End Semester Examination
**PHI455: Philosophical Logic**
HSS, IIT Kanpur

**General Instructions:** Read carefully each question. Fill in your with a pen and circle the correct answer on paper as well. All your work must be done in these pages.

- You have up to 180 minutes.
- You must attach rough work with your answers.
- Extra 45 minutes will be given for uploading answers and scanned copies.
- For each Wrong answer 0.5 marks will be deducted.
- Please ensure that you keep a copy of your rough work and save it somewhere for future reference. Save it with the course number and and your roll number. You need to attach scanned copy of your rough work.
- Every item on the test of part-A awards 2 points for each correct answer, PartB is for 40M. for a maximum possible score of 80 points.
- Multiple choice questions may have more than one answer. Circle each of the correct answer. Rough work needs to be attached.
- Each Question in part-B consists of 10 marks each. For each wrong answer 1.5 M will be deducted.

$\boxed{\text{GP – Graham Priest}}$

Syllabus:

→ Conditional Logic (GP)

→ Many-valued Logic (GP or Bergman)

→ Epistemic logic (Halpern – Reasoning about knowledge) + Slides

# Conditional logic : (Examples)

$p \to (q \to p)$

$\neg p \to (p \to q)$

① Following wff is an instance of Paradox of Material Implication

$p \to (q \to p)$

$\boxed{\text{a. True}}$ b. False

② $A \wedge B \models A \supset B$ holds in 'S'

1. True 2. False ✓ it is valid in C

(See p.90)

Pg 90:

If $w \in [A]$ and $w' \in f_A(w)$

then $w = w'$

$A \wedge B \models A > B$

$A \wedge B, i$

$\neg(A > B)$

$i r_A j$

$A, j$

$\neg B, j$

$A, i$

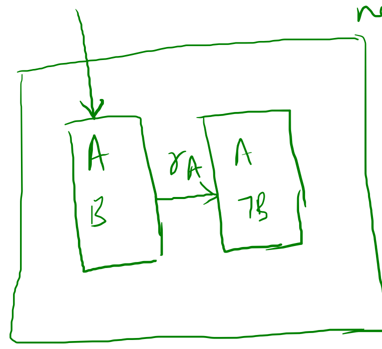
$B, i$

$\neg A, i$ ×  |  $A, i$

$i r_A i$

This is ruled out in $C_2$ and $S$

nearest world is $A$ $\neg B$ world.

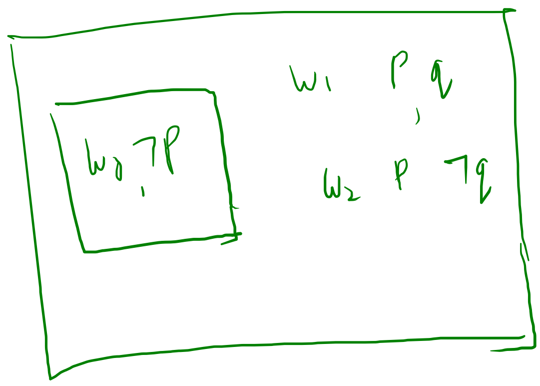



In the nearest worlds where LHS is True but in the nearest world

Very special case in $C_1$

But often (counter intuitive)

Ex 2 : $(p \supset q) \vee (p \supset \neg q)$




wrt to the actual world which we fixed it in the box

we have $w_1$ $w_2$ where 'p' - which is the antecedent, and it is true:

holds in $C_2$ but not in $C_1$ nor in '$S$'

Let us consider some more Examples → (See p.97) GP

1. $p > q$
2. $\neg(p > r)$
3. $\neg((p \wedge r) > q)$

$\models (p \wedge r) > q$

D.C.

