**Examples 8.2, 8.7, 8.12, and 8.15 in the textbook**

Romeo and Juliet start dating, but Juliet will be late on any date by a random amount $X$, uniformly distributed over the interval $[0, \theta]$. The parameter $\theta$ is unknown and is modeled as the value of a random variable $\Theta$, uniformly distributed between zero and one hour.

(a) Assuming that Juliet was late by an amount $x$ on their first date, how should Romeo use this information to update the distribution of $\Theta$?

(b) How should Romeo update the distribution of $\Theta$ if he observes that Juliet is late by $x_1, \ldots, x_n$ on the first $n$ dates? Assume that Juliet is late by a random amount $X_1, \ldots, X_n$ on the first $n$ dates where, given $\theta$, $X_1, \ldots, X_n$ are uniformly distributed between zero and $\theta$ and are conditionally independent.

(c) Find the MAP estimate of $\Theta$ based on the observation $X = x$.

(d) Find the LMS estimate of $\Theta$ based on the observation $X = x$.

(e) Calculate the conditional mean squared error for the MAP and the LMS estimates. Compare your results.

(f) Derive the linear LMS estimator of $\Theta$ based on X.

(g) Calculate the conditional mean squared error for the linear LMS estimate. Compare your answer to the results of part (e).