# Lecture 05 = Hash methods

- Division Method → $k = 1276$, $n = 10$
- Multiplication Method.

$h(1276) = 1276 \bmod 10 = 6$

$$\boxed{h(k) = k \bmod n}$$

↳ $h(k) = \text{floor}(n\,(kA \bmod 1))$

given, $k = 123$, $n = 100$, $A = 0.618033$

$h(123) = (100\,(123 \times 0.618033 \bmod 1))$

$= 100 \times (76.018059 \bmod 1))$

$= 100 \times (0.018059)$

$= 1$.

floor(6.1) = 7

floor(6.4) = 4

## Collisions Solution : Linear Probing.

$$h(k,i) = (h'(k) + i) \bmod m$$

exam: $m = 11$,

key( 20, 2)

$20 \bmod 11 = 9$

$2 \bmod 11 = 2$

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 2 4 |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | 20 4 |
| 10 | |

## Collisions Solutions : Quadratic Probing.

- $h(k,i) = (h'(k) + i^2) \bmod m$

$h'(k) = k \bmod m$

## Double Probing

$$h(k,i) = (h_1(k) + i h_2(k) \bmod m)$$

where, $h_1(k) = k \bmod m$

$h_2(k) = 7 - (k \bmod 7)$

example - Hash function

given, key = 58, 14, 91, 69, 80, 102, 25, 113, 124, 13

table size = 12 (0 - 11)

Solve = 58 mod 12 = 10

14 mod 12 = 2

91 mod 12 = 7

69 mod 12 = 9

80 mod 12 = 8

102 mod 12 = 6

25 mod 12 = 1

113 mod 12 = 5

124 mod 12 = 4

13 mod 12 = 1

| | |
|---|---|
| 0 | |
| 1 | 25 |
| 2 | 14 |
| 3 | 13 |
| 4 | 124 |
| 5 | 113 |
| 6 | 102 |
| 7 | 91 |
| 8 | 80 |
| 9 | 69 |
| 10 | 58 |
| 11 | |

$h_1(13) = 1$

$h_2(13) = 7 - (13 \bmod 7)$

$= 7 - 6$

$\boxed{= 1}$

$\therefore h(13, 1) = (1 + (1 \times 1 \bmod m))$

$h(k, i) = (1 + (1 \bmod 12))$

$= 1 + 1$

$= 2$

again, $h(13, 2) = (1 + 2 \times 1 \bmod 12)$

$= (1 + 2)$

$= 3$

Problem : Solve the characteristic equation giving us the eigenvalues, $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$.

Solve: $\left| \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$

$\Rightarrow \left| \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$

$\Rightarrow \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow (1-\lambda)\,[(1-\lambda)^2 - 1] - 1\,[(1-\lambda) - 1] + 1\,[1 - 1 - \lambda)] = 0$

$\Rightarrow (1-\lambda)(1 - 2\lambda + \lambda^2 - 1) + \lambda + \lambda = 0$

$\Rightarrow (1 - \lambda - 2\lambda + 2\lambda^2 + \lambda^2 - \lambda^3 - 1 + \lambda + \lambda + \lambda) = 0$

$\Rightarrow 3\lambda^2 - \lambda^3 = 0$

$\Rightarrow \lambda^3 - 3\lambda^2 = 0$

$\Rightarrow \lambda^2(\lambda - 3) = 0$

$\therefore \lambda^2 = 0 \qquad \lambda - 3 = 0$

$\lambda = 0, 0 \qquad \lambda = 3$

$\boxed{\lambda = 0, 0, 3}$

$\lambda = 0$ is $[A - 0I]\,x = 0$.

where $x$ is column matrix of order 3, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

so, implies that, $x + y + z = 0$.

let, $z = 0$, $y = 1$, then, $x = -1$. | $z = 1$, $y = 1$. $x = -2$.

Thus, eigenvalue $\lambda = 0$ are $(-1, 1, 0)$ and $(-2, 1, 1)$.

and $\lambda = 3$ is $[A - 3I]X = 0$.

$\therefore [A - 3I]\, X = 0$.