4. $i \quad p \wedge r \quad j$

$p, j$

$r, j$

$\neg q, j$

$i \ r_p \ k$

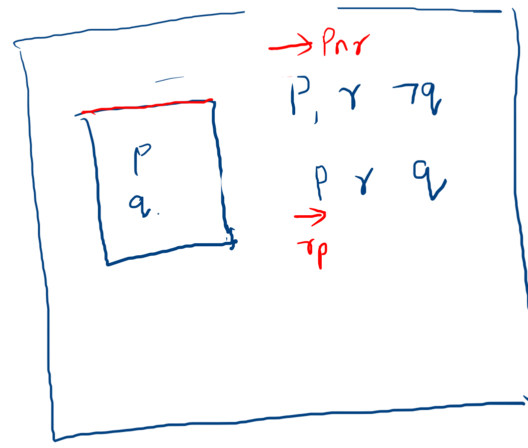
$p, k$

$r, k$

$q, k$

Open branch ⇒ invalid in C+

$\neg p, i \qquad p, i$ from 1 $\quad i\, r_p\, i \quad q, i$




PS: $f_A(w) \cap [B] \neq \emptyset$

$f_{A \cap B}(w) \subseteq f_A(w)$

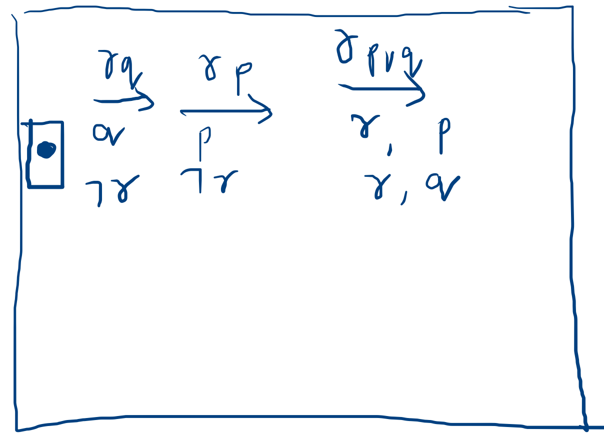
7. Show that the following is false in $C_2$

$$(p \vee q) \supset r \vdash (p \supset r) \wedge (q \supset r)$$

$(p \vee q) \supset r$

$\neg((p \supset r) \wedge (q \supset r))$

| | |
|---|---|
| $\neg(p \supset r)$ | $\neg(q \supset r)$ |
| $i\, r_p\, j$ | $i\, r_q\, k$ |
| $p, j$ | $q, k$ |
| $\neg r, j$ | $\neg r, k$ |
| $i\, r_{p \vee q}\, j$ | $i\, r_{p \vee q}\, j$ |
| $p \vee q, j$ | $p \vee q, j$ |
| $r, j$ | |
| $p, j$    $q, j$ | $p, j$    $q, j$ |




false in $C_2$ ✓

even in $S$

$C_1$ ✓ as well

$C_1 / C_2$

1. $p \supset q, i$
2. $\neg q, i \quad / \quad \neg q \supset \neg p$
3. $\neg(\neg q \supset \neg p) \quad$ DC
4. $i \, r_{\neg q} \, j$

$\neg q, j$

$p, j$

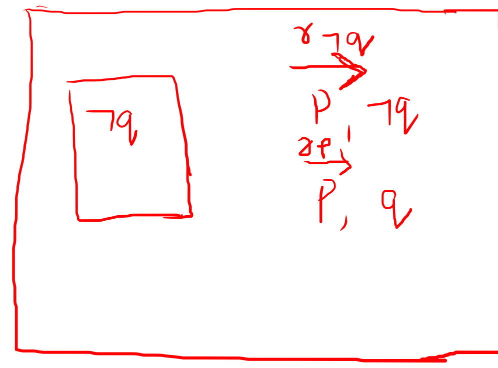
$i \, r_p \, k$

$p, k$

$q, k$

$\neg\neg q, i \qquad q, i$

$i \, r_{\neg q} \, i$




fails in $C_2$, $C_1$

(2)

$\Diamond p$
$p > q$ / $p > q$.
$\neg(p > q)$

1. $\Diamond p$
2. $p > q$ / $\neg(p > \neg q)$
3. $\neg\neg(p > \neg q)$ DC
4. $p > \neg q$
5. $i r j$ (as in Modal logic – Antecedent irrelevant)

$p, j$
$i r_p k$
$p, k$
$q, k$
$i r_p k$
$p$
$\neg q, k$

$\neg(p > q)$
$\equiv p > \neg q$
for slalnakers

$(p > q)$ $\neg(p > \neg q)$
are contradictory

$\neg(p > \neg q) = p > q$

It is valid in C itself
→ hence valid in $\boxed{\text{all}}$

③

$p \supset (q \supset r) \vDash q \supset (p \supset r)$ embedded conditionals

1 $p \supset (q \supset r), i$

2 $\neg(q \supset (p \supset r))$ DC

3 $i \, r_q \, j$

$q, j$ ✓

$\neg(p \supset r), j$

$j \, r_p \, k$

$p, k$

$\neg r, k$.

