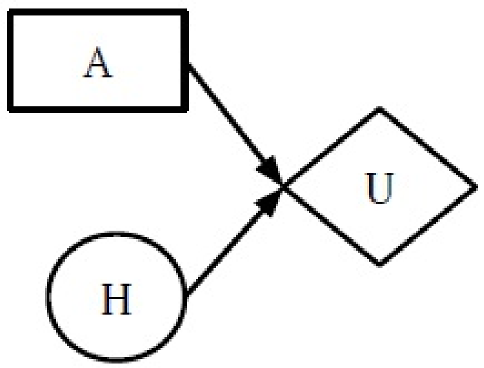
Q1. Decision Networks

After years of battles between the ghosts and Pacman, the ghosts challenge Pacman to a winner-take-all showdown, and the game is a coin flip. Pacman has a decision to make: whether to accept the challenge (*accept*) or decline (*decline*). If the coin comes out heads ($+h$) Pacman wins. If the coin comes out tails ($-h$), the ghosts win. No matter what decision Pacman makes, the outcome of the coin is revealed.



H	$P(H)$
+h	0.5
-h	0.5

H	A	U(H,A)
+h	*accept*	100
-h	*accept*	-100
+h	*decline*	-30
-h	*decline*	50

(a) Maximum Expected Utility

Compute the following quantities:

$EU(accept) =$ $\sum_h P(H=h) \cdot U(h, accept) = 0.5 \cdot 100 + 0.5(-100) = 0$

$EU(decline) =$ $\sum_h P(H=h) \cdot U(h, decline) = 0.5 \cdot (-30) + 0.5 \cdot 50 = 10$

$MEU(\{\}) =$ $\max_a EU(a) = \max(0, 10) = 10$

Action that achieves $MEU(\{\}) =$ $\arg\max_a EU(a) =$ decline

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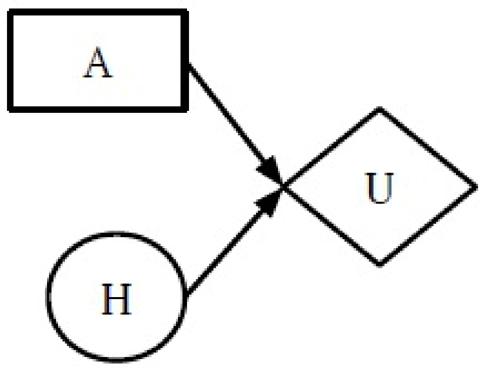

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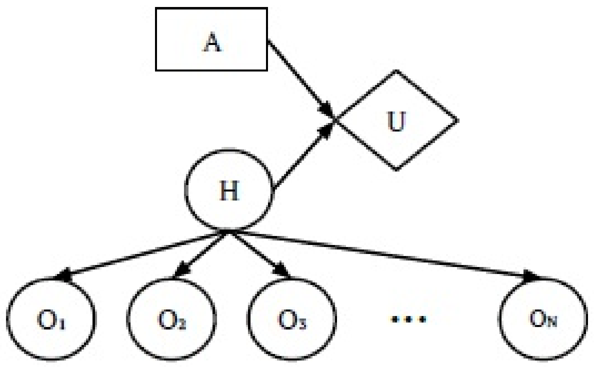
(b) **VPI relationships** When deciding whether to accept the winner-take-all coin flip, Pacman can consult a few fortune tellers that he knows. There are N fortune tellers, and each one provides a prediction O_n for H.

For each of the questions below, select **all** of the VPI relations that are guaranteed to be true, or select *None of the above.*

(i) In this situation, the fortune tellers give perfect predictions.
Specifically, $P(O_n = +h \mid H = +h) = 1$, $P(O_n = -h \mid H = -h) = 1$, for all n from 1 to N.



