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Theory and Methodology

# Planning and coordination of production and distribution facilities for multiple commodities

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**Abstract**

We study an integrated logistics model for locating production and distribution facilities in a multi-echelon environment. Designing such logistics systems requires two essential decisions, one strategic (e.g., where to locate plants and warehouses) and the other operational (distribution strategy from plants to customer outlets through warehouses). The distribution strategy is influenced by the product mix at each plant, the shipments of raw material from vendors to manufacturing plants and the distribution of finished products from the plants to the different customer zones through a set of warehouses. First we provide a mixed integer programming formulation to the integrated model. Then, we present an efficient heuristic solution procedure that utilizes the solution generated from a Lagrangian relaxation of the problem. We use this heuristic procedure to evaluate the performance of the model with respect to solution quality and algorithm performance. Results of extensive tests on the solution procedure indicate that the solution method is both efficient and effective. Finally a 'real-world' example is solved to explore the implications of the model. © 2001 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

The framework for decision analysis of a large-scale facilities' configuration has become an important line of research in private and public sectors. A configuration problem consists of simultaneous decisions about: (1) manufacturing plant and distribution center (warehouse) locations, (2) specification of plant and warehouse capacities, and (3) distribution systems for raw materials and goods in process. The purpose of this paper is to design and test a production and distribution systems model and evaluate its performance with respect to solution quality, model validation and algorithm performance. This problem has resulted from a study conducted by the authors for a large company manufacturing

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health-care products. The paper reports on the development of a model structure that can be used to predict the performance of the company with respect to its distribution and transportation costs and the level of service provided to its customers.

The model introduced in this paper belongs to the production–distribution, facility location–allocation problem class. This integrated production and distribution design problem has been developed from a strategic perspective in which an organization desires to evaluate the expansion, contraction, or relocation of its facilities and the associated production tasks and unique customer assignments to each facility. The model that is proposed can also be used at an operational setting and tackles both task and customer assignments to existing facilities. Material flow in production and distribution systems is managed by a variety of processes. In this paper, the inputs to various manufacturing plants consist of raw materials that can be sourced from different vendors. The output of finished products can then be transported to different warehouses that can distribute the multiple products to different outlets subject to customer demands for the products (see Fig. 1).

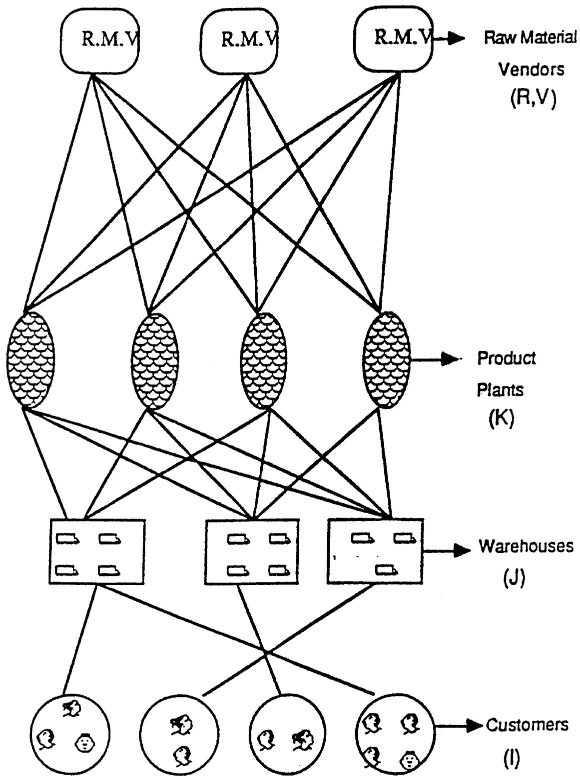

Fig. 1. Integrated multi-plant, multi-product distribution model.

A great deal of research has been carried out to develop models and exact algorithms in the area of facility location–allocation (Aikens, 1985; ReVelle and Laporte, 1996). We use the term facility here in its broadest sense. It is meant to include entities such as factories, distribution centers, warehouses, retail outlets, schools, hospitals, computer concentrators and terminals, emergency warning sirens, and day-care centers to name but a few that have been analyzed in the literature. When each facility has a limited capacity, the problem is referred to as the capacitated facility location problem. The single-source, capacitated facility location problem is a special case of the capacitated facility location problem in which each customer can only be supplied from exactly one facility.

A further extension of the location problem is the two-echelon facility location problem. Here deliveries are made from first-echelon facilities (such as plants) to second-echelon facilities (such as warehouses), and from there to customers. It is most commonly referred to as the plant location problem. The capacitated plant location problem has been solved by using both approximate and exact solutions (see, e.g., Lee, 1993; Tragantalerngsak et al., 1997). Geoffrion and Graves (1974) was one of the first to solve the version of the multi-commodity location problem that is characterized by multiple products, capacitated plants and warehouses, product flow and customer assignment. They used Benders Decomposition to solve their problem by considering the linear programming subproblem and decomposing it into as many independent transportation problems as there are commodities. The model dealt with facility location and customer assignment. However, the model does not employ any variable to include selection among alternative plant sites and also does not incorporate any fixed cost for annual possession and operating costs for plants. For a survey of work on the plant location problem, the interested reader is referred to some survey articles in this area (Krarup and Pruzan, 1983; ReVelle and Laporte, 1996).

The two-echelon facility location problem is further extended to incorporate vendors who supply raw materials to production plants. Many models in the literature that are concerned with material procurement, production or distribution activities treat each stage of the supply chain as a separate system and ignore complex supply chain interactions. Moon (1987) presented a brief overview of managerial aspects in production distribution facilities' design decisions and also reviewed a few mathematical models and solution algorithms for single and multiproduct problems. Brown et al. (1987) developed an optimization-based decision support system that involved facility selection, equipment location and utilization, and manufacture and distribution of products. They formulated a mixed integer model and applied a goal decomposition technique to decompose their problem into network subproblems for each product. Cohen and Lee (1988) presented a framework for linking decisions and performance throughout the supply chain. Bhatnagar et al. (1993) provide a literature review on the problem of integrating and coordinating parts of the logistics chain. Pyke and Cohen (1994) developed a Markov-chain model of an integrated production–distribution system. It consisted of a single factory, a stockpile of finished goods, and a single retailer. They provided results that yield insight into the behavior of their integrated system.

