

# Student Information

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## Answer 1

a)

$$\varphi = ((p \rightarrow \neg q) \iff (p \rightarrow q))$$

| $p$ | $q$ | $\neg q$ | $p \rightarrow \neg q$ | $p \iff q$ | $\varphi$ |
|-----|-----|----------|------------------------|------------|-----------|
| T   | T   | F        | F                      | T          | F         |
| T   | F   | T        | T                      | F          | F         |
| F   | T   | F        | T                      | F          | F         |
| F   | F   | T        | T                      | T          | T         |

b)

$$\varphi = (((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r)$$

| $p$ | $q$ | $r$ | $p \vee q$ | $p \rightarrow r$ | $q \rightarrow r$ | $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$ | $\varphi$ |
|-----|-----|-----|------------|-------------------|-------------------|--|-----------|
| T   | T   | T   | T          | T                 | T                 | T  | T         |
| T   | T   | F   | T          | F                 | F                 | F  | T         |
| T   | F   | T   | T          | T                 | T                 | T  | T         |
| T   | F   | F   | T          | F                 | T                 | F  | T         |
| F   | T   | T   | T          | T                 | T                 | T  | T         |
| F   | T   | F   | T          | T                 | F                 | F  | T         |
| F   | F   | T   | F          | T                 | T                 | F  | T         |
| F   | F   | F   | F          | T                 | T                 | F  | T         |

## Answer 2

|  |          |                                      |                            |
|--|----------|--------------------------------------|----------------------------|
| $\neg p \rightarrow (q \rightarrow r)$ | $\equiv$ | $\neg \neg p \vee (q \rightarrow r)$ | <i>table 7 - first row</i> |
| $\neg \neg p \vee (q \rightarrow r)$   | $\equiv$ | $p \vee (q \rightarrow r)$           | <i>double negation</i>     |
| $p \vee (q \rightarrow r)$             | $\equiv$ | $p \vee (\neg q \vee r)$             | <i>table 7 - first row</i> |
| $p \vee (\neg q \vee r)$               | $\equiv$ | $(\neg q \vee r) \vee p$             | <i>commutative law</i>     |
| $(\neg q \vee r) \vee p$               | $\equiv$ | $\neg q \vee (p \vee r)$             | <i>associative law</i>     |
| $\neg q \vee (p \vee r)$               | $\equiv$ | $q \rightarrow (p \vee r)$           | <i>table 7 - first row</i> |

## Answer 3

a)  $\forall x L(x, Burak)$

$$b) \forall y L(Hazal, y)$$

$$c) \forall x \exists y L(x, y)$$

$$d) \neg \exists x \forall y L(x, y)$$

$$e) \exists x \forall y L(x, y)$$

$$f) \neg \exists x (L(x, Burak) \wedge L(x, Mustafa))$$

$$g) \exists x \exists y \exists z (L(Ceren, x) \wedge L(Ceren, y) \wedge (x \neq y) \wedge (L(Ceren, z) \rightarrow (z = x \vee z = y)))$$

$$h) \forall x \exists y \exists z (L(x, y) \wedge (L(x, z) \rightarrow (z = y)))$$

$$i) \forall x \neg L(x, x)$$

$$j) \exists x \exists y \exists z (L(x, y) \wedge L(x, x) \wedge (x \neq y) \wedge (L(x, z) \rightarrow ((z = x) \vee (z = y))))$$

## Answer 4

|   |                                      |                       |
|---|--------------------------------------|-----------------------|
| 1 | $p$                                  | <i>premise</i>        |
| 2 | $p \rightarrow (r \rightarrow q)$    | <i>premise</i>        |
| 3 | $r \rightarrow q$                    | $\rightarrow_e, 1, 2$ |
| 4 | $\neg q$                             | <i>assumption</i>     |
| 5 | $\neg r$                             | $\rightarrow_e, 3, 4$ |
| 6 | $\neg q \rightarrow \neg r$          | $\rightarrow_i, 4, 5$ |
| 7 | $\neg q \rightarrow (s \vee \neg r)$ | $\vee_i, 4$           |

# Answer 5

|    |                                    |                             |
|----|------------------------------------|-----------------------------|
| 1  | $\forall x(p(x) \rightarrow q(x))$ | <i>premise</i>              |
| 2  | $\exists y p(y) \vee r(a)$         | <i>premise</i>              |
| 3  | $\neg \exists z(z)$                | <i>premise</i>              |
| 4  | $x_0 \quad r(x_0)$                 | <i>assumption</i>           |
| 5  | $\exists z r(x_0)$                 | $\exists_i, 4$              |
| 6  | $\perp$                            | <i>contradiction, 3, 5</i>  |
| 7  | $\neg r(x_0)$                      | $\neg_i, 4, 6$              |
| 8  | $\forall z \neg r(z)$              | $\forall_i, 7$              |
| 9  | $r(a)$                             | <i>assumption</i>           |
| 10 | $a \quad \neg r(a)$                | $\forall_e, 8$              |
| 11 | $\perp$                            | <i>contradiction, 9, 10</i> |
| 12 | $\exists y p(y)$                   | $\vee_e, 9 - 11$            |
| 13 | $x_1 \quad p(x_1)$                 | <i>assumption</i>           |
| 14 | $p(x_1) \rightarrow q(x_1)$        | $\forall_e, 1$              |
| 15 | $q(x_1)$                           | $\rightarrow_e, 9, 10$      |
| 16 | $\exists z q(z)$                   | $\exists_i, 9 - 11$         |