

Time Series Prediction Part I

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- Clustering
 - Dynamic Time Warping
- Segmentation
 - Speech recognition
- Classification
- Anomaly/Outlier Detection
- Imputation
- **Forecasting**
 - Trajectory prediction
 - Stock Market prediction
- **Univariate**
- Multivariate
- Zerovariate
 - Delphi Method



“Prediction is the essence of intelligence, and that’s what we’re trying to do.”

– Yann LeCun (2015)



“Prediction is very difficult, especially if it’s about the future!”

– *Niels Bohr*



“What we can’t predict, we call randomness.”

– Albert Einstein



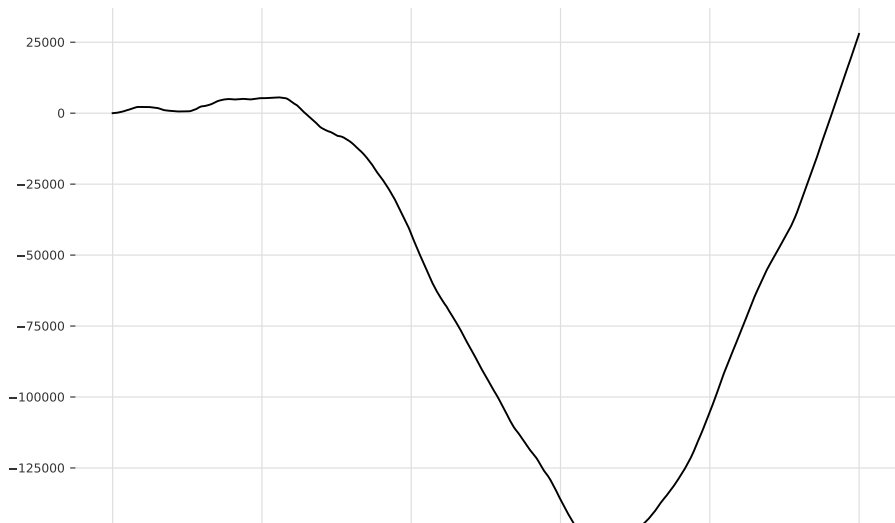
Understanding Randomness 1

Random walk cumulative sum of random numbers

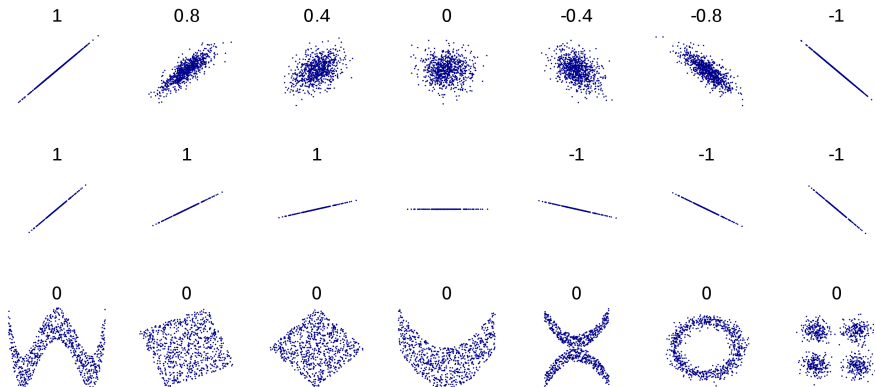


Understanding Randomness 2

Walk of random walk cumulative sum of a random walk



Pearson correlation coefficient r

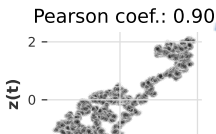
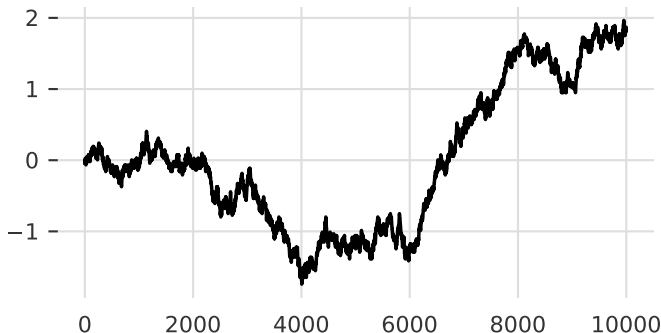


- Measures linear correlation between two sets of data
- Equal to the slope of a linear regression model (for standardized data $\sigma_x = \sigma_y = 1$)¹



