University of Asia Pacific

Department of Computer Science and Engineering

Course Code : CSE 313
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Assignment

Submitted by

Uniza Hossain Arpa
Registration No.: 19201061
Department: CSE

Submitted to

Rashik Rahman
Lecturer
Department of CSE, UAP

Bisection Method

Example 1:

$$0 \le x \le 2R$$
$$\Rightarrow 0 \le x \le 2(0.055)$$
$$\Rightarrow 0 \le x \le 0.11$$

$$f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-4}$$

Let's assume, $x_l = 0.00$
$x_u = 0.11$

$$\therefore f(x_l) = f(0) = 0^3 - 0.165 \times (0)^2 + 3.993 \times 10^{-4}$$
$$= 3.993 \times 10^{-4}$$
$$\therefore f(x_u) = f(0.11) = (0.11)^3 - 0.165(0.11)^2 + 3.993 \times 10^{-4}$$
$$= -2.662 \times 10^{-4}$$

~~So,~~ Hence,
$$f(x_l)\, f(x_u) = (3.993 \times 10^{-4})(-2.662 \times 10^{-4}) < 0$$

So, there is at least one root between x_l and x_u that is between 0 and 0.11

Iteration-1:

$$x_m = \frac{x_l + x_u}{2} = \frac{0 + 0.11}{2} = 0.055$$

$$f(x_m) = f(0.055)$$
$$= (0.055)^3 - 0.165(0.055)^2 + 3.993\times10^{-4}$$
$$= 6.655\times10^{-5}$$

$$f(x_l)f(x_m) = f(0)f(0.055)$$
$$= (3.993\times10^{-4})(6.655\times10^{-5}) > 0$$

Hence the root is bracketed between x_m and x_u, that is, between 0.055 and 0.11. So, the lower and upper limits of the new bracket are

$$x_l = 0.055 \quad \text{and} \quad x_u = 0.11$$

Iteration-2:

$$x_m = \frac{x_l + x_u}{2} = \frac{0.055 + 0.11}{2} = 0.0825$$

$$f(x_m) = f(0.0825) = (0.0825)^3 - 0.165(0.0825)^2 + 3.993\times10^{-4}$$
$$= -1.622\times10^{-4}$$

$$f(x_l)f(x_m) = f(0.055)f(0.0825)$$
$$= (-1.622\times10^{-4})\times(6.655\times10^{-5}) < 0$$

Hence the root is bracketed between x_l and x_m, that is, between 0.055 and 0.0825. So, the lower and upper limits of the new bracket are

$$x_l = 0.055, \quad x_u = 0.0825$$

The absolute relative approximate error,

$$|\epsilon_a| = \left|\frac{x_m^{new} - x_m^{old}}{x_m^{new}}\right| \times 100$$

$$= \left|\frac{0.0825 - 0.055}{0.0825}\right| \times 100$$

$$= 33.333\%$$

Iteration -3 :

$$x_m = \frac{x_l + x_u}{2} = \frac{0.055 + 0.0825}{2} = 0.06875$$

$$f(x_m) = f(0.06875) = (0.06875)^3 - 0.165(0.06875)^2 + 3.993 \times 10^{-4}$$

$$= -5.563 \times 10^{-5}$$

$$f(x_l)f(x_m) = f(0.055)\,f(0.06875) = (6.655 \times 10^{-5})(-5.563 \times 10^{-5}) < 0$$

Hence the root is bracketed between x_l and x_m, that is between 0.055 and 0.06875. So, the ~~root~~ lower and upper limits are,

$$x_l = 0.055 \quad \text{and} \quad x_u = 0.06875$$

The absolute relative approximate error,

$$|\epsilon_a| = \left|\frac{0.06875 - 0.0825}{0.06875}\right| \times 100$$

$$= 20\%$$

Newton Raphson Method

Ex. 1

$$f(x) = x^3 - 0.165x^2 + 3.993\times10^{-4}$$

$$f'(x) = 3x^2 - 0.33x$$

$$x_0 = 0.05$$

Iteration-1 :

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.05 - \frac{(0.05)^3 - 0.165(0.05)^2 + 3.993\times10^{-4}}{3(0.05)^2 - 0.33(0.05)}$$

$$= 0.05 - \frac{1.18\times10^{-4}}{-9\times10^{-3}}$$

$$= 0.05 - (-0.01242) = 0.06242$$

$$|\epsilon_a| = \left|\frac{x_1 - x_0}{x_1}\right| \times 100\,\%$$

$$= \left|\frac{0.06242 - 0.05}{0.06242}\right| \times 100 = 19.90\%$$

Iteration-2 :

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.06242 - \frac{(0.06242)^3 - 0.165(0.06238)^2 + 3.993\times10^{-4}}{3(0.06238)^2 - 0.33(0.06238)}$$

$$= 0.06238 - \frac{4.44\times10^{-11}}{-8.9117\times10^{-3}}$$

$$= 0.06238$$

$$|\epsilon_a| = \left|\frac{x_2 - x_1}{x_2}\right| \times 100$$

$$= \left|\frac{0.06238 - 0.06238}{0.06238}\right| \times 100$$

$$= 0\%$$

~~Secant Method~~.

