

Modal Logic in a nutshell

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Outline



- 1 What is Modal Logic?
- 2 Some Logical Problems

Sentences of Modal Logic



- 1 can, perhaps, must, certain
- 2 It is necessary that, it is possible that, it is contingent that
- 3 Intension of a sentence is called **proposition**.



What is Modal Logic?

- 1 Extension of classical first order logic, intensional logic, branch of Philosophical Logic (1918),
- 2 Modes of truth: $\Box p$, $\Diamond p$ and p , where p (What has been the case) is an atomic sentence referring to actual facts.
- 3 Logic of **Necessity** and **Possibility**
- 4 Modal Arguments and reasoning: reasoning about knowledge, Free Will, existence of God, Understanding nomic necessity in Laws, analyzing counterfactual conditionals.
- 5 interests computer scientist (program behaviour), express dynamical properties of transitions between states, AI (how agents reason about other agents), Economists (Game theory- knowledge games).

Why Modal Logic?



- 1 Meaning need not necessarily be equated with reference (departure from referential theory of meaning). Context and Multiple reference- play a role.
- 2 Dissatisfaction concerning **Material Implication** in understanding **deduction**
- 3 Distinguishing $\Box p$, p and $\Diamond p$, inferences based on these modal operators
- 4 Representing Future contingent sentences.
- 5 Analysis of conditionals.

Flavors of Modality



- 1 Epistemic: It is possible for all they knew that... It is known by the police that...
- 2 Temporal: It will sometime be that... It will always be that ...
- 3 Alethic: It could have been that... It is necessary that ...
- 4 Deontic: It is permissible that ... It is obligatory that ..

Syllabus



- 1 Crash Course on Propositional Logic.
- 2 Normal propositional Modal Logic: Syntax and Semantics
- 3 Conditional Logic (Application of Modal Logic)



Thank You!



Gold in an Island

It is rumoured that there is gold buried on the island. You ask one of the natives, A, whether there is gold on the island.

He makes the following response: **There is gold on this island equivaless I am a knight.** The problem is

- 1 Can it be determined whether A is a knight or a knave?
- 2 Can it be determined whether there is gold on the island?



Solution

Solution

Let G denote the proposition **There is gold on the island.**

A 's statement is $A \leftrightarrow G$. So what we are given is:

$$A \leftrightarrow A \leftrightarrow G.$$

This simplifies to G . So we deduce that there is gold on the island but it is not possible to tell whether A is a knight or a knave



Tourist at the fork

Tourist and inhabitants of Island

Each inhabitant of a remote village always tells the truth or always lies. A villager will only give a **Yes** or **No** response to a question the tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the restaurant(or ruins) you want to visit; the other branch leads deep into the jungle.

A villager is standing at the fork in the road. What one question can you ask the villager to determine which branch to take?

Direct questioning like does the right fork leads to the restaurant, will not work here

Formulating Questions



If native A is asked a yes/no question Q then the response to the question is $A \leftrightarrow Q$.

That is, the response will be yes if A is a knight and the answer is really **yes**, or A is a knave and the answer is really **no**.

Otherwise the response will be no. For example, asked the question **are you a knight** all natives will answer yes, as $A \leftrightarrow A$.

Asked the question **is B a knight** A will respond yes if they are both the same type, otherwise no.

That is, A's response is yes or no depending on the truth or falsity of $A \leftrightarrow B$

Solution



- 1 Let Q be the question.
- 2 Let A be **the native is a knight**. Let L be the proposition **the left fork leads to the restaurant**.
- 3 We require that L equivalence the response to the question is **yes**.
- 4 But the response to the question Q is yes equivalence $Q \leftrightarrow A$.
- 5 So we require that $L \leftrightarrow (Q \leftrightarrow A)$.
- 6 Equivalently, $Q \leftrightarrow (L \leftrightarrow A)$.
- 7 The question is thus: **Is it the case that the statement that the left fork leads to the restaurant(ruins) is equivalent to your being a knight?**



References

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