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A Risk Management Model for the Capacitated Continuous Location Allocation Problem

M. Saeed Jabalameli, S. Javad Hosseininezhad+ and S. Gholamreza Jalali Naini

Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

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Abstract

This paper proposes a risk management model for the facility location problems in fuzzy environment. We investigate the capacitated continuous location allocation problem in continuous space as a risk model. Two risk types are considered in the proposed model: customer risk and financial risk. The risks are caused because of unsatisfied demands and budget constraint, respectively. The introduced model is extension of the continuous location allocation model by adding fixed cost and customer risk concept. A facility belongs to a zone when is located in a predetermined radius from center of the zone. Because of uncertain budget and demand, the model is considered in fuzzy environment. Finally, a risk management model is proposed with presentation of degree satisfaction concept of each risk as objective function. Also, a numerical example is expressed to illustrate the proposed model.

Keywords: capacitated continuous location allocation problem, risk management, financial risk, customer risk, satisfaction degree, fuzzy set theory.

1. Introduction

Several researches have been concentrated on considering risk in the facility location problem. Guillen *et al.* [8] considered the design and retrofit problem of a supply chain consisting of several production plants, warehouses, markets and the associated distribution systems. They constructed a

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+ Corresponding author

Tel.: +98--9124098244; P.O. Box 16846-13114, Tehran, Iran; E-mail address: Hosseininezhad@iust.ac.ir

two-stage stochastic model in order to take into account the effects of the uncertainty in the production scenario. Snyder *et al.* [17] proposed a stochastic version of the location model with risk pooling which optimizes location, inventory and allocation decisions under random parameters described by discrete scenarios. The goal of their model was to find solutions that minimize the expected total cost of the system across all scenarios. They presented a Lagrangian-relaxation–based exact algorithm for the model. Ozsen *et al.* [14] introduced the capacitated warehouse location model with risk pooling. The model provided a logistics system in which a single plant shipped one type of product to a set of retailers, each with an uncertain demand. Also, the model was solved by a Lagrangian relaxation solution algorithm. Azaron *et al.* [1] developed a multi-objective stochastic programming approach for supply chain design under uncertainty. Demands, supplies, processing, transportation, shortage and capacity expansion costs were all considered as the uncertain parameters. They used the goal attainment technique to obtain the Pareto-optimal solutions. Afterwards, Wagner *et al.* [18] considered a location-optimization problem where the classical incapacitated facility location model was recast in a stochastic environment with several risk factors that made demand at each customer site probabilistic and correlated with demands at the other customer sites. They considered "Value-at-Risk" (VaR) measure and designed a branch-and-bound algorithm to solve the problem. Considering the risk management for mid-term planning of a global multi-product chemical supply chain under demand and freight rate uncertainty, You *et al.* [21] proposed a two-stage stochastic linear programming approach within a multi-period planning model. Furthermore, they developed an algorithm based on the multi-cut L-shaped method in order to solve the resulting large scale industrial size problems. Mete and Zabinsky [12] developed a stochastic optimization approach for the storage and distribution problem of medical supplies to be used for disaster management under a wide variety of possible disaster types and magnitudes. Wang *et al.* [19] presented location model of risk pooling with variable construction cost. They applied a square nonlinear integer-programming model and used particle swarm optimization algorithm to find suboptimum solutions. Cui *et al.* [5] investigated reliable facility location models considering unexpected failures with site dependent probabilities, as well as possible customer reassignment. They proposed a compact mixed integer program formulation which was solved using a custom-designed Lagrangian relaxation algorithm. Liu *et al.* [11] presented a location model that assigns online demands to the capacitated regional warehouses currently serving in-store demands in a multi-channel supply chain. The model explicitly considered the trade-off between the risk pooling effect and the transportation cost in a two-echelon inventory/logistics system. They formulated the assignment problem as a non-linear integer programming model.

