Mathematical Notation and Definitions

1. Definitions

   (a) Set: well defined collection of objects, consisting of elements. Denoted by capital letters. Order of elements does not matter.

   (b) Cardinality: size of a set, $|A|$, can be any number from 0 to infinity. Empty set B has cardinality of 0, $|B| = 0$

   (c) Equal Sets: two sets that have the same elements

   (d) Subset A: every element in A is contained in B.
   Proper subset A: every element in A is contained in B and B has elements that are not in A, $|B| > |A|$ since A excludes at least one element of B.

   (e) Complement of A: all the elements that are in the universal set but not in A Relative complement of A in B: all elements in B that are not in A.

2. Notation and Important Properties

   (a) subset: $A \subseteq B$

   (b) proper subset: $A \subset B$

   (c) intersection: A $\bigcap$ B

      i. $A \bigcap B = B \bigcap A$
      ii. $A \bigcap B = \emptyset$, then A and B are disjoint (no common elements)
      iii. $A \bigcap B = A$, then A is a subset of B
      iv. $A \bigcap \emptyset = \emptyset$
      v. $A \bigcap U = A$ where U is the universal set

   (d) union: A $\bigcup$ B

      i. $A \bigcup B = B \bigcup A$
      ii. $A \bigcup \emptyset = A$
      iii. $A \bigcup U = U$ where U is the universal set
      iv. $A \bigcup B = A$, then B is a subset of A (A contains all elements in B and possibly more)

   (e) relative complement: $A \backslash B$

      i. If relative complement of A in B has cardinality of non zero, then A is a proper subset of B.
      ii. $A \backslash A = \emptyset$
      iii. $A \backslash \emptyset = A$
      iv. $\emptyset \backslash A = \emptyset$

3. Important Sets

   (a) N: natural numbers (0,1,2,3, ...)

   (b) Z: integers (... -3, -2, -1, 0, 1, 2, 3, ...)

   (c) Q: rational numbers ($\frac{a}{b}$ such that $a, b \in Z$)

   (d) R: real numbers

   (e) C: complex numbers

Quantifiers

1. $\forall x$: universal quantifier, "for all x"
   $\forall x, P(x) \equiv P(x_1) \bigwedge P(x_2) \bigwedge P(x_3) \bigwedge$ ... for all x_i values

2. $\exists x$: existential quantifier, "there exists x", can refer to one or more x (at least one)
   $\exists x, P(x) \equiv P(x_1) \bigvee P(x_2) \bigvee P(x_3) \bigvee$ ... for all x_i values