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Robust supply chain network design with service level against disruptions and demand uncertainties: A real-life case

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A B S T R A C T

We have developed a stochastic mathematical formulation for designing a network of multi-product supply chains comprising several capacitated production facilities, distribution centres and retailers in markets under uncertainty. This model considers demand-side and supply-side uncertainties simultaneously, which makes it more realistic in comparison to models in the existing literature. In this model, we consider a discrete set as potential locations of distribution centres and retailing outlets and investigate the impact of strategic facility location decisions on the operational inventory and shipment decisions of the supply chain. We use a path-based formulation that helps us to consider supply-side uncertainties that are possible disruptions in manufacturers, distribution centres and their connecting links. The resultant model, which incorporates the cut-set concept in reliability theory and also the robust optimisation concept, is a mixed integer nonlinear problem. To solve the model to attain global optimality, we have created a transformation based on the piecewise linearisation method. Finally, we illustrate the model outputs and discuss the results through several numerical examples, including a real-life case study from the agri-food industry.

1. Introduction

The fierce competition in today's markets and the swift changing of customers' preferences, together with the rapid development of technology and globalisation, have forced organisations to operate as members of a supply chain (SC) instead of acting as individual enterprises. The success of an SC depends on the integration and coordination of its all entities to form an efficient network structure; an efficient network leads to cost-effective operations throughout the chain and helps it to react quickly in response to customers' needs. According to Simchi-Levi and Kaminsky (2004), SC network design is the most basic decision of SC management, which influences all other decisions concerning an SC and has the most extensive effect on the chain's return on investment and its overall performance. Lin and Wang (2011) define SC network design as an integrated configuration of supply, manufacturing and demand side sub-systems. SC network design deals with strategic decisions of the chain, such as the number, location and capacity of entities in each echelon of the chain. However, the network structure of the chain strongly influences the later operational decisions of flow management throughout the chain, so in addition to strategic locating and capacity setting costs,

the resulting operational inventory holding and transportation costs should be considered at the network design stage. Ignoring operational costs at this stage leads to sub-optimality of the network structure.

As mentioned earlier, SC network design mainly deals with strategic decisions that are usually long-term. Considering changes over time also has an important role in making suitable strategic decisions. This fact necessitates considering uncertainties of the environment at the strategic decision making stage, as uncertainty is an undeniable part of today's business environment. SCs are important entities of today's markets and also their decentralised nature makes them vulnerable against these uncertainties. Thus, SC risk management is a major part of SC management that involves designing a robust SC network structure and managing the product flow throughout this network in a manner which enables them to be able to predict, cope with and recover from disruptions. Today, many experts believe that numerous risk sources are involved in an SC, and these chains remain ill-equipped to handle them. There are many examples of disruption in real-world SC problems.

According to Sarkar et al. (2002), during the labour strike in 2002, 29 ports on the West coast of the United States were shut down, which led to the closure of the New United Motor Manufacturing production factory. During the recent destructive earthquake of Japan in 2011, the Toyota Motor Company had to

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cease manufacture at its twelve assembly plants, leading to a production loss of 140,000 vehicles. The main cause of this problem was disruption of its chain's manufacturing subsystem. In addition to the impairment of production facilities and factories throughout Japan, many Japanese companies had problems with the supply of required materials, fuel and power. In these kinds of catastrophes, supply and manufacturing disruptions are huge problems for companies. As mentioned by Norrman and Jansson (2004), a fire at one of the major suppliers of the Ericsson Company resulted in several serious problems for this company and the shutdown of its manufacturing plants for several days.

There are many other examples, for instance: Dole suffered revenue declines after their banana plantations were destroyed by Hurricane Mitch in 1998; Ford was forced to close five plants for several days after the terrorist attacks of September 11 caused a suspension in air traffic in 2001; the 1999 earthquake in Taiwan displaced power lines to the semiconductor fabrication facilities responsible for more than 50% of world wide supplies of memory chips, circuit boards, flat-panel displays and other computer components and many hardware manufacturers including HP, Dell, Apple, IBM, Gateway and Compaq suffered as a consequence; and a Motorola cell phone factory in Singapore closed after an employee became infected with the SARS disease. For more details, see Martha and Subbakrishna (2002) and Monahan et al. (2003). Further examples include Ericsson's loss of 400 million Euros after their supplier's semiconductor plant caught fire in 2000, and Apple losing many customer orders during a supply shortage of DRAM chips after an earthquake hit Taiwan in 1999;. The 2002 longshoremen's union strike at a US West Coast port interrupted trans-shipments and deliveries to many US-based firms, with port operations and schedules not returning to normal until 6 months after the strike had ended: for more detail see Cavinato (2004). Network-related risk sources arise from interactions between organisations within a supply chain, and interested readers should refer to Juttner et al. (2003) for a discussion of the relationships between categories of risk sources in a network.

Hendricks and Singhal (2005) quantified the negative effects of supply chain disruption through empirical analysis, which is useful for modelling purposes. They found 33–40% lower stock returns relative to their benchmarks over a 3-year time period that started 1 year before and ended 2 years after a disruption, large negative effects on profitability, a 107% drop in operating income, 7% lower sales growth and an 11% growth in costs, 2 years at a lower performance level after a disruption and a negative effect on stock prices, profitability, and share price volatility.

Chopra and Sodhi (2004) categorised potential supply chain risks into nine categories: (a) Disruptions (e.g. natural disasters, terrorism, war, etc.), (b) Delays (e.g. inflexibility of supply source), (c) Systems (e.g. information infrastructure breakdown), (d) Forecast (e.g. inaccurate forecast, bullwhip effect, etc.), (d) Intellectual property (e.g. vertical integration), (e) Procurement (e.g. exchange rate risk), (f) Receivables (e.g. number of customers), (g) Inventory (e.g. inventory holding cost, demand and supply uncertainty, etc.), and (h) Capacity (e.g. cost of capacity). They also discussed the mitigation strategy against each kind of risk.

Tang (2006) considers two types of risks: (a) *operational risks* which are inherent uncertainties such as uncertain customer demands, uncertain supply, and uncertain costs, and (b) *disruption risks* which are major disruptions caused by natural and man-made disasters such as earthquakes, floods, hurricanes, terrorist attacks, or economic crises such as currency fluctuations or strikes.

Waters (2007) divides SC risk sources into *internal risks* (which can be controlled) and *external risks* (which cannot be controlled). Internal risks appear in normal operations, such as late deliveries,

excess stock, poor forecasts, human errors, and faults in IT systems. External risks come from outside of an SC, such as earthquakes, hurricanes, industrial action, wars, terrorist attacks, price rises, problems with trading partners, shortages of raw materials and crime.

Moreover, Waters (2007) introduces a further three categories of risk sources: (a) Environmental risk sources, which comprise any uncertainties arising from the interaction of the SC with the environment and these may be the result of accidents (e.g. fire), socio-political actions (e.g. fuel protests or terrorist attacks) or acts of God (e.g. extreme weather or earthquakes); (b) Organisational risk sources which lie within the boundaries of SC parties and range from labour (e.g. strikes) or production uncertainties (e.g. machine failure) to IT-system uncertainties; and (c) Network-related risk sources which arise from interactions between organisations within an SC.

Kar (2010) believed the risks of a chain can also be categorised into two groups:

Systematic risks: This risk is related to environmental factors which are unavoidable. Companies do not have any control over factors such as: demand-side uncertainty; supply-side disruption; regulatory, legal, and bureaucratic changes; the occurrence of catastrophic events; and infrastructure disruption.
Non-systematic risks: This risk is related to factors that can be controlled to a large extent by a company, such as facility disruption of a manufacturing subsystem.

There are several ways to mitigate, model, quantify and solve uncertainty problems. Uncertainties can be demonstrated using concepts of fuzzy theory and also distribution function in stochastic models and this paper considers that supply and demand are probabilistic with known distribution functions. Another uncertainty we aim to cover is related to disruption and consequently closure of our manufacturing facilities due to disasters. From a modelling and solution point of view there are different strategies to cope with these problems. One of these is robust optimisation and here we try to use its concepts within the solution section.

We consider both demand and supply uncertainties in our model. Demand uncertainty will be demonstrated in the form of a demand distribution function and supply uncertainty will be modelled in terms of the probability of the occurrence of disruptions in manufacturers, distribution centres, and the connecting links of an SC network.

A real-life case study motivated the development of the model presented in this paper. The case was the designing of a network structure for a chain in the rice industry of a Middle Eastern country. Rice is one of the most important essential goods within Asia and especially in the Middle East. Most of the rice in the world is produced and consumed in this area and rice has an important role in the food basket of local people. Since the service level has an important role in the competitive rice markets in this country, so we have to consider the related costs of providing an appropriate service level in our modelling and propose a creative and straightforward solving method to dominate the computational difficulties of this model.

2. Literature review

We summarise the detailed specifications of some of the recent research in the field of stochastic SC network design in Table 1. As can be seen, most of the research that has been undertaken in the field of probabilistic SC network design only considers

Table 1
Literature of stochastic SC network design.

