Facility Location and Relocation Problem: Models and Decomposition Algorithms

A Dissertation

Presented to

the Faculty of the Department of Industrial Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

in Industrial Engineering

by

Ayse Durukan Sonmez

May 2012

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To my parents and sisters

&

my husband

...

with all my love

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Abstract

We consider the facility location and relocation problem (FLRP). Due to demand change in this problem, we may need to close some existing facilities from low demand areas and open new ones in newly emerging areas. Specifically, we discuss three optimization problems in FLRP. The first problem is to locate a certain number of facilities at a point in time, knowing that demand is subject to change and the total number of facilities may increase in the future. We develop a binary integer programming (BIP) model to find a set of initial and future facility locations. Utilizing the block-angular structure of the model, a decomposition algorithm is proposed to solve the problem. The second problem is the robust facility relocation problem. Suppose we already have a set of facilities and the demand distribution over the network has changed, however, we do not know the actual changes of demand. Therefore, different scenarios with known probabilities are used to capture such demand changes. We present two approaches to solve this problem. In the first approach, we develop a BIP model that can determine α-reliable relocations that minimize the maximum regret associated with a set of scenarios whose cumulative probability is at least α. In the second approach, we develop a BIP model that minimizes the expected weighted distance and ensures that relative regret for each scenario is no more than γ. We propose a Lagrangean decomposition algorithm to solve this problem. The third problem, which is the dynamic facility location and relocation problem, is designed to find locations for facilities in the aftermath of disasters such as hurricanes and earthquakes, where the population is in need of essential commodities due to the lack of infrastructure. We develop three MIP models, each having different objectives, and propose a heuristic algorithm to solve this problem. Numerical experiments are made to show the efficiency and complexity of our optimization models.

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Chapter 1 Introduction

1.1 Background and Motivation

Facility location problems have been widely studied by many researchers in a variety of sectors. Examples can include public facilities such as schools and public libraries that are located to best serve the communities. Most of the time, a community can be viewed as a group of people, where the initial demand for such facilities is known or estimated [Min, 1988]. Locations of these facilities are not intended for a very short term. These facilities should be able to serve the communities for long durations, where we can expect changes in the demand of the communities.

As an example, we consider the population of 256 counties in the state of Texas. When we looked at the U.S. Census Data and compared the population in those counties for years 1990 and 2000, we observed that 22 counties had a population change - both increase and decrease - of more than 40%. Also, 73 counties had more than a 20% population change during these years [U.S. Census Bureau, n.d.]. Similar changes are observed all around the U.S., as demonstrated in Figure 1.1.

Let us assume that we initially located facilities to serve those counties based on the 1990 demand. When we came to the year 2000, we would observe that population change over a decade made some of the existing facilities closer to lower demand areas, and some high demand areas far away from the existing facilities. Therefore, because of the demand change over time, existing facilities were not able to provide adequate service, which yields to an intolerable increase in total weighted distance traveled by the customers.

A similar example was discussed in Wang et al. [2003]. There are several branches of a bank in the City of Amherst, New York. Since the demand distribution has

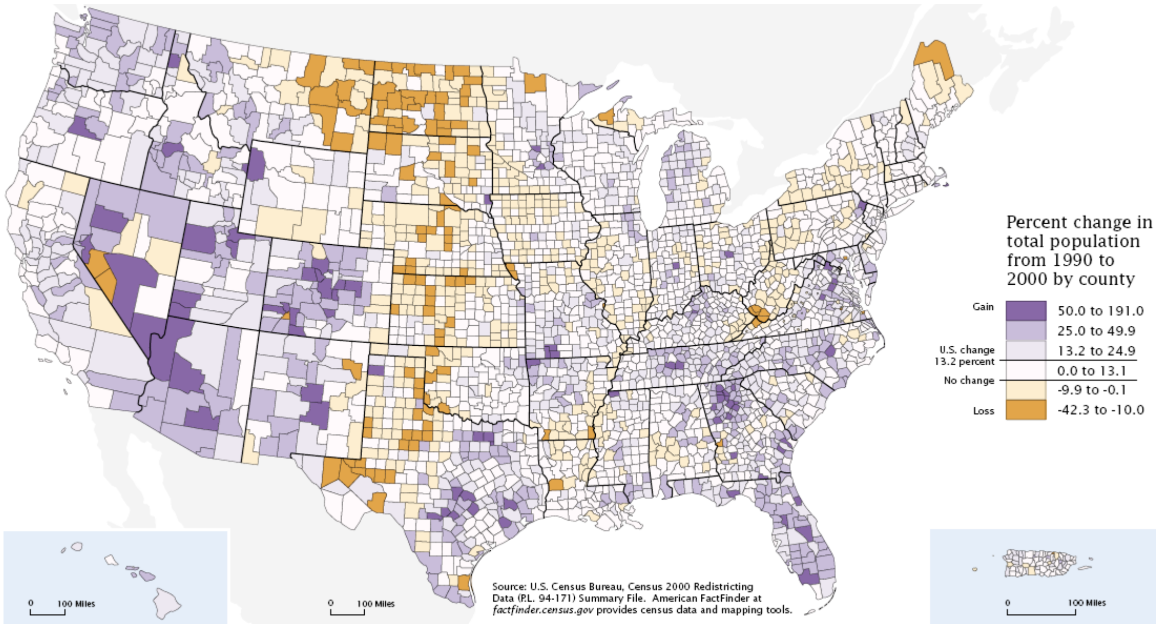


Figure 1.1: Percent change in population from 1990 to 2000 by county

changed, there are many problems associated with the performance of existing branches. Some of the branches are visited by very few people and some customers complain that they do not have a branch closeby. In such situations, relocating facilities is essential and inevitable.

The idea of relocation can be considered in two cases. Either we consider relocating some facilities that are already located, or we determine the initial locations for facilities and then consider relocating them in the future. In both cases, the relocation process can be controlled in different ways. We can set a limit on the maximum number of relocations [Min, 1988], minimize the cost associated with relocations [Wesolowsky and Trusctott, 1975], or set a budget limit for relocations [Melo and SaldanhadaGama, 2005, Wang et al., 2003].

Locating facilities initially and relocating them in the future is a long term decision. In such long term decisions, the number of future facilities is usually not known when locating initial facilities. For example, suppose that the available budget or company policies limit the number of initial facilities to open. However, an additional budget or policy changes may affect the number of facilities in the future.

Also, the number of facilities in the future may depend on the success of the initial ones [Berman and Drezner, 2008*b*]. In such a case, the initial locations of the facilities should be determined considering the probability of additional facilities as well. When we take both facts into account, the problem becomes more complicated, in which we need to handle both the demand change and the uncertainty in the number of future facilities.

As we have mentioned, we may also have some existing facilities and consider relocating them. In the cases where we have an accurate and reliable forecast for the demand, those values can be utilized for making the optimal relocation decision. However, there could be many instances where it is hard to obtain exact figures of the new demand. For instance, Census data is collected every ten years. For the years between any two Census surveys, we can only obtain a forecast for the population. However, we cannot assume that they are 100% accurate because of the difficulties in predicting mobility as well as in and out migration [Gregg et al., 1988]. Even if the demand is not correlated with population, it is still hard to obtain exact numbers due to the demand variability day by day, month by month or year by year. For such instances, it is more reasonable to treat demand as an uncertain parameter. Demand uncertainty is usually modeled in two different ways [Owen and Daskin, 1998]. The first approach assumes possible values for the demand with probabilities associated with those values. The second approach considers upper and lower bound values for the demand. In both cases, the demand can be represented with scenarios.

Facilities that are facing demand changes are not limited to public facilities or bank branches. Similar challenges of uncertainty, location and relocation are observed also in the service industry, military bases, supply chain, and humanitarian relief operations, to name a few [Balcik and Beamon, 2008, Dell, 1998, Melo and SaldanhadaGama, 2005, Sathe and Miller-Hooks, 2005].

When we talk about demand change, some of the most important locational decisions arise in humanitarian relief operations. Humanitarian relief has become a significant area of research because of financial damage and loss of life caused by natural disasters. For example, Hurricane Katrina had an estimated damage of $81 billion U.S. dollars, and took the lives of nearly 2000 people [Pielke et al., 2008, Sneath et al., 2009]. It caused hundreds of miles of traffic jams during evacuation, and it took over two hours to drive 10-20 miles [Litman, 2006]. After the incident, there was a delayed response which caused extreme human suffering.

The number of people that are affected by natural disasters has been increasing and is forecasted to increase 54% by the year 2015 [Ganeshan and Diamond, 2009]. Unfortunately, the number of people and assets located in high risk areas is also forecasted to increase by more than 100% [International Strategy for Disaster Reduction, 2012].

In order to reduce such human suffering and mitigate the losses after a disaster, shelters as well as facilities supplying food, water, and medical supplies needed to be located in the disaster areas, both before and after the incidents. Since the impact of disasters are enormous, more attention and importance should be given to locational decisions associated with the disaster response.

1.2 Problem Statement

1.2.1 Facility Location and Relocation Problem (FLRP) with Demand Change and Uncertain Number of Future Facilities

In this problem, we consider location and relocation of uncapacitated facilities in order to minimize the total initial and future weighted distance traveled by customers to their assigned facilities. This problem is significant because it takes into account some of the important aspects such as the demand changes and uncertain number of future facilities that we discussed in Section 1.1.

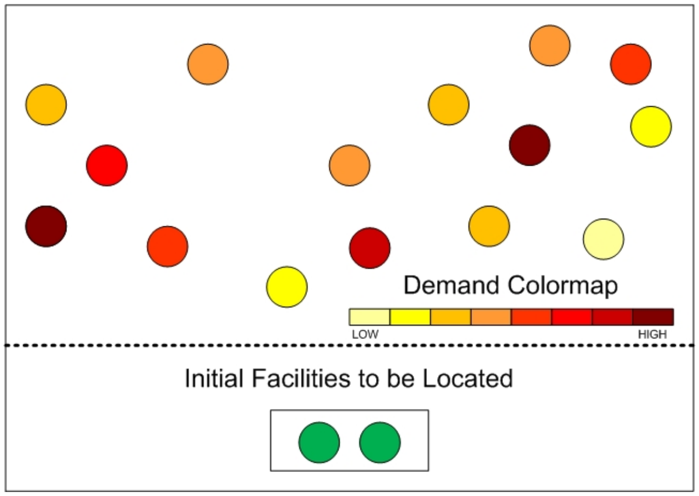


Figure 1.2: Illustration of the initial case for FLRP

Suppose that we need to locate p facilities now, and may add up to q facilities in the future. The probability of increasing the total number of facilities by r in the future is given by α_r. We have the current demand data for every node in the network. We are also given the forecasted future demand. Figures 1.2 and 1.3 illustrate the problem on a small network, with $p = 2$ and $q = 2$. Green nodes demonstrate the facilities to be located at any node in the network.

Initial and future demand values are demonstrated with colors, where we have light yellow for low demand points and dark red for high demand points. Due to the changes in the demand pattern among the network, we are allowed to perform relocations; we can close some of the existing p facilities and open new facilities to be able to have a total of $p + r$ facilities in the future. However, the relocations should be done within a certain budget, where we have opening and closing costs for every facility. Figure 1.4 demonstrates a possible solution for the problem with initial location decisions and possible future locations for each scenario.

The facility location and relocation problem with demand change and uncertain

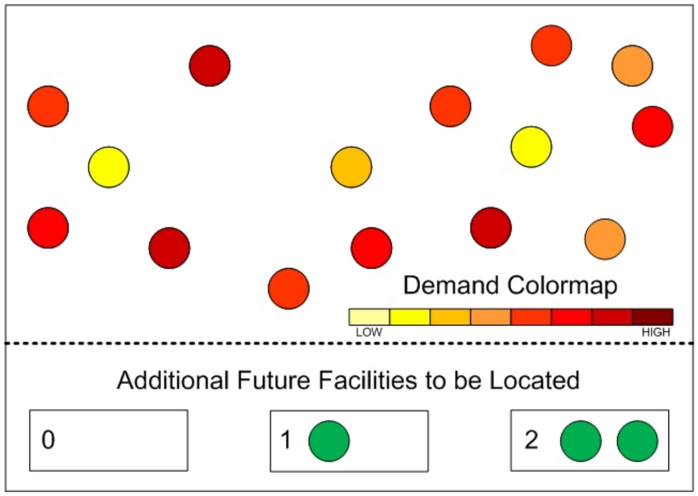


Figure 1.3: Illustration of the future case for FLRP

number of future facilities determines the initial locations and future relocations of facilities, knowing that the demand is subject to change and the total number of facilities may increase in the future.

1.2.2 Robust Facility Relocation Problem (RFRP)

As we have mentioned in Section 1.1, demand for a facility may not necessarily remain the same all the time. In order to provide enough service to the customers, relocation of facilities needs to be performed based on the new demand. However, in reality, demand at a point depends on many factors, such as community growth and economic vitality [Serra and Marianov, 1998], or the demand itself varies within different time periods. Therefore, while planning relocations, it is important to consider demand uncertainty. In most of the literature, demand is assumed to be known exactly or deterministic approximations are made. This is because deterministic models are easier to formulate and solve compared to the models under uncertainty. If the decisions are made without considering uncertainty, the solutions may be far from optimal. They may also result in poor performance, low customer satisfaction and economic losses. In order to avoid such complications, we need models that can reflect

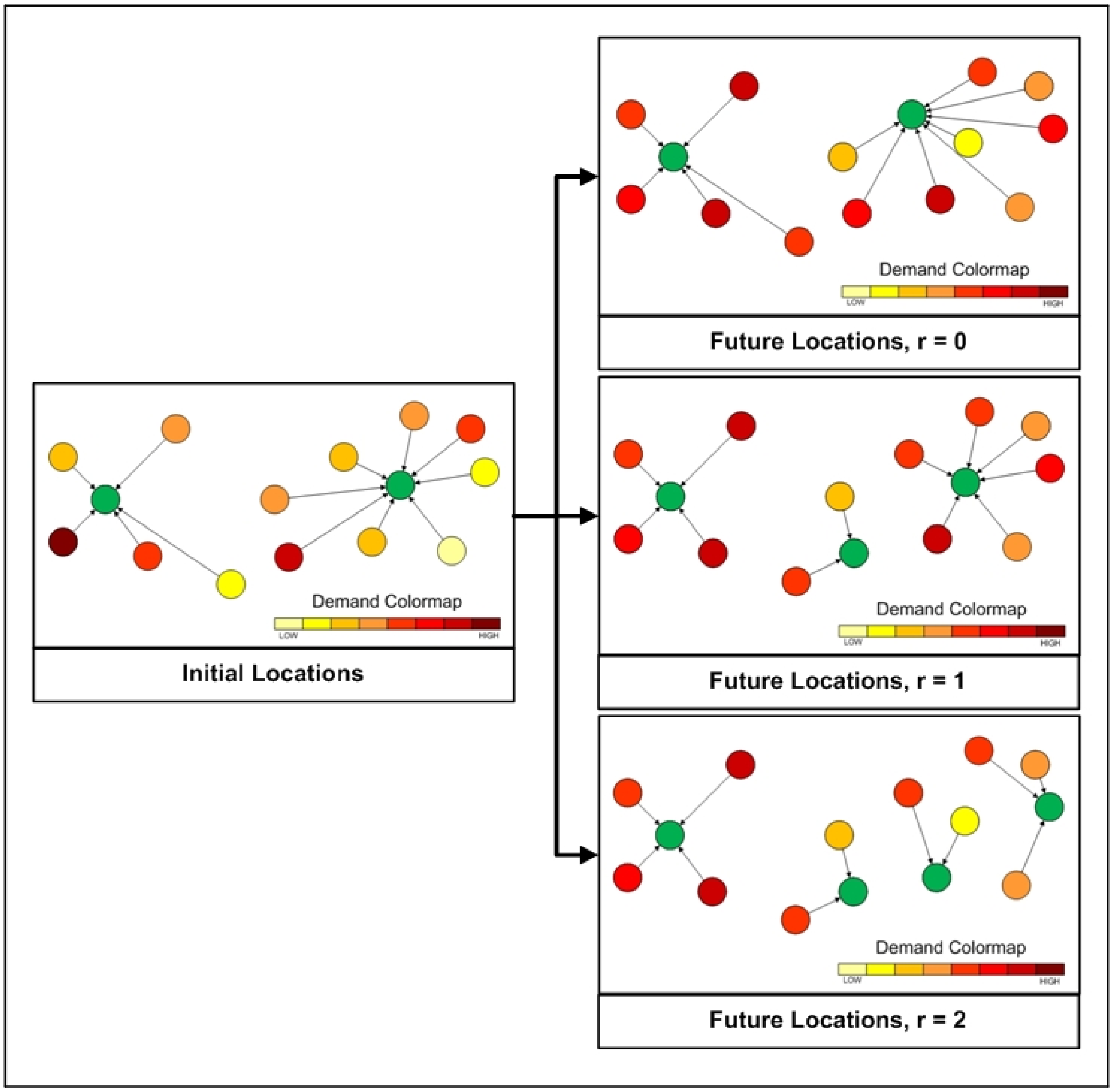


Figure 1.4: Illustration of the possible location decisions for FLRP

real life situations better and provide solutions that perform well under different scenarios . Therefore, decisions about opening and closing facilities should be considered in an uncertain environment.

Suppose that we have some existing facilities (green nodes) as shown in Figure 1.5. We know that demand distribution has changed over time. In order to handle this change, we can close some of the existing facilities and open new ones. We have opening and closing costs for each facility and the total cost of relocations should not exceed a given budget. The relocations have to be performed considering

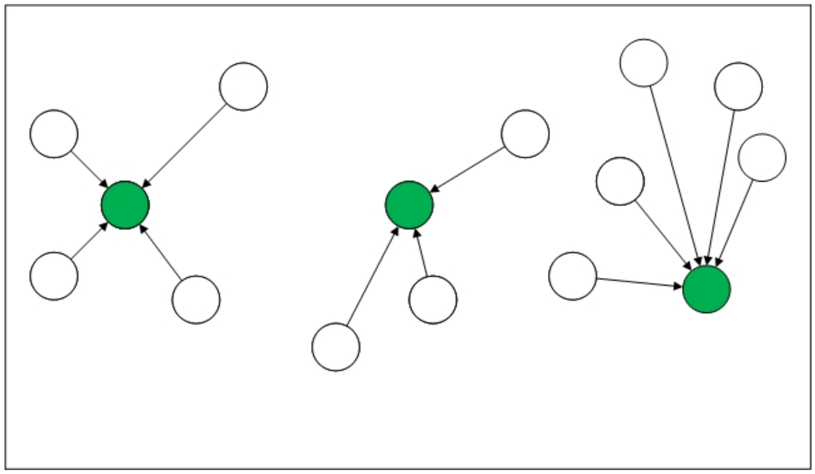


Figure 1.5: Illustration of existing facilities for RFRP

the new demand. However, we do not know the exact value of the demand at each node. Therefore, different scenarios with known probabilities are used to capture such demand changes. An illustration of three demand scenarios can be seen in Figures 1.6, 1.7, and 1.8. Demand in the first scenario, which is shown in Figure 1.6, is very high at some nodes that are represented with dark red. Figures 1.7 and 1.8 represent more evenly distributed scenarios.

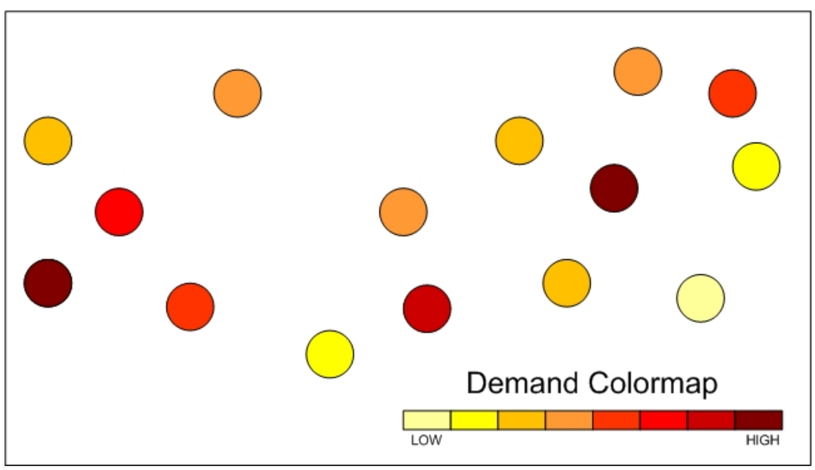


Figure 1.6: Illustration of demand scenario 1 for RFRP

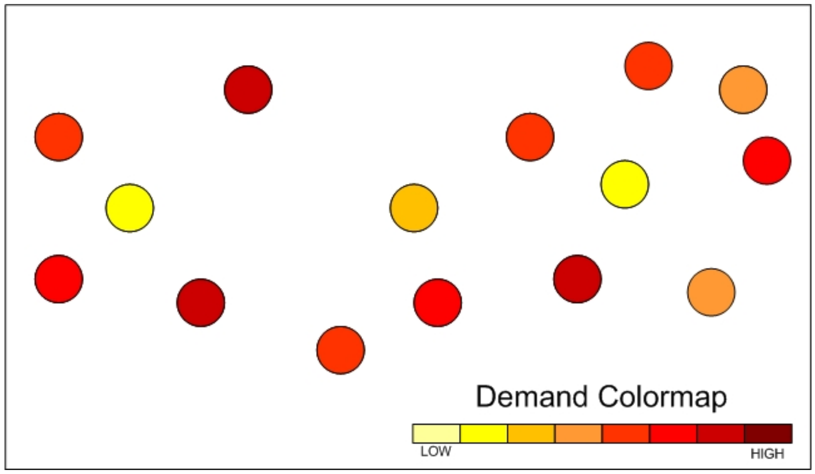


Figure 1.7: Illustration of demand scenario 2 for RFRP

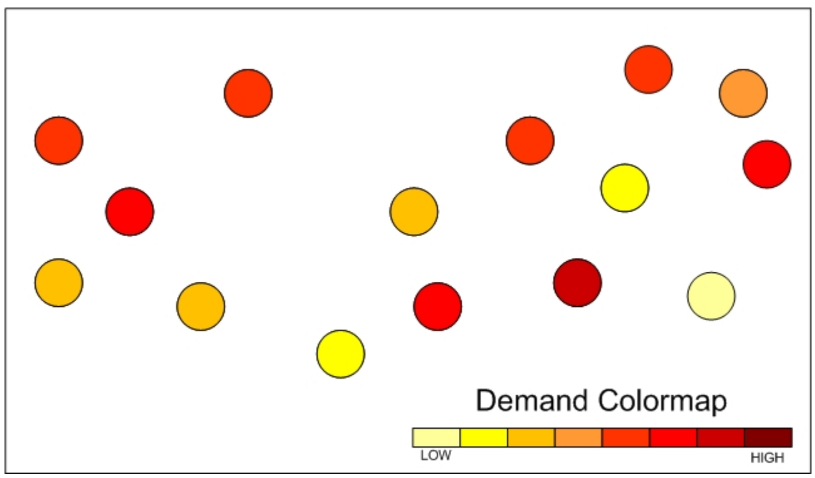


Figure 1.8: Illustration of demand scenario 3 for RFRP

The goal is to find robust facility relocations that can perform well under those scenarios. We need to identify which existing facilities to close, and which facilities to open, in order to obtain a reasonable total weighted travel distance among the network under the given scenarios.

Figures 1.9, 1.10, and 1.11 demonstrate a possible solution for the problem with final locations of facilities after the relocations. These locations are not intended to perform well under a specific scenario; rather, they are aimed to be robust. The relocations provide a moderate total weighted distance under all scenarios.

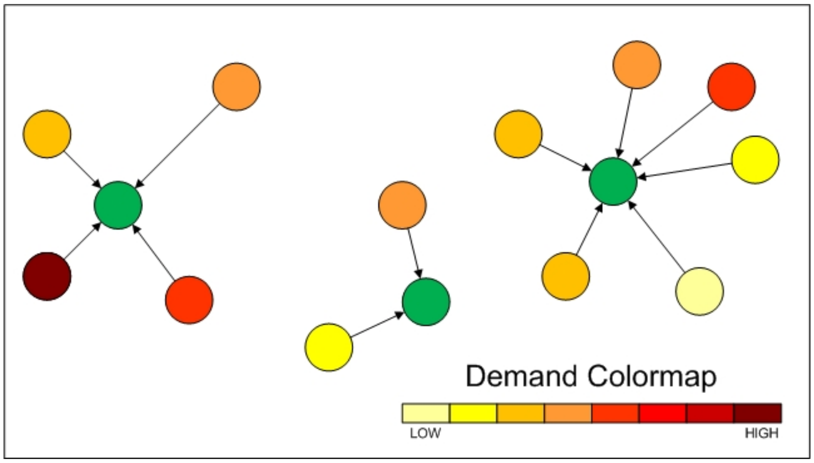


Figure 1.9: Illustration of robust relocations for scenario 1

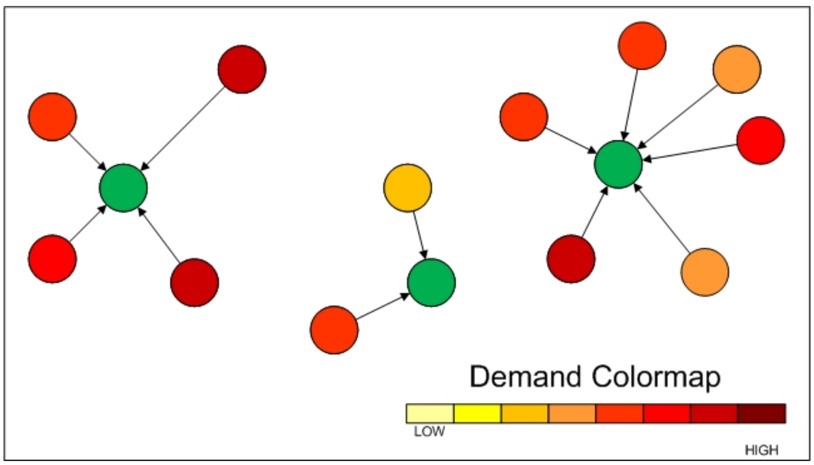


Figure 1.10: Illustration of robust relocations for scenario 2

1.2.3 Dynamic Facility Location and Relocation Problem in Disaster Response

Research in a variety of disciplines as well as administrative regulations put too much effort on decreasing the risks and hazards of both man-made and natural disasters. Unfortunately, it is impossible to consider a community that is not exposed to any of those disasters or to totally eliminate their outcomes. Therefore, it is important to make plans for response and recovery operations following a disaster.

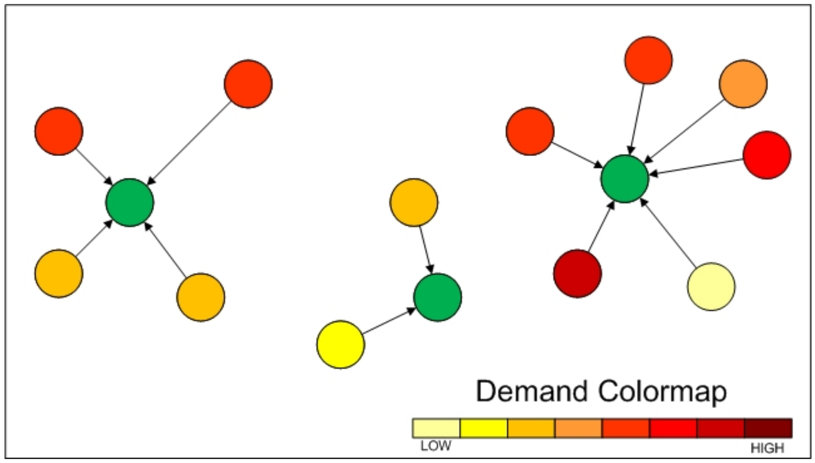


Figure 1.11: Illustration of robust relocations for scenario 3

An important aspect of response and recovery operations is providing the community with the necessary commodities such as foods, water, and ice when local infrastructure is compromised and retail stores are not operable. FEMA has a multiple-layer supply chain for performing these operations [FEMA, 2011]. The components of this supply chain are explained below and their relation is illustrated in Figure 1.12.

- FEMA Logistics Centers: permanent facilities that receive, store, ship, and recover disaster commodities and equipment.

- Other Federal Agencies Sites: representing vendors from whom commodities are purchased and managed; e.g., Defense Logistics Agency (DLA) and the United States Army Corps of Engineers (USACE).

- Mobilization (MOB) Centers: temporary federal facilities at which commodities, equipment, and personnel can be received and pre-positioned for deployment as required.

- Federal Operations Staging Areas (FOSAs): temporary facilities at which commodities, equipment and personnel are received and pre-positioned for distribution or deployment within a designated state as required.

- State Staging Areas (SSAs): temporary facilities at which commodities, equipment, and personnel are received and pre-positioned for distribution or deployment within a state.

- Points of Distribution Sites (PODs): temporary local facilities at which commodities are distributed directly to disaster victims. PODs are operated by the affected state and county.

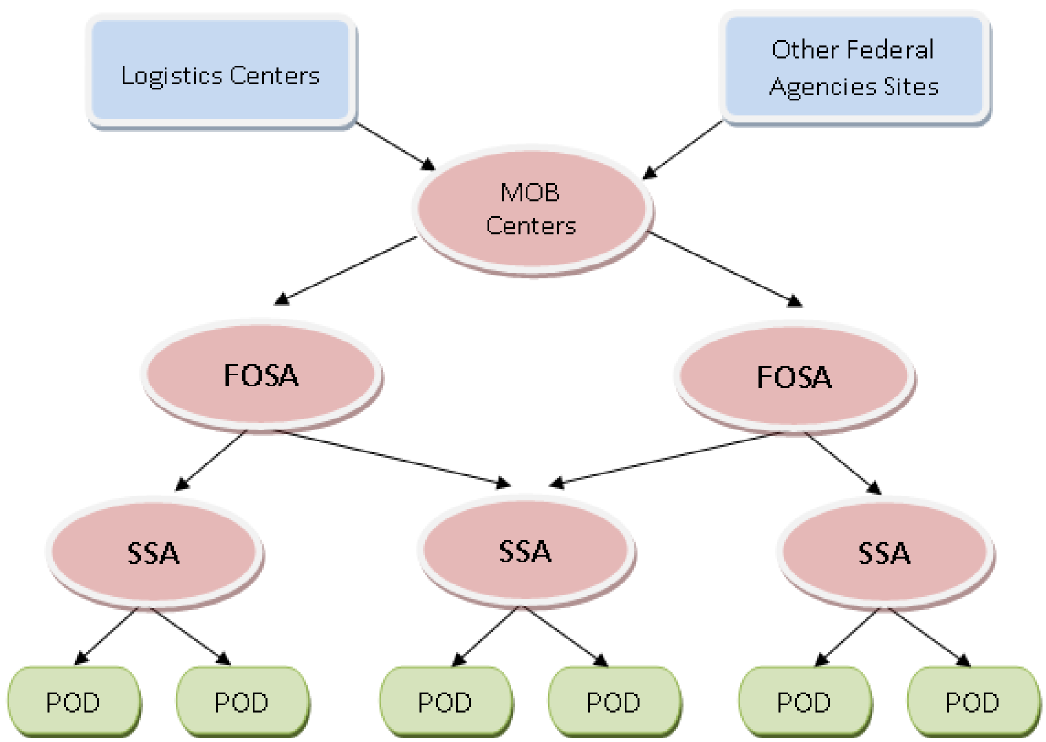


Figure 1.12: Illustration of FEMA Logistics Supply Chain

In this multi-layer supply chain, locating the PODs itself is an important research question because it is the main contact with the disaster victims. Another aspect that makes the POD location problem complicated is demand change over time. The demand may decrease or vanish totally as the infrastructure is available and retail stores reopen with time. On the other hand, citizens returning to their homes after an evacuation may increase the demand at some locations. Therefore, PODs should be opened, closed, or relocated considering these demand changes [TxDPS, 2011]. Another fact is demand prioritization, which implies that people living in more damaged areas should have a higher priority among others while considering POD locations [Harris County Judge Ed Emmett, 2011].

