313 -

1. Bisection - root এর condition, algorithm 4step
*
math, [upper, . . .]

2. Newton's divided difference -
theory - linear, Quardatic ; general formula
দিয়ে math.

3. lagrangian interpolation -
- 5 page equation theory ।টা
- math Linear, Qardatic, Cubic.

4. integration - 2 slide
- trapezoidal, simpson theory.
- math 2seg., multiple (4seg).

5. Regression - 2 page theory.
- math form table + do final prediction + Ca

25, - linear Regression (16 + prediction + ta)

6. Euler, Range-kutta কোনটা ভাল - theory

math - Heun's, midpoint

9 -

7. Golden-Section = algorithm :

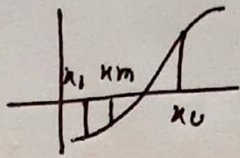
Math - 7

Bisection Method

→ As $f(x_l)\, f(x_u) < 0$,
then a root may or may not exist in the range
$[x_l, x_u]$. therefore further calculation is not needed

- has five step.

$\boxed{\text{step 2}}$ = $x_m = \dfrac{x_l + x_u}{2}$



$\boxed{\text{step 3}}$ = check following estimate the root of x_m

3. statement.

a. $f(x_l)\, f(x_m) < 0$; $x_l = x_l$; $x_u = x_m$

b. $f(x_l)\, f(x_m) > 0$, ; $x_m = x_l$
$x_u = x_u$

c. $f(x_l)\, f(x_m) = 0$; then the root is x_m.
stop the algorithm if the's's true

$\boxed{\text{step 4}}$ = absolute relative approximate error

$$|\epsilon_a| = \left| \frac{x_m' \text{ new} - x_m \text{ old}}{x_m \text{ new}} \right| \times 100$$

$\boxed{\text{step 5}}$ - if $|\epsilon_a| > \epsilon_s$; then go to step 2
else → stop.

exam: $f(t) = te^{-t} - 0.3$

Ans: let, $x_l = 1$, $x_u = 5$

$f(x_1) = 1e^{-1} - 0.3$

$f(x_u) = 5e^{-5} - 0.3$

hence, $f(x_1) f(x_u) = (e^{-1} - 0.3)(5e^{-5} - 0.3)$

$= 0.067879 \times (-0.2663)$

$= -0.018 \boxed{<0}$

so, there at least on root between x_l and x_u

Iteration 1

the estimate of the root is, $x_m = \frac{x_1 + x_u}{2} = \frac{1+5}{2} = \frac{6}{2} = \boxed{3}$

$f(x_m) = 3e^{-3} - 0.3 = -0.1506$.

Thus, $f(x_1) f(x_m) = f(1) f(3) = (0.0679)(-0.1506)$

$= -0.0102 < 0$

the root lies between x_1 and x_m.

so, the new upper and lower guesses for the root

are - $x_l = x_1 = 1$

$x_u = x_m = 3$.

iteration 2

estimate of the root is, $x_m = \frac{x_1 + x_u}{2} = \frac{1+3}{2} = 2$.

$f(x_m) = 2e^{-2} - 0.3 = -0.029$

the root lies between x_1 and x_m.

$f(x_1) f(x_m) = f(1) f(2) = (0.0679)(-0.029) = -1.9691 \times 10^{-3} < 0$.

The absolute relative app. error, $|\epsilon_a|$ at the end of iteration 2 is

$$|\epsilon_a| = \left|\frac{2-3}{2}\right| \times 100$$

$$= \left|\frac{-1}{2}\right| \times 100$$

$$= 50\ \%$$

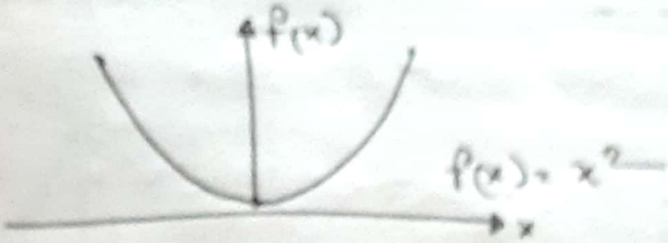
None of the significant digits are at least correct in the estimate root of $x_m = 2$ because the ϵ_a is greater than 5%

advantages

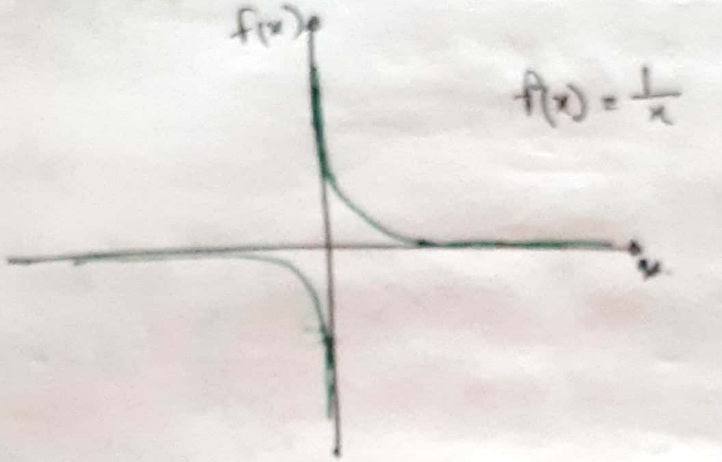
- always convergent
- the root bracket gets halved (divide into two parts of equal) with each iteration - guaranteed.