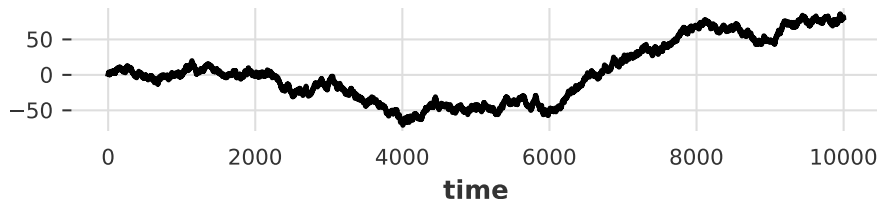
Autocorrelation 1

Pearson coefficients between time series and time shifted version of itself

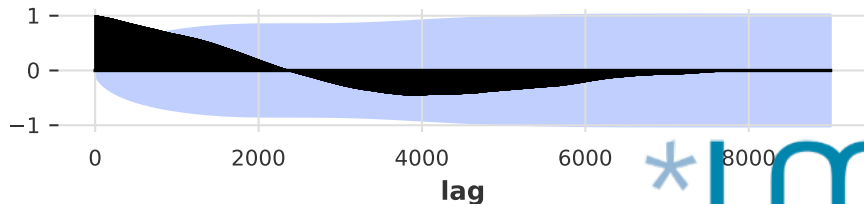


Autocorrelation 2

Random walk



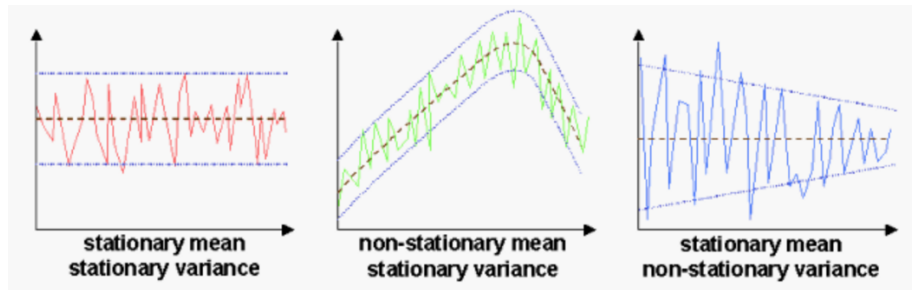
Autocorrelation (Darts)



• ACF (auto correlation function) of non-stationary data decreases

Stationarity

Statistical properties don't change over time

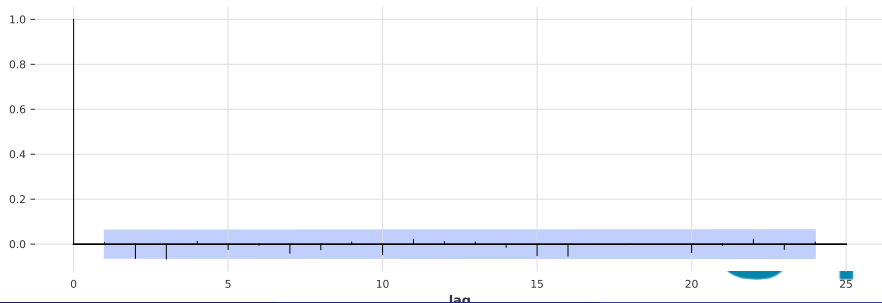
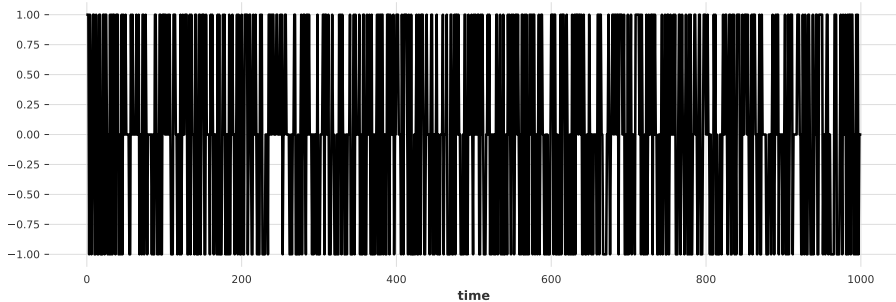


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- A stationary time series has no seasonality or trend.
- Many tools, statistical tests and models (ARIMA!) rely on stationarity.