Secant Method

Ex: 1

$$f(x) = x^3 - 0.165x^2 + 3.993\times10^{-4}$$

$x_{-1} = 0.02$ and $x_0 = 0.05$

Iteration-1:

$$x_1 = x_0 - \frac{f(x_0) \cdot (x_0 - x_1)}{f(x_0) - f(x_{-1})}$$

$$= 0.05 - \frac{\left((0.05)^3 - 0.165(0.05)^2 + 3.993\times10^{-4}\right)(0.05 - 0.02)}{\left((0.05)^3 - 0.165(0.05)^2 + 3.993\times10^{-4}\right) - \left((0.02)^3 - 0.165(0.02)^2 + 3.993\times10^{-4}\right)}$$

$$= 0.06461$$

$$|\epsilon_a| = \left|\frac{x_1 - x_0}{x_1}\right| \times 100$$

$$= \left|\frac{0.06461 - 0.05}{0.06461}\right| \times 100$$

$$= 22.62\%$$

Iteration-2:

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$= 0.06461 - \frac{(0.06461^3 - 0.165(0.06461)^2 + 3.993\times10^{-4})(0.0641 - 0.05)}{(0.06461^3 - 0.165(0.06461)^2 + 3.993\times10^{-4}) - (0.05^3 - 0.165(0.05)^2 + 3.993\times10^{-4})}$$

$$= 0.06241$$

$$|\epsilon_a| = \left|\frac{x_2 - x_1}{x_2}\right| \times 100$$

$$= \left|\frac{0.06241 - 0.06461}{0.06241}\right| \times 100$$

$$= 3.525\%$$

Iteration-3:

$$x_3 = x_2 - f(x_2)(x_2 - x_1)$$

$$= 0.06241 - \frac{(0.06241^3 - 0.165(0.0624)^2 + 3.993\times10^{-4})(0.0624 - 0.0646)}{0.06241^3 - 0.165(0.06241)^2 + 3.993\times10^{-4}) - (0.05^3 - 0.165(0.06461)^2 + 3.993\times10^{-4})}$$

$$= 0.06238$$

$$|\epsilon_a| = \left|\frac{x_3 - x_2}{x_3}\right| \times 100$$

$$= \left|\frac{0.06238 - 0.06241}{0.06238}\right| \times 100$$

$$= 0.0595\%$$

Gauss-Seidal Method

Ex-1:

$$v(t) = a_1 t^2 + a_2 t + a_3 \qquad 5 \le t \le 12$$

Using a matrix template of a form

$$\begin{bmatrix} t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \\ t_3^2 & t_3 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

Assume an inital gauss of

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

Rewriting each equation,

$$a_1 = \frac{106.8 - 5a_2 - a_3}{25}$$

$$a_2 = \frac{177.2 - 64a_1 - a_3}{8}$$

$$a_3 = \frac{279.2 - 144a_1 - 12a_2}{1}$$

Applying the initial guass and solving a_1

$$a_1 = \frac{106.8 - 5(2) - (5)}{25} = 3.6720$$

$$a_2 = \frac{177.2 - 64(3.6720) - (5)}{8} = -7.8510$$

$$a_3 = \frac{279.2 - 144(3.6720) - 12(-7.8510)}{1} = -155.36$$

$$|\epsilon_a|_i = \left| \frac{x_i^{new} - x_i^{old}}{x_i^{new}} \right| \times 100$$

$$\therefore |\epsilon_a|_1 = \left| \frac{3.6720 - 1}{3.6720} \right| \times 100 = 72.76\%$$

$$\therefore |\epsilon_a|_2 = \left| \frac{-7.8310 - 2}{-7.8510} \right| \times 100 = 125.47\%$$

$$\therefore |\epsilon_a|_3 = \left| \frac{-155.36 - 5}{-155.36} \right| \times 100 = 103.22\%$$

At the end of first iteration,

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 3.6720 \\ -7.8510 \\ -155.36 \end{bmatrix}$$