Drawbacks

- Slow convergence
- if one of the initial guesses is close to the root, the convergence is slower.
- if a function $f(x)$ is such that it just touches the x-axis it will be unable to find the lower and upper guesses.



- function changes sign but root does not exist.



Ct # 02.

Newton's Divided Difference Method

Linear interpolation : given, (x_0, y_0), (x_1, y_1)

$$\boxed{f_1(x) = b_0 + b_1 (x - x_0)}$$

where, $b_0 = f(x_0)$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

example:

$$v(t) = b_0 + b_1 (t - t_0)$$

Let, $t_0 = 15$ $\quad t_1 = 20$

$v(t_0) = 362.78$ $\quad v(t_1) = 517.35$

$$b_0 = v(t_0) = 362.78.$$

$$b_1 = \frac{v(t_1) - v(t_0)}{t_1 - t_0} = \frac{517.35 - 362.78}{20 - 15}$$

$$= 30.914.$$

$$v(t) = b_0 - b_1 (t - t_0)$$

$$= 362.78 + 30.914 (\cancel{16}\, t - 15)$$

$$\cancel{= 393.69\ ms^{-1}}, \quad \boxed{15 \le t \le 20}$$

At, $t = 16$

$$v(16) = 362.78 + 30.914 (16 - 15)$$

$$= 393.69\ ms^{-1}$$

Quadratic interpolation

given, (x_0, y_0), (x_1, y_1) and (x_2, y_2).

$$\boxed{f_2(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)}$$

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\frac{f(x_2)-f(x_1)}{x_2-x_1} - \frac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2 - x_0}$$

let,

$t_0 = 10$, $v(t_0) = 227.04$

$t_1 = 15$, $v(t_1) = 362.78$

$t_2 = 20$, $v(t_2) = 517.35$

example: $b_0 = v(t_0) = 227.04$

$$b_1 = \frac{v(t_1) - v(t_0)}{t_1 - t_0} = \frac{362.78 - 227.04}{15-10} = 27.148 \quad f(x_1, x_0)$$

$$\rightarrow \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{517.35 - 362.78}{20-15} = 30.914 \quad f(x_1, x_2)$$

$$b_2 = \frac{30.914 - 27.148}{20 - 10} = 0.37660.$$

$$v(t) = b_0 + b_1(t-t_0) + b_2(t-t_0)(t-t_2)$$

$$= 227.04 + 27.148(t-10) + 0.37660(t-10)(t-20).$$

at $t = 16$;

$$v(16) = 227.04 + 27.148(16-10) + 0.37660(16-10)(16-20)$$

$$= 392.19 \text{ ms}^{-1}$$

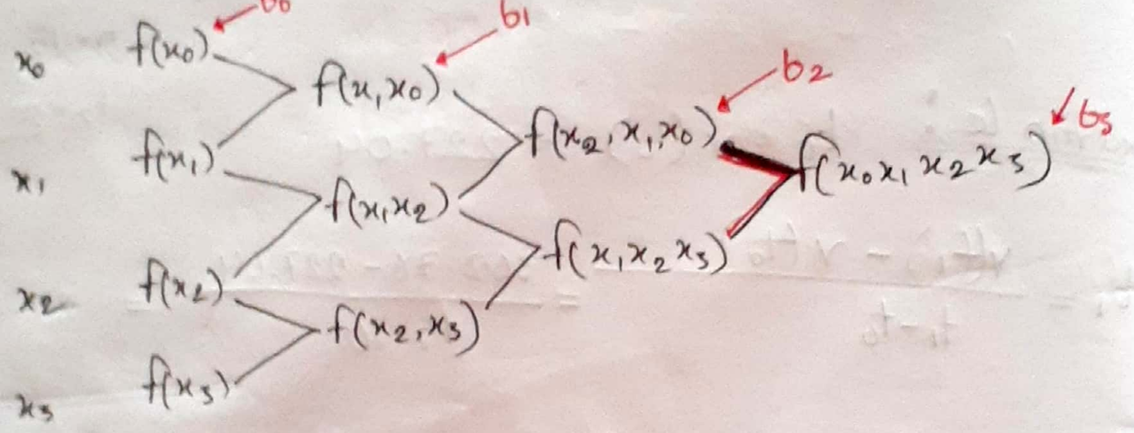
$$|\epsilon_a| = \left|\frac{392.19 - 393.69}{392.19}\right| \times 100\% = 0.38302\%$$

General from

the third order polynomial.

given (x_0, y_0), (x_1, y_1), (x_2, y_2) and (x_3, y_3).

$$f_3(x) = f[x_0] + f[x_1, x_0](x - x_0) + f[x_2, x_1, x_0](x - x_0)(x - x_1)$$
$$+ f[x_3, x_2, x_1, x_0](x - x_0)(x - x_1)(x - x_2).$$



example - do next.