$i \, r_p \, k$

$p, k$

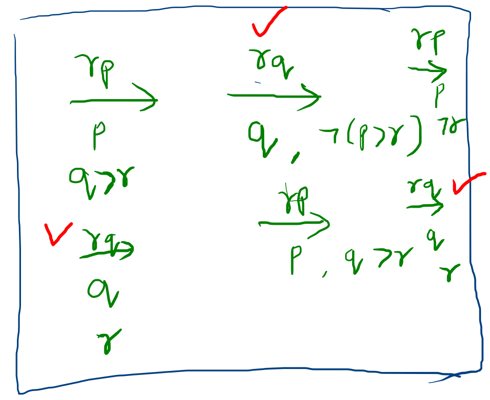
$q \supset r, k$

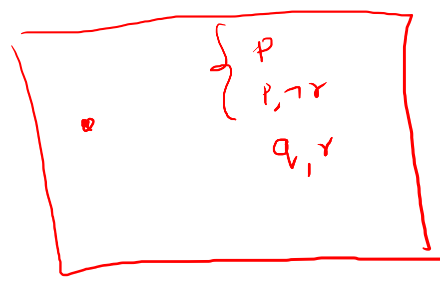
$k \, r_q \, l$

$q, l$

$r, l$.


Invalid in $C_1, C_2$



Eg: $p \supset (p \supset q) \models p \supset q$

1 $\quad p \supset (p \supset q)$

2 $\quad \neg(p \supset q)$

3 $\quad i \; r \; p \; j$

$p, j$

$\neg q, j$

$p \supset q, j$ $\quad$ from 1

$q, j$

$\times$

It is valid in C

hence valid in all

$=$

$X = (p \supset (q \supset (p \wedge q)))$, i

$\neg (p \supset (q \supset (p \wedge q)))$, i

$i \, r \, p \, j$

$p$, j

$\neg (q \supset (p \wedge q))$, j

$i \, r \, q \, k$

$q$, k ✓

$\neg (p \wedge q)$, k

$\neg p$, k | $\neg q$, k ×

$\neg p$, j

$j \, r \, p \, j$

$p$, j

$p$, i $\xrightarrow{rp}$ $p$ $\xrightarrow{rq}$ $q$, $\neg p$

Invalid in $C_1$, $C_2$

Ex: $A \wedge B \vdash A > B$

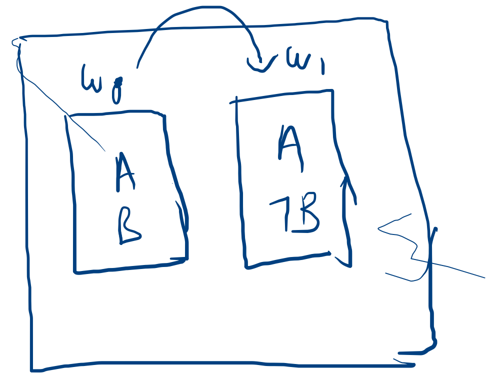
$A \wedge B \;, i$

$\neg(A > B) \;, i$

$i \; r_A \; j$

$A \;, j$

$\neg B \;, j$




in Lewis ($C_1$)

w0 is also taken into consideration as A is ~~true~~ there - hence the conditional holds in $C_1$

It does not hold in S'

② b sees 'a'

$K_b(K_a A \vee K_a \neg A)$

a sees b

$K_a(K_b B \vee K_b \neg B)$

Public announcement

$K_{a|b}(A \vee B)$

④

| P | Inter | External |
|---|---|---|
| 1 | 1 | 1 |
| 1/2 | 1/2 | 0 |
| 0 | 0 | 0 |

Other instances of LO:

$K_a\phi$
$K_a(\phi \to \psi)$
$\overline{\quad K_a\psi \quad}$

② $\vdash \phi \Rightarrow\ \vDash K_a\phi$

③ $\vDash \phi \to \psi \Rightarrow\ \vDash K_a\phi \to K_a\psi$

④ $K_a\phi \wedge K_a\psi \Rightarrow\ \vDash K_a(\phi\wedge\psi)$

⑤ $K_a\phi \to K_a(\phi \vee \psi)$

⑥ $S5 \nvDash \neg(K_a\phi \wedge K_a\neg\phi)$

SIDE A

**Part I.** TRUE OR FALSE QUESTIONS. 20M

1. The following argument is valid in Epistemic Logic- $KTD4$. $K_a(A \wedge B) \to K_a \wedge K_aB$

   A. True B. False

2. In Epstemic Logic *b sees a* is represented as $K_b(K_aB \vee K_a\neg B)$.

   A. True B. False

3. An instance of paradox of material implication, i.e., $p \to (q \to p)$, does not hold in Luckasewicz's three valued logic ($L_3^s$) Reason: in $L_3$ $P = 1/2$ $q = 1/2$ then $P \to q = 1$