A strategic supply chain management problem was studied by Peng *et al.* [15] to design reliable networks that perform as well as possible under normal conditions, while also performing relatively well when disruptions strike. They presented a mixed-integer programming model whose objective was to minimize the nominal cost while reducing the disruption risk using the *p*-robustness criterion which bounds the cost in disruption scenarios. Chen et al. [3] presented a multi-criteria decision analysis for environmental risk assessment with regard to avoiding and eliminating damages and loss under natural disasters in international airport projects. They used the ANP to demonstrate one of its utility modes in decision making support to location selection problems, which aims at an evaluation of different projects from different locations. Wang and Watada [20] studied a facility location model with fuzzy random parameters and its swarm intelligence approach. A VaR based fuzzy random facility location model was built in which both the costs and demands were assumed to be fuzzy random variables. The model was inherently a two-stage mixed 0–1 integer fuzzy random programming problem. A hybrid modified particle swarm optimization approach was proposed to solve the model. A corresponding framework for value-based performance and risk optimization in a single-stage supply chain problem was developed by Hahn and Kuhn [19]. They applied Economic Value Added as a prevalent metric of value-based performance to mid-term sales and operations planning. Due to the uncertainty of future events in a scenario based problem, they also used robust optimization methods to deal with operational risks in physical and financial supply chain management. Nickel *et al.* [13] provided a multi-period supply chain network design problem. In this problem, uncertainty was assumed for demand and interest rates, which was described by a set of scenarios. Accordingly, the problem was formulated as a multi-stage stochastic mixed-integer linear programming problem.

These researches have investigated risk in the location models with discrete space and have provided the models as a risk cost minimization model. In the next section, a risk management model in continuous space with uncertain demands and budget are provided and two risk types are investigated; customer risk because of unsatisfied customers and financial risk which is caused by budget constraint. Since a customer has an uncertain demand, once the customer was assigned to a facility, the facility may not service the customer properly and cause to some unsatisfied demand. Then, we introduce a model proposing a criterion named satisfaction degree of the risk so as to maximize the satisfaction degree in the location allocation model. In the rest of this section, aforementioned articles are classified based on Location model, Risk type, Space and Uncertainty as shown in *Table 1* in order to help the reader appreciate the symmetry associated with the facility location problems.

Therefore, the main differences of our research compared with other works are as follows:

- Providing a location allocation model with fixed cost and customer risk.
- Investigating risk in a continuous space
- Introducing satisfaction degree concept of risk

Table 1- Comparison between the works

Author(s)	Location model	Risk type	Space	Uncertainty
Guillen *et al.* [8]	Multi objective supply chain	Scenario based	Discrete	Stochastic
Snyder *et al.* [17]	Location with risk pooling	Scenario Based	Discrete	Stochastic
Ozsen *et al.* [14]	Warehouse location	Uncertain demand	Discrete	Stochastic
Azaron *et al.* [1]	Multi-objective stochastic	Scenario Based	Discrete	Stochastic
Wagner *et al.* [18]	Uncapacitated p-median	Value-at-Risk	Discrete	Stochastic
You *et al.* [21]	Multi-product supply chain	Uncertain demand	Discrete	Stochastic
Mete and Zabinsky [12]	Location with vehicle routing	Disaster	Discrete	Stochastic
Wang *et al.* [19]	Location with risk pooling	Stochastic demand	Discrete	Stochastic
Cui *et al.* [5]	Reliable facility location	Risk of disruption	Discrete	Stochastic
Liu *et al.* [11]	Two-echelon inventory/logistics	Stochastic demand	Discrete	Stochastic
Peng *et al.* [15]	Reliable logistics network design	Disruption	Discrete	Stochastic
Chen *et al.* [3]	Location selection	Disaster	Discrete	judgmental
Wang and Watada [20]	Fuzzy facility location	Value-at-Risk	Discrete	Fuzzy
Hahn and Kuhn [9]	Single-stage supply chain	Scenario Based	Discrete	Stochastic
Nickel S. *et al.* [13]	Multi-stage supply chain	Scenario Based	Discrete	Stochastic
This research	Capacitated continuous Location allocation	Uncertain demand and Budget	Continuous	Fuzzy

The remainder of the paper is organized as follows. In section 2 the capacitated continuous location allocation model is provided. In Section 3, we present the risk management model. A numerical example is given in Section 4 in order to illustrate the usability of the proposed model. Finally, Section 5 is devoted to the conclusion and future work.