Locations of all facilities in the FEMA disaster supply chain except PODs are determined by state or federal government. POD locations are determined by the local authorities given the locations for State Staging Areas. The goal of the dynamic location and relocation problem in disaster response is to solve this local level decision problem considering all challenges such as demand change, demand prioritization, multiple commodities, and some social considerations such as traveling distance from demand points to PODs.

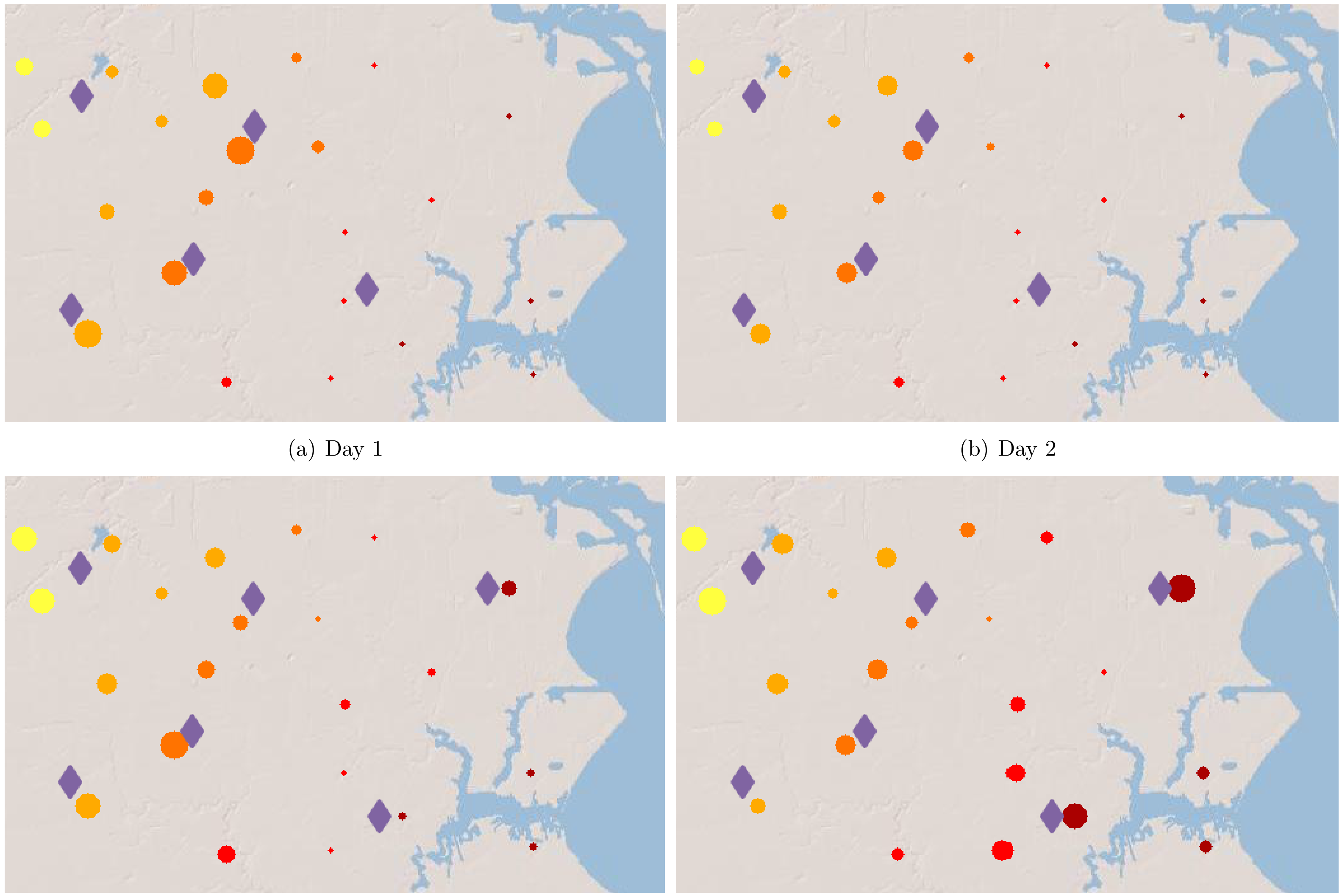
We define this problem using an eight day planning period for illustration purposes in Figures 1.13 and 1.14. We have a set of demand points that are demonstrated with circles, where demand amounts are represented with the size of the circles. As we mentioned, demand amount varies day by day due to various factors such as lack of infrastructure, people leaving for evacuation and coming back, restoration of the infrastructure, and availability of the retail stores. The colors are used to identify the priority indices of demand points, which are determined by the severity of the damage or any other local considerations. In our illustration light yellow represents demand points with low priority and dark red represents demand points with high priority. Locations of State Staging Areas (SSAs) are assumed to be given. Potential locations for PODs, their capacities and the number of PODs that will be active on each day are assumed to be given. PODs can be opened, closed, and relocated within a budget in order to cope with the changes in the demand. However, these operations should not happen every day because such frequent changes in POD locations may be confusing for people, and too costly for the planners. For illustration purposes, we allowed these operations every other day. Another important issue is the assignment of demand points to the POD locations. If we consider only demand satisfaction constraints, demand points are allowed to be assigned to multiple POD locations. In reality, it is hard to ensure that a fractional amount of the population goes to a particular POD location. Such assignments are likely to lead to over or under supply at some POD locations. Thus, it is important to assign each demand point to one POD location rather than satisfying the demand partially from different POD locations.

Figures 1.13 and 1.14 also demonstrate locations and relocations of PODs, which are represented with purple diamonds. A given number of facilities are located for days 1-2. Some of those facilities are relocated and some new facilities are opened for days 3-4 to handle the demand changes. Similarly, some existing facilities are closed or relocated for days 5-6 as well as 7-8 to handle the changes and decreases in the demand.

The dynamic facility location and relocation problem in disaster response aims to find where to locate initial facilities and relocate them throughout the planning horizon in order to serve people that are in need of essential commodities. We consider two important measures that can fulfill this objective: minimizing the total traveling cost of commodities and minimizing the maximum travel distance from demand points to their assigned POD locations.

1.3 Chapter Organization

This dissertation is composed of six chapters. We introduce the background and motivation as well as the problem definitions in Chapter 1. An extensive literature review is presented in Chapter 2. In Chapter 3, we formulate the facility location and relocation model which considers demand change and uncertain number of future facilities. We prove that the problem is *NP-hard* and introduce a solution algorithm. We then present the results of the numerical experiments. In Chapter 4, we discuss the robust facility relocation problem, develop mathematical models and propose a Lagrangean Decomposition Algorithm to solve the problem followed by the numerical results. In Chapter 5, we introduce the dynamic facility location and relocation problem in disaster response and present mathematical models and algorithms to solve the problem and then present their numerical results. We summarize our research and present future directions in Chapter 6.



(a) Day 1

(b) Day 2

(c) Day 3

(d) Day 4

Figure 1.13: Illustration of the daily demand for days 1-4

(a) Day 5

(b) Day 6

(c) Day 7

(d) Day 8

Figure 1.14: Illustration of the daily demand for days 5-8

Chapter 2 Literature Review

2.1 Deterministic Facility Location and Relocation Problems

Facility location problems are widely studied optimization problems, and different types of facility location problems discussed in the literature can be found in a recent book chapter by Verter [2011] and survey articles [Owen and Daskin, 1998, ReVelle and Eiselt, 2005]. Plant or warehouse location problems, which can also be named as uncapacitated facility location problems, aim to find the optimal number and location of facilities that minimize the total traveling costs and facility opening costs [Baumol and Wolfe, 1958, Kaufman et al., 1977, Khumawala, 1973]. On the other hand, p-median and p-center problems find the locations of a given number of facilities that minimize the total traveling cost (distance) and maximum traveling cost (distance), respectively [Hakimi, 1964, 1965]. Capacitated versions of these problems are also extensively studied in the literature [Ellwein and Gray, 1971, Sridharan, 1984]. Most of the facility location problems are proved to be NP-Hard, and a significant portion of the relevant literature attempted to develop algorithms to find good quality solutions in a timely manner [Arostegui et al., 2006, Geoffrion and Mcbridge, 1978, Lim et al., 2009, Reese, 2006].

Facility location literature is vast and we make no attempt to review it all. In this chapter we review only the relevant work that sets the foundations for the problems we introduced in Chapter 1. We present the literature that attempted to solve the same or similar problems, and we discuss advantages as well as limitations of their approaches. We also present the literature that helped us to develop solution methods for our problems.

2.1.1 Facility Location Problems

The facility location problems explained in the previous section have many extensions. Most of these extensions consider locating extra facilities. For example, the conditional median problem solved the issue of locating a certain number of new facilities, given that the locations of existing facilities are known [Berman and Drezner, 2008*a*, Berman and Simchi-Levi, 1990, Chen, 1988, Drezner, 1995*b*, Minieka, 1980]. The progressive p-median problem considered the demand changes as a function of time and located several facilities during a given time period [Drezner, 1995*a*].

Besides the literature about opening new facilities, only a handful papers deal with closing facilities. Roodman and Schwarz [1975] studied the problem of closing some of the existing facilities due to the demand shifts and declines over a planning horizon. The paper by Leorch et al. [1996] discussed closure of army bases after a force draw-down. On the other hand, Wang et al. [2003] proposed that it is not always realistic to consider closing existing facilities and opening new ones separate from each other. Therefore, researchers studied facility relocation problems that consider both opening and closing of facilities. Literature about facility relocation problems is discussed in the next section.

2.1.2 Facility Relocation Problems

The idea of relocating facilities is mostly considered in dynamic environments. A dynamic location allocation problem with facility relocation was presented in Wesolowsky and Trusctott [1975]. Even though demand values were not taken into account, they considered opening and closing costs of facilities and tried to minimize the overall relocation and allocation costs. A relocation problem for public facilities was introduced with the solution of a real life problem in Min [1988]. Demands were represented by the U.S Census population data. A fuzzy multi-objective model with constraints on a budget and the maximum number of relocations per period was constructed to solve the problem. Dell [1998] studied the problem of realigning or closing army installations considering their changing populations.

The supply chain point of view of the problem was also studied in literature. The gradual relocation of an existing manufacturing plant was discussed in Min and Melachrinoudis [1999] and Melachrinoudis and Min [2000]. Dias et al. [2006] and Thanha et al. [2008] considered opening, closing and reopening of facilities in a dynamic supply chain environment. Since they allowed opening and closing of a facility multiple times in the planning period, extra costs might be incurred due to unnecessary relocations. Melo and SaldanhadaGama [2005] introduced a Mixed Integer Programming (MIP) model to minimize the cost for a multi-commodity, multi-echelon, dynamic network by means of relocating facilities and capacity transfers which were all performed in a given budget. Canel et al. [2001] proposed a solution algorithm for a similar problem that does not have a budget constraints. Location and relocation of a single facility in a dynamic environment was studied in Bastian and Volkmer [1992] and Farahani et al. [2009], however they did not extend their model to a multi-facility relocation problem. All these problems assumed that the number of facilities for each time period are given.

Wang et al. [2003] aimed to minimize the total weighted distance between the facilities and customers by closing some of the existing facilities and opening new ones, where they incur opening and closing costs. The main reason behind this relocation was customer distribution change over time. One issue, which was not considered in Wang et al. [2003] was the uncertainty of the increase in the number of facilities in the future. They assumed that the number of facilities in the future is given, which may be less than, equal to or greater than the initial number of facilities. However, it is not uncommon that the number of facilities in the future is uncertain. Furthermore, for the problem explained in Section 1.2.1, this approach is one of the two appropriate methods in the literature. In order to handle such a situation, initial facilities are assumed to be located at the best possible sites based on the current demand. When the demand change occurs in the future, the technique in Wang et al. [2003] can be utilized. By observing the new demand pattern relocations may be performed within a given budget, in order to minimize the total weighted distance. Since this method does not consider the uncertainty of total number of facilities in the future, some facilities that are initially located might not be optimal for different future scenarios. This may lead to higher costs due to higher traveling distance, lower customer satisfaction and higher budget consumptions.

Another issue that needs to be addressed in a relocation problem is demand uncertainty. As mentioned before, the main reason behind relocations is demand distribution change. However, the new demand, for which we perform the relocations, is usually uncertain in reality. Using deterministic numbers for such a problem may yield poor solutions.

2.2 Facility Location and Relocation Problems Under Uncertainty

The uncertainty of input parameters has been an important aspect of optimization problems. While deterministic problems are easier to handle, uncertainty provides a better modeling of real life situations. For this purpose, uncertainty has been incorporated in facility location problems and handled by stochastic and robust optimization techniques.

Stochastic models assume uncertainty as a random parameter with known probabilities and some models aim to minimize or maximize the probability that the value of the objective function exceeds or does not exceed a given threshold value [Berman and Wang, 2004, 2010]. Some other models attempt to minimize the expected objective function value [Snyder, 2006]. Robust models aim to minimize worst-case or worst-case regret value of the objective function where uncertainty is demonstrated by discrete scenarios or interval data [Snyder, 2006]. Owen and Daskin [1998] and Snyder [2006] introduced an extensive review of these techniques. A recent review of min-max type robust optimization problems was discussed in Aissi et al. [2009].

2.2.1 Uncertain Demand and Travel Distance

Early work by Mirchandani and Odoni [1979] and Mirchandani [1980] are examples of uncertain demand and travel distance in facility location problems. Then, Weaver and Church [1983] proposed a reformulation of the stochastic p-median problem that yields multiple deterministic formulations and solved the resulting problem using a Lagrangean relaxation. Algorithms were developed for min-max regret 1-median problem on tree and general network with uncertain demand weights [Averbakh and Berman, 1998, Chen and Lin, 1998]. Another relevant paper addressed uncertainty in demand and travel distance in the p-median problem [Serra and Marianov, 1998]. They aimed to fulfill two objectives: minimizing the maximum average distance and minimizing the maximum regret. The more recent work of Conde [2007] attempted to minimize the maximum regret for a location-allocation problem.

Most papers in the literature discussed location of facilities under demand uncertainty, whereas a few of them considered relocations in such an environment [Berman and LeBlanc, 1984] and [Carson and Batta, 1990]. Carson and Batta [1990] defined a stochastic network in which demand at each node varies throughout the day. Due to this variation, one static location for the facility may increase the system-wide response time. In order to minimize the average response time, they considered relocation of the facility throughout the day. In their problem, facility relocation was perceived as an option to react to the changes in demand. Although relocation for each scenario could be acceptable for mobile facilities such as ambulances, for other types of facilities such as libraries, schools, bank branches or ATMs, it may not be reasonable to consider relocating facilities for every scenario. Furthermore, they did not consider costs for relocation in the model. This may allow very frequent but not necessarily required relocations.

For non-mobile facilities, the only approach to solve facility relocation problem with uncertain demand was proposed by Gregg et al. [1988]. They assumed a probability distribution for the demand and modeled a public facility system as a network with the objective of minimizing the summation of operating cost, travel distance, overage cost and underage cost. The model took a set of facilities as an input and found the capacity utilization of each facility while minimizing the aforementioned costs. They ran the model with all facilities open and obtained the utilization of each facility. Based on the capacity utilization, they chose to close some of the facilities and reran the model. They repeated this procedure until a satisfactory solution was found. A major drawback is that the facility opening and closing decisions are exogenous. A better approach could determine which facilities to open and close within an optimization model and find robust solutions.

In robust optimization, for the cases where we optimize the worst-case objective function or minimize the maximum regret, every single scenario may dominate the outcome, even though it may be less likely to occur in reality. This could be a good approach for location of facilities that deal with extreme situations such as nuclear reactors or ambulances. However, for other types of public or private facilities, decisions made by considering the worst case scenario may be too pessimistic because many non-extreme scenarios may happen with certain probabilities. Such decisions may lead to unnecessarily higher expected weighted distances. In order to overcome this issue of the traditional robust optimization approach, different approaches were suggested in the literature. Daskin et al. [1998] proposed an α-reliable minimax regret model for the p-median problem. They minimized the maximum regret of the total weighted distance over a set of scenarios whose total probability is at least α. Using this approach, the issue of the dominant but less likely scenarios can be solved and various solution alternatives can be obtained by modifying α values. Chen et al. [2006] proposed an α reliable mean-excess regret model. They minimize the expected regret with respect to the scenarios whose total probability of occurrence is no more than $(1 - \alpha)$. Snyder and Daskin [2006] introduced a model that finds a solution which minimizes the expected traveling cost while having a relative regret in each scenario of no more than a desired amount.

2.2.2 Uncertain Number of Facilities

Another uncertain parameter in facility location problems is the number of facilities to locate. The problem of locating p facilities currently and then locating new facilities in the future was discussed by [Current et al., 1998]. Because the number of facilities in the future was not known in advance, they used minimax regret criterion and compared it with the expected opportunity loss criterion.

Different than previously introduced in the literature, the p-median problem under uncertainty [Berman and Drezner, 2008*b*] aimed to locate p facilities initially, knowing that up to q additional facilities would be located in the future. The probabilities of locating r ($r = \{0, 1, 2, \ldots, q\}$) more facilities were assumed to be given. They formulated the problem on a graph, constructed the integer programming formulation, suggested heuristic solution techniques and extensively analyzed the case with $q = 1$.

A notable shortfall of both approaches is that they do not allow facility closings. Both initial and future locations are determined considering a constant demand. However, due to demand changes over time, it may be necessary to close some of the existing facilities. In order to handle such situations, we need a model that considers both the demand change and the uncertainty in number of future facilities.

2.3 Facility Location and Relocation in Disaster Response

Facility location in disaster response and humanitarian relief has been gaining significant attention due to the catastrophic impact of disasters on human life. Caunhye et al. [2011] reviewed the literature on operations research models that aim to solve emergency relief logistics problems.

As we explained in Section 1.2.3, the disaster response supply chain has multiple layers and multiple decision makers, i.e., federal and local government. In an effort to optimize the federal operations in this supply chain, Afshar and Haghani [2010] proposed a model that includes a facility location problem, vehicle routing problem, and capacity planning problem. They assumed that the POD locations are determined by the local government. Given these locations, they found initial locations and possible relocations of State Staging Areas and Federal Operation Staging Areas.

Similarly, many papers developed models and algorithms to determine the number and locations of the distribution centers in the disaster relief supply chain and the amount of relief supplies to be stocked at each distribution center [Balcik and Beamon, 2008, Campbell and Jones, 2011, Rawls and Turnquist, 2010].

Determination of POD locations is more of a local level decision and has to consider various factors. As we have mentioned, the need for PODs mostly arises due to the lack of infrastructure and it may be subject to change over the planning horizon. It is expressed in the State of Texas Commodity Distribution Plan that PODs should be relocated considering the demand changes [TxDPS, 2011]. Another fact is demand prioritization, which was mentioned by the Harris County Judge in his keynote speech in the Hurricane Conference, 2011 [Harris County Judge Ed Emmett, 2011]. He mentioned that not all demand points should have the same priority while considering POD locations; people that are living in more damaged areas should have a higher priority among others [Harris County Judge Ed Emmett, 2011]. However, these real-life facts are usually overlooked in most of the mathematical models in the literature.

Ekici et al. [2008] considered some of these facts and proposed a model that determines dynamic location of distribution centers after an Influenza Pandemic. They considered one commodity to distribute without the prioritization of the demand locations. They minimized the total cost of transporting humanitarian relief items in the network as well as the facility opening, closing and operating costs. Having both cost components - the transportation and the facility operation and location costs - in the objective function may yield solutions that are dominated by one of the cost components due to the scaling issue. In their paper, they used three combinations of coefficients to analyze the impact of these cost components. However, it is still hard to attribute the trade-off between them. Additionally, their model allows facility openings and closings but not relocations. In some instances, instead of closing a facility at location A and opening a new one at location B, it may be cheaper to relocate a facility from node A to node B.

Among the literature that considers locating PODs, many models have the objective to minimize the total cost of transporting humanitarian relief items. This is a logical objective from the planner's point of view. But this objective does not consider some social costs such as walking or driving costs to PODs [Jaller and Holguin-Veras, 2009]. None of the models in the literature has taken these aspects into account simultaneously.

2.4 Summary

In this chapter, we reviewed the literature about the problems we discussed in Chapter 1. To the best of our knowledge, there is no problem in facility location literature that considers both demand change and uncertain number of facilities in the future. However, two approaches can be adapted to solve the problem. In the first approach, the initial facility locations are optimized based on the current demand. When the demand change occurs in the future, the technique in [Wang et al., 2003] can be utilized to determine the relocations within a given budget in order to minimize the total weighted distance. Since the uncertainties are not considered, this method may lead to higher total traveling distances and budget consumptions. The second approach is to find the best locations for initial facilities considering the possibility of adding more facilities [Berman and Drezner, 2008*b*]. However, this method does not consider closing any of the existing facilities and is unable to handle the future demand change. Therefore, this approach is not capable of dealing with the systems that are inherent to such a change. Thus, we propose a binary integer programming model and a decomposition based solution algorithm to find the locations and relocations for facilities that minimize the total of initial and expected future weighted distance considering the demand changes and uncertain number of future facilities.

Facility relocation problems are mostly studied within static or dynamic concepts. However, uncertainty of the new demand was not incorporated in those work. Robust optimization is a good methodology to solve such problems, but classical regret methods could be very conservative since they provide solutions considering the worst case scenarios. Therefore, we utilize variations of robust optimization discussed in the literature to solve the facility relocation problem. In the first approach, we introduce a BIP model that finds α-reliable relocations that minimize the maximum regret associated with a set of scenarios whose cumulative probability is at least α. In the second approach, we develop a BIP model that minimizes the expected weighted distance and ensures that relative regret for each scenario is no more than γ. We also propose a Lagrangean decomposition based solution algorithm to solve this problem.

We discussed some of the real-life facts of disaster response and mentioned that most of them are not fully addressed in the literature. Therefore, we introduce BIP models to solve the dynamic POD location and relocation problem that considers the total cost of transporting humanitarian relief items as well as the maximum prioritized traveling distance from demand points to their closest facilities. Our models incorporate multiple commodities and priorities of demand points. Our models also allow facility opening, closing and relocation within a budget; and this helps decision makers to judge the outcome of different alternatives. Since this is a large scale problem, we also propose a decomposition based solution algorithm and a heuristic algorithm to solve the problem in a timely fashion.

Chapter 3 Facility Location and Relocation Problem with Demand Change and Uncertain Number of Future Facilities

In this chapter, we introduce a mathematical model for the facility location and relocation problem with demand change and uncertain number of facilities. We will present complexity analysis and some insights about the model. We will then introduce a solution algorithm that solves the problem with near optimal solutions in a reasonable time.

3.1 Methodology

We will discuss two approaches to handle the facility location and relocation problem presented in Section 1.2.1. The first approach is adapted from Wang et al. [2003] and because of its deterministic nature it is named Facility Location and Relocation Problem - Deterministic (FLRP-D). Our proposed approach solves the problem under uncertainty and is called Facility Location and Relocation Problem - Uncertainty (FLRP-U).

3.2 Input Parameters and Notation

In order to formulate both models, we use the following input parameters:

$\mathbb{V}$: Set of all vertices

λ_i : Initial demand at vertex $i \in \mathbb{V}$,

λ'_i : Future demand at vertex $i \in \mathbb{V}$,

d_{ij} : Distance between vertex $i \in \mathbb{V}$ and vertex $j \in \mathbb{V}$,

p : Number of initial facilities,

q : Upper limit for the increase in number of future facilities,

ρ_r : probability that r facilities are added in the future,
where $r \in [0, q]$ and $\sum_{r=0}^{q} \rho_r = 1$,

o_j : Opening cost of facility at vertex $j \in \mathbb{V}$,

c_j : Closing cost of facility at vertex $j \in \mathbb{V}$,

b : Available budget for relocations.

3.3 Problem Formulation for FLRP-D

For FLRP-D formulation, we need to define some extra parameters. The notation $\mathbb{V}_1$ is the set of existing facilities and $\mathbb{V}_2$ is the set of potential facility sites, where $\mathbb{V} = \mathbb{V}_1 \cup \mathbb{V}_2$. In this method, $(p + r)$ will be the total number of desired facilities where the problem will be solved for all $r \in [0, q]$. We assume that initial facilities are located at the best possible sites, based on the initial weights. After the demand change occurs, relocations can be performed. We individually optimize each scenario, and calculate the expected weighted distance and cost. FLRP-D can be formulated as a Binary Integer Program (BIP) with two decision variables which are:

$$\nu_j = \begin{cases} 1, & \text{if facility at } v_j \in \mathbb{V} \text{ is open,} \\ 0, & \text{otherwise.} \end{cases}$$

$$\phi_{ij} = \begin{cases} 1, & \text{if demand at } v_i \in \mathbb{V} \text{ is assigned to facility at } v_j \in \mathbb{V}, \\ 0, & \text{otherwise.} \end{cases}$$

Then the problem formulation for FLRP-D for a given scenario r is

$$\min Z = \sum_{i \in \mathbb{V}} \lambda_i' \sum_{i \in \mathbb{V}} d_{ij} \phi_{ij} \tag{3.1}$$

$$\text{s.t.} \sum_{j \in \mathbb{V}_1} c_j (1 - \nu_j) + \sum_{j \in \mathbb{V}_2} o_j \nu_j \leq b, \tag{3.2}$$

$$\sum_{j \in \mathbb{V}} \nu_j = p + r, \tag{3.3}$$

$$\sum_{j \in \mathbb{V}} \phi_{ij} = 1, \qquad \forall i \in \mathbb{V}, \tag{3.4}$$

$$\phi_{ij} \leq \nu_j, \qquad \forall i, j \in \mathbb{V}, \tag{3.5}$$

$$\nu_j, \phi_{ij} \in \{0, 1\}, \quad \forall i, j \in \mathbb{V}. \tag{3.6}$$

The objective of the formulation is to minimize the total weighted distance. Constraint 3.2 is to ensure that opening and closure of the facilities are performed within the given budget. Constraint 3.3 makes sure that exactly $p + r$ facilities are located. Constraint 3.4 states that demand at each v_i is assigned to a facility. Constraint 3.5 makes sure that we cannot assign the demand at v_i to v_j unless a facility is located at v_j. The rest of the constraints are binary constraints for the decision variables.

3.4 Problem Formulation for FLRP-U

In FLRP-U, our aim is to find the best locations for the initial facilities and identify the possible relocations in the future knowing that the number of future facilities may increase by $r, r \in [0, q]$. The relocations are maintained by closing some of the

facilities that were located initially and opening new ones, and the objective is minimizing the total of the initial weighted distance and the expected future weighted distance traveled by customers. Unlike FLRP-D, location and relocation of facilities are optimized considering the future demand change and uncertain number of future facilities, simultaneously. FLRP-U can also be formulated as BIP. For FLRP-U formulation, there are six set of decision variables and they are defined as follows:
The number of additional facilities is given as r, for $r = 0, \ldots, q$,

$$\xi_j = \begin{cases} 1, & \text{if one of the initial } p \text{ facilities is located at } v_j, \text{ for } j = 1, \ldots, n, \\ 0, & \text{otherwise.} \end{cases}$$

$$\chi_{ij} = \begin{cases} 1, & \text{if demand at } v_i \text{ is assigned to facility at } v_j, \text{ for } i, j = 1, \ldots, n, \\ 0, & \text{otherwise.} \end{cases}$$

$$y_j^r = \begin{cases} 1, & \text{if facility at } v_j \text{ is selected to open, for } j = 1, \ldots, n, \\ 0, & \text{otherwise.} \end{cases}$$

$$z_j^r = \begin{cases} 1, & \text{if facility at } v_j \text{ is selected to close, for } j = 1, \ldots, n, \\ 0, & \text{otherwise.} \end{cases}$$

$$w_j^r = \begin{cases} 1, & \text{if facility at } v_j \text{ is open (facility exists), for } j = 1, \ldots, n, \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ij}^r = \begin{cases} 1, & \text{if demand at } v_i \text{ is assigned to facility at } v_j,\ i, j = 1, \ldots, n, \\ 0, & \text{otherwise.} \end{cases}$$

Then, the problem formulation is

$$\min Z = \sum_{r=0}^{q} \rho_r \sum_{i=1}^{n} \lambda'_i \sum_{j=1}^{n} d_{ij} x_{ij}^r + \sum_{i=1}^{n} \lambda_i \sum_{j=1}^{n} d_{ij} \chi_{ij} \tag{3.7}$$

$$\text{s.t.} \sum_{j=1}^{n} \left(c_j z_j^r + o_j y_j^r \right) \leq b, \qquad r = 0, \ldots, q, \tag{3.8}$$

$$\sum_{j=1}^{n} \xi_j = p, \tag{3.9}$$

$$\sum_{j=1}^{n} w_j^r = p + r, r = 0, \ldots, q, \tag{3.10}$$

$$w_j^r - \xi_j - y_j^r + z_j^r = 0, \qquad j = 1, \dots, n,\ r = 0, \dots, q, \tag{3.11}$$

$$y_j^r + z_j^r \leq 1, \qquad j = 1, \dots, n,\ r = 0, \dots, q, \tag{3.12}$$

$$\sum_{j=1}^{n} \chi_{ij} = 1, \qquad i = 1, \dots, n, \tag{3.13}$$

$$\sum_{j=1}^{n} x_{ij}^r = 1, \qquad i = 1, \dots, n,\ r = 0, \dots, q, \tag{3.14}$$

$$\chi_{ij} \leq \xi_j, \quad i, j = 1, \dots, n, \tag{3.15}$$

$$x_{ij}^r \leq w_j^r, \quad i, j = 1, \dots, n,\ r = 0, \dots, q, \tag{3.16}$$

$$\xi_j, \chi_{ij} \in \{0, 1\},\ \ i, j = 1, \dots, n, \tag{3.17}$$

$$y_j^r, z_j^r, w_j^r, x_{ij}^r, \xi_j, \chi_{ij} \in \{0, 1\},\ \ i, j = 1, \dots, n,\ r = 0, \dots, q. \tag{3.18}$$

The objective of the problem is to minimize the total weighted distance which is the summation of initial weighted distance and expected future weighted distance. Constraint 3.8 is the budget limitation. Constraints 3.9 and 3.10 make sure that p facilities are located initially, and $p + r$ facilities are located in the future. Constraint 3.11 sets the conditions for existence of a facility in future scenarios. Constraint 3.12 prevents simultaneous opening and closing of a facility in each scenario. Constraint 3.13 and 3.14 make sure that we assign the demand at each v_i to a facility in the initial case and future scenarios, respectively. Constraint 3.15 and 3.16 ensure that we do not assign the demand at v_i to a facility unless a facility is located at v_j in the initial case and future scenarios, respectively. The rest of the constraints are binary constraints for the decision variables.

