Designing a Reliable Supply Chain Network with Random Disruption Consideration

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Abstract
Supply chain design models in the literature typically assume that facilities never fail. However, in the real world cases facilities are always subject to disruptions of various sorts due to natural disasters, strikes, machine breakdowns, power outages, and other factors. This paper presents an integrated supply chain network design model with the consideration of facility disruptions. Specifically, a mathematical model is proposed in order to determine supply chain design decisions such as location, assignment, inventory and shipment decisions. The model maximizes total profit for the whole supply chain, while simultaneously considers costs of location, inventory, transportation and disruption. In order to solve the proposed model, an effective solution approach based on Lagrangian relaxation is presented. Finally, computational results for several instances of the problem are proposed to imply the effectiveness of the developed solution approach.

Keywords
Integrated supply chain design model, Uncertainty, Disruption, Location, Inventory, Lagrangian relaxation.

1. Introduction

Traditional supply chain design models in the literature treat location and inventory decisions separately. However, ignoring interaction between long term decisions of location and short term decisions of inventory can lead to sub-optimality (Shen and Qi, 2007). Thus, integrated supply chain design models incorporating location and inventory decisions have gained much attention in recent years (Shen, 2007).

A limitation of most existing studies on integrated supply chain design is that they assume that facilities are always available and perfectly reliable. However, facilities in real world are always vulnerable to partial or complete disruptions. In fact, different factors such as natural disasters, labor strikes, power outages, parts shortages, quality rejections, transportation damages, and machine breakdowns can lead to unreliable performance of facilities and huge losses. For instance, many companies like Intel, Wal-Mart, Ford, Isuzu Motors and Suzuki, had to stop production because of cut-off of electricity and water supply at their suppliers in 2008 (Sweet, 2008). Furthermore, smaller-scale disruptions take place much more frequently. For example, Wal-Mart’s Emergency Operations Center receives a call almost every day from a store with some sort of crisis (Leonard, 2005).

These examples show that disruptions can significantly impact firm's operations and highlight the need to account for disruptions during design of supply chain network. Facility location models with disruptions consideration have emerged in the literature recently (Snyder and Daskin, 2005). Snyder and Daskin (2005) present two facility location models based on classic location problems, in which facilities are disrupted with the same probabilities. Their models rely on the assumption that the disruption probabilities of the facilities are equal. This assumption is relaxed by Berman et al. (2007), Lim et al. (2010) and Cui et al. (2010).

Aryanezhad et al. (2010), Qi et al. (2010) and Chen et al. (2011) analyze location-inventory models when facilities are subject to disruption risks. The models developed by Aryanezhad et al. (2010) and Chen et al. (2011), however, are based on the restrictive assumption that each facility fail independently with an equal probability. Qi and Shen (2007) and Aryanezhad et al. (2011) focus on integrated supply chain design under yield uncertainty; however, do not consider disruptions. Snyder et al. (2006) and Snyder and Daskin (2007) review the broad range of facility location models under presence of disruptions.

This paper examines an integrated supply chain design problem with multiple distribution centers subject to different sorts of disruptions. The proposed model in this study builds upon recent developments of integrated supply chain design models that simultaneously consider location, inventory, and shipment decisions in the same model. In order to solve the integrated model a solution method based on Lagrangian relaxation is developed.

The proposed model in this study differs from the earlier works in literature of integrated supply chain design. First, unlike the most of joint location-inventory models in the literature the model takes into account the different disruptions scenarios at facilities during decision making. Also, the profit-maximizing model in this study does not restrictively assume that every potential demand has to be satisfied. This article is also different from the literature on facility location problems with disruptions. First, the model does not ignore nonlinear inventory costs. Also, it takes into account the possibility of both partial and complete facility disruptions in the same model.

The remainder of the paper is organized as follows. Section 2 states the problem and presents the formulation model for the problem. In section 3 the solution approach based on Lagrangian relaxation is proposed. Computational results are presented in section 4. Finally, section 5 concludes the paper along with directions for future research.

