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A new method for finding an initial solution for the transportation problem

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Overview

- Transportation problem
  - Definition
  - Solving
- Previous methods
  - Explanation
  - weaknesses
- Proposed method
  - Explanation
  - Examples
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  - Runtime Comparing
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  - Results and tables
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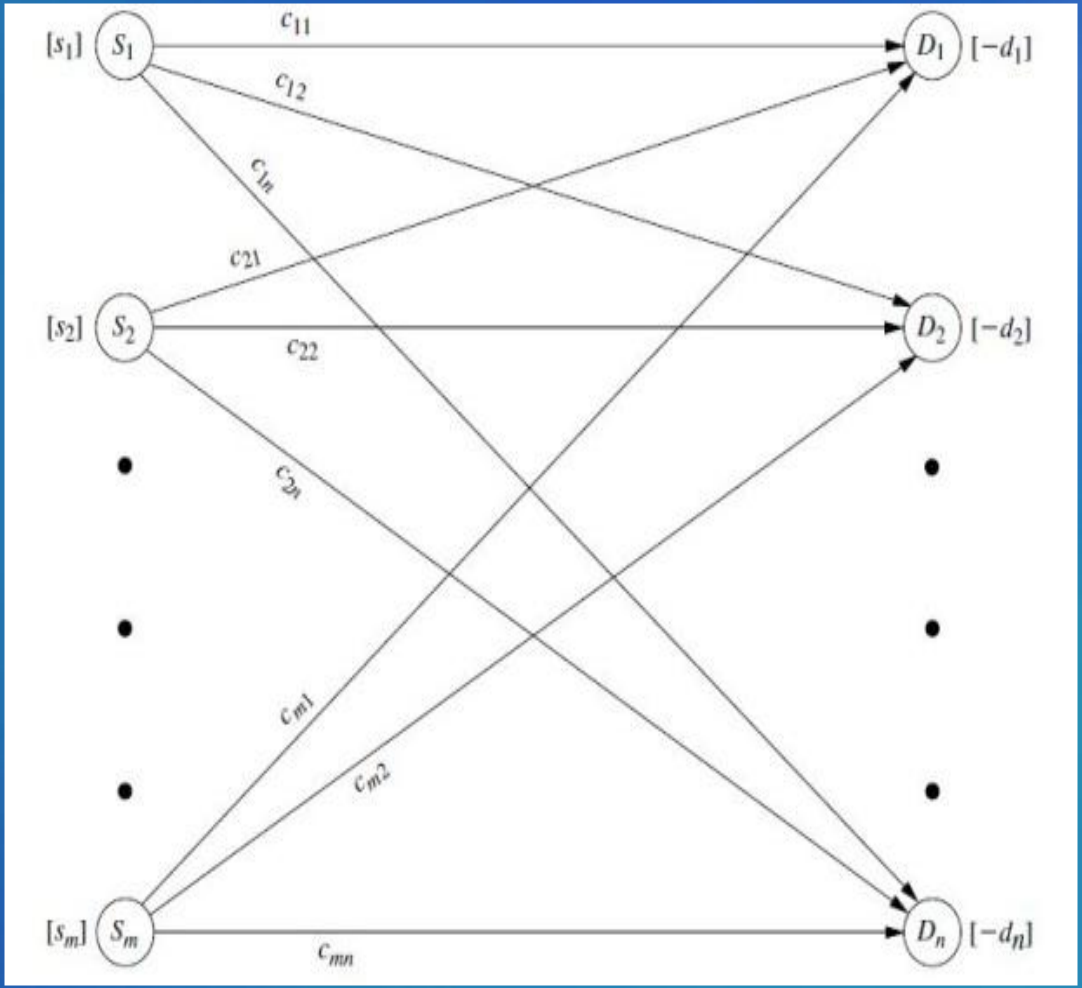
Definition

Transportation problem

- one of the well-known and useful models in linear optimization
- Has many applications in determining how to optimally transport goods, require a very large number of constraints and variables
- Solving with simplex method may require an exorbitant computational effort
- Special solving method

