Mid Semester Examination: Summer Term

PHI455: Philosophical Logic

HSS, IIT Kanpur

General Instructions: Read carefully each question. Fill in your with a pen and circle the correct answer on paper as well. All your work must be done in these pages.

- You have up to 180 minutes.
- For each Wrong answer 0.25% marks will be deducted.
- Please ensure that you keep a copy of your rough work and save it somewhere for future reference. Save it with the course number and and your roll number.
- Every item on the test awards 2 points for each correct answer, for a maximum possible score of 60 points.
- Multiple choice questions may have more than one answer. Circle each of the correct answer.
- Each Question in part-B consists of 5 marks each. For each wrong answer(0.25%) 1.25 M will be deducted.

"SORRY FOR THE TYPOS"

SIDE A

Part I. TRUE OR FALSE QUESTIONS. 20M

1. The following argument is valid in K. *1. If Narendra Modi is the prime minister of India, then Narendra Modi is an Indian citizen. 2. Narendra Modi is possibly an Indian citizen. So, 3. Narendra Modi is necessarily an Indian citizen.*
   A. True B. False

2. The class of transitive frames is characterized by the formula $\Diamond\Diamond A \to \Diamond A$.
   A. True B. False

3. $\Box(\Box A \to A) \to \Box A$ is a theorem in B modal logic system.
   A. True B. False

4. The statement *Food and water are required for survival* is translated into Modal logic as $\Box((F \wedge W) \to S)$.
   A. True B. False

5. The following argument is valid in $S5$. *Necessarily, if snow is white(S), then snow is white or Green($S \vee G$). Snow is necessarily white. Therefore, necessarily grass is green(G).*
   A. True B. False

6. The statement *If today is Friday, then it is impossible that today is not Friday* is better translated as $(F \to \neg\Diamond F)$.
   A. True B. False

7. A formula $\Diamond p \leftrightarrow \Box p$ is not valid in all Kripke models and hence it is not valid in K.
   A. True B. False

8. It is not possible that it is raining if and only if it is necessary that it is not raining, is translated as $\neg\Diamond p \leftrightarrow \Box\neg p$.
   A. True B. False

9. *God's being merciful(M) is inconsistent with your imperfection(I) being incompatible with your going to heaven(H)* is translated in to modal logic as $\neg\Diamond(M \wedge \neg\Diamond(I \wedge H))$
   A. True B. False

10. The wff $\neg\Diamond(p \wedge (q \vee \Diamond r))$ and $\Box(\neg p \vee \neg q) \wedge (\Box\neg p \vee \Box\neg r)$ are logically equivalent to each other.
    A. True B. False

No F, W - no survival.

$\equiv \Box(\neg(F \wedge W) \to \neg S)$

$\equiv \Box(S \to (F \wedge W))$

Law of contraposition

For logical equivalence you need to check or show

$\neg(X \leftrightarrow Y) \to \bot$

$\Rightarrow \neg\neg(X \leftrightarrow Y)$

$\Rightarrow X \equiv Y$

Typo in the second formula → 1, 2- are clearly not logically equivalent.

1. F
2. T
3. F
4. F
5. F
6. F
7. T
9. T
10. F

1. Which of the following are well formed formulas in Modal Propositional Logic $\mathcal{L}$:
   - A. $\Box\Diamond\Box\Diamond p \wedge \Diamond(p \to q)$
   - B. $(p \to \Diamond\Box\Diamond(r \vee \Box\top))$
   - C. $p \neg\Diamond \leftrightarrow \neg\Diamond p \vee q \wedge r$
   - D. $(\Box\Diamond\Box\Diamond\Diamond\ (p \to\ q) \leftrightarrow \neg\Box\Diamond\Box\Diamond\Diamond(p \vee q)) \wedge \Box(p \to (q \to p))$
   - E. none of the above

2. Which of the following are true in minimal modal logic K:
   1. If P is necessary, then P is true.
   2. If P is possible then P is true.
   3. If P is true, then P is possible
   4. If P is not true, then P is not possible.
   5. If P is necessary, then it is impossible for P to be false
   6. If P is possible, then P is not a necessary falsehood.