2. Capacitated Continuous Location Allocation Model

The location-allocation (LA) problem is to locate a set of new facilities such that the transportation cost from facilities to customers is minimized and an optimal number of facilities have to be placed in an area of interest in order to satisfy the customer demand. This problem occurs in many practical settings such as the determination and location of warehouses, distribution centers, communication centers and production facilities. Since LA problem was proposed by Cooper [4] and spread to a weighted network by Hakimi [10]. For a review, on the continuous location problem, see Drezner *et al.* [6], Salhi and Gamal [17], and Brimberg and Salhi [2].

Assuming, there are n customers (demand points) indexed by i and m facilities indexed by j and following notations,

Notations	Description
Parameters	
A_i	*coordinate of customer i*
d_i	*demand of customer i*
s_j	*capacity of facility j*
Decision variable	
X_j	*coordinate of new facility j*
$D(X_j,A_i)$	*distance between customer i and facility j*
y_{ji}	*quantity supplied to customer i by facility j*

Then the capacitated continuous location-allocation problem is also known as the capacitated multisource Weber problem (CMWP) [7] and is as following,

$$P_1: min \sum_{i=1}^{n}\sum_{j=1}^{m} y_{ji} \cdot D(X_j, A_i) \tag{1}$$

S.t.

$$\sum_{j=1}^{m} y_{ji} = d_i, \qquad \forall i = 1,2, \dots, n \tag{2}$$

$$\sum_{i=1}^{n} y_{ji} = s_j, \qquad \forall j = 1,2, \dots, m \tag{3}$$

$$y_{ji} \geq 0, \qquad \forall i = 1,2, \dots, n \text{ and } \forall j = 1,2, \dots, m \tag{4}$$

Equation (1) is objective function consists of transportation cost. Constraint set (2) is demand constraint. Constraint set (3) guarantees capacity constraint of each facility. Constraint set (4) is standard constraint.

As seen, the model and other location allocation problems are not considered fixed cost in continuous space, but, there are cases in real world which may be included some zones with high installation cost or forbidden zones and it needs to consider fixed cost. So, we introduce a continuous location allocation model with fixed cost in this paper; also, customer risk concept is introduced in the model. we are interested in finding the location of m facilities in continuous space with allocation of each facility to each customer in n points so that the total cost of transportation, installation and unsatisfied demands of customers is minimized. At first, the space is divided into n zones. The proposed model is extension of model P_1. This paper introduces a new concept; a facility will be located in a zone if the distance between the facility and the center of the zone is smaller than a predetermined radius. So, a new variable z_{ji} is introduced. In this regard, the z_{ji} shows whether or not the facility j is located in the zone i. It is assumed that distance between each customer and the facility is Euclidean. Accordingly, the *0-1* Nonlinear programming model P_2 is as follows:

$$P_2: min \sum_{i=1}^{n}\sum_{j=1}^{m} y_{ji}\left(\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2}\right) + \sum_{j=1}^{m}\sum_{i=1}^{n} z_{ji} f_{ji} + M \sum_{i=1}^{n}\left(\frac{C_i}{C}\right)\left(\frac{q_i}{d_i}\right) \tag{5}$$

S.t.

$$\sum_{i=1}^{n} z_{ji}\left(\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2}\right) \leq D, \qquad \forall j = 1,2,\dots,m \tag{6}$$

$$\sum_{i=1}^{n} z_{ji} = 1, \qquad \forall j = 1,2,\dots,m \tag{7}$$

$$\sum_{j=1}^{m} z_{ji} \leq 1, \qquad \forall i = 1,2,\dots,n \tag{8}$$

$$\sum_{j=1}^{m} y_{ji} + q_i = d_i, \quad i = 1,2,\dots,n \tag{9}$$

$$\sum_{i=1}^{n} y_{ji} \leq \sum_{i=1}^{n} z_{ji}\, s_{ji}, \qquad \forall j = 1,2,\dots,m \tag{10}$$

$$z_{ji} \in \{0,1\}, \qquad q_i \in \mathbb{Z} \text{ and } x_j, y_j \in \mathcal{R},, \qquad \forall i = 1,2,\dots,n, \forall j = 1,2,\dots,m$$