<u>Iteration-2:</u>

$$a_1 = \frac{106.8 - 5(-7.8510) - 155.36}{25}$$

$$= 12.056$$

$$a_2 = \frac{177.2 - 64(12.056) - 155.36}{8}$$

$$= -54.882$$

$$a_3 = \frac{279.2 - 144(12.056) - 12(-54.882)}{1}$$

$$= -798.34$$

$$|\epsilon_a|_1 = \left| \frac{12.056 - 3.6720}{12.056} \right| \times 100 = 69.542\%$$

$$|\epsilon_a|_2 = \left| \frac{-54.882 - (-7.8510)}{-54.882} \right| \times 100 = 85.695\%$$

$$|\epsilon_a|_3 = \left| \frac{-798.34 - (-155.36)}{-798.34} \right| \times 100 = 80.540\%$$

Direct Method

Ex-1:

$$v(t) = a_0 + a_1 t$$

$$v(15) = a_0 + 15a_1 = 362.78$$

$$v(20) = a_0 + 20a_1 = 517.35$$

Solving the above two equation give,

$$a_0 = -100.93 \quad , \quad a_1 = 30.914$$

Hence,

$$v(t) = -100.93 + 30.914t$$

$$v(16) = -100.93 + 30.914 \times 16 = 393.7 \text{ m/s}$$

Ex-2:

$$v(t) = a_0 + a_1 t + a_2 t^2$$

$$v(10) = a_0 + 10\,a_1 + 10^2 a_2 = 227.04$$

$$v(15) = a_0 + 15a_1 + 15^2 a_2 = 362.78$$

$$v(20) = a_0 + 20a_1 + 20^2 a_2 = 517.35$$

Solving above three eqn gives

$$a_0 = 12.05 \qquad a_1 = 17.733 \qquad a_2 = 0.3766$$

$$v(t) = 12.05 + 17.73t + 0.3766t^2$$

$$v(16) = 12.05 + 17.73(16) + 0.3766(16) = 392.19 \text{ m/s}$$

$$|\epsilon_a| = \left|\frac{392.19 - 393.70}{392.12}\right| \times 100$$

$$= 3841\,\%$$

Newton Divided Difference Method

Ex.-1:

$$v(t) = b_0 + b_1 (t - t_0)$$

$$t_0 = 15, \quad v(t_0) = 362.78$$

$$t_1 = 20, \quad v(t_1) = 517.35$$

$$b_0 = v(t_0) = 362.78$$

$$b_1 = \frac{v(t_1) - v(t_0)}{t_1 - t_0} = 30.914$$

$$v(t) = b_0 + b_1 (t - t_0)$$

$$= 362.78 + 30.914\,(t - 15)$$

$$t = 16 \quad v(t) = 362.78 + 30.914\,(16 - 15)$$

$$= 393.69$$

$$t(x) = b_0 + b_1 (x - x_0) + b_2 (x - x_0)(x - x_1)$$

$$b_0 = t(x_0)$$

$$b_1 = \frac{t(x_1) - t(x_0)}{x_1 - x_0}$$

Ex:

$$t_0 = 10 \quad v(t_0) = 227.04$$

$$t_1 = 15 \quad v(t_1) = 362.78$$

$$t_2 = 20 \quad v(t_2) = 517.35$$

$$b_0 = v(t_0) = 227.04$$

$$b_1 = \frac{v(t_1) - v(t_0)}{t_1 - t_0} = \frac{362.78 - 227.04}{15 - 10} = 27.148$$

$$b_2 = \frac{\frac{v(t_2) - v(t_1)}{t_2 - t_1} - \frac{v(t_1) - v(t_0)}{t_1 - t_0}}{t_2 - t_0}$$

$$= \frac{\frac{517.35 - 362.78}{20-15} - \frac{362.78 - 227.04}{15-10}}{20-10}$$

$$= \frac{30.914 - 27.148}{10} = 0.37660$$

$$v(t) = b_0 + b_1(t - t_0) + b_2(t - t_0)(t - t_1)$$

$$= 227.04 + 27.148(t - t_0) + 0.37660(t - \cancel{t_0})(t - 15)$$

$$v(10) = 227.04 + 27.148(16-10) + 0.37660(16-10)(16-15)$$

$$= 392.19$$

$$|E_a| = \left|\frac{392.19 - 393.69}{392.19}\right| \times 100$$

$$= 0.38502\ \%$$

Langrange Method

Ex-1:

