September 30, 2022

Istanbul Technical University

Department of Computer Engineering

BLG 231E - Digital Circuits

Assignment 1

Due Date: Thursday, October 6, 2022, 23:59.

- Please prepare your homework using a computer. Points will be taken off for handwritten submissions.
- **Consequences of plagiarism:** Any cheating will be subject to disciplinary action.
- **No late submissions** will be accepted. **Do not send your solutions by e-mail.** We will only accept files that have been uploaded to the official Ninova e-learning system before the deadline. Do not risk leaving your submission to the last few minutes.
- **Submissions:** Submit your solution PDFs to Ninova. Please **write your full name** (first name and last name) **and Student ID** inside the box below.

Student ID	:
Full Name	:

If you have any questions, please e-mail teaching assistant

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Part 1 – Computer Arithmetic

1) **a)** Using **signed** 2's-complement representation, convert the decimal numbers $(-\mathbf{102})$ and $(-\mathbf{27})$ to **8-bit binary** integers. Show ALL work.

b) Carry out the **binary** operations given below, and explain your answers using terms such as *carry*, *borrow*, and *overflow*. To interpret the results, use only binary numbers.

i. (-102) + (-27)

ii. (-102) – (-27)

iii. (27) – (102)

Part 2 – Boolean Algebra

2) Simplify each Boolean expression using algebraic manipulation (axioms and theorems). Show ALL work.
Show the steps of the simplification, and **write which axiom/theorem you used in each step** next to the simplification.

1) a) * Start with the absolute value of the number: 102

102 / 2 = 51 remainder 0 .... LSB (Least Significant Bit)

51 / 2 = 25 remainder 1

25 / 2 = 12 remainder 1

12 / 2 = 6 remainder 0

6 / 2 = 3 remainder 0

3 / 2 = 1 remainder 1

1 / 2 = 0 remainder 1 .... MSB (Most Significant Bit)

8-bit $+102_{10}$	:	0110 0110
1's complement	:	1001 1001
Add 1	:	+ 1
Result -102_{10}	:	1001 1010

*Start with the absolute value of the number: 27

27 / 2 = 13 remainder 1 .... LSB (Least Significant Bit)

13 / 2 = 6 remainder 1

6 / 2 = 3 remainder 0

3 / 2 = 1 remainder 1

1 / 2 = 0 remainder 1 .... MSB (Most Significant Bit)

8-bit $+27_{10}$	:	0001 1011
1's complement	:	1110 0100
Add 1	:	+ 1
Result -27_{10}	:	1110 0101

b)

i.

-102_{10}	:	1001 1010
-27_{10}	:	+ 1110 0101
		1 0111 1111

Note: (n+1)st bit is one.

It is ignored.

The sign bit (n^{th} bit) is 0 (green). So, this number is positive.

Remember: In an addition operation, overflow can occur in two cases:

$$pos. + pos. \rightarrow\ neg.\ and\ neg. + neg. \rightarrow\ pos.$$

Both operands are negative, and the result is positive. This means that there is overflow here.

So, $(-102) + (-27)$ cannot be represented using 8 bits.

ii. $(-102) - (-27) = (-102) + (+27)$

-102_{10}	:	1001 1010
$+27_{10}$	:	+ 0001 1011
-75_{10}	:	1011 0101

Since this is a signed number operation, we do not consider carry and borrow.

neg – neg -> neg: There is no overflow.

iii. *If we assume that 27 and 102 are signed numbers:*

$(27) - (102) = 27 + (-102)$

$+27_{10}$	:	0001 1011
-102_{10}	:	+ 1001 1010
-75_{10}	:	1011 0101

Since this is a signed number operation, we do not consider carry and borrow.

pos – pos -> neg: There is no overflow.

If we assume that 27 and 102 are unsigned numbers:

$(27) - (102) = 27 + (-102)$

27_{10}	:	0001 1011
-102_{10}	:	+ 1001 1010
	:	1011 0101

There is no carry. So, there is borrow. We cannot represent this result using unsigned integers.

2)

a) $(Y+\bar{X}\bar{Z})(X+\bar{Y}+\bar{Z})(\bar{X}+Y)$

$$= YX\bar{X} + Y\bar{Y}\bar{X} + Y\bar{Z}\bar{X} + \bar{X}\bar{Z}X\bar{X} + \bar{X}\bar{Z}\bar{Y}\bar{X} + \bar{X}\bar{Z}\bar{Z}\bar{X} + YXY + Y\bar{Y}Y + Y\bar{Z}Y + \bar{X}\bar{Z}XY + \bar{X}\bar{Z}\bar{Y}Y + \bar{X}\bar{Z}\bar{Z}Y$$

$$= YX\bar{X} + Y\bar{Y}\bar{X} + Y\bar{Z}\bar{X} + \bar{X}\bar{Z}X + \bar{X}\bar{Z}\bar{Y} + \bar{X}\bar{Z} + YX + Y\bar{Y} + Y\bar{Z} + \bar{X}\bar{Z}XY + \bar{X}\bar{Z}\bar{Y}Y + \bar{X}\bar{Z}Y$$

$$= Y\cdot 0 + \bar{X}\cdot 0 + Y\bar{Z}\bar{X} + \bar{Z}\cdot 0 + \bar{X}\bar{Z}\bar{Y} + \bar{X}\bar{Z} + YX + 0 + Y\bar{Z} + \bar{Z}Y\cdot 0 + \bar{X}\bar{Z}\cdot 0 + \bar{X}Y\bar{Z}$$

