Algebra Review

Harrell Biostat Book Club

Adam Sonty

What is Algebra?

$$2 + 3 = 5$$

$$2 + \boxed{x} = 5$$

Algebra is the part of mathematics in which letters or symbols are used to represent numbers in equations

Warm Up

$$\frac{(2x+1)(8x-6)-2(-2x-5)}{4(4x^2+1)} = ?$$

Distribute the “2”

Distribute the “-”

$$\frac{\boxed{(2x+1)(8x-6)}-\boxed{2(-2x-5)}}{4(4x^2+1)} = \frac{(16x^2-12x+8x-6)\boxed{-(-4x-10)}}{16x^2+4}$$

“FOIL”

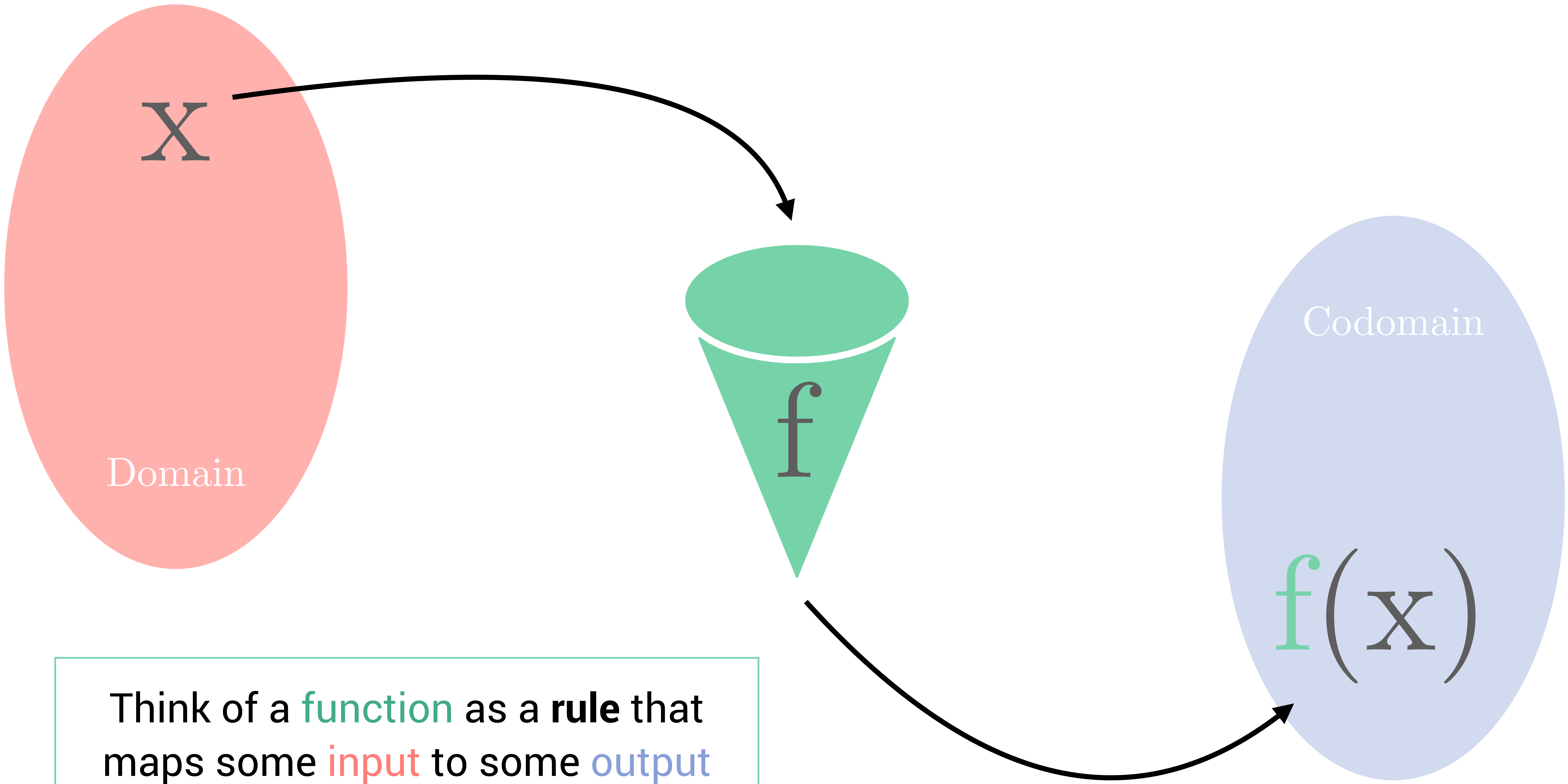
$$= \frac{\boxed{16x^2-12x+8x-6+4x+10}}{16x^2+4}$$