Solution approach	Multi-product problem	Multi-period planning	Facilities with limited capacity	Uncertainty of model: Parameters	Supply	Demand	Network Flow: Backward	Forward	Obj: Other	service level	income	cost: Extra inventory	Shortage cost	Purchase/Manufacturing/Operating/…	Transportation	Ordering	Inventory holding	Quality/Capacity expansion of facilities	Locating	Facility Loc: Plane	Network	Discrete set	Q/C 3th tier	Q/C 2th tier	Q/C 1th tier	#fac 3th tier	#fac 2th tier	#fac 1th tier	FL 3th tier	FL 2th tier	FL 1th tier	# of tiers	Article
ε-constraint method	*		*	*		*			*	*				*	*							*	P	P	P	P	P	P	U	U	P	3	Sabri & Beamon (2000)
Branch & Bound	*	*					*			*				*	*							*	-	-	-	P	U	U	P	U	U	3	Shih (2001)
Mixed-integer linear programming	*	*			*	*								*	*	*						*	U	U	P	U	U	P	U	U	P	3	Tsiakis et al. (2001)
Lagrangian relaxation					*	*								*	*	*						*	-	-	-	P	U	P	P	U	P	3	Daskin et al. (2002)
Heuristic solution methodology		*		*										*	*							*	-	-	-	P	U	P	P	U	P	3	Jayaraman et al. (2003)
Column generation					*	*								*	*	*						*	-	-	-	P	U	P	P	U	P	3	Shen et al. (2003)
ε-constraint method	*	T	*		*	*	*	*	*					*	*							*	P	U	U	P	U	U	P	U	U	3	Guillén et al. (2005)
Branch & Bound	*		*	*	*	*				*				*	*			*				*	-	-	-	U	U	P	U	U	P	4	Listes & Dekker (2005)
Sample average approximation (SAA) and accelerated Benders decomposition algorithm	*	*	*	*	*		*							*	*						*	*	P	P	P	P	P	P	P	U	P	3	Santoso et al. (2005)
Lagrangian relaxation embedded in a branch-and bound procedure	*					*								*	*							*	-	-	-	P	U		P	U		2	Shen (2005)
Weighting method and genetic algorithms					*	*			*					*	*	*						*	-	-	-	P	U	P	P	U	P	3	Shen and Daskin (2005)
Column generation					*	*								*	*	*						*	-	-	-	P	U	P	P	U	P	3	Shu et al. (2005)
Genetic algorithm		T			*									*		*						*	P	P	P	P	P	P	P	P	P	3	Daniel & Rajendran (2006)[#]
Lagrangian relaxation embedded in a branch-and bound procedure					*	*								*	*	*		*				*	-	-	-	P	U		P	U		2	Shen & Qi (2006)
Heuristic solution methodology (Decomposition based approach)		*	*		*	*			*					*								*	-	P	P	-	P	P	-	P	U	2	Goh et al. (2007)
Genetic algorithm-based heuristic	*	*	*			*	*							*	*			*	*			*	-	P	P	P	U	P	P	U	P	3	Ko & Erans (2007)
LINDO	*	*			*									*								*	P	P	P	P	P	P	P	P	P		Leung et al. (2007)
L-shaped method		*		*	*	*	*		*		*	*	*	*								*	-	-	-	U	U	P	U	U	P	3	Listes (2007)
Lagrangian relaxation					*	*				*	*	*	*	*		*						*	-	-	-	P	U		P	U		2	Qi & Shen (2007)
Column generation					*	*									*	*						*	-	P	P	-	P	U	-	U	U	2	Romeijn et al. (2007)
Branch & Bound	*		*		*	*	*			*	*			*	*							*	-	-	-	P	U	U	P	U	U	3	Salema et al. (2007)
Benders decomposition approach with multiple cut					*	*								*	*							*	-	-	-	P	U	U	P	U	U	3	Uster et al. (2007)
Goal attainment technique	*		*	*	*		*							*		*		*	*			*	P	P	P	P	P	P	P	U	P	3	Azaron et al. (2008)
STEP method	*		*				*	*						*	*		*					*	P	P	P	P	P	P	P	P	P	3	Pokharel (2008)
Fuzzy approach	*		*	*		*	*	*						*								*	P	P	P	-	P	P	-	P	P	2	Torabi & Hassini (2008)
Spanning tree-based genetic algorithms		*	*	*		*			*					*		*						*	P	P	P	U	U	P	U	U	P	3	Xu et al. (2008)
ε-constraint method		*			*		*	*		*	*			*	*							*	P	P	P	P	P	P	U	U	P	3	You & Grossmann (2008)[#]
STEM method	*	*	*	*	*		*							*		*		*	*			*	P	P	P	P	P	P	P	U	P	3	Azaron et al. (2009)
Fuzzy approach	*	*	*	*	*		*							*	*		*					*	P	P	P	P	P	P	P	P	P	3	Peidro et al. (2009)
Standard Branch & Bound techniques	*	*	*				*			*				*	*							*	P	P	P	P	P	P	-	U	U	2	Salema et al. (2009)
SAA and Dual decomposition	*	*		*	*	*	*							*	*	*					*	*	P	P	P	P	P	P	P	U	U	3	Schütz et al. (2009)
Two-Phase Ant Colony Algorithm		*	*			*								*	*							*	P	P	P	P	P	P	P	P	P	n	Wang (2009)
Decomposition algorithm					*		*							*		*						*	-	-	-	P	U	P	P	U	P	3	You & Grossmann (2009)
GAMS	*	*	*		*				*					*	*		*					*	-	-	-	-	-	-	P	P	P	3	Bassett & Gardner (2010)
L-shaped algorithm within an ε-optimality framework		*			*		*									*						*	-	P	P	-	U	P	-	U	P	2	Cardona-Valdés et al. (2010)
ε-constraint method	*	*	*		*		*	*		*				*	*							*	-	-	-	P	P	P	U	U	U	3	Franca et al. (2010)[#]
Game Theory				*		*			*				*		*							*	P	P	P	P	P	P	P	P	P	3	Li et al. (2010)
ε-constraint method		*			*		*	*		*					*							*	-	-	-	U	P	-	U	P	-	2	Olivares et al. (2010)
Modified Benders decomposition algorithm		*	*		*		*	*				*		*		*						*	P	P	P	P	P	P	P	P	P	3	Osman & Demirli (2010)
Heuristic method		*			*		*							*	*	*					*	*	P	P	P	U	U	U	P	P	P	n	Pan and Nagi (2010)
Fuzzy approach		*	*			*		*	*	*				*	*							*	-	-	-	P	P	P	P	U	P	6	Pishvaee & Torabi (2010)[#]
CPLEX	*	*	*	*			*			*				*	*							*	P	P	P	P	P	P	P	P	P		Zanjani et al. (2010)
Standard branch-and-bound techniques	*		*				*							*	*		*					*	-	-	-	P	P	P	U	U	P	3	Georgiadis et al. (2011)
L-shaped based solution algorithm	*		*		*	*						*		*	*							*	-	-	-	P	P	P	P	P	P	3	Lin & Wang (2011)
Standard Branch & Bound techniques	*	*	*			*								*	*		*					*	U	U	P	U	U	P	U	U	P	4	Longinidis & Georgiadis (2011)[#]
CPLEX		*	*		*	*	*							*	*							*	-	-	-	U	U	U	U	U	P	5	Pishvaee et al. (2011)[#]
CPLEX			*			*						*	*									*	-	P	P	-	P	P	-	P	P	2	Sawik (2011)
Normalized normal constraint method	*		*				*		*					*								*	-	-	-	P	U	P	P	U	P	3	Wang et al. (2011)
Using regression to linearize the NILP model and solve it by commercial optimization softwares	*		*	*	*		*		*	*	*	*	*	*	*		*	*	*			*	U	U	P	U	U	P	U	U	P	3	*This paper*
# The considered chain of this paper has more than three tiers																																	
U means unknown part																																	
P means predetermined part																																	

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demand-side uncertainty (Daskin et al., 2002; Shen and Qi, 2006; Shen and Daskin, 2005; Shu et al., 2005; Shen et al., 2003; Salema et al., 2007; Tsiakis et al., 2001; Leung et al., 2007; Romeijn et al., 2007; Pan and Nagi, 2010; Cardona-Valdés et al., 2011).

Santoso et al. (2005) considered a three-tiered multi-product SC network comprising of supplier, processing facilities and customers. Processing facilities include manufacturing centres, finishing facilities and warehouses. Their model determined the configuration of an SC which consisted of deciding which of the processing centres to build (major configuration decisions) and which processing and finishing machines to procure (minor configuration decisions). The operational decisions of this model consisted of routing the flow of the product from the suppliers to the customers. The objective function of their model consists of minimising the total investment and operational costs. They assumed that processing/transportation costs, demands, supplies, and capacities are stochastic parameters with known joint distribution functions. They integrated a sample average approximation (SAA) scheme with an accelerated Benders decomposition algorithm to quickly compute high quality solutions to large-scale stochastic SC design problems for a large number of scenarios.

Azaron et al. (2008) developed a multi-objective stochastic programming approach for SC design under uncertainty. Demands of markets, supplies of suppliers, processing, transportation and capacity expansion costs were all considered as uncertain parameters. Their multi-objective model included (I) minimisation of the sum of current investment costs and the expected future processing, transportation, shortage and capacity expansion costs, (II) minimisation of the variance of the total cost and (III) minimisation of the financial risk or the probability of not meeting a specific budget. They used the goal attainment technique to obtain Pareto-optimal solutions. Azaron et al. (2009) developed a different method for solving the model presented by Azaron et al. (2008).

Peidro et al. (2009) introduced a novel tactical SC planning model by integrating procurement, production and distribution planning activities into a multi-echelon, multi-product, multi-level and multi-period SC network. The model was formulated as a fuzzy mixed-integer linear programming (FMILP) model. The objective function was to minimise the total cost that is formed by the production costs with the differentiation between regular and overtime production. The costs corresponding to idleness, raw material acquisition, inventory holding and transport were also considered and Peidro et al. (2009) defined an approach to transform the FMILP model into an equivalent auxiliary crisp MILP model.

Schütz et al. (2009) formulated the multi-commodity SC design problem as a two-stage stochastic model. The first decision stage of this two-stage model consisted of strategic location decisions, whereas the second stage consists of operational decisions. The objective was to minimise the sum of investment costs and expected operating costs of the SC. In this problem, short-term uncertainty was considered as well as long-term uncertainty in demand. They also examined how the use of a stochastic model influences the decisions as compared to a deterministic model. They solved the problem by SAA in combination with dual decomposition.

Demand uncertainty, which reflects the uncertainty of customer demand for a product, can change the manufacturing operations and consequently the procurement plan of production facilities. Delays or inabilities in the procurement process, called supply disruptions, lead to extra inventory and lost sales in retailing outlets. Therefore considering both demand-side uncertainty and supply-side disruption seem necessary in the SC network design stage. Lin and Wang (2011) studied the SC

network design problem under supply and demand uncertainty in a system with both build-to-order and build-to-stock manufacturing processes. They emphasised the manufacturing postponement with downward substitution mitigation and discussed the procurement operations plan in the 1st stage under supply-side disruptions and the recourse problem in the 2nd stage under a demand scenario. They then integrated the procurement, manufacturing and distribution plans and formulated the build to order the SC network design with supply disruptions and demand uncertainty in its deterministic equivalent formulation. An L-shaped decomposition with an additional decomposition step in the master problem was proposed for solving this model.

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Considering uncertainties and disruptions helps companies to supply a better service level to their customers. Service level is one of the most important competitive factors in today's fierce markets (Bernstein and Federgruen, 2004a,b, 2007; Tsay and Agrawal, 2000; Boyaci and Gallego, 2005; Xiao and Yang, 2008). Increasing the inventory level of the retailers improves the service level, but also increases the risk of unsold products. The service level is usually considered by unit extra and shortage costs in the inventory systems. However, probabilistic features of these costs and their computational complexities in the mathematical models mean that these important cost components have been ignored from the set of operational costs considered previously in network design problems. We include these cost components in our model.

The contribution of this paper in comparison to the existing literature is as follows:

- Consideration of demand-side uncertainty in the SC network design problem.
- Consideration of supply-side uncertainty, which includes possible disruptions in manufacturers, distribution centres, and the connecting links of the network.
- Incorporation of service level and its related costs in the model.
- Modelling the problem mathematically. We use a completely different way to model the SC network design problem: instead of defining distinct flow variables between the facilities of sequential echelons, we define the concept of potential paths in the chain; the concept of paths helps us to consider the different uncertainties simultaneously.
- Linearisation of the model using a piecewise linear transformation.
- Implementation of the model in a real-life case study from the agri-food industry.

These contributions have been partially demonstrated in the last row of Table 1 in terms of the model's specification and the literature classification.

This paper is organised as follows: In Section 3 we will describe the problem to be studied in this paper in full detail; Section 4 presents the mathematical model of the problem; Section 5 contains the method of linearising and solving the model and in Section 6 we apply the presented model and solving method for a real-life case study. Section 7 also includes a sensitivity analysis of the model. Section 8 discusses computational efficiency of the developed technique. Finally, we conclude the paper in Section 9 and suggest several areas for future research.

3. Problem description

In this paper we consider a model for multi-product SC network design in markets under stochastic demands and with a disruption probability for manufacturers. We consider both demand- and supply-side uncertainties in this problem. This chain consists of: (a) several manufacturers, (b) distribution centres (DCs) and (c) retailing outlets respectively in its first, second and third echelons. Each retailer of this chain orders its goods early (as long as a given time unit) before the beginning of each period, which is called its lead time. DCs integrate the received orders of a retailer and pass them to the manufacturers with limited capacity. Produced goods of manufacturers are delivered to retailers through DCs. DCs are in charge of the packaging and labelling of the goods of the chain.

