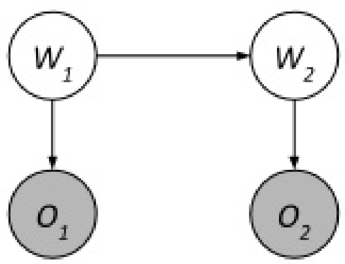
1 HMMs

Consider the following Hidden Markov Model.



W_1	$P(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$P(W_{t+1}\|W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t\|W_t)$
0	A	0.9
0	B	0.1
1	A	0.5
1	B	0.5

Suppose that we observe $O_1 = A$ and $O_2 = B$.
Using the forward algorithm, compute the probability distribution $P(W_2|O_1 = A, O_2 = B)$ one step at a time.

1. Compute $P(W_1, O_1 = A)$.

$$= P(O_1=A \mid W_1) \cdot P(W_1)$$

$$\Rightarrow P(W_1=1, O_1=A) = 0.5 \cdot 0.7 = 0.35$$

$$P(W_1=0, O_1=A) = 0.9 \cdot 0.3 = 0.27$$

2. Using the previous calculation, compute $P(W_2, O_1 = A)$.

$$= \sum_{W_1} P(W_2 \mid W_1)\, P(W_1, O_1=A)$$

$$\Rightarrow P(W_2=1, O_1=A) =$$

$$= P(W_2=1 \mid W_1=1) \cdot P(W_1=1, O_1=A)$$

$$+ P(W_2=1 \mid W_1=0) \cdot P(W_1=0, O_1=A)$$

$$= 0.2 \cdot 0.35 + 0.6 \cdot 0.27$$

$$= 0.07 + 0.162$$

3. Using the previous calculation, compute $P(W_2, O_1 = A, O_2 = B)$.

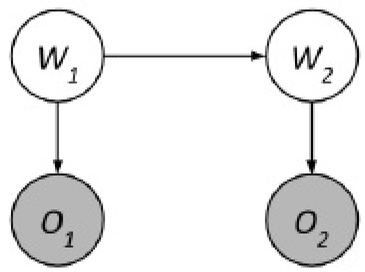
$$= P(W_2, O_1=A) \cdot P(O_2=B \mid W_2)$$

4. Finally, compute $P(W_2|O_1 = A, O_2 = B)$.

$$= \frac{P(W_2, O_1=A, O_2=B)}{\sum_{W_2} P(W_2, O_1=A, O_2=B)}$$

2 Particle Filtering

Let's use Particle Filtering to estimate the distribution of $P(W_2|O_1 = A, O_2 = B)$. Here's the HMM again:



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1	1	0.2

W_t	O_t	$P(O_t\|W_t)$
0	A	0.9
0	B	0.1
1	A	0.5
1	B	0.5

We start with two particles representing our distribution for W_1.
$P_1 : W_1 = 0$
$P_2 : W_1 = 1$
Use the following random numbers to run particle filtering:

[~~0.22~~, ~~0.05~~, ~~0.33~~, ~~0.20~~, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96]

1. **Observe:** Compute the weight of the two particles after evidence $O_1 = A$.

   $w(P_1) = P(O_1=A|W_1=0) = 0.9 \rightarrow \frac{0.9}{0.9+0.5} = \frac{0.9}{1.4} = 0.64$

   $w(P_2) = P(O_1=A|W_1=1) = 0.5 \rightarrow \frac{0.5}{1.4} = 0.34$

2. **Resample:** Using the random numbers, resample P_1 and P_2 based on the weights.

   weights = $\{0.64, 0.34\}$

   P_1 = sample(weights, 0.22) = 0

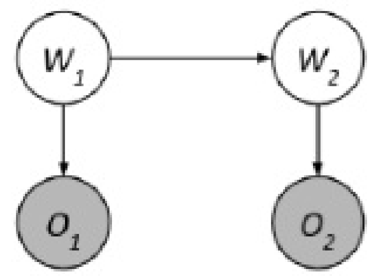
   P_2 = sample(weights, 0.05) = 0

3. **Elapse Time:** Now let's compute the elapse time particle update. Sample P_1 and P_2 from applying the time update.

   P_1 = sample($P(W_2|W_1=0)$, 0.33) = 0

   P_2 = sample($P(W_2|W_1=0)$, 0.20) = 0

Let's use Particle Filtering to estimate the distribution of $P(W_2|O_1 = A, O_2 = B)$. Here's the HMM again:



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W_t	O_t	$P(O_t\|W_t)$
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0	B	0.1
1	A	0.5
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We start with two particles representing our distribution for W_1.
$P_1 : W_1 = 0$
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Use the following random numbers to run particle filtering:

[~~0.22~~, ~~0.05~~, ~~0.33~~, ~~0.20~~, ~~0.84~~, 0.54, 0.79, 0.66, 0.14, 0.96]

4. **Observe:** Compute the weight of the two particles after evidence $O_2 = B$.

$w(P_1) = P(O_2=B|W_2=0) = 0.1 \rightarrow \frac{0.1}{0.2} = 0.5$

$w(P_2) = P(O_2=B|W_2=0) = 0.1 \rightarrow$ —"— $= 0.5$

5. **Resample:** Using the random numbers, resample P_1 and P_2 based on the weights.

weights = $\{0.5, 0.5\}$

P_1 = sample(weights, 0.84) = 0

P_2 = —"— , 0.54) = 0

6. What is our estimated distribution for $P(W_2|O_1 = A, O_2 = B)$?

$$P(W_2=0|O_1=A, O_2=B) = \frac{\Sigma w(W_2=0)}{\Sigma w(all)} = \frac{0.2}{0.2} = 1$$

$$P(W_2=1|O_1=A, O_2=B) = \frac{\Sigma w(W_2=1)}{\Sigma w(all)} = \frac{0}{0.2} = 0$$