Propositional Logic

Week1: Assignment1

Ravishankar Sarma

Last compiled on 11 June, 2021

1. Which of the following formulas are **well formed formulas** (wffs)

- $v1v2 \rightarrow \wedge q)$
- $v1 \rightarrow (v2 \rightarrow v3) \wedge v3$
- $(v1 \wedge v2 \wedge v2 \leftrightarrow v3)$
- $v1 \wedge \vee \rightarrow q$

2. Which of the following are declarative sentences:

- Shree Narendra Damodhar Modi is the current prime minister of republic of India.
- Dirty cockroach!
- 2+2 = 4
- What a beautiful painting!
- This sentence is *false*
- Ram is rich.
- Hyderabad is the capital of Lucknow.
- Turn right at the traffic light.
- What time is it?
- Happy Birthday!

3. Consider a propositional language with three proposition constants - mushroom, purple, and poisonous - each indicating the property suggested by its spelling. Using these proposition constants, encode the following English sentences as Propositional Logic sentences.
   - All purple mushrooms are poisonous.
   - A mushroom is poisonous only if it is purple.
   - A mushroom is not poisonous unless it is purple.
   - No purple mushroom is poisonous.
4. Construct trees for the following formulas:

- $(p \wedge q) \rightarrow \neg q$
- $p \leftrightarrow (q \leftrightarrow \neg r)$
- $q \wedge \neg r \vee s \rightarrow t$ (draw all possible trees: think of bracket arrangements).

5. Check if the following are valid consequences:

- $\neg(q \wedge r), q \models \neg r$
- $\neg p \vee \neg q \vee r, q \vee r, p \models r.$
- $\neg p \rightarrow q$, $\neg p \wedge s$, $\neg r \vee s$, $\neg s \models \neg p \vee s$

6. Which of the following pairs are logically equivalent? Confirm your answer using truth tables:

- $\phi \rightarrow \psi$ and $\psi \rightarrow \phi$
- $\phi \rightarrow \psi$ and $\neg\psi \rightarrow \neg\phi$
- $(\phi \wedge \psi) \leftrightarrow (\phi \vee \psi)$ and $\phi \leftrightarrow \psi$

7. You want to throw a party, respecting people's incompatibilities. You know that:

- Jayant comes if Madhuri or Abhishek comes.
- Abhishek comes if Madhuri does not come.
- If Abhishek comes, Jayant does not.

Can you invite people under these constraints?

Knights and Knaves:

- The island has two types of inhabitants, knights *who always tell the truth*, and knaves *who always lie.* Suppose A is the proposition person A is a knight and suppose A makes a statement S. Then A is true is the same as S is true. That is, $A \leftrightarrow S$. A says *I am a knight* is translated as $A \leftrightarrow A$. If A says: I am the same type as B. we infer $A \leftrightarrow (A \leftrightarrow B)$ which simplifies to B. If native A is asked a yes/no question Q then the response to the question is: $A \leftrightarrow Q$.

Robbery

- here was a robbery in which a lot of goods were stolen. The robber (s) left in a truck. It is known that :

1. Nobody else could have been involved other than A, B and C. 2. C never commits a crime without A's participation.
2. B does not know how to drive. Is A innocent or guilty?

Tourist and Fork:

- A tourist comes to a fork in the road, where one branch leads to a restaurant and one doesn't. A native of the island is standing at the fork. Formulate a single {**yes/no question**} **that the tourist can ask such that the answer will be yes if the left fork leads to the restaurant, and otherwise the answer will be no.**

Gold in an Island

- It is rumoured that there is gold buried on the island. You ask one of the natives whether there is gold on the island. The natives replies: {**There is gold on this island is the same as I am a knight.**}
  1. **Can it be determined whether the native is a knight or a knave?**
  2. **Can it be determined whether there is gold on the island?**
  1. **Let A be *the native is a knight* and G be *there is gold on the island.***
  2. $A \leftrightarrow G$**.**
  3. $A \leftrightarrow (A \leftrightarrow G)$ **is true.**
- **It is about a king and 12 prisoners. The King wants to get rid of them without executing them but he wants only the smart ones to survive. Therefore every prisoner face a logical puzzle he has to solve before he may enjoy freedom.**

Lady or Tiger:

1. **P1:** D1: In this room there is a lady, and in the other room there is a tiger. D2: In one of these rooms there is a lady, and in one of these rooms there is a tiger. **one of the signs is true and other is false**.
2. **P2:**D1:At least one of these rooms contains a lady. D2: A tiger is in the other room. Signs are both true and both false.
3. **P3:** Door 1: Either a tiger is in this room or a lady is the other room; Door 2: A lady is in the other room

- The king explained that in the lefthand room (room 1), if a lady is in it, then the sign on the door is true, but if a tiger is in it, the sign is false. In the righthand room (room 2), the situation is the

opposite: a lady in the room means the sign on the door is false, and a tiger in the room means the sign is true \begin{enumerate}

- Both rooms contain ladies; Door 2:Both rooms contain ladies
- Door 1: At least one room contains a lady. Door 2: The other room contains a lady

\end{enumerate}

1. You meet two people from this island: A and B. A claims that B is a knave, but B says, "Neither A nor I are knaves." Can you tell what A and B are?
2. On the island of knights and knaves, you come to a fork in the road with one man standing before each path. You know that one of them is a knight, and the other is a knave. You also know that one path leads to freedom, and the other path leads to certain death. You can ask one of the men one question. What do you ask to determine the path to freedom?
3. Dan, Ellen, Fred, Grace, and Hugh are from the island of knights and knaves. You hear them say the following: Dan: "Fred and Hugh are knights."

Ellen: "Fred and I are different."

Fred: "Ellen is a knave."

Grace: "Dan is not a knave."

Hugh: "Fred and Dan are either both knights or both knaves."

Who is who?