There are some available manufacturers with specific costs and reliability characteristics which can be selected to use as the facilities of the first echelon. The DCs and retailers of this chain will be located in a number of pre-determined candidate locations. In this problem, we are to determine the optimal network structure of this chain in a manner which maximises its whole profit, which is calculated as the difference between its income and the total cost. The total cost of the chain is the sum of the current investment costs and expected future shortages, production and transportation costs. Various decisions, namely the number, location, and capacity of entities in each tier and material/product flow throughout the network are made.

As stated before, the exact demand realisation at the retailer level is not known in advance, thus demand will be considered as a random variable with a known distribution function. Supply-side uncertainty is described through various scenarios.

We use a new method for designing the network structure of the chain. Instead of defining flow variables among the facilities of the chain's consequent echelons, we define the concept of a path in the chain's potential network. Each path starts from a facility of the first echelon and by passing through the intermediate facilities ends in a facility of the last echelon. Each of these potential paths of the chain can have a disorderliness probability. This disorderliness can be due to the impairment of a path's facilities or disruption of connecting links of the path among its facilities. This kind of modelling helps us to consider different types of possible breaks in the chain: (I) impairment of facilities, and (II) disruption of connecting links. Each set of simultaneous breakable potential paths is called a scenario and is associated with a finite probability of occurrence. This kind of formulation was originally based upon the cut-set concept in the reliability theory of networks.

There are several products in a new chain and each market is served by one retailer. Defining the scenarios and calculating their probability of occurrence will be described further later.

4. Model

In order to formulate the mathematical model the notation described below is utilised.

4.1. Sets and index

$i \in I$	set of potential manufacturers available for use by the chain
$j \in J$	set of potential locations for DCs
$m \in M$	set of markets and set of potential locations for locating retailers. Both are represented by one set because in each market there is only one candidate location for locating retailing facilities. Selecting each of these candidate locations also implies the willingness of the chain to supply its products to the market of that retailer
$p \in P$	set of products
$s \in S$	set of scenarios
$t \in T$	set of potential routes (starting from manufacturers, passing through DCs and ending at retailers) in the network structure of the chain with a unit handling cost less than the price of the goods in the markets
T^i	set of potential routes started from manufacturer i
T^j	set of potential routes passing through DC j
T^m	set of potential routes ended in retailer m

4.2. Parameters

B_j	fixed cost of opening a DC at candidate location j
C_m	fixed cost of opening a retailer at candidate location m
D_{mp}	demand of product p in market m
$E(D_{mp})$	expected demand of product p in market m
$F(D_{mp})$	cumulative distribution function of variable D_{mp}
cap_{ip}	capacity of manufacturer i for product p
$budg$	certain budget limit available for opening facilities in the chain
P_{mp}	price of one unit of product p in market m
SV_{mp}	salvage value of one unit of the residual product p in market m
SC_{mp}	shortage cost of one unit of unmet demand for product p in market m
M_{pt}	handling cost of a unit of product p in route t which can be calculated as the sum of: (a) unit production cost of the path's manufacturer, (b) unit transportation cost from the manufacturer to the DC of that path, (c) unit inventory holding cost of the path's DC, (d) unit transportation cost from the DC to the retailer of that path and (e) unit handling cost of the path's retailer
Pr_s	probability of the occurrence of scenario s; this corresponds to each type of disruption on the nodes (the manufacturers and the DCs) and their connecting links which disconnects the path
BP_{st}	binary parameter equal to 1 if route t is useable in scenario s and 0 otherwise

$a^+ = max\{a, 0\}.$

$a \wedge b = min\{a, b\}.$

4.3. Decision variables

W_j	binary variable equal to 1 if a DC is located in candidate location j and equal to 0 otherwise
V_m	binary variable equal to 1 if a retailer is located in candidate location m and equal to 0 otherwise
X_{pst}	amount of product p flows in scenario s through route t

4.4. Mathematical model

The mathematical formulation of the addressed problem is described in the following lines. First we compute the profit of each market in scenario s.

$$profit\ of\ market\ m = \sum_{p \in P} P_{mp} \cdot \left(\sum_{t \in T^m} X_{pst} \Lambda D_{mp} \right) + \sum_{p \in P} SV_{mp}$$
$$\cdot \left(\sum_{t \in T^m} X_{pst} - D_{mp} \right)^+ - \sum_{p \in P} SC_{mp}$$
$$\cdot \left(D_{mp} - \sum_{t \in T^m} X_{pst} \right)^+ - \sum_{p \in P} \sum_{t \in T^m} M_{pt} \cdot X_{pst} \qquad (1)$$

The first term in Eq. (1) represents the income of the SC in market m which is equal to the price of one unit multiplied by the amount of sold product. The second term exhibits the salvage value of extra product and the third term explains the shortage cost for unmet demand. The last term describes the handling cost of the product along the routes. We are exploiting the newsboy problem style for calculating the inventory managing systems of the retailers: each retailer anticipates its demand for the next period of time and orders it at an appropriate time before the beginning of the next period; unsold products and lost sales at the end of the period lead to the salvage value and shortage cost respectively. In this way, demand uncertainty is considered in the model. We simplify this function by using the following equations:

$$\left(\sum_{t \in T^m} X_{pst} \Lambda D_{mp} \right) = \sum_{t \in T^m} X_{pst} - \left(\sum_{t \in T^m} X_{pst} - D_{mp} \right)^+ \qquad (2)$$

where

$$\left(\sum_{t \in T^m} X_{pst} - D_{mp} \right)^+ = \int_0^{\sum_{t \in T^m} X_{pst}} F(D_{mp}) \cdot dD_{mp} \qquad (3)$$

$$\left(\sum_{t \in T^m} X_{pst} - D_{mp} \right)^+ - \left(D_{mp} - \sum_{t \in T^m} X_{pst} \right)^+ = \sum_{t \in T^m} X_{pst} - E(D_{mp}) \qquad (4)$$

By substituting the above equations, the profit function of each market can be simplified as follows:

$$profit\ of\ market\ m = \sum_{p \in P} (P_{mp} + SC_{mp}) \cdot \sum_{t \in T^m} X_{pst} - \sum_{p \in P} (P_{mp} + SC_{mp} - SV_{mp})$$
$$\cdot \int_0^{\sum_{t \in T^m} X_{pst}} F(D_{mp}) \cdot dD_{mp} - \sum_{p \in P} SC_{mp} \cdot E(D_{mp})$$
$$\cdot V_m - \sum_{p \in P} \sum_{t \in T^m} M_{pt} \cdot X_{pst} \qquad (5)$$

Since we want to consider the supply disruption in this model, we define several scenarios for the chain. In each scenario, some of the potential routes of the chain are unusable due to the facilities' and connection links' disorderliness. For example, if n different impairments of facilities and m different disruptions of links are possible

situations, then 2^{n+m} different scenarios are definable in the model. In this way, the expected profit of the whole chain can be calculated easily. Now the mathematical model of the addressed problem can be presented as follows:

$$Max\ f = \sum_{s \in S} pr_s \left(\sum_{m \in M} \sum_{p \in P} (P_{mp} + SC_{mp}) \sum_{t \in T^m} X_{pst} \right.$$
$$- \sum_{m \in M} \sum_{p \in P} (P_{mp} + SC_{mp} - SV_{mp}) \int_o^{\sum_{t \in T^m} X_{pst}} F(D_{mp})$$
$$\left. - \sum_{m \in M} \sum_{p \in P} SC_{mp} \cdot E(D_{mp}) \cdot V_m - \sum_{p \in P} \sum_{t \in T^m} M_{pt} \cdot X_{pst} \right)$$
$$- \sum_{j \in J} B_J \cdot W_J - \sum_{m \in M} C_m \cdot V_m \qquad (6)$$

Subject to:

$$\sum_{p \in P} X_{pst} \leqslant N \cdot PB_{st} \quad \forall s \in S, \forall\, t \in T \qquad (7)$$

$$\sum_{p \in P} \sum_{s \in S} \sum_{t \in T} X_{pst} \leqslant N \cdot W_j \quad \forall\, j \in J \qquad (8)$$

$$\sum_{p \in P} \sum_{s \in S} \sum_{t \in T} X_{pst} \leqslant N \cdot V_m \quad \forall\, m \in M \qquad (9)$$

$$\sum_{j \in J} B_j \cdot W_j + \sum_{m \in M} C_m \cdot V_m \leqslant bug \qquad (10)$$

$$\sum_{t \in T^i} X_{pst} \leqslant cap_{ip} \quad \forall\, i \in I, \forall p \in P, \forall s \in S \qquad (11)$$

$$X_{pst} \geqslant 0 \quad \forall p \in P, \forall s \in S, \forall t \in T \qquad (12)$$

$$W_j, V_m \in \{0, 1\} \quad \forall j \in J, \forall m \in M \qquad (13)$$

Constraint (7) explains that in each scenario shipment is only possible along the routes which are usable in that scenario (N is a big constant). Constraints (8) and (9) impose where facilities along the active routes of the chain should be located. According to constraint (10), the investment cost of the chain is less than the predetermined budget limit. Constraint (11) enforces the capacity constraints of the manufacturers. Constraints set (12) is for continuous non-negative variables and binary variables are required in constraint (13).

4.5. A robust mathematical model

Note that the previous model tries to maximise the expected profit of the chain. However, after realisation of markets' demand and the supply scenario, the profit of the chain can be very different from the value of this model's objective function. To reduce the amount of this bias, here we try to make our model more robust. Several different concepts related to robustness have been previously introduced in the literature, such as solution robustness and model robustness (Mulvey et al., 1995). In solution robustness, the optimal solution of the model remains "close" to optimum for any occurrence of the scenario, but in model robustness, the solution of the model remains "almost" feasible for any occurrence of the scenario. In the robust optimisation model, there are two types of variables: design and control variables. Design variables are decided before the realisation of stochastic parameters and cannot be adjusted after the realisation. Control variables are subject to adjustment when one specific occurrence of uncertain parameters is realised (Pan and Nagi, 2010).

We consider solution robustness in our model. We have added a term to represent model robustness that is weighted by λ. A high variance for benefit function means that the solution is a high-risk decision. In other words, a small change of the values of uncertainties can cause a big change in the value of the measure function. Thus we added this term to the model with a negative sign.

The amount of λ depends on the decision maker. Some decision makers are risk-neutral and select a lower amount for λ and some are risk-averse and select a higher amount for λ. We undertook a comprehensive sensitivity analysis with respect to λ, which is presented at the end of this paper to provide a wide range of choices for the decision maker to select. In fact we are faced with two objective functions: maximising profit and minimising risk and all the produced solutions are non-dominated and the decision maker can select among them according to his/her situation. He/she should decide how much profit he/she are willing to sacrifice for a robust design. However, when the probability of the normal condition is low, then selecting high amounts for λ seems more logical and vice versa.

