\}$. The first value resembles a situation where an affected demand will be served by competitors if the service cost of using survived facilities is more than 15. The second value represents a situation where all demands must be served (if capacity is sufficient) in any disruptive scenario.

C&CG algorithm is our primary solution method and we apply it to solve each type of problems. For the comparison purpose, we also implement Benders decomposition method (BD) and benchmark it with C&CG on the RO-PMP$_l$ model with $|I| = 25$ and $M = 15$. For all instances, the optimality tolerance $\epsilon = 0.1\%$ and time limit is 7200s. The master problems and the subproblems are solved by a mixed integer programming solver, CPLEX 12.1. All algorithms are implemented in C++ and tested on a Dell Optiplex 760 desktop computer (Intel Core 2 Duo CPU, 3.0GHz, 3.25GB of RAM) in Windows XP environment.

5.1 Algorithm Performance

Table 2 presents the performance of the BD methods on instances of RO-PMP$_l$. Table 3 - 6 summarize the computational results of C&CG algorithm on the four reliable models. In those tables, the column *Time(s)* presents the computational time in seconds; the column *Iter* indicates the number of iterations; the column *Obj* shows the best objective value ever found; the column *Gap*(%) provides the relative gap in percentage if it is larger than ϵ. If an instance can not be solved due to extensive computation time or lack of memory, we use T or M, respectively, to indicate the reason in the *Time(s)* column.

Based on those tables, we observe that

(*i*) C&CG algorithm performs hundreds of times faster and takes much fewer iterations than the classical Benders decomposition method. This result confirms the observations made in Zhao and Zeng (2010) for the robust power system scheduling problem and Zeng and Zhao (2011) for the location-transportation network design problem. Actually, compared to results in Zhao and Zeng (2010) and Zeng and Zhao (2011), a more significant superiority of C&CG algorithm is demonstrated in solving reliable p-median problems.

(*ii*) The computation complexity of C&CG algorithm increases with the problem size $|I|$ and k, as well as the weight coefficient ρ. In all four types of models, the most challenging instances are those with largest $|I|$, k, and ρ. Note that all instances with $k = 1$ are easy to compute. Most small size instances with $|I| = 25$ can be solved to optimality or with a small optimality gap while some instances with $|I| = 49$ are difficult. A closer analysis shows that SP can be computed easily and the actual bottleneck is to solve MP, which will grow into a large MIP problem over iterations. As CPLEX, a general-purpose MIP solver, is currently called to solve MP, one possible direction of future research is to develop a specialized algorithm that takes advantage of the structure of MP for a faster computation.

(*iii*) Including additional features does not incur significant computational expense. Compared with models without capacity restrictions or demand losses, capacitated ones are slightly harder while models with demand losses could be easier. Hence, our two-stage RO formulations of reliable p-median problems are computationally robust to additional features or restrictions.

(*iv*) Although for many instances the optimal objective values are the same for the different M values, the large penalty coefficient M generally negatively impacts the computational performance, which is more significant for instances in capacitated models. One explanation is that large penalty coefficient M forces demand that was served by a disrupted facility to be served by survived ones, instead of being simply treated as unmet demand. As a result, the optimization complexity increases.

5.2 Impact of the reliability

In this section, we investigate effect of the inclusion of the worst disruptive scenarios on the system configuration and operations. Specifically, for different ρ values, after deriving an optimal solution, we compute the corresponding operation costs under the normal situation (NOC) and under the worst disruptive scenario (WOC). Then, we plot those operation costs with respect to ρ. It is obvious that the classical p-median model can be obtained by setting ρ to 0 and the formulation to minimize only the worst case cost can be obtained by setting ρ to 1. Figure 1 and Figure 2 present our results for 25-node models ($M = 15$). Note that the weighted objective function values are also included.

Clearly, the two cost functions demonstrate a monotone property, or a "staircase pattern", over ρ. The pattern is more obvious in capacitated models. It can be seen that within a single stair the optimal system configurations are the same for different ρ values. Actually, the small number of stairs implies that the optimal system configuration based on two-stage RO is not very sensitive to different ρ. However, when ρ keeps increasing, WOC would decrease while NOC would increase. Sometimes a slight increase in NOC will lead to a significant decrease in WOC. Such a phenomenon is also observed in the stochastic programming based reliable facility location models (Snyder and Daskin, 2005). Overall, a desired trade-off between NOC and WOC can be achieved by selecting a configuration of an appropriate stair. Another observation is that the WOCs and hence the objective values of the models with demand losses are much smaller than those of the models without considering demand losses. It is reasonable since in RO-PMP$_l$/RO-CPMP$_l$, under a disruptive scenario, the demands of disrupted clients will not incur any cost, which counteracts the cost increase due to failed facilities.

