Istanbul Technical University

Department of Computer Engineering

BLG 231E - Digital Circuits

Assignment 1

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1.i) 1st canonical form of **f**

Row Num.	a	b	c	d	f	minterms
0	0	0	0	0	0	m0 = a'b'c'd'
1	0	0	0	1	1	m1 = a'b'c'd
2	0	0	1	0	0	m2 = a'b'cd'
3	0	0	1	1	0	m3 = a'b'cd
4	0	1	0	0	1	m4 = a'bc'd'
5	0	1	0	1	1	m5 = a'bc'd
6	0	1	1	0	1	m6 = a'bcd'
7	0	1	1	1	1	m7 = a'bcd
8	1	0	0	0	0	m8 = ab'c'd'
9	1	0	0	1	1	m9 = ab'c'd
10	1	0	1	0	0	m10 = ab'cd'
11	1	0	1	1	0	m11 = ab'cd
12	1	1	0	0	1	m12 = abc'd'
13	1	1	0	1	1	m13 = abc'd
14	1	1	1	0	1	m14 = abcd'
15	1	1	1	1	1	m15 = abcd

"True" (1-generating) combinations: **0001**, **0100**, **0101**, **0110**, **0111**, **1001**, **1100**, **1101**, **1110**, **1111**

f = $\sum_{a,b,c,d}$ (1,4,5,6,7,9,12,13,14,15) (sum of minterms)

f(**a**,**b**,**c**,**d**) = $\sum$m(1,4,5,6,7,9,12,13,14,15)

f(**a**,**b**,**c**,**d**) = **m1 + m4 + m5 + m6 + m7 + m9 + m12 + m13 + m14 + m15**

f(**a**,**b**,**c**,**d**) = **a'b'c'd + a'bc'd' + a'bc'd + a'bcd' + a'bcd + ab'c'd + abc'd' + abc'd + abcd' + abcd**

2.i) 2nd canonical form of **f**

Row Num.	a	b	c	d	f	MAXTERMS
0	0	0	0	0	0	M0 = a+b+c+d
1	0	0	0	1	1	M1 = a+b+c+d’
2	0	0	1	0	0	M2 = a+b+c’+d
3	0	0	1	1	0	M3 = a+b+c’+d’
4	0	1	0	0	1	M4 = a+b’+c+d
5	0	1	0	1	1	M5 = a+b’+c+d’
6	0	1	1	0	1	M6 = a+b’+c’+d
7	0	1	1	1	1	M7 = a+b’+c’+d’
8	1	0	0	0	0	M8 = a’+b+c+d
9	1	0	0	1	1	M9 = a’+b+c+d’
10	1	0	1	0	0	M10 = a’+b+c’+d
11	1	0	1	1	0	M11 = a’+b+c’+d’
12	1	1	0	0	1	M12 = a’+b’+c+d
13	1	1	0	1	1	M13 = a’+b’+c+d’
14	1	1	1	0	1	M14 = a’+b’+c’+d
15	1	1	1	1	1	M15 = a’+b’+c’+d’

“False” (0-generating) combinations: **0000**, **0010**, **0011**, **1000**, **1010**, **1011**

f = ∏ **a,b,c,d** (0,2,3,8,10,11) (product of maxterms)

f(**a**,**b**,**c**,**d**) = ∏M(0,2,3,8,10,11)

f(**a**,**b**,**c**,**d**) = **M0 · M2 · M3 · M8 · M10 · M11**

f(**a**,**b**,**c**,**d**) = **(a+b+c+d)(a+b+c’+d)(a+b+c’+d’)(a’+b+c+d)(a’+b+c’+d)(a’+b+c’+d’)**

2. Convertion from the 2nd canonical form expression to 1st canonical form expression

f(a,b,c,d,) = (a+b+c+d)(a+b+c'+d)(a+b+c'+d')(a'+b+c+d)(a'+b+c'+d)(a'+b+c'+d')

$\boxed{\text{2nd Canonical Form}}$

1. Distributive: (a+b+c+d)(a'+b+c+d) = (aa')+(b+c+d)

2. Distributive: (a+b+c'+d)(a'+b+c'+d) = (aa')+(b+c'+d)

3. Distributive: (a+b+c'+d')(a'+b+c'+d') = (aa')+(b+c'+d')

= [(aa')+(b+c+d)] [(aa')+(b+c'+d)] [(aa')+(b+c'+d')]

4. Inverse: aa' = 0

5. Inverse: aa' = 0

6. Inverse: aa' = 0

= [(0)+(b+c+d)][(0)+(b+c'+d)][(0)+(b+c'+d')]

7. Identity: (0)+(b+c+d) = (b+c+d)

8. Identity: (0)+(b+c'+d) = (b+c'+d)

9. Identity: (0)+(b+c'+d') = (b+c'+d')

= (b+c+d)(b+c'+d)(b+c'+d')

10. Distributive: (b+c+d)(b+c'+d) = (cc') + (b+d)

= [(cc') + (b+d)] (b+c'+d')

11. Inverse: cc' = 0

= [(0) + (b+d)] (b+c'+d')

12. Identity: (0) + (b+d) = b+d

= (b+d)(b+c'+d')