³Palachy (2019): Towards Data Science | Stationarity in time series analysis

Differencing



Partial autocorrelation 1

- Direct linear correlation r_k between y and x_k

$$\hat{y} = \phi_0 + 0 \cdot x_1 + \dots + r_k x_k$$

- Partial correlation p_k between y and x_k

$$\hat{y} = \phi_0 + \phi_1 x_1 + \dots + p_k x_k$$

Dependency on x_1, x_2, \dots is absorbed into ϕ_1, ϕ_2, \dots

Partial correlation is linear correlation between y and x_k with indirect correlation to x_1, x_2, \dots removed.⁴

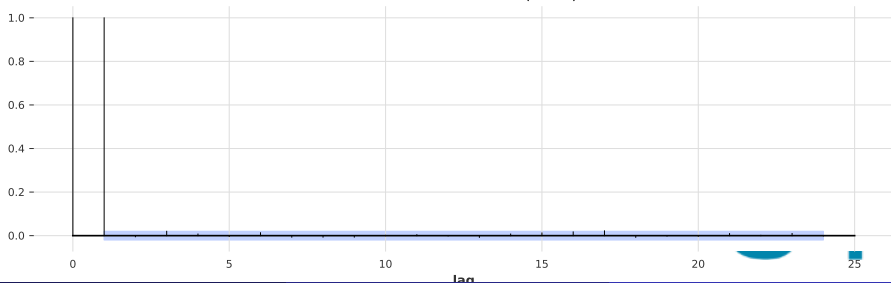


⁴Pen State: STAT 510 | Applied Time Series Analysis, chapter 2.2

Partial autocorrelation 2



Partial autocorrelation (Darts)



Autoregression model

- forecast y_t using a linear combination of past values

$$\hat{y}_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p}$$

- Autoregression means linear regression with lagged version of itself (similar to autocorrelation)
- AR(p) means $\phi_0, \dots, \phi_p \neq 0$
- Autoregressive process can be used to generate data from random white noise ϵ_t and fixed parameters ϕ_k

$$y_t = \epsilon_t + \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p}$$

- For $\phi_0 = 0$ and $\phi_1 = 1$ an AR(1) process is a random walk



Moving average model

- The name “moving average” is technically incorrect ⁵
- Better would be lagged error regression
- forecast y_t using a linear combination of past forecast errors

$$\hat{y}_t = \theta_0 + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \dots + \theta_q \epsilon_{t-q}$$

- MA(q) cannot be fitted like an ordinary least square (OLS), because the forecast errors are not known
- Example algorithm: Set initial values for θ_k and ϵ_k , then
 - For $i=1..N$ do
 - 1 Compute error terms for all t : $\epsilon_t = y_t - \hat{y}_t$
 - 2 Run a regression of y_t against \hat{y}_t and update θ_k
 - 3 Repeat
- Like(?) Iteratively_reweighted_least_squares or similar iterative process to estimate θ_k

⁵Stackexchange: why-are-maq-time-series-models-called-moving-averages

- Autoregressive integrated moving average model combines AR(p), differencing/integrating I(d) and MA(q)

$$\hat{y}_t = c + \phi_1 y_{t-1}^{(d)} + \dots + \phi_p y_{t-p}^{(d)} + \theta_1 \epsilon_{t-1} \dots + \theta_q \epsilon_{t-q}$$

Model	equivalent to
ARIMA(0,1,0)	Random Walk (with drift)
ARIMA(0,1,1)	simple exponential smoothing ETS(A,N,N)
ARIMA(1,0,0)	discrete Ornstein-Uhlenbeck process

- To find the best fitting ARIMA model one can use the Box-Jenkins-Method

⁶Wikipedia: Autoregressive_model



Box-Jenkins Method

- ① Make the time series stationary (e.g. standardization, differencing d -times, ...)
- ② Use ACF plot and PACF plot to identify the parameter p and q
- ③ Fitting the parameters of the ARIMA(p,d,q) model. This can be done with Hannan–Rissanen (1982) algorithm
 - ① AR(m) model (with $m > \max(p, q)$) is fitted to the data
 - ② Compute error terms for all t : $\epsilon_t = y_t - \hat{y}_t$
 - ③ Regress y_t on $y_{t-1}^{(d)}, \dots, y_{t-p}^{(d)}, \epsilon_{t-1}, \dots, \epsilon_{t-q}$
 - ④ To improve accuracy optimally regress again with updated ϕ, θ from step 3

Other algorithms (maximizing likelihood) are often used in practice ^{7 8}

- ④ Statistical model checking (analysis of mean, variance, (partial) autocorrelation, Ljung–Box test of residuals)
- ⑤ Once the residuals look like white noise, do the forecast

Nowadays all these steps are automated by tools like AutoARIMA etc.

⁷Brockwell, Davis (2016) Introduction to Time Series and Forecasting, chapter 5

⁸Hudon (2009). Automatic Time Series Forecasting: The Forecast Package for R

Why???

- Why combine AR and MA ?
 - Wold's theorem ?
- AR is analogous to linear regression, but what is MA analogous to outside of time series analysis?
- Why is ARIMA better than AR alone?

