1. Let Z be a continuous random variable with probability density function

$$f_z(z) = \begin{cases} \gamma(1+z^2), & \text{if } -2 < z < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) For what value of γ is this possible?

(b) Find the cumulative distribution function of Z.

$$\gamma\left(z + \frac{1}{3}z^3\right)$$

a) $$1 \overset{?}{=} \gamma \int_{-2}^{1} (1+z^2)\,dz = \int_{-2}^{1} \gamma\,dz + \gamma \int_{-2}^{1} z^2\,dz = \underbrace{\left.\gamma z + \gamma \frac{1}{3} z^3 \right|_{-2}^{1}} = 3\gamma + \gamma \frac{1}{3}(1+2^3)$$

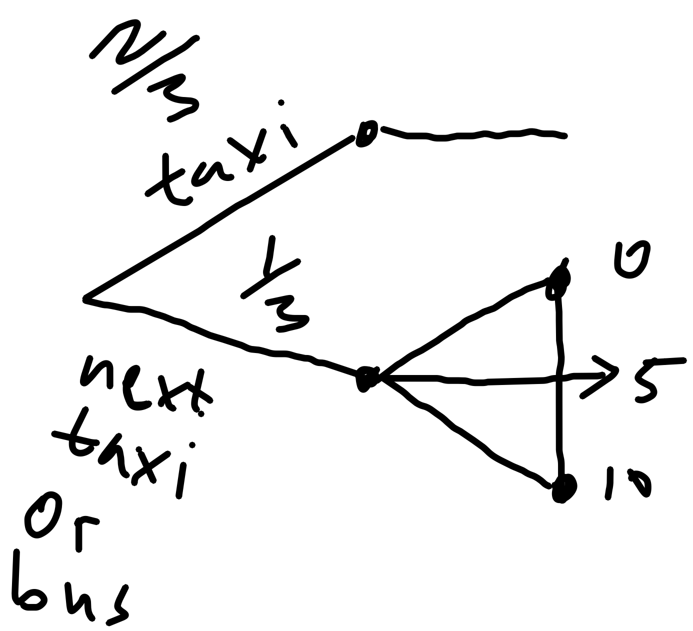
$$1 = 3\gamma + \frac{1}{3}\gamma 9 = 3\gamma + 3\gamma = 6\gamma$$

$$\Rightarrow \gamma = \frac{1}{6} = \frac{1}{6}$$

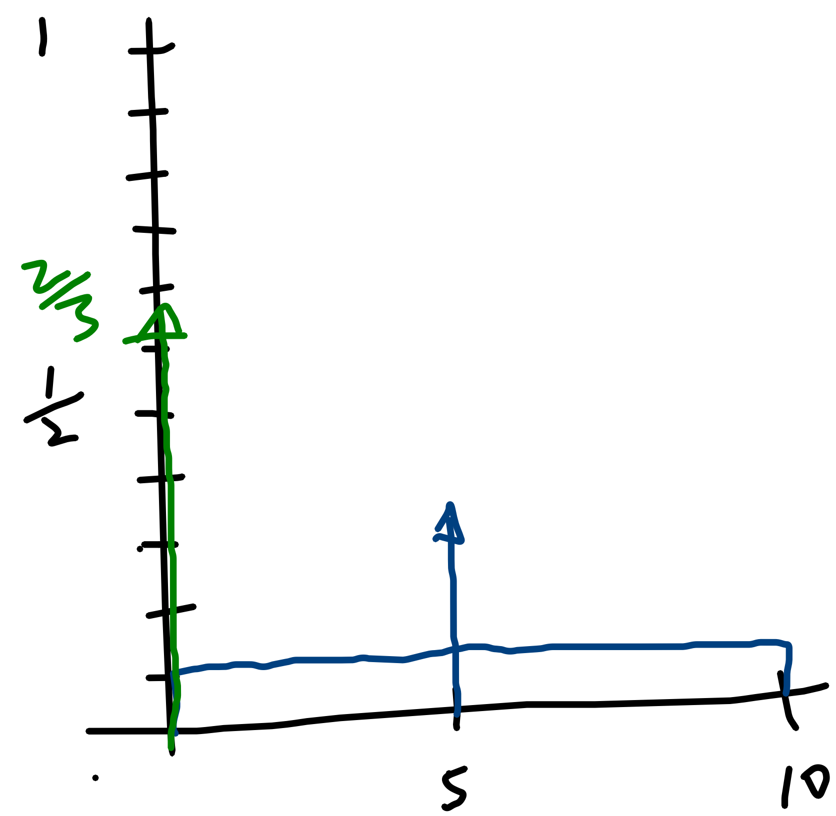
b) $$F_z(z) = \begin{cases} & z < -2 \\ \frac{1}{6}\left(1 + \frac{1}{3}z^3\right), & \text{if } -2 \leq z \leq 1 \\ 1 & z > 1 \end{cases}$$

2. Problem 3.9, pages 186–187 in the text.

The taxi stand and the bus stop near Al's home are in the same location. Al goes there at a given time and if a taxi is waiting, (this happens with probability 2/3) he boards it. Otherwise he waits for a taxi or a bus to come, whichever comes first. The next taxi will arrive in a time that is uniformly distributed between 0 and 10 minutes, while the next bus will arrive in exactly 5 minutes. Find the CDF and the expected value of Al's waiting time.



$$E[T] = \frac{2}{3}\cdot 0 + \frac{1}{3}\int_0^{10} t\cdot f_t(t)\,dt + \frac{1}{3}\cdot 5 = \frac{6}{3} = 2$$



3. Let λ be a positive number. The continuous random variable X is called **exponential** with parameter λ when its probability density function is

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function (CDF) of X.

(b) Find the mean of X.

(c) Find the variance of X.

(d) Suppose X_1, X_2, and X_3 are independent exponential random variables, each with parameter λ. Find the PDF of $Z = \max\{X_1, X_2, X_3\}$.

(e) Find the PDF of $W = \min\{X_1, X_2\}$.

a) $F_X(x) = \int_0^x \lambda e^{-\lambda x}\, dx = \frac{\lambda e^{-\lambda x}}{-\lambda}\Big|_0^x = -e^{-\lambda x}\Big|_0^x = -e^{-\lambda x} + e^0 = 1 - e^{-\lambda x}$

b) $E[X] = \int^{\infty} x \cdot \lambda e^{-\lambda x}\, dx = x \cdot \int_0^{\infty} \lambda e^{-\lambda x} dx + \int_0^{\infty} e^{-\lambda x} dx = -x e^{-\lambda x} +$

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