Time Series Prediction Part I

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Time Series

- Clustering
  - Dynamic Time Warping
- Segmentation
  - Speech recognition
- Classification
- Anomaly/Outlier Detection
- Imputation
- **Forecasting**
  - Trajectory prediction
  - Stock Market prediction

- **Univariate**
- Multivariate
- Zerovariate
  - Delphi Method

“Prediction is the essence of intelligence, and that’s what we’re trying to do.”

– Yann LeCun (2015)

"Prediction is very difficult, especially if it's about the future!"

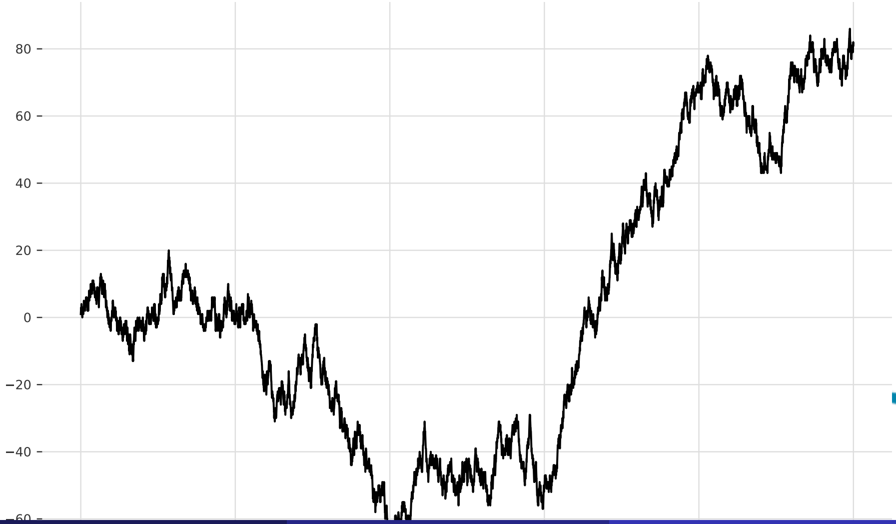
– Niels Bohr

“What we can’t predict, we call randomness.”