Notations of the model are as follows,

Notations		Description
Sets/Indices		
	N	*set of zones (demand points) in a continuous space indexed by i , {i=1,2,…,n}*
	K	*set of new facilities to be located indexed by j, {j=1,2,…,m}*
Parameters		
	a_i	*x coordinate of demand point i*
	b_i	*y coordinate of demand point i*
	p_j	*Unit production cost of facility j in zone i*
	f_{ji}	*Installation cost of facility j in zone i*
	D	*Maximum distance a facility can be located from center of a zone for belonging to that zone*
	s_{ji}	*Production capacity of facility j in zone i*
	d_i	*Demand amount of demand point i*
	C_i	*importance of demand point i*
	M	*Penalty of unsatisfied demand which is a large amount*
Decision variable		
	x_j	*x coordinate of facility j*
	y_j	*y coordinate of facility j*
	y_j	*transported product from facility j to demand point i*
	z_{ji}	*Binary variable; equal to1 if facility j is located in zone i; otherwise equal to 0*
	q_i	*Unsatisfied demand of demand point i*

Equation (5) is objective function of the model and constitutes of three terms. The first term is transportation cost; the second term is installation cost and the third term named customer risk cost is percent of the unsatisfied demand based on importance of each demand point. Since, there is a supply constraint in the model, all of the demand could not be supplied and demand of some customers may not be supplied, so the third term is added. Constraint set (6) guarantees that if the distance between the facility j and the zone i be greater than D, $z_{ji} = 0$, so the facility j does not belong to the zone i and the facility j will be located in the zone i, if the distance between the facility j and the zone i be smaller than D. in contrast, if $z_{ji} = 1$, the distance between the facility j and the zone i could not be greater than D. These constraints are applied to add fixed cost to the continuous models. Constraint set (7) guarantees that the facility j is installed only in one zone. Constraint set (8) guarantees that at most one facility could be located in the zone i. Constraint set (9) is a capacity constraint which guarantees that the transportation amount of the facility j should be less than the limitation of production in the zone i. Constraint set (10) indicates the demand constraint which guarantees that the satisfied and the unsatisfied demand of each demand point is equal to the demand of that demand point.

Since, we want to introduce a risk management model, use the model P_2 with some changes. For more adopting on real world, since demand of customers may not be certain, we consider the model in uncertain conditions Supposing mean and deviation of demand in the zone i are d_i and dd_i , respectively. Therefore, consider $\tilde{d}_i = (d_i - dd_i, d_i, d_i + dd_i)$ as fuzzy demand of the zone i. Because of the uncertain demand, once a customer was assigned to a facility, the facility may not service the customer properly and cause to some unsatisfied demand. Also, we set risk cost of the unsatisfied customers as objective function and consider the transportation and the fixed cost as a budget constraint. Assuming Ω, is the budget which is assigned to the transportation and the fixed costs, If the costs are greater than Ω, financial risk will be caused. Therefore, a budget constraint is added to the model and the transportation and the fixed costs are eliminated from the objective. Then a fuzzy risk management $P_3$is introduced as follows,

$$P_3: min \sum_{i=1}^{n} \left(\frac{C_i}{C}\right)\left(\frac{q_i}{\tilde{d}_i}\right) \tag{11}$$

S.t.

$$\sum_{i=1}^{n}\sum_{j=1}^{m} y_{ji}\left(\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2}\right) + \sum_{j=1}^{m}\sum_{i=1}^{n} z_{ji} f_{ji} \le \Omega$$

$$\sum_{i=1}^{n} z_{ji}\left(\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2}\right) \le D, \qquad \forall j = 1,2,\dots,m$$

$$\sum_{i=1}^{n} z_{ji} = 1, \qquad \forall j = 1,2,\dots,m$$

$$\sum_{j=1}^{m} z_{ji} \le 1, \qquad \forall i = 1,2,\dots,n$$

$$\sum_{j=1}^{m} y_{ji} + q_i = \tilde{d}_i, \quad i = 1,2,\dots,n$$

$$\sum_{i=1}^{n} y_{ji} \le \sum_{i=1}^{n} z_{ji}\, s_{ji}, \qquad \forall j = 1,2,\dots,m$$

$$z_{ji} \in \{0,1\}, \qquad q_i \in \mathbb{Z} \text{ and } x_j, y_j \in \mathcal{R},, \qquad \forall i = 1,2,\dots,n, \forall j = 1,2,\dots,m$$

In the next section, by considering fuzzy concepts, the final risk management model with crisp parameters is introduced.