3.4.1 Complexity Analysis

Theorem 1 below states that FLRP-D is *NP-Hard*. We now attempt to show the complexity of FLRP-U by reducing the problem to FLRP-D or its equivalent formulation.

Theorem 1. *FLRP-D is NP-hard. [Wang et al., 2003].*

To find an equivalent formulation of FLRP-D, we first reformulate FLRP-D with different variables and parameters following a similar structure to the formulation of FLRP-U.

Let ξ_j be a parameter that assumes the value of 1 if one of the p facilities is located at v_j initially and 0 otherwise. Then the decision variables are as follows:

$$y_j = \begin{cases} 1 & \text{if facility at } v_j \text{ is selected to open in the future,} \\ & \text{for } j = 1, \ldots, n, \\ 0 & \text{otherwise.} \end{cases}$$

$$z_j = \begin{cases} 1 & \text{if facility at } v_j \text{ is selected to close in the future,} \\ & \text{for } j = 1, \ldots, n, \\ 0 & \text{otherwise.} \end{cases}$$

$$w_j = \begin{cases} 1 & \text{if facility at } v_j \text{ is open (facility exists) in the future,} \\ & \text{for } j = 1, \ldots, n, \\ 0 & \text{otherwise.} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if demand at } v_i \text{ is assigned to facility at } v_j \text{ in the future,} \\ & \text{for } j = 1, \ldots, n, \\ 0 & \text{otherwise.} \end{cases}$$

Then the reformulation for FLRP-D for a given scenario r is:

$$\min Z = \sum_{i \in \mathbb{V}} \lambda_i' \sum_{i \in \mathbb{V}} d_{ij} x_{ij} \tag{3.19}$$

$$\text{s.t.} \sum_{j=1}^{n} (c_j z_j + o_j y_j) \leq b, \tag{3.20}$$

$$\sum_{j=1}^{n} w_j^r = p + r, \quad r = 0, \ldots, q, \tag{3.21}$$

$$w_j - \xi_j - y_j + z_j = 0, \qquad j = 1, \ldots, n, \tag{3.22}$$

$$y_j + z_j \leq 1, \qquad j = 1, \ldots, n, \tag{3.23}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1, \ldots, n, \tag{3.24}$$

$$x_{ij} - w_j \leq 0, \qquad i, j = 1, \ldots, n, \tag{3.25}$$

$$w_j, y_j, z_j, x_{ij} \in \{0, 1\}, \quad i, j = 1, \ldots, n. \tag{3.26}$$

Propositon 1. The original formulation and the reformulation of FLRP-D are equivalent.

Proof. The objective functions of the original formulation of FLRP-D and its reformulation are the same. Therefore, we will show the equivalence of the two formulations by performing the following steps. If a facility is open at node j, variables w_j and ν_j are equal to 1, and 0 otherwise, in their corresponding formulations. Therefore, we can put s_j instead of ν_j in the original formulation. By using the constraint 3.22 in the reformulation, we can rewrite the constraint 3.2 in the original formulation. In the first part of the constraint, for $\mathbb{V}_1$, since the only possible operation in $\mathbb{V}_1$ is closing a facility, $w_j = \xi_j - z_j$. In the second part, for $\mathbb{V}_2$, since the only possible operation in $\mathbb{V}_2$ is opening a new facility, $w_j = \xi_j + y_j$. Then, the constraint in the original formulation can be modified as follows:

$$\sum_{j \in \mathbb{V}_1} c_j \left(1 - \xi_j + z_j\right) + \sum_{j \in \mathbb{V}_2} o_j(\xi_j + y_j) \leq b. \tag{3.27}$$

As we know that $\xi_j = 1$ for $\mathbb{V}_1$ and $\xi_j = 0$ for $\mathbb{V}_2$, the constraint becomes

$$\sum_{j \in \mathbb{V}_1} c_j \left(z_j\right) + \sum_{j \in \mathbb{V}_2} o_j(y_j) \leq b. \tag{3.28}$$

In order to combine the two parts, we add the constraint $y_j + x_j \leq 1$ to make sure that at most one of those actions (opening and closing) can be performed, which ensures that there is no intersection between $\mathbb{V}_1$ and $\mathbb{V}_2$. $\square$

Corollary 1. *FLRP-U is NP-hard.*

Proof. If we formulate FLRP-U for $r = 0$ and relax the constraints 3.9, 3.14 and 3.16, the problem reduces to the reformulation of FLRP-D. Due to the increase in the number of variables and constraints, solving FLRP-U for multiple scenarios is much harder than solving the problem for a single scenario, when $r = 0$. Therefore, by Theorem 1 and Proposition 1 we claim that FLRP-U is NP-hard. □

3.4.2 Lower Bound Generation

The problem formulation for the FLRP-U is computationally hard when the number of facilities and future scenarios increase. However, relaxing the binary constraints decreases the computational burden and mostly yields binary solutions [Rosing et al., 1979]. If we relax the binary constraints for the variables x_{ij}^r, χ_{ij} and w_j^r, we can obtain a lower bound for the problem. Relaxing only the binary constraint for variable w_j^r, will also improve the computational time slightly, however it will guarantee an integer solution. This is because even though we relax w_j^r, binary variables x_j^r, y_j^r, z_j^r and the Constraint 3.11 in FLRP-U formulation force w_j^r to be binary.

3.5 Solution Method

As we have proved, FLRP-U is *NP-hard*, and the problem size rapidly grows when we increase the number of facilities as well as future scenarios. Therefore, BIP formulation of the problem becomes hard to solve especially for large instances. To be able to solve the problem in a timely manner, we analyzed the formulation and observed the block-angular structure of the problem, which is suitable for a decomposition approach [Sweeney and Murphy, 1979]. Then, we developed a decomposition algorithm to solve FLRP-U, where initial and future scenarios for facility locations demonstrate the blocks and the rest of the constraints form the bridge constraints.

The block-angular structure of the problem can be observed better in the following formulation. The first three sets of constraints are bridge constraints, and facility location problems for initial and future scenarios are the blocks connected by the bridge constraints.

$$
\begin{array}{llllll}
\text{Min} \ \sum\limits_{i=1}^{n} \lambda_i \sum\limits_{j=1}^{n} \chi_{ij} d_{ij} & +\rho_0 \sum\limits_{i=1}^{n} \lambda'_i \sum\limits_{j=1}^{n} x^0_{ij} d_{ij} & + \dots & +\rho_q \sum\limits_{i=1}^{n} \lambda'_i \sum\limits_{j=1}^{n} x^q_{ij} d_{ij} & & \\
\text{s.t.} & & & & & \\
-\xi_j & +w^0_j & & & -y^0_j + z^0_j & = 0 \\
 & & \ddots & & & \vdots \\
-\xi_j & & & +w^q_j & -y^q_j + z^q_j & = 0 \\
 & & & & \sum\limits_{j=1}^{n} o_j y^r_j + \sum\limits_{j=1}^{n} c_j z^r_j & \leq b \\
 & & & & y^r_j + z^r_j & \leq 1 \\
\sum\limits_{j=1}^{n} \xi_j & & & & & = p \\
\sum\limits_{j=1}^{n} \chi_{ij} & & & & & = 1 \\
-\xi_j + \chi_{ij} & & & & & \leq 0 \\
 & \sum\limits_{j=1}^{n} w^0_j & & & & = p \\
 & \sum\limits_{j=1}^{n} x^0_{ij} & & & & = 1 \\
 & -w^0_j + x^0_{ij} & & & & \leq 0 \\
 & & \ddots & & & \vdots \\
 & & & \sum\limits_{j=1}^{n} w^q_j & & = p + q \\
 & & & \sum\limits_{j=1}^{n} x^q_{ij} & & = 1 \\
 & & & -w^q_j + x^q_{ij} & & \leq 0.
\end{array}
$$

For the ease of illustration, the following variable and parameter substitutions will be used for the decomposition algorithm. Let t_k and ϵ be binary decision variables. The variable t_k is substituted for variables $\xi_j, \chi_{ij}, w^r_j, x^r_{ij}$, where k is the index for each block i.e., $t_1 = [\xi_1, \dots, \xi_n, \chi_{11}, \dots, \chi_{nn}]$, $t_2 = [w_1^0, \dots, w_n^0, x_{11}^0, \dots, x_{nn}^0]$, $\dots$, $t_{q+2} = [w_1^q, \dots, w_n^q, x_{11}^q, \dots, x_{nn}^q]$. The variable ϵ is substituted for variables y_j^r and z_j^r i.e., $\epsilon = [y_1^0, \dots, y_n^q, z_1^0, \dots, z_n^q]$. Let A and B represent the constraint coefficients and right hand side values of the constraints, respectively, and C represent the objective function coefficients.

Then FLRP-U formulation can be re-written as

$$
\begin{array}{lcccccccc}
\text{Min} & C_1 t_1 & +C_2 t_2 & +\dots & +C_{q+2} t_{q+2} & & & \\
\text{s.t.} & & & & & & & \\
& A_{011} t_1 & +A_{012} t_2 & +\dots & A_{01(q+2)} t_{q+2} & +A_{01(q+3)}\epsilon & = & B_{01} \\
& & & & & A_{02(q+3)}\epsilon & \leq & B_{02} \\
& & & & & A_{03(q+3)}\epsilon & \leq & B_{03} \\
& A_{11} t_1 & & & & & = & B_{11} \\
& A_{12} t_1 & & & & & = & B_{12} \\
& A_{13} t_1 & & & & & \leq & B_{13} \\
& & A_{21} t_2 & & & & = & B_{21} \\
& & A_{22} t_2 & & & & = & B_{22} \\
& & A_{23} t_2 & & & & \leq & B_{23} \\
& & & \ddots & & & \vdots & \\
& & & & A_{(q+2)1} t_{q+2} & & = & B_{(q+2)1} \\
& & & & A_{(q+2)2} t_{q+2} & & = & B_{(q+2)2} \\
& & & & A_{(q+2)3} t_{q+2} & & \leq & B_{(q+2)3}.
\end{array}
$$

If we apply Lagrangean relaxation to the bridge constraints with coefficient u, we will obtain $(q+2)$ sub-problems as

$$(SP_k) \qquad \begin{aligned} \text{Min} \quad & (C_k - uA_{01k})t_k \\ \text{s.t.} \quad & \\ & A_{k1}t_k = B_{k1} \\ & A_{k2}t_k = B_{k2} \\ & A_{k3}t_k \leq B_{k3} \\ & t_k \in \{0,1\}. \end{aligned}$$

Let t^* denote the optimal solution for each subproblem, and then a lower bound for the original problem can be obtained by the following equation as stated in Sweeney and Murphy [1979],

$$LB = (C_1 - uA_{011})t_1^* + (C_2 - uA_{012})t_2^* + \ldots + (C_{q+2} - uA_{01(q+2)})t_{q+2}^* + uB_{01}. \quad (3.29)$$

A good lower bound can be obtained by solving the LP relaxation of the sub-problems since LP relaxation of p-median problems usually leads to optimal or very close to optimal solutions [Rosing et al., 1979].

Let s_k be the set of columns that have been included in the master problem for subproblems $k = 1, \ldots, (q+2)$. Let $\tau_{s_k}^k$ be 1 when the corresponding column is in the optimal master problem solution and 0 otherwise. Then we can construct the master problem (MP) as

$$
(MP)\quad
\begin{aligned}
&\text{Min } \sum_{s_1}(C_1 t^1_{s_1})\tau^1_{s_1} + \sum_{s_2}(C_2 t^2_{s_2})\tau^2_{s_2} + \ldots + \sum_{s_{q+2}}(C_{q+2} t^{q+2}_{s_{q+2}})\tau^{q+2}_{s_{q+2}}\\
&\text{s.t.}\\
&\sum_{s_1} A_{011} t^1_{s_1} + \sum_{s_2} A_{012} t^2_{s_2} + \ldots + \sum_{s_{q+2}} A_{01(q+2)} t^{q+2}_{s_{q+2}} + A_{01(q+3)}\epsilon = B_{01},\\
&A_{02(q+3)}\epsilon \leq B_{02},\\
&A_{03(q+3)}\epsilon \leq B_{03},\\
&\sum_{s_1}\tau^1_{s_1} = 1,\\
&\sum_{s_2}\tau^2_{s_2} = 1,\\
&\ddots\\
&\sum_{s_{q+2}}\tau^{q+2}_{s_{q+2}} = 1.
\end{aligned}
$$

The parameter B_{01}, which is the right-hand-side value of the bridge constraints that connect the subproblems, is equal to 0. Therefore, equation (3.29) implies that selecting the Lagrangean coefficient (u) as a nonzero value may not have a big impact on the lower bound quality. So we select u to be 0 and each subproblem becomes a weighted p-median problem. Weights in the objective function of the first subproblem are the initial weights of the vertices (λ_i, for $i = 1, \ldots, n$). Weights in the objective function of the other subproblems are the multiplication of scenario probability by future weights of the vertices ($\rho_k \lambda'_i$, for $k = 2, \ldots, (q+2)$, $i = 1, \ldots, n$).

The column sets for the master problem are obtained by solving each subproblem using a modified version of the Discrete Lloyd Algorithm (DLA), a heuristic solution technique that can generate good upper bounds for the p-median problem [Lim et al., 2009]. DLA can be used for problems that are on a real network with uniform vertex weights. The algorithm starts with an initial set of medians and divides the network into p clusters by assigning each vertex to its closest median. For each cluster, center of gravity is determined and projected to the closest vertex in the network, which will construct an updated set of medians. The procedure is repeated until there is no change in the median locations. Since our subproblems are weighted p-median problems, we make a slight modification to their algorithm when calculating the center of gravity by incorporating the vertex weights.

The structure of our proposed solution algorithm is explained in Figure 3.1.

Initialize: $i \leftarrow$ 0,
$m \leftarrow$ Maximum number of iterations
$\delta^* \leftarrow$ Desired dual gap

1. Solve the LP relaxation of (SP_k) for $k = 1, \ldots, q+2$ and calculate the lower bound, $\underline{z}$, (3.29)

do{
2. Create multiple feasible solutions for (SP_k) for $k = 1, \ldots, q+2$ using DLA
Add those feasible solutions as columns to (MP)
3. Solve (MP) and obtain the upper bound, $\overline{z}$
4. Calculate the actual dual gap
$\delta_i = (\overline{z} - \underline{z})/\underline{z}$
5. $i \leftarrow i + 1$

}while$\{(\delta_i > \delta^*) and (i \leq m)\}$

Figure 3.1: Decomposition algorithm for FLRP-U

The algorithm starts by initializing the parameters. The lower bound ($\underline{z}$) for FLRP-U is calculated by adding up the objective function values of the LP relaxation of each subproblem. Then, multiple feasible solutions for each subproblem are created using DLA. An upper bound for FLRP-U ($\overline{z}$) is obtained by solving the master problem and the actual dual gap is calculated by $(\overline{z} - \underline{z})/\underline{z}$. If the actual dual gap is greater than the desired dual gap and maximum number of iterations has not been reached, we create more solutions for the subproblems and continue the algorithm.

3.6 Numerical Results

In this section, we present our numerical results for the experiments that are run to test the performance of the methods discussed in this chapter. We will first illustrate FLRP-U on a small network and present the numerical results that are used to compare FLRP-U and FLRP-D. Then, we will perform sensitivity analysis for FLRP-U. We will also show the results that compare the performance of our decomposition algorithm with the exact solution technique, the mathematical model we introduced in Section 3.4. All numerical results presented in this section were run on a Pentium 4 Xeon 3.6 Ghz machine with 4 GB RAM.

3.6.1 An Illustration with 1-median problem

A small example for FLRP is illustrated in Figure 3.2. The network has six nodes and nine arcs. Numbers near the nodes are the initial and forecasted future demand $(initial, future)$. Numbers on the arcs represent the distances between the nodes. Suppose we need to locate one facility initially, and we may locate up to

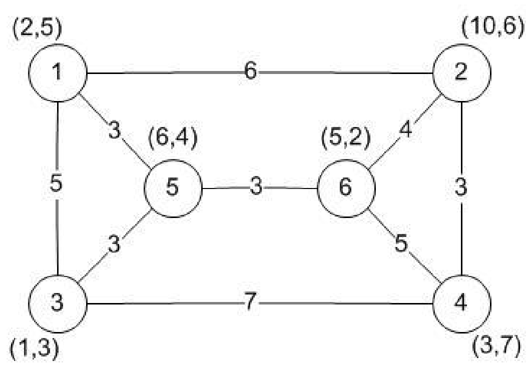


Figure 3.2: An illustrative 6 node network

two more facilities in the future, i.e., $q = 2$. The probability of adding one facility and two facilities are assumed to be 0.3, respectively, while the probability of adding no facilities is 0.4. A budget of 500 is available for the facility opening and closing operations. Based on the initial weights, optimum one median is located at node 6. Based on FLRP-D, the optimal decision when $r = 0$ is to close the facility at node 6 and open a new facility at node 2. The optimal solution for $r = 1$ is to keep the existing facility at node 6 and open another one at node 4. For $r = 2$, it is to keep the existing facility at node 6 and to open two new facilities at nodes 3 and 4. On the other hand, FLRP-U proposes to open the initial facility at node 2. In the future, when $r = 0$, the optimal decision is to keep that facility. When $r = 1$, FLRP-U suggests that we keep the existing facility and open a new facility at node 5. When $r = 2$, we should again keep the existing facility at node 2, and open new facilities at nodes 1 and 3.

The total weighted distance comparison for FLRP-D and FLRP-U is shown in Table 3.1. Table 3.2 shows the expected budget consumption for both methods. Expected weighted distance and expected budget consumption is calculated by multiplying the probability of each scenario by the corresponding weighted distance and budget consumption, respectively.

Table 3.1: Total weighted distance for FLRP-D and FLRP-U

Method	Weighted Distance					
	initial	r=0	r=1	r=2	expected	total
FLRP-D	91	117	78	55	86.7	177.7
FLRP-U	93	117	51	41	74.4	167.4

Table 3.2: Budget consumption for FLRP-D and FLRP-U

Method	Budget Consumption			
	r=0	r=1	r=2	expected
FLRP-D	280	259	494	337.9
FLRP-U	0	293	497	237

As can be seen from these tables, FLRP-U yields a better objective function value and lower budget consumption. Since this is a small example with a small budget, FLRP-U did not propose closing of the existing facility in any scenario. However, for larger cases, it considers the relocation opportunity to find the best solutions.

3.6.2 FLRP-U v.s. FLRP-D

As we mentioned in Section 3.1, FLRP-D assumes that initial facilities are located at the best possible sites, based on the initial weights. For comparison purposes, future weighted distances for each scenario are used to calculate the expected weighted distance. Summation of expected weighted distance and initial weighted distance yields the total weighted distance, which forms the main objective function.

The FLRP-U was tested on 20 randomly generated networks with 250 nodes in each network. Table 3.3 shows the parameters that are generated randomly from their corresponding uniform distributions. Table 3.4 shows different instances for q values and corresponding probability for each instance. The number of facilities that will initially be located, p, is set to 5.

Table 3.3: Parameters

Parameter	Uniform Distribution
Initial Demand (λ)	[100, 200]
Future Demand (λ')	[50, 250]
Opening Cost (o)	[200, 300]
Closing Cost (c)	[50, 100]

Table 3.4: Future instances and scenario probabilities

Upper Limit on	Scenarios		
Number of Future Facilities (q)	$r = 0$	$r = 1$	$r = 2$
0	1	-	-
1	0.5	0.5	-
2	0.4	0.3	0.3

Our experiments consist of five budget levels $500, 750, 1000, 1500$ and 3000. We also ran the experiments with the same parameters for FLRP-D for comparison purposes. All experiments are coded in GAMS [Brooke et al., 2009] and solved by CPLEX. Percent difference of the objective function values are calculated by subtracting average objective function values of FLRP-U from FLRP-D and dividing by FLRP-U for all scenarios and budget values.

Figures 3.3, 3.4, and 3.5 illustrate initial weighted distance, expected weighted distance, and the total weighted distance differences between FLRP-U and FLRP-D when $q = 0$, $q = 1$, and $q = 2$, respectively.

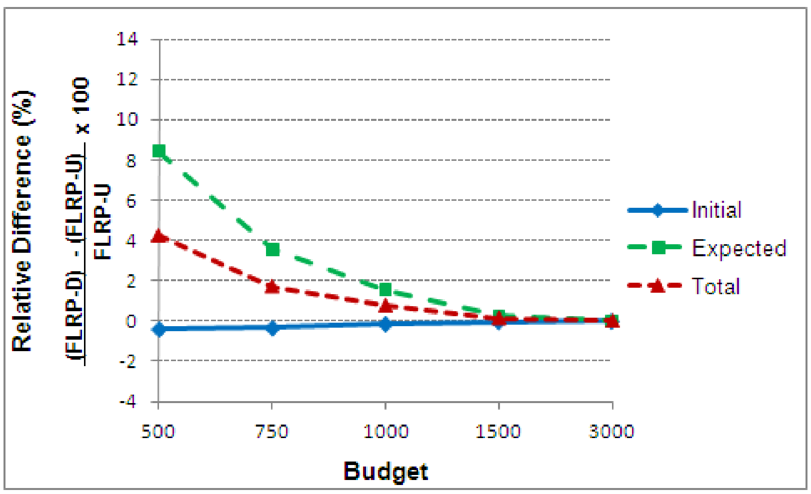


Figure 3.3: Weighted distance difference between FLRP-D and FLRP-U, $q = 0$

The initial weighted distance of FLRP-U is always higher than the initial weighted distance of FLRP-D for all cases. This is not surprising because initial weighted distance for FLRP-D is the optimal solution based on the initial weights. With respect to the initial weights, the difference between FLRP-D and FLRP-U becomes smaller for all scenarios as we increase the budget. This gap also decreases when the upper bound for the number of future facilities decreases.

For the expected weighted distance, we observe that FLRP-U produces better results in all instances. Also, it performs better when there is less available budget.

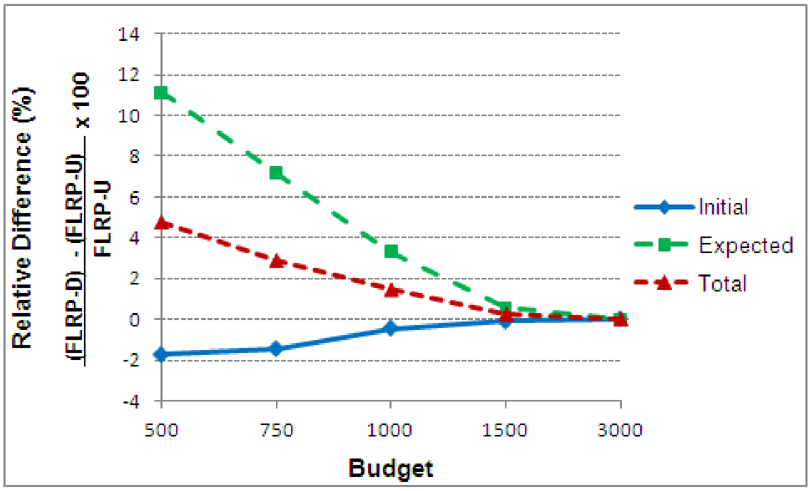


Figure 3.4: Weighted distance difference between FLRP-D and FLRP-U, $q = 1$

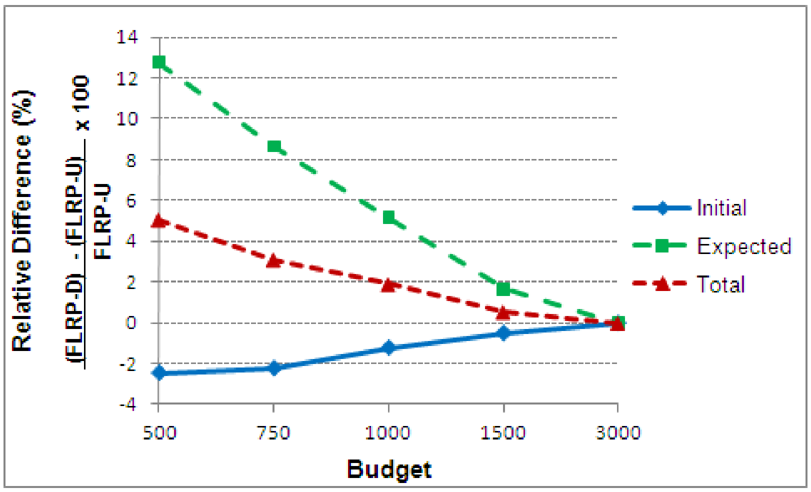


Figure 3.5: Weighted distance difference between FLRP-D and FLRP-U, $q = 2$

Opposite to the initial weighted distance values, the gap between the methods increases when the upper bound for the future facilities increases. We observe a larger difference between the results of FLRP-U and FLRP-D for $q = 2$, compared to $q = 1$ or $q = 0$. In FLRP-U, we let the initial weighted distance increase slightly in order to handle the demand changes and uncertain number of future facilities. However, in FLRP-D, initial decisions are made without considering the future uncertainties, and it is assumed that necessary relocations will be performed after observing the demand change. Figure 3.6 shows the difference of total weighted distance between FLRP-U and FLRP-D for different budget values.

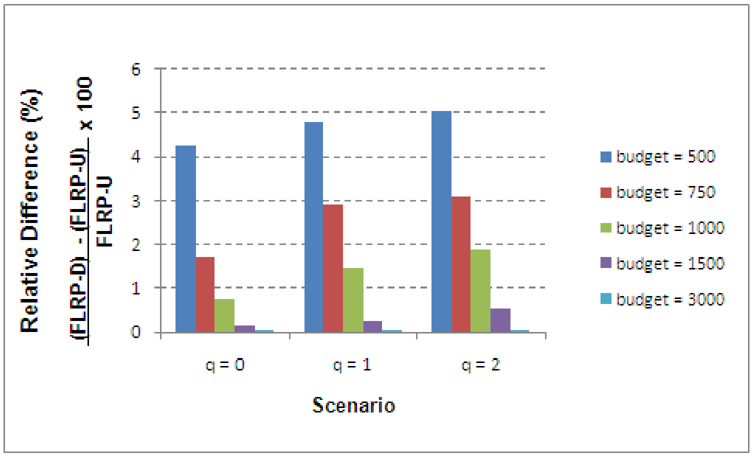


Figure 3.6: Total weighted distance difference between FLRP-D and FLRP-U wrt future scenarios for different budget values

We make three observations based on these experiments. First, the initial weighted distance for FLRP-D is always lower than or equal to FLRP-U. This gap decreases when either the budget is increased or the upper limit of the number of future facilities is decreased. Second, expected future weighted distance for FLRP-U is lower than or equal to that of FLRP-D. This gap increases when we either decrease the budget or increase the upper limit of the number of future facilities. Third, the two objectives explained above are contradicting. However, this contradiction results in favoring FLRP-U. Since the difference of the two methods for expected future weighted distance and initial weighted distance is higher in FLRP-D, FLRP-U yields better total weighted distance for the test problem instances. This gap also follows a similar pattern with the expected future weighted distance; it increases when either the budget is decreased or the upper limit of the number of future facilities is increased.

3.6.3 Computational Performance of FLRP-U

As we mentioned in section 3.4.1, FLRP-U is computationally intractable when we have larger networks and scenarios. In order to improve solution time, we utilized different options in CPLEX such as pricing options for primal and dual simplex methods that are used in branch and bound to solve the mixed integer problem. The same instances as in Section 3.6.2 are tested, and average solution times of the instances are reported.

CPLEX has five pricing options for primal simplex which are reduced cost pricing, hybrid reduced cost and devex pricing, devex pricing, steepest edge pricing, steepest edge pricing with slack initial norms, and full pricing. Options for dual simplex are standard dual pricing, steepest edge pricing, steepest edge pricing in slack space, steepest edge pricing unit initial norms, and devex pricing. We have also tried the barrier method which is another LP solution technique. The average solution time (cpu seconds) of 20 instances for $q = 0, 1$, and 2 using default options, barrier method and best pricing options for primal and dual simplex method are shown in Table 3.5.

Table 3.5: Average solution times for the FLRP-U using CPLEX options

	Default	Primal Simplex	Dual Simplex	Barrier
q=0	56.61	56.46	51.94	58.66
q=1	179.53	180.91	178.37	125.83
q=2	441.31	438.33	438.02	467.46

Pricing options for primal simplex yielded similar results, where hybrid reduced cost and devex pricing performed the best among all others. For dual simplex, the best method is full pricing. Overall, considering all pricing techniques, primal simplex yields equal or better results than dual simplex.

Another option that can be utilized to improve the solution time is multiple threads with deterministic or opportunistic parallel mode. However, in our experiments, we did not observe any improvement with neither opportunistic nor deterministic mode. Compared to opportunistic, deterministic parallel mode works better. However, in both options, no matter whether two or four threads are used, solution time increases. Utilizing pricing options lowers the solution time for both techniques, but they are still slower compared to the original solution time.

As we mentioned in Section 3.4.2, relaxing the binary constraint for s_j^r guarantees integer solution for the problem. The solution time for this relaxation is shorter than the original problem solution time when we have more than one scenario. We also tested pricing and multi-thread options for this formulation. We did not observe any improvement in using multiple threads. The average solution time (cpu seconds) of 20 instances for $q = 0, 1$, and 2 using default options, barrier method and best pricing options for primal and dual simplex method are shown in Table 3.6.

Table 3.6: Average solution times for the relaxed FLRP-U using CPLEX options

	Default	Primal Simplex	Dual Simplex	Barrier
q=0	61.47	61.28	53.43	61.31
q=1	129.53	128.51	127.92	128.81
q=2	320.76	317.88	277.38	318.64

As we observe in Table 3.6, results for primal simplex with different options are very close to each other and also to the solution time with default options. Results for dual simplex vary significantly. However, full pricing outperforms the others.