Although techniques like Branch and Bound and Benders Decomposition-based algorithms are well known, they do have some inherent disadvantages (Lee, 1993). Benders Decomposition methods exploit only the primal structure of the problem. However, it has been observed that many mixed-integer programming problems have both easy-to-solve primal and dual subproblems. Branch and Bound techniques expend considerable amount of time and effort to find an optimal solution for realistic size problems. Any production–distribution model must also capture all product flows within the firm's supply chain. In the past, the multi-product problem has been treated primarily by modeling the problem as if a single product mix is shipped. This drastically reduces the number of variables and facilitates model solution but lessens the model's accurate portrayal of the problem.

The paper makes three primary contributions that essentially differentiates this work from past literature. First, in the proposed model, we consider issues related to sourcing strategy like vendor source of raw materials inputs to manufacturing plants together with costs associated with production, distribution, and transportation. We look at the entire supply chain and make decisions simultaneously. The resulting multi-echelon problem is difficult to solve optimally, especially if capacity constraints are imposed on both plants and warehouses. Second, we develop a model that is characterized by multiple products, raw material vendors, plant locations, warehouse sites and customer zones and present an efficient solution procedure. Thirdly, extensive testing of the model is also provided. We also provide information about all input parameters used. This would help future researchers and practitioners more closely replicate the experimental conditions and compare the performance of their model.

The overall system developed to implement the model is computationally efficient for large scale formulations and generates near optimal production–distribution system design and utilization strategies. In Section 2, a formulation for the multi-echelon problem is presented. We then provide a solution procedure based on Lagrangian relaxation. We introduce a heuristic solution procedure to provide a feasible solution to the problem and report computational results as well. Conclusions and summary are then provided.

## 2. The integrated model formulation

The integrated model presented in this section represents a cost minimization problem subject to constraints associated with locating and operating the firm's production and distribution facilities. The main objective in this formulation is to minimize the total fixed and variable cost associated with the multiple products subject to constraints imposed on the demand, production capacity, warehouse capacity, raw material supply and requirements and the geography of customer zone outlets. As part of its production and distribution system, the integrated model deals with three major cost structures: production costs which incorporate both the fixed cost of operating the plants and variable cost associated with production, cost for transporting raw materials from vendors to plants and the fixed and variable costs of distributing the finished product from the plants to the customer zones through warehouses.

We use the following notations to define our model:

| | |
|---|---|
| $I$ | set of customer zones |
| $J$ | set of warehouses |
| $K$ | set of manufacturing plants |
| $L$ | set of product groups |
| $R$ | set of raw materials |
| $V$ | set of vendors |
| $o_j$ | annual fixed cost for operating a warehouse $j$ |
| $g_k$ | annual fixed cost for operating a plant $k$ |
| $v_j$ | unit cost of throughput for a warehouse at site $j$ |
| $v_{lk}$ | unit production cost for product $l$ at plant $k$ |
| $t_{vkr}$ | unit transportation and purchasing cost for raw material $r$ from vendor $v$ to plant $k$ |
| $c_{ijkl}$ | unit transportation cost for product $l$ from plant $k$ via warehouse $j$ to customer zone $i$ |
| $a_{il}$ | demand for product $l$ at customer zone $i$ |
| $W_j$ | annual throughput at warehouse $j$ |
| $D_k$ | capacity of plant $k$ |
| $\mathrm{SUP}_{vr}$ | supply capacity of vendor $v$ for raw material $r$ |
| $u_{rl}$ | utilization rate of raw material $r$ per unit of finished product $l$ |
| $s_l$ | capacity utilization rate per unit of product $l$ |
| $W$ | maximum number of warehouses that |

The following decision variables are used in the model:

$$z_j = \begin{cases} 1 & \text{if warehouse } j \text{ is open,} \\ 0 & \text{otherwise.} \end{cases}$$

$$p_k = \begin{cases} 1 & \text{if plant } k \text{ is open,} \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if warehouse } j \text{ serves customer zone } i, \\ 0 & \text{otherwise.} \end{cases}$$

| | |
|---|---|
| $b_{vkr}$ | quantity of raw material $r$ shipped from vendor $v$ to plant $k$ |
| $x_{lk}$ | quantity of product $l$ produced at plant $k$ |
| $q_{ijkl}$ | quantity of product $l$ shipped from plant $k$ via warehouse $j$ to customer zone $i$ |

Problem $P$

min

$$Z = \sum_j o_j\, z_j + \sum_i \sum_j \sum_l v_j a_{il} y_{ij} + \sum_k g_k p_k + \sum_l \sum_k v_{lk} x_{lk} + \sum_v \sum_k \sum_r t_{vkr} b_{vkr} + \sum_i \sum_j \sum_k \sum_l c_{ijkl} q_{ijkl}$$

subject to

$$\sum_j y_{ij} = 1 \quad \text{for all } i, \tag{1}$$

$$\sum_i \sum_l a_{il} y_{ij} \leqslant W_j z_j \quad \text{for all } j, \tag{2}$$

$$\sum_j z_j \leqslant W, \tag{3}$$

$$\sum_k q_{ijkl} = a_{il} y_{ij} \quad \text{for all } i, j, l, \tag{4}$$

$$\sum_k b_{vkr} \leqslant \mathrm{SUP}_{vr} \quad \text{for all } v \text{ and } r, \tag{5}$$

$$\sum_l u_{rl} x_{lk} \leqslant \sum_v b_{vkr} \quad \text{for all } r \text{ and } k, \tag{6}$$

$$\sum_l s_l x_{lk} \leqslant D_k p_k \quad \text{for all } k, \tag{7}$$

$$\sum_i \sum_j q_{ijkl} \leqslant x_{lk} \quad \text{for all } l \text{ and } k, \tag{8}$$