2. Problem description and model formulation

This paper addresses a supply chain design network problem under probabilistic disruptions. As typically considered in the supply chain design literature, a three-tiered supply chain consisting of a single supplier, distribution centers (DCs) and customers, is studied. The supplier ships one type of product to a set of customers in order to satisfy their demands. It is recalled that the supply chain is flexible in determining which customers to serve. In other words, if the cost of serving some customers is prohibitive, they are not served at all. DCs function as the direct intermediary between the plant and customers for shipment of the product. That is, DCs combine the orders from different customers and then order to the supplier. Similar to Berman et al. (2007), Lim et al. (2010) Qi et al. (2010) , Cui et al. (2010), Aryanezhad et al. (2010, 2011) and Chen et al. (2011) we assume that the lead time for order delivery is negligible and the capacities of DCs are infinite.

The key problem is that each DC may face different amounts of disruptions from time to time. The amounts of disruptions at DCs are probabilistic. DCs are not required to be the same in the problem; as a result, the probabilities of disruptions for each DC can be different from the others. To formulate the disruptions at the DCs, a scenario based modeling approach is used. Each scenario specifies the percentages of disruptions for each DC. Note that the scenario based modeling framework is flexible enough to consider both complete and partial disruptions. Also, it allows us to model the complex situation in which the probability failures of DCs are dependant. If a customer is assigned to DC but the DC is disrupted, the unmet demands are backlogged, at a penalty cost of disruptions.

The problem lies in simultaneously determining: 1) where DCs are located; 2) which subsets of customers are served; 3) which DCs are assigned to which customers; 4) how much and how often to order at each DC. The problem is formulated as a mixed-integer nonlinear program which maximizes the expected total profit. That is, the objective is to maximize the expected difference between total revenue and total cost. The total cost includes three main components: 1) the fixed cost to locate DCs, 2) the working inventory cost (including order costs, shipment costs from supplier to DCs, holding costs and penalty costs of disruptions) at the located DCs, and 3) shipment cost from located DCs to customers.

Following notations are used throughout the paper. Additional notations will be given out when required.

Parameters

- I : set of customers indexed by *i*;
- J : set of candidate DC locations indexed by *j*;
- S : set of disruption scenarios indexed by *s*;
- λ_i : demand rate at customer *i*;
- R_i : selling price at customer *i*, per unit of demand;
- f_j : fixed cost of locating a DC at *j*;
- a_j : fixed cost of placing an order at *j*;
- b_j : fixed cost per shipment from the supplier to DC at *j*;

- c_j : per-unit shipment cost from the supplier to DC at j;
- h : inventory holding cost per unit of product;
- d_{ij} : per-unit cost to ship from distribution center j to customer i;
- β: weight factor associated with the shipment cost;
- θ: weight factor associated with the inventory cost;
- r_{sj} : percentage of supply disruption at distribution center j in scenario s;
- q_s : probability that scenario s occurs;

Decision variables

- $X_j = 1$, if j is selected as a DC location, and 0, otherwise;
- $Y_{ij} = 1$, if customer i is assigned to a DC based at j, and 0 otherwise;
- $Z_i = 1$, if customer i is not selected to be served, and 0 otherwise.

2.1. Working inventory cost

This subsection formulates the working inventory cost at each DC including costs of ordering, shipment from supplier to DCs, holding and penalty of disruptions. For the moment, let D_j denote the unknown total demand that is assigned to the DC at j (it is obvious that $D_j = \sum_{i \in I} \lambda_i Y_{ij}$). Also, let n be the unknown number of orders in per year. In this case, the expected shipment size per shipment from the supplier to DC at j is equal to $\frac{D_j}{n}$ and the working inventory cost at distribution center j can be obtained by:

$$a_j n + \beta (b_j + \frac{c_j D_j}{n})n + \sum_s q_s \left(\frac{(1-r_{sj})D_j}{2n} \theta h \right) + \sum_s q_s \left(\frac{r_{sj} D_j}{n} \pi_j \right) n \quad (1)$$