3.6.4 Sensitivity Analysis for FLRP-U

In this section we present sensitivity analysis for FLRP-U based on different budget values. This analysis provides an insight about the trade-off between the allocated budget for relocations and the total weighted distance traveled by customers to their closest facilities. As we discussed in Section 2, extra sources for the budget can be utilized in real life. Therefore, this analysis also helps a decision maker to explore various budget options considering the possibility of additional resources.

Sensitivity analysis was conducted on 20 randomly generated networks, each one having 250 nodes. The same parameters in Tables 3.3 and 3.4 were used. The budget for opening and closing facilities are set to be 500, 750, 1000, 1500, 3000, and 5000. Figure 3.7 shows a summary of the results.

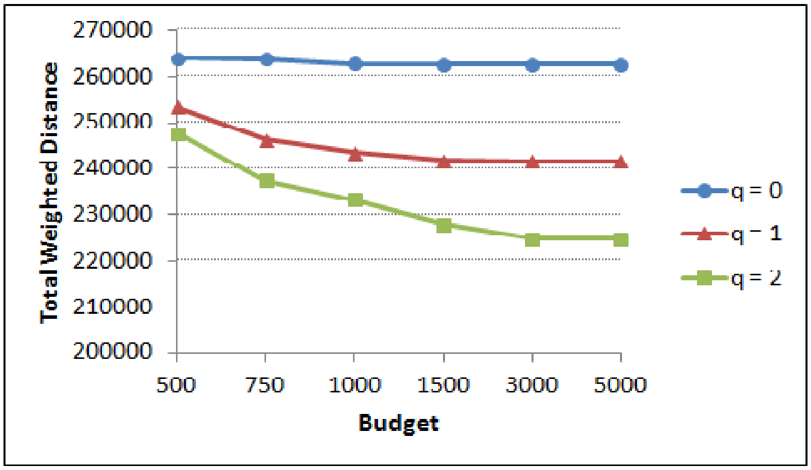


Figure 3.7: Trade-off curves between the budget and total weighted distance for q values

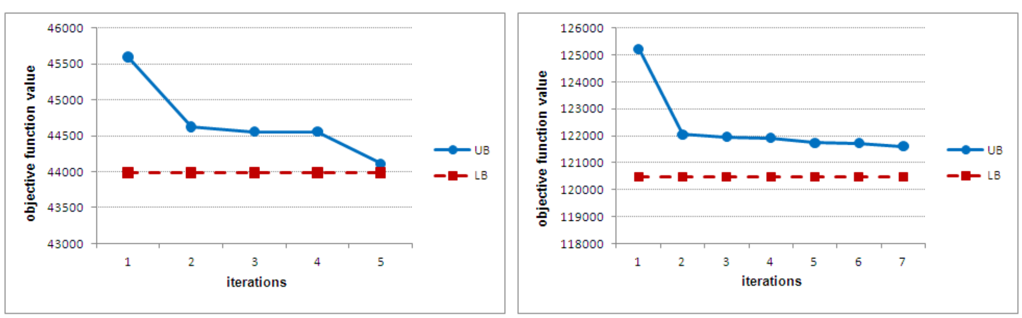
For $q = 0$, the line is almost flat, which means that allocating extra budget for such situations does not provide much improvement. This shows that, when there is no increase in the number of future facilities, the total of initial and expected future weighted distance does not depend too much on the available budget for relocations due to the fact that there is no uncertainty in the number of future facilities. For $q = 1$ and $q = 2$, the impact of extra budget is more evident, especially from 500 to 750. The rest of the improvement converges to a constant. This shows that allocating extra budget for relocations beyond 3000 may not further improve the objective function value.

Overall, the increase in the budget from 500 to 750 provides significant decrease in the objective function value, especially for $q > 0$. Further increases up to 3000 are also beneficial; however, the slope of the lines are getting smaller compared to the former. The impact of budget increase on the total weighted distance is more apparent for larger value of q.

3.6.5 Decomposition Algorithm for FLRP-U

In order to test the computational performance of the Decomposition Algorithm, we generated 20 random networks of size 100, 250 and 500, respectively. The same parameters in Table 3.3 were used. The initial number of facilities, p, is assumed to be five and the instance $q = 2$ in Table 3.4 is considered since it is more complicated than the other two instances. The budget for opening and closing facilities was set to 1200 for all scenarios. The number of initial columns created for the master problem and additional columns throughout the iterations are selected to be 10, 30, and 50 for network sizes 100, 250, and 500, respectively. This column size selection was made in order to have feasible master problems. For each instance, the algorithm is run for desired dual gap values 1%, 2%, 3%, 4%, 5%, and 10%. We coded the algorithm in C++ and used CPLEX [IBM, 2009] to obtain the LB and solve the MP. Experiment instances are also solved by an exact solution method, which is the BIP formulation introduced in Section 3.4. The exact method is formulated in GAMS [Brooke et al., 2009] and solved by CPLEX [IBM, 2009], and for each instance we set the relative termination tolerance to the same desired dual gap values.

Figures 4.3(a) and 4.3(b) illustrate the convergence of the algorithm for two instances of network sizes 100 and 250. First, the lower bound is calculated using Equation 3.29 once and it is fixed throughout the iterations. Then the rest of the algorithm attempts to minimize the upper bound by adding columns to the MP in each iteration.



(a) n = 100 (b) n = 250

Figure 3.8: Convergence of the decomposition algorithm

Table 3.7 shows the average actual dual gap for all network sizes and desired dual gaps for both the exact solution method and decomposition algorithm.

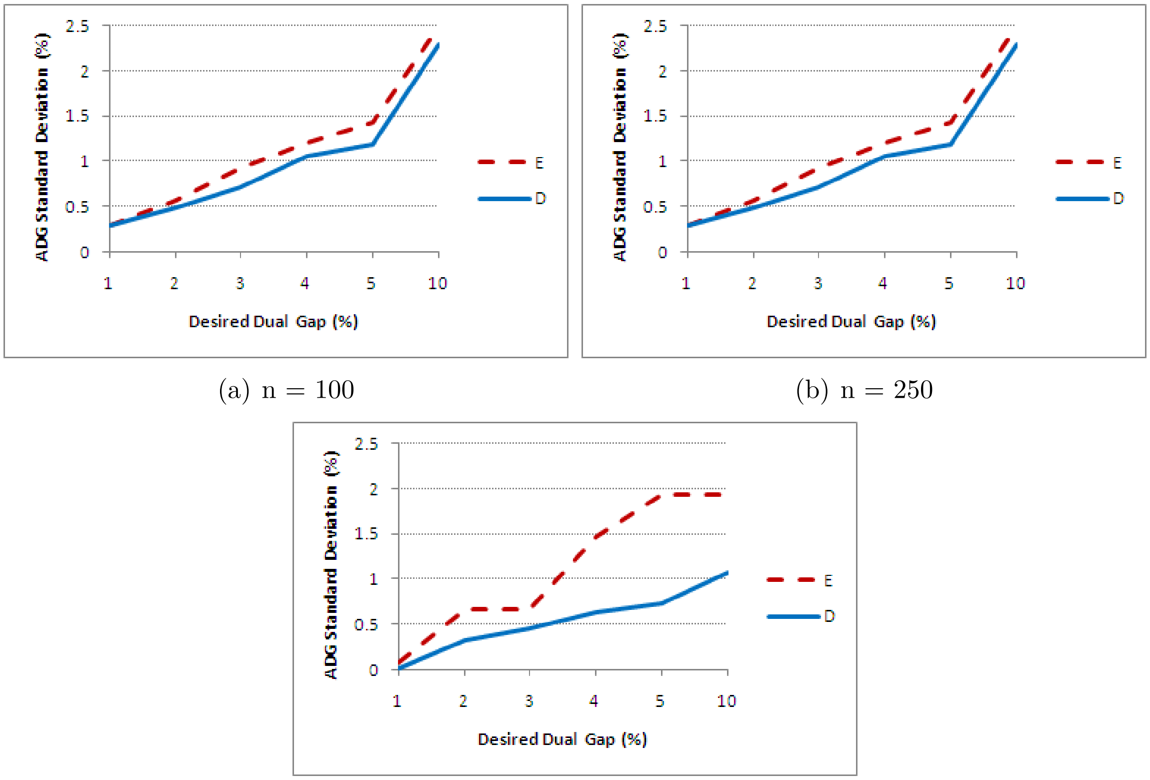
The average solution time in CPU seconds and percent time gain for each case are also compared. The percent time gain is calculated by subtracting the average solution time of the exact method from the decomposition algorithm and dividing it by the average solution time of the decomposition algorithm.

Table 3.7: Decomposition Algorithm and Exact Solution Method Comparison

	Stopping	Dual Gap (Average)		Solution Time (Average)		
	Criterion	Exact	Decomp	Exact	Decomp	Gain (%)
n=100	1%	0.17%	0.38%	13.08	3.97	69.63
	2%	0.38%	1.20%	12.00	3.61	69.94
	3%	0.59%	1.78%	11.45	3.33	70.91
	4%	0.83%	2.30%	11.21	3.36	70.01
	5%	0.92%	2.36%	10.66	3.27	69.28
	10%	1.64%	2.76%	10.83	3.32	69.35
n=250	1%	0.18%	0.51%	468.04	117.53	74.89
	2%	0.30%	1.10%	443.59	96.73	78.19
	3%	0.56%	1.47%	414.83	95.62	76.95
	4%	0.76%	1.50%	402.38	94.84	76.43
	5%	0.81%	1.73%	402.08	95.16	76.33
	10%	1.46%	1.78%	373.74	95.66	74.41
n=500	1%	0.14%	0.76%	10028.00	1378.50	86.25
	2%	0.43%	1.11%	9049.90	1358.90	84.98
	3%	0.43%	1.72%	9102.60	1356.90	85.09
	4%	1.07%	1.64%	8583.20	1355.00	84.21
	5%	1.92%	2.01%	7756.40	1356.30	82.51
	10%	1.92%	2.32%	7503.54	1357.30	81.91

As we can observe from Table 3.7, objective function values for both methods are within the desired dual gap, which we used as a stopping criterion for both the exact method and decomposition algorithm. However, there is a substantial gain in the average CPU time of the decomposition algorithm compared to the exact approach, up to 86.5 %.

Standard deviation of the actual dual gap of both methods for each network size is plotted with respect to the desired dual gap values in Figure 3.9. This figure shows that variability in the actual dual gap for both methods is less in the decomposition algorithm for most of the instances.



(a) n = 100

(b) n = 250

(c) n = 500

Figure 3.9: Standard Deviation of δ for Exact Solution Method (E) and Decomposition Algorithm (D) for network sizes $n = 100$, 250 and 500.

From Table 3.7 and Figure 3.9, we can conclude that the average dual gap and standard deviation of both methods increase when we increase the desired dual gap value. We have also observed that the standard deviation for the decomposition algorithm is usually smaller than the exact solution method.

3.6.6 Performance of FLRP-U for Large p Values

FLRP-U can be classified as a combinatorial optimization problem. Combinatorial optimization problems try to find the optimal solution among an extremely large number of possible solutions that increase with the problem size. Therefore, we analyzed the impact of increasing the p value on the FLRP-U solution time. For these experiments, we created 25 random networks of size 100 and 250, and parameters in Table 3.3 are used. The upper limit on the increase in the number of future facilities, q, is set to be five with the following scenario probabilities: $\rho = 0.25, 0.25, 0.2, 0.15, 0.1, 0.05$. The p values are increased up to the $N/2$ for both cases. Average solution times in CPU seconds for all instances are reported in Table 3.8.

Table 3.8: Solution time of FLRP-U for large p values

Number of facilities	N = 100	N = 250
5	25	929
25	21	864
50	19	579
100	-	376
125	-	326

We observed in our experiments that an increase in the p value does not have a negative impact on solution time. This follows the conclusion driven by Senne et al. [2005]. They state that the difficulty for solving the p-median problem increase with the increase of the N/p ratio. The same conclusion can be made for FLRP-U; the solution time increases for larger N/p ratios.

3.7 Summary

In this chapter, we introduced the facility location problem that considers future demand changes as well as uncertain number of future facilities. The objective is to minimize the sum of the initial weighted distance and the expected future distance traveled by customers to their closest facilities without exceeding a given budget for opening and closing facilities. As we discussed in Chapter 2, there are few approaches that consider closing of facilities, which is necessary to handle demand changes. Therefore, we presented FLRP-U; a method that determines the initial locations and future relocations of facilities based on the fact that the number of future facilities is not known exactly. We introduced a BIP formulation for FLRP-U. We have also discussed an alternative method, which is adapted from another method found in the literature, that can be used to handle such situations. The alternative model was named FLRP-D. We have proved that the BIP formulation of FLRP-U is *NP-hard* and we have developed a decomposition based solution algorithm to solve the problem. Then, we first presented the numerical results that compare the performance of FLRP-U against FLRP-D. Based on the average total weighted distance, FLRP-U outperformed FLRP-D in all scenarios, which shows that FLRP-U methodology is a valuable contribution to solve such problems. We also performed sensitivity analysis for FLRP-U to observe the trade off between the budget and total weighted distance. We then presented numerical results that compare the objective function value and solution time of our decomposition algorithm with the BIP formulation of FLRP-U, concluding that our proposed method yields a significant time gain while satisfying the desired dual gap level. Finally, we tested FLRP-U with larger p values. We observed that increasing the p value does not increase the solution time.

Chapter 4 Robust Facility Relocation Problem

In this chapter, we present two approaches to solve the robust facility relocation problem that considers relocation of facilities under uncertain demand. As we have discussed in Section 2.2.1, there are various robust approaches to handle demand uncertainty. Classical regret methods are very conservative because the worst-case scenario dominates the outcome, even though it may be less likely to occur in reality. For most of the public and private facilities, decisions made by considering the worst case scenario may be too pessimistic because many non-extreme scenarios may happen with certain probabilities. Therefore, we utilize variations of robust optimization models discussed in the literature.

In the first approach, we introduce a mathematical model that minimizes the maximum regret associated with a set of scenarios whose cumulative probability is at least α. This approach will be named α-reliable facility relocation problem, α-ReFRP in short. In the second approach, we present a mathematical model that minimizes the expected weighted traveling distance while making sure that relative regret for each scenario is less than γ. This approach will be named γ-robust facility relocation problem, γ-RoFRP. We will also introduce a Lagrangean Decomposition based solution algorithm to solve γ-RoFRP.

The regret associated with a scenario, which is utilized in both approaches, is calculated as the difference between the total weighted distance corresponding to a location decision and the optimal total weighted distance under that scenario. An optimal weighted distance for each scenario is obtained by solving the deterministic facility relocation problem introduced by Wang et al. [2003], which will be referred to as $dFRP$ in Chapters 4 and 4.3. The following notations and input parameters are used to formulate $dFRP$, α-ReFRP, and γ-RoFRP:

$\mathbb{V}_1$	: Set of existing facilities
$\mathbb{V}_2$	: Set of potential facilities
$\mathbb{V} = \{\mathbb{V}_1 \cup \mathbb{V}_2\}$	: Set of all vertices
$\mathbb{S}$	: Set of all demand scenarios
λ_{ik}	: Demand at vertex $i \in \mathbb{V}$ at scenario $k \in \mathbb{S}$
d_{ij}	: Distance between vertex $i \in \mathbb{V}$ and vertex $j \in \mathbb{V}$
p	: Number of initial facilities
q	: Number of final facilities
o_j	: Opening cost of facility at vertex $j \in \mathbb{V}_2$
c_j	: Closing cost of facility at vertex $j \in \mathbb{V}_1$
b	: Available budget for relocations
β_k	: Probability of scenario $k \in \mathbb{S}$
ζ_k^*	: Optimal objective function value of $dFRP$ for scenario $k \in \mathbb{S}$
ψ_k	: A large constant specific to scenario $k \in \mathbb{S}$
α	: Desired robustness level
γ	: Maximum value of relative regret permitted for each scenario

We have two sets of decision variables for $dFRP$.

$$\nu_j = \begin{cases} 1, & \text{if facility at } v_j \text{ is open, } \forall j \in \mathbb{V}, \\ 0, & \text{otherwise.} \end{cases}$$

$$\phi_{ij} = \begin{cases} 1, & \text{if demand at } v_i \text{ is assigned to facility at } v_j, \forall i, j \in \mathbb{V}, \\ 0, & \text{otherwise.} \end{cases}$$

Then the problem formulation for $dFRP$ for a given scenario k is

$$P1: \qquad \min \zeta_k = \sum_{i \in \mathbb{V}} \sum_{i \in \mathbb{V}} \lambda_{ik} d_{ij} \phi_{ijk} \tag{4.1}$$

$$\text{s.t.} \sum_{j \in \mathbb{V}_1} c_j (1 - \nu_j) + \sum_{j \in \mathbb{V}_2} o_j \nu_j \leq b, \tag{4.2}$$

$$\sum_{j \in \mathbb{V}} \nu_j = q, \tag{4.3}$$

$$\sum_{j \in \mathbb{V}} \phi_{ijk} = 1, \qquad \forall i \in \mathbb{V}, \tag{4.4}$$

$$\phi_{ijk} \leq \nu_j, \qquad \forall i, j \in \mathbb{V}, \tag{4.5}$$

$$\nu_j, \phi_{ij} \in \{0, 1\}, \quad \forall i, j \in \mathbb{V}. \tag{4.6}$$

The objective of the formulation is to minimize the total weighted distance. Constraint (4.2) is to ensure that opening and closing of the facilities are performed within the given budget. Constraint (4.3) makes sure that exactly q facilities are located. Constraint (4.4) states that demand at each v_i is assigned to a facility. Constraint (4.5) makes sure that demand at v_i to v_j is satisfied by v_j unless a facility is located at v_j.

It is known that $dFRP$ is *NP-hard* as stated in Theorem 2.

Theorem 2. *dFRP is NP-hard [Wang et al., 2003].*

4.1 α-Reliable Facility Relocation Problem

In this model we utilize a similar approach taken by Daskin et al. [1998]. We find α-reliable relocations of facilities. Different α values imply the significance of each scenario. For example, if the network is related to an emergency situation, and every single scenario is very essential to consider, α should be set to a high value such as 100% or 99%. If we can tolerate ignoring some of the scenarios that are less likely to occur, we can set α to 90% or 80%.

4.1.1 Problem Formulation for α-ReFRP

For this formulation, we use the optimal total weighted distance value ζ_k of each scenario, as an input parameter. We also have three decision variables as

$$\upsilon_k = \begin{cases} 1, & \text{if scenario } k \in \mathbb{S} \text{ is included in the maximization set} \\ 0, & \text{otherwise.} \end{cases}$$

$$w_j = \begin{cases} 1, & \text{if facility at } v_j \in \mathbb{V} \text{ is open} \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ijk} = \begin{cases} 1, & \text{if demand at } v_i \in \mathbb{V} \text{ is assigned to facility at } v_j \in \mathbb{V}, \\ & \text{in scenario } k \in \mathbb{S} \\ 0, & \text{otherwise.} \end{cases}$$

The problem formulation for robust facility relocation is as

$$P2: \qquad \min R \tag{4.7}$$

$$\text{s.t.} \sum_{k \in S} \beta_k \upsilon_k \geq \alpha \tag{4.8}$$

$$\Big(\sum_{i \in \mathbb{V}} \sum_{j \in \mathbb{V}} \lambda_{ik} d_{ij} x_{ijk} - \zeta_k^*\Big) - \psi_k (1 - \upsilon_k) \leq R, \qquad \forall k \in \mathbb{S} \tag{4.9}$$

$$\sum_{j \in \mathbb{V}_1} c_j \,(1 - w_j) + \sum_{j \in \mathbb{V}_2} o_j w_j \leq b \tag{4.10}$$

$$\sum_{j \in \mathbb{V}} w_j = q \tag{4.11}$$

$$\sum_{j \in \mathbb{V}} x_{ijk} = 1, \qquad \forall i \in \mathbb{V}, \forall k \in \mathbb{S} \tag{4.12}$$

$$x_{ijk} \leq w_j, \qquad \forall i, j \in \mathbb{V}, \forall k \in \mathbb{S} \tag{4.13}$$

$$w_j, x_{ijk} \in \{0, 1\}, \forall i, j \in \mathbb{V}, \forall k \in \mathbb{S}. \tag{4.14}$$

The objective of the formulation is to minimize the maximum regret which is denoted by R. Constraint 4.8 ensures that cumulative probability of scenarios, included in the maximization set, is at least α. Constraint 4.9 makes sure that R is greater than regrets for all scenarios that are included in the maximization set. Constraint 4.10 ensures that opening and closing of facilities are performed within the given budget. The number of open facilities is set to q in constraint 4.11. Constraint 4.12 ensures that demand at each vertex in each scenario is assigned to one and only one facility. Constraint 4.13 makes sure that demand at v_i in each scenario is assigned to v_j that has a facility located. Constraint 4.14 is the binary constraints for the decision variables.

4.1.2 Complexity Analysis

It is trivial to show that α-ReFRP is *NP-hard* as stated in Corollary 2.

Corollary 2. *α-ReFRP is NP-hard.*

Proof. In order to solve α-ReFRP, we solve $dFRP$ for each scenario to obtain ζ_k^*. Therefore, from Theorem 2, α-ReFRP is *NP-hard.* $\square$

An important issue in the formulation is the parameter ψ_k for each scenario. The parameter ψ_k is a big number that prevents the R from being greater than the regret for the scenarios that will not be included in the maximization set. Any big number could serve as ψ_k. However, the value will have an impact on the feasible solution space and the computational time to reach the optimal solution. Daskin et al. [1998] proposed a method to obtain reasonable ψ_k's. They calculate the total weighted distance for each scenario based on the optimal facility relocation decisions corresponding to other scenarios and take the maximum value among them. This maximum value becomes the ψ_k value for scenario $k \in \mathbb{S}$.

While this seems to be a reasonable approach, we can utilize another property of the model that was not pointed out before. In this problem, our aim is to minimize the maximum regret over a set of scenarios whose cumulative probability is at least

α. This implies that any scenario k that has a probability β_k greater than $1-\alpha$ has to be in the maximization set. Therefore, we can pre-process parameter μ and instead of assigning a big number for such scenarios, we can assign a very small number that can reduce the feasible solution space and computational effort required to solve the problem.

4.2 γ-Robust Facility Relocation Problem

We present a mathematical formulation for the γ-RoFRP. Our goal is to minimize the expected weighted distance while making sure that relative regret for each scenario is less than γ.

4.2.1 Problem Formulation for γ-RoFRP

γ-RoFRP has two sets of decision variables as

$$w_j = \begin{cases} 1, & \text{if facility at } v_j \in \mathbb{V} \text{ is open} \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ijk} = \begin{cases} 1, & \text{if demand at } v_i \in \mathbb{V} \text{ is assigned to facility at } v_j \in \mathbb{V}, \text{ in scenario } k \in \mathbb{S} \\ 0, & \text{otherwise.} \end{cases}$$

The problem formulation for γ-RoFRP is

$$P3: \qquad \min \sum_{k\in\mathbb{S}} \sum_{i\in\mathbb{V}} \sum_{j\in\mathbb{V}} \beta_k \lambda_{ik} d_{ij} x_{ijk} \tag{4.15}$$

$$\text{s.t.} \sum_{i\in\mathbb{V}} \sum_{j\in\mathbb{V}} \lambda_{ik} d_{ij} x_{ijk} \leq (1+\gamma)\zeta_k^*, \qquad \forall k \in \mathbb{S} \tag{4.16}$$

$$\sum_{j\in\mathbb{V}_1} c_j (1-w_j) + \sum_{j\in\mathbb{V}_2} o_j w_j \leq b \tag{4.17}$$

$$\sum_{j\in\mathbb{V}} w_j = q \tag{4.18}$$

$$\sum_{j \in \mathbb{V}} x_{ijk} = 1, \qquad \forall i \in \mathbb{V}, \forall k \in \mathbb{S} \tag{4.19}$$

$$x_{ijk} \leq w_j, \qquad \forall i, j \in \mathbb{V}, \forall k \in \mathbb{S} \tag{4.20}$$

$$w_j, x_{ijk} \in \{0, 1\}, \qquad \forall i, j \in \mathbb{V}, \forall k \in \mathbb{S}. \tag{4.21}$$

The objective of the formulation is to minimize the expected weighted distance. Constraint (4.16) makes sure that relative regret for each scenario is no more than γ. Constraint (4.17) limits that opening and closing of facilities are performed within the given budget. The number of open facilities is set to q in constraint (4.18). Constraint (4.19) ensures that demand at each v_i in each scenario is assigned to one facility. Constraint (4.20) makes sure that demand at v_i is satisfied by v_j that has a facility located.

It is trivial to show that γ-RoFRP is also *NP-hard* as stated in Corollary 3.

Corollary 3. *γ-RoFRP is NP-hard.*

Proof. In order to solve γ-RoFRP, we solve $dFRP$ for each scenario to obtain ζ_k^*. Therefore, from Theorem 2, γ-RoFRP is *NP-hard.* □

We proved that γ-RoFRP is *NP-hard.* The problem size rapidly grows when we increase the number of facilities as well as future scenarios. We have empirically verified that the computational time increases with the growth of the problem size. Therefore we propose Lagrangean Decomposition Algorithm to solve our problem in a timely manner.

4.2.2 Lagrangean Decomposition Algorithm for γ-RoFRP

There exists various solution techniques applied for large scale problems such as Bender's Decomposition [Kouvelis and Yu, 1997], Lagrangean Relaxation[Beasley, 1993] and Lagrangean Decomposition [Guignard and Kim, 1984]. Bender's Decomposition is more useful for problems where we have both binary and non-binary integer variables. In our problem, all variables are binary integers, which makes this method not a favorable candidate. If we attempt to relax some of the constraints using Lagrangean Relaxation, the relaxed problem cannot be separated into subproblems and it is not easier to solve. However, Lagrangean Decomposition Algorithm (LDA), which is also known as Variable Splitting Algorithm provides equal or better lower bounds than Lagrangean relaxation [Barcelo et al., 1991, Guignard and Kim, 1984, Snyder and Daskin, 2006]. LDA allows separation of variables by introducing a new set of variables that are made to be equal to the existing ones in the model. Then, two subproblems are obtained by relaxing this equality constraint. In our application, we utilize Lagrangean relaxation by adding the equality constraint and one of the complicated constraints to the objective function by multiplying with Lagrangean coefficients. Then, we utilize this solution to generate an upper bound and use a subgradient algorithm to optimize the multipliers. Details of these procedures are explained in the following sections.

In order to apply LDA, we modify our model by adding a new set of binary variables, τ_{ijk}, that should be equal to x_{ijk} by constraint (4.28), in the following formulation, P4. The objective functions of P3 and P4 are made to be equal by using the parameter σ, $0 \leq \sigma \leq 1$. The following transformations explain this process.

Define $z_1 = \min\ f(x)$, then

$z_1 = \min\ [\sigma + (1-\sigma)]f(x) = \min\ [\sigma f(x) + (1-\sigma)f(x)]$.

Let $z_2 = \min\ [\sigma f(x) + (1-\sigma)f(\tau)]$,

If $x = \tau$, then $f(x) = f(\tau)$

Hence, $z_1 = z_2$.

Then the new formulation becomes:

$$P4 : \min \sigma \sum_{i\in\mathbb{V}} \sum_{j\in\mathbb{V}} \sum_{k\in\mathbb{S}} \beta_k \lambda_{ik} d_{ij} x_{ijk} + (1-\sigma) \sum_{i\in\mathbb{V}} \sum_{j\in\mathbb{V}} \sum_{k\in\mathbb{S}} \beta_k \lambda_{ik} d_{ij} \tau_{ijk} \tag{4.22}$$

$$\text{s.t.} \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}} \lambda_{ik} d_{ij} x_{ijk} \leq (1+\gamma)\zeta_k^*, \qquad \forall k \in \mathbb{S} \tag{4.23}$$

$$\sum_{j\in\mathbb{V}_1} c_j\,(1-w_j) + \sum_{j\in\mathbb{V}_2} o_j w_j \leq b \tag{4.24}$$

$$\sum_{j\in\mathbb{V}} w_j = q \tag{4.25}$$

$$\sum_{j\in\mathbb{V}} x_{ijk} = 1, \qquad \forall i \in \mathbb{V}, \forall k \in \mathbb{S} \tag{4.26}$$

$$x_{ijk} \leq w_j, \qquad \forall i,j \in \mathbb{V}, \forall k \in \mathbb{S} \tag{4.27}$$

$$x_{ijk} = \tau_{ijk}, \qquad \forall i,j \in \mathbb{V}, \forall k \in \mathbb{S} \tag{4.28}$$

$$w_j, x_{ijk}, \tau_{ijk} \in \{0,1\}, \qquad \forall i,j \in \mathbb{V}, \forall k \in \mathbb{S}. \tag{4.29}$$

4.2.2.1 Lower Bound Generation

We obtain a lower bound for the γ-RoFRP, by adding the constraints (4.24) and (4.28) to the objective function by multiplying with Lagrangean coefficients u and l, respectively. The optimal solution for the relaxed problem provides a lower bound for P4. Moreover, relaxing those constraints allows us to decompose P4 into two subproblems. Solving these subproblems separately is easier than solving P4 itself and the summation of the objective function values of the subproblems provides a lower bound for P4. The two subproblems are demonstrated in the following formulations:

Subproblem 1:

$$\begin{aligned} \min \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} \sigma\beta_k\lambda_{ik}d_{ijk}x_{ijk} + u[\sum_{j\in\mathbb{V}_1} c_j(1-w_j) + \sum_{j\in\mathbb{V}_2} o_j w_j - b] - \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} l_{ijk}x_{ijk} \\ \text{s.t.} \sum_{j\in\mathbb{V}} w_j \quad = \quad q \\ x_{ijk} \quad \leq \quad w_j, \quad \forall i,j \in \mathbb{V}, \quad \forall k \in \mathbb{S} \\ w_j, x_{ijk} \quad \in \quad \{0,1\}, \quad \forall i,j \in \mathbb{V}, \quad \forall k \in \mathbb{S}. \end{aligned}$$

Subproblem 2:

$$
\begin{aligned}
\min \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}}(1-\sigma)\beta_k\lambda_{ik}d_{ijk}\tau_{ijk} &+ \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} l_{ijk}\tau_{ijk} \\
\text{s.t.} \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\lambda_{ik}d_{ij}\tau_{ijk} \quad &\leq \quad (1+\gamma)\zeta_k^*, \qquad \forall k\in\mathbb{S} \\
\sum_{j\in\mathbb{V}}\tau_{ijk} \quad &= \quad 1, \qquad \forall i\in\mathbb{V}, \quad \forall k\in\mathbb{S} \\
\tau_{ijk} \quad &\in \quad \{0,1\}, \quad \forall i,j\in\mathbb{V}, \quad \forall k\in\mathbb{S}.
\end{aligned}
$$

In order to solve the first subproblem, we reorganize its objective function as follows:

$$
\min \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}}(\sigma\beta_k\lambda_{ik}d_{ijk}-l_{ijk})x_{ijk} - \sum_{j\in\mathbb{V}_1} uc_jw_j + \sum_{j\in\mathbb{V}_2} uo_jw_j + \sum_{j\in\mathbb{V}_1} uc_j - bu. \quad (4.30)
$$

For each vertex j, the contribution of opening a facility at vertex j to the objective function can be denoted as

$$
\rho_j(u,l) = \begin{cases} \sum_{k\in\mathbb{S}}\sum_{j\in\mathbb{V}} \min\{0, (\sigma\beta_k\lambda_{ik}d_{ijk}-l_{ijk})\} - uc_j, & \text{if } j\in\mathbb{V}_1 \\ \sum_{k\in\mathbb{S}}\sum_{j\in\mathbb{V}} \min\{0, (\sigma\beta_k\lambda_{ik}d_{ijk}-l_{ijk})\} + uo_j, & \text{if } j\in\mathbb{V}_2. \end{cases}
$$

Since the last two terms of Equation (4.30) are constant, we rank the ρ_j's in ascending order and we set $w_j = 1$ for each of the q smallest ρ_j to find the optimal solution for the first subproblem. Consequently, the solution for x_{ijk} can be obtained as follows:

$$
x_{ijk} = \begin{cases} w_j, & \text{if } \sigma\beta_k\lambda_{ik}d_{ijk}-l_{ijk} < 0 \\ 0, & \text{otherwise.} \end{cases}
$$

The second subproblem can be divided into k instances, and for each $k \in \mathbb{S}$:

$$\begin{aligned} \min \sum_{i\in\mathbb{V}} \sum_{j\in\mathbb{V}} \sum_{k\in\mathbb{S}} ((1-\sigma)\beta_k \lambda_{ik} d_{ijk} + l_{ijk}) \tau_{ijk} & & & \\ \text{s.t.} \sum_{i\in\mathbb{V}} \sum_{j\in\mathbb{V}} \lambda_{ik} d_{ij} \tau_{ijk} & \leq & (1+\gamma)\zeta_k^* & \\ \sum_{j\in\mathbb{V}} \tau_{ijk} & = & 1, & \quad \forall i \in \mathbb{V} \\ \tau_{ijk} & \in & \{0,1\}, & \quad \forall i,j \in \mathbb{V}. \end{aligned}$$

Each instance is similar to 0-1 Multiple Choice Knapsack Problem (MCKP). In 0-1 MCKP, we need to select exactly one item from multiple disjoint subsets. The goal is to maximize (minimize) the objective function while satisfying the $\leq$ ($\geq$) knapsack constraints [Martello and Toth, 1990]. In the second subproblem of our decomposition, the assignment of $j \in \mathbb{V}$ facilities to each customer $i \in \mathbb{V}$ can be considered as a subset. The objective function coefficient and the constraint coefficient for each facility j in each subset i is $((1-\sigma)\beta_k \lambda_{ik} d_{ijk} + l_{ijk})$ and $\lambda_{ik} d_{ij}$, respectively.

