Lecture 10, Ensemble Learning, DD2421

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(with contributions from J. Sullivan)

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Background: Ensemble Learning

We will describe and investigate algorithms to

train weak classifiers/regressors and how to combine them

to construct a classifier/regressor more powerful than any of the individual ones.

They are called Ensemble learning, Commitee machine, etc.

Outline of the lecture

Wisdom of Crowds – why combine classifiers?

Bagging: static structure, parallel training

Forests: an extension of bagging

Boosting: static structure, serial training

(Example: face detection)

Crowd wiser than **any individual**

The **collective knowledge** of a *diverse* and *independent* body of people typically **exceeds** the knowledge of **any single individual** and can be harnessed by voting.

- When ?
- For which questions ?

See **The Wisdom of Crowds** by *James Surowiecki* published in 2004 to see this idea applied to business.

What makes a crowd wise?

Four elements required to form a wise crow (*J. Surowiecki*):

- **Diversity of opinion.** People in crowd should have a range of experiences, education and opinions.
- **Independence.** Prediction by person in crowd is not influenced by other people in the crowd.
- **Decentralization** People have specializations and local knowledge.
- **Aggregation.** There is a mechanism for aggregating all predictions into one single prediction.

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Combining classifiers

Will exploit *Wisdom of crowd* ideas for specific tasks by

- combining classifier predictions **and**
- aim to combine independent and diverse classifiers.

But will use labelled training data

- to identify the **expert** classifiers in the pool;
- to identify **complementary** classifiers;
- to indicate how to best combine them.

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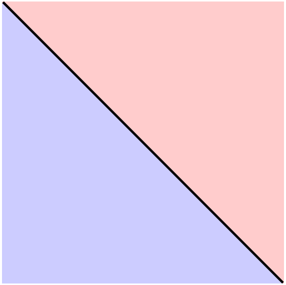
Ensemble method: **Bagging**

Bootstrap **Agg**regat**ing**

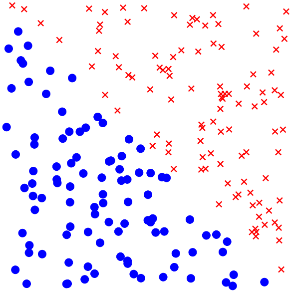
Use bootstrap replicates of training set by sampling with replacement.

On each replicate learn one model – combined altogether.

Binary classification example



True decision boundary



Training data set $\mathcal{S}_i$

Estimate the true decision boundary with a *decision tree* trained from some labeled training set $\mathcal{S}_i$.

High variance, Low bias classifiers

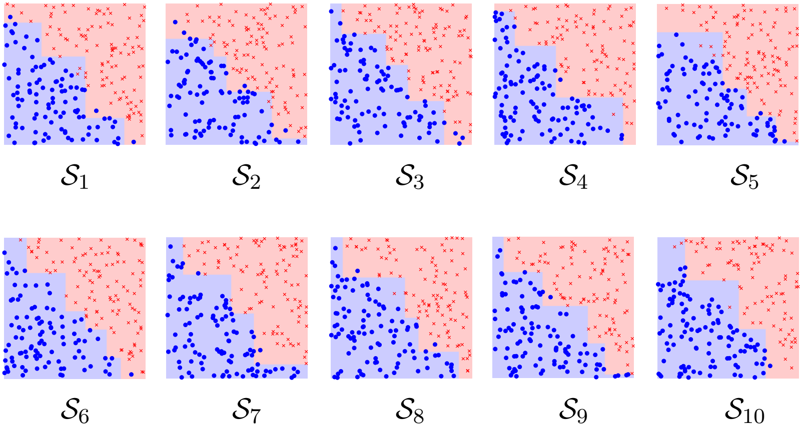
E.g. *decision trees*

High variance classifiers produce differing decision boundaries which are highly dependent on the training data.

Low bias classifiers produce decision boundaries which on average are good approximations to the true decision boundary.

Ensemble predictions using diverse high-variance, low-bias classifiers reduce the variance of the ensemble classifier.

Estimated decision boundaries found using bootstrap replicates:



Property of instability

Bagging - Bootstrap Aggregating

Input: Training data

$$\mathcal{S} = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_m, y_m)\}$$

of inputs $\mathbf{x}_j \in \mathbb{R}^d$ and their labels or real values y_j.