5.3 Effect of demand losses

As we mentioned earlier, demand losses due to disruptions have not been included or investigated in any existing reliable facility location models. So, it remains unknown that how demand losses will affect system design and operations, or how approximate the results we have if we ignore the demand loss factor when it does exist. To explore the impact of demand losses, Table 7 - 8 present optimal configurations of the models considering and ignoring the demand loss factor (M=15). The column OL represents the optimal locations of facilities and the columns NOC and WOC denote the normal operation cost and the worst case operation cost corresponding to the optimal facility locations of the models with demand losses. We also insert the configuration derived from the model $RO\text{-}PMP_0/RO\text{-}CPMP_0$ into $RO\text{-}PMP_l/RO\text{-}CPMP_l$ and compute the operation costs in normal and in the worst disruptive situation, denoted by $I\text{-}NOC$ and $I\text{-}WOC$. Their relative changes with respect to NOC and WOC are denoted by $\Delta_{NOC}(\%)$ and $\Delta_{WOC}(\%)$. For a case, if the optimal facility locations are different for the models with and without demand losses, we will highlight the locations by underlines.

We observe that the demand loss plays a significant role in determining system configuration. In all 36 instances, including uncapacitated and capacitated ones, there are 18 instances where optimal facility locations are different from those derived from models ignoring demand losses. In fact, when we put more weight on the worst disruptive situations, the impact of demand losses becomes more significant. For example, when $\rho = 0.4$, there are 11 instances (out of 18 ones) on which the models with and without demand losses yield different solutions. Figure 3 shows different optimal facility locations of two capacitated models with $|I| = 25$, $\rho = 0.4$, $p = 8$, and $k = 2$. If we consider demand losses, the facilities originally located in Boton Rouge and Lincoln will move to St. Paul and Denver, which are quite different in terms of geographical positions. Furthermore, once the facility locations are different, they present very different performances in both normal and the worst disruptive situations (in the environment with demand losses). From the column $\Delta_{NOC}(\%)$, we note that the system configuration derived without demand losses could incur more or less cost in the normal situation, which can hardly be predicted beforehand. In fact, the difference can be as low as -14.92% or as high as 18.97%, definitely a non-trivial value. Nevertheless, in the worst disruptive situation, it is generally observed that the former system configuration will incur much more operation cost, which can be up to 34.97% in a capacitated instance. Therefore, we can conclude that the demand loss factor, if it exists in the application, should not be ignored in system design, especially when the weight coefficient ρ is large and capacity needs to be considered.

5.4 A correlated disruption set

The disruption set with a simple cardinality restriction in equation (1) indicates that all nodes are of the identical failure possibility and there is little correlation among them. In this section, given the adaptability of our modeling framework, we investigate a different disruption set that carries some correlations. Specifically, we partition the node set into a few subsets and assume that nodes in each subset are temporally or spatially correlated. Hence, the number of disruptions in each subset can be better estimated. Also, we have an overall budget constraint to manage the total number of disruptions. The description of the disruption set takes the following form:

$$A_1 = \{\mathbf{z} \in \{0,1\}^{|J|} : \sum_{j \in J_1} z_j \leq k_1, \sum_{j \in J_2} z_j \leq k_2, ..., \sum_{j \in J_L} z_j \leq k_L, \sum_{l=1}^{L} \sum_{j \in J_l} a_j^l z_j \leq b\}. \tag{40}$$

In our numerical study, we let $L = 2$, and nodes are randomly assigned to J_1 and J_2, $k_1 = 2$ and $k_2 = 1$, a_j^1 takes the value of 10 for all $j \in J_1$ and $a_j^2 = 15$ for all $j \in J_2$. Experiments are performed with $n = 25, 49$, $\rho = 0.2, 0.4$, $p = 8, 10, 12$, $b = 15, 30, 40$, and $M = 15$. We mention that it is rather a simple set just for the demonstration of the impact of correlation. The computational performance of C&CG algorithm is presented in Table 9 and Table 10.

Note that with $b = 15, 30$, and 40 in A_1, the number of disruptions over the entire node set can be $1, 2$ and 3 respectively, which resembles the set we study in (1) with $k = 1, 2$ and 3. Nevertheless, comparing Table 9 with Table 3, and Table 10 with Table 5, we observe that: (i) the algorithm performance is generally faster in the correlated set with less iterations; (*ii*) the objective function value, i.e., the weighted operation cost, is often smaller. Both points can be explained by the fact that the disruption set in (40) is a tighter and more precise description of all kinds of disruptive scenarios, if correlations exist; and A_1 is a subset of that defined in (1) with an appropriate k. So, with more structural information available, both the master and subproblems are easier or smaller to compute than those with the cardinality set (1). Clearly, the cardinality set (1) is an overestimation of disruptions if disruptions actually happen according to the pattern defined in (40). Hence, in terms of system design and operations, the cardinality set (1) could lead to a more conservative system configuration with a higher objective function value as it is overprotective towards some *unrealistic* disruptive situations. So, with a proper description of the correlation, a less conservative system design can be achieved with the desired reliability level.

6 Conclusion

In this paper, we propose two-stage robust optimization based models for the reliable p-median facility location problem. In particular, we demonstrate the strong modeling capability of two-stage robust optimization framework by considering two practical features, i.e., facility capacity and demand loss (due to disruptions), in a compact fashion, which otherwise would demand for complicated stochastic programming formulations and have received little attention. We study those models and develop exact computing algorithms, i.e., column-and-constraint generation and Bender decomposition methods, to solve them. From a set of computational experiments, we note that (*i*) the column-and-constraint generation algorithm drastically outperforms the other method. Instances with up to 49 nodes, including those with demand losses and capacities, can be solved exactly or with a reasonable gap; (*ii*) by assigning an appropriate weight to the operation cost from the worst disruptive situations in the objective function, a considerable decrease of that cost could be achieved by a small increase in the operation cost in the normal situation; (*iii*) demand losses due to disruptions should not be ignored in system design. Otherwise, a different network configuration could result in a huge increase of the operation cost in the worst disruptive situation; (*iv*) a description of disruption correlations, even in a simple form, makes those models less challenging and leads to less conservative system designs with the desired reliability.