Definition

Transportation problem



$$min \sum_{j=1}^{n}\sum_{i=1}^{m} c_{ij}x_{ij}$$

Subject to: $\sum_{j=1}^{n} x_{ij} = s_i$

$\sum_{i=1}^{m} x_{ij} = d_j$

Definition

Parameter table for transportation problem

		Cost per Unit Distributed				
		Destination				
		1	2	⋯	n	Supply
Source	1	c_{11}	c_{12}	⋯	c_{1n}	s_1
	2	c_{21}	c_{22}	⋯	c_{2n}	s_2
	⋮	⋯	⋯	⋯	⋯	⋮
	m	c_{m1}	c_{m2}	⋯	c_{mn}	s_m
Demand		d_1	d_2	⋯	d_n	

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Solving the transportation problem

- **goal**
  - Find the optimal answer to satisfy the conditions with minimum cost
  - Select n+m-1 (or less) edges for transportation

- **Solving method**
  - Find an initial feasible solution
  - Run the iterative algorithm to improve the answer and finally find the optimal solution

- **Importance of initial solution**
  - Finding a better initial solution results in an impressive decrease in the number of iterations to reach the optimal solution in the main algorithm

This paper aims to introducing a novel method, to find an appropriate initial solution for the transportation problem

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Previous methods

Northwest corner method

56	21	4	87	97	57		1
17	32	39	7	67	56		
45	79	4[illegible]	55	58	11		
182	58	2[illegible]	5	17	6[illegible]		~~162~~ ~~71~~ 42
	42	29	91	83	55		
		~~82~~ ~~65~~ 20					

- Begin by selecting x_{11}
- if x_{ij} was the last basic variable selected, then next select $x_{i,j+1}$ if source i has any supply remaining. Otherwise, next select $x_{i+1,j}$.

57*55 + 97*1 + 67*17 + 38*45 + 5*91 + 28*29 + 58*42= 9748

Previous methods

Least Cost Method

56	21	4	87	97	57	27
17	32	39	7	67	56	2
45	79	46	55	58	11	
182	58	28	5	17	63	~~61~~ 8
	42 ~~10~~ 8	29	91	83	55 15	

select the cheapest edge and transfer as much as possible

Total cost:
4*29 + 5*91 + 11*45 + 17*83 + 21*27 + 32*15 + 56*2 + 63*8
= 4140

Previous methods

Vogel's method

- *Calculate the difference between the smallest and next-to-the-smallest unit cost c_{ij} still remaining in every rows or columns.*
- In that row or column having the *largest difference,* select the variable having the *smallest remaining unit cost.*

56	21	4	87	97	57	14
17	32	39	7	67	56	
45	79	46	55	58	11	
182	58	28	5	17	63	10
	42 10	29	91 74	83 15	55	

21-4 = 17
32-7 = 25
46-11 = 35
17-5 = 12

32-21 =11
28-4 =24
7-5 = 2
58-17 = 41
56-11 = 45

21-4 = 17
28-5 = 12

58-21 = 37
28-4 = 24
87-5 = 82
63-57= 6

21-4 = 17
32-7 = 25
17-5 = 12

32-21 =11
28-4 =24
7-5 = 2
67-17 = 50
57-56= 1

21-4 = 17
58-28= 30

58-21 = 37
28-4 = 24
63-57= 6
63-28= 35

21-4 = 17
32-7 = 25
28-5 = 12

32-21 = 11
28-4 = 24
7-5 = 2
57-56= 1

57-4 = 53
63-28= 35

28-4 = 24
63-57= 6

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weaknesses

- ***Northwest corner*** method choose the edges, independent of their costs
- ***Least Cost*** *Method* choose the minimum cost available, a greedy algorithm, but our choice in one iteration may force bad choices in the next iterations
- ***Vogel's*** *method* calculate a penalty to considering bad choices in the future, but only considering one level penalty, not enough to make a good choice.