Iterate: for $b = 1, \ldots, B$

1. Sample training examples, *with replacement*, m' times from $\mathcal{S}$ to create $\mathcal{S}_b$ ($m' \leq m$).
2. Use this bootstrap sample $\mathcal{S}_b$ to estimate the regression or classification function f_b.

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Output: The bagging estimate for

Classification:

$$f_{\text{bag}}(\mathbf{x}) = \underset{1 \leq k \leq K}{\operatorname{argmax}} \sum_{b=1}^{B} \operatorname{Ind}\left(f_b(\mathbf{x}) = k\right)$$

Regression:

$$f_{\text{bag}}(\mathbf{x}) = \frac{1}{B} \sum_{b=1}^{B} f_b(\mathbf{x})$$

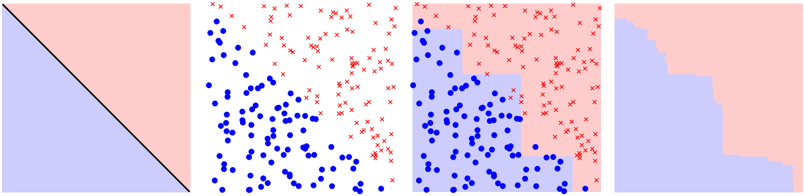
Note: $\operatorname{Ind}(x) = 1$ if $x = \text{TRUE}$
otherwise, $\operatorname{Ind}(x) = 0$

Bagging is a procedure to reduce the variance of our classifier (when labelled training data is limited).

$\rightarrow$ Powerful algorithm for controling overfitting.

Note: it only produces good results for **high variance**, **low bias** classifiers.

Apply bagging to the original example



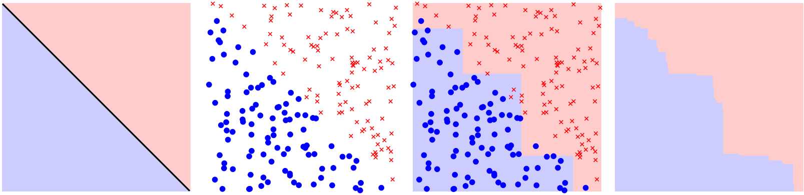
Ground truth

$\mathcal{S}_1$

Classifier from $\mathcal{S}_1$

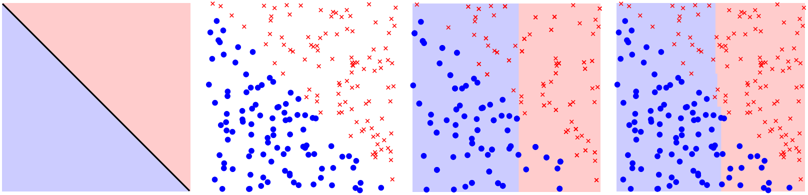
Bagged classifier $B = 100$.

Apply bagging to the original example



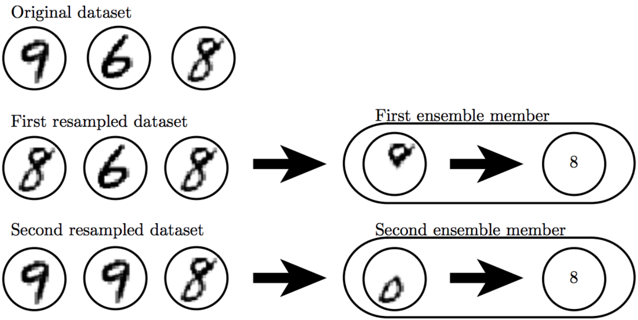
Ground truth | $\mathcal{S}_1$ | **Classifier from $\mathcal{S}_1$** | **Bagged classifier** $B = 100$**.**

If we bag a **high bias, low variance** classifier - *oriented horizontal and vertical lines* - we don't get any benefit.



A cartoon depiction of how bagging works

We train an 8 detector on the dataset, containing {6,8,9}.



Construct two different resampled dataset, each by sampling with replacement.

A cartoon depiction of how bagging works (cont.)

The first resampled dataset omits the 9 and repeats the 8.
$\rightarrow$ The detector learns a loop on top of the digit for an 8.

The second resampled dataset omits the 6 and repeat the 9.
$\rightarrow$ The detector learns a loop on the bottom for an 8.

Each rule is brittle.