$$\sum_k p_k \leqslant P, \tag{9}$$

$$z_j = \{0,1\} \quad \text{for all } j, \tag{10}$$

$$p_k = \{0,1\} \quad \text{for all } k, \tag{11}$$

$$y_{ij} = \{0,1\} \quad \text{for all } i \text{ and } j, \tag{12}$$

$$b_{vkr} \geqslant 0 \quad \text{for all } v, k \text{ and } r, \tag{13}$$

$$x_{lk} \geqslant 0 \quad \text{for all } k \text{ and } l, \tag{14}$$

$$q_{ijkl} \geqslant 0 \quad \text{for all } i, j, k \text{ and } l. \tag{15}$$

The objective function minimizes the total cost of the supply chain. This includes the fixed cost of operating and opening plants and warehouses, the variable cost of production and distribution, costs of transportation of raw materials from vendors to plants and the transportation of the finished products from plants to customer outlets through warehouses. Constraint (1) represents the unique assignment of a warehouse to a customer. The warehouse annual throughput is imposed by constraint (2). Constraint (3) limits the number of warehouses that can be open. Satisfaction of customer demand for all products are imposed by constraint (4). Constraint (5) describes the raw material supply restriction. Raw material requirements for production are represented by constraint (6). The plant production capacity constraint is described by constraint (7). In constraint (8), the total quantity of product shipped from a manufacturing plant to customer outlets through warehouses cannot exceed the amount of that product that we produce in that plant. Finally, constraint (9) limits the number of plants that are opened. Constraints (10)–(12) impose the integrality restriction on the decision variables $z_j, p_k, y_{ij}$ and constraints (13)–(15) impose the non-negativity restriction on the decision variables $b_{vkr}$, $x_{lk}$, $q_{ijkl}$.

We have presented a mixed-integer programming formulation to the integrated, multi-commodity production and distribution problem. In this model, customer zones are supplied products from a single warehouse. Single-source models have been used by several researchers in the past literature (Beasley, 1993; Geoffrion and Graves, 1974; Klicenwicz and Luss, 1986; Neebe and Rao, 1983; Sridharan, 1993). In Section 3 we describe a procedure to solve problem $P$.

## 3. Solution procedure

The decision analog of problem $P$ belongs to the notably difficult NP-complete class of problems (Garey and Johnson, 1979). A restricted version of problem $P$ without considering vendor source of raw material input reduces to a capacitated plant location problem. The capacitated plant location problem is NP-complete, and as such problem $P$ is also NP-complete. The use of conventional mixed-integer linear programming tools for solving problem $P$ is limited due to the complexity of the problem and the large number of variables and constraints, particularly for realistically sized, even fairly small, problems. In our study of the company manufacturing healthcare products, example problems represented large mixed integer programming models with well over 30,000 variables. Therefore, it is unlikely that standard mathematical programming packages can find optimal solution to all instances of this problem expending acceptable levels of computational effort. Further, there are no obvious heuristic procedures that can be adapted to the model. Development of an efficient solution procedure for this problem is dependent on whether one can effectively exploit the structure of this problem.

*General overview of the solution procedure*. The Lagrangian relaxation scheme is applied to the model. To solve the dual problem arising in this approach, we make use of a subgradient optimization method. These relaxations will produce lower bounds on the optimal objective function value for the problem. In order to find feasible solutions, and an upper bound, we apply a heuristic procedure. We present numerical results to compare the performance of the heuristic procedure to the quality of solutions that are obtained from the lower bounds.

### 3.1. A Lagrangian relaxation to problem P

The Lagrangian relaxation scheme has been used successfully in various operation management problems (For example, see Agnihotri et al., 1990; Pirkul and Schilling, 1991). It is computationally manageable and retains sufficient features from the original problem. It further provides good feasible bounds to the original problem. The reader is referred to a survey article on Lagrangian relaxation by Fisher (1981) for further details. It is well known that different relaxation of the same problem can lead to drastically different results in terms of performance. In this section we choose to relax constraints (1), (6) and (8) to form subproblem LR. Other relaxations were also tried. For example, we tried to relax constraint sets (2), (6) and (8). The resulting subproblem was a generalized assignment problem and was harder to solve than the subproblem that was generated by relaxing constraint set (1).

Problem LR

$$
\begin{aligned}
Z_{\mathrm{LR}} = \mathrm{MIN}\sum_j o_j z_j &+ \sum_i \sum_j \left[\sum_l v_j a_{il} + \lambda_i\right] y_{ij} \\
&+ \sum_i \sum_j \sum_k \sum_l [c_{ijkl} - \beta_{lk}] q_{ijkl} + \sum_k g_k p_k \\
&+ \sum_v \sum_k \sum_r (t_{vkr} + \gamma_{rk}) b_{vkr} \\
&+ \sum_l \sum_k \left( v_{lk} - \sum_r \gamma_{rk} u_{rl} + \beta_{lk} \right) x_{lk} - \sum_i \lambda_i
\end{aligned}
$$

subject to constraint sets (2)–(5),(7),(9)–(15).

Here $\lambda_i$, $\gamma_{rk}$ and $\beta_{lk}$, are the Lagrangian multipliers corresponding to constraint sets (1), (6) and (8), respectively.

We further decompose Problem LR into three subproblems, LR1, LR2 and LR3.

Subproblem LR1

$$
\begin{aligned}
Z_{\mathrm{LR1}} = \mathrm{MIN}\sum_j o_j z_j &+ \sum_i \sum_j \left[\sum_l v_j a_{il} + \lambda_i\right] y_{ij} \\
&+ \sum_i \sum_j \sum_k \sum_l [c_{ijkl} - \beta_{lk}] q_{ijkl} - \sum_i \lambda_i
\end{aligned}
$$

subject to constraints (2)–(4), (10), (12) and (15).

Subproblem LR2

$$
Z_{\mathrm{LR2}} = \sum_k g_k p_k + \sum_l \sum_k \left( v_{lk} - \sum_r \gamma_{rk} u_{rl} + \beta_{lk} \right) x_{lk}
$$

subject to constraints (7), (9), (11) and (14).

Subproblem LR3

$$
Z_{\mathrm{LR3}} = \sum_v \sum_k \sum_r (t_{vkr} + \gamma_{rk}) b_{vkr}
$$

subject to constraints (5) and (13).

### 3.2. Solution to subproblems

*Solution to subproblem LR1.* Ignoring constraint (3), subproblem LR1 decomposes into $j$ subproblems, one for each warehouse $j$. For each of these subproblem, $z_j = 0$ or 1. If $z_j = 0$, then $y_{ij} = 0$ which would imply that $q_{ijkl} = 0$. However, if $z_j = 1$, and using the equality constraints (4), the subproblem of LR1 for each warehouse $j$ can be written as the 0/1 knapsack problem

$$
\mathrm{SP1}_j = \mathrm{MIN}\left( o_j + \sum_i [\lambda_i + \sigma_{ij}] y_{ij} \right)
$$

subject to

$$
\sum_i \left( \sum_l a_{il} \right) y_{ij} \leqslant W_j, \quad y_{ij} \in \{0,1\} \text{ for all } i,
$$

where

$$
\sigma_{ij} = \sum_l a_{il} \left( v_j + \min_k [c_{ijkl} - \beta_{lk}] \right).
$$

Now LR1 can be rewritten as

$$
Z_{\mathrm{LR1}} = \min \sum_j \mathrm{SP1}_j z_j - \sum_i \lambda_i.
$$

We consider the linear relaxation of the 0/1 knapsack problem defining $\mathrm{SP1}_j$. Given the knapsack structure of the subproblems, we solve subproblem LR1 in the following fashion.