The first term of equation (1) is the fixed cost of placing n orders. The second term indicates the cost of shipping n orders of size $\frac{D_j}{n}$, assuming the shipment cost from the supplier to distribution center j has a fixed cost b_j and volume dependent cost c_j . The third term represents the cost of holding average of $\sum_s q_s \left(\frac{(1-r_{sj})D_j}{2n} \right)$ units. Recall that if scenario s is occurred r_{sj} % of supply is disrupted at distribution center j and the average units to hold will be $\frac{(1-r_{sj})D_j}{2n}$. Also, each scenario s is occurred with probability of q_s .

The last term in (1) indicates the penalty cost of disruption at distribution center j, where r_{sj} % of supply is disrupted in scenario s. Note that the penalty cost of disruption can be interpreted as lost sales cost, or the cost of providing the amount of disrupted supply with extra expense. In order to determine the optimal number of orders, we take derivative of (1) respect to n and set the derivative to zero:

$$a_j + \beta b_j - \sum_s q_s \left(\frac{(1-r_{sj})D_j}{(2n)^2} \theta h \right) = 0 \quad (2)$$

Solving the equation (2) for n , and plugging the result into (1), working inventory cost at distribution center j can be calculated as follows:

$$\sqrt{2\theta h (a_j + \beta b_j) \sum_s q_s (1-r_{sj}) D_j} + \beta c_j D_j + \pi_j \sum_s q_s r_{sj} D_j \quad (3)$$

Since $D_j = \sum_{i \in I} \lambda_i Y_{ij}$, (3) can be rewritten as follows:

$$\sqrt{2\theta h (a_j + \beta b_j) \sum_s q_s (1-r_{sj}) \sum_{i \in I} \lambda_i Y_{ij}} + \beta c_j \sum_{i \in I} \lambda_i Y_{ij} + \pi_j \sum_s q_s r_{sj} \sum_{i \in I} \lambda_i Y_{ij} \quad (4)$$

2.2. Integrated Model

The problem is formulated as follows:

$$\text{Max} \sum_{i \in I} R_i \lambda_i (1-Z_i) - \left(\sum_{j \in J} f_j X_j \right) - \beta \sum_{j \in J} \sum_{i \in I} d_{ij} \lambda_i Y_{ij} - \sum_{j \in J} \left(\sqrt{2\theta h (a_j + \beta b_j) \sum_s q_s (1-r_{sj}) \sum_{i \in I} \lambda_i Y_{ij}} + \beta c_j \sum_{i \in I} \lambda_i Y_{ij} + \pi_j \sum_s q_s r_{sj} \sum_{i \in I} \lambda_i Y_{ij} \right) \quad (5)$$

subject to:

$$\sum_{j\in J} Y_{ij} + Z_i = 1 \quad \forall i \in I \tag{6}$$

$$Y_{ij} \le X_j \quad \forall i \in I,\ \forall j \in J \tag{7}$$

$$X_j \in \{0,1\} \quad \forall j \in J \tag{8}$$

$$Y_{ij} \in \{0,1\} \quad \forall i \in I,\ \forall j \in J \tag{9}$$

The first component in objective (5) indicates the total sales revenue which is gained by serving the customers. The second component represents the fixed cost of locating DCs. The third component indicates the expected shipment cost from the DCs to customers. Finally, the forth component represents the working inventory cost. Constraints (6) require each customer to be assigned to exactly one DC or not to be served at all. Constraints (7) state that customers can only be assigned to candidate sites that are selected as DCs.

3. Solution approach

In order to solve the model, a solution approach based on Lagrangian relaxation is developed. Lagrangian relaxation approach provides both upper and lower bounds on the optimal value of the objective function. In following we explain how to derive the lower bound and upper bound for the model.