If we average their output, then the detector is robust; maximal confidence only when both loops of the 8 are present.

See Chapter 7.11 of Deep Learning in www.deeplearningbook.org (2016) by *Ian Goodfellow*.

Ensemble method: **Forest**

Decision/Random/Randomized Forest

Bagging + Random *feature selection* at each node

Decision Forests / Random Forests

Two kind of randomnesses involved in:
- Sampling training data (the same as in Bagging)
- Feature selection at each node

Trees are less correlated, i.e. even **higher variance** between weak learners.

A classifier suited to multi-class problem.

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Ensemble method: **Boosting**

Started from a question:

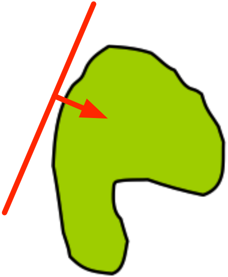
Can a set of weak learners create a single strong classifier where a weak learner performs only slightly better than a chance? (Kearns, 1988)

Loop:

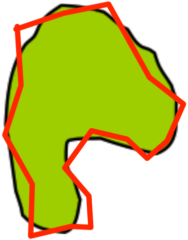
- Apply learner to weighted samples
- Increase weights of misclassified examples

Example: Ensemble Prediction

Voting of oriented hyper-planes can define convex regions.
Green region is the true boundary.



High-bias classifier



Low-bias classifier

Low model complexity (small # of d.o.f.) $\implies$ High-bias
High model complexity (large # of d.o.f.) $\implies$ Low-bias

Example (cont.)

A **diverse** and **complementary** set of high-bias classifiers, with performance better than chance, combined by **voting**

$$f_V(\mathbf{x}) = \operatorname{sign}\left(\sum_{t=1}^{T} h_t(\mathbf{x})\right)$$

can produce a classifier with a low-bias.

$h_t \in \mathcal{H}$ where $\mathcal{H}$ is a family of possible weak classifiers functions.

Ensemble Method: Boosting

Input: Training data $\mathcal{S} = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_m, y_m)\}$ of inputs $\mathbf{x}_i$ and their labels $y_i \in \{-1, 1\}$ or real values.

$\mathcal{H}$: a family of possible weak classifiers/regression functions.

Output: A strong classifier/regression function

$$f_T(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})\right) \text{ or } f_T(\mathbf{x}) = \sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})$$

weighted sum of weak classifiers

$h_t \in \mathcal{H} \quad t = 1, ..., T$

α_t: confidence/reliability

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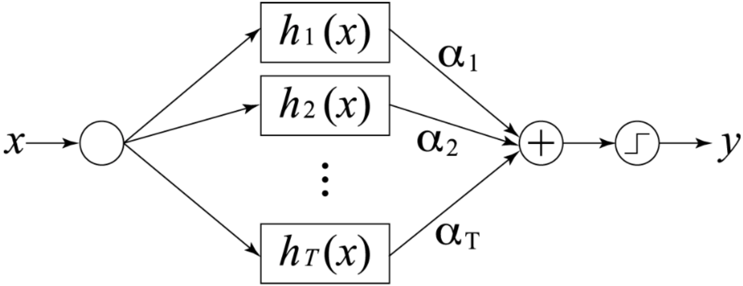
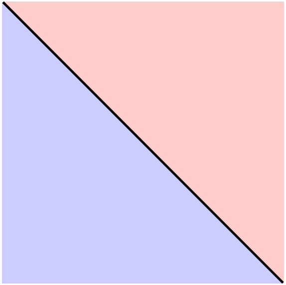


Ensemble Method: Boosting

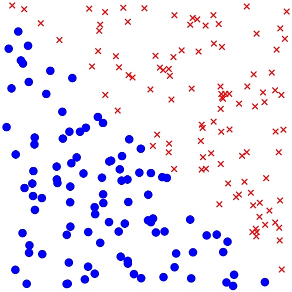
Core ideas (Just consider case of classification here.)

- Performance of classifiers $h_1, \ldots, h_t$ helps define h_{t+1}.
  Remember: Each $h_t \in \mathcal{H}$
- Maintain **weight** $w_i^{(t)}$ for each training example in $\mathcal{S}$.
- Large $w_i^{(t)} \implies \mathbf{x}_i$ has greater influence on choice of h_t.
- Iteration t: $w_i^{(t)}$ gets increased / decreased
  if $\mathbf{x}_i$ is wrongly / correctly classified by h_t.