☐ $\text{VPI}(O_1,\ O_2) \geq \text{VPI}(O_1) + \text{VPI}(O_2)$

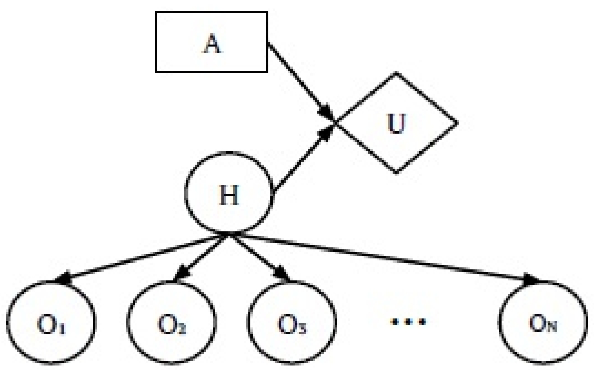
☐ $\text{VPI}(O_i) = \text{VPI}(O_j)$ where $i \neq j$

☐ $\text{VPI}(O_3 \mid O_2,\ O_1) > \text{VPI}(O_2 \mid O_1)$.

☐ $\text{VPI}(H) > \text{VPI}(O_1, O_2, \ldots O_N)$

☐ None of the above.

(ii) In another situation, the fortune tellers are pretty good, but not perfect.
Specifically, $P(O_n = +h \mid H = +h) = 0.8$, $P(O_n = -h \mid H = -h) = 0.5$, for all n from 1 to N.



☐ $\text{VPI}(O_1,\ O_2) \geq \text{VPI}(O_1) + \text{VPI}(O_2)$

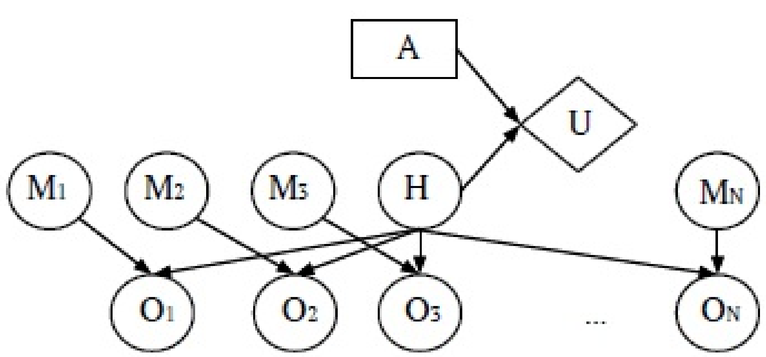
☐ $\text{VPI}(O_i) = \text{VPI}(O_j)$ where $i \neq j$

☐ $\text{VPI}(O_3 \mid O_2,\ O_1) > \text{VPI}(O_2 \mid O_1)$.

☐ $\text{VPI}(H) > \text{VPI}(O_1, O_2, \ldots O_N)$

☐ None of the above.

(iii) In a third situation, each fortune teller's prediction is affected by their mood. If the fortune teller is in a good mood ($+m$), then that fortune teller's prediction is guaranteed to be correct. If the fortune teller is in a bad mood ($-m$), then that teller's prediction is guaranteed to be incorrect. Each fortune teller is happy with probability $P(M_n = +m) = 0.8$.



☐ $\text{VPI}(M_1) > 0$

☐ $\forall i\ \text{VPI}(M_i \mid O_i) > 0$

☐ $\text{VPI}(M_1,\ M_2,\ \ldots,\ M_N) > \text{VPI}(M_1)$

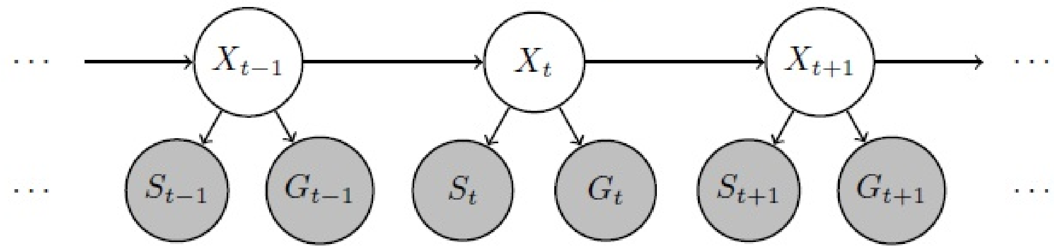
☐ $\forall i\ \text{VPI}(H) = \text{VPI}(M_i, O_i)$

☐ None of the above.

Q2. HMM: Where is the Car?

Transportation researchers are trying to improve traffic in the city but, in order to do that, they first need to estimate the location of each of the cars in the city. They need our help to model this problem as an inference problem of an HMM. For this question, assume that only *one* car is being modeled.