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Proposed method

- Target : better greedy choice in each iteration

Definition:

- ***Effective number of a source node (Ens_i)*** is an approximation of the number of destination nodes which will transfer with the $\boldsymbol{i_{th}}$ source in next iterations.

$$ENsi = \max\{ \left\lceil \frac{Csi}{Cst} * Nd \right\rceil , 2 \}$$

Csi: remaining capacity of $\boldsymbol{i_{th}}$ node
Cst: total remaining capacity of all sources
Nd: number of remaining destination

- ***Effective number of a destination node(End_i)*** defines similarly for the destinations.

$$ENdj = \max\{ \left\lceil \frac{Cdj}{Cdt} * Ns \right\rceil , 2 \}$$

Proposed method

- ***Effective value of a source node(EVs_i)*** is a approximation of the total penalty cost we will pay if we do not choose the lowest edge of $\boldsymbol{i_{th}}$ source.

$$EVsi = \frac{P[1]*(ENsi-1)+P[2]*(ENsi-2)+\cdots+P[ENsi-1]*(1)}{\left(\frac{ENsi(ENsi-1)}{2}\right)} - P[0]$$

array p[] is increasing order of $ENsi$ reaming cheapest edges of the $\boldsymbol{i_{th}}$ source node.

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10	2	17	32	39	7	67	56	15
35	2	45	79	46	55	58	11	
22.5	4	182	53	28	5 8	17	63	30
	10		42 21	29	91	83	55	
	2	3	2 2	2 2	2	2	2	
	1 41	28.6	11 24	24 11	2	41	45	

$$ENs4 = \max\{\left\lceil \frac{182}{300} * 5 \right\rceil, 2\} = 4$$

$$EVs_4 = \frac{17 * (3) + 28 * (2) + 58 * (1)}{(6)} - 5 = 22.5$$

$$EVd_1 = \frac{56}{(1)} - 11 = 45$$

$$ENd_1 = \max\left\{\left\lceil \frac{55}{300} * 4 \right\rceil, 2\right\} = 2$$

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comparing

Properties of proposing method

- by using the *effective number*, estimating the number of next transitions of each node
- by calculating *effective value* choosing a better greedy choice in each iteration which will provide better initial solutions
- We can implement this method by O(n^2) in time complexity, like Vogel's method.

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implementation

- We compared our method and Vogel’s method using 13 different size problems.

- For each problem instance, 1000 balanced transportation problem was implemented and solved using both method.

- Comparison criteria
  - average of total cost of the solution
  - Number of the better answers in each size of the problem