Find the velocity at $t = 16$ s.

for cubic interpolation

eq^n, $V(t) = b_0 + b_1(t - t_0) + b_2(t - t_0)(t - t_1) + b_3(t - t_0)(t - t_1)(t - t_2)$

from previous, $b_0 = 227.04$

$b_1 = 27.148$

$b_2 = 0.37660$

$b_3 = 5.4347 \times 10^{-3}$.

data points. $t_0 = 10$, $V(t_0) = 227.04$

$t_1 = 15$, $V(t_1) = 362.78$

$t_2 = 20$, $V(t_2) = 517.35$

$t_3 = 22.5$, $V(t_3) = 602.97$

$$\frac{34.248 - 30.914}{22.5 - 20} = 0.444$$

	$v(t)$	b_0	b_1	b_2	b_3
$t_0 = 10$,	227.04				
		27.148			
$t_1 = 15$,	362.78		0.37660		
		30.914		5.4347×10^{-3}	
$t_2 = 20$,	517.35		0.44453		
		34.248			
$t_3 = 22.5$	602.97				

hence, $v(t) = 227.04 + 27.148(16-10) + 0.37660(16-10)(16-15)$

$v(16)$ $\quad + 5.4347 \times 10^{-3}(16-10)(16-15)(16-20)$.

$= 392.06 \ ms^{-1}$

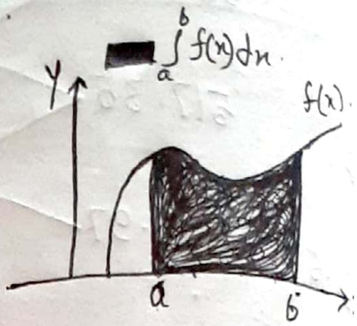
$$|\epsilon_a| = \left| \frac{392.06 - 392.19}{392.06} \right| \times 100$$

$= 0.033427 \ \%$

lecture 18 = ~~Lagrangian~~ Interpolation

Trapezoidal_ Rule

= the process of measuring the area under function plotted on a graph.



$$\boxed{I = \int_a^b f(x)\, dx}$$

where, f(x) = integrand.
a = lower limit of integration.
b = upper limit of integration.

example 01

t = 8 to t = 30 seconds.

$$x = \int_8^{30} \left(2000 \ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t\right) dt$$

Basis of trapezoidal Rule

= it based on the Newton- Cotes Formula that states if one can approximate the integrand as an $\boxed{n^{th}}$ order polynomial....

$$I = \int_a^b f(x)\, dx$$

where, $f(x) \approx f_n(x)$

$$f_n(x) = a_0 + a_1 x + \dots + a_{n-1} x^{n-1} + a_n x^n$$

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Basis of Trapezoidal Rule

$$\int_a^b f(x)\,dx = (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

$= \frac{1}{2} \times$ (sum of the length of the parallel sides)

$\times$ (perpendicular distance between the parrallel sides)

for under area,

Area of trapezoid

$= \frac{1}{2}$ (sum of parallel sides) (height)

$= \frac{1}{2}\left(f(b) + f(a)\right)(b-a)$

$= (b-a)\left[\frac{f(a)+f(b)}{2}\right]$ → some approx, value of f in a,b

exam 1

$$x = \int_8^{30} \left(2000 \ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t\right) dt$$

t = 8 to t = 30 s

ⓐ Use single segment t. rule to find the distance covered.

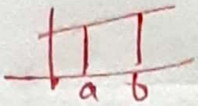
ⓑ Find true Error, E_t for ⓐ.

ⓒ Find abs. relative true Error, $|\epsilon_t|$.

$$f(x) = \int \left(2000 \ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t\right)$$

$a = 8$

$b = 30$



ⓐ $I = (b-a)\left[\frac{f(a)+f(b)}{2}\right]$

$f(a) = f(8) = 2000\ln\left[\frac{140000}{140000-2100(8)}\right] - 9.8(8)$

$= 2000\ln\left[\frac{140000}{140000-16800}\right] - 78.4$

$= 2000\ln(1.1363) - 78.4$

$= 255.554 - 78.4$

$= 177.154 \ ms^{-1}$

$f(30) = 2000\ln\left[\frac{140000}{140000-2100(30)}\right] - 9.8\times(30)$

$= 2000\ln\left[\frac{140000}{140000-63000}\right] - 294$

$= 2000\ln\left(\frac{140000}{77000}\right) - 294$

$= 2000\ \cancel{\ln(1.0471)} - 294$
0.5978

$= 119\cancel{5}.674 - 294$

$= 901.674 \ ms^{-1}$

$\therefore I = (30-8)\left[\frac{177.27 + 901.674}{2}\right]$

$= 22 \times 539.472$

$= 11868.384 \ m$

ⓑ True value,

$$x = \int_8^{30} \left(2000 \ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t\right) dt$$

$$= (30-8)\left[177.27\right.$$

$=$

$= \cancel{11868}$ m
11061

b E_t = True value − Previouse approximate value

$= 11061 - 11868 = -807$ m.

