Partial Differential Equations - MSO 203B

Assignment 4 - HEAT and WAVE Equation

Tutorial Problems

Please always assume smoothness of known functions, until otherwise mentioned.

1. Solve

$$\begin{aligned} u_{tt} - 4u_{xx} &= 0;\ x > 0,\ t > 0 \\ u(x,0) &= |\sin(x)|,\ x > 0 \\ u_t(x,0) &= 0,\ x > 0 \\ u(0,t) &= 0,\ t \geq 0 \end{aligned}$$

2. Solve

$$\begin{aligned} u_{xx} - u_{yy} &= 1 \\ u(x,0) &= \sin(x), \\ u_y(x,0) &= x \end{aligned}$$

3. Solve

$$\begin{aligned} u_{tt} - c^2 u_{xx} &= 0;\ x > 0,\ t > 0 \\ u(x,t) &= f(x) \text{ on } \{x + ct = 0\} \\ u(x,t) &= g(x) \text{ on } \{x - ct = 0\} \\ f(0) &= g(0) \end{aligned}$$

4. Solve

$$\begin{aligned} u_{tt} - c^2 u_{xx} &= F(x);\ l > x > 0,\ t > 0 \\ u(x,0) &= f(x),\ l > x > 0 \\ u_t(x,0) &= g(x),\ l > x > 0 \\ u(0,t) &= A,\ u(l,t) = B,\ t > 0 \end{aligned}$$

5. Solve

$$\begin{aligned} u_{tt} - c^2 u_{xx} &= h(x,t);\ l > x > 0,\ t > 0 \\ u(x,0) &= f(x),\ l > x \geq 0 \\ u_t(x,0) &= g(x),\ l > x \geq 0 \\ u(0,t) &= 0,\ u(l,t) = 0,\ t > 0 \end{aligned}$$

6. Let

$$u(x,t) = \Sigma_{n=1}^{\infty} B_n \sin(\frac{n\pi x}{l}) \exp(-k(\frac{n\pi}{l})^2 t)$$

be the formal solution of the heat equation

$$\begin{aligned} u_t - u_{xx} &= 0;\ l > x > 0,\ t > 0 \\ u(0,t) &= 0,\ u(l,t) = 0,\ t \geq 0 \\ u(x,0) &= f(x),\ l \geq x > 0 \end{aligned}$$

If the series $f(x) = \Sigma_{n=1}^{\infty} B_n \sin(\frac{n\pi x}{l})$ converges uniformly on $[0, L]$, then show that $u(x,t)$ is the classical solution of the heat equation.

7. Find a particular solution of $u_t - u_{xx} = \sin(t) + x^2$.

Practice Problems

1. Use SOV to solve the eigenvalue provlems for the Laplce equation.
2. Find a particular solution of $u_{tt} - u_{xx} = \sin(t) + x^2$.
3. Solve

$$\begin{aligned} u_t - u_{xx} &= h(x,t);\ l > x > 0,\ t > 0 \\ u(x,0) &= f(x),\ l > x \geq 0 \\ u(0,t) &= 0,\ u(l,t) = 0,\ t > 0 \end{aligned}$$

END