$t_0 = 15 \qquad v(t_0) = 362.78$

$t_1 = 20 \qquad v(t_1) = 517.35$

$$L_0(t) = \prod_{j=0}^{1} \frac{t - t_j}{t_0 - t_j} = \frac{t - t_1}{t_0 - t_1}$$

$$L_1(t) = \frac{t - t_0}{t_1 - t_0}$$

$$v(t) = \frac{t - t_1}{t_0 - t_1} \cdot v(t_0) + \frac{t - t_0}{t_1 - t_0} v(t_1)$$

$$= \frac{t - 20}{15 - 20}(362.78) + \frac{t - 15}{20 - 15}(517.35)$$

$$v(16) = \frac{16-20}{15-20}(362.78) + \frac{16-15}{20-15}(517.35)$$

$$= 0.8(362.78) + 0.2(517.35)$$

$$= 393.7 \text{ m/s}$$

Ex-2:

$t_0 = 10 \qquad v(t_0) = 227.04$

$t_1 = 15 \qquad v(t_1) = 362.78$

$t_2 = 20 \qquad v(t_2) = 517.35$

$$v(t) = \left(\frac{t-t_1}{t_0-t_1}\right)\left(\frac{t-t_2}{t_0-t_2}\right)v(t_0) + \left(\frac{t-t_0}{t_1-t_0}\right)\left(\frac{t-t_2}{t_1-t_2}\right)v(t_1) + \left(\frac{t-t_0}{t_2-t_0}\right)\left(\frac{t-t_1}{t_2-t_1}\right)v(t_2)$$

$$v(16) = \left(\frac{16-15}{10-15}\right)\left(\frac{16-20}{10-20}\right)(227.04) + \left(\frac{16-10}{15-10}\right)\left(\frac{16-20}{15-20}\right)(362.78) + \left(\frac{16-10}{20-10}\right)\left(\frac{16-15}{20-15}\right)(517.35)$$

$$= 392.19 \text{ m/s}$$

$$|\epsilon_a| = \left|\frac{392.19 - 393.70}{392.19}\right| \times 100$$

$$= 0.384\%$$

Trapezodial rule

Ex-1:

a) $f(t) = 2000 \ln \left[\frac{140000}{140000 - 2100t}\right] - 9.8t$

$f(8) = 2000 \ln \left[\frac{140000}{140000 - 2100(8)}\right] - 9.8 \times 8 = 177.27$

$f(30) = 2000 \ln \left[\frac{140000}{140000 - 2100 \times 30}\right] - 9.8 \times 30 = 901.7$

$I = (b-a) \left[\frac{f(a) - f(b)}{2}\right] \cdot \left[\begin{matrix} a = 8 \\ b = 30 \end{matrix}\right]$

$= (30-8) \left[\frac{177.27 + 901.67}{2}\right]$

$= 11868 \text{ m}$

b) the exact value,

$\lambda = \int_{8}^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t}\right] - 9.8t\right) dt$

$= 11061 \text{ m}$

$E_t = 11061 - 11868 = -807 \text{ m}$

c) $|\varepsilon_t| = \left|\frac{11061 - 11868}{11061}\right| \times 100 = 7.29\%$

Simpson 1/3 rule

Ex-1:

a) $x = \int_8^{30} f(t)\,dt$

$x = \left(\frac{b-a}{6}\right)\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

$= \frac{30-8}{6}\left[f(8) + 4f(19) + f(30)\right]$

$= 22/6\left[177.2667 + 4(484.7455) + 901.67\right)$

$= 11065.72\ m$

b) $x = \int_8^{30} \left(2000\ln\left[\frac{140000}{140000-2100t}\right] - 9.8t\right) dt$

$= 11061.34\ m$

$E_t = 11061.34 - 11065.72 = -4.38\ m$

c) $|\epsilon_t| = \left|\frac{11061.34 - 11065.72}{11061.34}\right| \times 100$

$= 0.0396\%$

Lineare Regression

Ex-1:

θ	T	θ^2	$T\theta$
0.698132	0.188224	0.4874	0.131405
0.959931	0.209138	0.921468	0.200758
1.134464	0.230052	1.2870	0.260986
1.570796	0.250965	2.4674	0.394215
1.919862	0.313707	3.6859	0.682274
$\Sigma =$ 6.2831	1.1921	8.8491	1.5896

$$k_2 = \frac{n\sum_{i=1}^{5}\theta_i T_i - \sum_{i=1}^{5}\theta_i \sum_{i=1}^{5} T_i}{n\sum_{i=1}^{5}\theta^2 - \left(\sum_{i=1}^{5}\theta\right)^2}$$

$$= \frac{5(1.5896)-(6.2831)(1.1921)}{5(8.8491)-(6.2831)^2}$$

$$= 9.6091\times10^{-2} \text{ N-m/rad}$$

$$\bar{T} = \frac{\sum^{5} T}{n} = \frac{1.1921}{5} = 2.3842\times10^{-1}$$

$$\bar{\theta} = \frac{\sum^{5}\theta}{n} = \frac{6.2831}{5} = 1.2566$$

$$k_1 = \bar{T} - k_2\bar{\theta} = 2.3842\times10^{-1} - (9.6091\times10^{-2})(1.2566)$$

$$= 1.1767\times10^{-1} \text{ N-m}$$

Eulerc's Method

Ex-1:

Step-1:

$$\frac{d\theta}{dt} = -2.2067 \times 10^{-12} (\theta^4 - 81 \times 10^8)$$

$$f(t, \theta) = -2.2067 \times 10^{-12} (\theta^4 - 81 \times 10^8)$$

$$\theta_{i+1} = \theta_i + f(t_i, \theta_i) h$$

$$\theta_1 = \theta_0 + f(t_0, \theta_0) h$$

$$= 1200 + f(0, 1200) \cdot 240$$

$$= 1200 + (-2.2067 \times 10^{-12} (1200^4 - 81 \times 10^8)) 240$$

$$= 106.09 K$$

θ_1 is the approximate temperature at ~~$t = t_1$~~

$$t = t_1 = t_0 + h = 0 + 240 = 240$$

$$\theta(240) = \theta_1 = 106.09 K$$

Step-2:

For $i = 1$, $t_1 = 240$, $\theta_1 = 106.09$

$$\theta_2 = \theta_1 + f(t_1, \theta_1) h$$

$$= 106.09 + f(240, 106.09) 240$$

$$= 106.09 + (-2.2067 \times 10^{-12} (106.09^4 - 81 \times 10^8)) 240$$

$$= 110.32 K$$

θ_2 is approx temperature at

$$t = t_2 = t_1 + h = 240 + 240 = 480$$

$$\theta(480) \approx \theta_2 = 110.32\,K$$

The exact solution of the ordinary differential equation is given by the solution of a non-linear equation. as

$$0.92593 \ln \frac{\theta - 300}{\theta + 300} - 1.8519 \tan^{-1}(0.003333\theta)$$

$$= -0.22067 \times 10^{-3}$$

The solution to this non-linear equation at $t = 480\,s$ is

$$\theta(480) = 647.57\,K$$

Runge-Kutta 2nd Order

Step-1:

$$i = 0, \quad t_0 = 0, \quad \theta_0 = \theta(0) = 1200K$$

$$K_1 = f(t_0, \theta_0)$$
$$= f(0, 1200)$$
$$= -2.2067 \times 10^{-12}(1200^4 - 81 \times 10^8)$$
$$= -4.5579$$

$$K_2 = f(t_0 + h, \theta_0 + k_1 h)$$
$$= f(0 + 240, 1200 + (-4.5579)240)$$
$$= f(240, 106.09)$$
$$= -2.2067 \times 10^{-12}(106.09^4 - 81 \times 10^8) = 0.017595$$

$\theta_1 = \theta_0 + (\frac{1}{2}K_1 + \frac{1}{2}K_2)h$

$= 1200 + (\frac{1}{2}(-4.5579) + \frac{1}{2}(0.017595))\,240$

$= 1200 + (-2.2702)\,240$

$= 655.16\text{ K}$

Step-2:

$i = 1,\ t_1 = t_0 + h = 0 + 240 = 240$

$\theta_1 = 655.16\text{ K}$

$K_1 = f(t_1, \theta_1)$

$= f(240, 655.16)$

$= -2.2067 \times 10^{-12}(655.16^4 - 81 \times 10^8)$

$= -0.38869$

$k_2 = f(t_1 + h,\ \theta_1 + K_1 h)$

$= f(240 + 240,\ 655.16 + (-0.38869)\,240)$

$= f(480,\ 561.87)$

$= -2.2067 \times 10^{-12}(561.87^4 - 81 \times 10^8)$

$= -0.20206$

$\theta_2 = \theta_1 + (\frac{1}{2}K_1 + \frac{1}{2}K_2)h$

$= 655.16 + (\frac{1}{2}(-0.38869) + \frac{1}{2}(-0.20206))\,240$

$= 655.16 + (-0.029538)\,240$

$= 584.27\text{ K}$

The exact solution of the ordinary, differential equation is given by the solution of a non-linear equation as,

$$0.92593 \ln \frac{\theta-300}{\theta+300} - 1.8519 \tan^{-1}(0.0033338\,\theta) = -0.22067\times10^{-3} t - 2.9282$$

The solution to this equation at $t=480$ s is

$$\theta(480) = 647.57 \text{ K}$$