ⓒ $|\epsilon_t| = \left|\frac{11061 - 11868}{11061}\right| \times 100$

$= |-0.072959| \times 100$

$= 7.2959\%$

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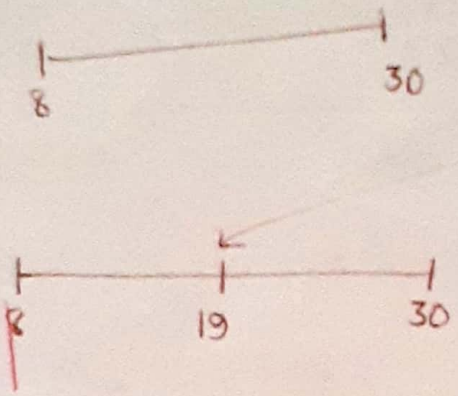
*** Important

multiple Segment Trapezoidal Rule

example | এই Rule টা large. এর error কমানোর জন্য double segment Trapezoidal rule use করব।

n = 2 $\qquad \frac{8+30}{2} = 19$

n = 3



so, we can divide the interval [8,30] into [8,19] and [19,30] intervals

$$\int_{8}^{30} f(t)\,dt = \int_{8}^{19} f(t)\,dt + \int_{19}^{30} f(t)\,dt$$

$$= (19-8)\left[\frac{f(8)+f(19)}{2}\right] + (30-19)\left[\frac{f(30)+f(19)}{2}\right]$$

$$= 11\left[\frac{177.27+484.75}{2}\right] + 11\left[\frac{901.67+484.75}{2}\right]$$

$$= 3641.11 + \cancel{8078.81}\ 7625.31$$

$$= 11266.42 \text{ m}$$

(b) $E_t = 11061 - 11266 = -205$ m.

The true error is reduced from -807 m to -205 m.

$$f(19) = 2000 \ln\left[\frac{140000}{140000 - 2100 \times 19}\right] - 9.8 \times 19$$

$$= 2000 \ln(1.3986) - 186.2$$

$$= 2000 \times 0.33547 - 186.2$$

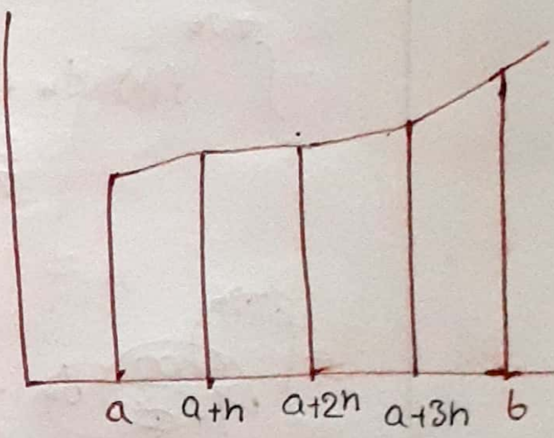
$$= 670.9454 - 186.2$$

$$= 484.74547 \ ms^{-1}$$

$f(30) = 901.67 \ ms^{-1}$

$f(8) = 177.27 \ ms^{-1}$ [from example 1]

if multiple is 4-segment



$n = 4$

$h = \frac{b-a}{n}$ → no. of segment

↳ mid value of a, b

→ 2 segment step: a, a+h, b

→ 4 segment step: a, a+h, a+2h, a+3h, b

multisegment এর ক্ষেত্রে Error, exact করা লাগবে না। but single seg. এ করা লাগবে with I

$I = \frac{1}{2} \cdot h \{f(a) + f(b)\}$

$$= \frac{1}{2} \cdot \frac{b-a}{n} \{f(a) + f(b)\}$$

$$= \frac{1}{2} \cdot \frac{b-a}{4} \cdot \{f(a) + f(b)\}$$

→ 3 segment $= \frac{b-a}{2 \cdot 3}\{f(a) + f(a+ih) + f(b)\}$

ADD

Question : $\int_7^{10} \frac{4000}{1+e^x} dx$

using 3 segment.

<u>solve:</u> for 3 seg.

a	a+h	a+2h	b
7	7.75	8.5	10

$\frac{10+7}{3} = \frac{17}{3}$

$n = 3$

$$h = \frac{b-a}{n} = \frac{10-7}{3} = \frac{3}{3} = 1$$

$$I = \frac{h}{2} \cdot \cancel{\frac{b-a}{2n}} \left\{ f(a) + 2\left\{ \sum_{i=1}^{n-1} f(a+ih) \right\} + f(b) \right\}.$$

$$= \frac{1}{2} \cdot \left\{ f(7) + 2f(7+1) + f(10) \right\}$$

$$= \frac{1}{2} \left\{ f(7) + 2f(8) + f(10) \right\}.$$

$$f(7) = \frac{4000}{1+e^7} = 3.6442$$

$$f(8) = \frac{4000}{1+e^8} = 1.3414$$

$$f(10) = \frac{4000}{1+e^{10}} = 0.1816$$

$$= \frac{1}{2} \left\{ (3.6442) + 2 \times (1.3414) + 0.1816 \right\}$$

$$= \frac{1}{2} \left\{ 6.50859 \right\}$$

$$= 3.254$$

But, we won't study error in multiple segment trapezoidal rule.

