1 Probability

Use the probability table to calculate the following values:

X_1	X_2	X_3	$P(X_1, X_2, X_3)$
0	0	0	0.05
1	0	0	0.1
0	1	0	0.4
1	1	0	0.1
0	0	1	0.1
1	0	1	0.05
0	1	1	0.2
1	1	1	0.0

1. $P(X_1 = 1, X_2 = 0)$ $= 0.1 + 0.05 = 0.15$

2. $P(X_3 = 0)$ $= 0.05 + 0.1 + 0.4 + 0.1 = 0.65$

3. $P(X_2 = 1 | X_3 = 1)$ $= \dfrac{P(X_2=1, X_3=1)}{P(X_3=1)}$

$$= \frac{0.2}{0.1+0.5+0.2+0.0} = \frac{0.2}{0.35} = 0.57$$

4. $P(X_1 = 0 | X_2 = 1, X_3 = 1)$ $= \dfrac{P(X_1=0, X_2=1, X_3=1)}{P(X_2=1, X_3=1)}$

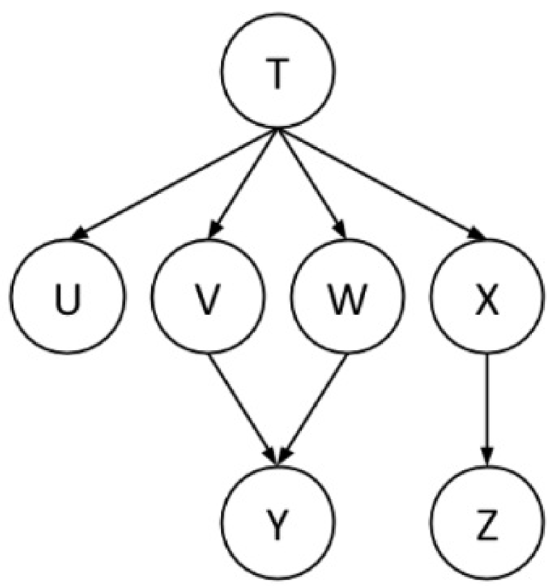
$$= \frac{0.2}{0.2+0.0} = 1$$

5. $P(X_1 = 0, X_2 = 1 | X_3 = 1)$ $= \dfrac{P(X_1=0, X_2=1, X_3=1)}{P(X_3=1)}$

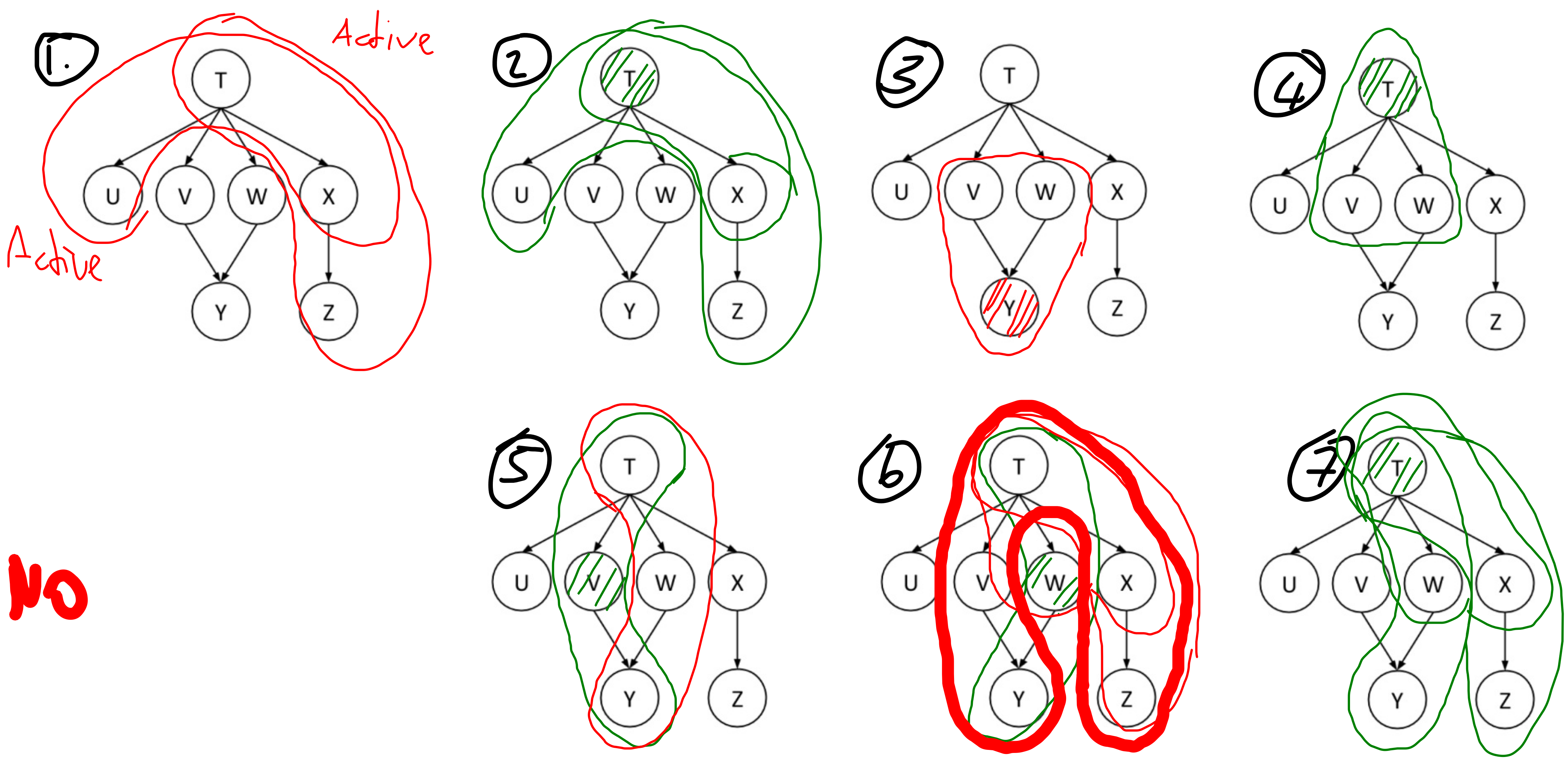
$$= \frac{0.2}{0.1+0.05+0.2+0.0} = \frac{0.2}{0.35} = 0.57$$