   - A. 2
   - B. 1
   - C. 3
   - D. 4
   - E. 1,3,4,5,6
   - F. None of the above
   - G. All

Correct answer is (5,6) - but it's not there

3. Which of the following is false.
   1. $\vdash_K \Box(p \wedge q) \to (\Box p \wedge \Box q)$
   2. $\vdash_K \Box(p \to q) \to (\Box p \to \Box q)$
   3. $\vdash_K \Box(p \vee q) \to (\Box p \vee q)$
   4. $\vdash_K \Diamond(p \wedge q) \to (\Diamond p \wedge \Diamond q)$

   - A. 1
   - B. 2
   - C. 3
   - D. 4
   - E. None of the above
   - F. All are false.

4. A formula ϕ is satisfiable if it is satisfiable in some model, and valid if it is globally true in every model(valid in K). Which of the following formulas are satisfiable? Which ones are valid? Write your answer below.
   1. $\Box\top \vee \Diamond\neg p$
   2. $\Diamond p \to \Diamond\Diamond p$
   3. $(\Diamond p \wedge \Box q) \to \Box(p \wedge q)$

4: All are satisfiable, except the last one only Formula '1' is valid.

4. $\Diamond\Box\Diamond p \rightarrow \Box\Diamond\Box p$
5. $\Diamond\Box p \rightarrow \Box\Diamond p$
6. $\Box p \leftrightarrow \Diamond \neg p$

A. Satisfiability:
B. validity:
C. Unsatisfiable:

5. The following formulae are S4-theorems:

1. $\Box\Box p \leftrightarrow p$
2. $\Diamond\Diamond p \leftrightarrow \Diamond p$
3. $\Box\Diamond\Box\Diamond p \leftrightarrow \Box\Diamond p$
4. $\Diamond\Box\Diamond\Box p \leftrightarrow \Diamond\Box p$
5. $\Box\Diamond\Box p \leftrightarrow p$
6. $\Diamond\Box p \rightarrow \Box\Diamond p$

A. 5
B. 6
C. 1, 2, 3,
D. None of the above
E. All

6. Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authorities. Ajay said *Chetan did it.* Jayanth said *I did not do it.* Chetan said *Digvijay* did it. Digvijay said *Chetan lied when he said that I did it.*

1. If the authorities also know that exactly one of the four suspects is telling the *truth*, who did it?
   A. Jayanth
   B. Digvijay
   C. Chetan, Jayanth
   D. Chetan
   E. None of them
   F. It cannot be determined
   G. All

2. If the authorities also know that exactly one is *lying*, who did it?
   A. Jayanth
   B. Digvijay
   C. Chetan, Jayanth
   D. Chetan
   E. None of them
   F. It cannot be determined
   G. All

1. $D_1 \leftrightarrow \neg p$

2. $D_2 \leftrightarrow q$

3. $D_3 \leftrightarrow \neg q$

p, q, r .... Lady in D_1, D_2, D_3 respectively

$\neg p, \neg q, \neg r$ ... Tiger in $D_1 D_2 D_3$

7. There once was a king in a far off island who read , in which a prisoner must choose between two rooms, one containing a lady and the other a tiger. If he chooses the former, the prisoner marries the lady; while if he chooses the latter, he (probably) gets eaten by the tiger. The following signs placed on the two doors.There could be ladies in both rooms, tigers in both rooms, or a lady in one room and a tiger in the other. Additionally, At most only one statement is true. Where's the Lady?

