INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
DEPARTMENT OF MATHEMATICS & STATISTICS

MSO 202/A COMPLEX ANALYSIS
ASSIGNMENT 5

Q1. Expand the given function in a Laurant series that converge for $0 < |z| < R$ and determine the precise region of convergence.
(i) $1/(z^4 - z^5)$ (ii) e^{-z}/z^3 (iii) $z^{-3}e^{1/z^2}$.

Q2. Expand the given function in a Laurant series that converge for $0 < |z - z_0| < R$ and determine the precise region of convergence.
(i) $e^z/(z-1)$, $z_0 = 1$ (ii) $1/z^2 + 1$, $z_0 = i$ (iii) $(z^2 - 4)/(z-1)$, $z_0 = 1$..

Q3. Find all Taylor and Laurant series with center $z = z_0$ and determine the precise region of convergence.
(i) $1/(1 - z^3)$, $z_0 = 0$, (ii) $(z^3 - 2iz^2)/((z-i)^2$, $z_0 = i$ (iii) $\sin z/(z + \frac{\pi}{2})$, $z_0 = -\pi/2$.

Q4. Does $\tan(1/z)$ have a Laurant series that converges in a region $0 < |z| < R$.

Q5. Expand the following in a Laurant sereis that converges for $|z| > 0$:

$$\frac{1}{z^2}\int_0^z \frac{e^t - 1}{t}\, dt.$$

Q6. Determine the location and kind of singularities of the functions:
(i) $\frac{\sin 3z}{(z^4-1)^4}$ (ii) $\cosh\left(\frac{1}{z^2+1}\right)$

Q7. Show that the points at which a nonconstant analytic function $f(z)$ has a given value k are isolated.

Q8. Let $f_j(z)$, $j = 1, 2$ are analytic in a domain $D \subseteq \mathbb{C}$ and $f_1(z_n) = f_2(z_n)$ for a sequence $\{z_n\}_{n=1}^{\infty} \subset D$ then $f_1(z) \equiv f_2(z)$ in D.