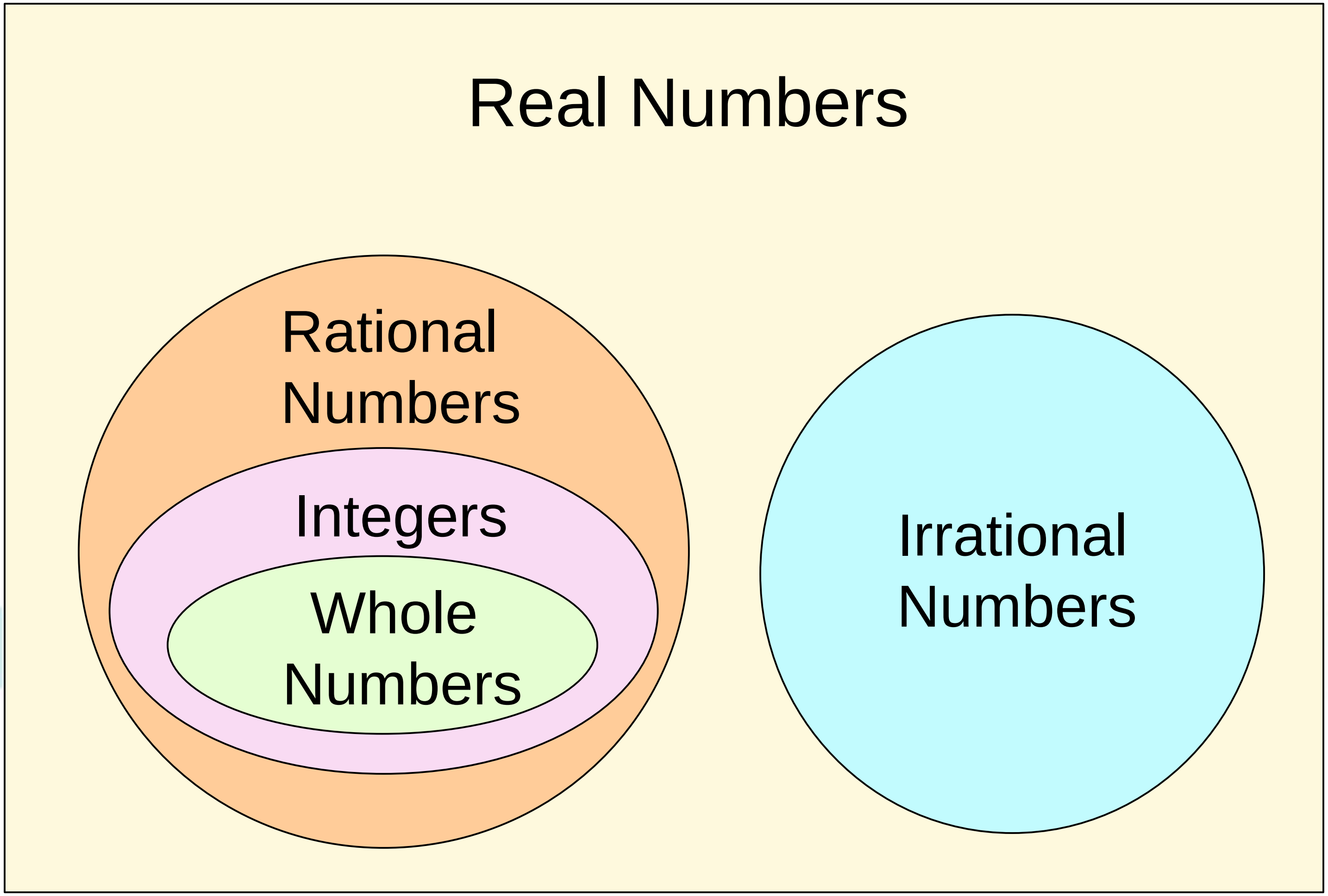
Chapter Summary

INTRODUCTION



REAL NUMBERS

REAL NUMBERS

- **RATIONAL NUMBERS**
  - $\frac{1}{2}$, 0.23, 3.333, ..., $3\frac{1}{4}$
- **IRRATIONAL NUMBERS**
  - $\sqrt{3}, 5\sqrt{2}+3, \pi$

Example

Rahul has some CDs. He does not have more than 50 CDs. When his friends asked him how many CDs he has got, he replied:

If counted in fives, four will remain.
If counted in fours, two will remain.
If counted in threes, one will remain.
If counted in pairs, nothing will remain.

Example - Solution

Let us try to find the number of CDs.

Let the number of CDs be a. Then a is less than or equal to 50.