2 D-Separation

Indicate whether each of the following conditional independence relationships is guaranteed to be true in the Bayes Net below. If the independence relationship does not hold, identify all active (d-connected) paths in the graph.



1. $U \perp X$ no
2. $U \perp X | T$ Yes
3. $V \perp W | Y$ no
4. $V \perp W | T$ yes
5. $T \perp Y | V$ no
6. $Y \perp Z | W$ ~~yes~~ NO
7. $Y \perp Z | T$ yes



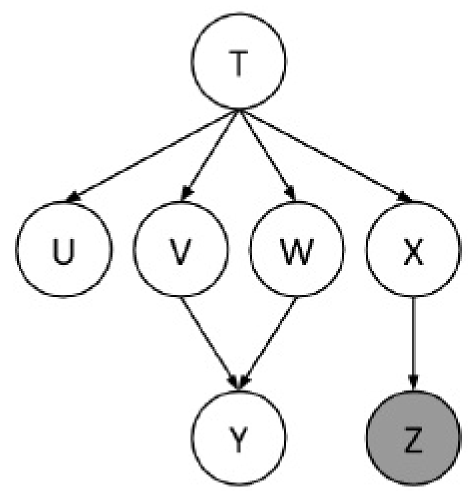
3 Variable Elimination

Using the same Bayes Net (shown below), we want to compute $P(Y \mid +z)$. All variables have binary domains. Assume we run variable elimination to compute the answer to this query, with the following variable elimination ordering: X, T, U, V, W.

Complete the following description of the factors generated in this process:

After inserting evidence, we have the following factors to start out with:

$$P(T), P(U|T), P(V|T), P(W|T), P(X|T), P(Y|V,W), P(+z|X)$$



(a) When eliminating X we generate a new factor f_1 as follows, which leaves us with the factors:

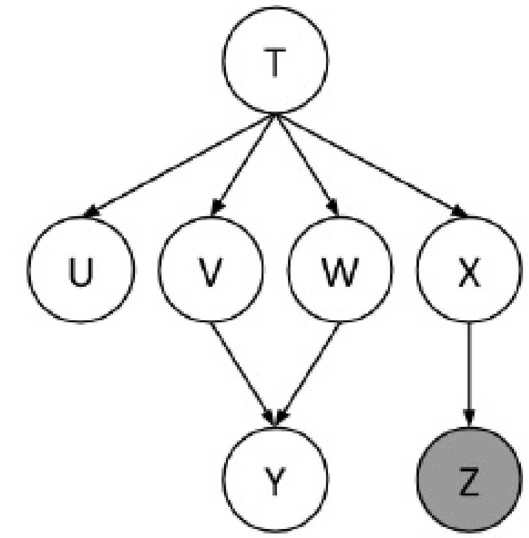
$$f_1(+z|T) = \sum_x P(x|T)P(+z|x) \qquad P(T), P(U|T), P(V|T), P(W|T), P(Y|V,W), f_1(+z|T)$$

(b) When eliminating T we generate a new factor f_2 as follows, which leaves us with the factors:

$$f_2(u,v,w,+z) = \sum_t P(t) \cdot P(u|t) \cdot P(v|t) \cdot P(w|t) \cdot f_1(+z|t)$$

$$P(Y|V,W), f_2(u,v,w,+z)$$

$P(Y \mid V, W),\ f_2(u, V, W, +z)$



(c) When eliminating U we generate a new factor f_3 as follows, which leaves us with the factors:

$$f_3(V, W, +z) = \sum_u f_2(u, W, V, +z)$$

$P(Y \mid V, W),\ f_3(V, W, +z)$

(d) When eliminating V we generate a new factor f_4 as follows, which leaves us with the factors:

$$f_4(W, +z, Y) = \sum_v f_3(v, W, +z) \cdot P(Y \mid v, W)$$

(e) When eliminating W we generate a new factor f_5 as follows, which leaves us with the factors:

$$f_5(+z, Y) = \sum_w f_4(w, +z, Y)$$

(f) How would you obtain $P(Y \mid +z)$ from the factors left above:

$$P(Y \mid +z) = \frac{f_5(+z, Y)}{P(+z)} = \frac{f_5(+z, Y)}{\sum_y f_5(+z, y)}$$

(g) What is the size of the largest factor that gets generated during the above process?

2^3

(m) Does there exist a better elimination ordering (one which generates smaller largest factors)?

X, V, W, T, U