The upward velocity of a rocket is given as a function of time in the Table 1. Find the velocity at $t = ☻$ seconds using the **Lagrangian method** for Quadratic interpolation.

Table 1: Velocity as a function of time

t (s)	$v(t)$ (m/s)
8	227.04
36	1004.597
65.75	1902.249
95.5	2799.901
125.25	3697.553
155	4595.205
184.75	5492.857

Note: Please replace the value of t (☻) in the question with the addition of your roll number (e.g. xxxxxx**51**) and 10 (i.e. **51 + 10**).

How will you calculate the absolute relative approximate error $|\epsilon_a|$ obtained between the results from the first order (Linear interpolation) and second order (Quadratic interpolation) polynomial?

Note: You have to solve question **2. (a)** using the **Lagrangian method** for Linear interpolation to answer question **2. (b)**.

2(a)

~~Given~~

$t = (3+10)s$
$= 13\,s$

For our calculation, Given

$t_0 = 8$, $t_1 = 36$, $t_2 = 65.75$

$v(t_0) = 227.04$ m/s, $v(t_1) = 1004.597$ m/s

$v(t_2) = 1902.219$ m/s

We know that

According to Lagrangian Interpolation,

$$f_n(x) = \sum_{i=0}^{n} L_i(x)\, f(x_i)$$

where $L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x-x_j}{x_i-x_j}$

$(t-t_1) = (13-36)$
$= -23$

$(t_0-t_1) = (8-36)$
$= -28$

$(t-t_2) = (13-65.75)$
$= -52.75$

$(t_0-t_2) = (8-65.75)$
$= -57.75$

$(t-t_0) = (13-8)$
$= 5$

Now for quadratic interpolation $n = 2$

So, $v(t) = L_0(t)\,v(t_0) + L_1(t)\,v(t_1) + L_2(t)\,v(t_2)$

$(t_1-t_0) = (36-8)$
$= 28$

$(t-t_2)$
$= (13-65.75)$
$= -52.75$

(t_1-t_2)
$= (36-65.75)$
$= -29.75$

Here

$$L_0(t) = \left(\frac{t-t_1}{t_0-t_1}\right)\left(\frac{t-t_2}{t_0-t_2}\right)$$

$$= \left(\frac{-23}{-28}\right)\left(\frac{-52.75}{-57.75}\right)$$

$$= 0.7503$$

$$L_1(t) = \left(\frac{t-t_0}{t_1-t_0}\right)\left(\frac{t-t_2}{t_1-t_2}\right)$$

$$= \left(\frac{5}{28}\right)\left(\frac{-52.75}{-29.75}\right)$$

$$= 0.3166$$

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$$L_2(t) = \left(\frac{t-t_0}{t_2-t_0}\right)\left(\frac{t-t_1}{t_2-t_1}\right)$$

$$= \left(\frac{5}{57.75}\right)\left(\frac{-23}{29.75}\right)$$

$$= -0.0669$$

$$(t_2-t_0) = (65.75-8) = 57.75$$

$$(t_2-t_1) = (65.75-36) = 29.75$$

Now,

$$v(t) = L_0(t)\,v(t_0) + L_1(t)\,v(t_1) + L_2(t)\,v(t_2)$$

$$= (0.7503 \times 227.04) + (0.3166 \times 1004.597) + (-0.0669 \times 1902.219)$$

$$= 361.1431$$

(Ans)

2(b)

Given :

$v(t_0) = 227.04$ m/s

$v(t_1) = 1004.597$ m/s

For linear Interpolation :

$t = 13$ s

$t_0 = 8$ s, $t_1 = 36$ s

Now according to ~~linear~~ Lagrangian Interpolation,

For 1st order ~~interpo~~ interpolation :

$$v(t) = L_0(t)\,v(t_0) + L_1(t)\,v(t_1)$$

Here

$$L_0(t) = \frac{t - t_1}{t_0 - t_1} = \frac{-23}{-28} = 0.8214$$

$(t - t_1) = (13 - 36) = -23$

$(t_0 - t_1) = (8 - 36) = -28$

$$L_1(t) = \frac{t - t_0}{t_1 - t_0} = \frac{5}{28} = 0.1786$$

$(t - t_0) = (13 - 8) = 5$

$(t_1 - t_0) = (36 - 8) = 28$

$$\therefore v(t) = L_0(t)\,v(t_0) + L_1(t)\,v(t_1)$$

$$= (0.8214 \times 227.04) + (0.1786 \times 1004.597)$$

$$= 365.911$$

$\therefore$ approximate relative error, $|\varepsilon_a| = \left|\dfrac{\text{2nd order} - \text{1st order}}{\text{2nd order}}\right| \times 100$

$$= \left|\frac{361.1931 - 365.9117}{361.1931}\right| \times 100$$

$$\approx 0.013204184 \times 100$$

$$= 1.3204\ \%$$