Our perception is that some SCs, e.g. SCs within the computer industry, accept higher risks, whilst some SCs e.g. SCs in consumer goods, are fairly risk averse. This attitude is due to the nature of the product, demand stability and competition level. For example, competition within the computer industry is severe as the product is high tech; in contrast the market is not as stable as in consumer products. Therefore, logically an SC network designer for a computer SC should consider a higher λ compared to a consumer goods SC.

Such a decision should be made by an SC network steering committee which includes representatives from partners of the SC with equal and unequal power in decision making.

The robust mathematical model of the addressed problem can be presented as follows:

$$
\begin{aligned}
Max\ f = \sum_{s\in S} pr_s &\left(\sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp})\sum_{t\in T^m}X_{pst} \right.\\
&- \sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp}-SV_{mp})\int_0^{\sum_{t\in T^m}X_{pst}}F(D_{mp}) \\
&\left. - \sum_{m\in M}\sum_{p\in P}SC_{mp}\cdot E(D_{mp})\cdot V_m - \sum_{p\in P}\sum_{t\in T^m}M_{pt}\cdot X_{pst} \right)\\
&- \lambda\cdot\sum_{s\in S}pr_s\left[\sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp})\sum_{t\in T^m}X_{pst}\right.\\
&- \sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp}-SV_{mp})\int_0^{\sum_{t\in T^m}X_{pst}}F(D_{mp})\\
&- \sum_{m\in M}\sum_{p\in P}SC_{mp}\cdot E(D_{mp})\cdot V_m - \sum_{p\in P}\sum_{t\in T^m}M_{pt}\cdot X_{pst}\\
&- \sum_{s\in S}pr_s\cdot\left(\sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp})\sum_{t\in T^m}X_{pst}\right.\\
&- \sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp}-SV_{mp})\int_0^{\sum_{t\in T^m}X_{pst}}F(D_{mp})\\
&\left.\left.- \sum_{m\in M}\sum_{p\in P}SC_{mp}\cdot E(D_{mp})\cdot V_m - \sum_{p\in P}\sum_{t\in T^m}M_{pt}\cdot X_{pst}\right)\right]^2\\
&- \sum_{j\in J}B_J\cdot W_J - \sum_{m\in M}C_m\cdot V_m
\end{aligned}
\tag{14}
$$

λ denotes the weight placed on solution variance. As can be seen, there is a quadratic term in Eq. (14). Yu and Li (2000) discussed the computational effort required due to a quadratic term and proposed an absolute deviation instead of the quadratic term, which is shown as follows:

$$
\begin{aligned}
MAX\ f = \sum_{s\in S} pr_s &\left(\sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp})\sum_{t\in T^m}X_{pst}\right]\\
&- \sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp}-SV_{mp})\int_0^{\sum_{t\in T^m}X_{pst}}F(D_{mp})
\end{aligned}
$$

$$
\begin{aligned}
&\left.- \sum_{m\in M}\sum_{p\in P}SC_{mp}\cdot E(D_{mp})\cdot V_m - \sum_{p\in P}\sum_{t\in T^m}M_{pt}\cdot X_{pst}\right)\\
&- \lambda\cdot\sum_{s\in S}pr_s\cdot\left|\sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp})\sum_{t\in T^m}X_{pst}\right.\\
&- \sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp}-SV_{mp})\int_0^{\sum_{t\in T^m}X_{pst}}F(D_{mp})\\
&- \sum_{m\in M}\sum_{p\in P}SC_{mp}\cdot E(D_{mp})\cdot V_m - \sum_{p\in P}\sum_{t\in T^m}M_{pt}\cdot X_{pst}\\
&- \sum_{s\in S}pr_s\cdot\left(\sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp})\sum_{t\in T^m}X_{pst}\right.\\
&- \sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp}-SV_{mp})\int_0^{\sum_{t\in T^m}X_{pst}}F(D_{mp})\\
&\left.\left.- \sum_{m\in M}\sum_{p\in P}SC_{mp}\cdot E(D_{mp})\cdot V_m - \sum_{p\in P}\sum_{t\in T^m}M_{pt}\cdot X_{pst}\right)\right|\\
&- \sum_{j\in J}B_J\cdot W_J - \sum_{m\in M}C_m\cdot V_m
\end{aligned}
\tag{15}
$$

Although Eq. (15) includes an absolute value, two additional variables, Q_s^+ and Q_s^- can be introduced to linearise the function (Wagner, 1975). The justification for the equivalence is a simple change of variables. The absolute value is substituted by $\lambda\sum_{s\in S}pr_s(Q_s^+ + Q_s^-)$. Q_s^+ is interpreted as the amount by which the term is greater than zero, while Q_s^- as the amount by which the term is less than zero. Through this substitution, Eq. (15) is transformed as follows:

$$
\begin{aligned}
MAX\ f = \sum_{s\in S} pr_s &\left(\sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp})\sum_{t\in T^m}X_{pst}\right.\\
&- \sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp}-SV_{mp})\int_0^{\sum_{t\in T^m}X_{pst}}F(D_{mp})\\
&\left.- \sum_{m\in M}\sum_{p\in P}SC_{mp}\cdot E(D_{mp})\cdot V_m - \sum_{p\in P}\sum_{t\in T^m}M_{pt}\cdot X_{pst}\right)\\
&- \lambda\cdot\sum_{s\in S}pr_s\cdot(Q_s^+ + Q_s^-) - \sum_{j\in J}B_J\cdot W_J - \sum_{m\in M}C_m\cdot V_m
\end{aligned}
\tag{16}
$$

And the following constraint is added to the model:

$$
\begin{aligned}
&\sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp})\sum_{t\in T^m}X_{pst} - \sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp}-SV_{mp})\\
&\times\int_0^{\sum_{t\in T^m}X_{pst}}F(D_{mp}) - \sum_{m\in M}\sum_{p\in P}SC_{mp}\cdot E(D_{mp})\cdot V_m - \sum_{p\in P}\sum_{t\in T^m}M_{pt}\cdot X_{pst}\\
&- \sum_{s\in S}pr_s\cdot\left(\sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp})\sum_{t\in T^m}X_{pst} - \sum_{m\in M}\sum_{p\in P}(P_{mp}+SC_{mp}\right.\\
&\left.- SV_{mp})\int_0^{\sum_{t\in T^m}X_{pst}}F(D_{mp}) - \sum_{m\in M}\sum_{p\in P}SC_{mp}\cdot E(D_{mp})\cdot V_m\right.\\
&\left.- \sum_{p\in P}\sum_{t\in T^m}M_{pt}\cdot X_{pst}\right) = Q_s^+ - Q_s^- \quad (\forall\ s\in S)
\end{aligned}
\tag{17}
$$

$$Q_s^+ \cdot Q_s^- = 0 \quad (\forall s\in S) \tag{18}$$

$$Q_s^+, Q_s^- \geqslant 0 \quad (\forall s\in S) \tag{19}$$

5. Solution method

The proposed model in the previous section is a mixed integer nonlinear (MINL) model with a nonlinear objective function and $|I| \times |P| \times |S| + |S| + |T| + |J| + |K| + 1$ linear constraints. It also contains $|I| + |J|$ and $|P| \times |S| \times |T|$ binary and continuous variables, respectively.

This model is seriously nonlinear due to the integral of cumulative distribution function in objective function. According to the type of distribution function of the markets, the form of this term can be different and for some cases, such as a normal distribution, calculating the amount of this term is not straightforward and it cannot be a closed form. Therefore, we need to use a numerical technique to figure out the form of this function and we used a piecewise linear transformation breaking the range of this nonlinear function into several intervals and substituted the convex function with straight lines (with a unique constant and coefficient) in each interval. For the uniform and normal distribution functions which are more common in the demand of markets, the approximations used can be seen in Figs. 1 and 2. In each of these figures, the ranges of the functions have been broken into five and four intervals respectively.

This approximation converts our nonlinear model into a linear one. Due to the approximation in the piecewise linear model of a nonlinear model, we reached an optimal solution for the linearised model; however, this linearised model is not exactly the same as the nonlinear model. On the other hand, since the nonlinear model is convex we have reached a solution which is close to the global optimum but there is an error with it. By increasing the number of intervals, the error decreases.

Therefore, we considered several approximation intervals for each market and defined one index and one binary variable for interval selection.

5.1. Index

$n^m \in N^m$	set of approximation intervals for each market m

5.2. Parameter

$coeff_{n^m}$	coefficient for interval n^m of market m
$const_{n^m}$	constant for interval n^m of market m
$lower_{n^m}$	lower bound for interval n^m of market m
$upper_{n^m}$	upper bound for interval n^m of market m

5.3. Variable

$Y_{n^m ps}$	binary variable equal to 1 if interval n^m is selected in scenario s for product p in market m and 0 otherwise
$X_{n^m pst}$	amount of product p flows in scenario s through route t in interval n^m

After defining the above notations, the model is linearised as follows:

Fig. 1. Approximation of accumulative standard normal distribution function.

$$MAX\ f = \sum_{s \in S} pr_s \Bigg(\sum_{m \in M} \sum_{p \in P} (P_{mp} + SC_{mp}) \sum_{t \in T^m} \sum_{n^m \in N^m} X_{n^m pst}$$
$$- \sum_{m \in M} \sum_{p \in P} (P_{mp} + SC_{mp} - SV_{mp})$$
$$\times \sum_{n^m \in N^m} (coeff_{n^m} \cdot \sum_{t \in T^m} X_{n^m pst} + const_{n^m} \cdot Y_{n^m ps})$$
$$- \sum_{m \in M} \sum_{p \in P} SC_{mp} \cdot E(D_{mp}) \cdot V_m - \sum_{p \in P} \sum_{t \in T^m} M_{pt} \cdot \sum_{n^m \in N^m} X_{n^m pst} \Bigg)$$
$$- \sum_{j \in J} B_J \cdot W_J - \sum_{m \in M} C_m \cdot V_m \tag{20}$$

$$\sum_{n^m \in N^m} \sum_{p \in P} \sum_{s \in S} \sum_{t \in T^j} X_{n^m pst} \leqslant M \cdot W_j \quad (\forall j \in J) \tag{21}$$

$$\sum_{n^m \in N^m} \sum_{p \in P} \sum_{s \in S} \sum_{t \in T^m} X_{n^m pst} \leqslant M \cdot V_m \quad (\forall\ m \in M) \tag{22}$$

$$\sum_{n^m \in N^m} \sum_{t \in T^i} X_{n^m pst} \leqslant cap_{ip} \quad (\forall i \in I, \forall\ p \in P, \forall\ s \in S) \tag{23}$$

$$\sum_{j \in J} B_J \cdot W_j + \sum_{m \in M} C_m \cdot V_m \leqslant bug \tag{24}$$

$$\sum_{n^m \in N^m} Y_{n^m ps} = 1 \quad (\forall\ p \in P, \forall\ s \in S) \tag{25}$$

$$\sum_{t \in T^m} X_{n^m pst} \leqslant upper_{n^m} \cdot Y_{n^m ps} \quad (\forall\ n^m \in N^m, \forall\ s \in S, \forall\ p \in P) \tag{26}$$
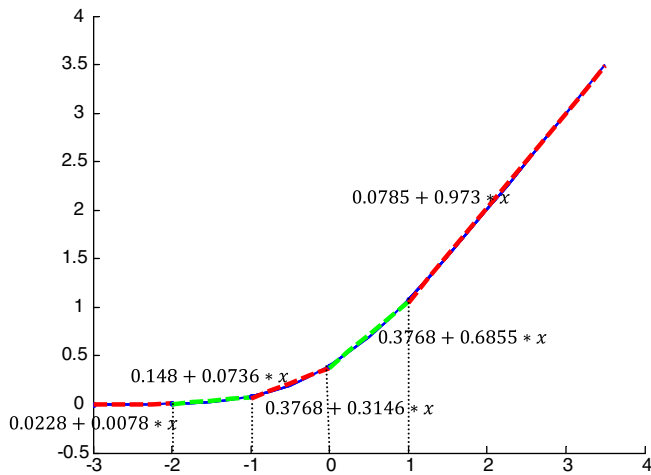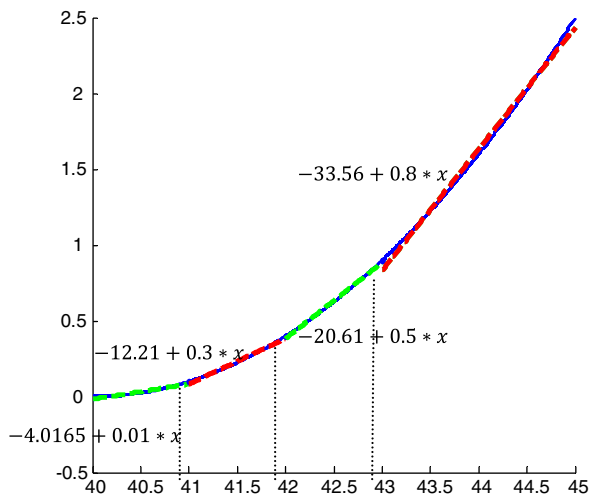