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$$\rightarrow \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\rightarrow \boxed{\begin{aligned} -2x + y + z &= 0 \\ x - 2y + z &= 0 \\ x + y - 2z &= 0. \end{aligned}}$$

$$\begin{vmatrix} -2x & y & z \\ x & -2y & z \\ x & y & -2z \end{vmatrix} = 0$$

$\rightarrow -2x(4yz - yz) - y(-2xz - xz) + xy + 2xy$

$\rightarrow -2x \cdot (3yz) - y(-3xz) + 3xy$

$\rightarrow -6xyz + 3xyz + 3xyz$

$\rightarrow 0$.

$\rightarrow \cancel{-2x(}4yz$

Taking last 2 equation;

$$\frac{x}{4-1} = -\frac{y}{-2-1} = \frac{z}{1+2}$$

$$\rightarrow \frac{x}{3} = \frac{y}{3} = \frac{z}{3}$$

$\lambda = 3$. given,

$$\boxed{(x, y, z) = (3, 3, 3)}$$

Problem 3: $A = \begin{vmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 0$

$\Rightarrow [A - \lambda I] = 0$

$\rightarrow \begin{vmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$

$\rightarrow \begin{vmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$

$\rightarrow (2-\lambda)(-15-3\lambda+5\lambda+\lambda^2) + 3(6-2\lambda) + 0 = 0$

$\rightarrow -30 + 15\lambda - 6\lambda + \cancel{3\lambda^2} + 10\lambda - \cancel{5\lambda^2} + \cancel{2\lambda^2} - \lambda^3 + 18 - 6\lambda = 0$

$\rightarrow -12 + 13\lambda - \lambda^3 = 0$

$\rightarrow \lambda^3 - 13\lambda + 12 = 0$

$\rightarrow \lambda^3 - \lambda^2 + \lambda^2 - \lambda - 12\lambda + 12 = 0$

$\rightarrow \lambda^2(\lambda-1) + \lambda(\lambda-1) - 12(\lambda-1) = 0$

$\rightarrow (\lambda-1)(\lambda^2+\lambda-12) = 0$

$\rightarrow (\lambda-1)(\lambda^2+4\lambda-3\lambda-12) = 0$

$\rightarrow (\lambda-1)(\lambda(\lambda+4) - 3(\lambda+4)) = 0$

$\rightarrow (\lambda-1)(\lambda-3)(\lambda+4) = 0$

$\rightarrow \lambda = 1, 3, -4$ , 2

eigen vector for $\lambda = 2$ is $(A - \lambda I)\bar{x} = 0$.

wrong work.

$$\rightarrow \left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right| = 0.$$

$$\rightarrow \begin{vmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{vmatrix} - \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$$\rightarrow \begin{vmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{vmatrix} - \begin{vmatrix} 2x & 0 & 0 \\ 0 & 2y & 0 \\ 0 & 0 & 2z \end{vmatrix} = 0.$$

$$\rightarrow \begin{vmatrix} 2-2x & -3 & 0 \\ 2 & -5-2y & 0 \\ 0 & 0 & 3-2z \end{vmatrix} = 0$$

$\rightarrow 2 - 2x - 3y = 0$ — ①

$2 - 5 - 2y = 0$ — (ii)

$3 - 2z = 0$ — (iii)

$\rightarrow$ (iii) $z = 2/3$

(ii) $y = -2/3$

(I) $2x = 2 - 3 \cdot (-\frac{2}{3})$

$= 2 + 2 = 4$

$\therefore x = 2.$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2/3 \\ 2/3 \end{bmatrix}$$

do as same as calculation for $\lambda = 1, 3, -4$.

Topic - Poisson Distribution

- Simeon Denis Poisson

- Probability of $x$, $f(x) = P(X=x) = \left(\frac{e^{-\lambda} \cdot \lambda^{x}}{x!}\right)$

where, $\lambda$ = average rate of value and $\lambda$ = variance.

$x = 0, 1, 2, \ldots$

$e$ = euler's number $= 2.718$

$e^{-\lambda} = \frac{1}{e^{\lambda}}$

Problem 01: $\lambda = 2.5$ goals, $k = ?$

Solve: $P(x=0) = \frac{2.5^{0}}{0! \cdot e^{-2.5}} = 0.082$

$P(x=1) = \frac{2.5^{1}}{1! \cdot e^{-2.5}} = 0.205$

$P(x=2) = \frac{2.5^{2}}{2! \, e^{-2.5}} = 0.257$

$P(x=3) = \frac{2.5^{3}}{3! \cdot e^{-2.5}} = 0.213$

$P(x=4) = \frac{2.5^{4}}{4! \cdot e^{-2.5}} = 0.133$

$P(x=5) = \frac{2.5^{5}}{5! \cdot e^{-2.5}} = 0.066$

And so on up to $X = k$.