(1) For each subproblem $j$, we form a list of cost ratio list $A_j$. List $A_j$ consists of the cost ratio $[\lambda_i + \sigma_{ij}]/[\sum_l a_{il}]$.

(2) In this step, List $A_j$ is sorted in ascending order of its cost ratio. Select that $i^*$ that provides the minimum (and negative) $[\lambda_i + \sigma_{ij}]/[\sum_l a_{il}]$ cost

ratio. This would correspond to solving the continuous relaxation of this knapsack subproblem. Then for $i^*$ fixed and for each product $l$, in order to satisfy the equality constraint (4), define $k(l) = \min_k(c_{i^*jkl} - \beta_{lk})$.

Assign the corresponding $q_{i^*jk(l)l} = a_{i^*l}$ if warehouse $j$ has enough capacity to handle the customer demand.

(3) Repeat step 2 until we either run out of capacity for warehouse $j$ or we satisfy demand of all customers.

Repeat steps (1)–(3) for every subproblem $j$. Note that in solving each subproblem $j$, $[\lambda_i + \sigma_{ij}]$ should be negative (or non-positive) in order to have $y_{ij} = 1$. Next $\mathrm{SP1}_j$ should be negative (or non-positive) in order to have $z_j = 1$. Constraint (3) can be enforced by choosing the $W$ best solutions obtained after solving the $J$ subproblems. We apply this procedure to obtain a lower bound on subproblem LR1.

*Solution to subproblem LR2*. Ignoring constraint (9), subproblem LR2 decomposes into $k$ subproblems, one for each plant $k$. For each of these subproblems, $p_k = 0$ or 1. If $p_k = 0$, then $x_{lk} = 0$. However, if $p_k = 1$, the subproblems reduces to solving $k$ continuous knapsack problems. The value of the continuous knapsack subproblem defined when $p_k = 1$ which is exactly $g_k + D_k \min_1\{[v_{lk} - \sum \gamma_{rk} u_{rl} + \beta_{lk}]/s_1\}$ needs to be negative in order to have $p_k = 1$. Constraint (9) can be enforced by choosing the $P$ best solutions obtained after solving the $K$ subproblems. This should provide a solution to subproblem LR2.

*Solution to subproblem LR3*. Subproblem LR3 is a trivial problem that can be solved in linear time. We decompose subproblem LR3 for each vendor and each raw material. In order to obtain a lower bound, we should only consider variables $b_{vkr}$ with negative objective coefficient $(t_{vkr} + \gamma_{rk})$. This procedure when repeated for all vendors and raw materials will provide a solution to subproblem LR3.

### 3.3. Determining Lagrangian multipliers

If we let $Z_\mathrm{L}(\lambda)$ be the value of the Lagrangian function with multiplier matrix $\lambda$, then the best bound is derived from calculating

$$Z_\mathrm{L}(\lambda^*) = \operatorname*{Max}_{\lambda}\{Z_\mathrm{L}(\lambda)\}.$$

Generally, the computation of a good set of multipliers is difficult (Gavish, 1978). Usually a good but not necessary optimal set of multipliers is obtained by subgradient optimization methods or various multiplier adjustment methods known as ascent or descent methods.

In this paper, we use the subgradient method to derive bounds for (LR). The subgradient method is an adaptation of the gradient method in which gradients are replaced by subgradients. The interested reader can refer to a paper by Held et al. (1974) which validates the use of subgradient optimization schema. In our implementation, the subgradient algorithm is terminated after 200 iterations or earlier if the gap between the lower bound and the feasible solution value is within 0.1% of the feasible solution value. These user values have been chosen in line with past literature (see, for e.g., Held et al., 1974; Beasley, 1993; Pirkul and Schilling, 1991).

## 4. A heuristic solution procedure

In this section, we outline a heuristic solution procedure that utilizes linear programming technique as well as results from the Lagrangian relaxation procedure that was outlined in Section 3. In obtaining a feasible solution for this problem (this serves as an upper bound to model $P$), information from the Lagrangian relaxation procedure is used to formulate and solve a production and distribution problem. The advantage of using a Lagrangian relaxation procedure is to provide a lower bound to check the quality of the upper bound solution obtained using the heuristic solution procedure. The overall solution procedure is described in Fig. 2. The flow chart explicitly describes the steps involved in the heuristic solution procedure.

The solution procedure first determines the $W$ warehouses and $P$ plants that need to be opened as those facilities most favored in the solution to the Lagrangian relaxation of the problem. The rationale here is that the relaxed problem will retain an approximation of the original problem structure and therefore provide direction that is useful in

