Forecasting Volatility

Multivariate Statistical Methods and Applications

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Outline

1. Why Forecasting Volatility?
2. Gentle Introduction to Neural Networks
3. Outlook

Forecasting Stock Return Volatility

Applications:

- risk management
- derivative pricing
- devising trading strategies

Realized volatility (RV) as proxy for unobserved volatility:

$$\mathrm{RV}_{i,t}^{(h)} := \sum_{s=t-h+1}^{t} r_{i,s}^2, \text{ for period } [t-h, t]$$

Model Comparison

Table 1: Models' out-of-sample performance (QLIKE loss function).

Model	Intraday Volatility Frequency		
	10-min	30-min	60-min
HAR	0.197	0.187	0.186
OLS	0.186	0.187	0.186
LASSO	0.191	0.187	0.186
XGBoost	0.177	0.173	0.173
NN (MLP)	0.171	0.171	0.172
NN (LSTM)	0.174	0.171	0.171

Source: Zhang et al. (2023). MPL: 3 hidden layers, LSTM: 2 hidden layers.

- neural networks (multilayer perceptron and long short-term memory) yield superior forecasts

(Artificial) Neural Networks

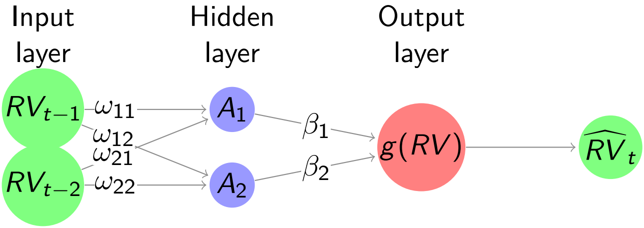
Figure 1: Human neural network

Source: Getty Images

- inspired by how human brains work
- universal function approximators

- $y_i = x_i^T \beta + \varepsilon_i$ (linear regression)
- $y_i = g(x_i^T \beta) + \varepsilon_i$
- $y_i = \sum_{j=1}^{r} g_j(x_i^T \beta_j) + \varepsilon_i$ (projection pursuit regression)

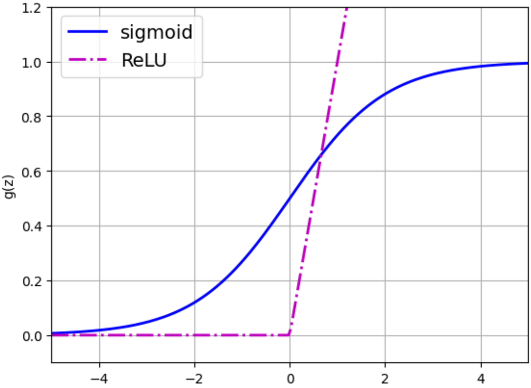
Single Hidden Layer Neural Network



- activations: $A_1 = g(\omega_{11} RV_{t-1} + \omega_{21} RV_{t-2} + b_1),$
  $A_2 = g(\omega_{12} RV_{t-1} + \omega_{22} RV_{t-2} + b_2)$
- $g(z)$: *nonlinear* activation function
- $\widehat{RV}_t = g(RV) = g(\beta_1 A_1 + \beta_2 A_2 + a_1)$
  $= \mathrm{g}\left(\beta_1 g(\omega_{11} RV_{t-1} + \omega_{21} RV_{t-2} + b_1) + \beta_2 g(\omega_{12} RV_{t-1} + \omega_{22} RV_{t-2} + b_2) + a_1\right)$