$$= 0 + 0 + Y\bar{Z}\bar{X} + 0 + \bar{X}\bar{Z}\bar{Y} + \bar{X}\bar{Z} + YX + 0 + Y\bar{Z} + 0 + 0 + \bar{X}Y\bar{Z}$$

$$= Y\bar{Z}\bar{X} + \bar{X}\bar{Z}\bar{Y} + \bar{X}\bar{Z} + YX + Y\bar{Z}$$

$$= \bar{X}Y\bar{Z} + \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Z} + XY + Y\bar{Z}$$

$$= \bar{X}Y\bar{Z} + \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Z} + XY + Y\bar{Z}(X+\bar{X})$$

$$= \bar{X}Y\bar{Z} + \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Z} + XY + XY\bar{Z} + \bar{X}Y\bar{Z}$$

$$= \bar{X}\bar{Z}(Y+\bar{Y}+1+Y) + XY(1+\bar{Z})$$

$$= \bar{X}\bar{Z} + XY$$

Distributive
Idemp. (4^{th}, 5^{th}, 6^{th}, 7^{th}, 8^{th}, 9^{th}, 12^{th})
Inverse (1^{st}, 2^{nd}, 4^{th}, 8^{th}, 10^{th}, 11^{th})
& Commutative (2^{nd}, 4^{th}, 10^{th}, 12^{th})
Annihilator/Dominance
Identity+Idempotency
Commutative
Distributive & Inverse & Identity, 5t
Distributive
Distributive
Annihilator/Dominance + Identity

So,

$$\boxed{(Y+\bar{X}\bar{Z})(X+\bar{Y}+\bar{Z})(\bar{X}+Y) = \bar{X}\bar{Z} + XY}$$

b) $\bar{X}\bar{Y}\bar{Z}\bar{T} + X\bar{Y}T + \bar{X}\bar{Y}Z + XZ\bar{T} + X\bar{Y}ZT + \bar{X}\bar{Y}\bar{Z}$

$$= \bar{X}\bar{Y}\bar{Z}\bar{T} + X\bar{Y}T + \bar{X}\bar{Y}Z + XZ\bar{T} + X\bar{Y}ZT + \bar{X}\bar{Y}\bar{Z}$$

$$= \bar{X}\bar{Y}\bar{Z}\bar{T} + X\bar{Y}T + \bar{X}\bar{Y}Z(T+\bar{T}) + XZ\bar{T} + X\bar{Y}ZT + \bar{X}\bar{Y}\bar{Z}$$

$$= \bar{X}\bar{Y}\bar{Z}\bar{T} + X\bar{Y}T + \bar{X}\bar{Y}ZT + \bar{X}\bar{Y}Z\bar{T} + XZ\bar{T} + X\bar{Y}ZT + \bar{X}\bar{Y}\bar{Z}$$

$$= \bar{X}\bar{Y}\bar{Z}\bar{T} + X\bar{Y}T + \bar{X}\bar{Y}ZT + \bar{X}\bar{Y}Z\bar{T} + XZ\bar{T} + X\bar{Y}ZT + \bar{X}\bar{Y}\bar{Z}(T+\bar{T})$$

$$= \bar{X}\bar{Y}\bar{Z}\bar{T} + X\bar{Y}T + \bar{X}\bar{Y}ZT + \bar{X}\bar{Y}Z\bar{T} + XZ\bar{T} + X\bar{Y}ZT + \bar{X}\bar{Y}\bar{Z}T + \bar{X}\bar{Y}\bar{Z}\bar{T}$$

$$= \bar{X}\bar{Y}\bar{Z}\bar{T} + X\bar{Y}T + \bar{X}\bar{Y}ZT + \bar{X}\bar{Y}ZT + \bar{X}\bar{Y}Z\bar{T} + XZ\bar{T} + X\bar{Y}ZT + \bar{X}\bar{Y}\bar{Z}T + \bar{X}\bar{Y}\bar{Z}T + \bar{X}\bar{Y}\bar{Z}\bar{T}$$

$$= \bar{X}\bar{Y}(\bar{Z}\bar{T} + ZT + Z\bar{T} + \bar{Z}T) + \bar{Y}T(X + \bar{X}Z + XZ + \bar{X}\bar{Z}) + XZ\bar{T}$$

$$= \bar{X}\bar{Y}[\bar{Z}(\bar{T}+T) + Z(\bar{T}+T)] + \bar{Y}T[X(1+Z) + \bar{X}(Z+\bar{Z})] + XZ\bar{T}$$

$$= \bar{X}\bar{Y}(\bar{Z}+Z) + \bar{Y}T(X+\bar{X}) + XZ\bar{T}$$

$$= \bar{X}\bar{Y} + \bar{Y}T + XZ\bar{T}$$

Distributive+Inverse+Identity (3^{rd})
Distibutive
Inverse (7^{th})
Distributive (7^{th})
Idempotency (3^{rd} , 7^{th})
Distributive+Commutative
Distributive
Inverse+Annihilator/Dominance
Inverse

So,

$$\boxed{\bar{X}\bar{Y}\bar{Z}\bar{T} + X\bar{Y}T + \bar{X}\bar{Y}Z + XZ\bar{T} + X\bar{Y}ZT + \bar{X}\bar{Y}\bar{Z} = \bar{X}\bar{Y} + T\bar{Y} + \bar{T}XZ}$$