Multiple Segment Simpson's 1/3rd Rule.

যদি ২ segment এর বেশি simpson's $\frac{1}{3}$ rule হবে।

- n needs to be even.

hence the segment width,

$$\int_a^b f(x)\,dx = \int_{x_0}^{x_n} f(x)\,dx$$

where, $x_0 = a$, $x_n = b$

- this rule is an extension of Trapezoidal rule where the integrand is approximated by a second order polynomial.

hence, $I = \int_a^b f(x)\,dx$

$$\approx \int_a^b f_2(x)\,dx$$

where, $f_2(x) =$ is a second order polynomial.

$$\boxed{f_2(x) = a_0 + a_1 x + a_2 x^2}$$

here, we consider 3 point,

a, b, $\frac{a+b}{2}$.

so we get those equations —

evaluate a_0, a_1 and a_2

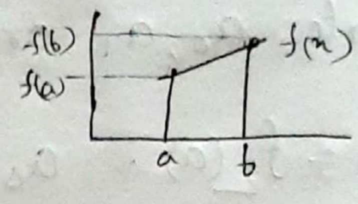
$$f(a) = f_2(a) = a_0 + a_1 \cdot a + a_2 \cdot a^2$$

$$f(b) = f_2(b) = a_0 + a_1 b + a_2 \cdot b^2$$

$$f\left(\frac{a+b}{2}\right) = f_2\left(\frac{a+b}{2}\right) = a_0 + a_1\left(\frac{a+b}{2}\right) + a_2\left(\frac{a+b}{2}\right)^2$$

Simpson's 1/3rd rule

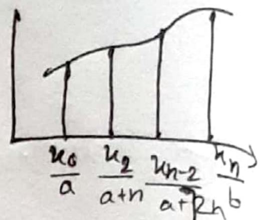
for single segment



$$x = \int_a^b f(x)\, dx$$

$$= \left(\frac{b-a}{6}\right)\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

for multiple segment / n segment.



$$\int_a^b f(x)\, dx$$

$$= \frac{b-a}{3.n}\left[f(x_0) + 4\sum_{\substack{i=1\\ i=odd}}^{n-1} f(x_i) + 2\sum_{\substack{i=2\\ i=even}}^{n-2} f(x_i) + f(x_n)\right]$$

math

* $\int_{0.2}^{2.2} e^x\, dx$ by using 2 segment simpson's 1/3 rule.

Solve: $a = 0.2$ $\quad n = 2$

$b = 2.2$ $\quad h = \frac{b-a}{n} =$

$$= \frac{2.2 - 0.2}{2}$$

$$= 1$$

$x_0 = 0.2$

$x_1 = x_0 + h = 0.2 + 1 = 1.2$

$x_2 = x_1 + h = 1.2 + 1 = 2.2$

$$\int_{0.2}^{2.2} e^x \, dx$$

$$= \frac{2.2 - 0.2}{3 \times 2} \left[f(0.2) + 4 \sum^{2-1 < 1} f(x_i) + 2 \sum^{2-2 < 0} f(x_i) + f(2.2) \right]$$

$$= 0.333 \left[f(0.2) + 4 \sum^{1} f(x_i) + 2 \sum_{0}^{0} f(x_i) + f(2.2) \right]$$

$$= 0.333 \left[f(0.2) + 4 f(1.2) + f(2.2) \right]$$

$$= 0.333 \left[e^{0.2} + 4 \times e^{1.2} + e^{2.2} \right]$$

$$= 0.333 \left[1.22 + 13.28 + 9.025 \right]$$

$$= 0.333 \times 23.525$$

$$= 7.833$$

Using one-segment trapezoidal rule →

$$\int_{0.2}^{2.2} xe^{x}\, dx$$

$$= (2.2 - 0.2)\left[\frac{f(0.2) + f(2.2)}{2}\right]$$

$$= 2\left[\frac{0.2 \times e^{0.2} + 2.2 \times e^{2.2}}{2}\right]$$

$$= 2\left[\frac{0.24428 + 19.855}{2}\right]$$

$$= 2 \times 10.0496$$

$$= 20.099 \text{ Ans}$$

* using 3 segment →

$$\int_{a}^{b} f(x)\, dx = \frac{b-a}{2n}\left[f(a) + 2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\} + f(b)\right]$$

$$= \frac{2.2-0.2}{2 \times 3}\left[f(0.2) + 2\left\{\sum_{i=1}^{3-1=2} f(0.2+ih)\right\} + f(2.2)\right]$$

$$= 0.333\left[f(0.2) + 2f(0.2+h) + 2f(0.2+2h) + f(2.2)\right]$$

$$h = \frac{2.2 - 0.2}{3} = 0.6667.$$

Cora-DX®

$= 0.333 \left[f(0.2) + 2f(0.8667) + 2f(1.5334) + f(2.2) \right)$

$= 0.333 \left(\dfrac{0.2\, e^{0.2}}{0.24428} + 2 \times 2.0618 + 2 \times 7.1048 + 19.855 \right.$