(a) The structure of this modified HMM is given below, which includes X, the location of the car; S, the noisy location of the car from the signal strength at a nearby cell phone tower; and G, the noisy location of the car from GPS.



We want to perform filtering with this HMM. That is, we want to compute the belief $P(x_t|s_{1:t}, g_{1:t})$, the probability of a state x_t given all past and current observations.

The **dynamics update** expression has the following form:

$$P(x_t|s_{1:t-1}, g_{1:t-1}) = \underline{\quad(\text{i})\quad}\ \underline{\quad(\text{ii})\quad}\ \underline{\quad(\text{iii})\quad}\ P(x_{t-1}|s_{1:t-1}, g_{1:t-1}).$$

Complete the expression by choosing the option that fills in each blank.

- (i) ○ $P(s_{1:t}, g_{1:t})$ ☒ $P(s_{1:t-1}, g_{1:t-1})$ ○ $P(s_{1:t-1})P(g_{1:t-1})$ ○ $P(s_{1:t})P(g_{1:t})$ ○ 1
- (ii) ○ $\sum_{x_t}$ ☒ $\sum_{x_{t-1}}$ ○ $\max_{x_{t-1}}$ ○ $\max_{x_t}$ ○ 1
- (iii) ○ $P(x_{t-1} \mid x_{t-2})$ ○ $P(x_{t-2}, x_{t-1})$ ○ $P(x_{t-1}, x_t)$ ☒ $P(x_t|x_{t-1})$ ○ 1

The **observation update** expression has the following form:

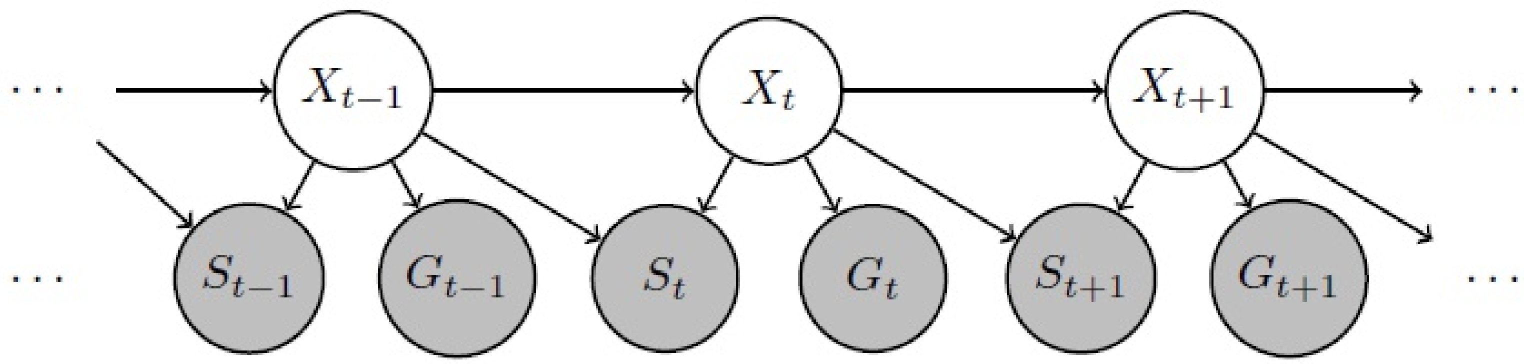
$$P(x_t|s_{1:t}, g_{1:t}) = \underline{\quad(\text{iv})\quad}\ \underline{\quad(\text{v})\quad}\ \underline{\quad(\text{vi})\quad}\ P(x_t|s_{1:t-1}, g_{1:t-1}).$$

$P(s_t, g_t | x_t) \cdot P(x_t | s_{1:t-1}, g_{1:t-1})$

Complete the expression by choosing the option that fills in each blank.