Fig. 2. Approximation of accumulative uniform distribution function.

$$\sum_{t \in T^m} X_{n^m pst} \leqslant lower_{n^m} \cdot Y_{n^m ps} \quad (\forall\, n^m \in N^m, \forall\, s \in S, \forall\, p \in P) \tag{27}$$

$$X_{n^m pst} \geqslant 0 \quad (\forall\, n^m \in N^m, \forall\, p \in P, \forall\, s \in S, \forall\, t \in T)) \tag{28}$$

$$Y_{n^m ps} \in \{0,1\} \quad (\forall\, n^m \in N^m, \forall\, p \in P, \forall\, s \in S) \tag{29}$$

$$W_j, V_m \in \{0,1\} \quad (\forall\, j \in J, \forall\, m \in M) \tag{30}$$

Eq. (25) indicates that only one interval should be selected for each market, Eqs. (26) and (27) determine boundaries for every interval, and other constraints are explained as before. Hence, the robust model can be modified as follows:

$$
\begin{aligned}
MAX\ f = \sum_{s \in S} pr_s \Bigg(& \sum_{m \in M} \sum_{p \in P} (P_{mp} + SC_{mp}) \sum_{t \in T^m} \sum_{n^m \in N^m} X_{n^m pst} \\
& - \sum_{m \in M} \sum_{p \in P} (P_{mp} + SC_{mp} - SV_{mp}) \\
& \sum_{n^m \in N^m} \left(coeff_{n^m} \cdot \sum_{t \in T^m} X_{n^m pst} + const_{n^m} \cdot Y_{n^m ps} \right) \\
& - \sum_{m \in M} \sum_{p \in P} SC_{mp} \cdot E(D_{mp}) - \sum_{p \in P} \sum_{t \in T^m} M_{pt} \cdot \sum_{n^m \in N^m} X_{n^m pst} \Bigg) \\
& - \lambda \cdot \sum_{s \in S} pr_s \cdot (Q_s^+ + Q_s^-) - \sum_{j \in J} B_j \cdot W_J - \sum_{m \in M} C_m \cdot V_m
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
& \sum_{m \in M} \sum_{p \in P} (P_{mp} + SC_{mp}) \sum_{t \in T^m} \sum_{n^m \in N^m} X_{n^m pst} - \sum_{m \in M} \sum_{p \in P} (P_{mp} + SC_{mp} - SV_{mp}) \\
& \quad \times \sum_{n^m \in N^m} \left(coeff_{n^m} \cdot \sum_{t \in T^m} X_{n^m pst} + const_{n^m} \cdot Y_{n^m ps} \right) \\
& \quad - \sum_{m \in M} \sum_{p \in P} SC_{mp} \cdot E(D_{mp}) - \sum_{p \in P} \sum_{t \in T^m} M_{pt} \cdot \sum_{n^m \in N^m} X_{n^m pst} \\
& \quad - \sum_{s \in S} pr_s \Bigg(\sum_{m \in M} \sum_{p \in P} (P_{mp} + SC_{mp}) \sum_{t \in T^m} \sum_{n^m \in N^m} X_{n^m pst} \\
& \quad - \sum_{m \in M} \sum_{p \in P} (P_{mp} + SC_{mp} - SV_{mp}) \\
& \quad \times \sum_{n^m \in N^m} \left(coeff_{n^m} \cdot \sum_{t \in T^m} X_{n^m pst} + const_{n^m} \cdot Y_{n^m ps} \right) \\
& \quad - \sum_{m \in M} \sum_{p \in P} SC_{mp} \cdot E(D_{mp}) - \sum_{p \in P} \sum_{t \in T^m} M_{pt} \cdot \sum_{n^m \in N^m} X_{n^m pst} \Bigg) \\
& = Q_s^+ - Q_s^- (\forall\, s \in S)
\end{aligned} \tag{32}
$$

$$Q_s^+, Q_s^- \geqslant 0 \quad (\forall\, s \in S) \tag{33}$$

Constraints (21)–(30) should be added to the above model. Since this model is a linear one, we may ignore $Q_s^+ \cdot Q_s^- = 0 (\forall s \in S)$ constraints which are nonlinear. As this model is based on linear programming theory, then obviously at least one of Q_s^+ and Q_s^- is going to be 0 in the optimal solution.

6. A real-life case study

The motivation for this problem, model and solution technique was a real-life case in the rice industry of a country in the Middle East. Rice is one of the most important essential goods in Asia and particularly within the Middle East. Most of the rice of the world is produced and consumed in this area and it has an important role in the food basket of local people. Consequently, the rice market is highly controlled by the governments of these countries.

In the case country, previously most of the harvested rice was purchased with a guaranteed minimum price (GMP) by the government, stored and gradually distributed in the country over the year among consumers; the government had complete control of the rice market. Recently, after implementing a privatisation scheme (PS) – also known as devolution – the role of the government in this industry decreased and all entities of the rice SC were transferred to the private sector. However, the price of this good is still controlled by local governments and it is sold with a fairly uniform price in all states of the country. In this country, paddy (unprocessed rice) is mostly cultivated in three states for a short season but it is consumed by all states for a whole year. In the harvested seasons, paddy milling factories buy the paddy from the farmers and store it in their warehouses, finally processing it gradually during the year. White rice is produced in these factories after passing through drying, husking, whitening, polishing and grading stages.

Since rice has a good and steady state demand in this country, after privatisation many companies became interested in this market and started to design their own rice SC networks. The model of this paper was developed for designing the rice SC network for the TAR-C Company. This company decided to choose one big factory in each of the rice-producing states as candidate producers of its
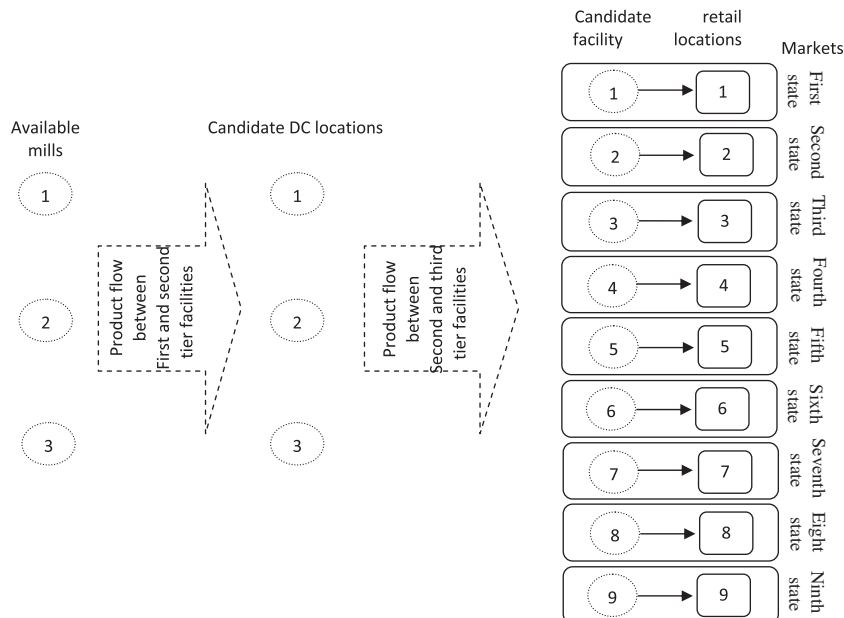

Fig. 3. Potential network structure of the TAR-C rice SC.

chain (three candidate producers in total). Since the processing facilities and equipments of these factories are mostly old, failure or breaking down of these facilities is a frequent event in these factories and TAR-C tried to select facilities with lower disruption probabilities.

TAR-C has three candidate DC locations. These DCs combine the orders received from different retailers and place orders with the producers. This company selected nine states as target markets and selected a candidate retailing facility location in each state for supplying its product. Fig. 3 illustrates the potential network structure of the rice SC for the TAR-C SC.

A description of the potential network structure of TAR-C is as follows:

According to the cost components, there are thirteen feasible routes in this potential network with a unit handling cost less than the price of the product in the markets: $T = \{t_{111}, t_{113}, t_{125}, t_{212}, t_{224}, - = \{t_{111}, t_{113}, t_{125}, t_{212}, t_{224}, t_{227}, t_{236}, t_{313}, t_{325}, t_{326}, t_{337}, t_{338}, t_{339}\}$. The handling costs of these paths are 9.25, 9, 9, 8.5, 8.75, 8.25, 8, 8.5, 8.75, 8.25, 7.5, 7.75 and 8 respectively. The distribution functions of demand in the related markets are considered to follow a normal distribution function:

$D_1 \sim N(E(D_1) = 300, \sigma(D_1) = 5), D_2 \sim N(E(D_2) = 220, \sigma(D_2) = 5),$
$D_3 \sim N(E(D_3) = 500, \sigma(D_3) = 10), D_4 \sim N(E(D_4) = 200, \sigma(D_4) = 4),$
$D_5 \sim N(E(D_5) = 500, \sigma(D_5) = 10), D_6 \sim N(E(D_6) = 250, \sigma(D_6) = 5),$
$D_7 \sim N(E(D_7) = 200, \sigma(D_7) = 5), D_8 \sim N(E(D_8) = 150, \sigma(D_8) = 3),$
$D_9 \sim N(E(D_9) = 150, \sigma(D_9) = 3).$

The unit price of this product in all markets is 10 and the shortage cost for unmet demand is 0.1 per unit. No salvage value for residual product is considered. The fixed cost of opening a DC at each candidate location is 500. The fixed cost of opening a retailer at a candidate location for retailing facilities are 90, 90, 120, 90, 120, 90, 90, 85 and 85 respectively for locations 1, 2, 3, 4, 5, 6, 7, 8 and 9. The capacity for every producer is 1000 and the budget for opening a facility is 2000.

