Пример. $z = xy$, $\underline{x+y=1} \Rightarrow \underbrace{x+y-1}_{g(x,y)}=0$

$$\Phi(x,y,\lambda) = xy + \lambda(x+y-1).$$

Необх. усл.

$$\begin{cases} \dfrac{\partial \Phi}{\partial x} = 0, \\ \dfrac{\partial \Phi}{\partial y} = 0, \\ \dfrac{\partial \Phi}{\partial \lambda} = 0 \end{cases} \Rightarrow \begin{cases} y+\lambda = 0, \\ x+\lambda = 0, \\ x+y-1=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} y = -\lambda, \\ x = -\lambda, \\ -\lambda-\lambda-1=0 \end{cases} \Rightarrow \begin{cases} y = -\lambda, \\ x = -\lambda, \\ -2\lambda = 1 \end{cases} \Rightarrow \begin{cases} y = 1/2, \\ x = 1/2, \\ \lambda = -1/2. \end{cases}$$

$$M_0\left(\frac{1}{2}; \frac{1}{2}\right), \quad \lambda_0 = -1/2.$$

$$d^2\Phi(M_0,\lambda_0) = \underbrace{\frac{\partial^2 \Phi}{\partial x^2}(M_0,\lambda_0)}_{0} dx^2 + 2\frac{\partial^2 \Phi}{\partial x \partial y}(M_0,\lambda_0)\,dx\,dy + \underbrace{\frac{\partial^2 \Phi}{\partial y^2}(M_0,\lambda_0)}_{0} dy^2 = 2\,dx\,dy = 2(-dy)\,dy = -2dy^2 < 0$$

$$\frac{\partial^2 \Phi}{\partial x^2} = 0, \quad \frac{\partial^2 \Phi}{\partial x^2}(x_0,y_0,\lambda_0) = 0,$$

$$\frac{\partial^2 \Phi}{\partial y^2} = 0, \quad \frac{\partial^2 \Phi}{\partial y^2}(x_0,y_0,\lambda_0) = 0,$$

$$\frac{\partial^2 \Phi}{\partial x \partial y} = 1, \quad \frac{\partial^2 \Phi}{\partial x \partial y}(x_0,y_0,\lambda_0) = 1$$

т. M_0 – усл. max.

$$g(x,y) = \underbrace{x+y-1} = 0, \quad dg(x_0,y_0) = 0$$

$$\underbrace{\frac{\partial g}{\partial x}(x_0,y_0)}dx + \underbrace{\frac{\partial g}{\partial y}(x_0,y_0)}dy = 0$$

$$\frac{\partial g}{\partial x} = 1, \quad \frac{\partial g}{\partial x}(x_0,y_0) = 1, \quad \Bigg| \quad dx + dy = 0$$

$$\frac{\partial g}{\partial y} = 1, \quad \frac{\partial g}{\partial y}(x_0,y_0) = 1 \quad \Bigg| \quad dx = -dy$$