We know that 0-1 MCKP is *NP-hard* [Martello and Toth, 1990], and using exact solution techniques for the second subproblem would be too time-consuming, especially for larger instances. As our goal in solving the second subproblem is to obtain a lower bound for the original problem, the second subproblem does not need to be solved optimally to obtain that lower bound. We used a linear programming based algorithm [Sinha and Zoltners, 1979] to solve the second subproblem. Two important but easy transformations are performed to apply their solution technique because the algorithm requires nonactive objective function coefficients and a greater than or equal to ($\geq$) sign in the knapsack constraint [Snyder, 2003].

$$
\begin{aligned}
P5: \quad & \min \sum_{i=1}^{m} \sum_{j \in N_i} g_{ij} x_{ij} & & \\
\text{s.t.} & \sum_{i=1}^{m} \sum_{j \in N_i} a_{ij} x_{ij} \leq e & & \\
& \sum_{j \in N_i} x_{ij} = 1, & & k = 1, \ldots, m \\
& x_{ij} \in \{0, 1\}, & & j \in N_i, i = 1, \ldots, m.
\end{aligned}
$$

Suppose P5 is a simplified version of each instance k of Subproblem 2, where $c_{ij} = ((1-\sigma)\beta_k \lambda_{ik} d_{ijk} + l_{ijk})$ and $a_{ij} = \lambda_{ik} d_{ij}$ for $\forall k \in \mathbb{S}$. Since the algorithm requires ($\geq$) constraint, we first calculate $\bar{a} = \max\{e/m, \max_{j \in N_i, i=1,\ldots,m} a_{ij}\}$. Then, we set $a_{ij} = \bar{a} - a_{ij}$ and $e = m\bar{a} - e$. In addition, negative objective function values may be incurred while applying the subgradient algorithm (Section 4.2.2.3). Therefore, in order to ensure non-negative coefficients, we make a simple adjustment to coefficients; we calculate $\bar{g} = |\min\{0, \min_{j \in N_i, i=1,\ldots,m} g_{ij}\}|$, and add $\bar{g}$ to each g_{ij}. After solving the problem, we subtract $m\bar{g}$ from the objective function value.

4.2.2.2 Upper Bound Generation

An upper bound can be obtained from the solution of the first subproblem. We set $w_j = 1$ for the facilities that are decided to be open in the optimal solution of the first subproblem, then we assign each customer to its closest facility. We first check if the solution is feasible with respect to the budget constraint. If it is feasible, we calculate the relative regret for each scenario and check if all regrets are smaller than or equal to γ. If so, we can say that the solution is feasible with respect to the robustness constraint and it provides an upper bound for the original problem.

If the solution is not feasible with respect to the robustness constraint, we apply a local neighborhood search (LNS) to obtain a local optimal solution. In LNS, we attempt to swap each facility with one of its closest f vertices. We first check if the swap satisfies the budget constraint. Then, we check if the solution after the swap satisfies the γ-robustness constraint by calculating the new relative regrets. If any of the swaps satisfy both constraints, the solution after the swap can be used as an upper bound for the original problem.

An initial and hypothetical upper bound for the algorithm can be obtained using the following proposition:

Propositon 2. $\sum_{k \in \mathbb{S}} \beta_k (1+\gamma)\zeta_k^*$ provides an upper bound for γ-RoFRP.

Proof. Let ζ_k^* be the optimal objective function value for each scenario. Based on the problem definition, each scenario can have a relative regret of at most γ. Therefore, the total travel distance in any scenario of γ-RoFRP can be at most $(1+\gamma)\zeta_k^*$ and the objective function value of γ-RoFRP is bounded above by $\sum_{k \in \mathbb{S}} \beta_k (1+\gamma)\zeta_k^*$. □

4.2.2.3 Subgradient Algorithm for Lagrangean Multipliers

In order to solve the decomposed problem, we use a subgradient algorithm to calculate the Lagrangean multipliers. The following notations are used for the procedure.

$\overline{z}^*$	: Best upper bound
$\underline{z}^*$	: Best lower bound
δ^{iter}	: Dual gap in each iteration
η, θ_{ijk}	: Subgradients of the Lagrangean multiplier u and l_{ijk}
π_1, π_2	: Step size coefficients for the Lagrangean multiplier u and l_{ijk}
μ_1, μ_2	: Step sizes for the Lagrangean multiplier u and l_{ijk}
$iter$	: Iteration index

There are two stopping criteria for the algorithm, where the algorithm terminates either one of them is reached:

δ : Dual gap value that terminates the algorithm

$iter_{max}$: Maximum number of iterations allowed for the algorithm

The initial value of the Lagrangean multiplier u is set to 0. For the multiplier l_{ijk}, we determine the closest f vertices for each customer $i \in \mathbb{V}$, and for each scenario $k \in \mathbb{S}$, assign the average demand of all customers multiplied by a closeness coefficient.

Step 0: Initialize the parameters

Set $iter = 0$, $\underline{z}^* = -\infty$, $\overline{z}^* = \sum_{k \in \mathbb{S}} \beta_k (1+\gamma)\zeta_k^*$, and $\delta^0 = \infty$,

Let $u^0 = 0$, and

$$l_{ijk}^0 = \begin{cases} \bar{\lambda}_{ik}\frac{f+2-\rho}{f+1}, & \text{if } j \text{ is the } \rho\text{th closest facility to customer } i, \text{ for } 1 \leq \rho \leq f, \\ 0, & \text{otherwise} \end{cases}$$

while $(\delta^{iter} > \delta) || (iter \leq iter_{max})$

{

Step 1: Solve subproblems 1 and 2 and obtain a lower bound, $\underline{z}^t$, by adding the objective function values of both problems.

If $\underline{z}^{iter} \geq \underline{z}^*, \underline{z}^* = \underline{z}^{iter}$.

Step 2: Check, if w_j satisfy the budget constraint

If yes, assign each customer to the closest facility and check if this assignment satisfy the robustness constraint.

If yes, calculate $\overline{z}^{iter}$

else, perform LNS to find a feasible solution and calculate the $\overline{z}^{iter}$

If $\overline{z}^{iter} \leq \overline{z}^*, \overline{z}^* = \overline{z}^{iter}$

Step 3: Calculate the dual gap:

$\delta^{iter} = (\overline{z}^* - \underline{z}^*)/\underline{z}^*$

Step 4: Calculate η, θ and step sizes and update the Lagrangean multipliers using the following equations:

$\eta = \sum_{j \in \mathbb{V}1} c_j(1 - w_j) + \sum_{j \in \mathbb{V}1} o_j w_j - b$

$\theta_{ijk} = -x_{ijk} + \tau_{ijk}$

$\mu_1^t = \pi_1(\overline{z}^* - \underline{z}^*)/\eta^2$

$\mu_2^t = \pi_2(\overline{z}^* - \underline{z}^*)/ \sum_{i \in \mathbb{V}} \sum_{j \in \mathbb{V}} \sum_{k \in \mathbb{S}} \theta_{ijk}^2$

$u^{iter+1} = \max\{0, u^{iter} + \eta\mu_1^{iter}\}$

$l_{ijk}^{iter+1} = l_{ijk}^{iter} + \theta_{ijk}\mu_2^{iter}$

}

4.2.2.4 Discussions on Problem Infeasibility

The γ-RoFRP has constraints on maximum allowable relative regret (γ) for each scenario and budget (b) for relocations. Due to these limitations on γ and b, some instances may not have feasible solutions [Snyder, 2006]. If these instances can be detected in advance, either no solution attempt should be made or, γ or b values should be increased to obtain feasible solutions. The following propositions explain this infeasibility.

Propositon 3. Let p and q be the number of existing and final facilities, respectively. Let o_j be the opening cost of facility $j \in \mathbb{V}$ and let $\mathbb{O}$ be the set of $(q - p)$ minimum values of o_j, $j \in \mathbb{V}$. Then γ-RoFRP will be infeasible if $b \leq \sum_{i \in \mathbb{O}} \mathbb{O}_i$.

Proof. Suppose that we have p facilities located initially and q will be the final number of facilities. Opening costs for potential facilities are given by o_j for each facility at vertex j. The minimum required budget to ensure a feasible solution is obtained by finding the minimum opening cost o_j of $(q - p)$ facilities and adding them up. If b is smaller than the summation of those opening costs, the problem is infeasible. □

Propositon 4. The γ-RoFRP will be infeasible for the following four conditions. Let $\eta = \sum_{j\in\mathbb{V}1} c_j(1-w_j) + \sum_{j\in\mathbb{V}2} o_j w_j - b$ and $\alpha_{ijk} = \beta_k \lambda_{ik} d_{ij}$,

1. The γ-RoFRP will be infeasible for $\tau_{ijk} = x_{ijk} \neq \pi_{ijk}$ if

   $\eta u > \sum_{(i,j,k)|(x_{ijk}\leq\pi_{ijk})} (1+\gamma)\alpha_{ijk} - \sum_{(i,j,k)|(x_{ijk}>\pi_{ijk})} \gamma\alpha_{ijk}.$

2. The γ-RoFRP will be infeasible for $\tau_{ijk} \neq x_{ijk}$ and $x_{ijk} = \pi_{ijk}$ if

   $$\eta u > \sum_{\substack{(i,j,k)|((x_{ijk}\geq\tau_{ijk})\\ \&\&(x_{ijk}=\pi_{ijk}))}} \gamma\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}>\tau_{ijk})\\ \&\&(x_{ijk}=\pi_{ijk}))}} (1-\sigma)\alpha_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}<\tau_{ijk})\\ \&\&(x_{ijk}=\pi_{ijk}))}} (1-\sigma)\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}>\tau_{ijk})\\ \&\&(x_{ijk}=\pi_{ijk}))}} l_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}<\tau_{ijk})\\ \&\&(x_{ijk}=\pi_{ijk}))}} l_{ijk}.$$

3. The γ-RoFRP will be infeasible for $\tau_{ijk} \neq x_{ijk}$ and $\tau_{ijk} = \pi_{ijk}$ if

   $$\eta u > \sum_{\substack{(i,j,k)|((x_{ijk}\leq\tau_{ijk})\\ \&\&(\tau_{ijk}=\pi_{ijk}))}} \gamma\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}<\tau_{ijk})\\ \&\&(\tau_{ijk}=\pi_{ijk}))}} (1-\sigma)\alpha_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}>\tau_{ijk})\\ \&\&(\tau_{ijk}=\pi_{ijk}))}} (1-\sigma)\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}>\tau_{ijk})\\ \&\&(\tau_{ijk}=\pi_{ijk}))}} l_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}<\tau_{ijk})\\ \&\&(\tau_{ijk}=\pi_{ijk}))}} l_{ijk}.$$

4. The γ-RoFRP will be infeasible for $\tau_{ijk} \neq x_{ijk}$, $x_{ijk} \neq \pi_{ijk}$ and $\tau_{ijk} \neq \pi_{ijk}$ if

   $$\eta u > \sum_{(i,j,k)|\pi_{ijk}>0} \gamma\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}<\pi_{ijk})\\ \&\&(\tau_{ijk}<\pi_{ijk}))}} \alpha_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}>\pi_{ijk})\\ \&\&(\tau_{ijk}>\pi_{ijk}))}} \alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}<\pi_{ijk})\\ \&\&(\tau_{ijk}=\pi_{ijk}))}} \sigma\alpha_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}>\pi_{ijk})\\ \&\&(\tau_{ijk}=\pi_{ijk}))}} \sigma\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}=\pi_{ijk})\\ \&\&(\tau_{ijk}<\pi_{ijk}))}} \sigma\alpha_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}=\pi_{ijk})\\ \&\&(\tau_{ijk}>\pi_{ijk}))}} \sigma\alpha_{ijk} + \sum_{(i,j,k)|x_{ijk}<\tau_{ijk}} l_{ijk} - \sum_{(i,j,k)|x_{ijk}>\tau_{ijk}} l_{ijk}.$$

Proof. See Appendix. □

4.3 Numerical Results

In this section, we present our numerical results for the experiments that we have run to test the performance of the RFRP models and Lagrangean Decomposition Algorithm. All numerical results presented in this section were run on a Pentium 4 Xeon 3.6 Ghz machine with 4 GB RAM.

Different demand scenarios are generated using an approach similar to the one discussed in Daskin et al. [1998]. In each scenario, we have the demand more intense in some area of the network. We first define some locations for each scenario, which are named attraction points. Demand of each vertex i in a scenario k is calculated based on the distance of the vertex and the attraction point of that scenario using Equation 4.31 given in Daskin et al. [1998]:

$$\lambda_{ik} = \lambda_i^0 + \lambda_{total} \left(\frac{1/d_{ik}}{\sum\limits_n 1/d_{in}} \right). \tag{4.31}$$

Then, the seed demand of vertices, λ_i^0, which are used as an input for Equation 4.31 are generated randomly from a uniform distribution over $[100, 200]$. The parameter λ_{total} is the total of λ_i^0's among the network. The parameter d_{ik} is the distance between vertex i and attraction point defined for scenario k.

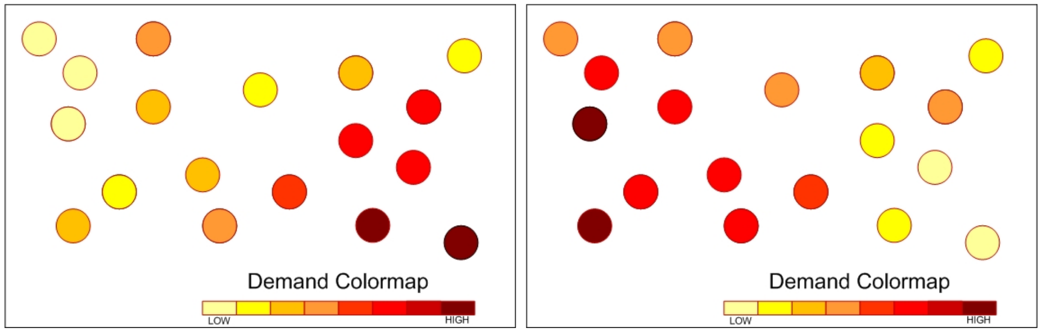
A similar setting with Daskin et al. [1998] was used for the scenario probabilities and attraction points. We designed two cases where we have five and nine scenarios respectively, and the probability of scenarios for $|\mathbb{S}| = 5$ are

$$\beta = [0.06, 0.22, 0.51, 0.14, 0.07],$$

and the probabilities of scenarios for $|\mathbb{S}| = 9$ are

$$\beta = [0.01, 0.04, 0.15, 0.02, 0.34, 0.14, 0.09, 0.16, 0.05].$$

Attraction points are located in the following regions of the network: southeast, northeast, southwest, northwest, center, south, north, west, and east. For example, in the first scenario, we will have more intense demand in the southeastern part of the network and in the eighth scenario, we will have more intense demand in the western part of the network as illustrated in Figures 4.1(a) and 4.1(b).



(a) Scenario 1 (b) Scenario 8

Figure 4.1: Illustration of two different demand scenarios

4.3.1 Numerical Results for α-Reliable FRP

The α-ReFRP was tested on 20 randomly generated networks with 100 nodes in each network. Opening and closing costs were randomly generated from uniform distributions over $[200, 300]$ and $[50, 100]$, respectively. The budget for opening and closing facilities was set to be 1500. The number of initial facilities was set to four, and locations for these facilities were randomly determined. The total number of final facilities, q, was set to eight. The α levels are set to be 80%, 90% and 100%.

All experiments in this section are coded in GAMS [Brooke et al., 2009] and solved by CPLEX [IBM, 2009]. For each instance, we first solved the deterministic facility relocation problem corresponding to each scenario $k \in \mathbb{S}$ to acquire input parameter ζ_k, $\forall k \in \mathbb{S}$. After the calculations explained in Section 4.1.2, we obtained the other input parameter, ψ_k, $\forall k \in \mathbb{S}$. Then, we solved each instance with the BIP formulation introduced in Section 4.1.1. We recorded the objective function value and solution time, and calculated their average values. Figure 4.2 shows the change in the maximum regret of the original problem with respect to the α value.

The maximum regret increases when we increase the α value. This means that if we want to maximize our regret over a larger reliability set, the maximum regret will be higher. The objective function value also increases when we increase the number of scenarios in the problem. This implies that the degree of uncertainty also

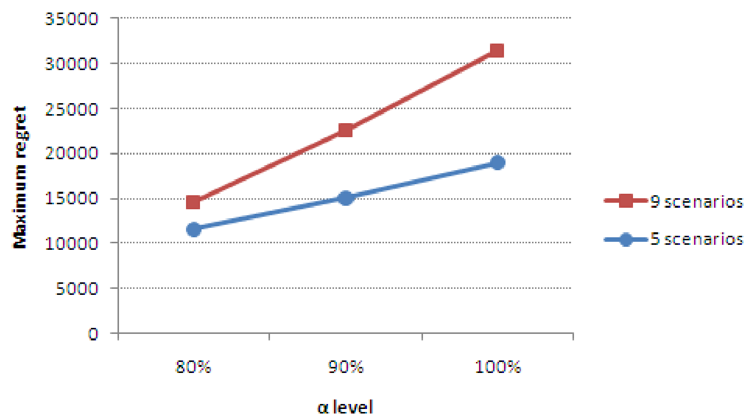


Figure 4.2: Maximum regret in α-ReFRP vs α for $|\mathbb{S}| = 5, 9$

impacts the maximum regret that we will obtain. We solved the same instances after pre-processing and modifying the parameter ψ_k. We recorded the objective function value and the solution time, and calculated their average values. Table 4.1 shows the comparison of the solution time (CPU seconds) for both methods.

Table 4.1: Solution time comparison for the α-ReFRP with original and pre-processed ψ

$\mathbb{S}$	α	Solution Time (CPU seconds)	
		α-ReFRP with original ψ	α-ReFRP with pre-processed ψ
5	80%	699	693
	90%	833	792
	100%	912	908
9	80%	8141	7274
	90%	9592	6836
	100%	10199	9357

As we can see in Table 4.1, the solution time for both methods increase when we increase either the number of scenarios or desired α level. Based on the experiment results, we can conclude that pre-processing for parameter ψ provides some gain in solution time. The gain is more significant in $|\mathbb{S}| = 9$, so we claim that the pre-processing approach is more helpful for larger scenarios.

4.3.2 Numerical Results for γ-Robust FRP

The γ-RoFRP was tested on 25 randomly generated networks with 100 and 250 nodes in each network for the comparison of the exact method and the proposed Lagrangean Decomposition Algorithm (LDA). For large scale problems, the LDA was tested on 25 randomly generated networks with $n = 500$. Opening and closing costs were randomly generated from uniform distributions over $[200, 300]$ and $[50, 100]$, respectively. The budget for opening and closing facilities were set to $1000, 1500$, and 3000. The number of initial facilities was set to four and locations for these facilities were randomly determined. The total number of final facilities, q, was set to eight.

For LDA, values of some parameters were determined after trial-and-error. The value of σ was set to 0.2. Initial values for step size coefficients π_1 and π_2 were set to 1.5 and 2, respectively. Both coefficients were decreased by 10% at every 30^{th} unimproved iteration after some parameter tuning.

For parameter tuning, we selected some instances and tried different values of parameters to observe their impact on lower bound (LB) quality and convergence of the algorithm. These parameters are π_1, π_2 as well as their reduction amount and speed. For Lagrangean relaxation, the initial value of the step size is usually chosen between zero and two. For three selected instances, we ran experiments with different combinations of initial values of π_1 and π_2, and recorded the final LB value. Table 4.2 shows the results of these experiments.

Table 4.2: Parameter tuning for π_1 and π_2

π_1	π_2	LB of Instance 1	LB of Instance 2	LB of Instance 3
1.5	2	409334	362853	434607
1.5	1.5	367250	329811	418404
1.5	1	388299	340308	425664
1.5	0.5	401492	363797	436731
1	0.5	405593	361739	434594

In this table, we observe that the combination $\pi_1 = 1.5$ and $\pi_2 = 2$ provides better solutions compared to other combinations. Therefore, we selected this combination as the initial value for Lagrangean multiplier coefficients. The other parameters, when and how much to reduce these coefficients, are also determined by a set of experiments that is run with different number of maximum iterations. Table 4.3 shows the LB values at the end of these experiments, where we reduce the π values at the ρ'th unimproved iteration by ω .

Table 4.3: Parameter tuning for ρ and ω values

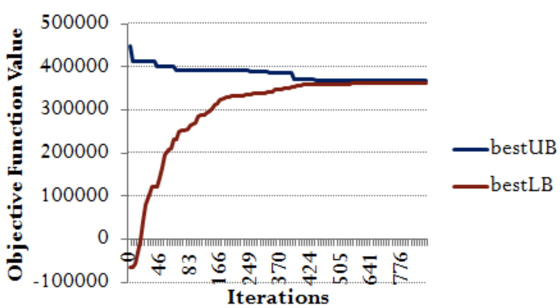
Instance	ρ	# of Iterations = 1000			# of Iterations = 2000		
		$\omega = 15\%$	$\omega = 10\%$	$\omega = 5\%$	$\omega = 15\%$	$\omega = 10\%$	$\omega = 5\%$
1	10	411329	409937	409676	413214	410273	409903
	15	403866	410948	412274	413435	411696	412650
	20	399619	410700	411209	413137	413887	413302
	30	403849	409547	411022	412436	413782	412850
2	10	364988	362926	361005	366280	363529	361195
	15	363884	365108	365197	367552	366801	366102
	20	358602	363610	365576	367464	367258	365882
	30	359234	362608	364564	366986	367335	366487
3	10	439171	438070	431786	440306	438222	431988
	15	440024	438217	439400	442872	439067	439726
	20	438842	442594	438736	444659	443937	430660
	30	443474	441821	439455	445123	443176	441741

Table 4.3 shows that different combinations may provide the best LB for each instance. However, we observed that if we reduce the parameters by 10% after every 30'th unimproved iteration, the lower bounds are consistently better than the others. Therefore we selected $\rho = 30$ and $\omega = 10\%$.

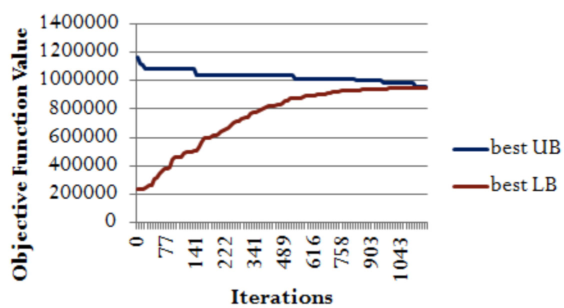
4.3.2.1 Exact Solution Approach vs. LDA

In this section we present the results of experiments that compare the performance of exact solution approach and LDA. The exact solution approach, which is the BIP model, was coded in GAMS [Brooke et al., 2009] and solved by CPLEX. The LDA was coded in C++. For each instance, we first solved the $dFRP$ corresponding to each scenario $k \in \mathbb{S}$, to acquire input parameter ζ_k, $\forall k \in \mathbb{S}$. Then, we solved each instance with both the exact method and the LDA, and recorded the objective function values and the solution time in CPU seconds. Both methods were stopped if the objective function value was within a given dual gap, i.e. 3% and 5% in our experiments.

Figures 4.3(a) and 4.3(b) illustrate the convergence of the LDA for two instances; $n = 100$ and $n = 250$, respectively. In LDA, the lower bound increases rapidly in the initial iterations and the increase becomes smaller after some point. On the other hand, the upper bound slowly decreases over the iterations. In both instances, these figures show that the dual gap eventually converged to 1.4% and 0.9%, respectively.



(a) n = 100



(b) n = 250

Figure 4.3: Convergence of the LDA

As we mentioned in Section 4.2.2.4, not all instances may be feasible due to limitations on b and γ values. Table 4.4 shows the number of feasible instances out of 25 instances we created for $n = 100$ and $n = 250$ for $budget = 1000, 1500$, and 3000, as well as $\gamma = 0.1, 0.15, 0.2$, and 0.25. The infeasible instances occur because of the limitations caused by either the budget or gamma values, or both.

Table 4.4: Number of feasible instances for γ-RoFRP

Network size	Budget	Gamma			
		0.25	0.2	0.15	0.1
100	1000	25	19	8	1
	1500	25	21	15	3
	3000	25	25	21	5
250	1000	21	21	20	6
	1500	24	24	24	15
	3000	25	24	24	17

Tables 4.5 and 4.6 show the average actual dual gap and desired dual gaps for both the exact solution method and LDA for $n = 100$ and 250, respectively. The average solution time and percent time gain of LDA over the exact method for each case are also compared. The percent time gain is calculated by subtracting the average solution time of the LDA from the exact method and dividing it by the solution time of the exact method.

Table 4.5: Comparison of LDA and Exact Solution Method for $n = 100$

Budget	γ	$\delta = 5\%$					$\delta = 3\%$				
		Dual Gap		Solution Time			Dual Gap		Solution Time		
		Exact	LD	Exact	LD	Gain	Exact	LD	Exact	LD	Gain
1000	0.25	1.6%	4.4%	75	42	44%	0.9%	2.7%	103	55	47%
	0.2	2.2%	4.3%	77	48	38%	1.7%	2.7%	87	66	24%
	0.15	2.0%	4.1%	114	62	46%	1.7%	2.6%	117	71	39%
	0.1	0.5%	2.4%	147	32	78%	0.5%	2.4%	147	38	74%
1500	0.25	2.2%	4.3%	261	55	79%	1.6%	2.7%	277	65	77%
	0.2	2.8%	4.1%	814	65	92%	2.0%	2.5%	862	87	90%
	0.15	1.8%	3.9%	871	77	91%	1.6%	2.3%	930	102	90%
	0.1	1.2%	4.2%	1676	104	94%	1.2%	2.4%	1677	118	93%
3000	0.25	0.3%	4.2%	59	47	20%	0.2%	2.5%	60	51	15%
	0.2	0.9%	3.7%	108	59	45%	0.6%	2.5%	109	64	41%
	0.15	1.0%	4.0%	205	62	70%	1.0%	2.5%	213	68	68%
	0.1	0.4%	4.1%	229	59	74%	0.4%	2.4%	229	63	72%

As we can observe from tables 4.5 and 4.6, objective function values for both methods are within the desired dual gap. Solution time gain in its average CPU time of LDA over the exact approach for $n = 100$ ranges from 15% to 94%. A substantial time gain, more than 84%, is observed for all cases with $n = 250$.

Table 4.6: Comparison of LDA and Exact Solution Method for $n = 250$

Budget	γ	$\delta = 5\%$					$\delta = 3\%$				
		Dual Gap		Solution Time			Dual Gap		Solution Time		
		Exact	LD	Exact	LD	Gain	Exact	LD	Exact	LD	Gain
1000	0.25	1.1%	4.6%	3127	229	93%	0.9%	2.8%	3420	301	91%
	0.2	1.1%	4.6%	3582	241	93%	0.9%	2.8%	4034	320	92%
	0.15	1.1%	4.5%	7639	282	96%	1.0%	2.8%	8329	403	95%
	0.1	0.9%	4.6%	30602	279	99%	0.9%	2.9%	30602	470	98%
1500	0.25	1.2%	4.1%	3362	296	91%	0.8%	2.5%	3709	340	91%
	0.2	0.9%	3.9%	2977	316	89%	0.8%	2.5%	3549	349	90%
	0.15	1.2%	3.9%	6825	345	95%	1.0%	2.5%	7040	418	94%
	0.1	0.9%	4.3%	3147	444	86%	0.7%	2.6%	3322	547	84%
3000	0.25	0.1%	3.1%	3353	354	89%	0.1%	2.0%	3353	365	89%
	0.2	0.3%	3.3%	6789	358	95%	0.3%	2.2%	6789	371	95%
	0.15	0.1%	3.2%	3082	407	87%	0.1%	2.2%	3082	428	86%
	0.1	0.4%	3.2%	6250	417	93%	0.4%	2.2%	6250	431	93%

4.3.2.2 Large Scale Experiments

In this section, we present the numerical results for larger scale problems. As we mentioned in the experiment setup, these experiments include 25 networks each having 500 nodes. We solved each instance using both methods. We stopped the algorithms after one hour. The exact method could not find an integer feasible solution at the end of an hour for any of the instances. In fact, no integer feasible solutions were found for several hours of run. Therefore, we could not make a comparison between two methods for large scale problems. We report the average dual gap obtained using LDA in Table 4.7.