3. 1. Finding a lower bound

First, we convert the model formulated in section 2 into a standard model for which an effective Lagrangian relaxation approach exists in the literature (Daskin et al., 2002; Qi et al., 2010). Replacing Z_i in (5) with $1-\sum_{j\in J} Y_{ij}$ according to (6), the original model formulated in section 2 can be written as follows:

$$\text{Max} \sum_{j\in J}\sum_{i\in I} R_i \lambda_i Y_{ij} - \left(\sum_{j\in J} f_j X_j\right) - \beta \sum_{j\in J}\sum_{i\in I} d_{ij}\lambda_i Y_{ij}$$
$$-\sum_{j\in J}\left(\begin{array}{l}\sqrt{2\theta\, h(a_j+\beta b_j)\sum_s q_s(1-r_{sj})\sum_{i\in I}\lambda_i Y_{ij}} \\ +\beta c_j \sum_{i\in I}\lambda_i Y_{ij} + \pi_j \sum_s q_s r_{sj} \sum_{i\in I}\lambda_i Y_{ij}\end{array}\right) \tag{10}$$

subject to:

$$\sum_{j\in J} Y_{ij} \le 1 \quad \forall i \in I \tag{11}$$

$$Y_{ij} \le X_j \quad \forall i \in I,\ \forall j \in J \tag{12}$$

$$X_j \in \{0,1\} \quad \forall j \in J \tag{13}$$

$$Y_{ij} \in \{0,1\} \quad \forall i \in I,\ \forall j \in J \tag{14}$$

Multiplying objective (10) by -1 and rearranging the terms, the problem can be converted to a minimizing model with following objective:

$$\text{Min} \sum_{j\in J}\left(f_j X_j + \sum_{i\in I} u_i Y_{ij} + \sqrt{\sum_{i\in I} v_i Y_{ij}}\right) \tag{15}$$

where:

$$u_{ij} = \left(\beta d_{ij} + \beta c_j + \pi_j \sum_s q_s r_{sj} - R_i\right)\lambda_i$$
$$v_{ij} = 2\theta\, h(a_j+\beta b_j)\sum_s q_s(1-r_{sj})\lambda_i$$

Relaxing constraints (11) with Lagrange multipliers, ω_i, leads to the following Lagrangian dual problem:

$$\underset{\omega}{\text{Max}}\ \underset{X,Y}{\text{Min}} \sum_{j\in J}\left\{f_j X_j + \sum_{i\in I} u_{ij} Y_{ij} + \sqrt{\sum_{i\in I} v_{ij} Y_{ij}}\right\}$$
$$+\sum_{i\in I}\omega_i\left(1-\sum_{j\in J} Y_{ij}\right) = \sum_{j\in J}\left\{f_j X_j + \sum_{i\in I}(u_{ij}-\omega_i) Y_{ij}\right.$$
$$\left.+\sqrt{\sum_{i\in I} v_{ij} Y_{ij}}\right\} + \sum_{i\in I}\omega_i \tag{16}$$

subject to:

$$Y_{ij} \le X_j \quad \forall i \in I,\ \forall j \in J \tag{17}$$

$$X_j \in \{0,1\} \quad \forall j \in J \tag{18}$$

$$Y_{ij} \in \{0,1\} \quad \forall i \in I,\ \forall j \in J \tag{19}$$

For given values of the Lagrange multipliers, ω_i, the objective is to minimize (16) over the decision variables X_j and Y_{ij}. This problem is decomposed by *j*; as a result, we need to solve the following subproblem for each candidate location $j \in J$:

$$\text{SP}_j: \widetilde{V}_j = \text{Min} \sum_{i\in I}(u_{ij}-\omega_i)P_i + \sqrt{\sum_{i\in I} v_{ij} P_i} \tag{20}$$

subject to:

$$P_i \in \{0,1\} \quad \forall i \in I \tag{21}$$

In (20)-(21), the assignment variables Y_{ij} have been replaced by P_i to simplify the notation, Subproblem SP_j, can be solved using the exact algorithm developed by Shen et al. (2003). After solving subproblem SP_j for each *j*, f_j is added to the optimal objective value of $\widetilde{V}_j$. If $\widetilde{V}_j + f_j < 0$, then we select the candidate distribution center *j* and set $X_j = 1$; otherwise, we set $X_j = 0$. For each selected distribution center *j* (those for which $X_j = 1$) the assignment variables Y_{ij} are the same as the optimal P_i values in subproblem SP_j; for each unselected distribution center *j* (those for which $X_j = 0$) $Y_{ij} = 0, \forall i \in I$. Having solved the Lagrangian problem, the optimal Lagrange multipliers are found using a standard subgradient optimization procedure in Fisher (1981, 1985). The optimal objective value of the Lagrangian dual problem (16) provides a lower bound on the optimal objective value of (15).

3. 2. Finding an upper bound

In the obtained lower bound solution, some customers can be assigned to more than one DC. Thus, in each iteration of Lagrangian relaxation the obtained lower bound solution is converted into feasible upper bound solution using the procedure adapted from Shen et al. (2003), as follows. If a customer is assigned to more than one DC, we find the DC which assigning it to the customer leads to the least objective (15). If the resulted objective (15) is less than the case that the customer is not served at all, we assign the customer to that DC. Otherwise, the customer is not assigned to any DC and is not served at all.

The DCs which no longer serve any customer are closed. If the obtained feasible solution results in a less value for (15) than the best known upper bound, it will be taken as the new upper bound solution. Also, it will be improved using customer reassignment algorithm as follows. For each customer, we check whether the objective value (15) is improved if the customer is assigned instead to another located DC or if it is not served at all. The best improving swap is performed. The DCs which no longer serve any customer are closed. At the end of Lagrangian procedure, a variable fixing technique proved by Shu et al. (2005) is employed.

4. Computational results

In this section the performance of the proposed solution approach is tested on the 49-node, 88-node, and 150-node data sets described in Daskin (1995). For all three data sets, the mean of demand was obtained by dividing the population data given in Daskin (1995) by 1000. Cost parameters including $a_j, b_j, c_j, d_{ij}, f_j$ and *h* were set similar to Daskin et al. (2002). The disruption scenarios were generated randomly. Specifically, for each $s \in S$ and for each $j \in J$, r_{sj} was set to 0 with probability 90% or was set to a random number between (0,1] with probability 10%. The probability of occurrence associated with each scenario is first generated with a uniform distribution Uniform (0,1] and then normalized such that the total probability of all the scenarios is equal to 1. The parameters of Lagrangian relaxation approach are set similar to Qi et al. (2010). The developed Solution approach were coded in Visual Basic.Net and executed on Pentium 5 computer with 1.00 GB RAM and 2.00 GHz CPU.

Table 1-3 summarize the results for our computational study on 49-node, 88-node, and 150-node problems with different number of scenarios and different values for the parameters R_i, θ and β. In these tables, the columns marked N indicate the number of scenarios. The columns labeled LB and UB represent the lower and upper bounds for (5), respectively. The last columns in the tables indicate the percentage gaps between the upper and lower bounds and are obtained by $\frac{(\text{UB-LB})}{\text{LB}} \times 100$.

It follows from Table 1-3 that gap does not exceed 0.0365 %, implying that the bounds provided by the Lagrangian relaxation approach are very tight and obtained solutions are so close to optimal values. The computational time varies between 3 to 60 seconds. Therefore, the proposed solution approach is effective to solve the model.