13. Distributive: (b+d)(b+c'+d') = b + d(c'+d')

= b + d(c'+d')

14. Distributive: d(c'+d') = c'd + dd'

= b + c'd + dd'

15. Inverse: dd' = 0

= b + c'd + (0)

16. Identity: b + c'd + (0) = b + c'd

= b + c'd

17. Identity: b = b(1)

= b(1) + c'd

18. Inverse: 1 = (c+c')

= b(c+c') + c'd

19. Distributive: b(c+c') = bc + bc'

= bc + bc' + c'd

20. Identity: bc = bc(1)

21. Identity: bc' = bc'(1)

22. Identity: c'd = c'd(1)

= bc(1) + bc'(1) + c'd(1)

23. Inverse: (1) = (d+d')

24. Inverse: (1) = (d+d')

25. Inverse: (1) = (b+b')

= bc(d+d') + bc'(d+d') + c'd(b+b')

26. Distributive: bc(d+d') = bcd + bcd'

27. Distributive: bc'(d+d') = bc'd + bc'd'

28. Distributive: c'd(b+b') = bc'd + b'c'd

= bcd + bcd' + bc'd + bc'd' + bc'd + b'c'd

29. Idempotency: bc'd + bc'd = bc'd

= bcd + bcd' + bc'd + bc'd' + b'c'd

30. Identity: bcd = (1)bcd

31. Identity: bcd' = (1)bcd'

32. Identity: bc'd = (1)bc'd

33. Identity: bc'd' = (1)bc'd'

34. Identity: b'c'd = (1)b'c'd

= (1)bcd + (1)bcd' + (1)bc'd + (1)bc'd' + (1)b'c'd

35. Inverse: (1) = (a+a')

36. Inverse: (1) = (a+a')

37. Inverse: (1) = (a+a')

38. Inverse: (1) = (a+a')

39. Inverse: (1) = (a+a')

= (a+a')bcd + (a+a')bcd' + (a+a')bc'd + (a+a')bc'd' + (a+a')b'c'd

40. Distributive: (a+a')bcd = abcd + a'bcd

41. Distributive: (a+a')bcd' = abcd' + a'bcd'

42. Distributive: (a+a')bc'd = abc'd + a'bc'd

43. Distributive: (a+a')bc'd' = abc'd' + a'bc'd'

44. Distributive: (a+a')ab'c'd = ab'c'd + a'b'c'd

= abcd + a'bcd + abcd' + a'bcd' + abc'd + a'bc'd + abc'd' + a'bc'd' + ab'c'd + a'b'c'd

45. Associative

1st Canonical Form

= a'b'c'd + a'bc'd' + a'bc'd + a'bcd' + a'bcd + ab'c'd + abc'd' + abc'd + abcd' + abcd = f(a,b,c,d)

3) Minimization of the expression in the 1st canonical form

f(a,b,c,d,) = a'b'c'd + a'bc'd' + a'bc'd + a'bcd' + a'bcd + ab'c'd + abc'd' + abc'd + abcd' + abcd

1st Canonical Form

1. Distributive: a'b'c'd + ab'c'd = (a+a')b'c'd

2. Distributive: a'bc'd' + abc'd' = (a+a')bc'd'

3. Distributive: a'bc'd + abc'd = (a+a')bc'd

4. Distributive: a'bcd' + abcd' = (a+a')bcd'

5. Distributive: a'bcd + abcd = (a+a')bcd

= (a+a')b'c'd + (a+a')bc'd' + (a+a')bc'd + (a+a')bcd' + (a+a')bcd

6. Inverse: (a+a') = (1)

7. Inverse: (a+a') = (1)

8. Inverse: (a+a') = (1)

9. Inverse: (a+a') = (1)

10. Inverse: (a+a') = (1)

= (1)b'c'd + (1)bc'd' + (1)bc'd + (1)bcd' + (1)bcd

11. Identity: (1)b'c'd = b'c'd

12. Identity: (1)bc'd' = bc'd'

12. Identity: (1)bc'd = bc'd

14. Identity: (1)bcd' = bcd'

15. Identity: (1)bcd = bcd

= b'c'd + bc'd' + bc'd + bcd' + bcd

16. Distributive: bcd' + bcd = bc(d+d')

= b'c'd + bc'd' + bc'd + bc(d+d')

17. Inverse: (d+d') = (1)

= b'c'd + bc'd' + bc'd + bc(1)

18. Identity: bc(1) = bc

= bc + b'c'd + bc'd' + bc'd

19. Distributive: bc'd' + bc'd = bc'(d+d')

= bc + b'c'd + bc'(d+d')

20. Inverse: (d+d') = (1)

= bc + b'c'd + bc'(1)

21. Identity: bc'(1) = bc'

= bc + bc' + b'c'd

22. Distributive: bc + bc' = b(c+c')

= b(c+c') + b'c'd

23. Inverse: $(c+c') = (1)$

$= b(1) + b'c'd$

24. Identity: $b(1) = b$

$= b + b'c'd$

25. Absorption Property: $b + b'c'd = b + c'd$

$= b + c'd$

Minimized Expression

4) The circuit for the minimized expression implemented with 2-input NAND gates only