Problem 02: average of 1.6 cars

probability that 3 or more cars.

$e = 2.718$

<u>Solve</u>: $\lambda = 1.6$, $P(X \geq 3)$

এটাও লিখিতে পার $= 1 - P(x \leq 2)$.

- $P(X \geq 3) = P(X = 3, 4, 5, 6, 7, \ldots)$ ।
- $P(X \leq 2) = P(X=0), P(X=1), P(X=2)$ ।

$$P(X=0) = \frac{1.6^0}{0!\, e^{1.6}} = 0.202$$

$$P(X=1) = \frac{1.6^1}{1!\, e^{1.6}} = 0.323$$

$$P(X=2) = \frac{1.6^2}{2!\, e^{1.6}} = 0.258$$

$$\therefore P(X \geq 3) = 1 - P(X \leq 2) = 0.217$$

45 = In a cafe, the customer arrives at a mean rate of 2 per min. Calculate the probability of arrival of 5 customers in 1 minute using the Poisson distribution formula.

<u>Solve</u>: given, $\lambda = 2$, $x = 5$, $e = 2.718$

known, $P(X=5) = \frac{2^5}{5! \cdot e^2} = 0.03609$

So, the P. of 5 customers in 1 minute is 0.03609 or 3.6%.

Example 01:

to predict whether we can pet an animal or not

→ test = (Cow, Medium, Black).

| Animal | Size of Animal | Body Color | Can We Pet them |
|---|---|---|---|
| Dog | Medium | Black | Yes |
| Dog | Big | White | No |
| Rat | Small | White | Yes |
| Cow | Big | White | Yes |
| Cow | Small | Brown | No |
| Cow | Big | Black | Yes |
| Rat | Big | Brown | No |
| Dog | Small | Brown | Yes |
| Dog | Medium | Brown | Yes |
| Cow | Medium | White | No |
| Dog | Small | Black | Yes |
| Rat | Medium | Black | No |
| Rat | Small | Brown | No |
| Cow | Big | White | Yes |

Solve:

Frequency table to likelihood

| Animal | Y | N | P(Y) | P(N) |
|---|---|---|---|---|
| Dog | 4 | 1 | 4/8 | 1/6 |
| Rat | 1 | 3 | 1/8 | 3/6 |
| Cow | 3 | 2 | 3/8 | 2/6 |

| Color | Y | N | P(Y) | P(N) |
|---|---|---|---|---|
| Black | 3 | 1 | 3/8 | 1/6 |
| White | 3 | 2 | 3/8 | 2/6 |
| Brown | 2 | 3 | 2/8 | 3/6 |

| size | Y | N | P(Y) | P(N) |
|---|---|---|---|---|
| Medium | 2 | 2 | 2/8 | 2/6 |
| Big | 3 | 2 | 3/8 | 2/6 |
| Small | 3 | 2 | 3/8 | 2/6 |

| total | Yes | No | P(Yes) | P(No) |
|---|---|---|---|---|
| 14 | 8 | 6 | 8/14 | 6/14 |

now, test = (cow, medium, Black)

$$\therefore P(Y|Test) = \frac{P(Animal=Cow \mid Y) \times P(size=Medium \mid Y) \times P(Color=Black \mid Y) \times P(Y)}{P(Test)}$$

$$\therefore P(N|test) = \frac{P(cow \mid N) \times P(Medium \mid N) \times P(Black) \times P(No)}{P(Test)}$$

known, P(test) = (P(Y | test) + P(N | test so, $P(Y|\text{test}) = 3/8 \times 2/8 \times 3/8 \times 8/14 = 0.02$

$P(N|\text{test}) = 2/6 \times 2/6 \times 1/6 \times 6/14 = 0.0079$

now, Normalize

$$P(\text{Yes}|\text{test}) = \frac{0.02}{0.02+0.0079} = \frac{0.02}{0.0279} = 0.7159 = 72\%$$

$$P(\text{No}|\text{test}) = \frac{0.0079}{0.02+0.0079} = \frac{0.0079}{0.0279} = 0.2831 = 28\%$$

we see that, $P(\text{Yes}|\text{test}) > P(\text{No}|\text{test})$

so, the predict that, we can pet this animal (cow) is 'Yes'.