2024 IUT Admission Test (SOCIE, Scholarship)

Math Examination (A TYPE)

<Multiple choice Types> There is only one correct answer per each question. Mark your answer choice on the OMR answer sheet.

○ For each correct answer, you will get the points indicated next to each question number.

○ No penalty point is applied to an incorrect answer.

1. [2 points]
Evaluate

$$\lim_{x \to 0} \frac{\sin^2 x}{1 - \cos 2x}.$$

① $\frac{1}{2}$ ② $\frac{3}{2}$ ③ 2
④ 1 ⑤ 0

2. [2 points]
Find the sum of solutions of the equation.

$$4^x - 2^{x+2} + 2 = 0$$

① 0 ② 1 ③ 2 ④ 3 ⑤ 4

3. [2 points]
Evaluate

$$\sum_{n=1}^{15} \frac{1}{\sqrt{n+1} + \sqrt{n}}.$$

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

4. [2 points]
When $x = 1 + \sqrt{2}$, find $x^4 + \frac{1}{x^4}$.

① 26 ② 28 ③ 30
④ 32 ⑤ 34

5. [2 points]
Simplify

$$\sin^2\theta + \sin^2\left(\theta + \frac{\pi}{3}\right) + \sin^2\left(\theta - \frac{\pi}{3}\right).$$

① $\frac{1}{2}$ ② 1 ③ $\frac{3}{2}$ ④ 2 ⑤ $\frac{5}{2}$

6. [3 points]
Let M, m be the maximum and the minimum value of $f(x)$. Find $M + m$.

$$f(x) = \cos^2 x - 2\sin x + 1$$

① −1 ② 0 ③ 1 ④ 2 ⑤ 3

7. [3 points]
When $g(x) = (x+1)\sqrt{f(x)}$ and $f(1) = 2$, $f'(1) = 3$ find $g'(1)$.

① $\frac{\sqrt{2}}{2}$ ② $\sqrt{2}$ ③ $\frac{3\sqrt{2}}{2}$
④ $2\sqrt{2}$ ⑤ $\frac{5\sqrt{2}}{2}$

[3 points]

8. When $w = \dfrac{-1+\sqrt{3}i}{2}$, find $\displaystyle\sum_{n=1}^{10} w^{2n}$.

① $\dfrac{3}{2}+\dfrac{\sqrt{3}}{2}i$ ② $\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$ ③ $\dfrac{3}{2}-\dfrac{\sqrt{3}}{2}i$

④ $-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$ ⑤ $-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

9. [3 points]

Calculate $\displaystyle\int_0^{\pi}(1+2\sin^2 x)\,dx$.

① $\dfrac{\pi}{2}$ ② π ③ $\dfrac{3\pi}{2}$ ④ 2π ⑤ $\dfrac{5\pi}{2}$

10. [3 points]

The sequence $\{a_n\}$ satisfies

$$\sum_{k=1}^{n}\frac{1}{ka_k}=n^2+3n$$

Compute $\displaystyle\sum_{n=1}^{10} a_n$.

① $\dfrac{2}{11}$ ② $\dfrac{4}{11}$ ③ $\dfrac{5}{11}$ ④ $\dfrac{10}{11}$ ⑤ $\dfrac{20}{11}$

11. [4 points]

When two graph of functions

$$y=x^3-2x^2+k,\ y=x^2-2$$

meet at two distinct points, find the positive value of k.

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

12. [4 points]

The function $f(x)$ is continuous at $x=0$.

$$f(x)=\begin{cases}\dfrac{e^x+ax+b}{x}, & x\neq 0\\ 2, & x=0\end{cases}$$

Find $a+b$. (a, b are constants)

① 0 ② −1 ③ −2 ④ 1 ⑤ 2

13. [4 points]

When $A=\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}2&0\\0&1\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}^{-1}$, find the maximum value of

$$\begin{pmatrix}\cos x & \sin x\end{pmatrix}A\begin{pmatrix}\cos x\\ \sin x\end{pmatrix}.$$

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

14. [4 points]

When $\dfrac{f'(x)}{1+f(x)}=x$ and $f(0)=1$, find $f(2)$.

① $2e^2-2$ ② $2e^2-1$ ③ $2e^2$

④ $2e^2+1$ ⑤ $2e^2+2$

15. [4 points]

The sequence $\{a_n\}$ with $a_5+a_6=10$ satisfies

$$a_{n+1}=\begin{cases}2a_n+1, & \text{If } a_n \text{ is odd}\\ \dfrac{a_n}{2}, & \text{If } a_n \text{ is even}\end{cases}.$$

Find a_8.

① 6 ② 7 ③ 15 ④ 31 ⑤ 32

16. [5 points]
Find the distance between $y = x^2$ and the curve $x^2 + y^2 + 10x + 2y + 25 = 0$.

① $2\sqrt{5} + 2$ ② $2\sqrt{5} + 1$ ③ $2\sqrt{5}$

④ $2\sqrt{5} - 2$ ⑤ $2\sqrt{5} - 1$

17. [5 points]
The function $f(x) = (ax^2 + b)e^{x-1}$ with $f(1) = 4$ satisfies $f(x) \geqslant 6\ln x + 4$. Find $a^2 + b^2$.

① 8 ② 10 ③ 12

④ 16 ⑤ 18

18. [5 points]
The polynomial $f(x)$ with $f(0) = f(3)$ satisfies

$$\lim_{x\to\infty} \frac{f(x)}{x^3+1} = 1,\ \lim_{x\to 0} \frac{f(x+3)}{f(x)\left[f^2(x)+1\right]} = 1\ .$$

Find the value of $f(4)$.

① 6 ② 8 ③ 10 ④ 12 ⑤ 14

19. [5 points]
Let $f(t)$ be the slope of a tangent line to the curve $y = e^x + t$ passing through $(0, -1)$. Find $f'(e^2 - 1)$.

① -1 ② $-\frac{1}{2}$ ③ 0

④ $\frac{1}{2}$ ⑤ 1

20. [5 points]
The continuous function $f(x)$ satisfies

$$f(x)\int_0^x (x-t)f(t)dt + \left(\int_0^x f(t)dt\right)^2 = 15x^4 + 40x^3 + 24x^2\ .$$

When $f(0) > 0$, find $f(2)$.

① 12 ② 14 ③ 16

④ 18 ⑤ 20