A clear direction of the future research is to explore the problem structure to enhance the column-and-constraint generation algorithm for larger scale instances. So, a customized procedure can be developed to replace the professional solver for a better performance. Another direction is to study other practical network design problems, e.g., the fixed charge facility location problem, to provide decision support tools for practitioners.

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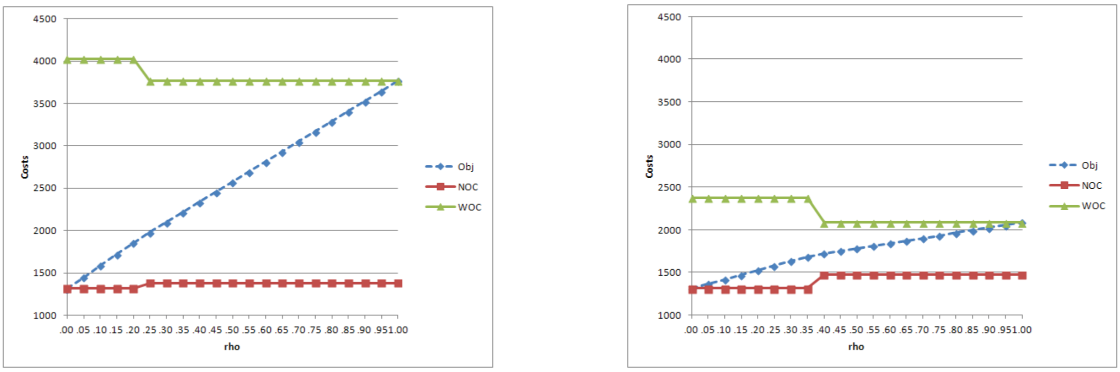
Table 1: Cities in the 25-node data set

No.	City	No.	City	No.	City	No.	City
0	Austin(TX)	7	St. Paul(MD)	14	Topeka(KS)	21	Pierre(SD)
1	Tallahassee(FL)	8	Baton Rouge(LA)	15	Charleston(WV)	22	Dover(DE)
2	Harrisburg(PA)	9	Frankfort(KY)	16	Salt Lake City(UT)	23	Washington(DC)
3	Columbus(OH)	10	Columbia(SC)	17	Lincoln(NE)	24	Montpelier(VT)
4	Richmond(VA)	11	Denver(CO)	18	Augusta(ME)		
5	Boston(MA)	12	Hartford(CT)	19	Boise City(ID)		
6	Annapolis(MD)	13	Des Moines(IA)	20	Helena(MT)		

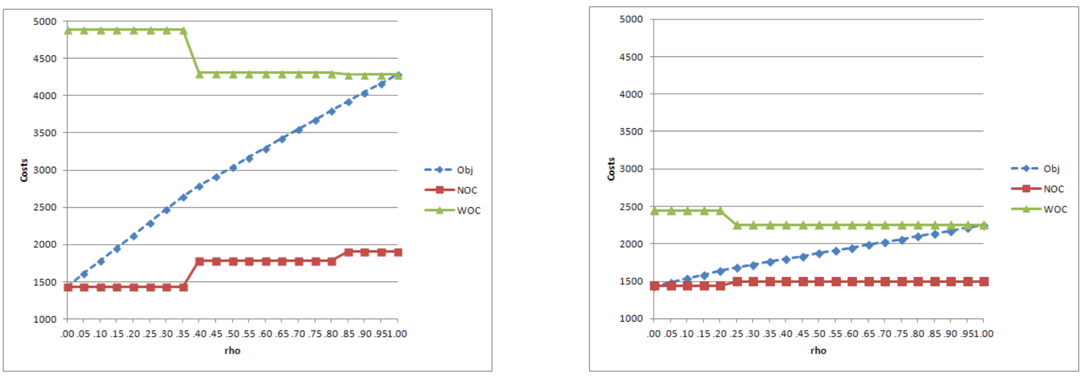
Table 2: Computational performance of the Benders decomposition method

			$\rho = 0.2$				$\rho = 0.4$			
$\|I\|$	p	k	*Time(s)*	*Iter*	*Obj*	*Gap(%)*	*Time(s)*	*Iter*	*Obj*	*Gap(%)*
25	8	1	M	2194	1426.76	1.50	M	2262	1539.77	31.39
		2	M	2259	1525.92	7.70	M	2257	1721.44	38.64
		3	M	2259	1649.93	14.64	M	2262	1821.58	42.00
	10	1	M	2259	1024.11	6.81	M	2259	1067.20	32.93
		2	M	2188	1066.29	10.67	M	2188	1151.57	37.97
		3	M	2257	1119.92	14.79	M	2259	1292.44	44.62
	12	1	555.7	732	622.54		M	2187	658.45	24.06
		2	3035.1	1614	653.32		M	2255	720.02	30.42
		3	M	1933	745.10	11.24	M	2257	893.52	43.93