– Albert Einstein

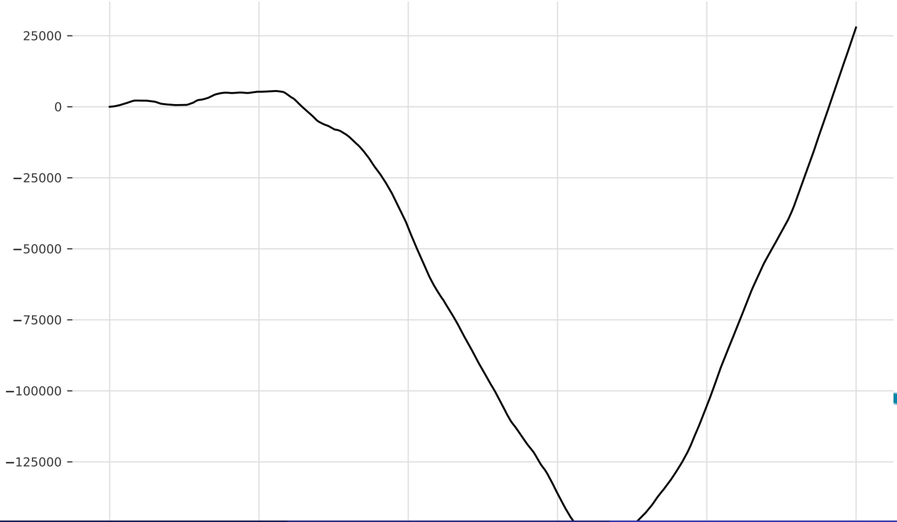
Understanding Randomness 1

Random walk cumulative sum of random numbers

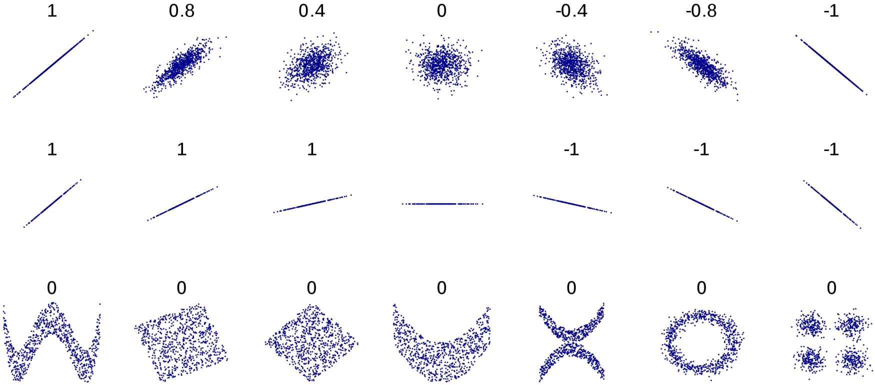


Understanding Randomness 2

Walk of random walk cumulative sum of a random walk



Pearson correlation coefficient r

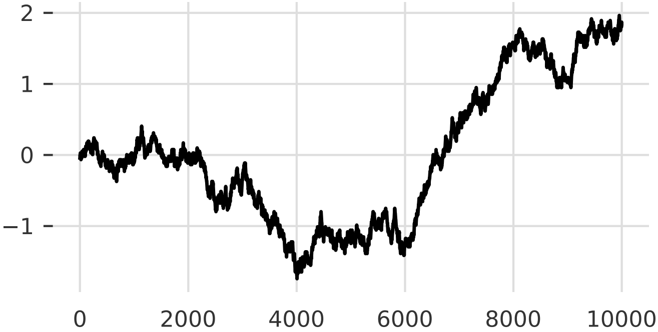


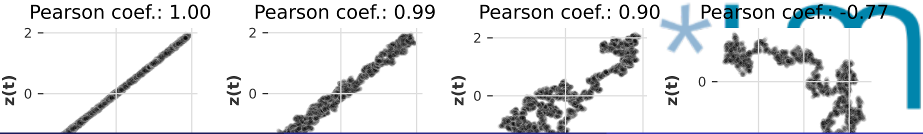
- Measures linear correlation between two sets of data
- Equal to the slope of a linear regression model (for standardized data $\sigma_x = \sigma_y = 1)^1$