$$= \frac{(16x^2)+(-12x+8x+4x)+(-6+10)}{16x^2+4}$$

$$= \frac{16x^2+4}{16x^2+4} = 1$$

Group “like” terms

Functions



Think of a function as a **rule** that maps some input to some output

Notable Functions

- $\min(a,\ b)$ or $\max(a,\ b)$
  - These functions say “choose the minimum/maximum of these 2 values”

- $\mathrm{I}_{[<\text{condition}>]}$
  - Indicator variables just say “if this condition is met, then the value is 1. If not, the value is 0”

Powers/Logarithms

Exponents & Roots

$$x^n = ?$$

What number do I get when I multiply x by itself n times?

$$\sqrt[n]{x} = ?$$

What number, when multiplied by itself n times, equals x ?

Logarithms

$$log_b x = ?$$

How many times do I need to multiply b by itself in order to get x ?

Memory Device

$$log_x \boxed{y = z} \iff \boxed{x^z} = y$$

Arrange x, y, & z in alphabetical order

Think of Xzibit and remember x^z

Xzibit a.k.a. “X-to-the-Z

Polynomials

“Degree” of the polynomial

$$\boxed{ax^{\boxed{3}}} + bx^2 + cx + d$$

Leading term of the polynomial

- Polynomials are written such that the exponents on variables are in **descending order**
- The term with the highest exponent is called the **leading term**
- The exponent of the leading term is called the **degree**, and indicates whether the polynomial is linear, quadratic, cubic, etc.
    - Degree = 1 : linear
    - Degree = 2 : quadratic
    - …

Dot/Inner Product

Dot Product

$$\begin{array}{cc} \mathrm{x} & \mathrm{y} \\ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \cdot & \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \end{array} = (x_1 y_1) + (x_2 y_2) + (x_3 y_3)$$

- The dot product of 2 vectors, x and y, is the sum of the products of their corresponding elements
- In order to take the dot product of 2 vectors, both vectors must have the same number of elements

Inner Product

$$\underset{1 \times 3}{\overset{\mathrm{x}^{\mathrm{T}}}{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}} \underset{3 \times 1}{\overset{\mathrm{y}}{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}} = \underset{1 \times 1}{\begin{bmatrix} x_1 y_1 + x_2 y_2 + x_3 y_3 \end{bmatrix}}$$

- The dot product of 2 vectors, x and y, is also sometimes expressed as the inner product of 2 matrices, x^{T} and y
- Note that the left matrix must be $1 \times n$ and the right matrix must be $n \times 1$ (the dimensions have to "agree")

Extension to Matrix Products

$$\underset{3\times 3}{\overset{\mathrm{X}}{\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}}} \underset{3\times 2}{\overset{\mathrm{Y}}{\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \\ y_{31} & y_{32} \end{bmatrix}}} = \underset{3\times 2}{\begin{bmatrix} x_{1\bullet} \cdot y_{\bullet 1} & x_{1\bullet} \cdot y_{\bullet 2} \\ x_{2\bullet} \cdot y_{\bullet 1} & x_{2\bullet} \cdot y_{\bullet 2} \\ x_{3\bullet} \cdot y_{\bullet 1} & x_{3\bullet} \cdot y_{\bullet 2} \end{bmatrix}}$$

- The product of 2 matrices, X and Y, can be expressed as an arrangement of the dot products of their component vectors and columns

- Note that $x_{i\bullet}$ represents the i-th row of X and $y_{\bullet j}$ represents the j-th column of Y