Binary classification example



True decision boundary

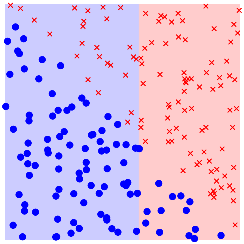


Training data

$\mathcal{H}$ is the set of all possible oriented vertical and horizontal lines.

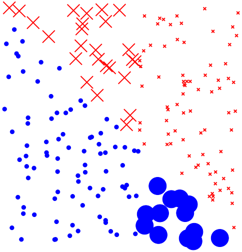
Example

Round 1



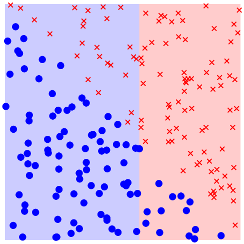
Chosen weak classifier

$\epsilon_1 = 0.19, \ \alpha_1 = 1.45$



Re-weight training points

$w_i^{(2)}$**'s**

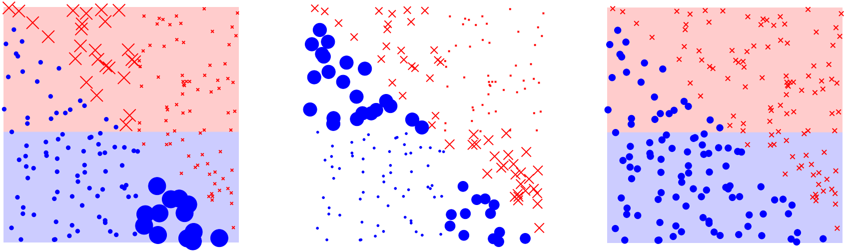


Current strong classifier

$f_1(\mathbf{x})$

Example

Round 2



Chosen weak classifier

$\epsilon_2 = 0.1512,\ \alpha_2 = 1.725$

Re-weight training points

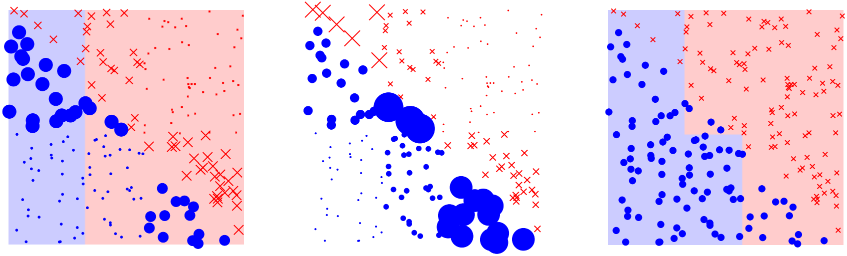
$w_i^{(3)}$**'s**

Current strong classifier

$f_2(\mathbf{x})$

Example

Round 3

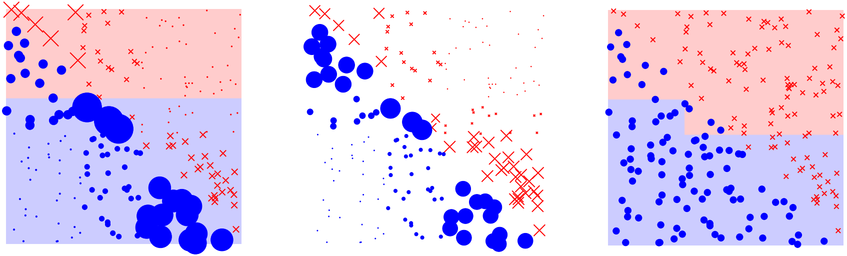


Chosen weak classifier | **Re-weight training points** | **Current strong classifier**

$\epsilon_3 = 0.2324,\ \alpha_3 = 1.1946$ | $w_i^{(4)}$**'s** | $f_3(\mathbf{x})$

Example

Round 4

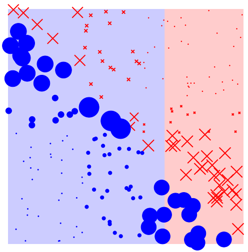


Chosen weak classifier | **Re-weight training points** | **Current strong classifier**

$\epsilon_4 = 0.2714$, $\alpha_4 = 0.9874$ | $w_i^{(5)}$**'s** | $f_4(\mathbf{x})$