- (iv) ○ $P(s_{1:t-1}|s_t)P(g_{1:t-1}|g_t)$ ☒ $\frac{1}{P(s_t, g_t|s_{1:t-1}, g_{1:t-1})}$ ○ $\frac{1}{P(s_{1:t-1}, g_{1:t-1}|s_t, g_t)}$
  ○ $P(s_t, g_t|s_{1:t-1}, g_{1:t-1})$ ○ $P(s_{1:t-1}, g_{1:t-1}|s_t, g_t)$ ○ $P(s_t|s_{1:t-1})P(g_t|g_{1:t-1})$
  ○ $\frac{1}{P(s_t|s_{1:t-1})P(g_t|g_{1:t-1})}$ ○ $\frac{1}{P(s_{1:t-1}|s_t)P(g_{1:t-1}|g_t)}$ ○ 1
- (v) ○ $\sum_{x_t}$ ○ $\sum_{x_{t-1}}$ ○ $\max_{x_t}$ ○ $\max_{x_{t-1}}$ ☒ 1
- (vi) ○ $P(x_{t-1}, s_{t-1})P(x_{t-1}, g_{t-1})$ ○ $P(x_{t-1}, s_{t-1}, g_{t-1})$ ○ $P(x_t|s_t)P(x_t|g_t)$
  ○ $P(s_{t-1}|x_{t-1})P(g_{t-1}|x_{t-1})$ ○ $P(x_t, s_t)P(x_t, g_t)$ ○ $P(x_t, s_t, g_t)$
  ○ $P(x_{t-1}|s_{t-1})P(x_{t-1}|g_{t-1})$ ☒ $P(s_t|x_t)P(g_t|x_t)$ ○ 1

(b) It turns out that if the car moves too fast, the quality of the cell phone signal decreases. Thus, the signal-dependent location S_t not only depends on the current state X_t but it also depends on the previous state X_{t-1}. Thus, we modify our original HMM for a new more accurate one, which is given below.



Again, we want to compute the belief $P(x_t|s_{1:t}, g_{1:t})$. In this part we consider an update that combines the dynamics and observation update in a *single* update.

$$P(x_t|s_{1:t}, g_{1:t}) = \underline{\quad\text{(i)}\quad}\ \underline{\quad\text{(ii)}\quad}\ \underline{\quad\text{(iii)}\quad}\ \underline{\quad\text{(iv)}\quad}\ P(x_{t-1}|s_{1:t-1}, g_{1:t-1}).$$

Complete the **forward update** expression by choosing the option that fills in each blank.

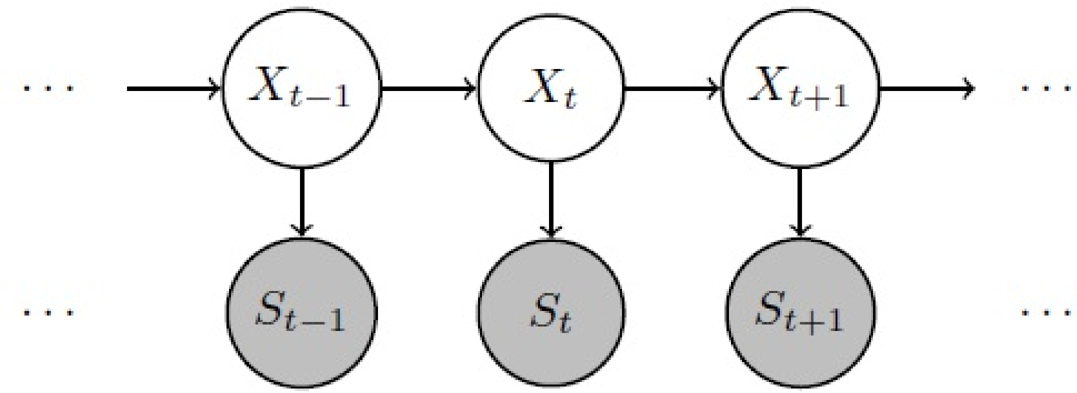
(i)
- ○ $P(s_{1:t-1}, g_{1:t-1}|s_t, g_t)$
- ○ $P(s_t, g_t|s_{1:t-1}, g_{1:t-1})$
- ○ $P(s_t|s_{1:t-1})P(g_t|g_{1:t-1})$
- ☒ $\frac{1}{P(s_t, g_t|s_{1:t-1}, g_{1:t-1})}$
- ○ $\frac{1}{P(s_{1:t-1}, g_{1:t-1}|s_t, g_t)}$
- ○ $P(s_{1:t-1}|s_t)P(g_{1:t-1}|g_t)$
- ○ $\frac{1}{P(s_t|s_{1:t-1})P(g_t|g_{1:t-1})}$
- ○ $\frac{1}{P(s_{1:t-1}|s_t)P(g_{1:t-1}|g_t)}$
- ○ 1