If counted in pairs, nothing will remain, i.e.,
$a = 2m + 0$, for some natural number m.

If counted in threes, one will remain, i.e.,
$a = 3n + 1$, for some natural number n.

If counted in fours, two will remain, i.e.,
$a = 4p + 2$, for some natural number p.

If counted in fives, four will remain, i.e.,
$a = 5t + 4$, for some natural number t.

Example - Solution

In each case we have '*a*' and a positive integer '*b*' (here *b* takes values 2, 3, 4 and 5, respectively) which divides '*a*' and leaves a remainder '*r*' (in this case *r* is 0, 1, 2 and 4, respectively) that is smaller than '*b*'.

By trial and error, we found that he has got 34 CDs.

Here, we have used **Euclid's Division Lemma** to write the equations.

In general, for each pair of positive integers *a* and *b*, we have whole numbers *q* and *r* such that: $a = bq + r$,

$$0 \le r < b$$

Euclid's Division Lemma

- For given positive integers *a* and *b*, there exists unique integers *q* and *r* satisfying

$$\boldsymbol{a = bq + r}, \qquad 0 \leq r < b$$

Example

Verify Euclid's division lemma for the integers 25, 7.

Solution:

$25 = 7 \times 3 + 4$

Comparing with $a = bq + r$,
we have $a = 25$, $b = 7$, $q = 3$ and $r = 4$, also $0 \leq 4 < 7$

Euclid's Division Algorithm

- Euclid's division algorithm is based on Euclid's Division Lemma.
- It is a technique to compute Highest Common Factor (HCF) of two given positive integers.
- HCF of two positive integers a and b is the largest positive integer d which divides both a and b.

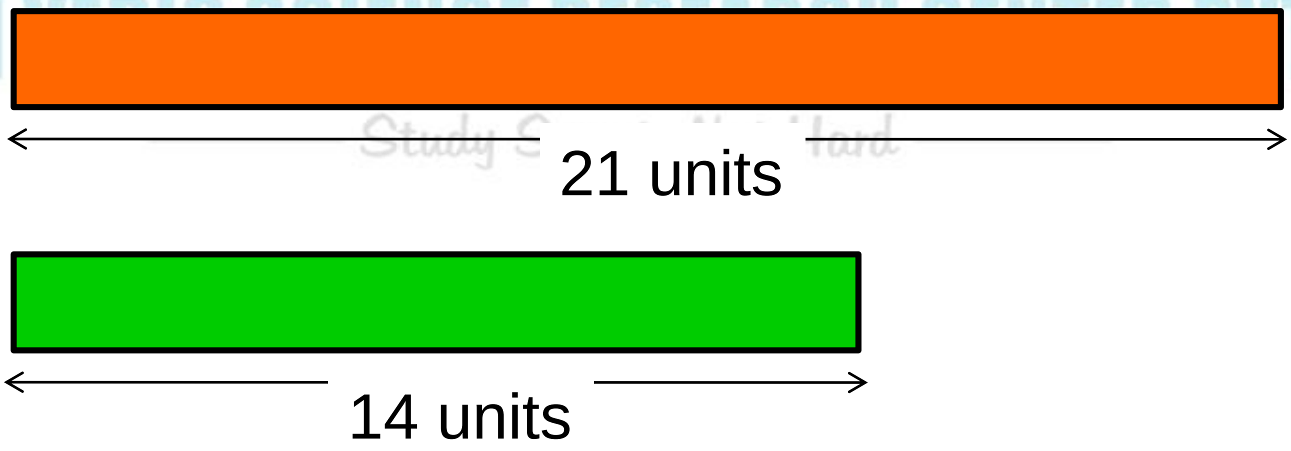
Euclid's Division Algorithm

Let us observe an activity to find HCF of two numbers which is based on Euclid's Division Algorithm.

Suppose we want to find the HCF of 21 and 14.