Table 4.7: Dual Gap using LDA for $n = 500$

Gamma	Budget		
	1000	1500	3000
0.25	2.9% (25)	2.7% (25)	2.3% (25)
0.2	2.9% (24)	2.8% (24)	2.3% (25)
0.15	2.9% (21)	2.6% (22)	2.1% (24)
0.1	3.0% (11)	2.9% (12)	2.5% (16)

Since some of the instances were infeasible, the numbers in parentheses indicate the number of feasible instances that are used to calculate the average value. As we can see from the numbers, infeasibility increases when we have smaller γ values and less budget available for relocations. Table 4.7 shows that LDA can generate good quality solutions within the given time limit for large scale problems.

4.3.2.3 Objective Function Value vs. γ and Budget Values

In γ-RoFRP problem, both the maximum relative regret permitted for each scenario, which is γ, and the available budget for relocations, are used as a constraint in the formulation where the objective is to minimize the expected weighted travel distance from each demand node to its closest facility. Therefore, any increase in those parameters is expected to decrease the objective function value. On the other hand, any decrease in those parameters may increase the objective function value or yield infeasible solution space.

In this section, we analyze the trade off between the objective function value and the parameters γ and budget values. The parameter values used for these experiments are $n = 250$, $\gamma = 0.15, 0.2, 0.25, 0.5$, and $budget = 1000, 1500, 3000$.

Figure 4.4 shows the change in the expected weighted distance with respect to the γ values for each budget level. Average objective function values of 20 feasible instances were calculated. In Figure 4.4, we can observe a decreasing pattern in the expected weighted distance when we increase the available budget as anticipated.

This is because a smaller budget allows less relocation opportunities and, when we have limited relocation opportunities, the travel distance from each customer to their closest facility may increase.

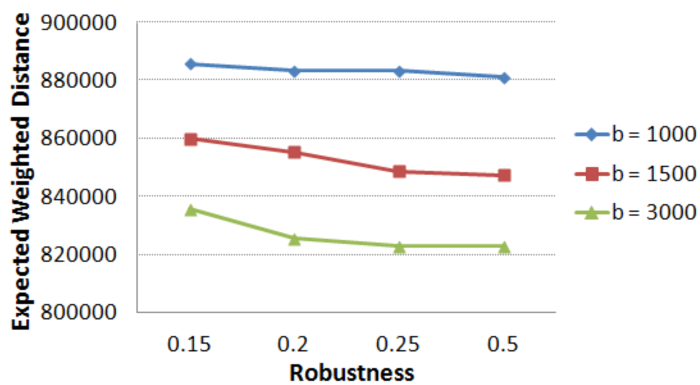
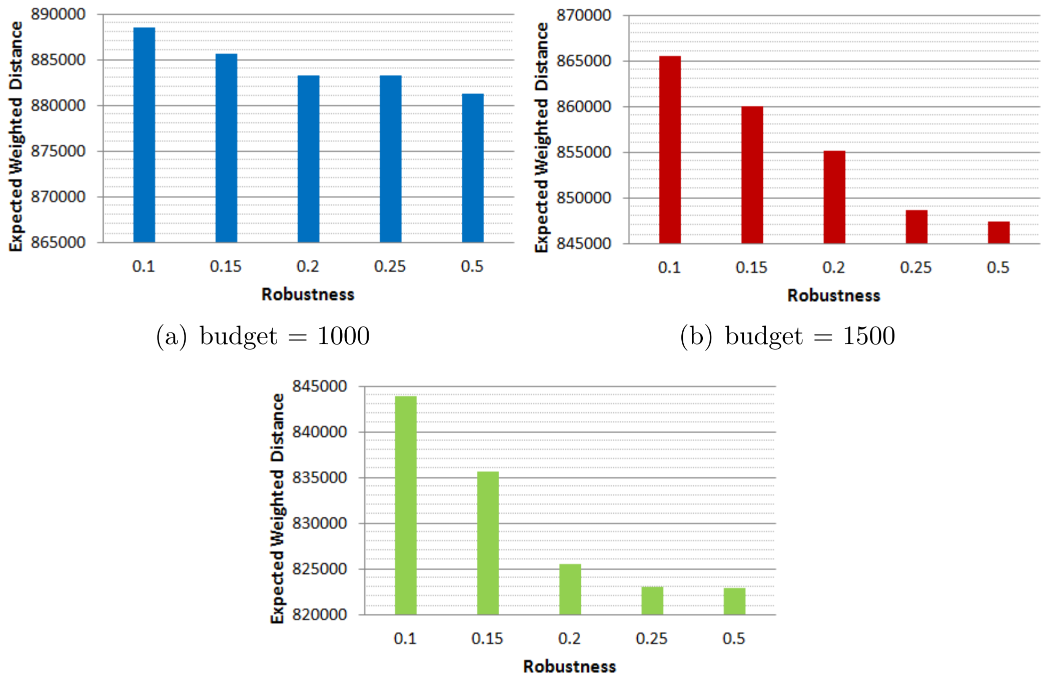


Figure 4.4: Expected weighted distance vs. robustness wrt budget

We can also observe the decrease in the expected weighted distance when we increase the γ value. The effect of γ on the objective function value for each budget level can be observed better in Figure 4.5. All three figures show that the objective function value decreases when we increase the value of γ as expected. Even though higher γ values may lead to less robustness for some scenarios, they allow the model to consider more location alternatives and this helps to decrease the total travel distance. This decrease becomes more apparent for higher budget levels because a higher budget gives more flexibility for relocations, which allows to find solutions with lower travel distances.

These figures help us to determine the trade-off between the objective function value and the γ value as well as the different budget levels. We can observe that, the more available budget we have or the less robustness we seek, the smaller our expected weighted distance will be. On the other hand, budget has an impact on the trade-off between the objective function value and the γ value. When there is a small amount of available budget, the γ value does not have a great effect on the objective function value because of limited relocation opportunities, i.e. an optimal solution



(a) budget = 1000

(b) budget = 1500

(c) budget = 3000

Figure 4.5: Expected weighted distance vs. robustness

for $\gamma = 0.15$ may be the only feasible solution for $\gamma = 0.2$ or 0.25. When there is an ample budget, the objective function value decreases as the γ value increases, because there are many relocation alternatives.

4.4 Summary

In this chapter, we introduced the robust facility relocation problem considering uncertain demand changes. We presented two approaches to solve this problem. In the first approach, we developed a mathematical model that minimizes the maximum regret associated with a set of scenarios whose total probability is at least α and named the model as α-ReFRP. In the second approach, we developed another model that minimizes the expected weighted distance and ensures that relative regret for each scenario is no more than γ. This model is named as γ-RoFRP. We analyzed properties of the models and proved that both models are *NP-hard*. In order to ease the computation time for γ-RoFRP, we proposed a Lagrangean Decomposition Algorithm.

We then discussed the results of the experiments that are run to test the performance of the methods explained in this chapter. As a result of α-ReFRP experiments, we observed that computational time increases when we increase either the uncertainty or the probability of the reliability set, α. We presented numerical results that compare the solution time and quality of LDA with the exact solution method for γ-RoFRP. Our experiments showed that LDA provides a significant time gain, while satisfying the desired dual gap value. When we increase the problem size, the exact method could not generate any integer feasible solution for hours of run whereas LDA could still find good quality solutions. Furthermore, we conducted sensitivity analysis that shows the impact of budget and γ values on the expected weighted traveling distance. Our results conclude that the objective function values decrease when we have more available budget for facility relocations. When we decrease the γ value, meaning that we allow less robustness, the expected weighted distance decreases.

Chapter 5 Dynamic Facility Location and Relocation Problem in Disaster Response

In this chapter, we introduce mathematical models for the facility location and relocation problem in disaster response. The goal of the models is to find optimal locations and relocations of PODs, given the set of SSA locations and potential POD locations. POD opening, closing and relocation operations are restricted to occur with a given frequency. Another important consideration is the assignment of each demand point to one POD location. Since this is a large scale optimization problem, we also introduce a decomposition based solution algorithm and a heuristic algorithm to solve the problem in a timely manner.

5.1 Problem Formulation

In order to find optimal locations and relocations of PODs through the planning horizon, we introduce three mathematical models each having different objective functions. Each model will be named as Dynamic Facility Location and Relocation Problem (DFLRP) continued with an acronym reflecting its objective function. The following input parameters are used to formulate these models:

$\mathbb{V}_1$: Set of demand points

$\mathbb{V}_2$: Set of potential Point of Distributions (PODs)

$\mathbb{V}_3$: Set of State Staging Areas (SSAs)

$\mathbb{R}$: Set of commodities distributed in PODs

$\mathbb{T}$: Set of days in the planning horizon

$\mathbb{T}'$: Set of days on which the facility opening, closing and relocation operations are performed, $\mathbb{T}' \subset \mathbb{T}$

τ : Interval between the days that the facility opening, closing or relocation operations are performed

d_{ij} : Distance between demand point $i \in \mathbb{V}_1$ and POD at vertex $j \in \mathbb{V}_2$

d_{jk} : Distance between POD at vertex $j \in \mathbb{V}_2$ and SSA location $k \in \mathbb{V}_3$

D_{it} : Number of people at demand point $i \in \mathbb{V}_1$ on day $t \in \mathbb{T}$

Demand for each commodity r at demand point i on day t, D_{itr}, is calculated by multiplying the required amount of commodity r per person.

S_{ktr} : Supply at SSA location $k \in \mathbb{V}_3$ for commodity $r \in \mathbb{R}$ on day $t \in \mathbb{T}$

C_{jtr} : Capacity of POD at vertex $j \in \mathbb{V}_2$ for commodity $r \in \mathbb{T}$ on day $t \in \mathbb{T}$

α_i : Priority of each demand point $i \in \mathbb{V}_1$ on a scale of 1 to 10

p_t : Number of facilities that needs to be located on day $t \in \mathbb{T}$

oc_j : Cost of opening a POD at vertex $j \in \mathbb{V}_2$

cc_j : Cost of closing a POD at vertex $j \in \mathbb{V}_2$

rc_{jl} : Cost of relocating a POD from vertex $j \in \mathbb{V}_2$ to vertex $l \in \mathbb{V}_2$

b : Available budget for opening, closing and relocating PODs

tc_r^1 : Unit traveling cost of commodity $r \in \mathbb{R}$ from PODs to demand points

tc_r^2 : Unit traveling cost of commodity $r \in \mathbb{R}$ from SSAs to PODs

Decision variables for the models are :

$$x_{ijt} = \begin{cases} 1, & \text{if demand point } i \in \mathbb{V}_1 \text{ is assigned to POD at vertex } j \in \mathbb{V}_2 \\ & \text{on day } t \in \mathbb{T} \\ 0, & \text{otherwise.} \end{cases}$$

$\pi_{jktr} =$ Amount of commodity $r \in \mathbb{R}$ transported from SSA $k \in \mathbb{V}_3$ to POD at vertex $j \in \mathbb{V}_2$ on day $t \in \mathbb{T}$

$$w_{jt} = \begin{cases} 1, & \text{if POD at vertex } j \in \mathbb{V}_2 \text{ provides service on day } t \in \mathbb{T} \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{jt'} = \begin{cases} 1, & \text{if POD at vertex } j \in \mathbb{V}_2 \text{ is selected to open on day } t' \in \mathbb{T}' \\ 0, & \text{otherwise.} \end{cases}$$

$$z_{jt'} = \begin{cases} 1, & \text{if POD at vertex } j \in \mathbb{V}_2 \text{ is selected to close on day } t' \in \mathbb{T}' \\ 0, & \text{otherwise.} \end{cases}$$

$$\gamma_{jlt'} = \begin{cases} 1, & \text{if POD at vertex } j \in \mathbb{V}_2 \text{ is relocated to vertex } l \in \mathbb{V}_2 \text{ on day } t' \in \mathbb{T}' \\ 0, & \text{otherwise.} \end{cases}$$

5.1.1 DFLRP-TC

In this model, our objective is to minimize the total weighted cost of transporting humanitarian relief items: the cost of transporting items from the SSAs to PODs and from PODs to demand points. Since the objective of this model is minimizing the total cost, we name it as DFLRP-TC.

$$\min \sum_{r\in\mathbb{R}} \sum_{t\in\mathbb{T}} \sum_{j\in\mathbb{V}_2} \sum_{i\in\mathbb{V}_1} tc_r^1 \alpha_i D_{itr} d_{ij} x_{ijt} + \sum_{r\in\mathbb{R}} \sum_{t\in\mathbb{T}} \sum_{k\in\mathbb{V}_3} \sum_{j\in\mathbb{V}_2} tc_r^2 d_{jk} \pi_{jktr} \tag{5.1}$$

$$\text{s.t.} \sum_{j\in\mathbb{V}_2} w_{jt} = p_t, \qquad \forall t \in \mathbb{T} \tag{5.2}$$

$$\sum_{j\in\mathbb{V}_2} x_{ijt} = 1, \qquad \forall i \in \mathbb{V}_1, \forall t \in \mathbb{T} \tag{5.3}$$

$$\sum_{i\in\mathbb{V}_1} D_{itr} x_{ijt} \leq C_{jtr} w_{jt}, \quad \forall j \in \mathbb{V}_2, \forall t \in \mathbb{T}, \forall r \in \mathbb{R} \tag{5.4}$$

$$\sum_{i\in\mathbb{V}_1} D_{itr} x_{ijt} = \sum_{k\in\mathbb{V}_3} \pi_{jktr}, \quad \forall j \in \mathbb{V}_2, \forall t \in \mathbb{T}, \forall r \in \mathbb{R} \tag{5.5}$$

$$\sum_{j\in\mathbb{V}_2} \pi_{jktr} \leq S_{ktr}, \qquad \forall k \in \mathbb{V}_3, \forall t \in \mathbb{T}, \forall r \in \mathbb{R} \tag{5.6}$$

$$\sum_{j\in\mathbb{V}_2} \sum_{t'\in\mathbb{T}'} (oc_j y_{jt'} + \sum_{l\in\mathbb{V}_2} rc_{jl} \gamma_{jlt'} + cc_j z_{jt'}) \leq b \tag{5.7}$$

$$w_{j1} = y_{j1}, \qquad \forall j \in \mathbb{V}_2 \tag{5.8}$$

$$w_{j(t'-1)} + y_{jt'} + \sum_{j \in \mathbb{V}_2} \gamma_{jlt'} - \sum_{l \in \mathbb{V}_2} \gamma_{jlt'} - z_{jt'} = w_{jt'}, \qquad \forall j \in \mathbb{V}_2, \forall t' \in \mathbb{T}' \backslash 1 \tag{5.9}$$

$$w_{j(t'+1)} = w_{jt'}, \qquad \forall j \in \mathbb{V}_2, \forall t' \in \mathbb{T}' \tag{5.10}$$

$$w_{j(t'+2)} = w_{jt'}, \qquad \forall j \in \mathbb{V}_2, \forall t' \in \mathbb{T}' \tag{5.11}$$

$$\vdots$$

$$w_{j(t'+\tau-1)} = w_{jt'}, \qquad \forall j \in \mathbb{V}_2, \forall t' \in \mathbb{T}'. \tag{5.12}$$

Constraint 5.2 sets the number of PODs on day t. Constraint 5.3 makes sure that demand at each vertex i is satisfied by one POD. Constraints 5.4 and 5.5 are the capacity and flow balance constraints for the PODs. Constraint 5.6 is the SSA capacity constraint. Constraint 5.7 is the budget constraint for facility opening, closing and relocations. Constraint 5.8 and 5.9 determine whether POD at vertex j is open or not on day t'. The rest of the constraints make sure that a facility is open on day $t' + \tau$, if it is open on day t'.

5.1.2 DFLRP-PD

As we mentioned in Section 1.2.3, PODs are the disaster response facilities that are in the first point of contact for people who are suffering after a disaster. In order to improve the quality and performance of disaster response operations, the mathematical models should also consider social merits such as individual walking or driving distance from demand points to their assigned POD locations. Therefore, the following model has the objective of minimizing the maximum prioritized distance traveled by people to their closest POD location. Since the objective of this model is minimizing the maximum prioritized distance, we name it as DFLRP-PD.

This model has the same constraints as in DFLRP-TC. The objective function of the model is:

$$\min \max_{i\in\mathbb{V}_1, j\in\mathbb{V}2, t\in\mathbb{T}} \{\alpha_i d_{ij} x_{ijt}\}. \tag{5.13}$$

5.1.3 DFLRP-TC&PD

Mathematical models which minimize the total traveling distance are likely to propose solutions such that some demand points are very far from their assigned facilities. Similarly, models that minimize the maximum distance from demand points to their assigned facilities might result in higher total distance. It is essential to maintain a balance between these two important objectives. Therefore, we propose a third model which is a combination of DFLRP-TC and DFLRP-PD, and named as DFLRP-TC&PD. The objective of the model is to minimize the summation of the total weighted cost of transporting humanitarian relief items and the maximum prioritized traveling distance of demand points to their closest POD locations. In order to overcome the scale difference between the total cost and the maximum prioritized distance, we multiply the maximum prioritized distance with a coefficient μ. The objective function of the model is:

$$\min \mu \max_{i\in\mathbb{V}_1, j\in\mathbb{V}2, t\in\mathbb{T}} \{\alpha_i d_{ij} x_{ijt}\} + \sum_{r\in\mathbb{R}} \sum_{t\in\mathbb{T}} \sum_{j\in\mathbb{V}_2} \sum_{i\in\mathbb{V}_1} tc_r^1 \alpha_i D_{itr} d_{ij} x_{ijt} + \sum_{r\in\mathbb{R}} \sum_{t\in\mathbb{T}} \sum_{k\in\mathbb{V}_3} \sum_{j\in\mathbb{V}_2} tc_r^2 d_{jk} \pi_{jktr}. \tag{5.14}$$

Note that, using a variable M, which denotes the maximum prioritized weighted distance, this model can be rewritten as:

$$\begin{aligned}
\min \quad & \mu M + \sum_{r\in\mathbb{R}} \sum_{t\in\mathbb{T}} \sum_{j\in\mathbb{V}_2} \sum_{i\in\mathbb{V}_1} tc_r^1 \alpha_i D_{itr} d_{ij} x_{ijt} + \sum_{r\in\mathbb{R}} \sum_{t\in\mathbb{T}} \sum_{k\in\mathbb{V}_3} \sum_{j\in\mathbb{V}_2} tc_r^2 d_{jk} \pi_{jktr} \\
\text{s.t} \quad & M \geq \alpha_i d_{ij} x_{ijt}, \forall i \in \mathbb{V}_1, \forall j \in \mathbb{V}_2, \forall t \in \mathbb{T} \\
& Constraints\ 5.2 - 5.12.
\end{aligned} \tag{5.15}$$

Dynamic facility location and relocation problem size rapidly grows with the increase of the parameters such as the number of demand nodes, potential POD locations and days in the planning horizon. In order to solve DFLRP-TC&PD in a timely fashion, the following two algorithms are developed.

5.2 Decomposition Algorithm for the Dynamic Facility Location and Relocation Problem in Disaster Response

As we mentioned, solving the MIP formulation of DFLRP-TC&PD is hard especially for large instances. Therefore, we analyzed the formulation and observed its block-angular structure, which is suitable for a decomposition approach [Sweeney and Murphy, 1979]. The following formulation demonstrates the block-angular structure of the problem, where Constraint 5.15 is the first block. The next $|\mathbb{T}|$ blocks are the capacitated p-median problem constraints for each day $t \in \mathbb{T}$, which are associated with Constraints 5.2, 5.3, and 5.4. Constraint 5.6, which is the POD capacity constraint for $|\mathbb{T}|$ days, constitutes the rest of the blocks. The rest of the constraints form the bridge constraints that connect the block constraints.

As can be seen from the formulation and explanation above, the DFRP-TC&PD consists of three problem groups. The first group is the constraints where we set the variable M to be greater than the prioritized weighted distance from all demand points to their assigned facilities. The second group is the constraints where we locate PODs and assign demand points to them. The third group is the constraints which set POD capacities. The most important problem among all is the second group, because both the first and third groups are strongly impacted by the decisions made for the second group, and most of the bridge constraints are related with this group.

$$
\begin{array}{lll}
\min & \mu M & + \sum_{r\in\mathbb{R}} \sum_{j\in\mathbb{V}_2} \sum_{i\in\mathbb{V}_1} tc_r^1 \alpha_i D_{i1r} d_{ij} x_{ij1} + \dots + \sum_{r\in\mathbb{R}} \sum_{j\in\mathbb{V}_2} \sum_{i\in\mathbb{V}_1} tc_r^1 \alpha_i D_{itr} d_{ij} x_{ijt} + \sum_{r\in\mathbb{R}} \sum_{k\in\mathbb{V}_3} \sum_{j\in\mathbb{V}_2} tc_r^2 d_{jk} \pi_{jk1r} + \dots + \sum_{r\in\mathbb{R}} \sum_{k\in\mathbb{V}_3} \sum_{j\in\mathbb{V}_2} tc_r^2 d_{jk} \pi_{jktr}
\end{array}
$$

s.t.

$$
\begin{array}{l}
M \geq \alpha_i d_{ij} x_{ijt} \\
\quad \sum_{j\in\mathbb{V}_2} w_{j1} = p_1 \\
\quad \sum_{j\in\mathbb{V}_2} x_{ij1} = 1 \\
\quad \sum_{i\in\mathbb{V}_1} D_{i1r} x_{ij1} \leq C_{j1r} w_{j1} \\
\qquad \ddots \\
\qquad\quad \sum_{j\in\mathbb{V}_2} w_{j|\mathbb{T}|} = p_{|\mathbb{T}|} \\
\qquad\quad \sum_{j\in\mathbb{V}_2} x_{ij|\mathbb{T}|} = 1 \\
\qquad\quad \sum_{i\in\mathbb{V}_1} D_{i|\mathbb{T}|r} x_{ij|\mathbb{T}|} \leq C_{j|\mathbb{T}|r} w_{j|\mathbb{T}|} \\
\qquad\qquad \sum_{j\in\mathbb{V}_2} \pi_{jk1r} \leq S_{k1r} \\
\qquad\qquad\quad \ddots \\
\qquad\qquad\qquad \sum_{j\in\mathbb{V}_2} \pi_{jk|\mathbb{T}|r} \leq S_{k|\mathbb{T}|r}
\end{array}
$$

$$
\begin{array}{l}
M \geq \alpha_i d_{ij} x_{ijt} \\
\sum_{i\in\mathbb{V}_1} D_{itr} x_{ijt} = \sum_{k\in\mathbb{V}_3} \pi_{jktr} \\
\sum_{j\in\mathbb{V}_2} \sum_{t'\in\mathbb{T}'} \left(oc_j y_{jt'} + \sum_{l\in\mathbb{V}_2} rc_{jl} \gamma_{jlt'} + cc_j z_{jt'} \right) \leq b \\
w_{j1} = y_{j1} \\
w_{j(t'-1)} + y_{jt'} + \sum_{j\in\mathbb{V}_2} \gamma_{jlt'} - \sum_{l\in\mathbb{V}_2} \gamma_{jlt'} - z_{jt'} = w_{jt'} \\
w_{j(t'+1)} = w_{jt'} \\
w_{j(t'+2)} = w_{jt'} \\
\quad \vdots \\
w_{j(t'+\tau-1)} = w_{jt'}.
\end{array}
$$

Therefore, in this decomposition algorithm, our main focus will be the second group of problems. The first and third group of problems will be incorporated in the algorithm by utilizing their relationship with the second group, which is established by the rest of the bridge constraints. Also, for the ease of expression, the rest of this section will assume $r = 1$, considering the fact that the algorithm can easily be modified to handle multiple commodities.

The second group of problems can be observed better in the following formulation:

$$
\begin{aligned}
\min \quad & \sum_{j\in\mathbb{V}_2}\sum_{i\in\mathbb{V}_1} tc^1\alpha_i D_{i1} d_{ij} x_{ij1} \quad + \ldots + \sum_{j\in\mathbb{V}_2}\sum_{i\in\mathbb{V}_1} tc^1\alpha_i D_{i\tau} d_{ij} x_{ij\tau} + \ldots \quad \ldots \quad + \sum_{j\in\mathbb{V}_2}\sum_{i\in\mathbb{V}_1} tc^1\alpha_i D_{it} d_{ij} x_{ijt} \\
\text{s.t.} \quad & \\
& \sum_{j\in\mathbb{V}_2} w_{j1} = p_1 \\
& \sum_{j\in\mathbb{V}_2} x_{ij1} = 1 \\
& \sum_{i\in\mathbb{V}_1} D_{i1} x_{ij1} \leq C_{j1} w_{j1} \\
& \qquad \ddots \\
& \sum_{j\in\mathbb{V}_2} w_{j\tau} = p_\tau \\
& \sum_{j\in\mathbb{V}_2} x_{ij\tau} = 1 \\
& \sum_{i\in\mathbb{V}_1} D_{i\tau} x_{ij\tau} \leq C_{j\tau} w_{j\tau} \\
& \qquad \ddots \\
& \qquad \ddots \\
& \sum_{j\in\mathbb{V}_2} w_{j|\mathbb{T}|} = p_{|\mathbb{T}|} \\
& \sum_{j\in\mathbb{V}_2} x_{ij|\mathbb{T}|} = 1 \\
& \sum_{i\in\mathbb{V}_1} D_{i|\mathbb{T}|} x_{ij|\mathbb{T}|} \leq C_{j|\mathbb{T}|} w_{j|\mathbb{T}|} \\
& \sum_{j\in\mathbb{V}_2}\sum_{t'\in\mathbb{T}'} (oc_j y_{jt'} + \sum_{l\in\mathbb{V}_2} rc_{jl}\gamma_{jlt'} + cc_j z_{jt'}) \leq b \\
& w_{j1} = y_{j1} \\
& w_{j(t'-1)} + y_{jt'} + \sum_{j\in\mathbb{V}_2} \gamma_{jlt'} - \sum_{l\in\mathbb{V}_2} \gamma_{jlt'} - z_{jt'} = w_{jt'} \\
& w_{j(t'+1)} = w_{jt'} \\
& w_{j(t'+2)} = w_{jt'} \\
& \qquad \vdots \\
& w_{j(t'+\tau-1)} = w_{jt'}.
\end{aligned}
$$

Using this formulation, we illustrate the concepts of subgroup and subproblem, which will be used in the decomposition algorithm. Each capacitated p-median problem represents a subproblem and every τ subproblems form a subgroup. Let $\mathbb{F}$ be the set of subproblems and we have a total of $\theta = |\mathbb{F}| = |\mathbb{T}|$ subproblems. Then, each subproblem is as follows:

$$
(SP_f) \qquad
\begin{aligned}
\text{Min} \quad & \sum_{j \in \mathbb{V}_2} \sum_{i \in \mathbb{V}_1} tc^1 \alpha_i D_{if} d_{ij} x_{ijf} \\
\text{s.t.} \quad & \sum_{j \in \mathbb{V}_2} w_{jf} = p_f \\
& \sum_{j \in \mathbb{V}_2} x_{ijf} = 1 \\
& \sum_{i \in \mathbb{V}_1} D_{if} x_{ijf} \leq C_{jf} w_{jf} \\
& w_{jf}, x_{ijf} \in \{0, 1\}.
\end{aligned}
$$

A good lower bound for DFLRP-TC&PD can be obtained by finding good lower bounds for the three groups of problems. Since the solutions of the first and third group of problems depend on the solutions of the second group of problems, we first find a lower bound for the subproblems in the second group, SP_f, and utilize them to obtain a lower bound for the other groups. Two methods can be used to find a lower bound for SP_f. The first method is to solve the LP relaxation of the capacitated p-median problem. The other method is to solve the LP relaxation of the uncapacitated p-median problem. Even though we omit the capacity constraints in the second method, lower bound quality will still be good because LP relaxation of p-median problems usually leads to optimal or very close to optimal solutions [Rosing et al., 1979]. Let x^*_{ijf} denote the optimal solution of the LP relaxation of each subproblem, M^* denote the corresponding maximum prioritized distance and π^*_{jkf} denote the corresponding total supply at POD j. Let objective function coefficients of the subproblems in the second and third group be A_{2f}, A_{3f}, respectively. The value of M^* is equal to $\max_{i \in \mathbb{V}_1, j \in \mathbb{V}2, f \in \mathbb{F}} \{\alpha_i d_{ij} x^*_{ijf}\}$. Variable π^*_{jkf} is equal to $\sum_{i \in \mathbb{V}1} D_{if} x^*_{ijf}$

and we assume that it is assigned to the closest SSA. Then the lower bound for DFLRP-TC&PD can be calculated as:

$$LB = \mu M^* + \sum_{f \in \mathbb{F}} A_{2f} x^*_{ijf} + \sum_{f \in \mathbb{F}} A_{3f} \pi^*_{jkf}. \tag{5.16}$$

After finding a lower bound for the problem, we construct the master problem (MP). The only bridge constraint included in the master problem formulation is Constraint 5.7, which is the budget constraint. Constraint 5.5 is satisfied by finding a solution for the third group using the solution of each column in the second group. Constraints 5.8, 5.10, 5.11, and 5.12 are also satisfied during the column generation process. For the ease of illustration, the following variable and parameter substitutions will be used for the master problem. Let q_f and u be binary decision variables. The variable q_f is substituted for variables w_{jt} and x_{ijt}, where f is the index for each block, $n = |\mathbb{V}_1|$, $m = |\mathbb{V}_2|$. The substitutions are as follows: $q_1 = [w_{11}, \ldots, w_{m1}, x_{111}, \ldots, x_{nm1}]$, $q_2 = [w_{12}, \ldots, w_{m2}, x_{112}, \ldots, x_{nm2}]$, $\ldots$, $q_\theta = [w_{1\theta}, \ldots, w_{m\theta}, x_{11\theta}, \ldots, x_{nm\theta}]$. The variable ν_f is substituted for variable π_{jkt}, where f is the index for each block and $s = |\mathbb{V}_3|$, i.e., $\nu_1 = [\pi_{111}, \ldots, \pi_{ms1}]$, $\nu_2 = [\pi_{112}, \ldots, \nu_{ms2}]$, $\ldots$, $\nu_\theta = [\pi_{11\theta}, \ldots, \pi_{ms\theta}]$. The variable u is substituted for variables y_{jt}, z_{jt} and γ_{jlt} i.e., $u = [y_{11}, \ldots, y_{m\theta}, z_{11}, \ldots, z_{m\theta}, \gamma_{111}, \ldots, \gamma_{mm\theta}]$.