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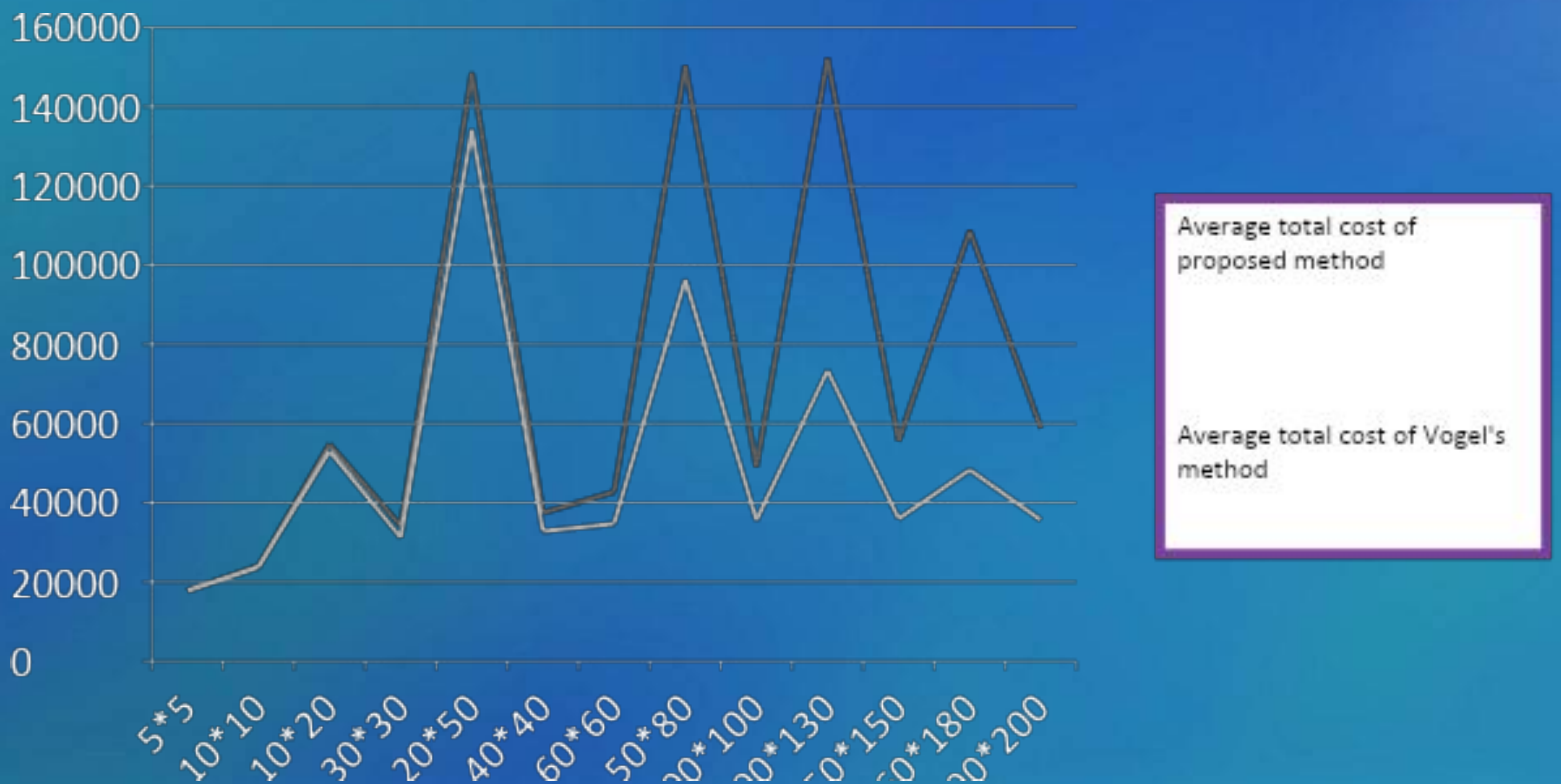
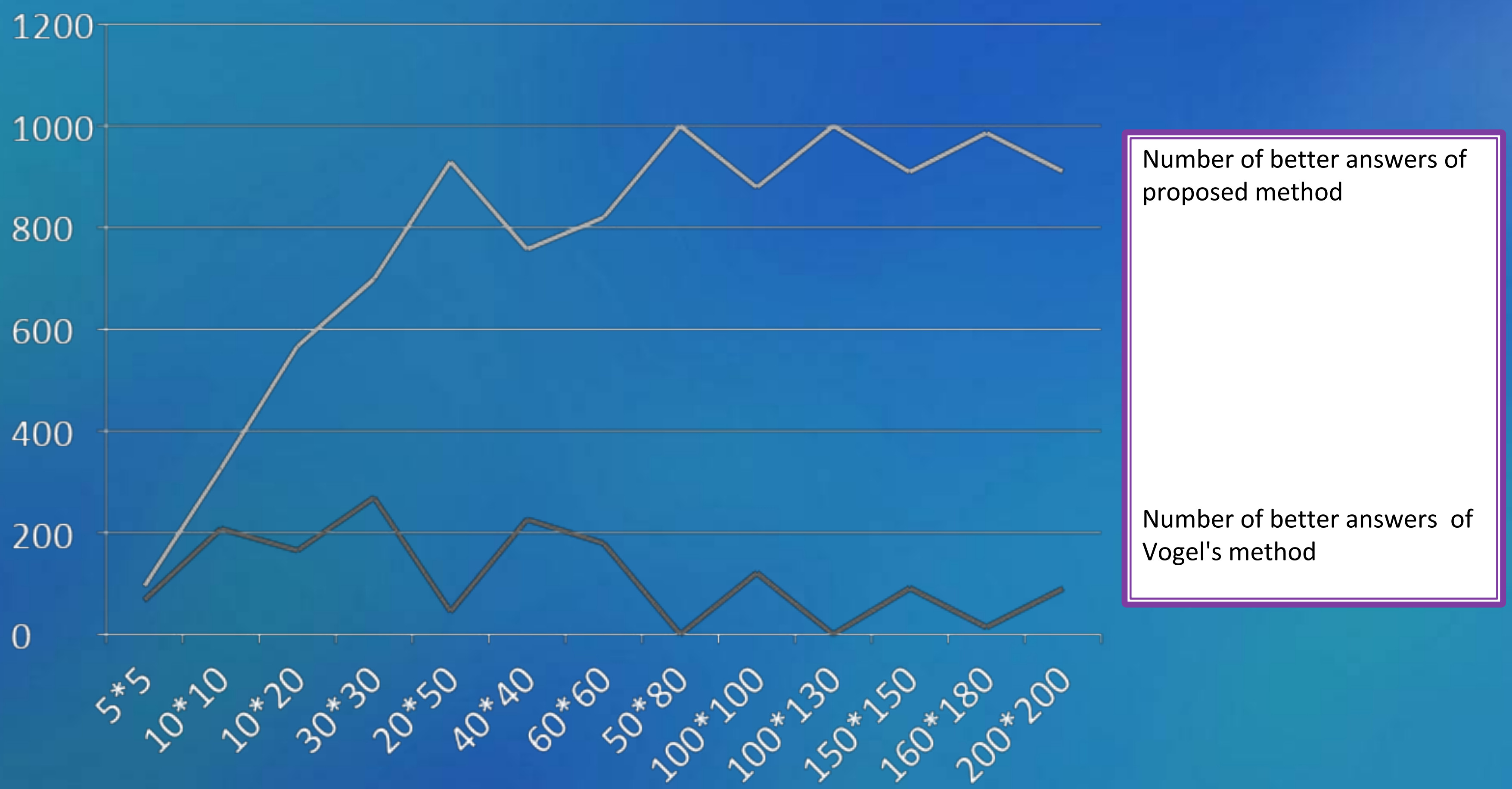
Experimental results



