# INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
# DEPARTMENT OF MATHEMATICS & STATISTICS

## MSO302A COMPLEX ANALYSIS
## ASSIGNMENT 3

**Q1.** Using the method of parametric representation, evalaute:
(i)

$$\oint_C (z + \frac{1}{z})dz, \quad C : |z| = 1 \text{ (counterclockwise)}.$$

(ii)

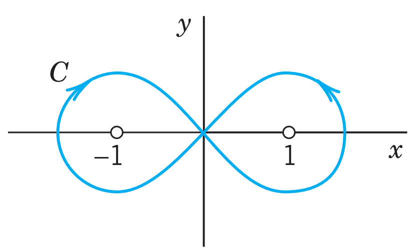
$$\int_C \bar{z}dz, \quad C : \text{parabola along } y = x^2 \text{ from } -1+i \text{ to } 1+i.$$

**Q2.** Integrate $f(z)$ counterclockwise around the unit circle indicating whether Cauchy's integral theorem applies.
(i) $f(z) = \text{Re } z$ (ii) $f(z) = 1/(2|z|^3)$.

**Q3.** Verify Cauchy's integral theorem for $f(z) = z^2$ over the boundary of the sqaure with vertices $1+i$, $-1+i$, $-1-i$ and $1-i$, counterclockwise.

**Q4.** Evalaute

$$\oint \frac{dz}{z^2 - 1}, \quad C \text{ as shown in the figure.}$$




**Q5.** Using Cauchy integral formula integrate counterclockwise:

$$\oint_C \frac{\text{Ln }(z+1)}{z^2+1}\, dz, \quad C: \ |z-4| = 2.$$

**Q6.** Integrate around $C$, taken counterclockwise.

$$\oint \frac{(1+2z)\cos z}{(2z-1)^2}\,dz, \quad C : |z| = 1.$$

**Q7.** If $f(z)$ is non constant entire function and $R$ and $M$ are any positive numbers (no matter how large), show that there exists $z$ such that $|z| > R$ and $|f(z)| > M$. Can you make this assertion for all $z$ outside the circle $|z| > R$?

**Q8.** If $f(z)$ is a polynomial of degree $n > 0$ and $M$ is any positive mumber. Show that there exists $R > 0$ such that $|f(z)| > M$ for all $|z| > R$.

**Q9.** If $f(z)$ is a polynomial in $z$, not a constant, then $f(z) = 0$ for at least one value of $z$. Thus conclude that a plynomial of degree $n > 0$ has exactly $n$ roots in the complex plane.