Let σ_f be the set of columns that have been included in the master problem for subproblems $f \in \mathbb{F}$. Let $\lambda^f_{\sigma_f}$ be 1 when the corresponding column is in the optimal master problem solution and 0 otherwise. The objective function coefficient of the variable $\lambda^f_{\sigma_f}$ is denoted with $B^f_{\sigma_f}$ and calculated as $(\mu M^f_{\sigma_f} + A_{2f} q^f_{\sigma_f} + A_{3f} \nu^f_{\sigma_f})$. Bridge constraint coefficients are denoted with E_{0f} and right hand side value of the constraint is denoted with G_0. Then we can construct the master problem (MP) as:

$$
\begin{array}{lllllll}
(MP) & & & & & & \\
\text{Min} & \sum\limits_{\sigma_1} B^1_{\sigma_1}\lambda^1_{\sigma_1} & + \ldots + \sum\limits_{\sigma_2} B^\tau_{\sigma_\tau}\lambda^\tau_{\sigma_\tau} + \ldots\ldots & + \sum\limits_{\sigma_{\theta-\tau}} B^{\theta-\tau}_{\sigma_{\theta-\tau}}\lambda^{\theta-\tau}_{\sigma_{\theta-\tau}} & + \ldots\ldots + \sum\limits_{\sigma_\theta} B^\theta_{\sigma_\theta}\lambda^\theta_{\sigma_\theta} & & \\
\text{s.t.} & & & & & & \\
 & \sum\limits_{\sigma_1} E_{01}q^1_{\sigma_1} & & + \sum\limits_{\sigma_{(\theta-\tau)}} E_{0(\theta-\tau)}q^{(\theta-\tau)}_{\sigma_{(\theta-\tau)}} & & +E_{0(\theta+1)}u & = G_0, \\
 & \sum\limits_{\sigma_1} \lambda^1_{\sigma_1} & & & & & = 1, \\
 & \ddots & & & & & \\
 & & \sum\limits_{\sigma_\tau} \lambda^\tau_{\sigma_\tau} & & & & = 1, \\
 & & \ddots & & & & \\
 & & & \sum\limits_{\sigma_{\theta-\tau}} \lambda^{\theta-\tau}_{\sigma_{\theta-\tau}} & & & = 1, \\
 & & & \ddots & & & \\
 & & & & \sum\limits_{\sigma_\theta} \lambda^\theta_{\sigma_\theta} & & = 1 \\
 & \lambda^1_{\sigma_1} & -\lambda^\tau_{\sigma_\tau} & & & & = 0, \\
 & & \ddots & & & & \\
 & & & \lambda^{\theta-\tau}_{\sigma_{\theta-\tau}} & -\lambda^\theta_{\sigma_\theta} & & = 0.
\end{array}
$$

The column sets for the master problem are obtained by finding feasible solutions for the subproblems. Solution algorithms for the capacitated p-median problem can provide such solutions in a timely manner compared to an exact solution. However, those algorithms are still very time consuming for our decomposition algorithm since we are creating multiple columns at each iteration. Therefore, we utilize a fast solution approach developed for the uncapacitated p-median problem, Discrete Lloyd Algorithm (DLA) [Lim et al., 2009]. Each subproblem, SP_f, is solved using a modified version of DLA which was explained in Section 3.5. Then, we use an assignment heuristic (AH) [Martello and Toth, 1990] to find a feasible solution for SP_f. In this heuristic, we define d_i^1 as the distance between the demand point i to its closest POD location and d_i^2 as the distance between the demand point i to its second closest POD location. For each demand point, we calculate $\eta_i = \alpha_i D_{if}(d_i^2 - d_i^1)$, and sort demand points in descending order based on their η_i values. Then, we assign each demand point to the closest POD location that has enough capacity.

During this column generation procedure, an important implication of the bridge constraints should also be taken into account. Constraints 5.10, 5.11, and 5.12 require that if a POD is open on the first day of a subgroup, it has to be open for the rest of the days in the same subgroup. In order to satisfy this requirement, we first create a column for a subproblem using DLA and AH. Utilizing the w_j information of that column, we use AH to find the assignment variable solutions, x_{ij}, for the rest of the subproblems in the same subgroup. Then, we obtain columns for those subproblems by combining the w_j variables and their corresponding x_{ij} variables.

The solution of each SP_f can be used to create solutions for the first and third group of problems in DFLRP-TC&PD. In order to obtain a feasible solution for the first group, we use the variable x_{ij} to calculate the prioritized weighted distance for each demand point to its assigned POD location. The maximum value among all distances is selected and it becomes the M_f. A feasible solution for the third group is found by using AH. For each POD j, we calculate the total amount of demand satisfied by this POD, which is $\sum_{i \in \mathbb{V}i} D_i x_{ij}$. We define d_j^1 as the distance between POD j to its closest SSA location and d_j^2 as the distance between POD j to its second closest SSA location. For each POD, we calculate $\gamma_j = \sum_{i \in \mathbb{V}i} D_i x_{ij} (d_i^2 - d_i^1)$, and sort PODs in descending order based on their γ_j values. Then, we assign each POD to the closest SSA that has enough capacity and this assignment yields the variable ν_f.

The structure of our proposed solution algorithm is explained in Figure 5.1. The algorithm starts by initializing the parameters. The lower bound (z_l) for DFLRP-TC&PD is calculated by adding up the lower bound values of three groups of problems. If the LP relaxation solutions of w_{jf} variables are integer, we solve Generalized Assignment Problem (GAP) [Martello and Toth, 1990] to obtain variable x_{ijf} and add those columns to (MP). Then, multiple feasible solutions for each subgroup and subproblem are created using modified DLA and AH. An upper bound for DFLRP-TC&PD (z_u) is obtained by solving the master problem and the actual dual gap is calculated by $(z_u - z_l)/z_l$. If the actual dual gap is greater than the desired dual

Initialize: $i \leftarrow 0$,
$m \leftarrow$ Maximum number of iterations,
$\delta^* \leftarrow$ Desired dual gap.

1. Solve the LP relaxation of capacitated/uncapacitated version of (SP_f) for $f = 1, \ldots, \theta$. Obtain the solution of variables w_{jf} and x_{ijf}, and calculate the lower bound, z_l, (5.16).
2. If variables w_{jf} are integer, solve GAP to obtain variable x_{ijf}. Add those columns to (MP).

do{

3. For each subgroup, create unique solutions for the uncapacitated version of (SP_f) using modified DLA.
   4. Find a feasible assignment of demand points to facilities using AH for all SP_f in the subgroup.
      Obtain the variable ν_f using AH and calculate M_f value

   Add those feasible solutions as columns to (MP).
5. Solve (MP) and obtain the upper bound, z_u.
6. Calculate the actual dual gap,
   $\delta_i = \frac{z_u - z_l}{z_l}$.
7. $i \leftarrow i + 1$.

}while$\{(\delta_i > \delta^*) and (i \leq m)\}$

Figure 5.1: Decomposition algorithm for DFLRP-TC&PD

gap and the maximum number of iterations has not been reached, we create more solutions for the subproblems and continue the algorithm.

5.3 Divide and Conquer Heuristic for the Dynamic Facility Location and Relocation Problem in Disaster Response

In this heuristic, we solve the problem by dividing it into smaller pieces. For example, if our planning period is 12 days and facility opening, closing and relocation operations occur every other day, we can solve 3 subproblems of 4 days or 6 subproblems of 2 days. Using the parameter τ which denotes the interval between the days that we perform those operations, we present two heuristics each having $|\mathbb{T}|/2\tau$ and $|\mathbb{T}|/\tau$ subproblems.

In both heuristics, budget is allocated to each subproblem based on the number of facilities that needs to be opened/closed, and possible combinations of relocations. Subproblems are solved iteratively. The first subproblem is solved and the POD condition information (whether the POD is open or not) on the last day of the subproblem, ρ_j, is recorded. Using ρ_j as an input parameter, the second problem is solved, and the procedure is continued until all subproblems are solved. While solving all subproblems, their objective functions (z_s), M values and solution times (t_s) are also recorded. For each subproblem, the cost of transporting humanitarian relief items from SSAs to PODs and from PODs to demand points, which is denoted by z_s^{tc}, is calculated by $z_s^{tc} = z_s - \mu_s M$. Then, the total cost of transporting humanitarian relief items from SSAs to PODs and from PODs to demand points is calculated by $\sum_{s=1}^{\mathbb{S}} z_s^{tc}$. Also, the maximum M value among all subproblems is recorded as the maximum prioritized traveling distance from demand points to their assigned POD locations.

5.3.1 Divide and Conquer Heuristic with Subproblems of 2τ days

In this heuristic, we have $\mathbb{S} = |\mathbb{T}|/(2\tau)$ subproblems. Mathematical formulations of these subproblems are similar to the formulation of DFLRP-TC&PD, where the sets of $\mathbb{T}$ and $\mathbb{T}'$ are split for each subproblem such that $\mathbb{T}_1 = \{1, 2, \ldots, 2\tau\}, \mathbb{T}_2 = \{2\tau+1, 2\tau+2, \ldots, 4\tau\}, \ldots, \mathbb{T}_{\mathbb{S}} = \{(2\mathbb{S}-2)\tau+1, (2\mathbb{S}-2)\tau+2, \ldots, 2\mathbb{S}\tau\}$ and $\mathbb{T}'_1 = \{1, \tau+1\}, \mathbb{T}'_2 = \{2\tau+1, 3\tau+1\}, \ldots, \mathbb{T}'_{\mathbb{S}} = \{(2\mathbb{S}-2)\tau+1, (2\mathbb{S}-1)\tau+1\}$.

For subproblems $s = 2, 3, \ldots, \mathbb{S}$, Constraint 5.8 is exchanged with the following constraint where ρ_j denotes the POD condition output of the predecessor subproblem:

$$\rho_j + y_{jt'} + \sum_{j \in \mathbb{V}_2} \gamma_{jlt'} - \sum_{l \in \mathbb{V}_2} \gamma_{jlt'} - z_{jt'} = w_{jt'}, \forall j \in \mathbb{V}_2, \forall t' \in \mathbb{T}'_s. \tag{5.17}$$

5.3.2 Divide and Conquer with Subproblems of τ days

In this heuristic, we have $\mathbb{S} = |\mathbb{T}|/\tau$ subproblems. Subproblems of this heuristic have a similar formulation as DFLRP-TC&PD, where the set of $\mathbb{T}$ is split for each subproblem such that $\mathbb{T}_1 = \{1, 2, \ldots, \tau\}, \mathbb{T}_2 = \{\tau+1, \tau+2, \ldots, 2\tau\}, \ldots, \mathbb{T}_{\mathbb{S}} = \{(\mathbb{S}-1)\tau+1, (\mathbb{S}-1)\tau+2, \ldots, \mathbb{S}\tau\}$. In each subproblem, the first day in $\mathbb{T}_s$ is the day that we perform facility opening, closing and relocation operations. Therefore Constraint 5.9 is omitted in the subproblem formulations.

For subproblems, $s = 2, 3, \ldots, \mathbb{S}$, Constraint 5.8 is exchanged with the following constraint where ρ_j denotes the POD condition output of the predecessor subproblem:

$$\rho_j + y_{j1} + \sum_{j \in \mathbb{V}_2} \gamma_{jl1} - \sum_{l \in \mathbb{V}_2} \gamma_{jl1} - z_{j1} = w_{j1}, \forall j \in \mathbb{V}_2. \tag{5.18}$$

5.4 Numerical Results

In this section, we present our numerical results for the experiments that are run to test the performance of the methods and algorithms discussed in this chapter. All experiments presented in this section are coded in GAMS [Brooke et al., 2009] and solved by CPLEX [IBM, 2009] on an Intel Xeon 2.53 Ghz machine with 24 GB RAM.

In order to test DFLRP, information about the lack of infrastructure and affected population can be estimated by using different methods. For example, Hazus-MH is a software that estimates potential losses from disasters and it can be used to calculate the number of people who require assistance for essential commodities. Models proposed by Qiao and Vipulanandan [2010] and Liu et al. [2007] can be used to estimate the power outage and restoration times after a hurricane. The number of people leaving their homes for evacuation can be estimated by evacuation models in the literature.

As a specific example, we use an experiment data that is created based on Hurricane Ike. Power outage data for a total of 150 zip codes for 18 days (September 13 to September 30, 2008) is obtained from the Center Point Energy's website [CenterPoint Energy, 2011]. The household at each location is assumed to be composed of three members [State of Florida, 2011] and the population without power at each zip code is calculated by multiplying the number of outages by three. In disaster response, not everyone requires assistance, and the typical planning factor is 60% [State of Florida, 2011]. Therefore, we considered 60% of the population without power as the demand amount at each zip code. Priorities of the demand points are assigned on a scale between 1 to 10, based on the priorities of the evacuation zones which are obtained from the Hurricane Evacuation Zip-Zones map [Houston Galveston Area Council, 2011].

FEMA and USACE POD Guide describes three types of PODs [FEMA and USACE, 2011]. For our experiments, we use POD Type I, which has the capacity to serve 20,000 people. Since the POD capacity is defined in terms of people, the amount of demand is considered as the population without power at each zip code instead of multiplying the population by the number of commodities per person for each commodity. SSA capacities are determined to be 1,500,000.

POD and SSA locations' surface area must accommodate large tractor trailer trucks and provide easy access to and through the site. Such facilities can be selected among schools, mall parking lots, and convention centers [TxDPS, 2011]. In our experiments, we selected two university campuses, University Center at Woodlands and University of Houston Cinco Ranch Campus as State Staging Area locations. For POD locations, we picked two or three elementary, middle or high school locations in each zip code and have a total of 345 potential POD locations. In Ekici et al. [2008], the opening cost of a facility is assumed to be twice of closing a facility. We followed a similar pattern and set opening and closing costs to 1,000 and 500, respectively. Facility relocation costs are calculated by $250 + 25d_{jl}$, where d_{jl} denotes the distance between two potential POD locations j and l, $j, l \in \mathbb{V}_2$. Per our discussions with a member of the International Association of Emergency Managers (IAEM) - USA Region 6, the frequency of opening, closing and relocation operations is assumed to be every third day.

The minimum number of facilities that is required for each day is calculated using a heuristic method. For each day, demand points are sorted in descending order. Each demand point is assigned to the first available POD location with available capacity. As a result, the capacity of this POD is updated by subtracting the current demand amount from the existing capacity, and this procedure is repeated until all demand points are assigned to a POD. The index of the last POD that has a demand point assigned is recorded as the minimum number of facilities required for that day. The number of facilities calculated using this heuristic is tight, and solving the problem with them becomes very hard and time consuming. Therefore, we round the number of facilities obtained from the heuristic to the nearest multiple of 5.

To test our models, we consider the first 12 days of the power outage data. Utilizing the above-mentioned heuristic, the number of facilities on day t is calculated as follows: $p_t = 145, 145, 145, 105, 105, 105, 75, 75, 75, 50, 50, 50$. The budget allocated to perform opening, closing and relocation operations is 197,500. The amount of 192,500 is required for mandatory facility openings and closings (opening 145 facilities and closing a total of 95 facilities through the planning horizon), and 5,000 is allocated for relocations or extra openings and closings. This setting will be named as Case 1. We also run experiments with a different setting, named as Case 2, which has 5 additional facilities per day. The number of facilities on day t for Case 2 is as follows:

$p_t = 150, 150, 150, 110, 110, 110, 80, 80, 80, 55, 55, 55$. The available budget is set to 202,500 in this case. For both cases, transportation costs are assumed to be one.

5.4.1 Comparison of the DFLRP Models

In this section, we test the performance of DFLRP-TC, DFLRP-PD, and DFLRP-TC&PD. We ran DFLRP-TC&PD with three different values of μ. Based on the formulations in Section 5.1, it is obvious that all three models have different objectives. However, our goal in conducting these experiments is to show the outcome of each model and compare them in terms of transportation costs, maximum prioritized distance and solution time. DFLRP-PD is a min-max problem and typically such problems take a very long time to solve, especially for large scale instances. Therefore, for these experiments, we consider a smaller version of the experiment data. We consider the same demand points and demand values for 12 days, but 50 potential POD locations each having higher capacities. Results of these experiments are reported in Table 5.1.

Table 5.1: Comparison of DFLRP models

	DFLRP-TC	DFLRP-PD	DFLRP-TC&PD (μ_1)	DFLRP-TC&PD (μ_2)	DFLRP-TC&PD (μ_3)
Maximum prioritized distance from PODs to demand points (M)	462	264	407	275	334
Total traveling cost from PODs to demand points	463 x 10^6	1589 x 10^6	469 x 10^6	476 x 10^6	459 x 10^6
Total traveling cost from SSAs to PODs	447 x 10^6	651 x 10^6	445 x 10^6	443 x 10^6	443 x 10^6
Solution Time	186	26844	462	1138	614

DFLRP-TC has the objective of minimizing total traveling cost. However, the maximum prioritized distance obtained as a result of this model is almost twice of DFLRP-PD. Similarly, DFLRP-PD, which minimizes the maximum prioritized weighted distance, yielded a total traveling cost of more than twice of DFLRP-TC. Another drawback of DFLRP-PD is very long computation time.

DFLRP-TC&PD has both goals in its objective function, and we tested three different values of μ for this model. When $\mu_1 = 1$, the total cost component of the objective function became highly dominant on the objective value because it was in millions, where the value of M was in hundreds as we observed in Table 5.1. Such μ values do not necessarily result in lower M values. When μ_2 was set to the average demand value, the impact of M on the objective function value increased and the problem behaved similar to a min-max problem; the solution time increased rapidly. Considering all these aspects, we set μ_3 to a half of the average demand value to make sure that M had an impact on the objective function value, yet the problem could be solved in a timely manner.

5.4.2 Sensitivity Analysis for DFLRP-TC&PD on Budget

In this section, we analyze the impact of budget on the total traveling cost among the network. In addition to Case 2 mentioned in the experiment setup, we created Case 3 with p_t = 155, 155, 155, 115, 115, 115, 85, 85, 85, 60, 60, 60 and Case 4 with p_t = 160, 160, 160, 120, 120, 120, 90, 90, 90, 65, 65, 65. All cases were tested with three values of the budget. The first budget value for Cases 2, 3, and 4 are denoted with b2, b3, and b4, respectively. Each budget is equal to the amount that is required for mandatory openings and closings. The second and third budget values for each case is obtained by increasing the first budget by 2.5% and 5%, respectively. Figure 5.2 illustrates the change in the total cost of transporting humanitarian items in the network with respect to different cases and budget values.

We observe in Figure 5.2 that total cost of transporting humanitarian relief items decreased when we had a greater number of PODs located. Case 3 had a 7.8% decrease in total cost compared to Case 2, whereas Case 4 had a 5.6% decrease compared to

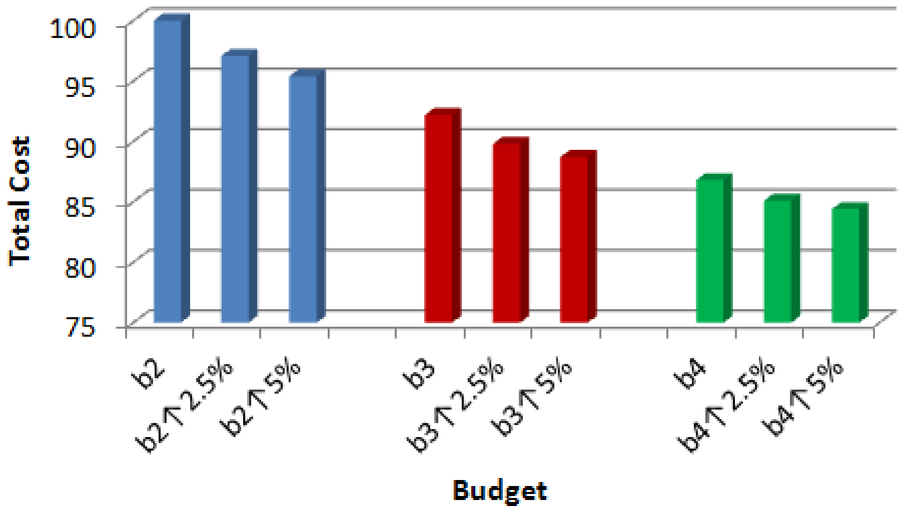


Figure 5.2: Sensitivity analysis on budget for the total cost of DFLRP-TC&PD

Case 3. This can help decision makers to determine the trade-off between the number of PODs and the total cost. Available budget also affected the total cost within each case. The impact of budget was more obvious for Case 2, where an increase in the budget by 2.5% resulted in a decrease of the total cost by 2.9%. A further increase of 2.5% provided an extra 1.7% decrease in the total cost. In Case 3, we observe that an increase in the budget by 2.5% resulted in a decrease of the total cost by 2.6%, whereas a further increase of 2.5% provided an extra 1.2% decrease in the total cost. For Case 4, an increase in the budget by 2.5% decreased the total cost by 1.9%, and a further increase of 2.5% provided an extra 0.9% decrease in the total cost. This shows that increasing the budget to perform relocations can help to reduce the total cost. However, the rate of decrease diminishes with an additonal budget. We can conclude that allocating some budget for relocations is helpful, however this benefit is not directly proportional to the additional budget and has more impact when there is a smaller number of PODs located.

5.4.3 Decomposition Algorithm for DFLRP-TC&PD

We tested the computational performance of the Decomposition Algorithm using Case 2 with a desired dual gap value set to 5%, and a number of maximum iterations set to 2000. We coded the algorithm in C++ and used CPLEX [IBM, 2009] to obtain the z_l and solve the MP. Experiment instances were also solved by the exact solution method, which is the MIP formulation introduced in Section 5.1.3. The exact method was formulated in GAMS [Brooke et al., 2009] and solved by CPLEX [IBM, 2009], and we set the relative termination tolerance to 5%.

As we mentioned in Section 5.2, the lower bound of the Decomposition Algorithm, z_l, is calculated with two different methods. In our experiments, we used LP relaxation of both the capacitated and uncapacitated version of SP_f. We compare the solution quality of these two methods in Table 5.2.

Table 5.2: Solution quality comparison of Decomposition Algorithm with capacitated and uncapacitated subproblems

	z_u	z_l	δ
DA - z_l with capacitated SP_f	956×10^6	827×10^6	13.4 %
DA - z_l with uncapacitated SP_f	955×10^6	841×10^6	11.9 %

The lower bound of the first method was less than the lower bound of the second method. This is due to the fact that LP relaxation of uncapacitated p-median problems usually provides integer solutions whereas LP relaxation of capacitated p-median problems yields fractional solutions. In our experiments, relaxing the capacity constraints but obtaining mostly integer solutions resulted in less loss in the lower bound value compared to complying with the capacity constraints but obtaining fractional solutions. Upper bound value of both methods were almost similar, but the dual gap obtained with the second method was better because it had a higher lower bound value. Even though the final dual gap value of the second method was better than the first method, it still did not satisfy the dual gap requirements. One reason behind this is the solution quality of DLA that heavily depends on the initial solution of the algorithm. To mitigate this problem Lim et al. [2009] proposed solving multiple DLAs and selecting the best solution among all, which is named as multiDLA. In Table 5.3, we compare the performance of the exact solution method with the decomposition algorithm that uses single DLA and multiDLA.

Table 5.3: Solution quality and time comparison of Decomposition Algorithm with the Exact Solution Method

	z_u	z_l	δ	Time (CPU seconds)
DA - with single DLA	955×10^6	841×10^6	11.9 %	347
DA - with multiDLA	916×10^6	841×10^6	8.2 %	971
Exact Solution Method	876×10^6	844×10^6	3.6 %	29507

We observe that multiDLA yielded better upper bound and dual gap value compared to single DLA. However, dual gap value for the decomposition algorithm still did not satisfy the desired dual gap value. The lower bound value of the decomposition algorithm was close to the objective function value of the exact solution method but the upper bound was much higher than the objective function value of the exact solution method. One reason behind this is that most of the algorithms for facility location problems assume that facilities are located at the same locations as demand points. In our experiment setting, demand points and potential POD locations are different. This difference resulted in low solution quality for the algorithms that were used for the column generation process. We should also note that decomposition algorithm was significantly faster than the exact method. For the cases where solution quality can be traded off for speed, the decomposition algorithm is a beneficial method.

5.4.4 Convergence of the Divide and Conquer Heuristic Algorithm

To test the convergence of the solution algorithm, we created 25 random instances with 50 demand points, 100 potential POD locations and 12 days. We set the parameter τ to 3 and used Divide and Conquer Heuristic with subproblems of 3 and 6 days. Experiment instances were also solved by the exact solution method, which is the MIP formulation of DFLRP-TC&PD introduced in Section 5.1.3. The relative termination tolerance for the exact solution was set to the desired dual gap value. After solving the problem we obtained the best possible solution (z^*), dual gap value and solution time. For Divide and Conquer Heuristic, we set the relative termination tolerance of the subproblems to the desired dual gap value and solved the subproblems. The results were recorded in terms of their objective function values (z_s) and the solution times. The overall dual gap of the heuristic algorithm was calculated by $(\sum_{s\in\mathbb{S}} z_s)/z^*$. Solution times of the subproblems were added up to obtain the total solution time. We calculated the average values of the dual gaps and solution times for 25 instances. Tables 5.4 and 5.5 represent a comparison of the average dual gap and solution time between the exact solution method and Divide and Conquer Heuristic with subproblems of 6 days, denoted by D&C 6 days.

Table 5.4: Dual gap and solution time comparison between the Exact Solution Method and Divide and Conquer Heuristic with subproblems of 6 days

Desired	Final Dual Gap		Solution Time (CPU seconds)	
Dual Gap	Exact	D&C 6 days	Exact	D&C 6 days
0.1	0.035	0.037	53	20
0.05	0.028	0.028	54	21
0.01	0.006	0.007	121	32
0.001	0.00007	0.0003	305	55

In Table 5.4, we observe that Divide and Conquer Heuristic with subproblems of 6 days resulted in solutions that are within the desired dual gap value set for subproblems. The heuristic algorithm took substantially less time than the exact method to solve the problems; we observed up to 81 % reduction in the solution time.

Table 5.5: Dual gap and solution time comparison between the Exact Solution Method and Divide and Conquer Heuristic with subproblems of 3 days

Desired	Final Dual Gap		Solution Time (CPU seconds)	
Dual Gap	Exact	D&C 3 days	Exact	D&C 3 days
0.1	0.035	0.056	53	16
0.05	0.028	0.041	54	19
0.01	0.006	0.007	121	25
0.001	0.00007	0.0009	305	33

Table 5.5 shows that Divide and Conquer Heuristic with subproblems of 3 days also yielded solutions that are within the desired dual gap value set for subproblems. D&C with subproblems of 3 days ran faster than both the exact solution method and D&C with subproblems of 6 days.

5.4.5 Comparison of the Exact Solution Method with the Divide and Conquer Heuristic

In this section, we present the results of the experiments that were made to compare the performance of the exact solution method and Divide and Conquer Heuristic with subproblems of 6 days as well as 3 days. All problems were solved with 3 different values of μ; $\mu_1 = 1$, $\mu_3 =$ average demand value, and $\mu_2 = 0.5\mu_3$. For Divide and Conquer Heuristic, each subproblem is denoted with $SP_s, s \in \mathbb{S}$. After solving each SP_s, we recorded its objective function value (z_s), M value and solution time (t_s). For each SP_s, the cost of transporting humanitarian relief items from SSAs to PODs and from PODs to demand points, which is denoted as z_s^{tc}, was calculated by $z_s^{tc} = z_s - \mu_s M$. Then, the total cost of transporting humanitarian relief items from SSAs to PODs and from PODs to demand points was calculated by $\sum_{s=1}^{\mathbb{S}} z_s^{tc}$.

All problems and subproblems were run with a relative termination gap set to 3% and 5% and were run up to 11 hours. Tables 5.6, 5.7, 5.8, 5.9, and 5.10 represent the results of the experiments.

The μ values used for the exact solution method are as follows: $\mu_1 = 1, \mu_2 = 47823, \mu_3 = 95646$. For Case 1, the exact solution method could not find an integer solution for the problem instances with μ_2 and μ_3. For the problem instance with μ_1, the relative gap for the solution was 6%, therefore we did not report it. The results of the experiments for Case 2 are reported in Table 5.6. Dashes in the table mean that no feasible solution was found for that instance within the time limit. For other instances, we observed that it took a long time to reach even a 5% dual gap for all μ values. Note that only the problem instance with μ_1 found a solution that is within 3%.

Table 5.6: Results of the exact solution method for Case 2

	Relative Gap = 3%				Relative Gap = 5%			
	Objective	Total Cost	M	Time	Objective	Total Cost	M	Time
μ_1	855×10^6	855×10^6	660	39316	874×10^6	874×10^6	688	27156
μ_2	-	-	-	-	876×10^6	864×10^6	394	29507
μ_3	-	-	-	-	884×10^6	872×10^6	191	34636

For Divide and Conquer Heuristic with 6 days, the following values are used for each subproblem s: $\mu_{11} = 1, \mu_{12} = 1, \mu_{21} = 31272, \mu_{22} = 16551, \mu_{31} = 62544, \mu_{32} = 33102$. For Case 1, the budget for SP_1 and SP_2 was set to 167,500 and 30,000, respectively. For Case 2, the budget for SP_1 and SP_2 was set to 172,500 and 30,000, respectively. In both cases, each subproblem was allocated the budget to perform mandatory openings and closings and an ample budget for relocations or extra openings and closings. For example, SP_1 in Case 1 required a budget of 145,000 to open 145 facilities on day 1, a budget of 20,000 to close 40 facilities on day 4, and an extra budget of 2,500 was allocated to perform relocations on day 4. The results of Divide and Conquer Heuristic with 6 days for Cases 1 and 2 are reported in Tables 5.7 and 5.8.

Dashes in Table 5.7 mean that no feasible solution was found for that instance within the time limit. Instances with a star (*) mean that a feasible solution was found, but it did not satisfy the relative gap requirements; consequently the total results obtained from those instances did not guarantee the specified relative gap. We observe that the heuristic could find feasible solutions for Case 1. However, it took a very long time to find those solutions and yet, they did not satisfy the relative gap requirements.

Divide and Conquer with 6 days results are more promising for Case 2 as we could find solutions within the relative gap. However, solution times for a 3% gap were still long, whereas solutions within 5% could be obtained within 1 or 2 hours. Another observation we made is that the solution time for SP_2 was always longer than SP_1. This could be due to the constraint difference that we explained in Section 5.3.1. SP_1 has a simpler constraint for the POD condition on the initial day compared to SP_2.

For Divide and Conquer Heuristic with 3 days, the following values are used for each subproblem s: $\mu_{11} = 1, \mu_{12} = 1, \mu_{13} = 1, \mu_{14} = 1$, $\mu_{21} = 17509, \mu_{22} = 13764, \mu_{23} = 10271, \mu_{24} = 6280$, $\mu_{31} = 35018, \mu_{32} = 27528, \mu_{33} = 20542, \mu_{34} = 12560$. The budget allocation for Case 1 is: SP_1 = 145,000, SP_2 = 22,500, SP_3 = 16,500, SP_4 = 13,500. For Case 2, the budget for SP_1 was increased to 150,000 and the rest of the budget amounts remained the same. The results of Divide and Conquer Heuristic with 3 days for Cases 1 and 2 are reported in Tables 5.9 and 5.10.

