TREE-BASED MULTIPLE IMPUTATION METHODS

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1. Motivation

- Standard MICE approach: conditional models to be specified for *all* variables with missing data
- Still may fail to capture interactive and nonlinear relations among variables as well as non-standard distributions
- Tree-based methods *automatically* capture interactions, nonlinear relations, and complex distributions with no parametric assumptions or data transformations needed (Burgette & Reiter 2010)
- Implementation in R: *mice* and *miceRanger* packages

2. Tree-based methods

Classification and regression trees (CART):

- seek to approximate conditional distribution of univariate outcome from multiple predictors
- segment predictor space into non-overlapping regions with relatively homogeneous outcomes
- segments found by recursive binary splits of predictors
- prediction for observations that fall into the same region is mean (or mode) of response values for training observations in region
- may be very non-robust and have lower predictive accuracy

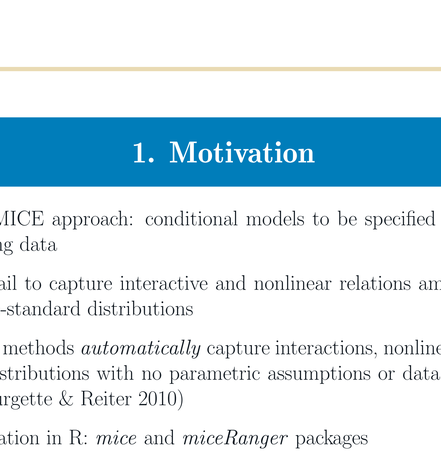

Fig. 1: Example of tree structure. Source: Hastie et al. (2009)

Random forest:

- *ensemble* method that addresses non-robustness and low predictive accuracy
- average predictions from B non-pruned trees constructed using B bootstrapped training sets
- *decorrelates* trees by performing each split on *randomly* chosen subset of predictors

3. Imputation algorithm

Following Burgette & Reiter (2009), let Y be $n \times p$ the data matrix arranged as $Y = (Y_p, Y_c)$, where

- Y_p consists of p_1 *partially observed* columns, such that moving from left to right, the number of missing elements in each column is nondecreasing
- Y_c remaining completely observed columns
- Y_{obs} set of observed and Y_{mis} set of missing elements

4-steps algorithm:

1. Initial values for the missing values filled in as follows:
   (a) Define a matrix Z equal to Y_c
   (b) Impute missing values in Y_i, where $i = 1, ...p_1$, using tree-based method on Z and append the completed version of Y_i to Z prior to incrementing i
2. Replace the originally missing values of Y_i, where $i = 1, ...p_1$, with tree-based methods on Y_{-i}
3. Repeat l times step 2
4. Repeat steps 1–3 m times and obtain m imputed sets.

4. Comparison mice/miceRanger packages

- both implement Stef van Buuren's Multivariate Imputation by Chained Equations
- *mice* supports variety of imputation methods, *miceRanger* only randomForest
- *mice* uses common R packages "rpart" and "randomForest" to implement tree based imputation methods (van Buuren 2023)
- *miceRanger* uses the "ranger"-package instead, which claims to be faster and more efficient with medium and large data sets (Wilson 2022)
  ⇒ core functions written in C++ (faster than R, compiled vs. interpreted code) (Wright und Ziegler 2017)
  ⇒ supports parallel computing (Wright und Ziegler 2017; Wright 2023)

5. Empirical simulation study

Objective:

- evaluate efficacy of tree-based imputation methods on missing data
- compare *mice* package methods against the extended methods in *miceRanger*

Empirical data set:

- RAND's Health Insurance Experiment

Monte Carlo simulation:

- simulations (R): 1000 cycles to ensure robustness
- iterations (niter): 10 iterations for chained equations.
- random forest trees (nrtree): 10 trees in each RandomForest model for depth

Comparison Metrics: determine the most accurate and efficient imputation method that best reconstructs true values while minimizing systematic errors

- bias: deviation of imputed values from true values.
- mean squared error: average squared difference between the imputed and true values.
- coverage: proportion of times true values fall within the calculated confidence intervals.

6. Results

Metric	Method	Bias	MSE	Coverage
mean(age)	BD	NA	NA	NA
mean(age)	CC	NA	NA	NA
mean(age)	CART	NA	NA	NA
mean(age)	RandomForest	NA	NA	NA
mean(age)	miceRanger	NA	NA	NA
mean(educ)	BD	NA	NA	NA
mean(educ)	CC	NA	NA	NA
mean(educ)	CART	NA	NA	NA
mean(educ)	RandomForest	NA	NA	NA
mean(educ)	miceRanger	NA	NA	NA
ρ(mdvis, hltg)	BD	NA	NA	NA
ρ(mdvis, hltg)	CC	NA	NA	NA
ρ(mdvis, hltg)	CART	NA	NA	NA
ρ(mdvis, hltg)	RandomForest	NA	NA	NA
ρ(mdvis, hltg)	miceRanger	NA	NA	NA
reg.(mhi)	BD	NA	NA	NA
reg.(mhi)	CC	NA	NA	NA
reg.(mhi)	CART	NA	NA	NA
reg.(mhi)	RandomForest	NA	NA	NA
reg.(mhi)	miceRanger	NA	NA	NA

Table 1: Simulation results

7. Conclusion

References

Burgette, Lane F, and Jerome P Reiter. "Multiple imputation for missing data via sequential regression trees". *American journal of epidemiology* 172, no. 9 (2010): 1070–1076.

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