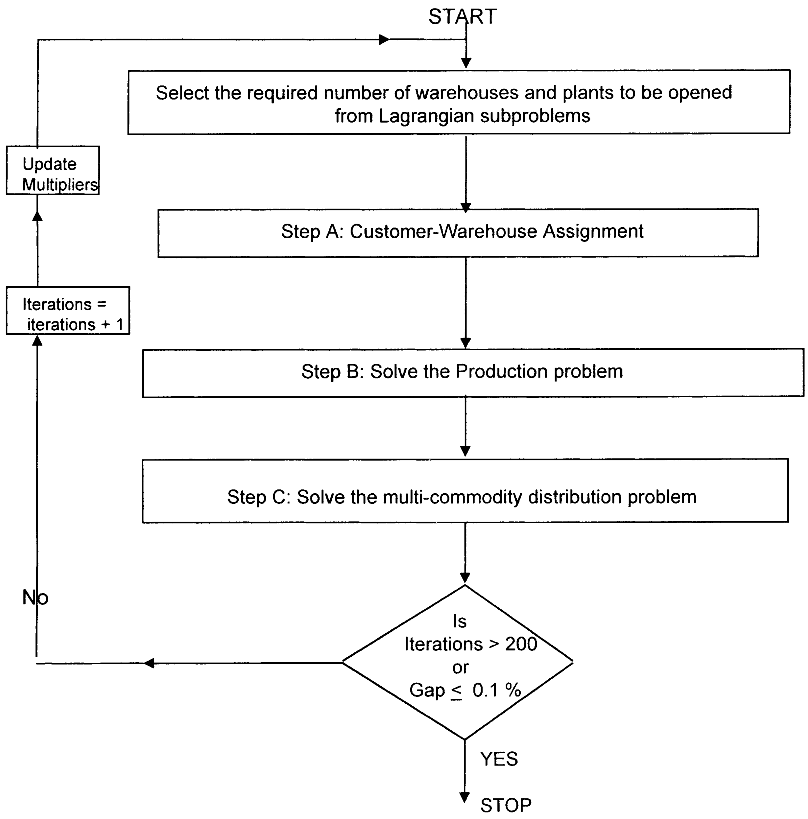


Fig. 2. Integrated distribution model heuristic solution procedure.

solving the original problem. This expectation is confirmed by the computational results reported in Section 5. We then proceed to step A that deals with assignment of customers to warehouses. We define $f_{ijl}$ as the unit transportation cost for product $l$ from warehouse $j$ to customer zone $i$ and $f_{jkl}$ as the unit transportation cost for product $l$ from plant $k$ to warehouse $j$. An initial solution that lists all the open warehouses is taken as an input for this step. In step B, we solve the production problem. We require the set of open plants and the demand at warehouse $j$ for product $l$ (which is obtained from step A) to start the problem. This step should provide the amount of raw material that we need to ship to manufacturing plants, the amount of finished products that we produce in the manufacturing plants and the amount of products that we transport to open warehouses. Step C of the heuristic procedure takes as its input the set of open warehouses, the customer–warehouse links and the amount of finished products that exist at the manufacturing plants and provides the amount of finished products that will be shipped from open manufacturing plants to customer outlets via open warehouses.

*Step A*: *Customer–warehouse assignment*

In this step, we attempt to find the 'best' set of customer–warehouse link to provide an initial assignment for the heuristic procedure.

1. Consider the set of open warehouses. This serves as an input to step A.
2. For each customer zone $i$, calculate the corresponding cost of supplying all of the customer's demand for product $l$ from the set of warehouses that are connected to the customer zone. Calculate $(v_j + f_{ijl})$

$$\text{Assignment cost } \mathrm{ASC}_{ij} = \sum_{l} (v_j + f_{ijl}) a_{il}.$$

Sort this assignment cost in ascending order. Pick the cheapest $(i, j)$ pair. Denote this as $(i\sim, j\sim)$. Make the assignment of customer zone $i\sim$ to warehouse $j\sim$. Adjust warehouse capacities, remove customer $i\sim$, and repeat until all customer zones have been uniquely assigned to one open warehouse.

The output to step A provides a feasible, unique assignment of customers to warehouses that can be used to provide a good input to the production problem that needs to be solved in step B.

Once the customer–warehouse assignments have been made, we can calculate the demand of warehouse $j$ for product $l$ by

$$D_{jl} = \sum_{i} a_{il} y_{ij} \quad \text{for all } j \text{ and } l.$$

*Step B*: *Production problem*

In this step, we consider the set of open plants and the demand at warehouse $j$ for product $l$ that was obtained at the end of step A.

The formulation is as follows:

min

$$\sum_{k} g_k p_k + \sum_{l} \sum_{k} v_{lk} x_{lk} + \sum_{v} \sum_{k} \sum_{r} t_{vkr} b_{vkr} + \sum_{j} \sum_{k} \sum_{l} f_{jkl} q_{jkl}$$

subject to

$$\sum_{k} q_{jkl} = D_{jl} \quad \text{for all } j \text{ and } l$$

and constraints (5)–(7) and (13)–(15).

Step B is a formulation of the production optimization problem. This is an optimization problem with constraints that are linear and variables that are non-negative. We apply the simplex procedure to generate a solution to the production problem. The set of open warehouses, open plants and the demand at open warehouses for product $l$ are taken as input values for this linear programming problem. In other words, we fix the values for $p_k$ and $D_{il}$ as inputs for this linear programming problem.

The output to the production problem provides the amount of product $l$ produced at plant $k$ ($x_{lk}$), the amount of product $l$ shipped from plant $k$ to warehouse $j$ ($q_{jkl}$) and the amount of raw materials $r$ shipped from vendor $v$ to plant $k$ ($b_{vkr}$).

From the output, we can determine the amount of supply of finished product $l$ that exist at plant $k$

$$\mathrm{AS}_{lk} = \sum_{j} q_{jkl} \quad \text{for all } k \text{ and } l.$$

*Step C*: *Distribution problem*

In this step we take the available supply of finished product $l$ that exist at plant $k$ and set of open warehouses and their corresponding links with customers.

Min

$$\sum_{i} \sum_{j} \sum_{k} \sum_{l} c_{ijkl} q_{ijkl} + \sum o_j z_j + \sum_{i} \sum_{j} \sum_{l} v_j a_{il} y_{ij}$$

subject to

$$\sum_{k} q_{ijkl} = a_{il} y_{ij} \quad \text{for all } i, j \text{ and } l,$$

$$\sum_{i} \sum_{j} q_{ijkl} < \mathrm{AS}_{lk} \quad \text{for all } l \text{ and } k$$

and constraints (1), (2), (10), (12) and (15).

This is the mixed-integer programming formulation to the distribution problem. We take the set of open warehouses and their customer–warehouse links together with the available supply of finished products as input values to this problem. In other words, the values for $z_j$ and $y_{ij}$ are fixed in this step C. We use the built-in MIP solver ZOOM, the portable solver available for mixed integer work using GAMS (1994), to find an optimal solution to the problem in step C.

The distribution problem provides the amount of product that is shipped from open plants to customer outlets via open warehouses. The solution from steps A–C provides a feasible solution to the integrated distribution problem (Model P). This also provides an upper bound to the original problem.

## 5. Computational results

The heuristic solution procedure was coded in Pascal as an integral part of the subgradient procedure. A series of computational experiments was carried out on an IBM RISC 6000 machine.