3. Risk management model

In this section a risk management model is proposed based on the model P_3 . We introduce a model which proposes a criterion named satisfaction degree of the risk so as to maximize the satisfaction degree in the model. Since, we investigate the model in uncertain condition; the satisfaction degree concept in fuzzy set theory is applied to provide the risk management model. We introduce the satisfaction degree of the customers risk and the financial risk and demand constraint, respectively. We use the satisfaction degree concept to provide a crisp risk management model. By utilization of the max–min operator introduced by Zimmermann [22], the problem convert to a single objective optimization problem, assuming

$$f(X)=\sum_{i=1}^{n}\sum_{j=1}^{m}y_{ji}\left(\sqrt{\left(x_j-a_i\right)^2+\left(y_j-b_i\right)^2}\right)+\sum_{j=1}^{m}\sum_{i=1}^{n}z_{ji}f_{ji}$$

Be total cost and Ω^1 be the existing budget and Ω^2 be at most value we could assign as the budget so that total cost could not be greater than Ω^2, Since $\tilde{d}_i=(d_i-dd_i,d_i,d_i+dd_i)$, we introduce fuzzy variable $\tilde{\mu}_\Omega$ in (12) as fuzzy satisfaction degree of financial risk which is shown in *Figure 1*,

$$\tilde{\mu}_\Omega=\left(\mu_\Omega^l,\mu_\Omega^m,\mu_\Omega^u\right)=\begin{cases}1, & f(X)\le\Omega^1\\ \dfrac{\Omega^2-f(X)}{\Omega^2-\Omega^1}, & \Omega^1\le f(X)\le\Omega^2\\ 0, & f(X)\ge\Omega^2\end{cases}\tag{12}$$

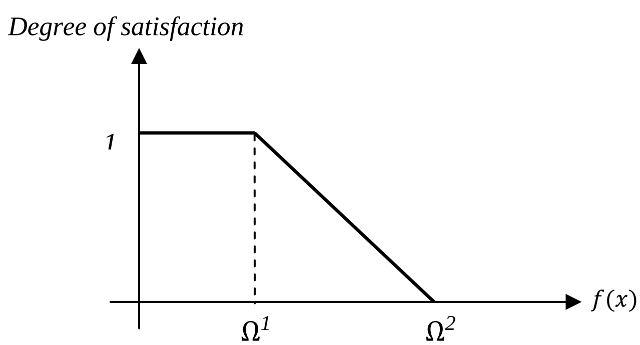

Figure 1- degree satisfaction of the financial risk

Where $\mu_\Omega^l,\mu_\Omega^m,\mu_\Omega^u$ are calculated when demand of each customer are d_i-dd_i,d_i,d_i+dd_i, respectively. As seen in constraint (12), if total cost is greater than Ω^2, then the satisfaction degree of financial risk is *0*, in other words, the financial risk is *1* or *100%*. Also if total cost is smaller than Ω^1, then the satisfaction degree of financial risk is *1*, in other words, the financial risk is *0* or *0%*. If $\lambda_\Omega=min\{\mu_\Omega^l,\mu_\Omega^m,\mu_\Omega^u\}$, which is named satisfaction degree of financial risk as shown in(13):

$$\lambda_\Omega\le\left(\Omega^2-\sum_{i=1}^{n}\sum_{j=1}^{m}y_{ji}\left(\sqrt{\left(x_j-a_i\right)^2+\left(y_j-b_i\right)^2}\right)+\sum_{i=1}^{n}\sum_{j=1}^{m}z_{ji}f_{ji}\right)\Big/\left(\Omega^2-\Omega^1\right),\forall d_i\in\{d_i-dd_i,d_i,d_i+dd_i\}\tag{13}$$