(ii)
- ○ $\max_{x_{t-1}}$
- ○ $\max_{x_t}$
- ☒ $\sum_{x_{t-1}}$
- ○ $\sum_{x_t}$
- ○ 1

(iii)
- ○ $P(s_{t-1}|x_{t-2}, x_{t-1})P(g_{t-1}|x_{t-1})$
- ○ $P(s_t|x_{t-1}, x_t)P(g_t|x_t)$
- ☒ $P(s_t, g_t|x_t)$
- ○ $P(x_{t-2}, x_{t-1}, s_{t-1})P(x_{t-1}, g_{t-1})$
- ○ $P(x_{t-1}, x_t, s_t)P(x_t, g_t)$
- ○ $P(s_{t-1}, g_{t-1}|x_{t-1})$
- ○ $P(x_{t-2}, x_{t-1}|s_{t-1})P(x_{t-1}|g_{t-1})$
- ○ $P(x_{t-1}, x_t|s_t)P(x_t|g_t)$
- ○ 1
- ○ $P(x_{t-2}, x_{t-1}, s_{t-1}, g_{t-1})$
- ○ $P(x_{t-1}, x_t, s_t, g_t)$

(iv)
- ○ $P(x_{t-1}, x_t)$
- ☒ $P(x_t|x_{t-1})$
- ○ $P(x_{t-2}, x_{t-1})$
- ○ $P(x_{t-1}|x_{t-2})$
- ○ 1

(c) The Viterbi algorithm finds the most probable sequence of hidden states $X_{1:T}$, given a sequence of observations $s_{1:T}$, for some time $t = T$. Recall the canonical HMM structure, which is shown below.



For this canonical HMM, the Viterbi algorithm performs the following dynamic programming computations:

$$m_t[x_t] = P(s_t|x_t) \max_{x_{t-1}} P(x_t|x_{t-1}) m_{t-1}[x_{t-1}].$$

We consider extending the Viterbi algorithm for the modified HMM from part **(b)**. We want to find the most likely sequence of states $X_{1:T}$ given the sequence of observations $s_{1:T}$ and $g_{1:T}$. The dynamic programming update for $t > 1$ for the modified HMM has the following form:

$$m_t[x_t] = \underline{\quad\textbf{(i)}\quad}\ \underline{\qquad\textbf{(ii)}\qquad}\ \underline{\qquad\textbf{(iii)}\qquad}\ m_{t-1}[x_{t-1}].$$

Complete the expression by choosing the option that fills in each blank.

(i) ◯ $\sum_{x_{t-1}}$ ◯ $\sum_{x_t}$ ◯ $\max_{x_t}$ ◯ $\max_{x_{t-1}}$ ◯ 1

(ii)
◯ $P(s_{t-1}|x_{t-2}, x_{t-1})P(g_{t-1}|x_{t-1})$ ◯ $P(s_t|x_{t-1}, x_t)P(g_t|x_t)$ ◯ $P(s_t, g_t|x_t)$
◯ $P(x_{t-2}, x_{t-1}, s_{t-1})P(x_{t-1}, g_{t-1})$ ◯ $P(x_{t-1}, x_t, s_t)P(x_t, g_t)$ ◯ $P(s_{t-1}, g_{t-1}|x_{t-1})$
◯ $P(x_{t-2}, x_{t-1}|s_{t-1})P(x_{t-1}|g_{t-1})$ ◯ $P(x_{t-1}, x_t|s_t)P(x_t|g_t)$ ◯ 1
◯ $P(x_{t-2}, x_{t-1}, s_{t-1}, g_{t-1})$ ◯ $P(x_{t-1}, x_t, s_t, g_t)$

(iii) ◯ $P(x_{t-1}, x_t)$ ◯ $P(x_t|x_{t-1})$ ◯ $P(x_{t-2}, x_{t-1})$ ◯ $P(x_{t-1}|x_{t-2})$ ◯ 1