We first solved a small set (between 10 and 20 customer zones) of problems where we compared the bound obtained from the Lagrangian relaxation for the problem with the optimal solution that we obtained using a commercial mathematical programming software. Providing an optimal solution for smaller problems would perhaps yield more information about the accuracy of the bounds. We report this comparison in Table 1. The gap between the Lagrangian bound and the optimal solution remained low and range 0.02–1.06%. However, the mean cpu time varied significantly. On an average, the time it took to obtain the optimal solution using the commercial software was at least 50–100 times more than the time it took to obtain a lower bound solution to the same set of problems. It is highly unlikely we can obtain optimal solution to reasonably large sized problems.

This experiment provided enough motivation for resorting to a heuristic procedure. The performance of the heuristic solution procedure is reported in Table 2. Many problems with the same input structure were solved in order to achieve a reasonable level of confidence about the performance and validation of the solution procedure. The Euclidean distance between nodes $i$ and $j$ was used to define cost coefficients $f_{ijl}$ and the Euclidean distance between nodes $j$ and $k$ was used to define cost coefficients $f_{jkl}$. Cost coefficients $c_{ijkl}$ was determined by adding the cost coefficients $f_{ijl}$ and $f_{jkl}$ with a positive constant ratio that was fixed at 10 units. The Euclidean distance between nodes $v$ and $k$ was used to define cost coefficients $t_{vkr}$. The demand requirements of the multiple products for the customer zones were drawn from a uniform distribution between 10 and 99 units and the utilization rates of raw material $r$ per unit of finished product $l$ was drawn from a uniform distribution between 0 and 10 units. The production capacity of product $l$ was drawn from a uniform distribution between 100 and 300 units. The unit cost of throughput for a warehouse and the unit production cost for products at plants was also drawn from a uniform distribution between 0 and 100 units. The fixed costs for opening of plants, warehouses and the capacities of warehouses and plants were varied to present a realistic flavor for the integrated problem.

We solved a wide range of problems for a number of customer zones, warehouses, plants, products, raw materials and vendors. The capacities of the potential warehouses and the plants as well as their fixed costs were also

Table 1
Comparison of Lagrangian bound with optimal solution[a]

| Customer zones ($I$) | Warehouses ($J$) | Plants ($K$) | Product groups ($L$) | Vendors ($V$) | Raw materials ($R$) | Gap (%) | Mean CPU (s) | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | Opt | Lag |
| 10 | 4 | 3 | 2 | 1 | 2 | 0.02 | 362.33 | 15.23 |
| 10 | 4 | 3 | 2 | 2 | 2 | 0.06 | 594.45 | 19.27 |
| 10 | 5 | 3 | 2 | 1 | 2 | 0.11 | 529.86 | 21.09 |
| 10 | 5 | 3 | 2 | 2 | 2 | 0.09 | 645.67 | 26.78 |
| 10 | 7 | 5 | 2 | 1 | 2 | 0.12 | 657.89 | 23.32 |
| 10 | 7 | 5 | 2 | 2 | 2 | 0.15 | 685.97 | 31.11 |
| 15 | 5 | 3 | 2 | 2 | 2 | 0.25 | 1645.33 | 36.64 |
| 15 | 7 | 5 | 2 | 2 | 2 | 0.31 | 1754.32 | 22.21 |
| 15 | 10 | 7 | 2 | 2 | 2 | 0.26 | 1884.56 | 39.96 |
| 20 | 10 | 7 | 2 | 2 | 2 | 0.87 | 2245.58 | 25.69 |
| 20 | 12 | 7 | 2 | 2 | 2 | 1.02 | 2578.92 | 20.06 |
| 20 | 12 | 10 | 2 | 2 | 2 | 0.93 | 3432.69 | 31.14 |
| 20 | 15 | 10 | 2 | 2 | 2 | 1.06 | 3365.41 | 29.63 |

[a] GAP = ((GAMS solution − lag lower bound)/GAMS solution) $*$ 100.

Table 2
Performance of solution procedure for integrated model[a]

| Customer zones (*I*) | Warehouses (*J*) | Plants (*K*) | Produt groups (*L*) | Vendors (*V*) | Raw materials (*R*) | Fixed cost (ware) | Fixed cost (plant) | Gap (%) | CPU (s) |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 2 | 2 | 2 | 2 | 0.31 | 0.34 | 0.05 | 12.39 |
| 5 | 3 | 2 | 2 | 2 | 2 | 0.33 | 0.29 | 0.12 | 14.32 |
| 5 | 3 | 2 | 2 | 2 | 2 | 0.35 | 0.31 | 0.33 | 11.76 |
| 5 | 3 | 2 | 2 | 2 | 2 | 0.29 | 0.34 | 0.28 | 15.47 |
| 5 | 3 | 2 | 2 | 2 | 2 | 0.32 | 0.33 | 0.23 | 18.85 |
| 10 | 5 | 3 | 2 | 2 | 3 | 0.3 | 0.33 | 0.14 | 21.94 |
| 10 | 5 | 3 | 2 | 2 | 3 | 0.31 | 0.29 | 0.58 | 26.58 |
| 10 | 5 | 3 | 2 | 2 | 3 | 0.34 | 0.33 | 0.77 | 37.93 |
| 10 | 5 | 3 | 2 | 2 | 3 | 0.29 | 0.31 | 0.68 | 29.48 |
| 10 | 5 | 3 | 2 | 2 | 3 | 0.31 | 0.34 | 0.74 | 27.99 |
| 20 | 5 | 3 | 2 | 2 | 3 | 0.29 | 0.27 | 1.15 | 23.29 |
| 20 | 5 | 3 | 2 | 2 | 3 | 0.31 | 0.29 | 0.13 | 26.01 |
| 20 | 5 | 3 | 2 | 2 | 3 | 0.32 | 0.31 | 0.64 | 35.45 |
| 20 | 5 | 3 | 2 | 2 | 3 | 0.29 | 0.33 | 0.88 | 33.27 |
| 20 | 5 | 3 | 2 | 2 | 3 | 0.33 | 0.29 | 1.12 | 36.65 |
| 30 | 10 | 5 | 3 | 2 | 3 | 0.28 | 0.37 | 0.62 | 30.72 |
| 30 | 10 | 5 | 3 | 2 | 3 | 0.41 | 0.33 | 1.84 | 26.89 |
| 30 | 10 | 5 | 3 | 2 | 3 | 0.25 | 0.39 | 1.37 | 41.11 |
| 30 | 10 | 5 | 3 | 2 | 3 | 0.31 | 0.24 | 0.94 | 38.76 |
| 30 | 10 | 5 | 3 | 2 | 3 | 0.33 | 0.32 | 2.06 | 34.58 |
| 40 | 10 | 5 | 3 | 2 | 3 | 0.33 | 0.32 | 1.12 | 31.57 |
| 40 | 10 | 5 | 3 | 2 | 3 | 0.31 | 0.33 | 0.98 | 33.98 |
| 40 | 10 | 5 | 3 | 2 | 3 | 0.29 | 0.34 | 2.34 | 29.76 |
| 40 | 10 | 5 | 3 | 2 | 3 | 0.35 | 0.31 | 1.77 | 32.25 |
| 40 | 10 | 5 | 3 | 2 | 3 | 0.34 | 0.32 | 2.86 | 33.76 |
| 50 | 15 | 10 | 3 | 2 | 2 | 0.31 | 0.33 | 0.21 | 37.75 |
| 50 | 15 | 10 | 3 | 2 | 2 | 0.38 | 0.32 | 0.54 | 42.39 |
| 50 | 15 | 10 | 3 | 2 | 2 | 0.37 | 0.35 | 0.92 | 46.67 |
| 50 | 15 | 10 | 3 | 2 | 2 | 0.35 | 0.37 | 1.21 | 33.27 |
| 50 | 15 | 10 | 3 | 2 | 2 | 0.32 | 0.38 | 2.28 | 39.95 |
| 75 | 15 | 10 | 3 | 2 | 2 | 0.39 | 0.33 | 0.81 | 29.84 |
| 75 | 15 | 10 | 3 | 2 | 2 | 0.37 | 0.32 | 0.97 | 33.23 |
| 75 | 15 | 10 | 3 | 2 | 2 | 0.39 | 0.29 | 1.09 | 38.46 |
| 75 | 15 | 10 | 3 | 2 | 2 | 0.35 | 0.31 | 0.96 | 36.70 |
| 75 | 15 | 10 | 3 | 2 | 2 | 0.31 | 0.33 | 1.63 | 41.06 |