5. Conclusion

This paper has addressed a supply chain design problem where distribution centers are subject to disruptions. The problem has been formulated as a nonlinear mixed-integer programming which maximizes the total profit for the whole supply chain. The model simultaneously determines the optimal number and location of DCs, the subset of customers to serve, the assignment of customers to DCs and the cycle order quantities at DCs. In order to solve the model, a solution approach based on Lagrangian relaxation has been presented. Computational results for different data sets have revealed that the proposed solution approach is quiet effective. In future it would be interesting to formulate the problem when supplier is unreliable too. Also, the model can be extended to consider constraints on the maximum capacity of inventory at DCs or on the maximum demand that can be supplied by the supplier. Finally, considering routing decisions in the model makes it more useful.

REFERENCES

[1] Aryanezhad MB, Jalali SG, Jabbarzadeh A (2010). An integrated supply chain design model with random disruptions consideration. African Journal of Business Management 4(12): 2393-2401.

[2] Aryanezhad MB, Jalali SG, Jabbarzadeh A (2011). An integrated model for designing supply chain network under demand and supply uncertainty. International Journal of the Physical Sciences, under press.

[3] Berman O, Krass D, Menezes MBC (2007). Facility reliability issues in network p-median problems: strategic centralization and co-location effects. Operations Research 55(1): 332-350.

[4] Chen Q, Li X, Ouyang Y (2011). Joint inventory-location problem under the risk of probabilistic facility disruptions, Transportation Research Part B 45(7): 991-1003.

[5] Cui T, Ouyang Y, Shen ZJM (2010). Reliable facility location design under the risk of disruptions. Operations Research 58 (4): 998-1011.

[6] Daskin MS (1995). Network and discrete location: models, algorithms, and applications. Wiley, New York.

[7] Daskin MS, Coullard C, Shen ZJ (2002). An inventory-location model: formulation, solution algorithm and computational results. Annals of Operations Research 110: 83-106.

[8] Fisher ML (1981). The Lagrangian relaxation method for solving integer programming problems. Management Science 27: 1–18.

[9] Fisher ML (1985). An applications oriented guide to Lagrangian relaxation. Interfaces 15: 2-21.

[10] Leonard D (2005). The only lifeline was the Wal-Mart. Fortune 152(7): 74-80.

[11] Lim M, Daskin MS, Bassamboo A, Chopra S (2009). A facility reliability problem: Formulation, properties and algorithm. Naval Research Logistics 57 (1): 58-70.

[12] Qi L, Shen ZJ (2007). A supply chain design model with unreliable supply. Naval Research Logistics 54: 829-844.

[13] Qi L, Shen ZJ, Snyder LV (2010). The effect of supply disruptions on supply chain design decisions. Transportation Science 44 (2) 274-289.

[14] Shen ZJ (2007). Integrated Supply Chain Design Models: A Survey and Future Research Directions. Journal of Industrial and Management Optimization 3: 1-27.

[15] Shen ZJ, Coullard C, Daskin MS (2003). A joint location-inventory model. Transportation Science 37: 40–55.

[16] Shen ZJ, Qi L (2007). Incorporating inventory and routing costs in strategic location models. European Journal of Operational Research 179: 372-389.

[17] Shu J, Teo CP, Shen ZJ (2005). Stochastic transportation-inventory network design problem. Operations Research 53: 48-60.

[18] Snyder LV, Daskin MS (2005). Reliability models for facility location: The expected failure cost case. Transportation Science 39: 400-416.

[19] Snyder LV, Daskin MS (2007). Models for reliable supply chain network design, in Critical Infrastructure: Reliability and Vulnerability. A.T. Murray and T.H. Grubesic (eds). Chapter 13. 257- 289, Springer-Verlag, Berlin.

[20] Snyder LV, Scaparra MP, Daskin MS, Church RL (2006). Planning for disruptions in supply chain networks. in TutORials in Operations Research. M.P. Johnson, B. Norman, and N. Secomandi (eds). Chapter 9, 234-257, INFORMS, Baltimore.