   A. True B. False

4. Let $a_p$ is an assertion or truth operator in Bochvar's semantics. Then the following ( $B_3^I$ or $B_3^E$ ) formula is a tautology: $a_PP \vee a_P\neg P$ In $B_3^I$ — F, $B_3^E$ — T

   A. True B. False

5. Let $\phi$, $\psi$ be formulas in $LK$, and let $Ka$ be an epistemic operator for $a \in A$. Let $M$ be the set of all Kripke models, and $S5$ the set of Kripke models in which the accessibility relation is an equivalence. The following is an instance of Logical omniscience: $M \models (Ka\,\phi \wedge Ka\,\psi) \to Ka\,(\phi \wedge \psi)$

   A. True B. False

   In the Final Column of truth table → we should see only 1 or 1/2

6. We will say that a formula is a *quasi tautology* if it is never false. Based on this definition, $X \equiv (A \vee \neg A) \to (B \wedge \neg B)$ is a quasi tautology in LP Logic. When $A = 1$ $B = 0$, $X$ taken value 0

   A. True B. False

7. If roses are red and violets are blue, then roses are not red. The above statement interpreted in RM3: $(R \wedge B) \to \neg R$ is a quasi contradiction. See Graham Priest Pp no. 125

   A. True B. False

   $(P \wedge q) \vee (P \to \neg q)$

8. In Bochvar's internal three valued Logic, the statement *Either roses are red(p) and violets are blue(q), or roses are red only if violets aren't blue.* has a value 1/2 when both $p$ and $q$ takes value 1/2. See P. 80 Bergman - Many valued Logic

   A. True B. False

9. The following argument is invalid. *I know he is either in his room or in the library.* It follows from this that *Either I know he in his room or I know he is in the library.*

   A. True B. False

10. In any three valued logic, we say that a set of formulas ($\Gamma$) entails a formula $P$, whenever all of the formulas in $\Gamma$ are true, $P$ is true as well. It means, there is no truth value assignment on which all the formulas in $\Gamma$ have the value $T$, while $P$ has the value $F$ or $N$. So If $\Gamma \models_{K_3^s} P$ , then $\Gamma \models P$.

    A. True B. False

11. In Epistemic logic the following wff is valid. $E_G(\phi \wedge C_G\,\phi) \to C_G\,\phi$.

    A. True B. False

$\vdash K_i(A \wedge B) \to K_iA \wedge K_iB$

$\neg(K_i(A \wedge B) \to (K_iA \wedge K_iB))$

$K_i(A \wedge B)$

$\neg(K_iA \wedge K_iB)$

$\neg K_iA$ $\quad$ $\neg K_iB$

$L_i\neg A$ $\quad$ $L_i\neg B$

$q \to$ True

$(R \wedge B) \to \neg R$

RM3:

$R = 1 \quad B = 0$

then $(R \wedge B) \to \neg R$ takes value '1'

RM3

Any thing implies less than its value is takes value 'zero'

| $\to$ | 1 | 1/2 | 0 |
|---|---|---|---|
| 1 | 1 | 0 | 0 |
| 1/2 | 1 | 1/2 | 0 |
| 0 | 1 | 1 | 1 |

It is not a quasi contradiction as we find (zero), in the above instance

12. The well formed formula $[p \to (q \to p)] \to p \to q$ is a tautology in Bochvar's three valued logic ($B_3^I$).

    A. True B. False

There are no tautologies in $B_3^I$

13. The well formed formula $p \to (q \to p)$ is not a tautology in Kleene's weak three valued Logic ($K_3^w$).

    A. True B. False

$K_3^w$ and $B_3^I$ are same.

14. If there is a convention among a group that $\phi$, then everyone knows $\phi$ , everyone knows that everyone knows $\phi$, everyone knows that everyone knows that everyone knows $\phi$, and so on ad- infinitum. In such a case, we say that the group has common knowledge of $\phi$.

    A. True B. False

$C_G\phi \leftrightarrow E_G(\phi \wedge C_G\phi)$

$C_G\phi = E_G^k\phi$

15. *A student thinks that he can secure A in PHI455 course* can be interpreted in Kripke semantics as $(M, w) \models K_i\phi$ iff$(M, w_i) \models \phi$ for all $w1$ such that $w\, R_i w1$.