Since the facilities at some of these mills are old, there is the possibility that they will not be able to deliver the orders on time due to production disruption. In our problem, the impairment of a second and third mill is likely, with a probability of 5% and 15% respectively. So four scenarios can be defined in this problem:

(I)**First scenario**: All mills of the chain are usable. Thus all of the thirteen potential feasible paths of the chain can be used. The probability of the occurrence of this scenario is 1-(0.05)-(0.15)-(0.05 × 0.15) = 0.725.
(II)**Second scenario**: Only the second mill is impaired. Thus, t_{111}, t_{113}, t_{125}, t_{313}, t_{325}, t_{326}, t_{337}, t_{338} and t_{339} are the only potential feasible paths of the chain. The probability of the occurrence of this scenario is 0.05.
(III)**Third scenario**: Only the third mill is impaired. Therefore, $t_{111}, t_{113},$ $t_{125}, t_{212},$ $t_{224},$ t_{227} and t_{236} are the only potential feasible paths of the chain. The probability of the occurrence of this scenario is 0.15.
(IV)**Fourth scenario**: Only the second and third mills are impaired. Hence, t_{111}, t_{113} and t_{125} are the only potential
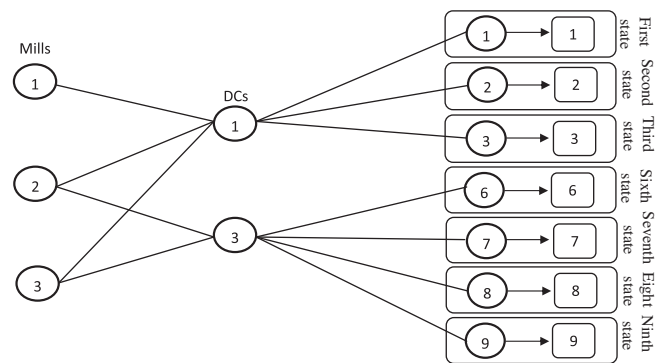

Fig. 4. Optimal network structure of the rice SC.

feasible paths of the chain. The probability of the occurrence of this scenario is 0.05 × 0.15 = 0.075.

For each market, we considered four intervals to linearise the nonlinear term in the objective function. This problem can be formulated and solved by commercial optimisation software; we used LINGO 10.00 software on a computer with an Intel(R)Core(TM)4 Duo CPU, 3.6 gigahertz, and 12276 megabytes RAM using the default settings. The time for solving the model was less than a minute (45 seconds).

Two DCs are located in the first and third candidate DC locations and retailers are established in all the candidate retailing locations except the fourth and the fifth (Fig. 4). The objective function value of this problem is 1024.4397.

The detailed computational results for solving the model of this problem are shown in Table 2.

7. Sensitivity analysis

In this section, we investigate the TAR-C Company problem again to illustrate the solutions' sensitivity in terms of important and changeable parameters of the problem. We try to demonstrate how changing these parameters may affect the network structure of this chain. This also validates the correctness of the proposed model because it reacts in a reasonable way to the parameter changes.

In the beginning, we investigated the effect of changes in the probability of the occurrence of a scenario. The probability can differ from time to time because it is completely influenced by the quality of the facilities and the reliability of the connection links in the chain. For instance, renovation of facilities and transportation devices may change these probabilities. We incrementally increased the probability for the third scenario from 0.15 to 0.85. By doing so the probability of the fourth and first scenario starts to rise and fall respectively. Other parameters of the model are assumed to be fixed. The results are summarised in Table 3.

By increasing the probability of the third scenario, the possibility of impairment of the third mill starts to increase. In this case, the potential routes originated from this mill are out of use most

Table 2
Computational results for solving the case study problem.

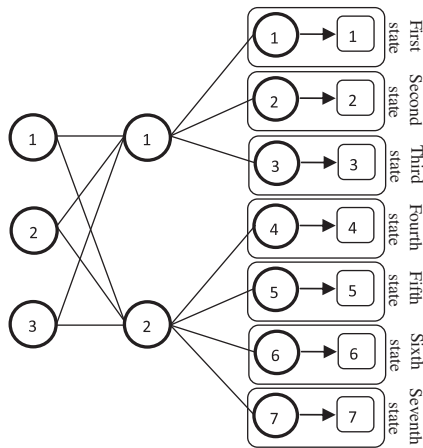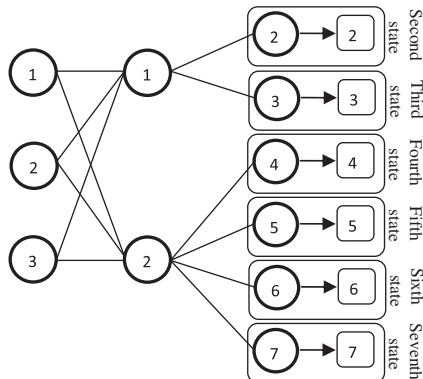| Scenarios	Flow of routes												
	t_{111}	t_{113}	t_{125}	t_{212}	t_{224}	t_{227}	t_{236}	t_{313}	t_{325}	t_{326}	t_{337}	t_{338}	t_{339}
1	300	45	–	220	–	–	265	485	–	–	215	150	150
2	300	45	–	–	–	–	–	485	–	–	215	150	150
3	300	530	–	220	–	–	265	–	–	–	–	–	–
4	300	530	–	–	–	–	–	–	–	–	–	–	–

Table 3
Computational results of the sensitivity analysis for the probability of the third scenario.

State	Probability of scenario 1	Probability of scenario 2	Probability of scenario 3	Probability of scenario 4	Profit	Revenue	Operational cost	Strategic cost	Total cost	Located facilities
1	0.7925	0.05	0.15	0.0075	1024.43	17130.34	14455.90	1650	16105.90	W1, W3 V1, V2, V3, V6, V7, V8, V9
2	0.6875	0.05	0.25	0.0125	891.14	20267.31	17686.17	1690	19376.17	W1, W2 V1, V2, V3, V4, V5, V6, V7
3	0.5825	0.05	0.35	0.0175	776.75	19602.71	17135.95	1690	18825.95	W1, W2 V1, V2, V3, V4, V5, V6, V7
4	0.4775	0.05	0.45	0.0225	662.37	18938.11	16585.74	1690	18275.74	W1, W2 V1, V2, V3, V4, V5, V6, V7
5	0.3725	0.05	0.55	0.0275	559.62	17002.53	14842.90	1600	16442.90	W1, W2 V2, V3, V4, V5, V6, V7
6	0.2675	0.05	0.65	0.0325	471.83	16652.23	14580.39	1600	16180.39	W1, W2 V2, V3, V4, V5, V6, V7
7	0.1625	0.05	0.75	0.0375	414.12	15770.75	13846.63	1510	15356.63	W1, W2 V2, V3, V4, V5, V7
8	0.0575	0.05	0.85	0.0425	368.28	15682.91	13804.63	1510	15314.63	W1, W2 V2, V3, V4, V5, V7

of the time. Since these paths usually pass through the third DC, it would not be rational for the model to spend money on locating the third DC (Fig. 5).

When the probability of the third scenario reaches 0.55, the first retailer is omitted from the network. Obviously the first potential path which serves the first retailer is the most expensive path of the chain, so in the event of disruption when the other mills are out of use, the model tries to use the cheaper potential paths of the chain to supply the products of the first mill to the markets.

In this case, the first potential path and the first retailer are only usable in the first scenario which now has a low probability of occurrence. Therefore, the expected income of this retailer cannot compensate its facility locating cost and is omitted from the chain's structure (Fig. 6).

When the probability of the third scenario reaches 0.75, the sixth retailer is also omitted from the network. In the previous states, this retailer was supplied through the tenth path by the third mill. However, by further increasing the impairment probability of the third mill, this path will be usable in fewer cases and since the potential demand of this retailer is less than the other ones, the model removes it from the set of located retailers (Fig. 7).

As can be seen, the potential routes originating from mill 3 are less expensive. Since these paths are now not usable, the profit of the chain starts to decrease (Fig. 8).
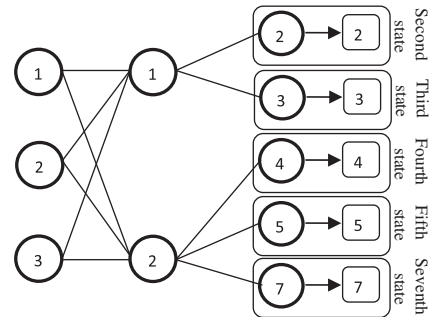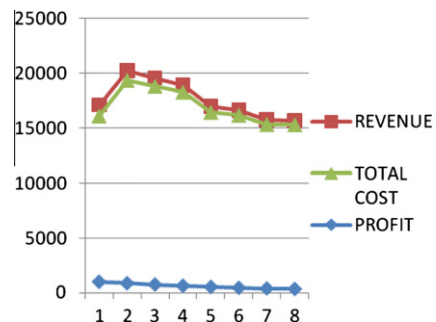

Fig. 5. Optimal network structure of SC for states 2–4.

Fig. 7. Optimal network structure of SC for states 7–8.

Fig. 6. Optimal network structure of SC for states 5–6.

Fig. 8. Cost, income and profit variations in terms of the probability of the third scenario.

Table 4
Computational results of the sensitivity analysis for the probability of the second scenario.

State	Probability of scenario 1	Probability of scenario 2	Probability of scenario 3	Probability of scenario 4	Profit	Revenue	Operational cost	Strategic cost	Total cost	Located facilities
1	0.7925	0.05	0.15	0.0075	1024.43	17130.34	14455.90	1650	16105.90	W1,W3 V1,V2,V3,V6,V7,V8,V9
2	0.6775	0.15	0.15	0.0225	892.97	16232.01	13689.03	1650	15339.03	W1,W3 V1,V2,V3,V6,V7,V8,V9
3	0.5625	0.25	0.15	0.0375	779.16	15868.01	13438.85	1650	15088.85	W1,W3 V1,V2,V3,V6,V7,V8,V9
4	0.4475	0.35	0.15	0.0525	656.52	15236.84	12930.32	1650	14580.32	W1,W3 V1,V2,V3,V6,V7,V8,V9
5	0.3325	0.45	0.15	0.0675	533.88	14605.68	12421.79	1650	14071.79	W1,W3 V1,V2,V3,V6,V7,V8,V9
6	0.2175	0.55	0.15	0.0825	441.58	11841.27	9929.69	1470	11399.69	W2,W3 V5,V6,V7,V8,V9
7	0.1025	0.65	0.15	0.0975	401.16	11713.02	9841.86	1470	11311.86	W2,W3 V5,V6,V7,V8,V9

Next we incrementally and gradually increased probability of the second scenario from 0.05 to 0.65. Along with this increasing, probabilities for the fourth and the first scenario start to increase and decrease, respectively. Other parameters of the model are assumed to be fixed. The results obtained are shown in Table 4.

In this case, the probability of impairment in the second mill increases and the potential paths originating from this mill are out of use for a major part of the time. Since a number of these paths pass through the second DC, this DC is omitted from the network structure of the chain. All the potential paths which ended at the fourth and fifth retailers pass through the second DC. Thus the omission of the second DC leads to the omission of these retailers (Fig. 9).