Activation Functions

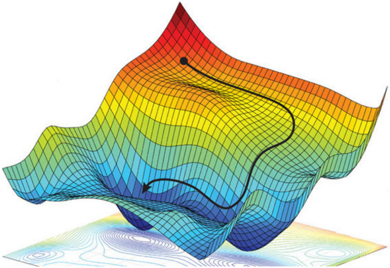
Figure 2: Common activation functions



- sigmoid: $g(z) = \frac{1}{1+e^{-z}}$, $\quad e.g.\ \mathrm{A}_1 = \frac{1}{1+e^{-(\omega_{11}RV_{t-1}+\omega_{21}RV_{t-2}+b_1)}}$
- ReLU (*rectified linear unit*): $g(z) = max(0, z)$

Training

Figure 3: Gradient descent

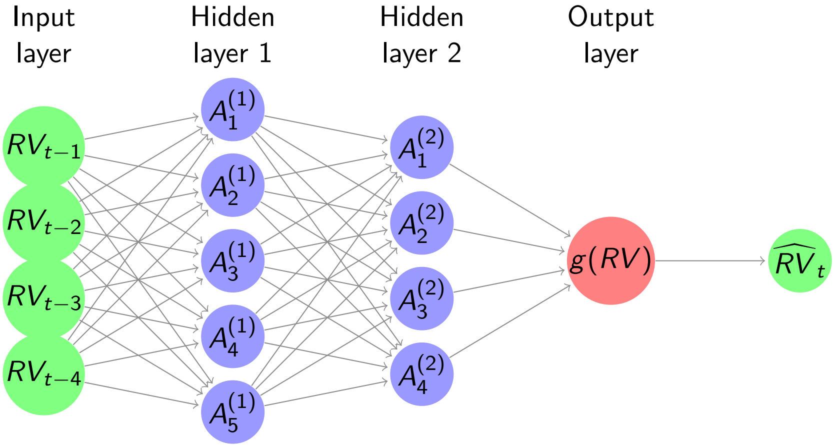


Source: Amini et al. (2018)

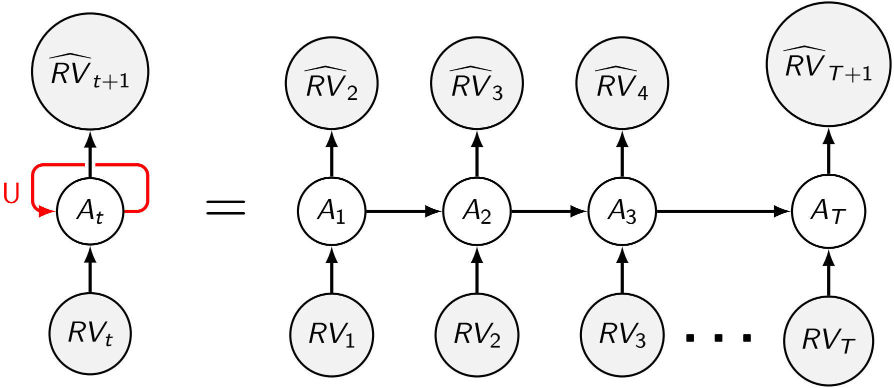
- weights and biases: random starting values near zero
- backpropagation (through gradient descent) until minimum of loss function is reached
- training epoch: one sweep through the entire training set

Multilayer Neural Network

Multilayer perceptron: *uni-directional* feedforward NN



Recurrent Neural Networks



- *bi-directional* NN
- feedback loops: $A_{tk} = g(b_k + \sum_{i=1}^{N} w_{ki} RV_{ti} + \sum_{s=1}^{K} u_{ks} A_{t-1,s})$
- previous steps remembered
- vanishing gradient problem

Long Short-Term Memory

- subtype of RNN
- tackles vanishing gradient problem

LSTM cell:

- adds new information (input gate)
- forgets irrelevant information (forget gate)
- passes updated information (output gate)

Pros & Cons

Pros:

- better performance
- capture complex non-linearities and interactions
- no assumption on data generating process needed

Cons:

- require big data sets
- require hyperparameter tuning
- harder to interpret and present ("black box")

Outlook

- realized volatility forecasts: random walk vs NN
- intraday data: Refinitiv Datastream

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