example 01

$$V(t) = a_1 t^2 + a_2 t + a_3 \quad , \quad 5 \leq t \leq 12$$

time	velocity
5	106.8
8	177.2
12	279.2

solve -

using Matrix formate -

$$\begin{bmatrix} t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \\ t_3^2 & t_3 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

1/ the system of equations -

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

assume $\rightarrow \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

2/ now appling the initial guess to solve for a_i

$$a_1 = \frac{106.8 - 5a_2 - a_3}{25} = \frac{106.8 - 5\times 2 - 3}{25} = 3.7520$$

$$a_2 = \frac{177.2 - 64a_1 - a_3}{8} = \frac{177.2 - 64\times(3.7520) - 3}{8} = -8.241$$

$$a_3 = \frac{279.2 - 144a_1 - 12a_2}{1} = \frac{279.2 - 144\times(3.7520) - 12\times(-8.241)}{1} = -162.196$$

3 at the end of the 1st iteration -

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 3.7520 \\ -8.2410 \\ -162.1960 \end{bmatrix}$$

finding the abs.. relative appoximate error -

$$|\epsilon_a|_i = \left| \frac{x_i^{new} - x_i^{old}}{x_i^{new}} \right| \times 100$$

$$|\epsilon_a|_1 = \left| \frac{3.7520 - 1}{3.7520} \right| \times 100 = 73.34754797\ \%$$

$$|\epsilon_a|_2 = \left| \frac{-8.2410 - 2}{-8.2410} \right| \times 100 = 124.2688994\ \%$$

$$|\epsilon_a|_3 = \left| \frac{-162.1960 - 3}{-162.1960} \right| \times 100 = 101.849614\ \%$$

here, the maximum absolute relative approximate error is 125.47%,

iteration 2 -

using iteration 1 values of a_i.

$$a_1 = \frac{106.8 - 5(-8.2410) + 162.1960}{25} = 12.40804$$

$$a_2 = \frac{177.2 - 64(12.40804) + 162.1960}{8} = -56.83982$$

$$a_3 = \frac{279.2 - 144(12.40804) - 12(-56.82982)}{1} = -825.47992$$

at the end of 2nd iteration -

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 12.40804 \\ -56.83982 \\ -825.47992 \end{bmatrix}$$

3 abs. rela. app. error find -

$$|\epsilon_a|_1 = \left| \frac{12.40804 - 3.7520}{12.40804} \right| \times 100\% = 69.7615\%$$

$$|\epsilon_a|_2 = \left| \frac{-56.83982 + 8.2410}{-56.83982} \right| \times 100\% = 85.5013\%$$

$$|\epsilon_a|_3 = \left| \frac{-825.47992 + 162.1960}{-825.47992} \right| \times 100\% = 80.35130\%$$

here, the maximum abs. rela. app. error is 85.5013%.

iteration 3

using iter. 2 values of a_i →

$$a_1 = \frac{106.8 - 5(-56.83982) + 825.47992}{25} = 48.6591$$

$$a_2 = \frac{177.2 - 64(48.6591) + 825.47992}{8} = -263.93781$$

$$a_3 = \frac{279.2 - 144(48.6591) - 12(-263.9378)}{1} = -3560.4568$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

find abs. -

$$|\epsilon_a|_1 = \left| \frac{48.6591 - 12.4080}{48.6591} \right| \times 100\% = 74.5001\%$$

$$|\epsilon_a|_2 = \left| \frac{-263.93781 + 56.83982}{-263.93781} \right| = 78.4646\%$$

$$|\epsilon_a|_3 = \left| \frac{-3560.4568 + 825.47992}{-3560.4568} \right| = 76.8153\%$$

pitfall using

here, need one with a diagonally dominant coefficient matrix.

diagonally dominant = $[A]\,[X] = [C]$

Example:

$$12x_1 + 3x_2 - 5x_3 = 1$$
$$x_1 + 5x_2 + 3x_3 = 28$$
$$3x_1 + 7x_2 + 13x_3 = 76$$

solve: the coefficient matrix is diagonally dominant -

$$[A] = \begin{bmatrix} 12 & 3 & -5 \\ 1 & 5 & 3 \\ 3 & 7 & 13 \end{bmatrix}$$

$\boxed{|a_{11}| \geq |a_{12}| + |a_{13}|}$

$|a_{11}| = |12| = 12 \geq |a_{12}| + |a_{13}| = |3| + |-5| = 8$ - true

$|a_{22}| = |5| = 5 \geq |a_{21}| + |a_{23}| = |1| + |3| = 4$ - true

$|a_{33}| = |13| = 13 \geq |a_{31}| + |a_{32}| = |3| + |7| = 10$ - true

1 assume that, $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

2 $b_1 = \dfrac{1 - 3b_2 + 5b_3}{12} = \dfrac{1 - 3\times 0 + 5\times 1}{12} = -0.5000$

$b_2 = \dfrac{28 - b_1 - 3b_3}{5} = \dfrac{28 + 0.5000 - 3\times 1}{5} = 4.9000$

$b_3 = \dfrac{76 - 3b_1 - 7b_2}{13} = \dfrac{76 + 3\times(0.5000) - 7\times 4.9000}{13} = \cancel{3.26667}$. 3.0923.