By further increasing the probability of the second scenario and consequently further decreasing the probability of the first scenario, the importance of the first and third mills as more reliable production facilities starts to increase. So the model tries to supply the products of these mills to the markets with more potential demands through more profitable paths. Since the major part of these paths pass through the second candidate DC location, so again is the second candidate DC location and its corresponding retailers located in the network structure of the chain (Fig. 10). Due to the high cost of producing and distributing through the potential paths passing through the first DC, this DC and the first, second and third retailers which can only be supplied by this DC are omitted from the network structure (Fig. 10).

Similar to the previous case, the paths originating from the second mill are more profitable than the ones starting form the first mill. By increasing the probability of the second scenario, the paths originating from the second mill become less usable, so the profit of the chain starts to decrease (Fig. 11).

At this stage, we investigated the effect of the robustness coefficient on the selected network structure of the chain. We increased this coefficient from 0 to 10. The probabilities of the scenarios are fixed for these problems and set as 0.7925, 0.05, 0.15 and 0.0075 for the first, second, third and fourth scenarios, respectively. The results are shown in Table 5.

By increasing the robustness coefficient in the objective function, the model prefers to choose more reliable paths, which are usually more costly; using these more costly paths reduces the profit of the chain (Table 5). In the initial model, all the potential facilities are established except the second DC and the fourth and fifth retailers. By increasing the robustness coefficient, since the
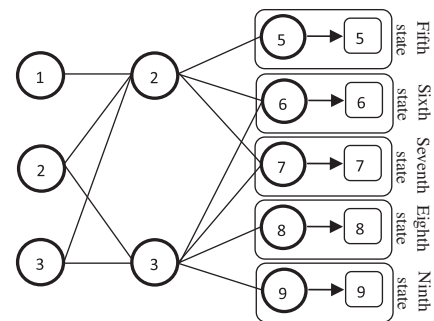

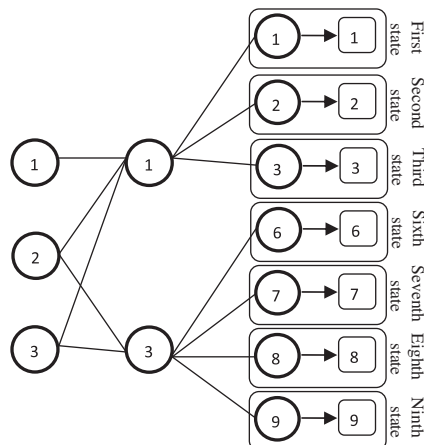**Fig. 10.** Optimal network structure of SC for states 6–7.

Fig. 9. Optimal network structure of SC for states 1–5.
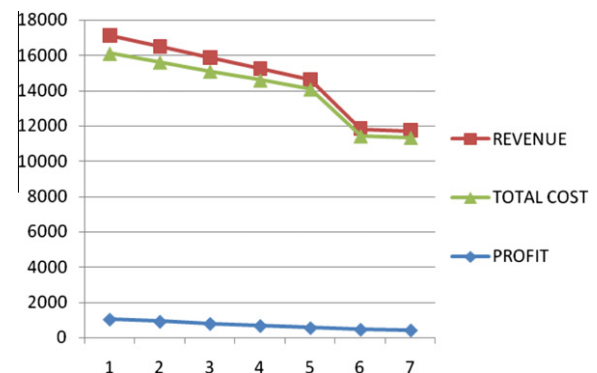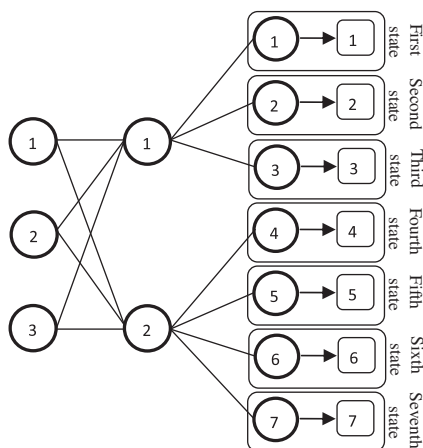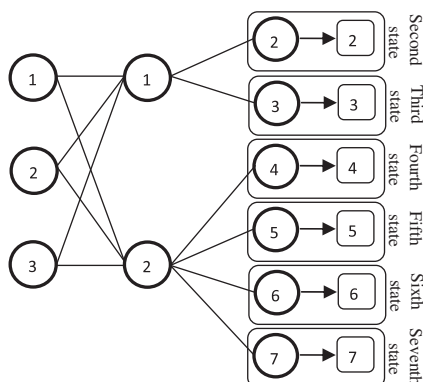

Fig. 11. Cost, income and profit variations with respect to the probability of the second scenario.

Table 5
Computational results of the sensitivity analysis on robustness weight.

Weight	Profit	Revenue	Operational cost	Strategic cost	Variation value	Total costs	Located facilities
0	1024.43	17130.34	14455.90	1650	0	16105.90	W1 W3 & V1 V2 V3 V6 V7 V8 V9
0.2	936.36	17130.34	14455.90	1650	88.07	16105.90	W1 W3 & V1 V2 V3 V6 V7 V8 V9
0.4	865.34	20931.92	18236.39	1690	140.19	19926.38	W1 W2 & V1 V2 V3 V4 V5 V6 V7
0.6	795.30	20931.94	18236.38	1690	210.25	19926.38	W1 W2 & V1 V2 V3 V4 V5 V6 V7
0.8	725.21	20931.94	18236.38	1690	280.34	19926.38	W1 W2 & V1 V2 V3 V4 V5 V6 V7
1	655.13	20931.94	18236.38	1690	350.43	19926.38	W1 W2 & V1 V2 V3 V4 V5 V6 V7
1.5	479.92	18403.74	15892.94	1600	430.88	17492.94	W1 W2 & V2 V3 V4 V5 V6 V7
2	336.29	18403.74	15892.94	1600	574.51	17492.94	W1 W2 & V2 V3 V4 V5 V6 V7
2.5	230.55	14710.03	12781.95	1510	187.53	14291.95	W1 W2 & V2 V3 V4 V5 V7
3	215.45	10026.24	8968.97	800	41.80	9768.97	W1 & V1 V2 V3
3.5	208.49	8892.66	7835.40	800	48.77	8635.40	W1 & V1 V2 V3
4	201.52	9015.12	7957.85	800	55.74	8757.85	W1 & V1 V2 V3
4.5	194.55	8098.67	7041.41	800	62.70	7841.41	W1 & V1 V2 V3
5	187.58	8132.14	7074.87	800	69.67	7874.87	W1 & V1 V2 V3
5.5	180.62	7943.06	6885.79	800	76.64	7685.79	W1 & V1 V2 V3
6	173.65	8889.59	7832.33	800	83.61	8632.33	W1 & V1 V2 V3
6.5	166.68	9054.72	7997.45	800	90.58	8797.45	W1 & V1 V2 V3
7	159.71	8892.66	7835.40	800	97.54	8635.40	W1 & V1 V2 V3
7.5	152.74	7968.43	6911.17	800	104.51	7711.17	W1 & V1 V2 V3
8	145.78	8949.12	7891.86	800	111.48	8691.86	W1 & V1 V2 V3
8.5	138.81	8025.97	6968.71	800	118.45	7768.71	W1 & V1 V2 V3
9	134.87	9585.10	8614.44	800	35.77	9414.44	W1 & V1 V2 V3
9.5	132.89	9403.68	8433.03	800	37.76	9233.03	W1 & V1 V2 V3
10	130.90	9497.90	8527.25	800	39.75	9327.25	W1 & V1 V2 V3

impairment probability of the third mill and its corresponding paths are high, the model now selects the second candidate location for establishing the DC instead of the third one. By deleting the third DC, the eighth and the ninth retailers are also omitted because the potential paths which can supply them only pass through this DC (Fig. 12). By gradually increasing the robustness coefficient, less profitable retailers are deleted from the network of the chain one by one (Figs. 13 and 14).

For $\lambda \geqslant 3$, since major paths passing through the second and the third DCs have higher disruption probabilities, these DCs and the retailers at the end of the paths passing through them are deleted from the network of the chain. In this case, the model selects the more reliable facilities and potential paths for the chain (Fig. 15).

These results make it clear that the proposed model reacts reasonably to changes in the model's parameters.

We next considered several problems with different scenario probabilities. We wanted to investigate the difference of the

Fig. 12. Optimal network structure of SC for $\lambda = 0.4, 0.6$ and 0.8.
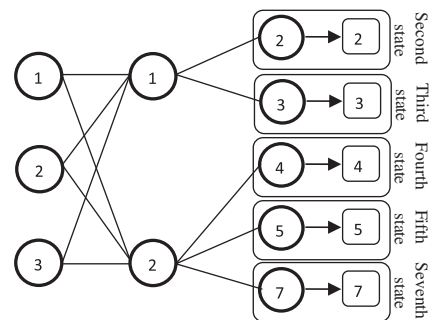
Fig. 14. Optimal network structure of SC for $\lambda = 2.5$.

Fig. 13. Optimal network structure of SC for $\lambda = 1.5$ and 2.
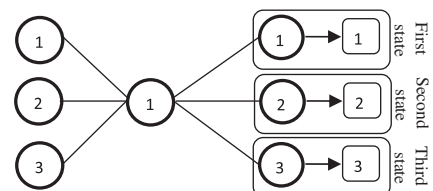
Fig. 15. Optimal network structure of SC for $\lambda \geqslant 3$.

model's behaviour with respect to raising the robustness coefficient. In the **first problem**, the probability of the first, second, third and fourth scenarios are changed to 0.31, 0.3, 0.3 and 0.09, respectively. In this situation, the probabilities of the second, third and fourth scenarios are increased so that even with the low values of the robustness coefficient, the third DC and the corresponding eighth and ninth retailers are not located in the SC network. This structure does not change for values up to $\lambda = 0.6$. By further increasing λ, the network of the chain changes. In this case, only the first DC and the first, second and third retailers are located. By increasing λ, the importance of robustness ascends as the result of the model; so the model tries to select more reliable paths. If we further increase λ, the second retailer which is supplied by the fourth path is omitted from the network of the chain because this path is supplied by the second producer with a relatively high disruption probability.

By increasing λ, the profit and robustness of the SC with a fixed network structure starts to decrease. By reducing the robustness of the SC, the model starts to change the network structure of the chain and selects more reliable paths and facilities.

In the **second problem**, the probability of the first, second, third and fourth scenarios are changed to 0.4375, 0.25, 0.25 and 0.0625, respectively. As can be seen, the network structure of the chain and its gradual changes are similar to the first problem. Since the probability of disruption is reduced in this problem, SC network changes occur for higher values of λ.

In the **third problem**, the probability of the first, second, third and fourth scenarios are changed to 0.56, 0.2, 0.2 and 0.04, respectively. In this case the probability of facility activeness (probability of the first scenario) has been increased compared to the second one. Obviously at $\lambda = 0$, the third DC and its corresponding retailers are located instead of the second DC because in this case only the profit of the chain is considered in the objective function and the increased facility activeness probability results in the model locating the third DC as the cost of the paths passing through it is relatively lower and using them can improve the objective function. However, at higher values of λ, the gradual changes in the network structure of the chain are similar to the previous ones.

