Final 2015 Statistics

[redacted]

Q1

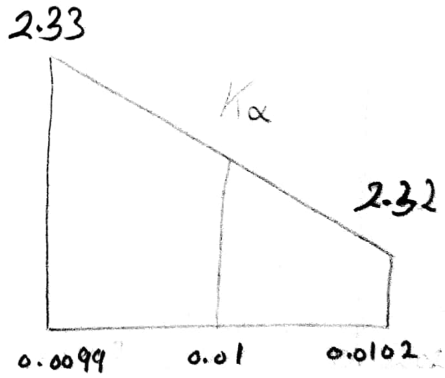
i) $P_r\{|\bar{X}-\mu| > 0.1\} < 0.02$

$\therefore \varepsilon = 0.1$ & $\frac{\sigma^2}{n\varepsilon^2} = 0.02$

$\therefore n = \frac{0.5^2}{0.02 \times 0.1^2} = 1250$

ii) $P_r\left\{ \frac{|\bar{X}-\mu|}{\sigma/\sqrt{n}} > \frac{0.1}{\sigma/\sqrt{n}} \right\} = 0.02$

$\therefore P_r\{ Z > \sqrt{n}/5 \} = 0.01$



$\therefore \frac{K_\alpha - 2.32}{2.33-2.32} = \frac{0.0102-0.01}{0.0102-0.0099}$

$\therefore K_\alpha = 2.32667$

$\therefore \sqrt{n}/5 = 2.32667$

$\therefore n = 135.33 \simeq 136$

Q2

a) $\mu = 100 \qquad S = 15$

$$P\left\{ \frac{\bar{X} - \mu}{S/\sqrt{n}} < \frac{10}{S/\sqrt{n}} \right\} = P\left\{ \frac{\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}{\sqrt{S^2/\sigma^2}} < \frac{2}{3}\sqrt{20} \right\}$$

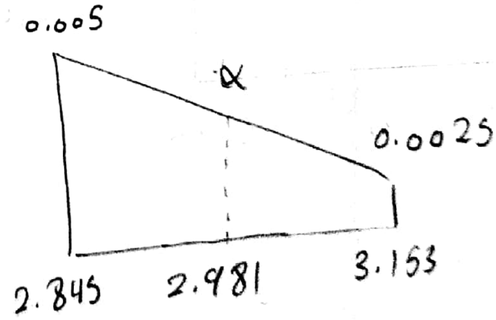
$$= P_r\left\{ \frac{Z}{\sqrt{\chi^2_n/n}} < \frac{2}{3}\sqrt{20} \right\}$$

$$= P_r\left\{ t_n < \frac{2}{3}\sqrt{20} \right\} = P_r\left\{ t_{20} < 2.981 \right\}$$

$$= 1 - P_r\left\{ t_{20} > 2.981 \right\} = 1 - \alpha = \boxed{0.9961}$$

$$\frac{\alpha - 0.0025}{0.005 - 0.0025} = \frac{3.153 - 2.981}{3.153 - 2.845}$$

$\therefore \alpha = 0.003896$



$\boxed{b}$ $\because$ $$F_{m,n} = \frac{\chi^2_m / m}{\chi^2_n / n}$$

$$\therefore \frac{1}{F_{m,n}} = \frac{\chi^2_n / n}{\chi^2_m / m} = F_{n,m}$$

$$\therefore \alpha = P(X \leq F_{\alpha,m,n})$$

$$= P\left(\frac{1}{X} \geq \frac{1}{F_{\alpha,m,n}}\right)$$

$\because X \sim F_{m,n}$

$\frac{1}{X} \sim F_{n,m}$

$$= 1 - P\left(\frac{1}{X} \leq \frac{1}{F_{\alpha,m,n}}\right)$$

$$= 1 - P\left(F_{n,m} \leq \frac{1}{F_{\alpha,m,n}}\right)$$

$$= 1 - F\left(\frac{1}{F_{\alpha,m,n}}\right)$$

$$\therefore F_{n,m}\left(\frac{1}{F_{\alpha,m,n}}\right) = 1 - \alpha$$

$$\therefore F_{1-\alpha,n,m} = \frac{1}{F_{\alpha,m,n}}$$

Q3

a) $\because \dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

$\therefore 1-\alpha = P_r\left\{ \dfrac{(n-1)S^2}{\sigma^2} \leq \chi^2_{\alpha\, n-1} \right\}$

$= P_r\left\{ \sigma^2 \geq \dfrac{(n-1)S^2}{\chi^2_{\alpha\, n-1}} \right\}$

$\therefore L. = \dfrac{(n-1)S^2}{\chi^2_{\alpha\, n-1}}$

Q4

i) $\because \mathcal{L} f(x,\mu) = \sum^{n} \ln 0.5 + 3\ln\mu + 2\ln x - \mu x$

$\therefore \dfrac{d.\mathcal{L} f(x,\mu)}{d\mu} = \sum^{n} 3/\mu - x = 0$

$\therefore 3n/\mu = n\bar{x} \quad \therefore \mu = \dfrac{3}{\bar{x}}$

$\because \ln f(x,\mu) = \ln 0.5 + 3\ln\mu + 2\ln x - \mu x$

$\therefore \dfrac{\partial}{\partial\mu} \ln f(x,\mu) = 3/\mu - x$

$\therefore \dfrac{\partial^2}{\partial\mu^2} \ln f(x,\mu) = -3/\mu^2 \qquad \therefore E\left(\dfrac{\partial^2}{\partial\mu^2} \ln f(x,\mu)\right) = -3/\mu^2$

$\therefore \sigma_b^2 = \dfrac{1}{-nE\left(\frac{\partial^2}{\partial\mu^2}\ln f(x,\mu)\right)} = \dfrac{\mu^2}{3n} \qquad \therefore C.I = \hat{\mu} \mp K_{0.05}\, \sigma_b$

evaluate

ii) MLE of $3/\mu = 3/\hat{\mu} = 3/(3/\bar{x}) = \bar{x} = \frac{1}{n}\sum_{i}^{n} x_i$

Q5

ii)

$$v = \frac{\left[\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right]^2}{\left[\left(\frac{s_x^2}{n_x}\right)^2 / n_x - 1\right] + \left[\left(s_y^2/n_y\right)^2 / n_y - 1\right]}$$

$= 20.05 \simeq 21$

$$\therefore C.I = \left[(84.46 - 76.40) - t_{0.025,21}\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}\right.,\ \left.(84.46 + 76.40) + t_{0.025,21}\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}\right]$$

Evaluate $t_{0.025,21}$