1. Consider an experiment in which a fair four-sided die (with faces labeled 0, 1, 2, 3) is thrown once to determine how many times a fair coin is to be flipped. In the sample space of this experiment, random variables N and K are defined by

- N = the result of the die roll
- K = the total number of heads resulting from the coin flips

(a) Determine and sketch $p_N(n)$

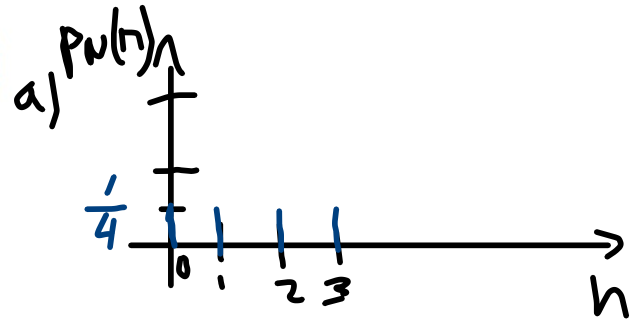
(b) Determine and tabulate $p_{N,K}(n,k)$

(c) Determine and sketch $p_{K|N}(k \mid 2)$

(d) Determine and sketch $p_{N|K}(n \mid 2)$

$p_K(k)$

a)



$p_K(k)$	k \ n	0	1	2	3
$\frac{1}{32}$	3	0	0	0	$\frac{1}{32}$
$\frac{5}{32}$	2	0	0	$\frac{1}{16}$	$\frac{3}{32}$
$\frac{3}{32} = \frac{11}{32}$	1	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{32}$
$\frac{8}{32} + \frac{4}{32} + \frac{2}{32} + \frac{1}{32} = \frac{15}{32}$	0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$
		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

b)

k \ n	0	1	2	3
3	0	0	0	$\frac{1}{32}$
2	0	0	$\frac{1}{16}$	$\frac{3}{32}$
1	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{32}$
0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

$$P_r(K=k, N=n) = P_r(k|n) \cdot P_r(n)$$

$$= p^k(1-p)^{n-k}\binom{n}{k} \cdot \frac{1}{n} = p^n \binom{n}{k} \cdot \frac{1}{n}$$

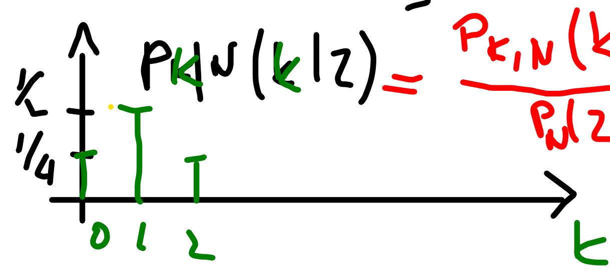
$$P_r(K=0, N=3) = \frac{1}{2^3}\binom{3}{0}\frac{1}{4} = \frac{1}{8}\frac{1}{4} = \frac{1}{32}$$

$$P_r(K=1, N=3) = \frac{1}{2^3}\binom{3}{1}\frac{1}{4} = \frac{1}{2^3}\frac{3!}{1!\,2!}\frac{1}{4} = \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{32}$$

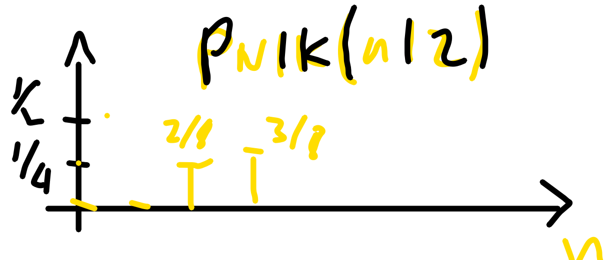
$$P_r(K=2, N=3) = \frac{1}{2^3}\binom{3}{2}\frac{1}{4} = \frac{3}{8}\frac{1}{4} = \frac{3}{32}$$

$$P_r(K=3, N=3) = \frac{1}{2^3}\binom{3}{3}\frac{1}{4} = \frac{1}{8} \cdot \frac{1}{4} = \frac{1}{32}$$

