$$f(x,y)=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\cdot(x-x_0)+\frac{\partial f}{\partial y}(x_0,y_0)\cdot(y-y_0)+\frac{1}{2}\Big[\frac{\partial^2 f}{\partial x^2}(x_0,y_0)\cdot(x-x_0)^2+$$

$$+2\frac{\partial^2 f}{\partial x\partial y}(x_0,y_0)\cdot(x-x_0)(y-y_0)+\frac{\partial^2 f}{\partial y^2}(x_0,y_0)(y-y_0)^2\Big]+R_2(x,y)$$

$\Big|_{(x_0,y_0)}$

Пример. $f(x,y)=e^{x+y}$, $(-1;1)$, т.е. $x_0=-1$, $y_0=1$.

$y(x)$

$f(-1,1)=e^0=1$; $\frac{\partial f}{\partial x}=e^{x+y}\cdot 1=e^{x+y}$

$\frac{\partial f}{\partial x}(-1,1)=1$, $\frac{\partial f}{\partial y}=e^{x+y}\cdot 1=e^{x+y}$; $\frac{\partial f}{\partial y}(-1,1)=1$

$\frac{\partial^2 f}{\partial x^2}=\frac{\partial}{\partial x}(e^{x+y})=e^{x+y}$; $\frac{\partial^2 f}{\partial x^2}(-1,1)=1$

$\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial}{\partial y}(e^{x+y})=e^{x+y}$; $\frac{\partial^2 f}{\partial x\partial y}(-1,1)=1$

$\frac{\partial^2 f}{\partial y^2}=\frac{\partial}{\partial y}(e^{x+y})=e^{x+y}$; $\frac{\partial^2 f}{\partial y^2}(-1,1)=1$

$$e^{x+y}=1+1\cdot(x+1)+1\cdot(y-1)+\frac{1}{2}\Big[1\cdot(x+1)^2+2\cdot 1\cdot(x+1)(y-1)+$$

$$+1\cdot(y-1)^2\Big]+R_2(x,y)=1+(x+1)+(y-1)+$$

$$+\frac{1}{2}(x+1)^2+(x+1)(y-1)+\frac{1}{2}(y-1)^2+R_2(x,y)$$

$$a_{11}=\frac{\partial^2 f}{\partial x^2}(x_0,y_0),\quad a_{12}=a_{21}=\frac{\partial^2 f}{\partial x\partial y}(x_0,y_0),$$

$$a_{22}=\frac{\partial^2 f}{\partial y^2}(x_0,y_0)$$

$$d^2 f(x_0,y_0)=\frac{\partial^2 f}{\partial x^2}(x_0,y_0)dx^2+2\frac{\partial^2 f}{\partial x\partial y}(x_0,y_0)\,dx\,dy$$

$$+\frac{\partial^2 f}{\partial y^2}(x_0,y_0)dy^2=a_{11}dx^2+2a_{12}\,dx\,dy+$$

$$+a_{22}dy^2=A(dx,dy)$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}$$

1) $a_{11}>0$, $\begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}=a_{11}a_{22}-a_{12}^2>0$

2) $a_{11}<0$, $\begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}>0$

Пример. $f(x,y)=x^2-xy+y^2-2x+y$

Необх. усл.

$$\begin{cases} \frac{\partial f}{\partial x}=0, \\ \frac{\partial f}{\partial y}=0 \end{cases} \Rightarrow \begin{cases} 2x-y-2=0, \\ -x+2y+1=0 \end{cases}$$

$$\begin{cases} y = 2x - 2, \\ -x + 4x - 4 + 1 = 0 \end{cases} \Rightarrow \begin{cases} y = 2x - 2, \\ 3x - 3 = 0 \end{cases} \Rightarrow \begin{cases} y = 0 \\ x = 1. \end{cases}$$

$$M_0(1;0)$$

$$a_{11} = \frac{\partial^2 f}{\partial x^2}(1,0) = 2,$$

$$a_{12} = a_{21} = \frac{\partial^2 f}{\partial x \partial y}\Big|(1;0) = -1.$$

$$a_{22} = \frac{\partial^2 f}{\partial y^2}(1;0) = 2$$

$$a_{11} = 2 > 0,$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3 > 0$$