   D1 A Tiger is in this room

   D2 A Lady is in this room

   D3 A Tiger is in room two

   A. Door 1

   B. both doors $D1 \wedge D2$

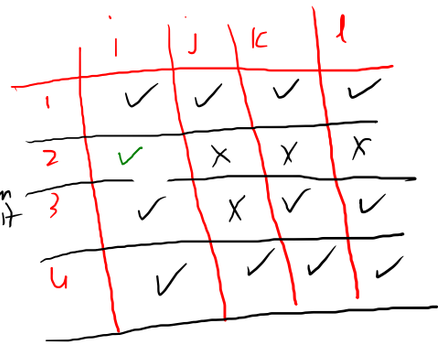
   C. Door 2

   D. Cant determine, Prisoner needs more information.

   E. None of the above

for answer in Page 6

Reflexive
→ But no harm in stating it



8. In the Kripke model $\{W, R, V\}$, Let $W = \{i, j, k, l\}$, $R = \{(i, j), (j, k), (k, l), (l, i), (l, j)(k, k), (l, j)\}$ and V is a valuation function defined as follows: $M, i \models \neg p, q, r$ and $M, j \models p, q, \neg r$ $M, k \models p, q, r$, $M, l \models \neg p, \neg q, \neg r$. Which of the following formulas are valid in the model (You need to check the truth of the formula under all worlds (i,j,k,l).

   1. $\Diamond(p \wedge q) \rightarrow \Diamond p \wedge \Diamond q$.
   2. $\Box\Box\Box\Diamond r$
   3. $\Diamond\Box p \vee \Diamond\Diamond\Box p$
   4. $\Box\Box\Diamond q$

   A. 1,2,3

   B. 1,3

   C. 2,3

   D. 3

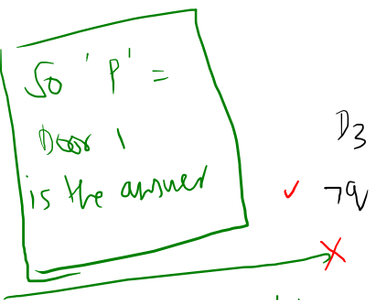
   E. none of the above

In options 3, 1 we can clearly say about 'p'

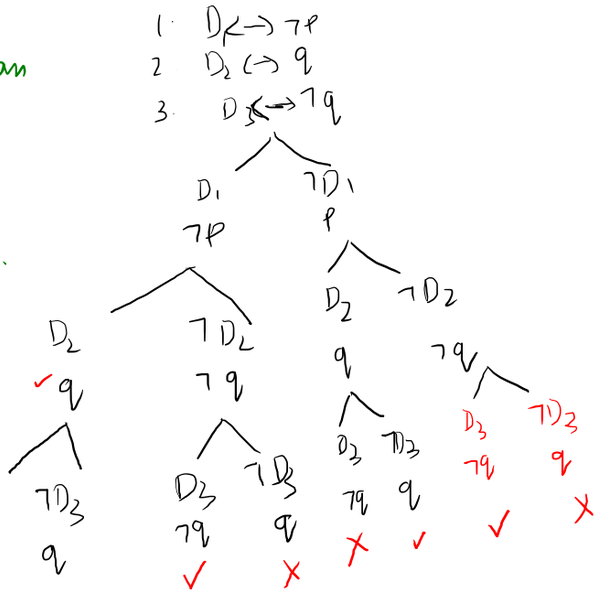
But we have q, ¬q

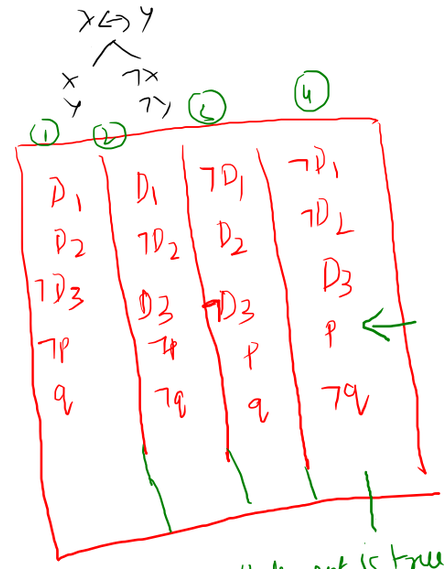
⇒ It cannot be determined.

So 'p' = Door 1 is the answer



Lot of reasoning!!





Atmost only one statement is true

we have Option 3, 4

option = both

Part III. Rough Work

Part III. Rough Work

Part I11. Rough Work