1) Write the first and second canonical forms of *f*.

1st canonical form: Sum of Products

"True" (1) combinations

Sum of Minterms:

$$F(a,b,c,d) = \sum m(1,4,5,6,7,9,12,13,14,15)$$

$$= a'b'c'd + a'bc'd' + a'bc'd + a'bcd' + a'bcd + ab'c'd + abc'd' + abc'd + abcd' + abcd$$

2nd canonical form: Product of Sums

"False" (0) combinations

Product of Maxterms:

$$F(a,b,c,d) = \prod M(0,2,3,8,10,11)$$

$$= (a+b+c+d)(a+b+c'+d)(a+b+c'+d')(a'+b+c+d)$$

$$\cdot (a'+b+c'+d)(a'+b+c'+d')$$

2) Convert <u>the second canonical form expression to first canonical form expression</u> using axioms and theorems of Boolean algebra

$$F(a,b,c,d) = (a+b+c+d)(a+b+c'+d)(a+b+c'+d')(a'+b+c+d)(a'+b+c'+d)(a'+b+c'+d')$$

$$= (a+b+d)(a+b+c')(a'+b+d)(a'+b+c')$$

Absorption+consensus

$$= (b+d)(b+c')$$

Absorption+consensus

Distributive

$$= b + c'd \quad \text{(Minimized expression)}$$

Need to expand this to get first canonical form:

$$= b(a+a')(c+c')(d+d') + c'd(a+a')(b+b')$$

expand by ANDing with both forms of missing literals

$$= abcd + abcd' + abc'd + abc'd' + a'bcd + a'bcd' + a'bc'd + a'bc'd'$$

$$+ abc'd + ab'c'd + a'bc'd + a'b'c'd$$

$$= a'b'c'd + a'bc'd' + a'bc'd + a'bcd' + a'bcd + ab'c'd + abc'd' + abc'd + abcd' + abcd$$

3) Minimize the expression for f(a,b,c,d) in the <u>first canonical form</u> using axioms and theorems of Boolean algebra. <u>Show all steps</u> in your minimization and <u>write the name of the axiom/theorem/property</u> you use on the right-hand side of the expression <u>at each step.</u>

$$F(a,b,c,d) = a'b'c'd + a'bc'd' + a'bc'd + a'bcd' + a'bcd + ab'c'd + abc'd' + abc'd + abcd' + abcd$$

distributive

$$= (a+a')b'c'd + a'bc'(d+d') + (a+a')bcd' + (a+a')bcd + abc'(d+d')$$

inverse, identity

$$= b'c'd + a'bc' + bcd' + bcd + abc'$$

distributive

$$= b'c'd + (a+a')bc' + bc(d+d')$$

inverse, identity

$$= bc + bc' + b'c'd$$

distributive

$$= b(c+c') + b'c'd$$

inverse, identity

$$= b + b'c'd$$

consensus

$$= b + b'c'd + c'd$$

distributive

$$= b + (b'+1)c'd$$

annihilator

$$= b + c'd$$

OR

Absorption

4) Draw the circuit for the <u>minimized</u> expression you found in Question 3 above using <u>2-input</u> NAND <u>gates only</u>. Show all steps, and explain your work leading up to the final circuit.

$$F(a,b,c,d) = \overline{\overline{F(a,b,c,d)}} = \overline{\overline{b + c'd}} = \overline{\overline{b}\ \overline{(c'd)}}$$