We carry out similar definitions for the customer risk as (14),

$$\tilde{\mu}_C = \left(\mu_C^l, \mu_C^m, \mu_C^u\right) = \begin{cases} 1, & \sum_{i=1}^{n} \left(\frac{C_i}{C}\right)\left(\frac{q_i}{\tilde{d}_i}\right) \leq 0 \\ 1 - \sum_{i=1}^{n} \left(\frac{C_i}{C}\right)\left(\frac{q_i}{\tilde{d}_i}\right), & 0 \leq \sum_{i=1}^{n} \left(\frac{C_i}{C}\right)\left(\frac{q_i}{\tilde{d}_i}\right) \leq 1 \\ 0, & \sum_{i=1}^{n} \left(\frac{C_i}{C}\right)\left(\frac{q_i}{\tilde{d}_i}\right) \geq 1 \end{cases} \tag{14}$$

As seen in constraint (14), if the satisfied demands are smaller than *0*, then the satisfaction degree of customer risk is *0*, in other words, the financial risk is *1* or *100%*. Also if the satisfied demands are greater than total demands, then the satisfaction degree of financial risk is *1*, in other words, the financial risk is *0%*. If $\lambda_C = min\{\mu_C^l, \mu_C^m, \mu_C^u\}$, which is named satisfaction degree of the customer risk as shown in (15):

$$\lambda_C \leq 1 - \sum_{i=1}^{n} \left(\frac{C_i}{C}\right)\left(\frac{q_i}{d_i}\right) \forall d_i \in \{d_i - dd_i, d_i, d_i + dd_i\} \tag{15}$$

Finally, degree satisfaction of demand constraint is introduced as shown in *Figure 2*, this concept is carried out for each *i*, μ_d^{i-}, μ_d^{i+} are satisfaction degree of Left Hand Side (LHS) and Right Hand Side (RHS) of demand constraint *i* as shown in (16):

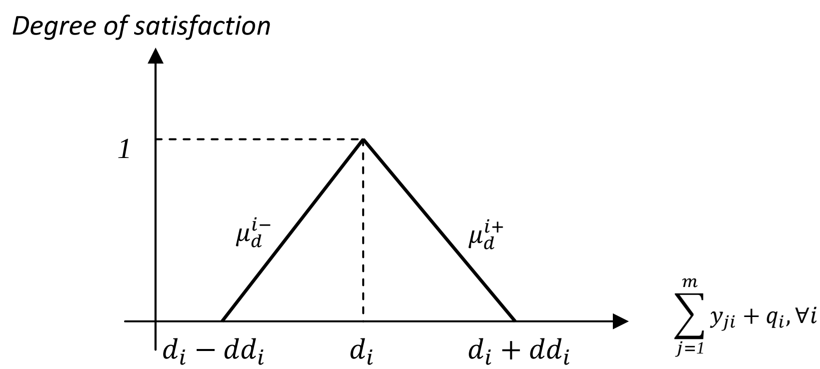

Figure 2- degree satisfaction of demand constraint

$$\mu_d^i = \left(\mu_d^{i-}, \mu_d^{i+}\right) = \begin{cases} 0, & \sum_{j=1}^{m} y_{ji} + q_i \leq d_i - dd_i \\ \left(d_i - \sum_{j=1}^{m} y_{ji} + q_i\right) \Big/ dd_i, & d_i - dd_i \leq \sum_{j=1}^{m} y_{ji} + q_i \leq d_i \\ \left(\sum_{j=1}^{m} y_{ji} + q_i - d_i\right) \Big/ dd_i, & d_i \leq \sum_{j=1}^{m} y_{ji} + q_i \leq d_i + dd_i \\ 0, & \sum_{j=1}^{m} y_{ji} + q_i \geq d_i + dd_i \end{cases} \tag{16}$$

If $\lambda_d = \min\{\mu_d^{i-}, \mu_d^{i+} | 1 = 1,2,\dots,n\}$, then $\lambda_d \leq \{\mu_d^{i-}, \mu_d^{i+}\}$, $\forall i = 1,2,\dots,n$ which is named satisfaction degree of demand constraint the constraints (17) and (18) are as follows,

$$d_i - dd_i - \sum_{j=1}^{m} y_{ji} - q_i + \lambda_d dd_i \leq 1\,, \quad i = 1,2,\dots,n \tag{17}$$

$$\sum_{j=1}^{m} y_{ji} + q_i - d_i - dd_i + \lambda_d dd_i \leq 1\,, \quad i = 1,2,\dots,n \tag{18}$$

