Summer 2018 Sinho Chewi and Vrettos Moulos Head Count Consider flipping a fair coin twice. (a) What is the sample space Ω generated from these flips? TT, TH, HT, HH ? sample points have no (b) Define a random variable X to be the number of heads. What is the distribution of X?

POINT maps to $\alpha \times = 0$ TT TH X = # heads = 0, 1, 2Single RV X = 2 + HH TH X = 1 $P(X = 0) = P(TT) = \frac{1}{4}$ X = 1 X = $Y = \begin{cases} 1 & \text{if } HT & P(H,T) = \frac{1}{4} \\ 0 & \text{els} \end{cases}$ Y= { 1 wp 0.75 (d) Define a third random variable Z = X + Y. What is the distribution of Z? $= \begin{cases} 0 \longrightarrow \mathcal{H} & x=0, Y=0 \\ 1 \longrightarrow \mathcal{H} & x=1, Y=0 \\ 2 \longrightarrow \mathcal{H} & x=1, Y=1 \\ 3 \longrightarrow \mathcal{H} & x=1,$ Maybe Lossy Maybe Not $P(Z=Z) = P(X=1 \cap Y=1) + P(P(X=Z) \cap Y=0)$ P(X=1) P(Y=1 | X=1) Let us say that Alice would like to send a message to Bob, over some channel. Alice has a message of length 4 and sends 5 packets. (a) Packets are dropped with probability p. What is probability that Bob can successfully reconneed poly. degree -> 4-1=3 -> need 4 points to construct poly.

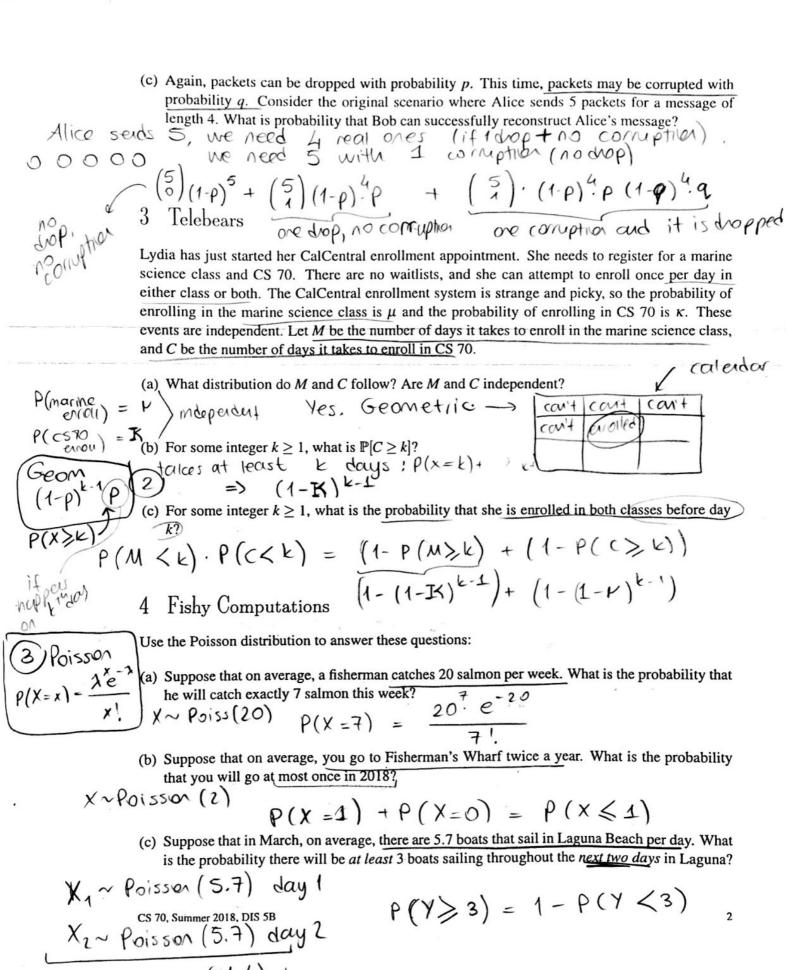
Goal: Probability that at most 1 is lost. (O or 1) (5) (1-p)4.p + (5) (1-p)5.p° (b) Again, packets can be dropped with probability p. The channel may additionally corrupt 1 packet. Alice realizes this and sends 3 additional packets. What is the probability that Bob receives enough packets to successfully reconstruct Alice's message? (1 corruption -> 2 extra needed)

Alice now sends 5+3 extra = 8 7 n + 2(1) = 6 at least original message = 4 packets } 4 one corruption

Some corruption -> Probability that 0, 1 or 2 padcages are lost. (the rest = 2 (8) (1-p)8-k, pk can be

Discrete Mathematics and Probability Theory

CS 70



Y ~ Poisson (11.4) days