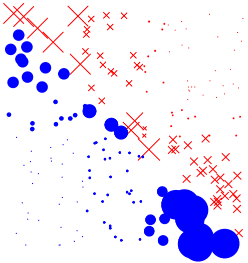
Example

Round 5



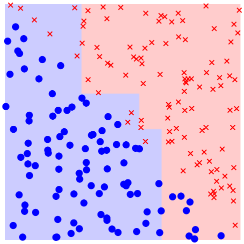
Chosen weak classifier

$\epsilon_5 = 0.2616,\ \alpha_5 = 1.0375$



Re-weight training points

$w_i^{(6)}$**'s**

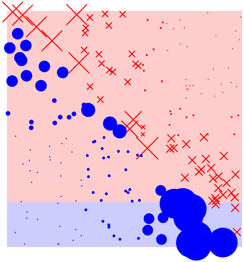


Current strong classifier

$f_5(\mathbf{x})$

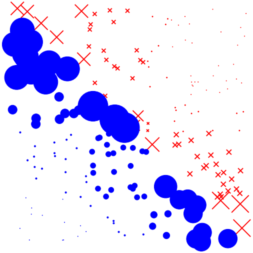
Example

Round 6



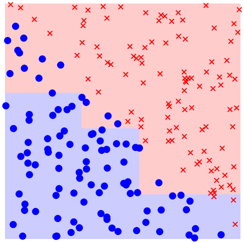
Chosen weak classifier

$\epsilon_6 = 0.2262,\ \alpha_6 = 1.2298$



Re-weight training points

$w_i^{(7)}$**'s**

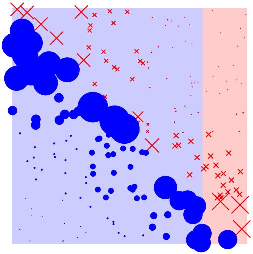


Current strong classifier

$f_6(\mathbf{x})$

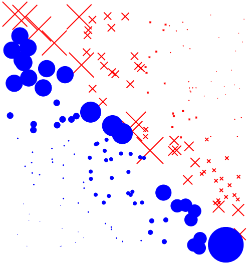
Example

Round 7



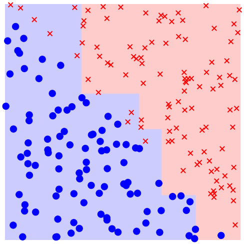
Chosen weak classifier

$\epsilon_7 = 0.2680,\ \alpha_7 = 1.0049$



Re-weight training points

$w_i^{(8)}$**'s**

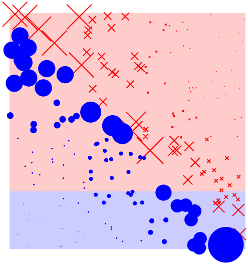


Current strong classifier

$f_7(\mathbf{x})$

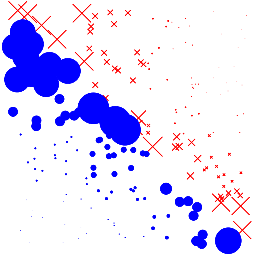
Example

Round 8



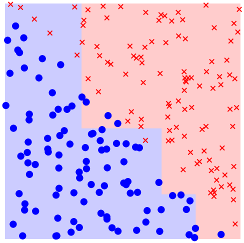
Chosen weak classifier

$\epsilon_8 = 0.3282$, $\alpha_8 = 0.7165$



Re-weight training points

$w_i^{(9)}$**'s**

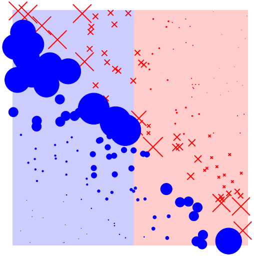


Current strong classifier

$f_8(\mathbf{x})$

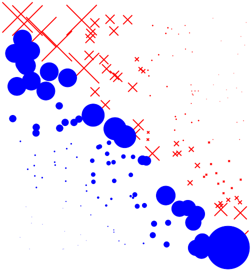
Example

Round 9



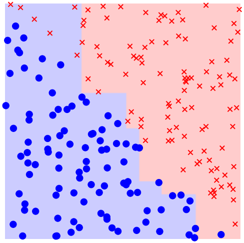
Chosen weak classifier

$\epsilon_9 = 0.3048$, $\alpha_9 = 0.8246$



Re-weight training points

$w_i^{(10)}$**'s**

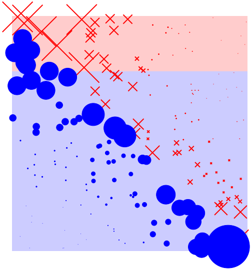


Current strong classifier

$f_9(\mathbf{x})$

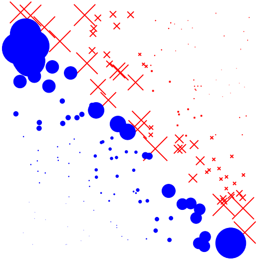
Example

Round 10



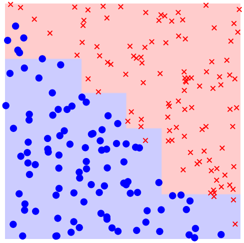
Chosen weak classifier

$\epsilon_{10} = 0.2943$, $\alpha_{10} = 0.8744$



Re-weight training points