In the next three problems, we increased the probability of disruption in the second and third facilities gradually. The probabilities of the scenarios in these problems are as follows:

- **The fourth problem**: Probabilities of the first, second, third and fourth scenarios are 67.5%, 15%, 15% and 2.25%, respectively.

Fig. 16. Profit functions of the investigated problems respect to λ.

- **The fifth problem**: Probabilities of the first, second, third and fourth scenarios are 79%, 10%, 10% and 1%, respectively.
- **The sixth problem**: Probability of the first, second, third and fourth scenarios are 89.75%, 5%, 5% and 0.25%, respectively.

The lower the occurrence probability for the disruption of facilities, the more the chain retains a similar network structure. Increasing the probability of facility activeness stabilises the model with respect to λ. The behaviour of the model makes sense and demonstrates the rationality of the developed model and solution technique. Problems with a lower disruption occurrence probability in facilities allows the model to choose more profitable paths usually with lower production costs and consequently these problems have higher profits. However, by increasing λ and consequently the importance of reliability in the chains, the model prefers to choose more reliable paths with higher production costs (Fig. 16).

8. Computational efficiency

8.1. Model size

The original model (6)–(13) contains $|I| + |J|$ binary variables, however after we linearised the model, as many as $\sum_{m=1}^{|M|} |N^m| \cdot |P| \cdot |S|$ new binary variables were added. Therefore, the solving time of this model would grow by increasing the number of candidate locations for retail outlets, scenarios and products. Solving the model for large scale problems with many retailers, scenarios and products will also be time-consuming. In these cases, using heuristic and meta-heuristic techniques would be more logical.

8.2. Real-life problem size

We need to discuss the application of the developed model to larger problems and its computational efficiency, although the definition of a "large" and "small" problem varies in different applications. In other words, the size of a "large" number in a vehicle routing problem (VRP), scheduling, sequencing, facility location, and so on is quite different from SC network design problems. For example, having more than 1000 retailers when making transportation-inventory decisions regarding an existing SC is acceptable (like Wal-Mart and Tesco SCs); because these are operational and tactical decisions for an existing SC which has already been formed over a long period of time. However, in SC network design 1000 retailers are not located from scratch as this is a high investment decision.

The size of an SC network design problem is identified by the number of products, suppliers, manufacturers, DCs, retailers and customers. In order to obtain an idea about the sizes of the largest SC network designs for risk and disruption problems, we investigated the literature and the findings are shown in Tables 6.

The last row of Table 6 illustrates the size of our case study against those reported in the literature which appears comparable. However, we also try to solve larger problems.

8.3. Run time

SC network design is a strategic decision and we have solved only one such problem over a 10-year time period. In contrast, when we design an initial SC network it is designed from scratch to exist for a long time. Later, we simply improve (usually grow) an existing network by adding individual entrants to the existing SC. In summary, when designing from scratch we face the largest problem in reality. We should note that for solving such a strategic
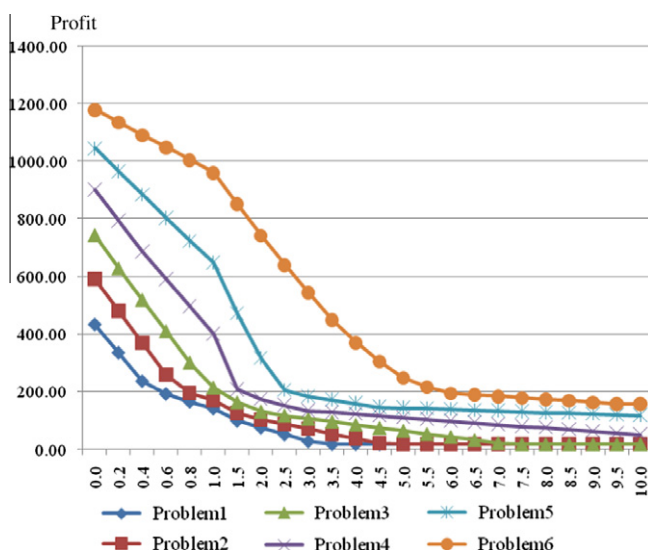
Table 6
The size of large SC networks for risk and disruption problems reported in the literature.

References	No. of products	No. of suppliers	No. of manufacturers	No. of DCs	No. of retailers/customers
Dada et al. (2007)	1	2	1	–	–
Chopra et al. (2007)	1	2	1	–	–
Wilson (2007)	1	1	1	1	1
Azaron et al. (2008)	1	4	4	–	3
Ross et al. (2008)	1	1	1	–	–
You and Grossmann (2008)	2	6	7	5	9
Chen and Xiao (2009)	1	–	1	–	10
Yu et al. (2009)	1	2	1	–	–
Li et al. (2010)	1	2	–	–	1
Schmitt et al. (2010)	1	–	1	4	–
Hsu and Li (2011)	1	4	5	–	5
Schmitt and Snyder (2010)	1	1–2	1	–	–
Lundin (2012)	1	–	2	5	3
This paper's case problem	1	–	3	3	9

Table 7
Solving time for randomly generated problems.

Problem	No. of manufacturers	No. of DCs	No. of retailers	No. of products	No. of scenarios	Solving time of problem
1	3	3	9	1	4	45″
2	3	3	9	1	8	9′:56″
3	3	4	15	1	4	4′:10″
4	3	4	15	1	8	2:41′:5″
5	3	5	20	1	4	18′:42″
6	3	5	20	1	8	9:1′:37″
7	3	6	30	1	4	3:2′:29″
8	3	6	30	1	8	22:11′:30″
9	3	8	40	1	4	>36 hours

problem, even 1 month of running time is still acceptable whereas to solve operational problems (like inventory, sequencing and scheduling) even 10 hours can be unacceptable!

For making sense of the solvable size of problems and computational time of this approach, we used the model presented in this paper and a problem-solving approach for some randomly generated numerical problems. The size of the problems and their solving times are summarised in Table 7.

For the last problem (last row in Table 7), we could not gain a solution even after 36 hours. However, compared to the size of the problems solved in the literature and also the nature of SC network design problems, the computational times are acceptable.

9. Conclusion and suggestions for further research

In this paper we develop a stochastic model for designing a network structure for an SC which includes several capacitated production facilities, DCs and retailing facilities by considering both demand-side and supply-side uncertainties. By considering extra and shortage costs in retailing facilities, this model guarantees to provide a predetermined service level for its customers.

The resultant model is a MINL problem and is difficult to tackle. In order to solve the model to attain global optimality, we linearised it via a multiple linear regression; in fact, we relied on the convexity of the relaxed model and used a piecewise linear approximation conversion. The resulting linear model could then be solved by using existing commercial software. We illustrated the model output for a real-life case study from the agri-food industry. Finally, we further investigated this problem to illustrate the solutions' sensitivity following changes to important and changeable parameters of the problem. We have shown how changing these parameters can affect the network structure of this chain and why this occurs. This also verifies the correctness of our

proposed model because it reacts in a reasonable way to the parameter changes.

According to Schmitt et al. (2010), two different groups of strategies have been used to manage disruptions in SCs: (a) proactive strategies, which are undertaken prior to disruption in order to reduce its impacts (such as holding strategic reserves) and (b) reactive strategies, which are applicable after a disruption occurs such as the one used in this paper. We can extend the model of this paper to consider proactive strategies too: this paper only considers demand and supply uncertainties. However, other sources of uncertainties can be found in chains, such as exchange rates, travel times, amounts of returns in reverse logistics and transportation costs. Considering these factors in designing a network structure for green and global SCs is essential. In addition, the competitive environment in which a firm operates may significantly affect the decisions that a firm makes with respect to risk mitigation. For many firms, the key objective may be ensuring that their post-disruption situation is not worse than that of their competitors. Embedding these objectives in a game theoretic environment is another important extension.

Another area of possible research is proactively running the model for the worst case scenario and being prepared for such a conservative scenario. Then, using general strategies for mitigating disruption consequences (Chopra and Sodhi, 2004) we can prescribe an optimum combination of adding capacity, adding inventory, having redundant suppliers, increasing responsiveness, increasing flexibility, aggregating or pulling demands, increasing capability, or having more customer accounts to manage such disruption.

The model of this paper is quite cost- or profit-based. Considering objective functions for the performance of a SC rather than traditional objectives could be another issue for research (Tromp et al., 2010). From a modelling point of view, considering parametric uncertainty and incorporating an acceptance threshold for the
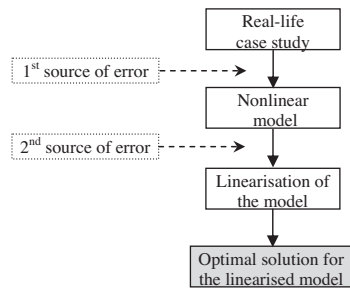
Fig. 17. Conceptual framework for the modelling and solution presented in this paper.
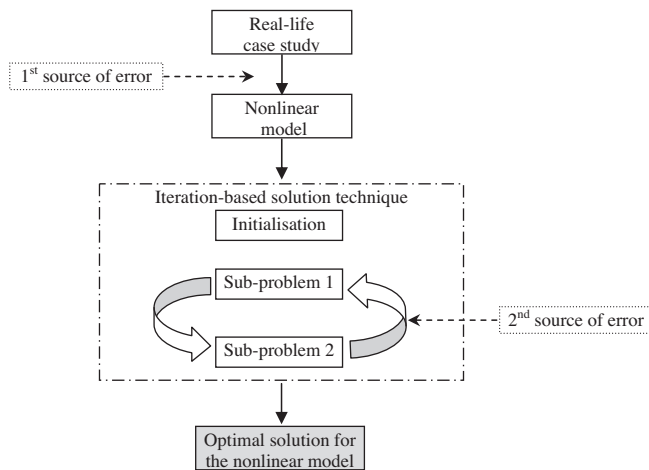

Fig. 18. Conceptual framework for further research on solution techniques.

owner of an SC could be another potential research area (Gerrard and Tsanakas, 2011).

We believe there are two different strategies for solving such nonlinear problems. Our method is similar to the Dantzig–Wolf decomposition method, which works as shown in Fig. 17.

There are other techniques, such as Branch and Bound, Outer Approximation (OA), Generalised Benders and Extended Cutting Plane methods, which work according to the system illustrated in Fig. 18.

These figures show that both ways of thinking have advantages and disadvantages as these two systems encounter two sources of errors. On the other hand, since the original nonlinear model is convex, the final solution of either method will be too close to the global optimum, rather than the local optimum, for this problem. However, when it comes to computational times, we need to investigate further to determine which one performs better for such problems.

Finally, we could investigate whether the developed model can capture time-varying uncertainty issues using our approach. In other words, sometimes it is possible that the parameters of the demand distribution function (its mean or variance) or the disruption possibility of the chain's facilities will change after some period in the future. In this case we should try to model this problem as a multi-period SC network design. Therefore, a new index representing the time period would need to be inserted into the variables and parameters of the model and some new parameters should be added to the model for the cost of shutting down DCs and retailers or the cancellation of contracts with producers. In each new period, it would be possible to get closer to some of the facilities and select or locate new ones within the network of

the chain. In such a case, our modified model would be flexible with respect to time-varying uncertainty, and in each period the model would make the best decision through a trade-off between the costs and benefits of changing the chain's network.

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