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Answer all questions. Write your answers in a separate booklet.
Max. Marks: 60 Time: 3 hours

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    The common knowledge assumption underlies all of the game theory
and much of economic theory.  Whatever be the model under discussion,
whether complete or incomplete information, consistent or inconsistent,
repeated or one shot, cooperative or non-cooperative, the model itself
must be assumed common knowledge; otherwise, the model is insufficiently
specified and the analysis incoherent.
.  .  .  .  .  .  Robert Aumann
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1. **Answer whether the following statements are True or False**[1] Justify it preferably with a short note while making your rough work reflected in the answer sheet. (20)

   1. The following argument is valid in conditional logic $C+$. *If I were to kick the door, my foot would hurt. If my foot were to hurt, I would be sad. So, if I were to kick the door, I would be sad.*
   2. An agent A knows that B knows whether ϕ is translated as $K_A\,(K_B\,\phi \wedge K_B\,\neg\phi)$.
   3. The following wff hold: $p \wedge \neg p \;\models_{LP}\; q$ in the three valued logic K_3^s (Kleene's three valued logic)
   4. Stalnaker's conditional logic denies the following assumption: *For every world w and proposition A of a conditional $A > B$ there is at most one A world minimally different from w.*
   5. The statement *I shall be in native place on May 25, 2018 if and only if I shall be in my native on May 25, 2018* is a tautology in Luckaseiwicz three valued logic (L3).
   6. The following well formed formulas are equivalent in Epistemic logic: $L_a\,p \wedge L_a\,\neg p$ is equivalent to $\neg\,L_a\,\neg p \wedge \neg\,K_a\,p$
   7. Given a model $M = \{W, R1 \ldots\ldots, Rn, v\}$, we can define the truth conditions for common knowledge $CG\,A$ as follows: $M, w \models CG\,A$ iff $M, w \models EG\,KA$ for every $k \geq 1$.
   8. In Epistemic logic, if it is the case that ϕ is self-evident, in the case that if it is true, then everyone knows it, and in addition, if ϕ is true, then everyone knows that ϕ, we can show by induction that that if ϕ is true, then so is$E_A^k\,(\phi \wedge \psi)$ for all k.
   9. Suppose an agent A knows some fact m $(K_A\,m)$, Now suppose$A$ emails this message to another agent B, and B reads it that evening. we then have $K_B\,m \wedge K_B\,K_A\,m$.

10. In RM3 valued Logic, $(\neg p \to \neg q) \to \neg(p \to q)$ takes the value 1/2 when both constituents takes value 1/2.

2. Answer the following questions (40)

    1. The following puzzle is a variant of Muddy-Children puzzle. Draw Kripke diagram for the following puzzle and analyze how wise men figure out what was there on their forehead.[4M]

       The king summons three of his wise men and pastes on their forehead a small black dot. The wise men are facing one another so that they can all see the dot on each others forehead, but they do not know the colour of the dot on their own. He tells them, "On each of your foreheads, I have placed a dot which is either white or black. I have put a black dot on at least one of your foreheads. Do any of you know what the colour on your forehead is?" The wise man all answer in unison, "No, your majesty".
       The king asks, "Do you now know what the colour on your forehead is?"
       They again answer in unison, "No, your majesty."
       The king asks, "Do you now know what the colour on your forehead is?"
       All three wise men answer, "Your majesty, I have a black dot on my forehead."
       How do the wise men determine this?

    2. Determine whether the following wffs hold in each of $C2$, $C1$[10M] [2]

       (a) $[p > (q > r)] \to (p \wedge q) > r$

       (b) $p > (q \wedge r) \to [(p > q) \wedge (p > r)]$

    3. Using truth-tables, determine which of the following formulas are tautologies in L3, K3 (note: they are all classical tautologies). What happens when the number of designated values are taken as 1, $\frac{1}{2}$? [10M]

       (a) $\neg p \to (p \to q)$

       (b) $(p \wedge q) \to (p \vee q)$

    4. Show how the following sentences are handled in many-valued Logic. You may choose any many-valued logic (L3, K3, LP etc) [10M]

       (a) Set(Z) of all sets that are not members of themselves ($Z \in Z \leftrightarrow Z \notin Z$).

       (b) Thiggledy piggledy if and only if I shall be in my native place on May 25, 2018.

Notes[1]

1. C: $A > B, i \; ir_A j \Downarrow B, j$
2. C: $\neg(A > B), i \Downarrow ir_A j \, \neg B, j.$
3. C+: $\neg(A > B), i \Downarrow ir_A j \, A, j \, B, j.$

4. C+: Either A is false in i or A is true provided $i\,rAi$.

2

1. $[A]$ is a set of worlds in which A is true. $f_A(w) = \{x \in W : wR_Ax\}$ and $f_A(w) \subseteq [A]$.
2. If $w \in [A]$, then $w \in f_A(w)$.
3. If $[A] \neq \emptyset$,then $f_A(w) \neq \emptyset$.
4. If $f_A(w) \subseteq [B]$ and $f_B(w) \subseteq [A]$ then $f_A(w) = f_B(w)$
5. If $f_A(w) \bigcap [B] \neq \emptyset$, then $f_{A \bigcap B}(w) \subseteq f_A(w)$.
6. **C2:** If $x \in f_A(w)$ and $y \in f_A(w)$, then $x = y$, where f_A is a singleton.
7. **Lewis (C1):** (a-e)+ **P**: If $[A] = [B]$, then $f_A(w) = f_B(w)$