

Salads

TOTAL POINTS 1

1. We have an unlimited supply of tomatoes, bell peppers and lettuce. We want to make a salad out of 4 units among these three ingredients (we do not have to use all ingredients). The order in which we use the ingredients does not matter. How many different salads we can make?

1 point

We do not have the formula to answer this question yet, so try to list all the salads first or create a program that will do that for you. Then you can count the number of salads by hand (note the answer to the problem should be the number).

Enter answer here

Upgrade to submit

~~Handwritten work:~~

$t \ b \ l$

~~$\left. \begin{array}{l} t t t t \\ t t t b \\ t t t l \\ t t b t \end{array} \right\} 3^4 = \frac{81}{4}$~~

~~$2^3 = 8$~~

~~What if 2 out of 3~~

~~$\{0, 1\}$~~

~~$\left\{ \begin{array}{l} 0 0 0 \\ 0 0 1 \\ 0 1 0 \\ 0 1 1 \\ 1 0 0 \\ 1 0 1 \\ 1 1 0 \\ 1 1 1 \end{array} \right\}$~~

~~$\Rightarrow 4 = \frac{2^3}{3}$~~

~~$\Rightarrow \frac{k^n}{n}$~~

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Distributing Assignments Among People

TOTAL POINTS 2

1. Suppose there are 4 people and 9 different assignments. Each person should receive one assignment. Assignments for different people should be different. How many ways are there to do it?

1 point

Enter answer here

2. There are 4 people and 9 different assignments. We need to distribute all assignments among people. No assignment should be assigned to two people. Every person can be given arbitrary number of assignments from 0 to 9. How many ways are there to do it?

1 point

Enter answer here

Distributing Candies Among Kids

TOTAL POINTS 2

→ order doesn't matter

1. There are 15 identical candies. How many ways are there to distribute them among 7 kids?

1 point

Enter answer here

2. There are 15 identical candies. How many ways are there to distribute them among 7 kids in such a way that each kid receives at least 1 candy?

1 point

Enter answer here

①

$$\underbrace{\begin{array}{cccccccccccccccc} 0 & 0 & | & 0 & 0 & | & 0 & 0 & | & 0 & 0 & | & 0 & 0 & | & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}}_{15 + 7 - 1, \text{ candies}} \quad \text{Kids} - 1$$

$$\Rightarrow k\text{-Subsets}$$

$$6\text{-Subsets}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$
$$\Rightarrow \binom{21}{6} = \frac{21!}{6!(21-6)!} =$$

Distributing Candies Among Kids

TOTAL POINTS 2

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② Give each kid a candy first $\rightarrow 15 - 7 = 8$ candies left

$$\binom{k+n-1}{n-1} = \binom{8+7-1}{7-1} = \binom{14}{6} = \frac{14!}{6!(14-6)!} = \frac{14!}{6!8!} = \underline{\underline{3003}}$$

Numbers with Fixed Sum of Digits

TOTAL POINTS 2

1. How many non-negative integer numbers are there below 10000 such that their sum of digits is equal to 9?

1 point

Enter answer here

2. How many non-negative integer numbers are there below 10000 such that their sum of digits is equal to 10?

1 point

Enter answer here

① $1 \ 1 \mid 1 \mid 1 \ 1 \mid 1 \ 1 \mid 1 \rightarrow \text{sum } 9$

$\rightarrow 4 \text{ digits } (< 10000)$

$(4-1) - \text{subsets of } (9+4-1)$
 $3 - \text{subsets of } 12$

$\rightarrow \binom{12}{3} = \frac{12!}{3!9!} = \frac{12 \cdot 11 \cdot 10}{6 \cdot 1} = \underline{\underline{220}}$

Numbers with Fixed Sum of Digits

TOTAL POINTS 2

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2. How many non-negative integer numbers are there below 10000 such that their sum of digits is equal to 10?

1 point

Enter answer here

Sum 10 is not possible because of decimal system (0-9)

$$\textcircled{2} \binom{10+4-1}{4-1} - 4 = \binom{13}{3} - 4 = \frac{13!}{3!10!} - 4 = \frac{13 \cdot 12 \cdot 11}{6} - 4 = 143 \cdot 2 - 4 = 286 - 4 = \underline{\underline{282}}$$

subtracting 4 which is the number of all 10's

Numbers with Non-increasing Digits

TOTAL POINTS 1

1. How many four-digit numbers are there such that their digits are non-increasing, that is each next digit is not greater than the previous one? Three-digit numbers are also four-digit, they just start with 0.

1 point

Enter answer here

Simplification:

1 Digit:

9 8 7 6 5 4 3 2 1 0 | 1

2 Digits:

99 88 77 66 55 44 33 22 11
98
:
:
:
91 81
90 80 70 60 50 40 30 20 10 00
10 9 8 7 6 5 4 3 2 1

3 Digits:

988 977 966 955 ... 911 900
!
980 970
9 8 7 6 5 4 3 2 1

Numbers with Non-increasing Digits

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Enter answer here

* * * *

3 7 8 5

8 7 5 3

4-Digits

random pick

each random pick
has one unique
way to order it in
a decreasing way

①

→ order is not counted
→ set
→ combination!

② repetition allowed

⇒ combination with
repetition

$$\binom{k + (n-1)}{n-1}$$

$$\binom{4 + (10-1)}{10-1} = \binom{13}{9}$$

why 10?

Splitting into Working Groups

TOTAL POINTS 1

1. There are 12 students in the class. How many ways are there to split them into working groups of size 2 to work on the same assignment?

1 point

Enter answer here

$$\begin{array}{cccccc} \frac{12 \cdot 11}{2} & \times & \frac{10 \cdot 9}{2} & \times & \frac{8 \cdot 7}{2} & \times & \frac{6 \cdot 5}{2} & \times & \frac{4 \cdot 3}{2} & \times & \frac{2 \cdot 1}{2} \\ \text{Group 1} & & 2 & & 3 & & 4 & & 5 & & 6 \\ \left(\begin{matrix} 12 \\ 2 \end{matrix} \right) \times \left(\begin{matrix} 10 \\ 2 \end{matrix} \right) \left(\begin{matrix} 8 \\ 2 \end{matrix} \right) \left(\begin{matrix} 6 \\ 2 \end{matrix} \right) \left(\begin{matrix} 4 \\ 2 \end{matrix} \right) \left(\begin{matrix} 2 \\ 2 \end{matrix} \right) \end{array}$$

Numbers with Non-increasing Digits

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* * * *

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4-Digits

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