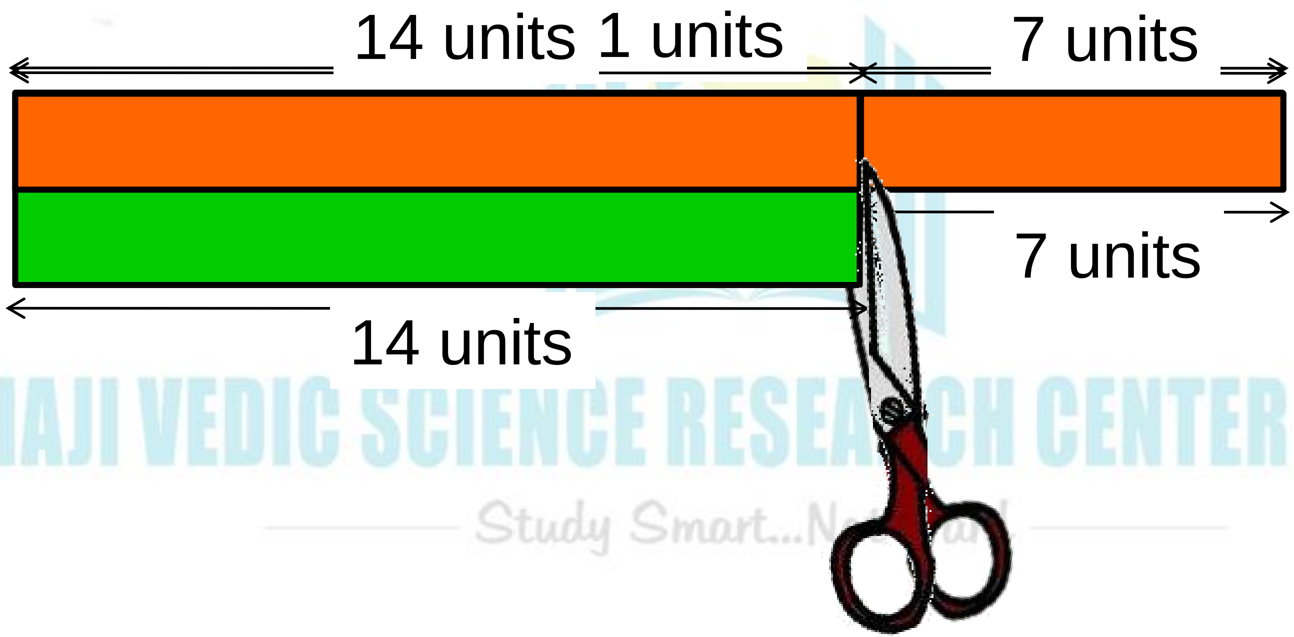
First, cut out a strip of length 21 units using a colour grid paper.

Now, cut out another strip of length 14 units using another colour grid paper.



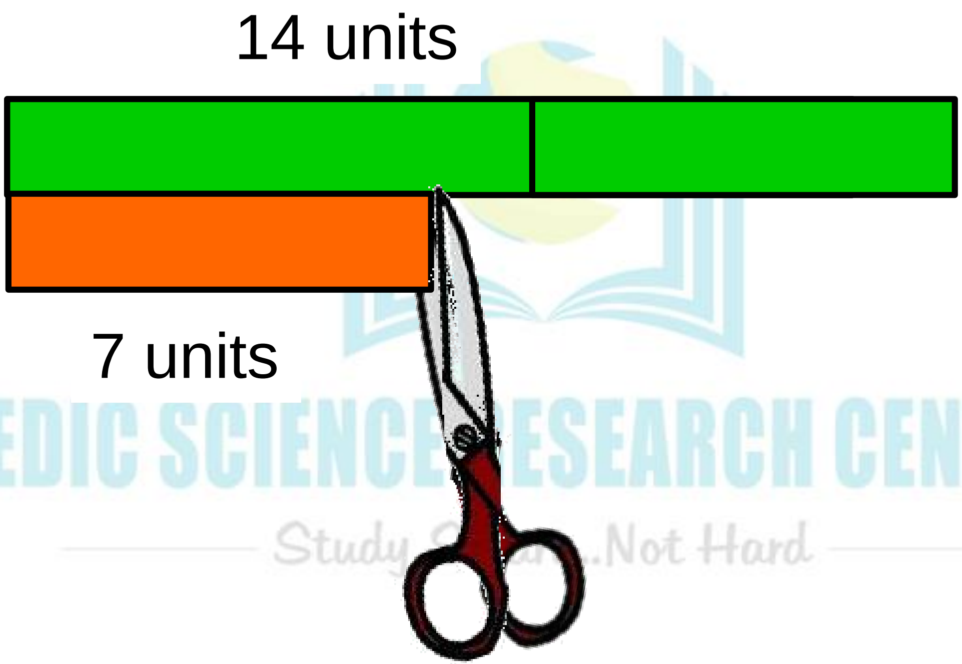
Euclid's Division Algorithm

- Place the green (shorter) strip along the orange (longer) strip as shown.
- Cut out the remaining part of the orange strip.



Euclid's Division Algorithm

Now place the smaller cut out strip (orange-7 units) along the larger cut out strip (green-14 units) and cut out the extra part (green strip). Repeat the activity until both strips become equal in length.



