

## Mathematical Notation and Definitions

### 1. Definitions

- (a) Set: well defined collection of objects, consisting of elements. Denoted by capital letters. Order of elements does not matter.
- (b) Cardinality: size of a set,  $|A|$ , can be any number from 0 to infinity. Empty set  $B$  has cardinality of 0,  $|B| = 0$
- (c) Equal Sets: two sets that have the same elements
- (d) Subset  $A$ : every element in  $A$  is contained in  $B$ .  
Proper subset  $A$ : every element in  $A$  is contained in  $B$  and  $B$  has elements that are not in  $A$ ,  $|B| > |A|$  since  $A$  excludes at least one element of  $B$ .
- (e) Complement of  $A$ : all the elements that are in the universal set but not in  $A$  Relative complement of  $A$  in  $B$ : all elements in  $B$  that are not in  $A$ .

### 2. Notation and Important Properties

- (a) subset:  $A \subseteq B$
- (b) proper subset:  $A \subset B$
- (c) intersection:  $A \cap B$ 
  - i.  $A \cap B = B \cap A$
  - ii.  $A \cap B = \emptyset$ , then  $A$  and  $B$  are disjoint (no common elements)
  - iii.  $A \cap B = A$ , then  $A$  is a subset of  $B$
  - iv.  $A \cap \emptyset = \emptyset$
  - v.  $A \cap U = A$  where  $U$  is the universal set
- (d) union:  $A \cup B$ 
  - i.  $A \cup B = B \cup A$
  - ii.  $A \cup \emptyset = A$
  - iii.  $A \cup U = U$  where  $U$  is the universal set
  - iv.  $A \cup B = A$ , then  $B$  is a subset of  $A$  ( $A$  contains all elements in  $B$  and possibly more)
- (e) relative complement:  $A \setminus B$ 
  - i. If relative complement of  $A$  in  $B$  has cardinality of non zero, then  $A$  is a proper subset of  $B$ .
  - ii.  $A \setminus A = \emptyset$
  - iii.  $A \setminus \emptyset = A$
  - iv.  $\emptyset \setminus A = \emptyset$

### 3. Important Sets

- (a)  $\mathbb{N}$ : natural numbers (0,1,2,3, ...)
- (b)  $\mathbb{Z}$ : integers (... -3, -2, -1, 0, 1, 2, 3, ...)
- (c)  $\mathbb{Q}$ : rational numbers ( $\frac{a}{b}$  such that  $a, b \in \mathbb{Z}$ )
- (d)  $\mathbb{R}$ : real numbers
- (e)  $\mathbb{C}$ : complex numbers