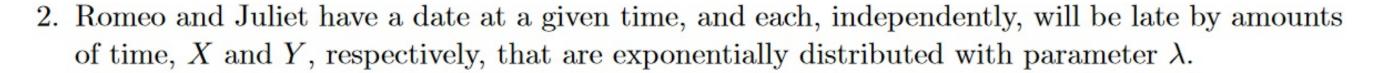
1. Show  $\rho(aX + b, Y) = \rho(X, Y)$ .



- (a) Find the PDF of Z = X Y by first finding the CDF and then differentiating.
- (b) Find the PDF of Z by using the total probability theorem.

3. Problem 4.16, page 248 in text.

Let X and Y be independent standard normal random variables. The pair (X,Y) can be described in polar coordinates in terms of random variables  $R \ge 0$  and  $\Theta \in [0,2\pi]$ , so that

$$X = R\cos\Theta, \quad Y = R\sin\Theta.$$

Show that R and  $\Theta$  are independent (i.e. show  $f_{R,\Theta}(r,\theta) = f_R(r)f_{\Theta}(\theta)$ ).

- (a) Find  $f_R(r)$ .
- (b) Find  $f_{\Theta}(\theta)$ .
- (c) Find  $f_{R,\Theta}(r,\theta)$ .

4. Problem 4.20, page 250 in text. Schwarz inequality. Show that for any random variables X and Y, we have

 $(\mathbf{E}[XY])^2 \le \mathbf{E}[X^2]\mathbf{E}[Y^2].$