For all instances of Case 1 and 2, Divide and Conquer Heuristic with 3 days was able to find solutions within the relative gap in a timely manner except for Case 1 with 3% dual gap. In that particular instance and some other instances, solving SP_3 took a substantially longer time, which also increased the total solution time. Since this did not happen when we solved the same instance with 5% dual gap, it is hard

Table 5.7: Results of the Divide and Conquer Heuristic with 6 days for Case 1

	Relative Gap = 3%				Relative Gap = 5%			
	Objective	Total Cost	M	Time	Objective	Total Cost	M	Time
SP_1, μ_{11}	547×10^6	547×10^6	226	1674	551×10^6	551×10^6	332	1672
$^*SP_2, \mu_{12}$	354×10^6	354×10^6	202	40000	351×10^6	351×10^6	380	40000
***Total**	$\mathbf{901\times10^6}$	$\mathbf{901\times10^6}$	**226**	**41674**	$\mathbf{903\times10^6}$	$\mathbf{903\times10^6}$	**380**	**41672**
SP_1, μ_{21}	-	-	-	-	562×10^6	553×10^6	313	854
SP_2, μ_{22}	-	-	-	-	351×10^6	346×10^6	333	40000
***Total**	**-**	**-**	**-**	**-**	$\mathbf{914\times10^6}$	$\mathbf{899\times10^6}$	**333**	**40854**
SP_1, μ_{31}	-	-	-	-	-	-	-	-
SP_2, μ_{32}	-	-	-	-	-	-	-	-
Total	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**

Table 5.8: Results of the Divide and Conquer Heuristic with 6 days for Case 2

	Relative Gap = 3%				Relative Gap = 5%			
	Objective	Total Cost	M	Time	Objective	Total Cost	M	Time
SP_1, μ_{11}	538×10^6	538×10^6	214	1033	539×10^6	539×10^6	235	821
SP_2, μ_{12}	326×10^6	326×10^6	436	12162	329×10^6	329×10^6	658	3705
Total	$\mathbf{865\times10^6}$	$\mathbf{865\times10^6}$	**436**	**13195**	$\mathbf{868\times10^6}$	$\mathbf{868\times10^6}$	**658**	**4526**
SP_1, μ_{21}	541×10^6	534×10^6	231	643	541×10^6	534×10^6	231	648
SP_2, μ_{22}	324×10^6	321×10^6	162	22100	335×10^6	330×10^6	318	3745
Total	$\mathbf{866\times10^6}$	$\mathbf{856\times10^6}$	**231**	**22743**	$\mathbf{876\times10^6}$	$\mathbf{864\times10^6}$	**318**	**3745**
SP_1, μ_{31}	537×10^6	529×10^6	125	475	549×10^6	537×10^6	187	513
SP_2, μ_{32}	322×10^6	315×10^6	114	37738	335×10^6	323×10^6	347	4666
Total	$\mathbf{860\times10^6}$	$\mathbf{845\times10^6}$	**125**	**38213**	$\mathbf{884\times10^6}$	$\mathbf{860\times10^6}$	**347**	**5179**

Table 5.9: Results of the Divide and Conquer Heuristic with 3 days for Case 1

	Relative Gap = 3%				Relative Gap = 5%			
	Objective	Total Cost	M	Time	Objective	Total Cost	M	Time
SP_1, μ_{11}	298×10^6	298×10^6	358	83	300×10^6	300×10^6	358	83
SP_2, μ_{12}	247×10^6	247×10^6	175	194	248×10^6	248×10^6	265	328
SP_3, μ_{13}	208×10^6	208×10^6	186	28436	212×10^6	212×10^6	355	1855
SP_4, μ_{14}	131×10^6	131×10^6	596	209	133×10^6	133×10^6	751	211
Total	$\mathbf{886\times10^6}$	$\mathbf{886\times10^6}$	**596**	**28922**	$\mathbf{895\times10^6}$	$\mathbf{895\times10^6}$	**751**	**2477**
SP_1, μ_{21}	300×10^6	294×10^6	290	85	301×10^6	296×10^6	290	81
SP_2, μ_{22}	251×10^6	250×10^6	114	887	254×10^6	251×10^6	244	352
SP_3, μ_{23}	210×10^6	208×10^6	140	13147	213×10^6	210×10^6	296	531
SP_4, μ_{24}	132×10^6	131×10^6	394	7426	135×10^6	133×10^6	455	556
Total	$\mathbf{894\times10^6}$	$\mathbf{885\times10^6}$	**394**	**21541**	$\mathbf{904\times10^6}$	$\mathbf{891\times10^6}$	**455**	**1520**
SP_1, μ_{31}	301×10^6	295×10^6	179	119	305×10^6	297×10^6	233	108
SP_2, μ_{32}	252×10^6	248×10^6	145	418	249×10^6	246×10^6	135	228
SP_3, μ_{33}	210×10^6	207×10^6	162	38027	215×10^6	211×10^6	187	394
SP_4, μ_{34}	133×10^6	129×10^6	268	4199	137×10^6	132×10^6	394	3473
Total	$\mathbf{897\times10^6}$	$\mathbf{880\times10^6}$	**268**	**42763**	$\mathbf{908\times10^6}$	$\mathbf{887\times10^6}$	**394**	**11158**

to attribute the long computational time to the formulation. It might be due to the network structure, where finding a good quality solution with that particular number of facilities could be difficult.

One common observation for Divide and Conquer Heuristic is that the maximum prioritized distance value, M, was usually identified by the last subproblems. For Divide and Conquer with 6 days, SP_2 usually had a higher M value compared to SP_1. Similarly, for Divide and Conquer with 6 days, SP_4 was the subproblem with the highest M value. This happens because last subproblems have a smaller number of facilities and their locations in the network tend to be more sparse. In such cases, the distance between a demand node and its closest facility might be larger compared to the cases with a higher number of facilities.

The performance of Divide and Conquer Heuristic with 6 days and 3 days for Case 1 is compared in Table 5.11. The exact solution method is not included in this comparison because we could not find any feasible instances for Case 1 using the exact solution method. Furthermore, solutions found by Divide and Conquer with 6 days could not reach the relative termination gap within the time limit. Therefore, it is hard to make an overall conclusion on the solution quality for Case 1. However, we observed the impact of μ value on the maximum prioritized distance while using Divide and Conquer with 3 days. When we increased the value of μ, we obtained a lower maximum prioritized distance. However, an increase in μ resulted in an increase in the impact of the maximum prioritized distance on the objective function, which also increased the solution time.

The performance of the exact solution method and Divide and Conquer Heuristics with 6 days and 3 days for Case 2 is compared in Table 5.12. For all three methods, we observed a similar impact of μ value on the maximum prioritized distance and solution time as in Case 1. From our experiments, it is clear that the exact solution method was very time consuming, whereas Divide and Conquer Heuristics with 6

Table 5.10: Results of the Divide and Conquer Heuristic with 3 days for Case 2

	Relative Gap = 3%				Relative Gap = 5%			
	Objective	Total Cost	M	Time	Objective	Total Cost	M	Time
SP_1, μ_1	288×10^6	288×10^6	108	48	288×10^6	288×10^6	108	49
SP_2, μ_2	243×10^6	243×10^6	192	121	243×10^6	243×10^6	192	122
SP_3, μ_3	197×10^6	197×10^6	352	2035	201×10^6	201×10^6	268	1882
SP_4, μ_4	125×10^6	125×10^6	660	120	126×10^6	126×10^6	566	153
Total	$\mathbf{856\times10^6}$	$\mathbf{856\times10^6}$	**660**	**2324**	$\mathbf{861\times10^6}$	$\mathbf{861\times10^6}$	**566**	**2206**
SP_1, μ_1	293×10^6	290×10^6	186	51	293×10^6	290×10^6	186	51
SP_2, μ_2	243×10^6	241×10^6	162	536	248×10^6	244×10^6	227	360
SP_3, μ_3	200×10^6	197×10^6	321	1000	203×10^6	199×10^6	360	609
SP_4, μ_4	127×10^6	125×10^6	353	1061	129×10^6	126×10^6	380	175
Total	$\mathbf{865\times10^6}$	$\mathbf{854\times10^6}$	**353**	**2648**	$\mathbf{874\times10^6}$	$\mathbf{861\times10^6}$	**380**	**1195**
SP_1, μ_1	295×10^6	290×10^6	162	64	301×10^6	294×10^6	179	63
SP_2, μ_2	241×10^6	238×10^6	128	162	248×10^6	243×10^6	195	159
SP_3, μ_3	200×10^6	195×10^6	224	1015	204×10^6	198×10^6	321	226
SP_4, μ_4	127×10^6	124×10^6	245	7019	130×10^6	125×10^6	455	706
Total	$\mathbf{865\times10^6}$	$\mathbf{848\times10^6}$	**245**	**8260**	$\mathbf{885\times10^6}$	$\mathbf{861\times10^6}$	**455**	**1154**

Table 5.11: Comparison of the Divide and Conquer Heuristics with 6 days and 3 days for Case 1

		Relative Gap = 3%			Relative Gap = 5%		
		Total Cost	M	Time	Total Cost	M	Time
D&C 6 days	$*\mu_1$	901×10^6	226	41674	903×10^6	380	41672
	$*\mu_2$	-	-	-	899×10^6	333	40854
	μ_3	-	-	-	-	-	-
D&C 3 days	μ_1	886×10^6	596	28922	895×10^6	751	2477
	μ_2	885×10^6	394	21541	891×10^6	455	1520
	μ_3	880×10^6	268	42763	887×10^6	394	11158

Table 5.12: Comparison of the Exact Solution Method and Divide and Conquer Heuristics with 6 days and 3 days for Case 2

		Relative Gap = 3%			Relative Gap = 5%		
		Total Cost	M	Time	Total Cost	M	Time
Original Formulation	μ_1	855×10^6	660	39316	874×10^6	688	27516
	μ_2	-	-	-	864×10^6	394	29507
	μ_3	-	-	-	872×10^6	191	34636
D&C 6 days	μ_1	865×10^6	436	13195	868×10^6	658	4526
	μ_2	856×10^6	231	22743	864×10^6	318	3745
	μ_3	845×10^6	125	38213	860×10^6	347	5179
D&C 3 days	μ_1	856×10^6	660	2324	861×10^6	566	2206
	μ_2	854×10^6	353	2648	861×10^6	380	1195
	μ_3	848×10^6	245	8260	861×10^6	455	1154

days and 3 days were able to find good quality solutions in a timely manner. With respect to the solution quality, Divide and Conquer with 6 days found the smallest maximum prioritized distance for relative termination gaps 3% and 5% whereas the total cost for Divide and Conquer with 3 days was equal or lower. Therefore, Divide and Conquer with 3 days outperformed other methods considering the fact that it was faster and could find the best total cost and reasonable maximum prioritized distance.

5.5 Summary

In this chapter, we proposed solution methodologies for dynamic facility location and relocation problem in disaster response. We introduced three MIP formulations each having different objective functions. Utilizing the block-angular structure of the problem, we developed a decomposition based solution algorithm for DFLRP-TC&PD. We also proposed a heuristic algorithm, in which we divide the problem into smaller subproblems and solve them iteratively by using the output of the predecessor problem as an input.

In order to test the performance of the methods that we ran numerical experiments using data from Hurricane Ike. We first presented the numerical results that compare the performance of DFLRP-TC, DFLRP-PD and DFLRP-TC&PD with different μ values. DFLRP-TC&PD can fulfill the objectives of both DFLRP-TC and DFLRP-PD, and setting μ to a half of the average demand value provided a good balance between these two objectives. We also performed sensitivity analysis for DFLRP-TC&PD to observe the trade off between the budget and total cost. Finally, we tested the performance of the proposed solution algorithms concluding that Divide and Conquer Heuristic with 3 days outperforms others.

We then presented numerical results of the decomposition algorithm and Divide and Conquer Heuristic algorithm. Results of the decomposition algorithm showed that the algorithm provided very fast results, however the dual gap was not within the desired value. Comparing the objective function value, total cost, M value and solution time of Divide and Conquer Heuristic with the exact method, we concluded that the heuristic algorithm could find good quality solutions in a timely manner.

Chapter 6 Summary and Future Work

6.1 Summary

In this dissertation, we studied the facility location and relocation problem with demand change. Due to demand changes in the network over time, we may need to close some of the existing facilities and open new ones. Specifically, we considered three optimization problems.

The first problem, which we named FLRP-U is to locate and relocate facilities considering future demand changes as well as uncertain number of future facilities. The objective is to minimize the sum of the current weighted distance and the expected future distance traveled by customers to their closest facilities without exceeding a given budget for opening and closing facilities. We introduced an integer programming model of the problem and presented some numerical results that compare the performance of our method against another method adapted from existing literature, which was named FLRP-D. Based on the average total weighted distance, FLRP-U outperformed FLRP-D in all scenarios. We conducted sensitivity analysis that shows the impact of budget increase on total weighted distance. This analysis can be utilized for making decisions about whether finding extra sources to increase the available budget for relocations is worthwhile or not. We developed a decomposition algorithm for FLRP-U to ease the time to solve the problem for large scale instances and high uncertainty. experiment results concluded that our proposed algorithm yields a significant time gain, while satisfying the desired dual gap level.

The second problem that we worked on was the robust facility relocation problem that considers uncertain demand changes. We presented two solution approaches for this problem. In the first approach, we introduced an integer programming model that provides α-reliable robust relocations that minimize the maximum regret associated with a set of scenarios whose cumulative probability is at least α. In the second approach, we presented an integer programming model that determines optimal relocations of facilities with respect to γ-robustness under uncertain demand changes. Due to the long computational time especially for larger instances, we developed a Lagrangean Decomposition Algorithm (LDA) to expedite the solution process. Our experiments showed that, LDA provides a significant time gain for moderate size problems. LDA is the clear winner if the problem size increased because for larger scale problems, the exact method could not generate any integer feasible solution for hours of run. We conducted sensitivity analysis that showed the impact of budget and γ values on the expected weighted traveling distance. This analysis can be utilized to determine the trade-off between the expected traveling distance from customers to their closest facilities and robustness for various budget levels.

In our third problem, we developed three optimization models that determine the locations and relocations of PODs after a disaster. The first model minimizes the total transportation cost of the commodities, whereas the second model minimizes the maximum distance traveled by people to their closest POD location. The third model, which is named as DFLRP-TC&PD incorporates both goals in its objective. We first compared the performance of all three methods and then conducted sensitivity analysis for DFLRP-TC&PD. This analysis can help decision makers to determine the trade-off between the budget and the total cost of transporting humanitarian relief items. We also proposed two approaches to solve DFLRP-TC&PD. The first approach, which is a decomposition algorithm, could provide very fast results, however the solution quality was not very satisfactory. The second approach, which we named as Divide and Conquer Heuristic, was able to find good quality solutions in a timely manner.

6.2 Future Work

In this section, we explain the potential research avenues that are created by the problems that we studied in this dissertation. We also discuss possible improvements for the methodologies that we proposed. The facility location and relocation problem under uncertainty is suitable for a two stage stochastic solution method. The initial location decisions represent the first stage whereas the relocation decisions in the future represent the second stage. Therefore, this problem can also be solved using two-stage stochastic solution techniques.

For the robust facility relocation problem, we did not consider any capacity limitations for the facilities. As a future work, this problem can be extended to a capacitated robust facility relocation problem to better reflect the reality. In such a case, for the γ-robust relocation approach, capacity limitations will contribute to the infeasibility of the problems caused by robustness and budget constraint. Therefore, infeasibility issues should further be investigated and solution algorithms should be developed. For α-reliable relocation approach, solution algorithms should be developed for both the capacitated and uncapacitated version of the problem.

The decomposition algorithm developed for dynamic facility location and relocation problem can be improved by developing a better solution algorithm for the subproblems, which can handle distinct locations for demand points and potential facility locations. This problem can be extended to a multi-level decision making problem where both federal and local governmental decisions take place. Another important issue in this problem is the demand uncertainty. A further extension of this problem can treat demand as an uncertain parameter and propose location and relocation of facilities throughout the planning horizon.

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Appendix - Infeasibility conditions for Lagrangean Decomposition Algorithm for γ-RoFRP

Let $\overline{z}^* \leq \sum_{k \in \mathbb{S}} \beta_k (1+\gamma) \zeta_k^*$ as discussed in Proposition 2. Let $f_1 = \sum_{i \in \mathbb{V}} \sum_{j \in \mathbb{V}} \sum_{k \in \mathbb{S}} \sigma \beta_k \lambda_{ik} d_{ijk} x_{ijk} + u[\sum_{j \in \mathbb{V}_1} c_j (1-w_j) + \sum_{j \in \mathbb{V}_2} o_j w_j - b] - \sum_{i \in \mathbb{V}} \sum_{j \in \mathbb{V}} \sum_{k \in \mathbb{S}} l_{ijk} x_{ijk}$, $f_2 = \sum_{i \in \mathbb{V}} \sum_{j \in \mathbb{V}} \sum_{k \in \mathbb{S}} (1-\sigma) \beta_k \lambda_{ik} d_{ijk} \tau_{ijk} + \sum_{i \in \mathbb{V}} \sum_{j \in \mathbb{V}} \sum_{k \in \mathbb{S}} l_{ijk} \tau_{ijk}$ at a given iteration that improves the previous lower bound and $\underline{z}^* = f_1 + f_2$. If $\underline{z}^* > \overline{z}^*$ the problem is proved to be infeasible and such infeasibility occurs in the following conditions:

Let $\eta = \sum_{j \in \mathbb{V}1} c_j (1 - w_j) + \sum_{j \in \mathbb{V}2} o_j w_j - b$ and $\alpha_{ijk} = \beta_k \lambda_{ik} d_{ij}$. Following the $dFRP$ formulation, $\zeta_k = \sum_{i \in \mathbb{V}} \sum_{i \in \mathbb{V}} \lambda_{ik} d_{ij} \phi_{ijk}$.

$\underline{z}^* = \eta u + \sum_{(i,j,k)|x_{ijk}=\phi_{ijk}} \alpha_{ijk} [\sigma x_{ijk} + (1-\sigma)\tau_{ijk}] + \sum_{(i,j,k)|x_{ijk}=\phi_{ijk}} l_{ijk} [x_{ijk} - \tau_{ijk}]$

$\overline{z}^* = \sum_{(i,j,k)|x_{ijk}=\phi_{ijk}} \alpha_{ijk} \phi_{ijk} + \sum_{(i,j,k)|x_{ijk}=\phi_{ijk}} \gamma \alpha_{ijk} \phi_{ijk}$.

Then, $\underline{z}^* = \eta u + \sum_{(i,j,k)_1} \alpha_{ijk} [\sigma x_{ijk} + (1-\sigma)\tau_{ijk}] + \sum_{(i,j,k)_1} l_{ijk} [x_{ijk} - \tau_{ijk}] + \sum_{(i,j,k)_2} \alpha_{ijk} [\sigma x_{ijk} + (1-\sigma)\tau_{ijk}] + \sum_{(i,j,k)_2} l_{ijk} [x_{ijk} - \tau_{ijk}] + \sum_{(i,j,k)_3} \alpha_{ijk} [\sigma x_{ijk} + (1-\sigma)\tau_{ijk}] + \sum_{(i,j,k)_3} l_{ijk} [x_{ijk} - \tau_{ijk}]$.

1. Case 1: $\tau_{ijk} = x_{ijk}$ and $x_{ijk} \neq \phi_{ijk}$

   Considering these inequalities and binary nature of the variables, the index set for the variables can be divided into 3 subsets as follows:

   (a) $(i,j,k)_1 = (i,j,k)|(x_{ijk} = \phi_{ijk})$

   (b) $(i,j,k)_2 = (i,j,k)|(x_{ijk} > \phi_{ijk})$

   (c) $(i,j,k)_3 = (i,j,k)|(x_{ijk} < \phi_{ijk})$

   Then, $\underline{z}^* = \eta u + \sum_{(i,j,k)_1} \alpha_{ijk} x_{ijk} + \sum_{(i,j,k)_2} \alpha_{ijk} x_{ijk}$ and

   $\overline{z}^* = \sum_{(i,j,k)_1} \alpha_{ijk} \phi_{ijk} + \sum_{(i,j,k)_1} \gamma \alpha_{ijk} \phi_{ijk} + \sum_{(i,j,k)_3} \alpha_{ijk} \phi_{ijk} + \sum_{(i,j,k)_3} \gamma \alpha_{ijk} \phi_{ijk}$.

   Therefore, the problem is infeasible if $\eta u > \sum_{(i,j,k)|(x_{ijk} \leq \phi_{ijk})} (1+\gamma) \alpha_{ijk} - \sum_{(i,j,k)|(x_{ijk} > \phi_{ijk})} \gamma \alpha_{ijk}$.

2. Case 2: $\tau_{ijk} \neq x_{ijk}$ and $x_{ijk} = \phi_{ijk}$

   Considering these inequalities and binary nature of the variables, the index set for the variables can be divided into 3 subsets as follows:

   (a) $(i,j,k)_1 = (i,j,k)|((\tau_{ijk} = x_{ijk})\&\&(x_{ijk} = \phi_{ijk})\&\&(\tau_{ijk} = \phi_{ijk}))$

   (b) $(i,j,k)_2 = (i,j,k)|((\tau_{ijk} > x_{ijk})\&\&(x_{ijk} = \phi_{ijk})\&\&(\tau_{ijk} > \phi_{ijk}))$

   (c) $(i,j,k)_3 = (i,j,k)|((\tau_{ijk} < x_{ijk})\&\&(x_{ijk} = \phi_{ijk})\&\&(\tau_{ijk} < \phi_{ijk}))$

   Then, $\underline{z}^* = \eta u + \sum\limits_{(i,j,k)_1} \alpha_{ijk} x_{ijk} + \sum\limits_{(i,j,k)_2} (1-\sigma)\alpha_{ijk}\tau_{ijk} + \sum\limits_{(i,j,k)_2} l_{ijk}\tau_{ijk} + \sum\limits_{(i,j,k)_3} \sigma\alpha_{ijk}x_{ijk} - \sum\limits_{(i,j,k)_3} l_{ijk}x_{ijk}$ and

   $\overline{z}^* = \sum\limits_{(i,j,k)_1} \alpha_{ijk}\phi_{ijk} + \sum\limits_{(i,j,k)_1} \gamma\alpha_{ijk}\phi_{ijk} + \sum\limits_{(i,j,k)_3} \alpha_{ijk}\phi_{ijk} + \sum\limits_{(i,j,k)_3} \gamma\alpha_{ijk}\phi_{ijk}$.

   Therefore, the problem is infeasible if

   $$\eta u > \sum_{\substack{(i,j,k)|((x_{ijk} \geq \tau_{ijk})\\ \&\&(x_{ijk} = \phi_{ijk}))}} \gamma\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk} > \tau_{ijk})\\ \&\&(x_{ijk} = \phi_{ijk}))}} (1-\sigma)\alpha_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk} < \tau_{ijk})\\ \&\&(x_{ijk} = \phi_{ijk}))}} (1-\sigma)\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk} > \tau_{ijk})\\ \&\&(x_{ijk} = \phi_{ijk}))}} l_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk} < \tau_{ijk})\\ \&\&(x_{ijk} = \phi_{ijk}))}} l_{ijk}.$$

3. Case 3: $\tau_{ijk} \neq x_{ijk}$ and $\tau_{ijk} = \phi_{ijk}$

   Considering these inequalities and binary nature of the variables, the index set for the variables can be divided into 3 subsets as follows:

   (a) $(i,j,k)_1 = (i,j,k)|((\tau_{ijk} = x_{ijk})\&\&(\tau_{ijk} = \phi_{ijk})\&\&(x_{ijk} = \phi_{ijk}))$

   (b) $(i,j,k)_2 = (i,j,k)|((\tau_{ijk} > x_{ijk})\&\&(\tau_{ijk} = \phi_{ijk})\&\&(x_{ijk} < \phi_{ijk}))$

   (c) $(i,j,k)_3 = (i,j,k)|((\tau_{ijk} < x_{ijk})\&\&(\tau_{ijk} = \phi_{ijk})\&\&(x_{ijk} > \phi_{ijk}))$

   Then, $\underline{z}^* = \eta u + \sum\limits_{(i,j,k)_1} \alpha_{ijk} x_{ijk} + \sum\limits_{(i,j,k)_2} (1-\sigma)\alpha_{ijk}\tau_{ijk} + \sum\limits_{(i,j,k)_2} l_{ijk}\tau_{ijk} + \sum\limits_{(i,j,k)_3} \sigma\alpha_{ijk}x_{ijk} - \sum\limits_{(i,j,k)_3} l_{ijk}x_{ijk}$ and

   $\overline{z}^* = \sum\limits_{(i,j,k)_1} \alpha_{ijk}\phi_{ijk} + \sum\limits_{(i,j,k)_1} \gamma\alpha_{ijk}\phi_{ijk} + \sum\limits_{(i,j,k)_2} \alpha_{ijk}\phi_{ijk} + \sum\limits_{(i,j,k)_2} \gamma\alpha_{ijk}\phi_{ijk}$.

   Therefore, the problem is infeasible if

$\eta u > \sum_{\substack{(i,j,k)|((x_{ijk}\leq\tau_{ijk}) \\ \&\&(\tau_{ijk}=\phi_{ijk}))}} \gamma\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}<\tau_{ijk}) \\ \&\&(\tau_{ijk}=\phi_{ijk}))}} (1-\sigma)\alpha_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}>\tau_{ijk}) \\ \&\&(\tau_{ijk}=\phi_{ijk}))}} (1-\sigma)\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}>\tau_{ijk}) \\ \&\&(\tau_{ijk}=\phi_{ijk}))}} l_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}<\tau_{ijk}) \\ \&\&(\tau_{ijk}=\phi_{ijk}))}} l_{ijk}.$

4. Case 4: $\tau_{ijk} \neq x_{ijk}$ and $x_{ijk} \neq \phi_{ijk}$ and $\tau_{ijk} \neq \phi_{ijk}$

   Considering these inequalities and binary nature of the variables, the index set for the variables can be divided into 7 subsets as follows:

   (a) $(i,j,k)_{11} = (i,j,k)|((\tau_{ijk} = x_{ijk})\&\&(x_{ijk} = \phi_{ijk})\&\&(\tau_{ijk} = \phi_{ijk}))$

   (b) $(i,j,k)_{12} = (i,j,k)|((\tau_{ijk} > x_{ijk})\&\&(x_{ijk} > \phi_{ijk})\&\&(\tau_{ijk} > \phi_{ijk}))$

   (c) $(i,j,k)_{13} = (i,j,k)|((\tau_{ijk} < x_{ijk})\&\&(x_{ijk} < \phi_{ijk})\&\&(\tau_{ijk} < \phi_{ijk}))$

   (d) $(i,j,k)_{21} = (i,j,k)|((\tau_{ijk} > x_{ijk})\&\&(x_{ijk} = \phi_{ijk})\&\&(\tau_{ijk} > \phi_{ijk}))$

   (e) $(i,j,k)_{22} = (i,j,k)|((\tau_{ijk} > x_{ijk})\&\&(x_{ijk} < \phi_{ijk})\&\&(\tau_{ijk} = \phi_{ijk}))$

   (f) $(i,j,k)_{31} = (i,j,k)|((\tau_{ijk} < x_{ijk})\&\&(x_{ijk} = \phi_{ijk})\&\&(\tau_{ijk} < \phi_{ijk}))$

   (g) $(i,j,k)_{32} = (i,j,k)|((\tau_{ijk} < x_{ijk})\&\&(x_{ijk} > \phi_{ijk})\&\&(\tau_{ijk} = \phi_{ijk}))$

   Then, $\underline{z}^* = \eta u + \sum_{(i,j,k)_{11}} \alpha_{ijk}x_{ijk} + \sum_{(i,j,k)_{12}} \alpha_{ijk}x_{ijk} + \sum_{(i,j,k)_{21}} (1-\sigma)\alpha_{ijk}\tau_{ijk} + \sum_{(i,j,k)_{21}} l_{ijk}\tau_{ijk} + \sum_{(i,j,k)_{22}} (1-\sigma)\alpha_{ijk}\tau_{ijk} + \sum_{(i,j,k)_{22}} l_{ijk}\tau_{ijk} + \sum_{(i,j,k)_{31}} \sigma\alpha_{ijk}x_{ijk} - \sum_{(i,j,k)_{31}} l_{ijk}x_{ijk} + \sum_{(i,j,k)_{32}} \sigma\alpha_{ijk}x_{ijk} - \sum_{(i,j,k)_{32}} l_{ijk}x_{ijk}$ and

   $\overline{z}^* = \sum_{(i,j,k)_{11}} \alpha_{ijk}\phi_{ijk} + \sum_{(i,j,k)_{11}} \gamma\alpha_{ijk}\phi_{ijk} + \sum_{(i,j,k)_{13}} \alpha_{ijk}\phi_{ijk} + \sum_{(i,j,k)_{13}} \gamma\alpha_{ijk}\phi_{ijk} + \sum_{(i,j,k)_{22}} \alpha_{ijk}\phi_{ijk} + \sum_{(i,j,k)_{22}} \gamma\alpha_{ijk}\phi_{ijk} + \sum_{(i,j,k)_{31}} \alpha_{ijk}\phi_{ijk} + \sum_{(i,j,k)_{31}} \gamma\alpha_{ijk}\phi_{ijk}.$

   Therefore, the problem is infeasible if

   $\eta u > \sum_{(i,j,k)|\phi_{ijk}>0} \gamma\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}<\phi_{ijk}) \\ \&\&(\tau_{ijk}<\phi_{ijk}))}} \alpha_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}>\phi_{ijk}) \\ \&\&(\tau_{ijk}>\phi_{ijk}))}} \alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}<\phi_{ijk}) \\ \&\&(\tau_{ijk}=\phi_{ijk}))}} \sigma\alpha_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}>\phi_{ijk}) \\ \&\&(\tau_{ijk}=\phi_{ijk}))}} \sigma\alpha_{ijk} + \sum_{\substack{(i,j,k)|((x_{ijk}=\phi_{ijk}) \\ \&\&(\tau_{ijk}<\phi_{ijk}))}} \sigma\alpha_{ijk} - \sum_{\substack{(i,j,k)|((x_{ijk}=\phi_{ijk}) \\ \&\&(\tau_{ijk}>\phi_{ijk}))}} \sigma\alpha_{ijk} + \sum_{(i,j,k)|x_{ijk}<\tau_{ijk}} l_{ijk} - \sum_{(i,j,k)|x_{ijk}>\tau_{ijk}} l_{ijk}.$