INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
DEPARTMENT OF MATHEMATICS & STATISTICS

MSO 202/A COMPLEX ANALYSIS
ASSIGNMENT 4

Q1. Show that the sequence, $z_n = a_n + ib_n$, where a_n and b_n are real numbers, is convergent if and only if a_n and b_n are convergent.

Q2. Are the following sequences convergent or divergent?
(i) $z_n = \sin(n\pi/4) + i^n$, (ii) $z_n = (0.9 + 0.1i)^{2n}$.

Q3. Are the following series convergent?
(i) $\sum_{n=1}^{\infty} 1/\sqrt{n}$ (ii) $\sum_{n=0}^{\infty}(n-i)/(3n+2i)$

Q4. Let $|z_{n+1}/z_n| \leqslant q < 1$, so that the series $\sum_{n=1}^{\infty} z_n$ converges by the ratio test. Show that the remainder $R_n = \sum_{j=n+1}^{\infty} z_j$ satisfies the inequality $|R_n| \leqslant |z_{n+1}|/(1-q)$.

Q5. Show thta if $\sum a_n z^n$ has the radius of convergence R then $\sum a_n z^{2n}$ has the radius of convergence $\sqrt{R}$.

Q6. Find the center and the radius of convergence of the following series.
(i) $\sum_{n=1}^{\infty}(z+i)^n/n^2$ (ii) $\sum_{n=0}^{\infty}(a/b)^n z^n$ (iii) $\sum_{n=1}^{\infty}(-1)^{n+1}(z^n/n)$.

Q7. Use the fact that the radii of the convergence of the series and its derived or integrated series are the same, find the radius of convergence of the series.

$$\text{(i)} \sum_{n=2}^{\infty} \frac{n(n-1)}{3^n}(z-2i)^n, \quad \text{(ii)} \sum_{n=1}^{\infty} \frac{-7)^n}{n(n+1)(n+2)} z^{2n}.$$

Q8. Find the Taylor or Maclaurin series of $f(z)$, with the center z_0, where
(i) $f(z) = e^{-z^2/2},\ z_0 = 0,$ (ii) $f(z) = \text{Ln }(1-z),\ z_0 = i,$ (iii) $f(z) = z^6 - z^4 + z^2 - 1,\ z_0 = 1.$

Q9. Let

$$\sec z = E_0 - \frac{E_2}{2!}z^2 + \frac{E_4}{4!}z^4 - + \cdots.$$

Show that $E_0 = 1$, $E_2 = -1$, $E_4 = 5$. These E_{2n} are called Euler numbers.

Q10. Developing $1/\sqrt{1-z^2}$ and integrating, show that

$$\sin^{-1} z = z + \left(\frac{1}{2}\right)\frac{z^3}{3} + \left(\frac{1\cdot 3}{2\cdot 4}\right)\frac{z^5}{5} + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)\frac{z^7}{7} + \cdots, \quad (|z| < 1).$$