hw3

1. $X \sim Geom(p)$ $Var(X) = ?$

$Var(X) = E[X^2] - E[X]^2 = \frac{1}{p^2} - \frac{1}{p} = \frac{1-p}{p^2}$

$E[X] = \sum_{t=1}^{+\infty} t\cdot p(1-p)^{t-1} = p\sum_{t=1}^{+\infty} t(1-p)^{t-1} = p\cdot(-A'(p)) = \frac{1}{p}$

$A(p) = \sum_{t=1}^{+\infty}(1-p)^t = \frac{(1-p)}{p} = \frac{1}{p} - 1$

$E[X^2] = \sum_{t=1}^{+\infty} t^2\cdot p(1-p)^{t-1} = p\cdot \underbrace{\sum_{t=1}^{+\infty} t^2(1-p)^{t-1}}_{B(p)} = \frac{2}{p^2} - \frac{1}{p}$

$A(p) = \sum_{t=1}^{+\infty}(1-p)^t$; $A'(p) = -\sum_{t=1}^{+\infty} t(1-p)^{t-1}$ $A'(p) = -\frac{1}{p^2}$ $B(p)$

$\| $

$\frac{1}{p} - 1$

$(p-1)A'(p) = \sum_{t=1}^{+\infty} t(1-p)^t$

$-((p-1)A'(p))' = \sum_{t=1}^{+\infty} t^2(1-p)^{t-1} = B(p)$

$\Rightarrow B(p) = -(pA'(p) - A'(p))' = -\left(-\frac{1}{p} + \frac{1}{p^2}\right)' = -\frac{1}{p^2} + \frac{2}{p^3}$

2. A_n – после n пар бросков число орлов на первой монете равно числу орлов на второй монете

$Pr(A_n) = \left(\frac{1}{4}\right)^n \sum_{i=0}^{n}\binom{n}{i}^2 = \left(\frac{1}{4}\right)^n\binom{2n}{n}$

$\overbrace{a_1 a_2 \dots a_n}^{k} \mid \overbrace{a_{n+1} a_{n+2} \dots a_n}^{n-k}$

$\binom{n}{k}$ $\binom{n}{n-k} = \binom{n}{k}$

X = общее число совпадений пар бросков

$X = \sum_{i=1}^{+\infty} Y_i$ Y_i – было ли совпадение на i броске

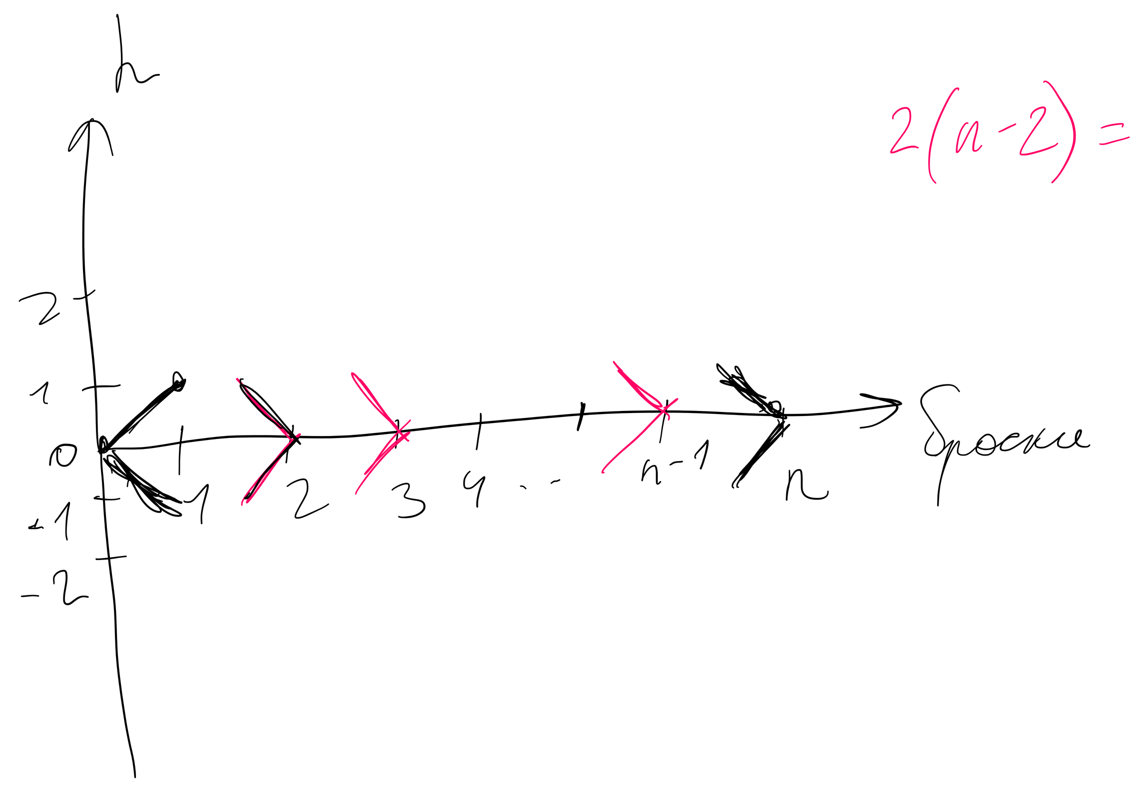
$E[X] = \sum_{i=1}^{\infty} E[Y_i] = \sum_{i=1}^{\infty}\left(\frac{1}{4}\right)^n\binom{2n}{n} \approx \sum_{i=1}^{\infty}\left(\frac{1}{4}\right)^n \frac{2\sqrt{\pi n}\,(2n)^{2n}\cancel{e^{2n}}}{\cancel{e^{2n}}\cdot 2\pi n\cdot n^{2n}} =$

стирлинг

$= \sum_{i=1}^{+\infty}\frac{1}{\sqrt{\pi n}}$ – расходится

3.

$2(n-2) = 2n-4$



4. $X_1 \sim Geom(p)$ $X_2 \sim Geom(p)$

$Y = \min\{X_1, X_2\}$ $Z = X_1 - X_2$

$p_Y(y) = 2\cdot q^{y-1}p\cdot\left(\sum_{i=y}^{+\infty} q^{i-1}p\right) - p^2q^{2y-2} = 2p^2q^{y-1}\cdot\frac{q^{y-1}}{p} - p^2q^{2y-2} = q^{2y-2}p(2-p)$

$p_Z(z) = Pr(X_1 - X_2 = z) = Pr(X_1 = X_2 + z) = \sum_{i=1}^{+\infty}(1-p)^{i-1}\cdot p\cdot(1-p)^{i+|z|-1}p =$

$= p^2\sum_{i=1}^{+\infty} q^{2i-2+|z|} = p^2\cdot q^{|z|}\sum_{i=1}^{+\infty}(q^2)^{i-1} = \frac{p^2\cdot q^{|z|}}{1-q^2} = \frac{p^2q^{|z|}}{1-1+2p-p^2} = \frac{p\,q^{|z|}}{2-p}$

$p_{Y,Z}(y,z) = Pr(\min\{X_1,X_2\} = y \cap X_1 = X_2 + z) = \begin{cases}(1-p)^{y-1}p\,(1-p)^{z+y-1}p, & z \geq 0\\ (1-p)^{y-1}p\,(1-p)^{y-z-1}p, & z < 0\end{cases} =$

$= p^2q^{2y+|z|-2}$ $\Rightarrow p_{Y,Z}(y,z) = p_Y(y)\cdot p_Z(z)$