[a] Fixed cost ratio for warehouses = total fixed cost of WMAX warehouses/total feasible cost; fixed cost ratio for plants = total fixed cost of PMAX plants/total feasible cost; gap = (feasible solution value − lower bound)/lower bound $*$ 100.

changed to present a realistic picture of the model together with its solution procedure. The gap between the best feasible solution and the lower bound is used to judge the quality of the solution procedure. The gaps together with the CPU time for execution of the solution procedure are presented in Table 2.

In Table 2 we fixed the warehouse load ratio (WLR) and the plant load ratio (PLR) to be 0.85. The maximum number of warehouses (*W*) and the maximum number of plants (*P*) that can be opened were set equal to the set of possible warehouse locations (*J*) and plant locations (*K*), respectively. The warehouse load ratio is defined to be the ratio of the total demand of customers for the multiple products to the total capacity of the $W$ open warehouses. The plant load ratio is defined to be the ratio of the total capacity of the $W$ open warehouses and the total capacity of the $P$ open plants. Based on demand for multiple products and change in the capacities for potential warehouses and plants, it is possible to change the load ratio as well. The fixed cost ratio for plants and warehouses are also reported in the table. We report low gaps for even large sized problems with 75 customers, 15 possible warehouse location, 10 possible plant location, 3 products and 2 suppliers. Gaps ranged from 0.05% to 2.86%. The computing times were also found to be quite stable.

In Tables 3 and 4, we conducted further experiments to test the model for changes in the warehouse load ratio and plant load ratio while also increasing the number of customer zones. For both these experiments we fixed the maximum number of open warehouses to not exceed 15, and the maximum number of plant locations to not exceed 5. We varied the warehouse load ratio between 0.81 and 0.95 while keeping the plant load ratio fixed at 0.90 in Table 3. In Table 4, we varied the plant load ratio between 0.86 and 1.00 while keeping the warehouse load ratio fixed at 0.90. The heuristic solution opened 15 warehouses and 5 plants. The problems reported in Tables 3 and 4 represent models with more than 139,000 variables and 20,000 functional constraints. We report low gaps and found the solution times not to exceed more than 182 seconds.

These computational results indicate that the proposed Lagrangian relaxation based procedure together with the heuristic solution procedure produced good and effective results within acceptable computing time. In all the experiments, $\mathrm{SUP}_{vr}$ the supply capacity of vendor $v$ for raw material $r$ was very tight. Otherwise, the lowest raw material cost can be simply added to the production cost. The detailed formulation and interactive nature of the integrated model enabled extensive sensitivity analysis to input

Table 3
Performance of solution procedure with changes in warehouse load ratio for integrated model (plant load ratio = 0.90)

| Customer zones ($I$) | Ware-houses ($J$) | Plants ($K$) | Product groups ($L$) | Vendors ($V$) | Raw materials ($R$) | WLR | Fixed cost (ware) | Fixed cost (plant) | Gap (%) | CPU (s) |
|---|---|---|---|---|---|---|---|---|---|---|
| 150 | 30 | 10 | 5 | 3 | 2 | 0.95 | 0.32 | 0.33 | 2.46 | 168.89 |
| 150 | 30 | 10 | 5 | 3 | 2 | 0.90 | 0.28 | 0.30 | 3.78 | 172.31 |
| 150 | 30 | 10 | 5 | 3 | 2 | 0.88 | 0.31 | 0.32 | 1.56 | 155.86 |
| 150 | 30 | 10 | 5 | 3 | 2 | 0.84 | 0.35 | 0.28 | 2.49 | 171.11 |
| 150 | 30 | 10 | 5 | 3 | 2 | 0.81 | 0.37 | 0.36 | 3.25 | 160.09 |

Table 4
performance of solution procedure with changes in plant load ratio for the integrated model (warehouse load ratio = 0.90)[a]

| Customer zones ($I$) | Ware-houses ($J$) | Plants ($K$) | Product groups ($L$) | Vendors ($V$) | Raw materials ($R$) | PLR | Fixed cost (ware) | Fixed cost (plant) | GAP (%) | CPU (s) |
|---|---|---|---|---|---|---|---|---|---|---|
| 150 | 30 | 10 | 3 | 2 | 2 | 0.86 | 0.31 | 0.33 | 1.23 | 165.43 |
| 150 | 30 | 10 | 3 | 2 | 2 | 0.89 | 0.29 | 0.37 | 2.09 | 171.48 |
| 150 | 30 | 10 | 3 | 2 | 2 | 0.92 | 0.32 | 0.36 | 2.35 | 173.52 |
| 150 | 30 | 10 | 3 | 2 | 2 | 0.95 | 0.30 | 0.31 | 1.97 | 169.27 |
| 150 | 30 | 10 | 3 | 2 | 2 | 1.00 | 0.36 | 0.30 | 2.85 | 181.44 |