FALSE

16. $p, q \to \neg p \models \neg q$ holds(valid) in $LP$ but not in $RM3$.

    A. True B. False

$p \to q$

$\neg q$

$\neg p$

For validity Designated values should match.

17. Let $W = \{w, v\}$ and $Ri = \{(w, v), (w, w), (v, v), (v, w)\}$ and $p$ is true in $w$ where as it is false in $v$. The following well formed formula is true in $w$, i.e., $M, w \models \neg Ki\, p$.

    A. True B. False

18. Every one knows $\phi$ is represented as $\bigcap_{i \in A} K_i\, A$.

    A. True B. False

$\bigwedge_{i \in A} K_i A$ = (It is Conjunction)

19. The following example is an instance of distributive knowledge. If $a$ knows that every political leader is interested in nationalism, and $b$ knows that ND Modi is a political mass leader, then there is distributed knowledge among $a$ and $b$ that ND Modi is interested in Nationalism, even if none of the agents needs to know this.

    A. True B. False

20. Not every entailment that holds in classical propositional logic holds in $B3^I$ as well.

    A. True B. False

See PPT1 Bergman "many-valued logic"

21. The following instances of paradox of material implication

    1. $\neg p \models (p \to q)$
    2. $q \models (p \to q)$ are valid in

       A. L3

       B. L3, K3, but not $B_3^I$

       C. K3, $B_3^I$

       D. None of the above.

       E. All systems mentioned above.

See the table in PP: 126 GP

22. Which of the following well formed formulas are Quasi tautologies or Quasi contradictions in the three valued logic: LP and RM3:

    1. $p \wedge \neg p$
    2. $p \vee \neg p$
    3. $p \leftrightarrow \neg p$
    4. $p \to (q \to p)$

       A. RM3

       B. LP

       C. neither of them

       D. Both LP, RM3

       E. Your Answer (if any)

Quasi tautologies → you can have 1, 1/2 but not '0'

Quasi contradiction → you can have 0, 1/2 but not '1'

RM3 $x \to y$ $x < y$ takes value 0

$1/2 \to 0 = 0$

$1 \to 1/2 =$

23. Let $P$ be a set of atomic propositions and $A$ a set of agents (both enumerable). An epistemic model $M = \{W, Ri, V\}$, is defined as where $W = \{w, u, v\}$ is the non-empty set of possible worlds, $Ri \subseteq (WxW)$ is agent i's indistinguishability relation, and $Rj$ is agent's j's indistinguishability relation. $V$ is a valuation function. For both i, j the following are true in state w, u, v.

$K_i\varphi \equiv \neg M_i \neg \varphi$

$M_i\varphi \equiv \neg K_i \neg \varphi$

This Problem is similar to that of one we had it in the modal logic.

1. $M, w \models \{p, q\}$ and $M, u \models \{\neg p, q\}$ and $M, v \models \{p, \neg q\}$
2. Accessibility relations for $i$, $j$ are as follows:

   (a) $R_i = \{(w, w), (u, u), (w, u), (u, w)\}$

   (b) $R_j = \{(w, w), (v, v), (w, v), (v, w)\}$

Which of the following wffs are ture in a state $w$.

1. $K_i\, p \vee \neg K_i\, p$
2. $K_j\, p \vee K_j\, p$
3. $K_i\,(p \vee \neg p)$
4. $K_j\, p \vee \neg K_a \neg p$
5. $E_G\,(p \vee \neg p)$
6. $C_G\,[(p \wedge \neg p) \to q]$

   A. 1

   B. 3

   C. 1,3,5

   D. 5

E. 6
F. Your Answer (in case you dont find any answer).

24. The following inference is valid in the following three valued logic.

$$\{p \to q, (p \to \neg p) \to q\} \models q$$

A. L3
B. K3(Weak)
C. B3 (internal)
D. B3 external
E. All
F. None of the above.

$L_3, K_3, B_3$ - $\{1\}$
designated values

Only LP, $RM_3$
we have 2 designated values

$D_i = \{1, 1/2\}$

Means in a Row
Premises can take value 1
conclusion can take value 1/2
but it should not be 'zero'

for validity
= designated values of LHS and RHS should match.

**Part III.** Rough Work

**Part III.** Rough Work

**Part I11.** Rough Work