$= 0.333 \times 38.433$

$= 12.811$ Ans

$\int$

using 4 segment =

$a = 0.2$

$b = 2.2$

$n = 4$

$$h = \frac{b-a}{n} = \frac{2.2 - 0.2}{4} = 0.5.$$

$$\int_{0.2}^{2.2} e^x \, dx = \frac{2.2-0.2}{3 \times 4}\left[f(x_0) + 4\sum_{i=odd}^{4-1=3} f(x_i) + 2\sum_{even}^{4-2=2} f(x_i) + f(x_4) \right]$$

$$= 0.1667 \left[f(x_0) + 4f(x_1) + 4f(x_3) + 2f(x_2) + f(x_4) \right]$$

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so, $f(x) = e^{x}$

~~f(x_0) = e^{fl}~~

$x_0 = 0.2$

$x_1 = 0.2 + 0.5 = 0.7$

$x_2 = 0.7 + 0.5 = 1.2$

$x_3 = 1.2 + 0.5 = 1.7$

$x_4 = 1.7 + 0.5$

$= 2.2$.

$f(x_0) = e^{0.2} = 1.22$

$f(x_1) = e^{0.7} = 2.01$

$f(x_2) = e^{1.2} = 3.32$

$f(x_3) = e^{1.7} = 5.47$

$f(x_4) = e^{2.2} = 9.025$.

$= 0.1667 [1.22 + 4 \times 5.47 + 4 \times 2.01 + 2 \times 3.32 + 9.025]$

$= 0.1667 [46.806]$

$= 7.802$

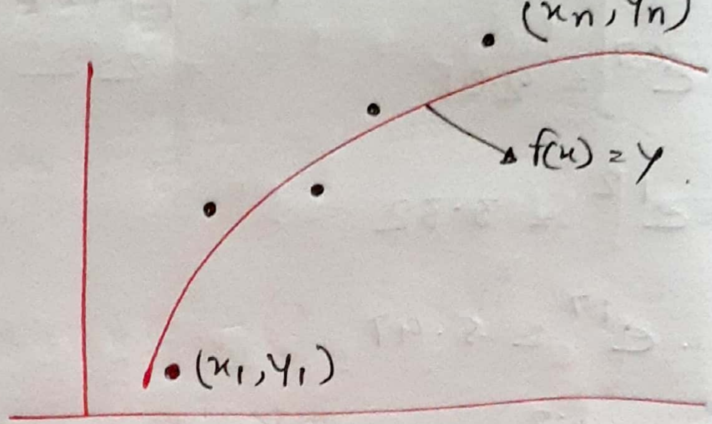
★ Regression

given, n data points হবে। যেখানে $y = f(x)$ হবে: এমন যেন Data best ফিট হয়,

best ফিট টা mainly minimizing the sum of the ~~residual~~ square of the residuals, S_r.

Residual at a point is,

$\varepsilon_i = y_i - f(x_i)$



and sum of square of the Residuals is

$$\boxed{S_r = \sum_{i=1}^{n} \left(y_i - f(x_i)\right)^2}$$

for linear regression - Criterion

$$y = a_0 + a_1 x = f(x)$$

$$\Rightarrow a_0 = y - a_1 x$$

$$\varepsilon_i = \underbrace{y_i}_{\text{observed value}} - \underbrace{(a_0 + a_1 x_i)}_{\text{Predicted value}}$$

$$a_1 = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$a_0 = \bar{y_i} - a_1 \overline{x_i}$$

Question

$T = k_1 + k_2 \theta$

$n = 5$

Table

Angle, θ Radians	Torque, T N-m	θ^2	θT
0.698132	0.188224	0.487388	0.1314
0.959931	0.209138	0.921468	0.20075
1.134464	0.230052	1.2870	0.2609
1.570796	0.250965	2.4674	0.3942
1.919862	0.313707	3.6859	0.60227
$\Sigma\theta =$ 6.2831	ΣT 1.1921	$\Sigma\theta^2$ 8.8491	$\Sigma\theta T$ 1.5896

$$k_2 = \frac{5\times(1.5896) - (6.2831)(1.1921)}{5\times(8.8491) - (6.2831)^2}$$

$$= \frac{7.948 - 7.49 = 0.458}{44.2455 - 39.4773 = 4.768}$$

$= 9.6091 \times 10^{-2}$ N-m/rad

$= 0.096$

now, use the average torque and angle to calculate k_1

$$\boxed{\bar{T}, \bar{\theta} \; ; \; k_1 = \bar{T} - k_2 \bar{\theta}}$$

$$\bar{T} = \frac{\sum_{i=1}^{5} T_i}{n} = \frac{1.1921}{5} = 2.3842 \times 10^{-1}$$

$$\bar{\theta} = \frac{\sum^{5} \theta_i}{n} = \frac{6.2831}{5} = 1.2566$$

$$\therefore k_1 = 2.3842 \times 10^{-1} - (9.6091 \times 10^{-2})(1.2566)$$

$$= 1.1767 \times 10^{-1} \text{ N-m}$$

Question : $x = 48$; $\frac{f(x)}{Y} = a_0 + a_1 x$

						$\sum_{i=1}^{n=5}$
x	1	5	25	37	48	116
Y	1	94	456	675	985	2211
$\Rightarrow xy$	1	470	11400	24975	47280	84126
x^2	1	25	625	1369	2304	4324

