Partial Differential Equations - MSO203B

Assignment 2 - Basics and 1st Order PDE

Tutorial Problems

1. Classify the following based on their linearity/semilinearity/quasilinearity and fully nonlineraity.
   - $u_x + u_y = 1$
   - $u_x + xu_y = u^2$
   - $u_x + uu_y = 0$
   - $\mathrm{div}(|\nabla u|^{p-2}\nabla u) = 0$ for $1 < p < 2$.
   - $\det(D^2 u) = 1$
2. Solve the following PDE:
   - $u_{xx} = u$
   - $u_{xy} + u_x = 0$
3. Show that for smooth functions u, v in a smooth, bounded domain one has the following

$$\int_{\Omega} (u\Delta v - v\Delta u)\, dx = \int_{\partial\Omega} (u\frac{\partial v}{\partial \gamma} - v\frac{\partial u}{\partial \gamma})\, dS$$

   where γ is the unit outward normal vector.

4. Verify that $u(x,t) = x^2 + t^2$ solves the wave equation given by $u_{tt} = u_{xx}$.
5. Show that the solutions $z(x,y)$ of $yz_x = xz_y$ represents surface of revolution.

6. Show that the family of spheres $x^2+y^2+(z-c)^2 = r^2$ satisfies the first order PDE: $xz_y - yz_x = 0$ where c is a constant.

7. Find the general solution of the Euler equation $xu_x + yu_y = nu$ where $n \in \mathbf{N}$.

8. Find the general solution of the linear equation $x^2u_x+y^2u_y = (x+y)u$.

9. Find the solution of the equation $(x + y)uu_x + (x - y)uu_y = x^2 + y^2$ with the Cauchy data $u = 0$ on the line $y = 2x$.

10. Use $v = \log(u)$ to reduce the problem $x^2u_x^2 + y^2u_y^2 = u^2$ and then solve it using $v = f(x) + g(y)$ where f, g smooth and arbitrary.

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