Istanbul Technical University

Department of Computer Engineering

BLG 231E - Digital Circuits

Assignment 1

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Part 1 – Computer Arithmetic

1) **a)** Using **signed** 2's-complement representation, convert the decimal numbers ($-\mathbf{102}$) and ($-\mathbf{27}$) to **8-bit binary** integers. Show ALL work.

(-102)

102 / 2 = 51 remainder 0
51 / 2 = 25 remainder 1
25 / 2 = 12 remainder 1
12 / 2 = 6 remainder 0
6 / 2 = 3 remainder 0
3 / 2 = 1 remainder 1
1 / 2 = 0 remainder 1

natural binary weighted coding:

$102_{10} = 1.2^6+1.2^5+0.2^4+0.2^3+1.2^2+1.2^1+0.2^0 = 1100110_2$

8-bit signed binary representation:

$(+102)_{10} = (01100110)_2$

2's complement operation:

$(-102)_2$ = [1's complement of $(+102)_2$] + 1
= 2's complement of $(+102)_2$

8-bit $(+102)_{10}$: 01100110
1's complement : 10011001
Add 1 : + 1
Result: 8-bit $(-102)_{10}$: 10011010

8-bit signed binary representation:

$(-102)_{10} = (10011010)_2$

(-27)

27 / 2 = 13 remainder 1
13 / 2 = 6 remainder 1
6 / 2 = 3 remainder 0
3 / 2 = 1 remainder 1
1 / 2 = 0 remainder 1

natural binary weighted coding:

$27_{10} = 1.2^4+1.2^3+0.2^2+1.2^1+1.2^0 = 11011_2$

8-bit signed binary representation:

$(+27)_{10} = (00011011)_2$

2's complement operation:

$(-27)_2$ = [1's complement of $(+27)_2$] + 1
= 2's complement of $(+27)_2$

8-bit $(+27)_{10}$: 00011011
1's complement : 11100100
Add 1 : + 1
Result: 8-bit $(-27)_{10}$: 11100101

8-bit signed binary representation:

$(-27)_{10} = (11100101)_2$

b) Carry out the **binary** operations given below, and explain your answers using terms such as *carry*, *borrow*, and *overflow*. To interpret the results, use only binary numbers.

P.S. I assumed that the operations are carried out on **8-bit** binary numbers and I used mentioned terms accordingly.

i. (-102) + (-27)

8-bit signed binary representation (from 1.a):
$(-102)_{10} = 10011010_2$
$(-27)_{10} = 11100101_2$

addition of 8-bit signed numbers

```
  10011010  : -102
+ 11100101  : -27
 101111111  : cannot be represented
```

8th bit is the sign bit. Sign of the result is positive.
9th bit is ignored.

While both operands are negative, result is positive (neg. + neg. → pos. overflow case)
Therefore there is an OVERFLOW.

ii. (-102) – (-27)

8-bit signed binary representation (from 1.a):
$(-102)_{10} = 10011010_2$
$(-27)_{10} = 11100101_2$

subtraction of 8-bit signed numbers

```
  10011010  : -102
- 11100101  : -27
```

2's complement operation:
2's complement of $(-27)_{10} = (+27)_{10}$

8-bit $(-27)_{10}$	11100101
1's complement	00011010
Add 1	+ 1
Result: 8-bit $(+27)_{10}$	00011011

```
  10011010  : -102
+ 00011011  : +27
  10110101  : Result
```

8th bit is the sign bit.
Sign of the result is negative.

Operation of subtracting two negative operands (neg. – neg.) is not one of the overflow cases.
So there is NO OVERFLOW.

iii. (27) – (102)

8-bit unsigned representation:
$27_{10} = 00011011_2$
$102_{10} = 01100110_2$

subtraction of 8-bit signed numbers

00011011	:	27
- 01100110	:	102

2's complement operation:
2's complement of $(102)_{10} = (-102)_{10}$

8-bit $(-27)_{10}$	01100110
1's complement	10011001
Add 1	+ 1
Result: 8-bit $(+27)_{10}$	10011010

00011011	:	27
+ 10011010	:	-102
010110101	:	cannot be represented

No Carry : BORROW

9th bit of the result is zero. There is NO CARRY. There is a BORROW.