[21] Sweet K (2008). China earthquake hits home for u.s. companies. Fox Business May 12.

Table 1 Computational results for 49-node problem

1	10	10	0.01	0.0004	2230788.1	2230806.9	0.0008
2	10	10	0.001	0.00001	2230809.9	2230896	0.0039
3	10	100	0.01	0.0004	24465403	24465437.2	0.0001
4	10	100	0.001	0.00001	24465451.9	24465537.8	0.0004
5	20	10	0.01	0.0004	1936072.9	1936107.6	0.0018
6	20	10	0.001	0.00001	1936104.9	1936277.2	0.0089
7	20	100	0.01	0.0004	24170690.4	24170737.7	0.0002
8	20	100	0.001	0.00001	24170747.2	24170918.9	0.0007
9	30	10	0.01	0.0004	1625960.2	1626008.2	0.0030
10	30	10	0.001	0.00001	1626000.9	1626259	0.0159
11	30	100	0.01	0.0004	23860579	23860638.6	0.0002
12	30	100	0.001	0.00001	24170747.2	24170918.9	0.0007
13	40	10	0.01	0.0004	1295846	1295910	0.0049
14	40	10	0.001	0.00001	1295896	1296242	0.0267
15	40	100	0.01	0.0004	23530464	23530540	0.0003
16	40	100	0.001	0.00001	23530539	23530883	0.0015
1	10	10	0.01	0.0004	2230788.1	2230806.9	0.0008

Table 2 Computational results for 88-node problem

	N	R_i	θ	β	LB	UB	Gap
1	10	10	0.01	0.0004	4291225	4291245	0.0005
2	10	10	0.001	0.00001	4291252	4291260	0.0002
3	10	100	0.01	0.0004	44647692	44647751	0.0001
4	10	100	0.001	0.00001	44647762	44647771	0.0000
5	20	10	0.01	0.0004	4046212	4046244	0.0008
6	20	10	0.001	0.00001	4046248	4046264	0.0004
7	20	100	0.01	0.0004	44402677	44402750	0.0002
8	20	100	0.001	0.00001	44402758	44402775	0.0001
9	30	10	0.01	0.0004	3783094	3783143	0.0013
10	30	10	0.001	0.00001	3783142	3783167	0.0007
11	30	100	0.01	0.0004	44139567	44139649	0.0002
12	30	100	0.001	0.00001	44139652	44139678	0.0001
13	40	10	0.01	0.0004	3504978	3505042	0.0018
14	40	10	0.001	0.00001	3505037	3505071	0.0010
15	40	100	0.01	0.0004	43861450	43861547	0.0002
16	40	100	0.001	0.00001	43861547	43861582	0.0001

Table 3 Computational results for 150-node problem

	N	R_i	θ	β	LB	UB	Gap
1	10	10	0.01	0.0004	5319622	5319629	0.0001
2	10	10	0.001	0.00001	5319650	5319821	0.0032
3	10	100	0.01	0.0004	57696436	57696478	0.0001
4	10	100	0.001	0.00001	57696522	57696693	0.0003
5	20	10	0.01	0.0004	4819612	4819622	0.0002
6	20	10	0.001	0.00001	4819647	4819990	0.0071
7	20	100	0.01	0.0004	57196423	57196471	0.0001
8	20	100	0.001	0.00001	57196519	57196863	0.0006
9	30	10	0.01	0.0004	3819594	3819609	0.0004
10	30	10	0.001	0.00001	3819641	3820326	0.0179
11	30	100	0.01	0.0004	56196406	56196463	0.0001
12	30	100	0.001	0.00001	56196513	56197199	0.0012
13	40	10	0.01	0.0004	2819570	2819599	0.0010
14	40	10	0.001	0.00001	2819633	2820662	0.0365
15	40	100	0.01	0.0004	55196388	55196451	0.0001
16	40	100	0.001	0.00001	55196505	55197535	0.0019