Partial Differential Equations - MSO 203B

Assignment 1 - Fourier Series and SL theory

Tutorial Problems

1. Find the Fourier series of $f(x) = \begin{cases} x;\ 0 \leq x \leq \pi \\ 0;\ -\pi \leq x < 0 \end{cases}$ and f is extended periodically to whole of $\mathbf{R}$.

2. Find the Fourier series of $f(x) = \begin{cases} 1;\ 0 \leq x < \frac{1}{2} \\ 0;\ \frac{1}{2} \leq x < 1 \\ -1;\ -\frac{1}{2} \leq x < 0 \\ 0;\ -1 \leq x < \frac{1}{2} \end{cases}$ and f is extended periodically to whole of $\mathbf{R}$.

3. An elastic string of length 4m with fixed ends is raised by 2m and then released from rest. Assume the displacement satisfies $u_{tt} - 25u_{xx} = 0$ for $0 \leq x \leq 4;\ t \geq 0$. Describe the motion of the string in terms of a Fourier series solution.

4. Prove that $\frac{1}{\pi}\int_{-\pi}^{\pi} f^2(x)\ dx = 2a_0^2 + \Sigma_{n\in\mathbf{N}}(a_n^2 + b_n^2)$ where a_i and b_i are Fourier coefficients of the square summable function f.

5. Comment on the convergence of $\Sigma_{n\in\mathbf{N}}\frac{(-1)^n}{n^2}$ using the Fourier series of $f(x) = x^2$ on $[-\frac{1}{2}, \frac{1}{2}]$.

6. Solve the equation $y''(x) = x\ ; x \in (0, \pi)$ using Fourier Series subject to the boundary condition $y(0) = y(\pi) = 0$.

7. Show that the Chebychev's equation given by $(1-x^2)y'' - xy' + n^2y = 0$ in $[-1, 1]$ is a singular SL-BVP.

8. Let (λ_1, y_1) and (λ_2, y_2) be the two distinct eigenpair of a regular SL-BVP such that $\lambda_1 < \lambda_2$. Show that y_2 admits a zero in between two consecutive zeroes of y_1.

9. Reduce the problem $y^{''} + xy' + \lambda y = 0$ into an appropriate SL-BVP e.q., in self-adjoint form.

10. Comment on the eigen pairs of the problem $y^{''} + \lambda y = 0$; $\lambda > 0$ subject to the boundary condition $y'(0) = h_1 y(0)$ and $y'(l) = -h_2 y(l)$ where $h_1, h_2 > 0$.

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