$cov(x, y)$ σ

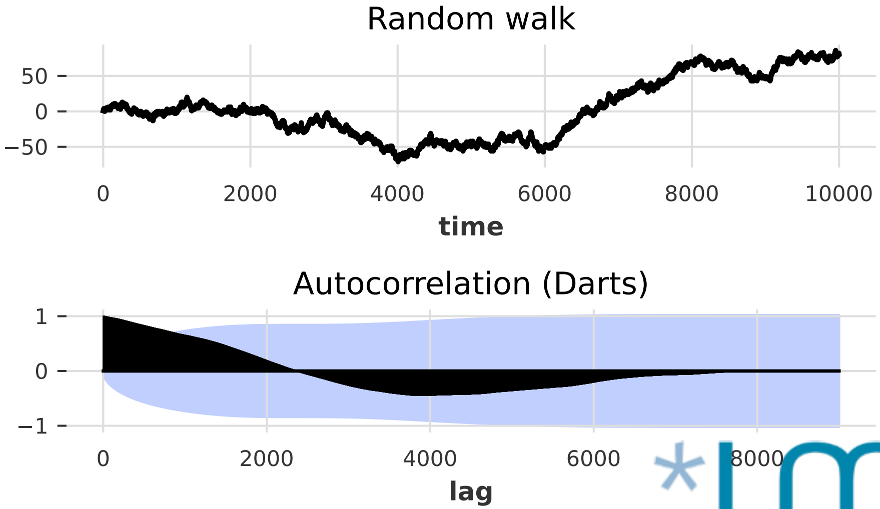
Autocorrelation 1

Pearson coefficients between time series and time shifted version of itself





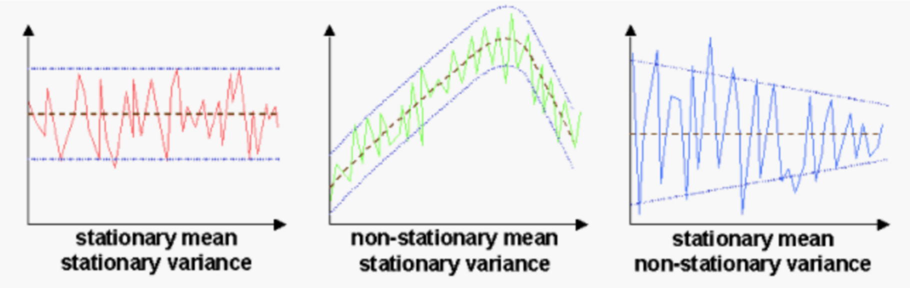
Autocorrelation 2



- ACF (auto correlation function) of non stationary data decreases

Stationarity

Statistical properties don't change over time

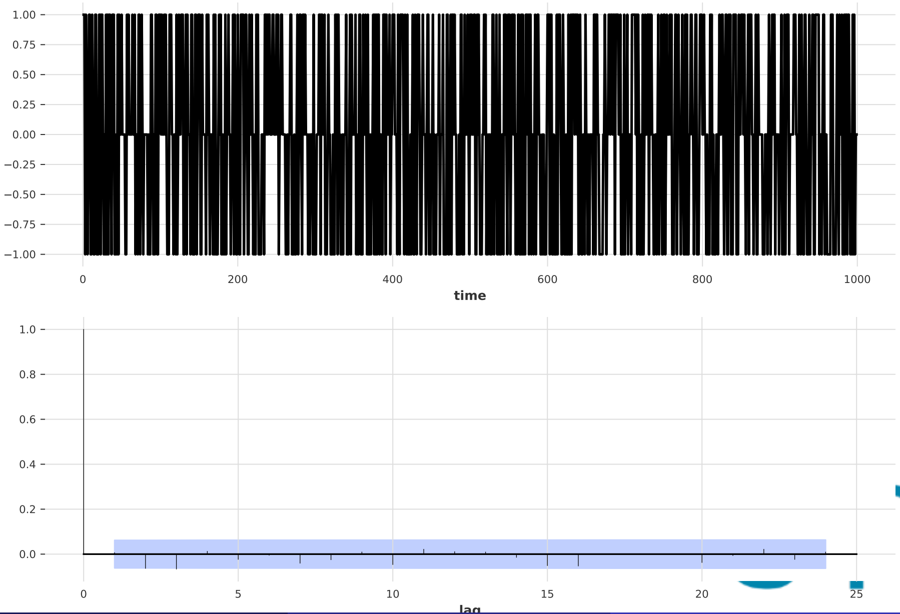


[3]

- A stationary time series has no seasonality or trend.
- Many tools, statistical tests and models (ARIMA!) rely on stationarity.

[3] Palachy (2019): Towards Data Science | Stationarity in time series analysis

Differencing



Partial autocorrelation 1

- Direct linear correlation r_k between y and x_k

$$\hat{y} = \phi_0 + 0 \cdot x_1 + ... + r_k x_k$$

- Partial correlation p_k between y and x_k

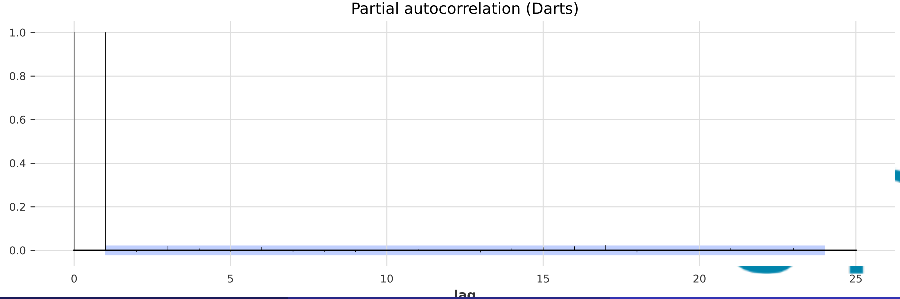
$$\hat{y} = \phi_0 + \phi_1 x_1 + ... + p_k x_k$$