Finally, if $\lambda = \min\{\lambda_\Omega, \lambda_C, \lambda_d\}$ we apply the Zimmermann Max-min operator to provide a crisp model. So, the final risk management model P_4 is as follows in (19),

$$P_4: max\ \lambda \tag{19}$$

$$S.t.$$

$$\lambda \leq 1 - \sum_{i=1}^{n} \left(\frac{C_i}{C}\right)\left(\frac{q_i}{d_i}\right), \quad \forall d_i \in \{d_i - dd_i, d_i, d_i + dd_i\}$$

$$\lambda \leq \left(\Omega^2 - \sum_{i=1}^{n}\sum_{j=1}^{p} y_{ji} d_i \left(\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2}\right) + \sum_{i=1}^{n}\sum_{j=1}^{p} z_{ji} f_{ji}\right) \Big/ (\Omega^2 - \Omega^1)$$

$$\sum_{i=1}^{n} z_{ji}\left(\sqrt{(x_j - a_i)^2 + (y_j - b_i)^2}\right) \leq D, \qquad \forall j = 1,2,\dots,m$$

$$\sum_{i=1}^{n} z_{ji} = 1, \qquad \forall j = 1,2,\dots,m$$

$$\sum_{j=1}^{m} z_{ji} \leq 1, \qquad \forall i = 1,2,\dots,n$$

$$d_i - dd_i - \sum_{j=1}^{m} y_{ji} - q_i + \lambda dd_i \leq 1\,, \quad i = 1,2,\dots,n$$

$$\sum_{j=1}^{m} y_{ji} + q_i - d_i - dd_i + \lambda dd_i \leq 1\,, \quad i = 1,2,\dots,n$$

$$\sum_{i=1}^{n} y_{ji} \leq \sum_{i=1}^{n} z_{ji}\, s_{ji}, \qquad \forall j = 1,2,\dots,m$$

$$0 < \lambda \leq 1, z_{ji} \in \{0,1\}, \qquad q_i \in \mathbb{Z}\ and\ x_j, y_j \in \mathcal{R}, \quad \forall i = 1,2,\dots,n, \forall j = 1,2,\dots,m$$

Where λ is the overall satisfaction degree of risk, Finally, (x_j, y_j) and y_{ji} provide the best location of the facility *j* and the best allocation values to the customers with the highest satisfaction degrees of the risk.

4. Numerical example

In this section, a numerical example is expressed to illustrate the introduced model**.** Suppose we want to locate *4* new facilities in a region including *16* zones (customers) as shown in *Figure 3.* Number of each zone is indicated in each cell. Specification of each zone consisting importance, mean and standard deviation demand of each customer, fixed cost and capacity of facility in each zone are shown in *Table 2*. Ω^1 and Ω^2 are 1150 and 1200, respectively and $D=0.50$.

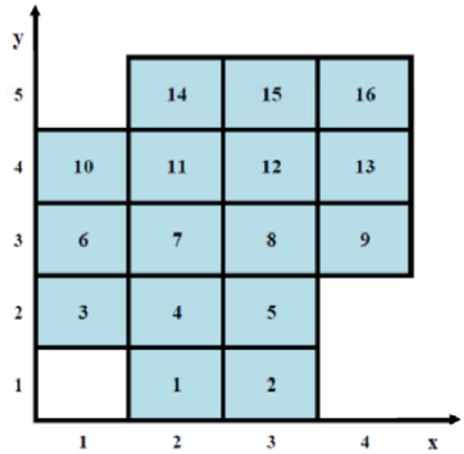

Figure 3- Numerical Example with 16 zones

The example was solved by *GAMS* software applying *Baron Solver* which uses *branch and reduce* algorithm. The Location and the allocation variables are shown in *Table 3* and *Table 4*, respectively. So, $q_2 = 20, q_{12} = 40, q_{16} = 5$ and others $= 0$, consumed budget is $f(x) = 1156$ and finally $\lambda = 0.80$. it means that the overall satisfaction degree of the risks is 0.80 so the overall risk is approximately *20%*. The final solution is shown in *Figure4*.