$w_i^{(11)}$**'s**

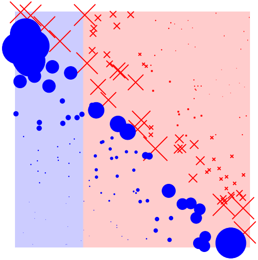


Current strong classifier

$f_{10}(\mathbf{x})$

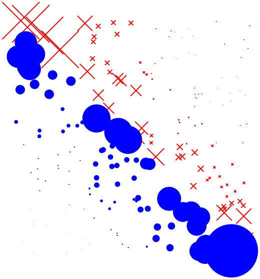
Example

Round 11



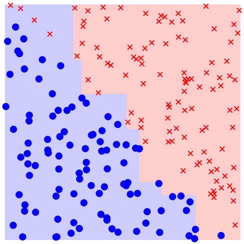
Chosen weak classifier

$\epsilon_{11} = 0.2876$, $\alpha_{11} = 0.9071$



Re-weight training points

$w_i^{(12)}$**'s**



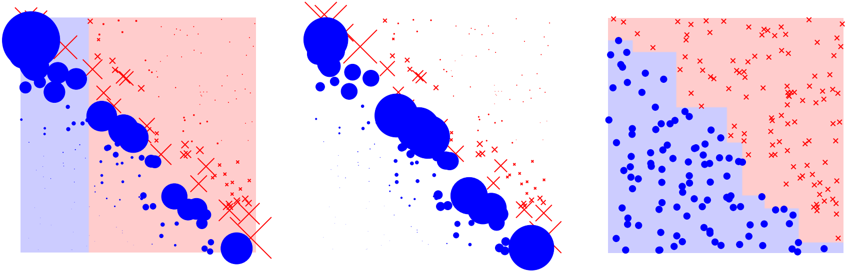
Current strong classifier

$f_{11}(\mathbf{x})$

Example

Example

Round 21



Chosen weak classifier

$\epsilon_{21} = 0.3491$, $\alpha_{21} = 0.6232$

Re-weight training points

$w_i^{(22)}$**'s**

Current strong classifier

$f_{21}(\mathbf{x})$

Adaboost Algorithm (Freund & Schapire, 1997)

Given:

- Labeled training data

$$\mathcal{S} = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_m, y_m)\}$$

  of inputs $\mathbf{x}_j \in \mathbb{R}^d$ and their labels $y_j \in \{-1, 1\}$.
- A set/class $\mathcal{H}$ of T possible weak classifiers.

Initialize:

- Introduce a weight, $w_j^{(1)}$, for each training sample.
- Set $w_j^{(1)} = \frac{1}{m}$ for each j.

Adaboost Algorithm (cont.)

Iterate: for $t = 1, \ldots, T$

1. Train weak classifier $h_t \in \mathcal{H}$ using $\mathcal{S}$ and $w_1^{(t)}, \ldots, w_m^{(t)}$; select the one that minimizes the training error:

$$\epsilon_t = \sum_{j=1}^{m} w_j^{(t)} \text{ Ind}\left(y_j \neq h_t(\mathbf{x}_j)\right)$$

(sum of the weights for misclassified samples)

2. Compute the reliability coefficient:

$$\alpha_t = \log_e \left(\frac{1 - \epsilon_t}{\epsilon_t}\right)$$

ϵ_t must be less than 0.5. Break out of loop if $\epsilon_t \approx .5$

3. Update weights by using: $w_j^{(t+1)} = w_j^{(t)} exp(-\alpha_t y_j h_t(\mathbf{x}_j))$
Note the sign of $y_j h_t(\mathbf{x}_j)$.