(a) Cost curves of the RO-PMP$_0$ model (b) Cost curves of the RO-PMP$_l$ model

Figure 1: Effect of ρ value on the robust uncapacitated facility location models



(a) Cost curves of the RO-CPMP$_0$ model (b) Cost curves of the RO-CPMP$_l$ model

Figure 2: Effect of ρ value on the robust capacitated facility location models

Table 3: Computational results for uncapacitated instances with $M = 15$

$\lvert I \rvert$	ρ	p	k	RO-PMP$_0$ *Time(s)*	*Iter*	*Obj*	*Gap(%)*	RO-PMP$_l$ *Time(s)*	*Iter*	*Obj*	*Gap(%)*
25	0.2	8	1	1.9	5	1558.09		1.9	5	1426.76	
			2	4.8	8	1855.51		4.2	7	1525.92	
			3	T	58	2292.82	0.46	27.3	13	1649.93	
		10	1	0.8	2	1139.08		3.4	7	1024.11	
			2	2.0	6	1374.09		10.9	10	1066.29	
			3	16.4	11	1601.93		6.9	8	1119.92	
		12	1	0.7	2	811.73		0.5	2	622.54	
			2	0.7	3	973.46		0.7	2	653.32	
			3	1.5	3	1110.90		6.1	9	745.10	
	0.4	8	1	1.9	5	1802.45		5.0	8	1539.77	
			2	30.3	15	2335.00		40.0	14	1721.44	
			3	T	84	3194.56	13.69	80.1	17	1821.58	
		10	1	0.6	2	1364.19		5.2	9	1067.20	
			2	9.8	10	1785.45		13.4	11	1151.57	
			3	181.9	23	2235.93		61.8	15	1292.44	
		12	1	0.7	2	1036.84		1.8	3	658.45	
			2	1.4	3	1360.29		0.6	3	720.02	
			3	0.7	3	1635.19		139.1	22	893.52	
49	0.2	8	1	9.1	5	6145.54		4.2	2	5684.45	
			2	187.2	10	6808.88		244.9	10	6020.47	
			3	T	35	7471.69	9.98	T	29	6364.45	3.26
		10	1	3.3	3	4950.32		34.7	6	4687.15	
			2	617.3	16	5534.76		1050.1	14	4982.95	
			3	T	27	6106.86	4.80	T	22	5131.20	0.64
		12	1	4.6	3	4181.94		3.2	3	3830.82	
			2	402.9	11	4582.71		56.4	9	4022.75	
			3	7174.4	25	5011.58		T	20	4217.79	0.27
	0.4	8	1	15.5	6	6625.50		32.5	6	5898.97	
			2	T	28	8326.04	7.60	T	26	6534.28	2.55
			3	T	36	9344.74	21.74	T	34	7215.81	13.62
		10	1	11.4	5	5351.47		34.5	7	4796.70	
			2	T	35	6815.54	12.80	T	24	5298.70	1.44
			3	T	37	7520.25	15.49	T	34	5827.05	11.91
		12	1	48.8	8	4583.09		7.9	4	3920.28	
			2	T	26	5397.18	2.35	2577.6	18	4239.83	
			3	T	39	6249.57	12.98	T	33	4694.21	9.68

Table 4: Computational results for uncapacitated instances with $M = \max_{i,j}\{c_{ij}\}$

$\|I\|$	ρ	p	k	RO-PMP$_0$ *Time(s)*	*Iter*	*Obj*	*Gap(%)*	RO-PMP$_l$ *Time(s)*	*Iter*	*Obj*	*Gap(%)*
25	0.2	8	1	2.9	5	1558.09		2.8	5	1426.76	
			2	5.6	7	1855.51		4.3	7	1525.92	
			3	T	54	2412.61	4.78	129.6	19	1734.64	
		10	1	0.6	2	1139.08		6.8	9	1026.99	
			2	3.8	6	1374.09		10.7	10	1083.22	
			3	30.5	12	1616.81		10.5	9	1119.92	
		12	1	0.8	2	811.73		0.8	2	622.54	
			2	1.4	3	973.46		1.8	2	653.32	
			3	1.6	3	1110.90		7.8	9	745.10	
	0.4	8	1	2.8	5	1802.45		4.7	7	1539.77	
			2	31.1	15	2335.00		82.6	17	1738.11	
			3	T	87	3414.34	18.51	1949.8	34	2010.03	
		10	1	0.8	2	1364.19		18.7	12	1094.38	
			2	7.6	8	1785.45		31.9	13	1183.81	
			3	409.4	27	2252.60		116.1	17	1292.44	
		12	1	0.6	2	1036.84		0.7	3	658.45	
			2	1.7	3	1360.29		1.7	3	720.02	
			3	2.9	4	1635.19		162.3	22	893.52	
49	0.2	8	1	14.5	5	6149.60		3.7	3	5686.38	
			2	T	30	7599.45	12.35	283.4	10	6026.61	
			3	T	36	8602.54	13.64	T	28	6432.67	4.05
		10	1	5.5	3	4950.32		35.6	7	4687.15	
			2	1027.1	15	5534.76		1581.2	16	4992.81	
			3	T	38	6748.05	17.17	T	22	5131.20	1.10
		12	1	5.5	3	4181.94		4.6	3	3830.82	
			2	377.8	10	4582.71		62.8	9	4022.75	
			3	T	25	5119.71	3.24	1458.1	14	4217.79	
	0.4	8	1	22.1	6	6633.61		11.3	5	5902.83	
			2	T	39	9502.10	22.51	T	23	6583.29	3.05
			3	T	37	10991.60	22.10	T	33	7474.85	17.14
		10	1	14.6	5	5351.47		22.4	6	4796.70	
			2	T	26	6365.19	3.74	T	29	5461.92	5.16
			3	T	40	8527.85	23.76	T	31	5888.55	14.73
		12	1	49.7	8	4583.09		15.7	5	3920.28	
			2	T	26	5397.18	2.42	1848.7	17	4239.83	
			3	T	33	6539.20	16.90	T	33	4694.21	9.47

