Proof Methods

Direct Proof	by Contraposition	Contradiction	by Cases
$P \Rightarrow Q$	$\neg Q \Rightarrow \neg P$	P	P
Assume P, tweak stuff therefore get Q.	Assume $\neg Q$, tweak stuff, therefore get $\neg P$	Assume $\neg P$, tweak stuff, show contradiction by showing that $\neg P \Rightarrow R$ and $\neg P \Rightarrow \neg R$. Since the individual implications has to be True, $\neg P \Rightarrow (R \wedge \neg R)$ has to hold. Notice $(R \wedge \neg R)) \equiv F$. So $\neg P \Rightarrow F$ can only hold if $\neg P$ is false, $(F \Rightarrow F \equiv T)$. So, $\neg P \equiv F \Rightarrow P \equiv T$. Thus P has to be T.	list all of exhaustive cases.
-When you an use the info given with P to derive info that eventually leads you to Q	- When the info given from P is vague or hard to derive info from, assuming $\neg Q$ gives you more information and easier to derive conclusions from	-When there is only a single statement to be proven without an implication	-Usually when nothing else works -But technically you can do all proofs using by cases, if you are determined enough.
Can use when: - specific definitions given in P such as "perfect square," "odd," "divisible by 5" can be used as stepping stones to get to next step - Assuming P immediately gives you information about other variables in the question (such as for odd n, $n = m^2$, means m has to be odd etc)	Can use when: - P doesn't give enough information to move forward, is to vague or hard to assume - $\neg Q$ is easier to assume, use and expand from - when P is "n bigger/smaller than 6" (covers all n bigger than 6, hard to build on) - when $\neg Q$ is a simple statement that can easily be negated	Can use when: - Mostly when there is a simple statement like "there exists," "there is," without an implicit implication - P is easy to negate and develop on: - "there are infinite prime numbers" $\rightarrow$ "there are finite prime numbers" $\rightarrow$ "there is a largest prime number" - "at least 2 people have same number of friends" $\rightarrow$ "nobody has same number of friends" $\rightarrow$ "everybody has a unique number of friends"	Can use when: - Either case a, or case b, or case c ... has to occur but it is not possible to know which one will. - Only can be used when all possible cases are known, so be careful with infinite sets. - For example, the values n can take in the set Natural Numbers can be separated into cases as n=0, n=1, n=2 ... (which is hard to take into account one by one) or as $n < 4, n = 4, n > 4$ etc, so define cases carefully.

Useful Notes

1. Basic Negation Keypoints
   (a) everybody does P $\rightarrow$ there exists (at least one) person that does not P
   (b) there exists (at least one) n such that P $\rightarrow$ all n are (not P)
   (c) $A > B \rightarrow A <= B$
2. To have a proof of "if and only if" have to show the original implication and its converse to be true. If converse holds for an implication $P \Rightarrow Q$, then $P \equiv Q$.
3. Proof by contraposition relies on the fact that contraposition of an implication is logically equivalent to the original implication.
4. Contraposition proof is essentially the direct proof of contrapositive, the only difference is that you start by assuming a the negation of RHS. So, if the goal is to prove an implication, decide between direct proof or contraposition by deciding whether P or $\neg Q$ is easier to assume.
5. Proof by contradiction especially works well with proving something does not exist. Start by assuming that it does exist and show why it would break the universe if it did exist. (Key idea: assume negation to show some nonsense/contradiction, but since all the steps were legit based on that assumption, the only explanation is that the assumption itself was wrong)