4. Normalize the weights so that they sum to 1.

Adaboost Algorithm (cont.)

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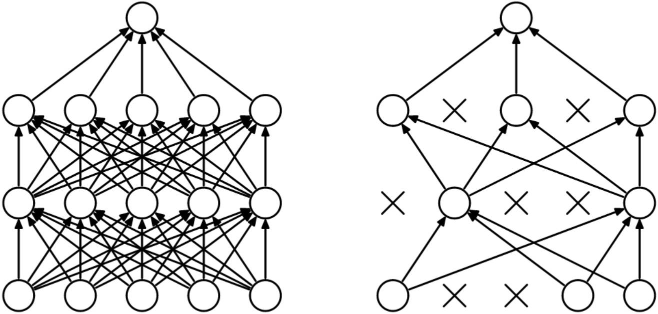
Dropout

A regularization method for Artificial Neural Networks

(can be seen also as an ensemble method)

Dropout: "ultimate" ensemble learning in ANN



(a) Standard Neural Net

(b) After applying dropout.

Srivastava, Hinton, Krizhevsky, Sutskever and Salakhutdinov, **Dropout: A Simple Way to Prevent Neural Networks from Overtting.** *Journal of Machine Learning Research* 15: 1929-1958, 2014.

Summary

Summary: Ensemble Prediction

Can combine many weak classifiers/regressors into a stronger classifier; voting, averaging, bagging

- if weak classifiers/regressors are better than random.
- if there is sufficient de-correlation (independence) amongst the weak classifiers/regressors.

Summary: Ensemble Prediction & Learning

Can combine many (high-bias) weak classifiers/regressors into a **strong** classifier; boosting

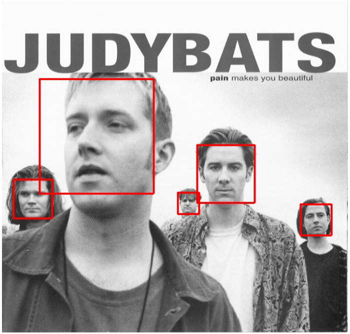
- if weak classifiers/regressors are **chosen** and **combined** using knowledge of how well they and others performed on the task on training data.

- The selection and combination encourages the weak classifiers to be complementary, diverse and de-correlated.

Appendix

P. Viola, M. J. Jones, **Robust real-time face detection**. *International Journal of Computer Vision* 57(2): 137-154, 2004.

Viola & Jones Face Detection



- Most state-of-the-art face detection on mobile phones, digital cameras etc. are/were based on this algorithm.
- Example of a classifier constructed using the Boosting algorithm.

Viola & Jones: Training data

Positive training examples: Image patches corresponding to faces - $(\mathbf{x}_i, 1)$.

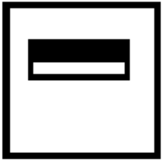
Negative training examples: Random image patches from images not containing faces - $(\mathbf{x}_j, -1)$.

Note: All patches are re-scaled to have same size.

$\Uparrow$

Positive training examples

Viola & Jones: Weak classifier



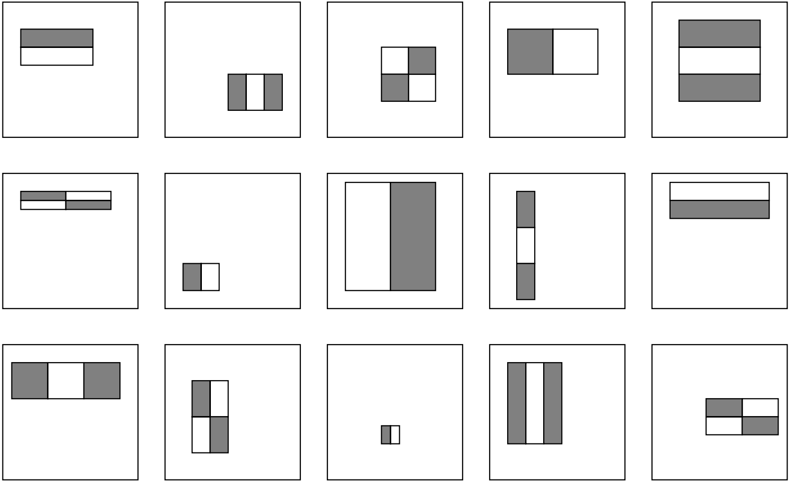
FACE or **NON-FACE**

Input: $\mathbf{x}$ **Apply filter:** $f^j(\mathbf{x})$ **Output:** $h(\mathbf{x}) = (f^j(\mathbf{x}) > \theta)$