Dependency on $x_1, x_2, ...$ is absorbed into $\phi_1, \phi_2, ...$

Partial correlation is linear correlation between y and x_k with indirect correlation to $x_1, x_2, ...$ removed.[4]

[4]Pen State: STAT 510 | Applied Time Series Analysis, chapter 2.2

Partial autocorrelation 2



Autoregression model

- forecast y_t using a linear combination of past values

$$\hat{y}_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p}$$

- Autoregression means linear regression with lagged version of itself (similar to autocorrelation)
- AR(p) means $\phi_0, ..., \phi_p \neq 0$
- Autoregressive process can be used to generate data from random white noise ϵ_t and fixed parameters ϕ_k

$$y_t = \epsilon_t + \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p}$$

- For $\phi_0 = 0$ and $\phi_1 = 1$ an AR(1) process is a random walk

Moving average model

- The name "moving average" is technically incorrect [5]
- Better would be lagged error regression
- forecast y_t using a linear combination of past forecast errors

$$\hat{y}_t = \theta_0 + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + ... + \theta_q \epsilon_{t-q}$$

- MA(q) cannot be fitted like an ordinary least square (OLS), because the forecast errors are not known
- Example algorithm: Set initial values for θ_k and ϵ_k, then
  - For i=1..N do
    1. Compute error terms for all t: $\epsilon_t = y_t - \hat{y}_t$
    2. Run a regression of y_t against $\hat{y}_t$ and update θ_k
    3. Repeat
- Like(?) Iteratively_reweighted_least_squares or similar iterative process to estimate θ_k

[5] Stackexchange: why-are-maq-time-series-models-called-moving-averages

ARIMA

- Autoregressive integrated moving average model combines AR(p), differencing/integrating I(d) and MA(q)

$$\hat{y}_t = c + \phi_1 y_{t-1}^{(d)} + \dots + \phi_p y_{t-p}^{(d)} + \theta_1 \epsilon_{t-1} \dots + \theta_q \epsilon_{t-q}$$

Model	equivalent to
ARIMA(0,1,0)	Random Walk (with drift)
ARIMA(0,1,1)	simple exponential smoothing ETS(A,N,N)
ARIMA(1,0,0)	discrete Ornstein-Uhlenbeck process [6]

- To find the best fitting ARIMA model one can use the Box-Jenkins-Method

[6] Wikipedia: Autoregressive_model

Box-Jenkins Method

1. Make the time series stationary (e.g. standardization, differencing d-times, ...)
2. Use ACF plot and PACF plot to identify the parameter p and q
3. Fitting the parameters of the ARIMA(p,d,q) model. This can be done with Hannan–Rissanen (1982) algorithm
   1. AR(m) model (with $m > max(p,q)$) is fitted to the data
   2. Compute error terms for all t: $\epsilon_t = y_t - \hat{y}_t$
   3. Regress y_t on $y^{(d)}_{t-1}, .., y^{(d)}_{t-p}, \epsilon_{t-1}, ..., \epsilon_{t-q}$
   4. To improve accurancy optinally regress again with updated ϕ, θ from step 3

   Other algorithms (maximizing likelihood) are often used in practice [7] [8]
4. Statistical model checking (analysis of mean, variance, (partial) autocorrelation, Ljung–Box test of residuals)
5. Once the residuals look like white noise, do the forecast

Nowadays all these steps are automated by tools like AutoARIMA etc.

[7] Brockwell, Davis (2016) Introduction to Time Series and Forecasting, chapter 5

[8] Hyndman (2008), Automatic Time Series Forecasting: The forecast Package for R

Why???

- Why combine AR and MA ?
    - Wold's theorem ?
- AR is analogeous to linear regression, but what is MA analogeous to outside of time series analysis?
- Why is ARIMA better than AR alone?