Part 2 – Boolean Algebra

2) Simplify each Boolean expression using algebraic manipulation (axioms and theorems). Show ALL work.
Show the steps of the simplification, and **write which axiom/theorem you used in each step** next to the simplification.

a)

$(Y + X'Z')(X + Y' + Z')(X' + Y)$

1. Distributive $((Y + X'Z')(X' + Y) = (Y + X'X'Z'))$

$= (Y + X'X'Z')(X + Y' + Z')$

2. Idempotency $(X'X' = X')$

$= (Y + X'Z')(X + Y' + Z')$

3. Distributive $((Y + X'Z')(X + Y' + Z') = Y(X + Y' + Z') + X'Z'(X + Y' + Z'))$

$= Y(X + Y' + Z') + X'Z'(X + Y' + Z')$

4. Distributive $(Y(X + Y' + Z') = YX + YY' + YZ')$
5. Distributive $(X'Z'(X + Y' + Z') = XX'Z' + X'Z'Y' + X'Z'Z')$

$= YX + YY' + YZ' + XX'Z' + X'Z'Y' + X'Z'Z'$

6. Inverse $(YY' = 0)$

$= YX + 0 + YZ' + XX'Z' + X'Z'Y' + X'Z'Z'$

7. Identity $(YX + 0 + YZ' + X'Z'X + X'Z'Y' + X'Z'Z' = YX + YZ' + X'Z'X + X'Z'Y' + X'Z'Z')$

$= YX + YZ' + XX'Z' + X'Z'Y' + X'Z'Z'$

8. Inverse $(XX' = 0)$
9. Idempotency $(Z'Z' = Z')$

$= YX + YZ' + Z'0 + X'Z'Y' + X'Z'$

10. Dominance $(Z'0 = 0)$

$= YX + YZ' + 0 + X'Z'Y' + X'Z'$

11. Identity $(YX + YZ' + 0 + X'Z'Y' + X'Z' = YX + YZ' + X'Z'Y' + X'Z')$

$= YX + YZ' + X'Z'Y' + X'Z'$

12. Absorption $(X'Z'Y' + X'Z' = X'Z')$

$= YX + YZ' + X'Z'$

13. Distributive $(YX + YZ' = Y(X + Z'))$

$= Y(X + Z') + X'Z'$

14. Identity $(Z' = 1Z')$

Concencus Theorem (SOP Form)
$(YX + YZ' + X'Z' = YX + X'Z')$

$= Y(X + 1Z') + X'Z'$

15. Inverse $(1 = X + X')$

$= Y(X + (X + X')Z') + X'Z'$

16. Distributive $((X + X')Z' = XZ' + X'Z')$

$= Y(X + XZ' + X'Z') + X'Z'$

17. Absorption $(X + XZ' = X)$

$= Y(X + X'Z') + X'Z'$

18. Distributive $(Y(X + X'Z') = YX + YX'Z')$

$= YX + YX'Z' + X'Z'$

19. Absorption $(YX'Z' + X'Z' = X'Z')$

$= YX + X'Z'$ RESULT

b)

$X'Y'Z'T' + XY'T + X'Y'Z + XZT' + XY'ZT + X'Y'Z'$

1. Absorption ($X'Y'Z'T' + X'Y'Z' = X'Y'Z'$)

$= X'Y'Z' + XY'T + X'Y'Z + XZT' + XY'ZT$

2. Absorption ($XY'T + XY'ZT = XY'T$)

$= X'Y'Z' + XY'T + X'Y'Z + XZT'$

3. Distributive ($X'Y'Z' + X'Y'Z = X'Y'(Z' + Z)$)

$= X'Y'(Z' + Z) + XY'T + XZT'$

4. Inverse ($Z' + Z = 1$)

$= X'Y'1 + XY'T + XZT'$

5. Dominance ($1 = 1 + T$)

$= X'Y'(1 + T) + XY'T + XZT'$

6. Distributive ($X'Y'(1 + T) = X'Y'1 + X'Y'T$)

$= X'Y'1 + X'Y'T + XY'T + XZT'$

7. Identity ($X'Y'1 = X'Y'$)

$= X'Y' + X'Y'T + XY'T + XZT'$

8. Distributive ($X'Y'T + XY'T = (X' + X)Y'T$)

$= X'Y' + (X' + X)Y'T + XZT'$

9. Inverse ($X' + X = 1$)

$= X'Y' + 1Y'T + XZT'$

10. Identity ($1Y'T = Y'T$)

$= X'Y' + Y'T + XZT'$ RESULT