Table 5: Computational results for capacitated instances with $M = 15$

$\|I\|$	ρ	p	k	RO-CPMP$_0$ *Time(s)*	*Iter*	*Obj*	*Gap(%)*	RO-CPMP$_l$ *Time(s)*	*Iter*	*Obj*	*Gap(%)*
25	0.2	8	1	2.9	4	1783.39		2.8	4	1550.58	
			2	10	6	2126.51		8.5	8	1638.56	
			3	38.7	10	2466.17		9.7	8	1725.95	
		10	1	0.5	2	1148.48		1.4	5	1036.93	
			2	1.7	3	1383.48		4.6	6	1094.58	
			3	2.9	6	1642.20		10.8	8	1180.54	
		12	1	0.7	2	821.12		0.7	2	642.54	
			2	1.9	3	982.85		2.5	5	715.66	
			3	8.7	8	1266.38		3.9	6	756.96	
	0.4	8	1	2.9	4	2024.04		2.5	6	1623.29	
			2	75.9	13	2791.94		23.3	10	1799.26	
			3	4172	35	3495.95		175.1	18	2015.52	
		10	1	0.8	2	1373.58		8.7	8	1083.45	
			2	1.2	3	1843.59		21.3	10	1224.17	
			3	16.6	8	2307.08		782.4	25	1403.52	
		12	1	0.4	2	1046.23		1.5	3	689.06	
			2	1.8	3	1369.69		27.6	12	835.30	
			3	37.7	16	1828.23		60.8	14	895.72	
49	0.2	8	1	10.6	4	6432.77		79.3	6	6180.25	
			2	T	26	7290.78	10.90	T	19	6525.89	2.01
			3	T	27	8302.32	19.43	T	22	7134.59	8.75
		10	1	17.2	5	5045.10		23.8	6	4778.10	
			2	T	23	5752.22	3.06	1262.5	14	5097.14	
			3	T	20	6187.69	6.55	T	19	5250.64	0.76
		12	1	8.3	4	4228.04		4.7	3	3920.98	
			2	T	18	4797.85	0.76	71.8	8	4104.87	
			3	T	36	5235.33	8.09	T	21	4379.91	1.60
	0.4	8	1	131.5	8	6939.44		502.5	11	6426.95	
			2	T	31	8677.54	17.51	T	25	7388.72	10.00
			3	T	34	10205.90	23.84	T	34	8291.22	17.83
		10	1	55.9	7	5434.73		34.9	7	4881.02	
			2	T	34	6888.70	14.20	T	24	5593.89	6.08
			3	T	25	7759.64	15.83	T	24	5885.55	10.76
		12	1	35.0	7	4617.67		9.4	4	4018.48	
			2	T	23	5399.02	2.68	2794.1	15	4321.95	
			3	T	41	6647.17	21.97	T	32	4906.77	12.87

Table 6: Computational results for capacitated instances with $M = \max_{i,j}\{c_{ij}\}$

