Notation

1. Definifitons

   (a) Axiom: known to be true statements, asserted as True, aren't proven.

   (b) Proposition: statements that have a truth value of either true or false, should be proven. We use axioms to determine truth values of propositions.

   (c) Propositional form: can create larger statements by putting propositions together using logical symbols, (i.e $\neg((P_1 \vee P_2) \wedge (P_3 \vee P_4))$

   (d) Logical Equivalence: $A \equiv \neg((P_1 \vee P_2) \wedge (P_3 \vee P_4))$ shows that A and the given complex proposition represent the same statement and their truth values are the same. Like an equal sign in logic notation (i.e can write $2 + 6 + 8 = 8 + 8 = 16$). Since we don't know the value of the proposition during the proof, we can't use the equal sign but we can step by step simplify the original statement by finding its logical equivalent at each step.

2. Logical Symbols

   (a) not: $\neg P$, unary operation

   (b) and: $P \wedge Q$, binary operation

   (c) or: $P \vee Q$, binary operation

   (d) implies: $P \Rightarrow Q$, binary operation, $(P \Rightarrow Q) \equiv \neg P \vee Q$

Distributive Laws and Inference

1. Distributing Negation: De Morgan's Laws

   (a) $\neg\neg P \equiv P$

   (b) $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$

   (c) $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$

   (d) $\neg(\forall x\ P(x)) \equiv \exists x\ \neg P(x)$

   (e) $\neg(\exists x\ P(x) \equiv \forall x\ \neg x P(x)$

2. Distributing Conjunctive and Disjunctive Forms

   (a) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$, similarly $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge (P \wedge R) \equiv P \wedge Q \wedge R$

   (b) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$, similarly $P \vee (Q \vee R) \equiv (P \vee Q) \vee (P \vee R) \equiv P \vee Q \vee R$

3. Inference: we can simplify statements using known truth values:

   (a) $T \wedge P \equiv P$

   (b) $F \wedge P \equiv F$

   (c) $T \vee P \equiv T$

   (d) $F \vee P \equiv P$

Understanding Implication

1. $P \Rightarrow Q$: If P, then Q. Gives a condition/background for Q.

2. Class example: P(x): student x is in cs70 discussion, Q(x): "student x is taking cs70." Consider then "if P, then Q" true (i.e it would make logical sense in this example)?

   (a) if student is in class, and she is taking cs70: $T \Rightarrow T \equiv T$
   (makes sense since she showed up for the discussion section)

   (b) if student is not in class, and she is taking cs70: $F \Rightarrow T \equiv T$
   (makes sense since she might be in a different discussion section)

   (c) if student is not in class, and she is not taking cs70: $F \Rightarrow F \equiv T$
   (makes sense since if she is not taking the class, there is no reason for her to be in discussion)

   (d) it would only not make sense if the student was in class but she is not enrolled in cs70: $T \Rightarrow F \equiv F$ (since why would she come to discussion for a class she is not enrolled in)

Truth Tables

P	Q	R	$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

1. Use truth tables to get truth values of complex propositions.

2. We specify functions by specifying their outputs for each possible output.

3. If final columns of two truth tables are the same, then two functions are logical equivalent of each other.