$[b_1 \quad b_2 \quad b_3] = [0.5 \quad 4.9 \quad 3.0923]$

3. $|\epsilon_a|_1 = \left|\frac{0.5 - 1}{0.5}\right| \times 100\% = 100\%$

$|\epsilon_a|_2 = \left|\frac{4.9 - 0}{4.9}\right| = 100\%$

$|\epsilon_a|_3 = \left|\frac{3.0923 - 1}{3.0923}\right| = 67.6616\%$

here, the max abs. rela. error is 100%.

Iteration 02

$b_1 = \frac{\cancel{12} - 3\times4.9 + 5\times3.0923}{12} = 0.14679$

$b_2 = \frac{28 + 0.14679 - 3\times3.0923}{5} = 3.773978$

$b_3 = \frac{76 - 3\times0.14679 - 7\times3.7790}{13} = 3.7801$

3 $|\epsilon_a|_1 = \left|\frac{0.14679 - 0.5}{0.14679}\right| \times 100\% = 240.622$

$|\epsilon_a|_2 = \left|\frac{3.773978 - 4.9}{3.773976}\right| = 29.84\%$

$|\epsilon_a|_3 = \left|\frac{3.7801 - 3.0923}{3.7801}\right| = 18.2\%$

here, the max. abs. rela. error is 240.622 %

it-3

$[b_1 \quad b_2 \quad b_3] = [0.14679 \quad 3.77397 \quad 3.7801]$

iteration 03 :

$$b_1 = \frac{1 - 3\times 3.773978 - 3\times 3.7801}{12} = -1.8051$$

$$b_2 = \frac{28+1.8051 - 3\times 3.7801}{5} = 3.69296$$

$$b_3 = \frac{76 + 3\times 1.8051 - 7\times 3.69296}{13} = 4.274198462$$

$$|\epsilon_a|_1 = \left|\frac{-1.8051 - 0.14679}{-1.8051}\right| \times 100\% = 108.13\%$$

$$|\epsilon_a|_2 = \left|\frac{3.69296 - 3.77397}{3.69296}\right| \times 100\% = 2.19\%$$

$$|\epsilon_a|_3 = \left|\frac{4.2742 - 3.7801}{4.2742}\right| \times 100\% = 11.56\%$$

here, the max. abs. rela. error is 108.13%

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6 iteration values

[b_1 b_2 b_3] = [

□ Spring 2022

10R.@. $[a, b, c] = [1, 0, 1]$

$$\text{matrix } [A] = \begin{bmatrix} 12 & 2 & 5 \\ -11 & 5 & 2 \\ -3 & -6 & 13 \end{bmatrix}$$

Will the solution converge using the Gauss-Seidel method? explain.

Solue: here, we need to check a diagonally dominant coefficient Matrix.

using $|a_{11}| \geq |a_{12}| + |a_{13}|$ formula to check -

$|a_{11}| = |12| = 12 \quad \geq |2| + |5| = 2+5 = 7$ — true

$|a_{22}| = |5| = 5 \quad \geq |-11| + |2| = 11+2 = 13$ — false

$|a_{33}| = |13| = 13 \quad \geq |-3| + |-6| = 3+6 = 9$ — true.

here, one $|a_{22}|$ is not true.

that's why is not converge the method.

Scanned with CamScanner

1. (b) $[x, y, z] = [1, 0, 0]$

Solve: $20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$.

$$\begin{array}{ccc}[A] & [x] & [z]\end{array}$$

$$\begin{vmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 17 \\ -18 \\ 25 \end{vmatrix}$$

1. check it's diagonally dominant -

$|a_{11}| = |20| = 20 \geq |a_{12}| + |a_{13}| = |1| + |-2| = 1 + 2 = 3$ - true

$|a_{22}| = |20| = 20 \geq |a_{21}| + |a_{23}| = |3| + |-1| = 3 + 1 = 4$ - true

$|a_{33}| = |20| = 20 \geq |a_{31}| + |a_{32}| = |2| + |-3| = 2 + 3 = 5$ - true

iteration 01

2. $x = \dfrac{17 - 1y + 2z}{20} = \dfrac{17 - 0 + 0}{20} = 0.85$

$y = \dfrac{-18 - 3x + z}{20} = \dfrac{-18 - 3 \times 0.85}{20} = -1.0275$

$z = \dfrac{25 - 2x + 3y}{20} = \dfrac{25 - 2 \times 0.85 + 3 \times (-1.0275)}{20} = 1.0108$

$[x \quad y \quad z] = [0.85 \quad -1.0275 \quad 1.0108]$

3. $|\epsilon a|_x = \left| \dfrac{0.85 - 1}{0.85} \right| \times 100\% = 17.65\%$

$|\epsilon a|_2 = \left| \dfrac{-10275 - 0}{-1.0275} \right| = 100\%$

$|\epsilon a|_3 = \left| \dfrac{1.0108 - 0}{1.0108} \right| = 100\%$.

here, the max. abs. rela. error is 100%.