[a] Warehouse load ratio = (total space demanded)/total capacity of WMAX warehouses; plant load ratio = (total capacity of WMAX warehouses)/total capacity of PMAX plants; fixed cost ratio for warehouses = total fixed cost of WMAX warehouses/total feasible cost; fixed cost ratio for plants = total fixed cost of PMAX plants/total feasible cost; gap = (feasible solution value − lower bound)/lower bound ∗ 100.

parameter variation in the context of solving real logistics design problems as reported in Tables 3 and 4.

### 5.1. Implementation of model and solution procedure

The model and solution procedure developed in this paper were applied to 'real-world' data obtained from a study by the authors conducted on a firm manufacturing health-care products in US (The data have been disguised to retain confidentiality). In terms of transportation and distribution costs, a bulk of the input to manufacturing plants consists of two raw materials and is from two major suppliers. Hence, we were primarily interested in modeling this problem based on these given inputs. The firm manufactures 10 major products in five manufacturing plants and transports these products via warehouses. Each customer zone is supplied their demand from a single warehouse. The company felt that it was a horrendous logistical problem to manage multiple shipment of products from different warehouses to a single customer. Hence they consolidated their shipments to maintain this single sourcing strategy. Further, the company took advantage of economies that could be achieved by combining such shipments of products to a customer zone. The example discussed here is intended to serve as an illustration of the applicability of the model to practical-sized problems and not as a solution to any specific problem.

The demand points consist of 75 customer zone outlets. Table 5 provides the performance of the algorithm with 30 possible warehouse location, 5 possible manufacturing plant locations and 10 major products. We fixed the maximum number of open warehouses to not exceed 15, and the maximum number of plant locations to not exceed 5. The warehouse load ratio and the plant load ratio were fixed at 90% in Table 5.

To present the decision-maker with a different flavor, we varied the warehouse load ratio and the plant load ratio in Table 5. We applied the 'real-world' data to the input structure provided in Table 5. We obtained interesting results. As the load ratios for the open warehouses and plants kept increasing, the solution times and the gaps also increased but were well within acceptable limits.

The example problem obviously represents a large mixed integer program with well over 30,000 variables and 20,000 functional constraints. The heuristic was able to find solutions that would have been practically unobtainable with commercial integer programming codes. For both cases

Table 5
Results for example problem[a]

| Customer zones ($I$) | Warehouses ($J$) | Plants ($K$) | Product groups ($L$) | Vendors ($V$) | Raw materials ($R$) | Fixed cost (ware) | Fixed cost (plant) | WLR | PLR | Gap (%) | CPU (s) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 75 | 30 | 5 | 10 | 2 | 2 | 0.28 | 0.37 | | | 1.36 | 55.63 |
| 75 | 30 | 5 | 10 | 2 | 2 | 0.33 | 0.32 | | | 2.24 | 61.15 |
| 75 | 30 | 5 | 10 | 2 | 2 | 0.35 | 0.39 | | | 2.65 | 67.82 |
| 75 | 30 | 5 | 10 | 2 | 2 | 0.29 | 0.35 | | | 1.02 | 46.89 |
| 75 | 30 | 5 | 10 | 2 | 2 | 0.32 | 0.31 | | | 1.77 | 62.27 |
| 75 | 30 | 5 | 10 | 2 | 2 | 0.29 | 0.31 | 0.79 | 0.81 | 1.76 | 52.21 |
| 75 | 30 | 5 | 10 | 2 | 2 | 0.31 | 0.33 | 0.81 | 0.82 | 2.21 | 50.68 |
| 75 | 30 | 5 | 10 | 2 | 2 | 0.32 | 0.32 | 0.84 | 0.85 | 2.09 | 67.79 |
| 75 | 30 | 5 | 10 | 2 | 2 | 0.34 | 0.35 | 0.86 | 0.87 | 2.43 | 72.24 |
| 75 | 30 | 5 | 10 | 2 | 2 | 0.38 | 0.39 | 0.89 | 0.88 | 2.59 | 87.73 |

[a] Warehouse load ratio = (total space demanded)/total capacity of WMAX warehouses; plant load ratio = (total capacity of WMAX warehouses)/total capacity of PMAX plants; fixed cost ratio for warehouses = total fixed cost of WMAX warehouses/total feasible cost; fixed cost ratio for plants = total fixed cost of PMAX plants/total feasible cost; gap = (feasible solution value − lower bound)/lower bound ∗ 100.

the heuristic solution opened 15 warehouses and 5 plant locations. The gap between the feasible solution and the Lagrangian bound ranged between 1.36% and 2.65% with solution times ranging between 45 and 88 s. These are well within the range of tractability.

## 6. Summary and conclusions

As discussed earlier, the strength of the model presented in this paper is that it has both a theoretical and practical appeal. The integrated model described in this paper has proven to be a cost-minimization procedure for analyzing facility logistics strategies in the context of production and distribution system design studies. Further, the model developed in this study has abstracted both strategic and tactical issues in the area of supply chain management and has significantly extended the scope of previous research on developing a fairly robust model to tackle production and distribution issues. The level of generality of the model coupled with the availability of an efficient solution procedure may be appealing to the practitioners. Our analysis of the different problems indicates that the proposed solution algorithm generates solutions with low gaps in stable/low computing times.

A number of research issues for extending the current model are under investigation. An interesting extension to our proposed model is to incorporate the idea of multitype–multilevel distribution centers (Fleischmann, 1993). Many large organizations that operate a variety of distribution centers or warehouses have established standardized facilities in order to save construction and start-up costs when new facilities are built. Hence, an organization can (by using a multitype distribution network) save additional costs through standardized building design. Each type of warehouse can offer a different capacity with different fixed set-up costs. Due to strong economies of scale exhibited by transportation tariffs, multilevel hierarchy of warehouses generally results in huge cost savings over the case of separate individual shipments from each manufacturing plant to each retail outlet through warehouses.

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