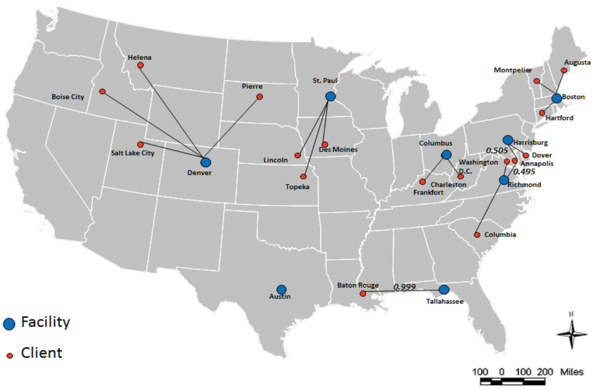
$\|I\|$	ρ	p	k	RO-CPMP$_0$				RO-CPMP$_l$			
				Time(s)	*Iter*	*Obj*	*Gap(%)*	*Time(s)*	*Iter*	*Obj*	*Gap(%)*
25	0.2	8	1	2.9	4	1783.39		3.1	5	1550.58	
			2	3.5	4	2126.51		58.7	12	1752.15	
			3	949.7	22	2772.17		261.8	16	1880.17	
		10	1	0.7	2	1148.48		3.6	6	1037.56	
			2	1.8	3	1383.48		4.1	6	1094.58	
			3	33.8	10	1726.22		63.7	13	1226.52	
		12	1	0.5	2	821.12		0.6	2	642.54	
			2	1.6	3	982.85		2.7	5	715.66	
			3	49.7	12	1282.01		5.5	6	761.97	
	0.4	8	1	2.5	4	2024.04		4.6	7	1656.87	
			2	85.4	13	2816.64		1043.6	25	2024.80	
			3	T	43	4066.84	13.32	T	33	2263.44	1.80
		10	1	0.8	2	1373.58		14.7	9	1103.78	
			2	1.5	3	1843.59		25.7	10	1224.17	
			3	107.4	15	2425.86		T	43	1529.67	2.30
		12	1	1.9	2	1046.23		1.5	3	689.06	
			2	0.8	3	1369.69		34.3	12	835.30	
			3	31.1	14	1840.08		102.9	16	895.72	
49	0.2	8	1	89.7	8	6517.80		304.6	8	6197.10	
			2	T	25	8627.90	17.62	T	22	6585.34	2.89
			3	T	36	11052.60	35.79	T	31	8191.92	22.44
		10	1	28.8	5	5045.10		44.6	6	4794.66	
			2	T	27	6026.77	8.64	1723.4	14	5097.14	
			3	T	31	7255.46	21.16	T	21	5292.58	3.29
		12	1	9.9	4	4228.04		37.1	5	3942.97	
			2	T	17	4797.85	0.76	1183.3	14	4160.83	
			3	T	36	5622.94	16.49	T	20	4391.66	2.58
	0.4	8	1	1053.1	11	7086.20		1164.2	11	6475.30	
			2	T	29	10323.50	23.98	T	29	7232.63	0.97
			3	T	33	15124.70	47.13	T	32	9137.95	9.36
		10	1	55.7	6	5434.73		88.5	8	4914.15	
			2	T	32	7416.03	19.57	T	23	5593.89	5.38
			3	T	39	10004.60	25.56	T	28	5969.43	13.27
		12	1	49.3	7	4617.67		24.0	5	4062.45	
			2	T	27	5572.89	7.27	T	20	4469.23	3.37
			3	T	42	7066.55	22.88	T	28	4959.84	14.41

Table 7: Comparison between RO-PMP$_l$ and RO-PMP$_0$ models ($|I| = 25$)

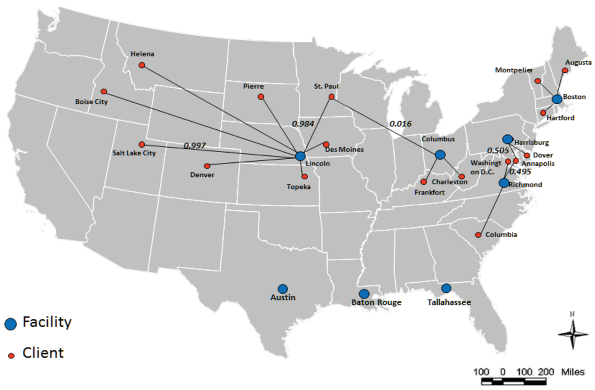
			RO-PMP$_l$			RO-PMP$_0$				
ρ	p	k	*OL*	*NOC*	*WOC*	*OL*	*I-NOC*	*I-WOC*	Δ_{NOC}(%)	Δ_{WOC}(%)
0.2	8	1	0 1 2 3 5 8 11 13	1313.74	1878.84	0 1 2 3 5 8 11 13	1313.74	1878.84	0.00	0.00
		2	0 1 2 3 5 8 11 13	1313.74	2374.64	0 1 2 3 5 8 11 13	1313.74	2374.64	0.00	0.00
		3	0 1 2 3 4 5 7 11	1478.28	2336.53	0 1 2 3 4 5 11 13	1417.77	2659.42	-4.09	13.82
	10	1	0 1 2 3 4 5 7 8 11 14	981.02	1196.49	0 1 2 3 4 5 8 11 13 16	913.98	1479.05	-6.83	23.62
		2	0 1 2 3 4 5 7 8 11 14	981.02	1407.39	0 1 2 3 4 5 8 11 13 16	913.98	1764.15	-6.83	25.35
		3	0 1 2 3 4 5 8 11 13 16	913.98	1943.70	0 1 2 3 4 5 8 10 11 13	967.93	2132.01	5.90	9.69
	12	1	0 1 2 3 4 5 7 8 10 11 14 16	586.62	766.19	0 1 2 3 4 5 7 8 10 11 14 16	586.62	766.19	0.00	0.00
		2	0 1 2 3 4 5 7 8 10 11 14 16	586.62	920.11	0 1 2 3 4 5 7 8 10 11 14 16	586.62	920.11	0.00	0.00
		3	0 1 2 3 4 5 7 8 10 11 14 16	586.62	1378.98	0 1 2 3 4 5 7 8 10 11 14 16	586.62	1378.98	0.00	0.00
0.4	8	1	0 1 2 3 5 8 11 13	1313.74	1878.82	0 1 2 3 5 8 11 13	1313.74	1878.82	0.00	0.00
		2	0 1 2 3 4 5 7 11	1478.28	2086.18	0 1 2 3 8 11 12 13	1380.83	2696.66	-6.59	29.26
		3	0 1 2 3 4 5 7 11	1478.28	2336.53	0 1 3 5 6 10 11 17	1546.41	2983.21	4.61	27.68
	10	1	0 1 2 3 4 5 7 8 11 14	981.02	1196.48	0 1 2 3 4 5 8 11 13 16	913.98	1479.04	-6.83	23.62
		2	0 1 2 3 4 5 7 8 11 14	981.02	1407.40	0 1 2 3 4 5 7 8 11 14	981.02	1407.40	0.00	0.00
		3	0 1 2 3 4 5 7 8 11 16	974.49	1769.37	0 1 2 3 4 5 8 10 11 13	967.93	2132.02	-0.67	20.50
	12	1	0 1 2 3 4 5 7 8 10 11 14 16	586.62	766.19	0 1 2 3 4 5 7 8 10 11 14 16	586.62	766.19	0.00	0.00
		2	0 1 2 3 4 5 7 8 10 11 14 16	586.62	920.11	0 1 2 3 4 5 7 8 10 11 14 16	586.62	920.11	0.00	0.00
		3	0 1 2 3 4 5 6 7 8 11 13 16	689.51	1199.54	0 1 2 3 4 5 7 8 10 11 14 16	586.62	1378.98	-14.92	14.96

