MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Fall 2010)

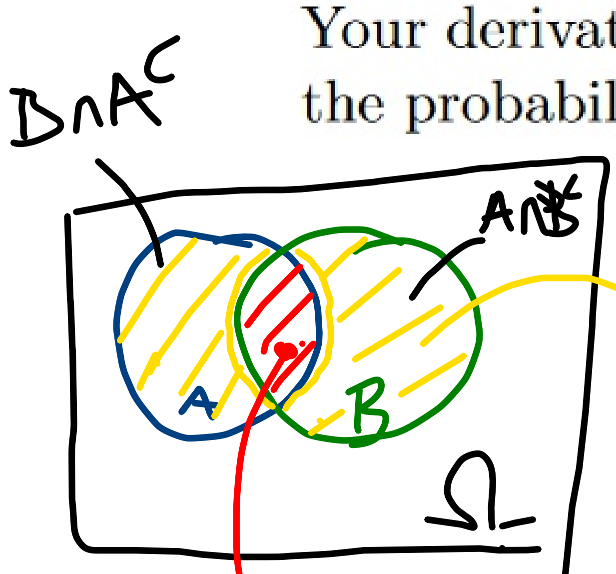
Recitation 1

September 9, 2010

1. Give a mathematical derivation of the formula

$$\mathbf{P}((A \cap B^c) \cup (A^c \cap B)) = \mathbf{P}(A) + \mathbf{P}(B) - 2\mathbf{P}(A \cap B).$$

Your derivation should be a sequence of steps, with each step justified by appealing to one of the probability axioms.

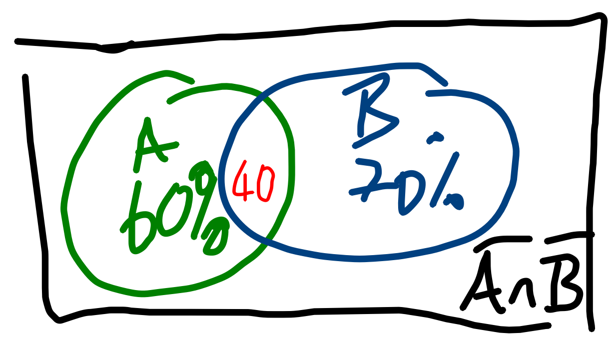


$(A \cap B^c) \cup (A^c \cap B) =$

$(A \cap B)$

2. Problem 1.5, page 54 in the text.

Out of the students in a class, 60% are geniuses, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a genius nor a chocolate lover.



$$\overline{X} = A\cup B = A + B - A\cap B$$

$$X = \Omega - (A+B-A\cap B)$$

$$= 1 - (0.6 + 0.7 - 0.4)$$

$$= 1 - (1.3 - 0.4)$$

$$= 1 - 0.9$$

$$= 0.1$$

3. A six-sided die is loaded in a way that each even face is twice as likely as each odd face. Construct a probabilistic model for a single roll of this die, and find the probability that a 1, 2, or 3 will come up.

$P(\text{even}) = \frac{2}{3}$

$P(\text{odd}) = \frac{1}{3}$

$P(2) = P(4) = P(6) = \frac{2}{9}$

$P(1) = P(3) = P(5) = \frac{1}{9}$

$P(1 \cup 2 \cup 3) = \frac{1}{9} \cdot 2 + \frac{2}{9} = \frac{4}{9}$

4. Example 1.5, page 13 in the text.

   Romeo and Juliet have a date at a given time, and each will arrive at the meeting place with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 15 minutes and will leave if the other has not yet arrived. What is the probability that they will meet?

G1[†]. Problem 1.13, page 56 in the text. **Continuity property of probabilities.**

(a) Let $A_1, A_2, \ldots$ be an infinite sequence of events that is "monotonically increasing," meaning that $A_n \subset A_{n+1}$ for every n. Let $A = \cup_{n=1}^{\infty} A_n$. Show that $\mathbf{P}(A) = \lim_{n\to\infty} \mathbf{P}(A_n)$. *Hint:* Express the event A as a union of countably many disjoint sets.

(b) Suppose now that the events are "monotonically decreasing," i.e., $A_{n+1} \subset A_n$ for every n. Let $A = \cap_{n=1}^{\infty} A_n$. Show that $\mathbf{P}(A) = \lim_{n\to\infty} \mathbf{P}(A_n)$. *Hint:* Apply the result of the previous part to the complements of the events.

(c) Consider a probabilistic model whose sample space is the real line. Show that

$$\mathbf{P}([0,\infty)) = \lim_{n\to\infty} \mathbf{P}([0,n]) \qquad \text{and} \qquad \lim_{n\to\infty} \mathbf{P}([n,\infty)) = 0.$$