Experimental results



Experimental results

Size of problem	Num of vogel 's better answers	Num of proposed method 's better answers	Num of equal answers
5*5	67	95	838
10*10	208	326	466
10*20	165	567	268
30*30	269	699	32
20*50	45	930	25
40*40	226	757	17
60*60	179	820	1
50*80	0	1000	0
100*100	121	879	0
150*130	0	1000	0
150*150	91	909	0
160*180	14	986	0
200*200	90	910	0

Size of problem	Avg cost vogel 's answer	Avg cost our answer
5*5	17836	17777
10*10	24052	23735
10*20	54814	53066
30*30	34213	31213
20*50	148577	134252
40*40	37486	32814
60*60	42730	34708
50*80	150351	96446
100*100	49125	35550
150*130	152166	73396
150*150	55631	35984
160*180	108752	48316
200*200	58805	35651

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conclusion

- This paper aims to introducing a novel method, to find an initial solution for transportation problem
- The proposed method, according to the remaining capacity of each node and calculating an estimation number of next transitions of the node, will reach better answer through making a better choice in each iteration
- The implementation also shows the prospering performance of this method

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Future works

- Focus on the main algorithm in order to reduce the number of iterations to solve the problem.
- Define a new formula for effective number and effective value .
- Take beneficiary of distributed and parallel systems to implement the algorithm.

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