Table 8: Comparison between RO-CPMP$_l$ and RO-CPMP$_0$ models ($|I| = 25$)

			RO-CPMP$_l$			RO-CPMP$_0$				
ρ	p	k	*OL*	*NOC*	*WOC*	*OL*	*I-NOC*	*I-WOC*	Δ_{NOC}(%)	Δ_{WOC}(%)
0.2	8	1	0 1 2 3 4 5 11 13	1436.38	2007.38	0 1 3 4 5 8 11 13	1518.49	2162.64	5.72	7.73
		2	0 1 2 3 4 5 11 13	1436.38	2447.28	0 1 2 3 4 5 11 13	1436.38	2447.28	0.00	0.00
		3	0 1 2 3 4 5 11 13	1436.38	2884.23	0 1 2 3 4 5 11 13	1436.38	2884.23	0.00	0.00
	10	1	0 1 2 3 4 5 7 8 11 14	990.41	1223.01	0 1 2 3 4 5 8 11 13 16	923.37	1494.32	-6.77	22.18
		2	0 1 2 3 4 5 8 11 13 16	923.37	1779.42	0 1 2 3 4 5 8 11 13 16	923.37	1779.42	0.00	0.00
		3	0 1 2 3 4 5 8 11 13 16	923.37	2209.22	0 1 2 3 4 5 8 10 11 13	977.33	2183.93	5.84	-1.14
	12	1	0 1 2 3 4 5 7 8 10 11 14 16	596.02	828.62	0 1 2 3 4 5 7 8 10 11 14 16	596.02	828.62	0.00	0.00
		2	0 1 2 3 4 5 7 8 10 11 14 16	596.02	1194.22	0 1 2 3 4 5 7 8 10 11 14 16	596.02	1194.22	0.00	0.00
		3	0 1 2 3 4 5 7 8 10 11 14 16	596.02	1400.72	0 1 2 3 4 5 7 8 10 11 14 16	596.02	1400.72	0.00	0.00
0.4	8	1	0 1 2 3 4 5 7 11	1496.89	1812.89	0 1 3 4 8 11 12 13	1573.83	2333.11	5.14	28.70
		2	0 1 2 3 4 5 7 11	1496.89	2252.82	0 1 2 3 4 5 8 17	1780.79	3040.72	18.97	34.97
		3	0 1 2 3 4 5 11 13	1436.38	2884.23	0 1 2 3 4 5 11 13	1436.38	2884.23	0.00	0.00
	10	1	0 1 2 3 4 5 7 8 11 14	990.41	1223.01	0 1 2 3 4 5 8 11 13 16	923.37	1494.35	-6.77	22.19
		2	0 1 2 3 4 5 7 8 11 16	983.88	1584.60	0 1 2 3 4 5 8 11 13 16	923.37	1779.45	-6.15	12.30
		3	0 1 2 3 4 5 7 8 10 11	1037.84	1952.04	0 1 2 3 4 5 8 10 11 13	977.33	2183.93	-5.83	11.88
	12	1	0 1 2 3 4 5 7 8 10 11 14 16	596.02	828.62	0 1 2 3 4 5 7 8 10 11 14 16	596.02	828.62	0.00	0.00
		2	0 1 2 3 4 5 7 8 10 11 14 16	596.02	1194.22	0 1 2 3 4 5 7 8 10 11 14 16	596.02	1194.22	0.00	0.00
		3	0 1 2 3 4 5 6 7 8 11 13 16	689.51	1205.04	0 1 2 3 4 5 7 8 9 10 11 14	724.38	1533.51	5.06	27.26



(a) System configuration of RO-CPMP$_l$



(b) System configuration of RO-CPMP$_0$

Figure 3: Comparison of Optimal Configurations of RO-CPMP$_l$ and RO-CPMP$_0$

Table 9: Computational results for uncapacitated instances with a correlated uncertainty set