Filters used compute differences between sums of pixels in adjacent rectangles. (These can be computed very quickly using **Integral Images**.)

Viola & Jones: Filters Considered

Huge **library** of possible Haar-like filters, $f^1, \ldots, f^n$ with $n \approx 16,000,000$.



Viola & Jones: AdaBoost training

Recap: define weak classifier as

$$h_t(\mathbf{x}) = \begin{cases} 1 & \text{if } f^{j_t}(\mathbf{x}) > \theta_t \\ -1 & \text{otherwise} \end{cases}$$

Use AdaBoost to efficiently choose the **best weak classifiers** and to **combine** them.

Remember: a weak classifier corresponds to a filter type and a threshold.

Viola & Jones: AdaBoost training (cont.)

For $t = 1, \ldots, T$

- for each filter type j
  1. Apply filter, f^j, to each example.
  2. Sort examples by their filter responses.
  3. Select best threshold for this classifier: θ_{tj}.
  4. Keep record of error of this classifier: ϵ_{tj}.
- Select the filter-threshold combination (weak classifier j^*) with minimum error. Then set $j_t = j^*$, $\epsilon_t = \epsilon_{tj^*}$ and $\theta_t = \theta_{tj^*}$.
- Re-weight examples according to the AdaBoost formualae.

Note: (There are many tricks to make this implementation more efficient.)

Viola & Jones: Sliding window

Remember: Better classification rates if use a classifier, f_T, with large T.

Given a new image, I, detect the faces in the image by:

- for each plausible face size s
  - for each possible patch centre c
    1. Extract sub-patch of size s at c from I.
    2. Re-scale patch to size of training patches.
    3. Apply detector to patch.
    4. Keep record of s and c if the detector returns positive.

This is a **lot** of patches to be examined. If T is very large processing an image will be very slow!

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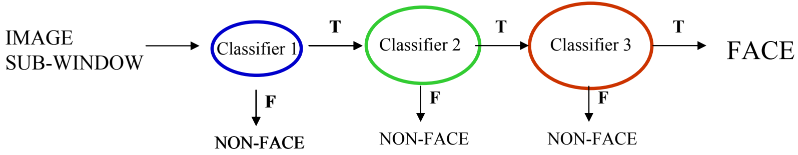
Viola & Jones: Cascade of classifiers

But:
only a tiny proportion of the patches will be faces **and** many of them will not look anything like a face.

Exploit this fact: Introduce a cascade of increasingly strong classifiers



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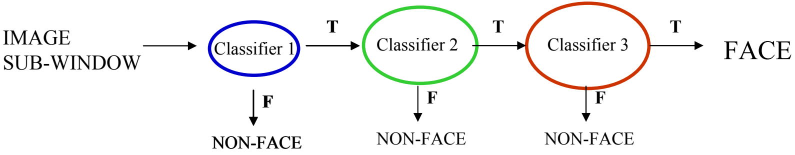
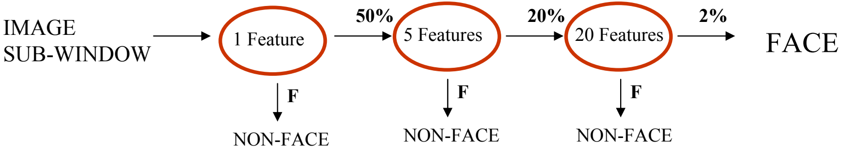


Viola & Jones: Cascade of classifiers



- A 1 feature classifier achieves 100% detection rate and about 50% false positive rate.
- A 5 feature classifier achieves 100% detection rate and 40% false positive rate (20% cumulative) - using data from previous stage.
- A 20 feature classifier achieves 100% detection rate with 10% false positive rate (2% cumulative).

Viola & Jones: Typical Results