$$a_1 = \frac{n \times 84126 - (2211 \times 116)}{n \times 4324 - (116)^2}$$

$$= \frac{12 \times 84126 - 256476}{12 \times 4324 - 13456}$$

$$= \frac{753036}{38432}$$

$$= 19.59398$$

$$a_0 = \bar{y} - a_1 \bar{x}$$

$$= \frac{2211}{12} - 19.59 \times \frac{116}{12} = 184.25 - 189.37 = -5.12$$

Runge-kutta 2nd order Method

From taylor series,

$$y_{i+1} = y_i + f(x_i, y_i)(x_{i+1} - x_i) + \frac{1}{2!} f'(x_i, y_i)(x_{i+1} - x_i)^2 + \frac{1}{3!} f''(x_i, y_i)(x_{i+1} - x_i)^3 + \ldots$$

$y_{i+1} = y_i + f(x_i, y_i) h$; it's Euler's Method.

Rung-kutta has 3 method

1. Heun's
2. midpoint 3. Ralston

Heun's Method

$$y_{i+1} = y_i + \left(\frac{1}{2} k_1 + \frac{1}{2} k_2\right) h$$

where, $k_1 = f(x_i, y_i)$

$k_2 = f(x_i + h, y_i + k_i h)$

Midpoint

$\Rightarrow y_{i+1} = y_i + k_2 h$

where, $k_2 = f\left(x_i + \frac{1}{2} h, y_i + \frac{1}{2} k_1 h\right)$

$k_1 = f(x_i, y_i)$

Ralston

$\Rightarrow y_{i+1} = y_i + \left(\frac{1}{3} k_1 + \frac{2}{3} k_2\right) h$

$k_2 = f\left(x_i + \frac{3}{4} h, y_i + \frac{3}{4} k_1 h\right)$

Heno's = $y_{i+1} = y_i + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)h$

example

$\theta_1 = \theta_0 + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)h$

$$\frac{d\theta}{dt} = -2.2067 \times 10^{-12}\left(\theta^4 - 81 \times 10^8\right)$$

step 01: $i = 0,\ t_0 = 0,\ \theta_0 = \theta(0) = 1200k,\ h = 240.$

$$k_1 = f(t_0, \theta_0) = f(0, 1200)$$

$$= -2.2067 \times 10^{-12}\left(1200^4 - 81 \times 10^8\right)$$

$$= -4.5579.$$

$$k_2 = f(t_0 + h,\ \theta_0 + k_1 h)$$

$$= f(0 + 240,\ 1200 + (-4.5579)\,240)$$

$$= f(240,\ \underbrace{106.08}_{\theta_r})$$

$$= -2.2067 \times 10^{-12}\left(106.08^4 - 81 \times 10^8\right)$$

$$= 0.017595$$

$$\therefore \theta_1 = \theta_0 + \left(\tfrac{1}{2}k_1 + \tfrac{1}{2}k_2\right)h$$

$$= 1200 + \left\{\frac{1}{2}(-4.5579) + \tfrac{1}{2}(0.017595)\right\} 240$$

$$= 1200 + (-2.2702) \times 240$$

$$= 655.16\ k$$

Step 2: $i=1$, $t_1 = t_0 + h = 0 + 240 = 240$

$\theta_1 = 655.16\,k$

$k_1 = f(t_1, \theta_1)$

$= f(240, 655.16k)$

$= -2.2067 \times 10^{-12}(655.16^4 - 81 \times 10^8)$

$= -0.38869$

$k_2 = f(t_1 + h, \theta_1 + k_1 h)$

$= f(240+240, 655.16 + (-0.38869)\,240)$

$= f(480, \underbrace{561.87}_{\theta_2})$

$= -2.2067 \times 10^{-12} \times (561.87^4 - 81 \times 10^8)$

$= -0.20206$

$\theta_2 = \theta_1 + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)h$

$= 655.16 + \frac{h}{2}\left(-0.38869 - 0.20206\right)$ $h=240$

$= 655.16 + \frac{240}{2}(-0.59075)$

$= 655.16 - 70.89$

$= 584.79\,k$

Question :

$$3\frac{dy}{dx} + 5y^2 = \sin x \quad , \quad y(0.3) = 5$$

$y(x) = 5$

$h = 0.3$

Solve: rewritten, $\frac{dy}{dx} = \frac{1}{3}(\sin x - 5y^2)$

$$f(x,y) = \frac{1}{3}(\sin x - 5y^2)$$

Huen's method use,

$a_2 = \frac{1}{2}$ is chosen,

giving, $a_1 = \frac{1}{2}$, $p_1 = 1$, $q_{11} = 1$.