$\|I\|$	ρ	p	b	RO-PMP$_0$ *Time(s)*	*Iter*	*Obj*	*Gap(%)*	RO-PMP$_l$ *Time(s)*	*Iter*	*Obj*	*Gap(%)*
25	0.2	8	15	2.2	5	1558.09		2.2	5	1426.76	
			30	3.9	7	1820.23		4.8	7	1525.92	
			40	48.7	16	1989.57		11.2	10	1592.05	
		10	15	0.9	2	1139.08		3.5	7	1024.11	
			30	2.1	6	1374.09		5.4	8	1060.02	
			40	22.3	12	1490.87		7.6	9	1088.26	
		12	15	0.7	2	811.73		1.2	2	622.54	
			30	0.8	3	973.46		0.8	2	653.32	
			40	1.4	4	1090.26		1.4	4	683.63	
	0.4	8	15	2.5	5	1802.45		5.4	8	1539.77	
			30	19.8	13	2306.74		26.7	13	1674.14	
			40	71.7	20	2458.94		43.2	14	1724.86	
		10	15	0.9	2	1364.19		5.1	9	1067.20	
			30	9.1	9	1785.45		5.0	8	1139.02	
			40	205.4	21	2055.38		53.8	15	1221.07	
		12	15	0.8	2	1036.84		1.5	3	658.45	
			30	1.4	3	1360.29		0.7	3	720.02	
			40	1.3	5	1593.91		6.6	8	776.20	
49	0.2	8	15	10.2	5	6145.54		4.4	3	5684.45	
			30	119.5	8	6632.02		1571.7	17	6011.43	
			40	73.7	9	6643.61		796.9	16	6046.56	
		10	15	3.4	3	4950.32		34.5	7	4687.15	
			30	30.8	8	5307.93		126.4	10	4887.14	
			40	288.7	11	5559.63		T	25	5055.11	0.93
		12	15	3.6	3	4181.94		3.5	3	3830.82	
			30	18.7	6	4478.13		26.1	6	4022.75	
			40	139.9	12	4705.29		4588.5	19	4157.57	
	0.4	8	15	15.5	6	6625.50		32.7	7	5898.97	
			30	1043.4	17	7393.31		4075.3	22	6284.88	
			40	331.1	13	7393.31		3154.2	20	6284.88	
		10	15	11.7	5	5351.47		34.5	8	4796.70	
			30	211.8	13	5947.01		5165.7	21	5200.39	
			40	767.2	15	6192.15		T	26	5395.59	5.83
		12	15	49.6	8	4583.09		7.8	4	3920.28	
			30	1781.2	15	5126.96		4170.7	19	4239.83	
			40	T	24	5485.15	0.53	T	27	4452.74	4.98

Table 10: Computational results for capacitated instances with a correlated uncertainty set

$\|I\|$	ρ	p	b	RO-CPMP$_0$ *Time(s)*	*Iter*	*Obj*	*Gap(%)*	RO-CPMP$_l$ *Time(s)*	*Iter*	*Obj*	*Gap(%)*
25	0.2	8	15	2.5	4	1783.39		2.4	4	1550.58	
			30	2.8	4	2126.51		5.5	6	1597.10	
			40	26.7	8	2331.96		8.7	8	1626.47	
		10	15	0.9	2	1148.48		2.6	5	1036.93	
			30	1.7	3	1383.48		3.4	6	1084.08	
			40	18.3	10	1612.96		14.5	10	1119.55	
		12	15	1.6	2	821.12		0.8	2	642.54	
			30	0.7	3	982.85		2.7	5	715.66	
			40	49.9	13	1224.47		3.3	6	756.96	
	0.4	8	15	2.5	4	2024.04		2.8	6	1623.29	
			30	52.3	12	2791.94		29.7	11	1737.18	
			40	33.4	10	2895.55		28.8	10	1791.85	
		10	15	0.9	2	1373.58		9.1	8	1083.45	
			30	1.7	3	1843.59		28.3	11	1224.17	
			40	103.8	14	2211.98		48.2	12	1291.03	
		12	15	0.6	2	1046.23		0.8	3	689.06	
			30	1.7	3	1369.69		26.4	12	835.30	
			40	124.5	18	1742.80		100.3	15	882.31	
49	0.2	8	15	10.7	4	6432.77		78.3	6	6180.25	
			30	T	23	7097.47	4.08	1438.4	13	6470.81	
			40	809.9	11	7062.09		2732.5	16	6470.81	
		10	15	17.3	5	5045.10		22.5	6	4778.10	
			30	1293.7	13	5466.82		1535.4	14	5066.65	
			40	T	22	5898.33	5.79	T	21	5215.26	2.31
		12	15	8.4	4	4228.04		4.8	3	3920.98	
			30	66.0	7	4546.14		47.8	6	4104.87	
			40	T	19	4833.22	0.93	T	21	4299.91	1.74
	0.4	8	15	125.5	8	6939.44		504.5	11	6426.95	
			30	T	23	7977.01	5.65	T	18	6791.59	1.14
			40	5668.7	17	7948.09		1254.8	24	6826.96	
		10	15	55.5	7	5434.73		33.4	7	4881.02	
			30	T	30	6276.43	4.55	T	20	5468.66	4.10
			40	T	22	6825.37	11.45	T	20	5519.86	5.16
		12	15	35.5	7	4617.67		9.7	4	4018.48	
			30	1564.9	13	5173.96		5312.5	16	4321.95	
			40	T	23	5531.67	0.20	T	23	4637.33	7.85