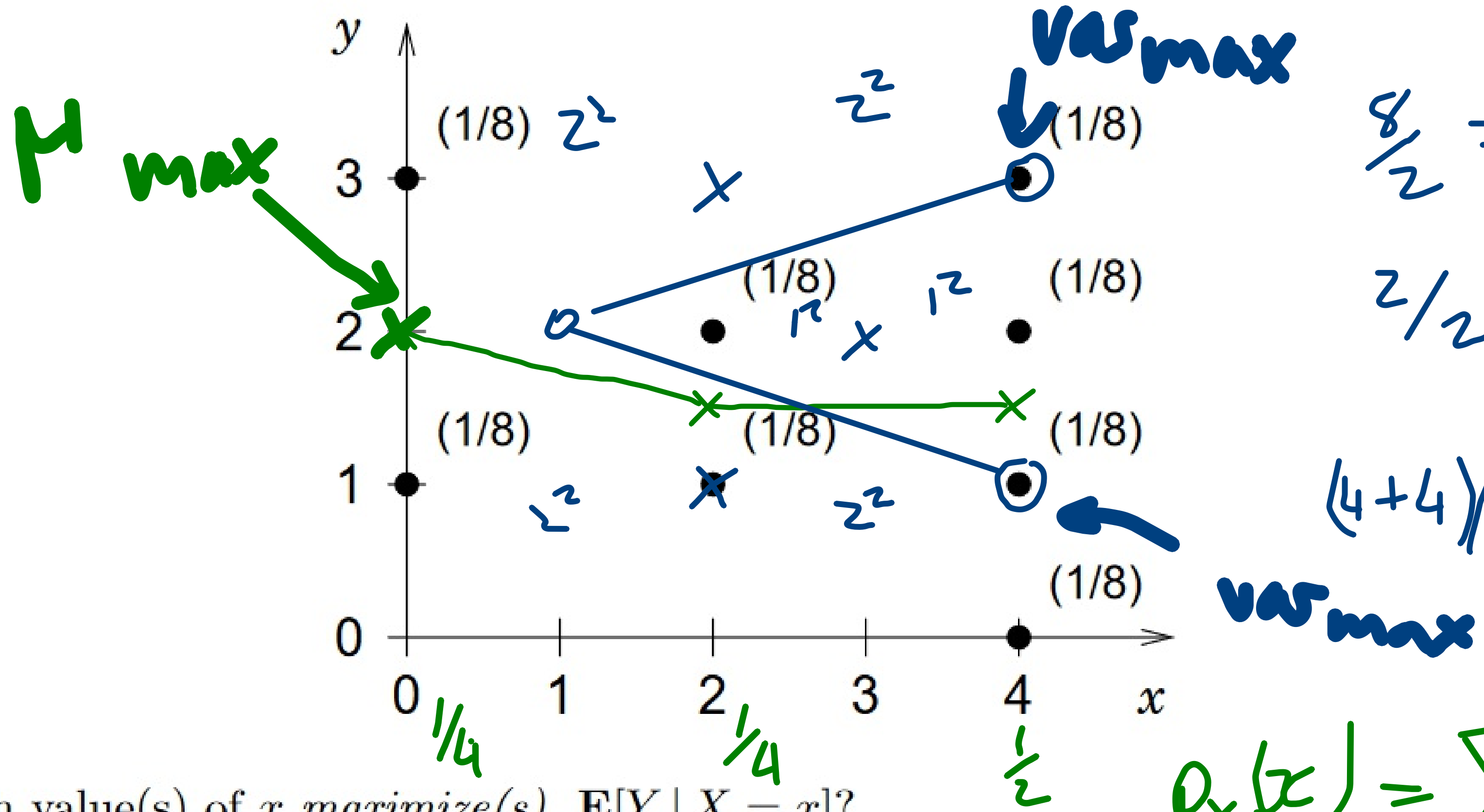
c) $p_{K|N}(k|2) = \frac{p_{K,N}(k,2)}{p_N(2)}$



d) $p_{N|K}(n|2)$



2. Consider an outcome space comprising eight equally likely event points, as shown below:



$8/2 = 4$

$2/2 = 1$

$(4+4)/2 = \frac{8}{2} = 4$

$p_X(x) = \sum_y p_{Y,X}(y,x)$

(a) Which value(s) of x *maximize(s)* $\mathbf{E}[Y \mid X = x]$?

(b) Which value(s) of y *maximize(s)* $\text{var}(X \mid Y = y)$?

(c) Let $R = \min(X, Y)$. Prepare a neat, fully labeled sketch of $p_R(r)$.

(d) Let A denote the event $X^2 \geq Y$. Determine numerical values for the quantities $\mathbf{E}[XY]$ and $\mathbf{E}[XY \mid A]$.

a) $E[Y|X=x] = \sum_y y \Pr(Y,x) \frac{1}{\Pr(x)}$

$E[Y|X=0] = \left(\frac{1}{8}+\frac{3}{8}\right)/\frac{1}{4} = \frac{4}{8}\cdot 4 = \underline{\underline{2}}$

b) $var(X) = E_X((X - E_X(X))^2) = E_X((X-\mu)^2)$

Tutorial 3

2. Problem 2.40, page 133 in the text.
   A particular professor is known for his arbitrary grading policies. Each paper receives a grade from the set $\{A, A-, B+, B, B-, C+\}$, with equal probability, independently of other papers. How many papers do you expect to hand in before you receive each possible grade at least once?