so, $\boxed{y_{i+1} = y_i + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)h}$

where, $k_1 = f(x_i, y_i)$

$k_2 = f(x_i + h, y_i + k_1 h)$

Step 1: for, $i = 0$, $x_0 = \boxed{0.3}$, $y_0 = 5$

$k_1 = f(x_0, y_0)$

$= f(0.3, 5)$

$= \frac{1}{3}(\sin 0.3 - 5\times 5^2)$

$= \frac{1}{3}(5.236\times 10^{-3} - 125)$

$= \frac{1}{3}\times(-124.99)$

$= -41.66$

$k_2 = f(x_0+h, y_0 + k_1 h)$

$= f(0.3+0.3,\ 5+(-41.66)\times 0.3)$

$= f(0.6,\ -7.4704)$

$= \frac{1}{3}\{\sin 0.6 - 5\cdot(-7.4704)^2\}$

$= \frac{1}{3}(0.56464 - 279.04)$

$= -92.824$

$y_1 = y_0 + \frac{h}{2}(k_1+k_2)$

$= 5 + \frac{0.3}{2}(-41.66 - 92.824)$

$= 5 + \frac{0.3}{2} \times (-134.484)$

$= 5 + (-20.1726)$

$= -15.1726$

$x_1 = x_0 + h = 0.3 + 0.3 = 0.6.$

giving $y(x_1) = y = (0.6) = y_1 = -15.17$

step 2 for $i=1$, $x_1 = 0.6$, $y_1 = -15.17$.

doing step 2

midpoint

<u>Question</u> $3\frac{dy}{dx} + 5\sqrt{y} = e^{0.1x}$, $y(0.3) = 5$.

$h = 0.3$.

$\Rightarrow \frac{dy}{dx} = \frac{1}{3}(e^{0.1x} - 5\sqrt{y}) = f(x,y)$. $\frac{dy}{dx} = (0.9)$.

using midpoint method, $a_2 = 1$.

$a_1 = 0$, $p_1 = \frac{1}{2}$, $q_{11} = \frac{1}{2}$

resulting in, $\boxed{y_{i+1} = y_i + k_2 h}$

where, $k_1 = f(x_i, y_i)$

$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h)$.

<u>step 1</u>: for $i = 0$, $x_0 = 0.3$, $y_0 = 5$, $h = 0.3$

$k_1 = f(x_0, y_0)$

$= \frac{1}{3}(e^{0.1\times0.3} - 5(\sqrt{5}))$

$= \frac{1}{3} \times (1.03 - 11.18)$

$= \frac{1}{3} \times (-10.15)$

$= -3.3834$

$k_2 = f\left(0.3 + \frac{0.3}{2}, 5 + \frac{-3.3834\times0.3}{2}\right)$

$= f(0.45, 4.49249)$

$= \frac{1}{3}(e^{0.1\times0.45} - 5\sqrt{4.49249})$

$= \frac{1}{3}(1.046 - 10.5977)$

$= -$ ~~3.1839~~ 3.1839

$y_1 = y_0 + k_2 h$

$= 5 + (-3.1839) \times 0.3$

$= \cancel{\theta}\ 4.0448$

$x_1 = x_0 + h = 0.3 + 0.3 = 0.6$

$y(0.6) = y_1 = -3.1839$.

step 2 -

for $i=1$, $x_1 = 0.6$, $y_1 = -3.1839$

$h = 0.3$.

- $k_1 = -2.9980$

- $k_2 = -2.8008$

$y_2 = y_1 + k_2 h$

$= 4.04483 + (-2.8008) \times 0.3$

$= 3.2046$

$x_2 = x_1 + h = 0.6 + 0.3 = 0.9$

$y(0.9) \approx y_2 = -3.2046$.

so, $\frac{dy}{dx}(0.9) = f(x,y)\Big|_{x=0.9}$

$\approx f(x_2, y_2)$

$= f(0.9, 3.2046)$

$= \frac{1}{3}\left(e^{0.1 \times 0.9} - 5\sqrt{3.2046}\right)$

$= \frac{1}{3}(1.0942 - 8.9507)$

$= -2.6188$

10.04.2013

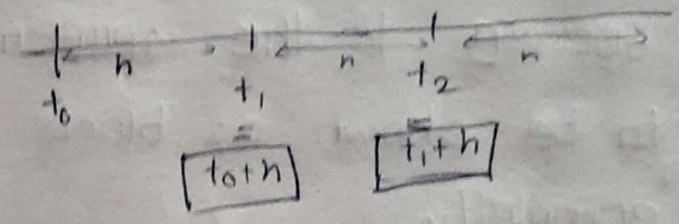
CSE 313 - Euler's Method [lec - 23]

$$y_{i+1} = y_i + f(x_i, y_i)\,h$$

where, $h = x_{i+1} - x_i$

□ iteration কিছু দেখা না থাকলে নিজে করবে step size and ~~temperature~~ time ধরে নেবে। $\text{itera} = \frac{T}{5.5}$

Q_1 at $t = t_1 = t_0 + h$



* 30 step size, $E_a = 2.2\%$

□ deri: second নিয়ম দেখে করবে, not come

Runge-kutta 2nd order [lec - 24]

□ বলে না দিলে ***Heun's method টা দিয়ে করবে, নয়তো Ralston's Method.

* 2nd order better for less error.

* which better?

A - Euler + graph + Heun's + graph + both equ^n + less error.

□ Practice all three term.

*** Theoretical Solution and Convergence. [lec - 25]