As the remaining strips is of 7 units, the HCF of 21 and 14 is 7.

Steps of Finding HCF

- To obtain the HCF of two positive integers, say *c* and *d* with $c > d$, we can use the following steps:

Step 1: Apply Euclid's division lemma to *c* and *d*. So, we find whole numbers *q* and *r* such that $c = dq + r$,

$$0 \le r < d.$$

Step 2: If $r = 0$, *d* is the HCF of *c* and *d*. If $r \neq 0$ apply the division lemma to *d* and *r*.

Step 3: Continue the process till the remainder is zero. The divisor at that stage will be required HCF.

This algorithm works because HCF (c, d) = HCF (d, r), where the symbol HCF (c, d) denotes the HCF of *c* and *d* and HCF(d, r) denotes the HCF of *d* and *r*.

Example

Use Euclid's division algorithm to find the HCF of 210 and 55.

Solution:

Given integers are 210 and 55. Clearly, 210 > 55. Applying Euclid's Division Lemma to 210 and 55, we get

$210 = 55 \times 3 + 45$ ...(i)

$$\because \begin{array}{r|l} & \;3 \\ \hline 55 & 210 \\ & \underline{165} \\ & 45 \end{array}$$

Example

Since the remainder 45 $\neq$ 0. So, we apply the division lemma to the divisor 55 and remainder 45 to get

55 = 45 $\times$ 1 + 10 ...(ii)

$$\because \begin{array}{r|l} & 1 \\ \hline 45 & 55 \\ & \underline{45} \\ & 10 \end{array}$$

Now, we apply division lemma to the new divisor 45 and new remainder 10 to get

45 = 10 $\times$ 4 + 5 ...(iii)

$$\because \begin{array}{r|l} & 4 \\ \hline 10 & 45 \\ & \underline{40} \\ & 5 \end{array}$$

Example

We now consider the new divisor 10 and the new remainder 5 and apply division lemma to get

$10 = 5 \times 2 + 0$ ...(iv)

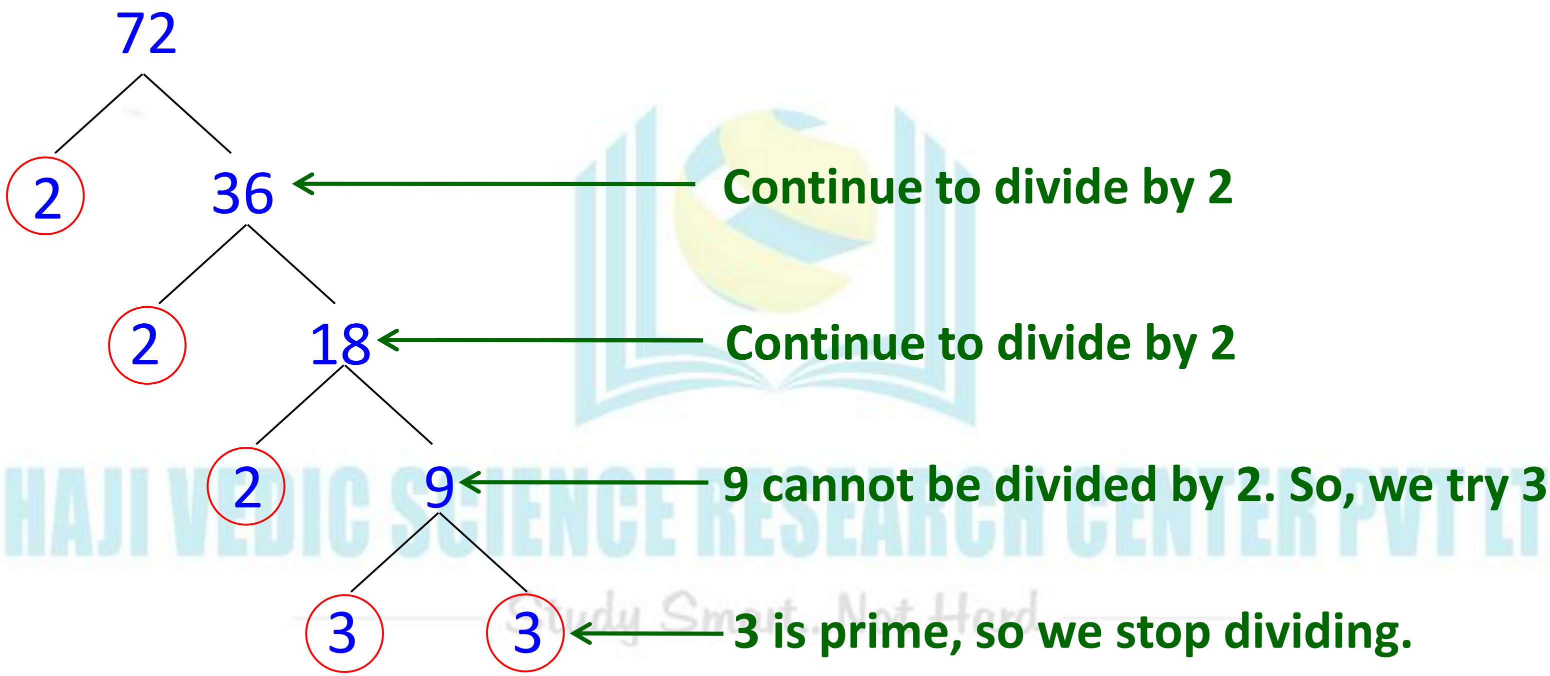
The remainder at this stage is zero. So, the divisor at this stage or the remainder at the previous stage, i.e., 5 is the HCF of 210 and 55.