Table 2- specification of customers and zones for the Numerical Example

Customer/Zone	*Coordinate*	*Mean of Demand*	*Dev. of Demand*	*Demand*	*Importance*	*Fixed cost*	*Capacity*
1	(2,1)	17	8	(9,17,25)	1	200	70
2	(3,1)	65	5	(60,65,70)	1.5	250	70
3	(1,2)	15	7	(8,15,22)	1.1	500	75
4	(2,2)	24	12	(12,24,36)	1.2	1500	70
5	(3,2)	10	2	(8,10,12)	1	1200	90
6	(1,3)	13	3	(10,13,16)	1.3	1500	80
7	(2,3)	22	3	(19,22,25)	1.4	150	60
8	(3,3)	38	5	(33,38,43)	1.6	400	100
9	(4,3)	23	5	(18,23,28)	1	300	50
10	(1,4)	20	5	(15,20,25)	1	400	80
11	(2,4)	17	6	(11,17,23)	1.1	400	60
12	(3,4)	70	25	(45,70,95)	2	600	30
13	(4,4)	20	3	(17,20,23)	1	600	40
14	(2,5)	15	14	(1,15,29)	1	300	70
15	(3,5)	11	5	(6,11,16)	1.5	600	60
16	(4,5)	25	1	(24,25,26)	1.3	200	90

Table 3- location of facilities

j(facility)	(x_j, y_j)	*zone*
1	(1.420,2.270)	3
2	(2.996,1.002)	2
3	(3.398,3.303)	8
4	(1.487,4.110)	10

Table 4- Allocation variables customers to facilities

j(facility)	$y_{ji}(i = 1, \dots, 16)$
1	$y_{1,3} = 14,\ y_{1,4} = 22,\ y_{1,6} = 13, y_{1,7} = 22, y_{1,8} = 4$
2	$y_{2,1} = 16, y_{2,2} = 44, y_{2,5} = 10$
3	$y_{3,8} = 33, y_{3,9} = 22, y_{3,12} = 5, y_{3,13} = 20, y_{3,16} = 20$
4	$y_{4,10} = 19, y_{4,11} = 16, y_{4,12} = 21, y_{4,14} = 13, y_{4,15} = 10$

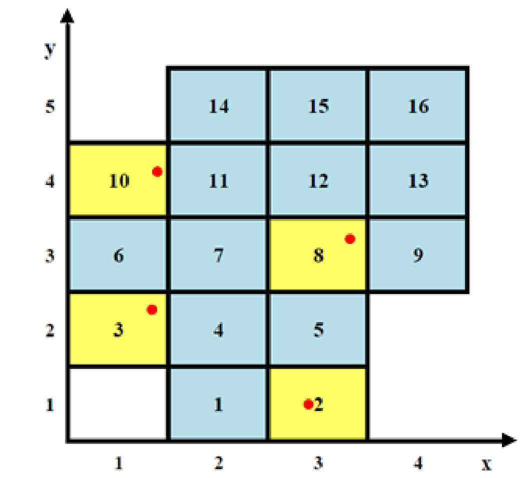

Figure 4- Location of facilities for the numerical example

Red circles are the location of the facilities and the selected zones are identified by yellow color

5. Conclusion

This paper proposed a risk management model for the facility location problems in fuzzy environment. We investigated the capacitated continuous location allocation problem in continuous space as a risk model. Two risk types were investigated in the proposed model: the customer and the financial risk. The risks are caused because of the unsatisfied demands and the budget constraint, respectively. The first presented model's advantage over the traditional models was presentation of the capacitated continuous location allocation model with fixed cost and the customer risk. The second was consideration of continuous space for risk management model in the location problems. Because of uncertain budget and demand, the risk management model was provided. Finally, the third advantage was presentation of the degree satisfaction concept of each risk as the objective function for the risk models. Considering other parameters such as capacity and fixed cost in uncertain condition, providing other continuous location models such as the covering and the p-median as a risk problem and applying a heuristic method to solve large scale cases are research issues which we think may need future investigations.

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