The Fundamental Theorem of Arithmetic

Theorem: Every composite number can be expressed as a product of primes and its factorisation is unique, apart from the order in which the prime factors occur.

- Find the prime factors of 72.

Solution:



The prime factorisation of 72 is the product of the circled factors.

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

An Important Result on LCM and HCF of Two Numbers

Let a and b be two positive integers, then

HCF (a, b) $\times$ LCM (a, b) = Product of two numbers (a, b)

Example

- Find the HCF and LCM of 135 and 225 by the prime factorisation method.

Solution:

We have,

$$135 = 3^3 \times 5$$
$$225 = 3^2 \times 5^2$$

Here, 3^2 and 5^1 are the smallest powers of the common factors 3 and 5 respectively.

So, HCF (135, 225) = $3^2 \times 5^1 = 45$

3^3 and 5^2 are the greatest power of the prime factors 3 and 5 respectively, involved in the two numbers.

So, LCM (135, 225) = $3^3 \times 5^2 = 675$

IRRATIONAL NUMBERS

- Numbers that cannot be expressed in the form of *p/q.*
- Decimal representation of irrational numbers are non-terminating, non-repeating.

For example,

4.6565565556…

8.235749378256…

THEOREM

Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

For example,

If $p = 6$ and $a = 12$, then

$12^2 = 144$ is divisible by 6.

Also, 12 is divisible by 6.

THEOREM

Revisiting Rational Numbers

Decimal representation of rational numbers

Terminating

Examples: 0.35, 7.56

Non-terminating, repeating

Examples: $0.\overline{8}$, $0.\overline{23}$

Remarks:

Any rational number of the form *p/q* where *p* and *q* are co-primes and the denominator contains factors of the form $2^n 5^m$, only then the decimal expression of the rational number will be terminating.

If the denominator contains factors other than $2^n 5^m$, then it becomes non-terminating repeating.

THEOREM

- **Check out without division whether the following will have a terminating or a non-terminating decimal representation.**

$$\frac{3}{5}$$ ✓ As the denominator contains only factor 5.

$$\frac{13}{3125}$$ ✓ As 3125 can be expressed in the form 5^4.

$$\frac{175}{210}$$ ✗ Here we need to reduce the rational number to the lowest term, we get 5/6 and the factors of 6 are 2 and 3.

$$\frac{35}{450}$$ ✗ On reducing to the lowest term, we get 7/90 and $90 = 2 \times 3^2 \times 5$.

Example

Show that $\sqrt{7}$ is irrational.

Solution:

Let us assume that $\sqrt{7}$ is rational.

Then there exist co-prime positive integers

a and *b* such that

$$\sqrt{7} = \frac{a}{b}$$

$$\Rightarrow \sqrt{7}b = a$$

$$\Rightarrow 7b^2 = a^2 \quad \text{squaring both sides}$$

Therefore, 7 divides a^2. Also 7 divides *a*.

So, we can write $a = 7c$ for some integer *c*.

Example

Substituting for a, we get $7b^2 = 49c^2$.

This means that 7 divides b^2, so 7 divides b.

Therefore, a and b have at least 7 as common factor.

But this contradicts the fact that a and b have no common factors other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{7}$ is rational.

So, we conclude that $\sqrt{7}$ is irrational.

Example

- Four bells ring at intervals 4, 6, 8 and 14 seconds. They start ringing simultaneously at 12.00 O'clock. At what time will they again ring simultaneously?

- Is the required time has any relation with the
- LCM of 4, 6, 8 and 14…